

Willingness to Pay:  
Referendum Contingent Valuation and  
Uncertain Project Benefits

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# ABSTRACT

The use of contingent valuation (CV) methods to estimate benefits has become increasingly common in project analysis. Ever since the NOAA Blue Ribbon Panel Report in 1993 (NOAA, 1993) recommended the use of the referendum form of CV, it seems to have become the method of choice in practical settings.

Referendum-type questions are thought to be easier to answer than the open-ended variety. But there is a downside: econometric techniques must be applied to the referendum data in order to infer the mean or median willingness to pay (WTP) of the sample and, thus, of the population of potential beneficiaries.

This is not, however, just a technical point. Its implications are demonstrated with data obtained from a referendum CV study done for a proposed sewer and wastewater treatment project designed to improve water quality in the Tietê River flowing through the city of São Paulo, Brazil. The results show that:

- ! A factor of 4 separates lowest from highest central tendency estimates of WTP, ignoring one implausible outlier that is 14 times larger than the largest of the other figures.
- ! This variation is ample enough to make a difference in the cost-benefit analysis results for the project under conservative assumptions.

Analysts that use referendum CV data must be sensitive to the problems they buy into, and decide how to deal with the resulting benefits uncertainty in their project analysis. If the principal use of CV survey data is to produce a mean or median estimate of WTP for Cost-Benefit analysis rather than to test for the factors influencing referendum choice responses and, by implication, WTP, nonparametric approaches have the advantage of simplicity over parametric approaches.

The text assumes the reader has some familiarity with contingent valuation and the statistical estimation of qualitative dependent variable models.

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# OVERVIEW

Cost-benefit (CB) analysis of proposed projects is an inherently uncertain enterprise because it involves the future, which we can never know. Costs and project performance can be different from our expectations. The economy in which the project is embedded may change, and the tastes, incomes and preferences of the population affected by the project may change as well in ways that are hard to predict. When the proposed project involves environmental public goods, such as improved air or water quality, another widely recognized source of uncertainty is the behavior of the natural system involved. Completing this familiar list is uncertainty about project benefits, the issue of concern in this paper.

An increasingly respectable and common way of estimating the benefits of environmental projects is to use the so-called contingent valuation (CV) method, which involves directly asking people about their willingness to pay (WTP) for the environmental effects to be provided by the project. Two broad alternative ways of asking the valuation question are available: “open-ended,” in which the respondent can name any amount s/he wishes when asked some version of “What are you WTP?”; and “dichotomous choice” (referendum or yes/no) in which the respondent is asked: “Are you willing to pay (at least) B (per period)?”

There is a vast and rapidly growing literature on these methods, especially the problems that arise in creating successful survey instruments, obtaining satisfactory response rates, and interpreting responses. See, for example, the relatively early seminal work by Mitchell and Carson (1989); and the exchange in *Economic Perspectives* occasioned by the huge damage-estimation efforts done by both sides in the Exxon Valdez case, Portney *et al.* (1994); Cummings and Harrison (1995); and Loomis *et al.* (1996).

One ambitious attempt to assess the usefulness of the method and to suggest ground rules for future applications was made by the National Oceanic and Atmospheric Agency (NOAA) in the U.S. in the early 1990s. This effort involved a panel of distinguished economists, and the panel’s report (NOAA 1993) has been very influential since its publication.

In particular, and most relevant to this paper, the panel recommended that CV studies be done using yes/no referendum format questions. This recommendation has been adopted by many practitioners who deal with real-world program or project evaluation rather than methods development. For example, at the Inter-American Development Bank (IDB) contingent valuation has become the method of choice for estimating the benefits of investment projects aimed at improving water quality. Over the past decade the Bank approved 18 projects with sewer provision and/or wastewater treatment components, and 13 of them employed CB analysis whose benefits came at least in part from CV estimates. Most of the stated preference CV surveys used the referendum format (Ardila *et al.* 1998).

The referendum CV approach opens up a new and substantial source of uncertainty in benefit estimation. That source is the choice of econometric technique and subsequent calculation rules used to translate yes/no responses into mean or median WTP numbers. In project analysis this source of uncertainty is easily

overlooked; almost none of the projects reviewed by Ardila *et al.* (1998) addressed it. Moreover, most analyses appear to have used an estimation formula that understates benefits.

This paper demonstrates the range alternative central tendency measures for WTP produced under alternative parametric and nonparametric approaches using data gathered from a recent referendum CV survey that was conducted in Brazil to analyze a large, multi-phase water quality improvement project. It explains why one of the most commonly used measures, the unrestricted mean of the conditional inverse distribution function of WTP, may be less desirable and more computationally intensive than simpler alternatives like the nonparametric mean of the marginal inverse distribution function.

The paper is organized as follows. First, the nature of the inference problem is described in what we hope is an intuitively appealing way. A simple stylized example shows how ambiguity about the correct measure of central tendency can arise, specifically the theoretically inconsistent phenomenon of a negative mean willingness to pay for a utility improving intervention. Moving closer to reality, alternative central tendency measures are proposed and illustrated using referendum contingent valuation survey data collected for a recent IDB project appraisal. The water quality impacts of the case study project that respondents were asked to value in a referendum CV survey are briefly described.<sup>1</sup> Then, eleven different versions of a central tendency measure of per household benefits from the project data are produced using methods suggested in the literature.<sup>2</sup> Six come from the economist's customary route of econometrically estimating a binary choice model relating the respondent's acceptance probability to the bid offered and socioeconomic characteristics, using a Logit specification of the inverse distribution function.<sup>3</sup> The rest are alternatives which either involve nonparametric measures that can be easily obtained without econometrics from the pooled data (i.e. the marginal rather than conditional distribution), or a more complex method that imposes lower and upper bounds on median WTP in econometric estimation. Finally, the effect that uncertainty about "actual" WTP has on the discounted net benefits of the case study project is explored and some general lessons are drawn.

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<sup>1</sup> Technically oriented readers can safely skip this section.

<sup>2</sup> The emphasis here is on function evaluation to extract a measure of central tendency, not on the prior steps of survey design or model estimation.

<sup>3</sup> The emphasis throughout the paper is on function evaluation to extract a measure of central tendency, not on the prior steps of CV survey design or choice model estimation. Instead, the survey is taken as a given. The specifications of the functional form and arguments in the econometric choice model follow the selections made by the Brazilian consultant who initially analyzed the data.



# AN INTRODUCTION TO THE PROBLEM

Contingent valuation approaches to project benefit estimation necessarily involve surveying samples of the population of interest. If the sample is representative of the population, the sample mean of willingness to pay per capita (or per household) can simply be attributed to everyone in the beneficiary population of size N, so total project benefits are obtained as the product of N and per capita WTP.

## Implications of the Referendum Format

In the early years of CV, the method of payment elicitation was direct and open ended. People were asked to reveal the specific monetary amount they would be willing to sacrifice for the provision of a non-marketed good such as an improvement in ambient environmental quality. Obtaining a measure of central tendency from this kind of data was as simple as calculating the mean or median of the WTP values provided by the survey respondents. The econometric analysis involved was minimal, usually being confined to plausibility checks undertaken by split sample comparisons or by regressing the payment amounts on income and other socioeconomic variables to see if the signs on the parameter estimates in the relationship were consistent with prior expectations (e.g. WTP increasing with income).

All of this changed with the advent of the referendum format, which only asks if the respondent would or would not be willing to pay a specific pre-selected amount. Under this format it is not possible to know the true WTP of any individual directly.<sup>4</sup> Because those who answer in the affirmative might actually be willing to pay even more, and those who answer in the negative might be willing to pay something less, econometric techniques have to be brought to bear to somehow interpolate and infer an expected value or other central tendency measure from the dichotomous choice information.

Simplicity of data analysis was sacrificed in the referendum method in order to construct what many felt was a more realistic choice game. The upshot of this change has been that the central tendency measure is no longer independent of manipulation by the analyst because, of necessity, it is the outcome of a sometimes complex process of survey design, choice model specification, model estimation, and function evaluation (Duffield and Patterson 1991).

In consequence, the notion that contingent valuation experiments of the referendum type can reveal a unique number which accurately and unambiguously represents individual willingness to pay for water quality improvement is unrealistic. Rather, there are several possible numbers, each dependent upon the way the initial survey was designed and administered and the way the resulting raw data was passed through the summarizing

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<sup>4</sup> The discussion leaves aside prior questions about whether a “true” value exists previous to the survey process, whether respondents bother to try to discover their own WTPs, and whether, even if they know them, interviewees try to conceal their true preferences by providing misleading answers that reflect strategic bias.

econometric sieve and reconstituted in the form of a central tendency measure. In short, such estimates are always uncertain when we acknowledge the existence of many routes that potentially can be taken to get at them and the several decision alternatives present at each step along the way. This is not a counsel of doom, or a suggestion that CB analysis based on referendum CV not be undertaken. But it is a fact that any benefit estimate to a greater or lesser degree is always a product of the analyst's protocol and judgement, something respectable analysts recognize and communicate to the users of their results.

## A Hypothetical Example

In a dichotomous choice referendum survey, a group of  $i=1\dots n$  different payment or bid levels is pre-selected and the total sample is split up into  $n$  groups or sub-samples. For each bid,  $B_i$ , dichotomous choice information can be summarized by the fraction of sub-sample respondents offered a given bid amount and saying "No, I am not willing to pay  $B_i$  for the public good" relative to the total number of respondents offered  $B_i$ . At each bid level surveyed there will therefore be a fraction,  $F_i$ , rejecting the offer, and a fraction  $(1-F_i)$  accepting it.<sup>5</sup>

If the project being investigated is a good idea, one would expect that everyone would be willing to pay some positive amount to have it, or at least would not require a payment to accept it (that is, not have a negative WTP).<sup>6</sup> But with a dichotomous choice survey instrument there will be uncertainty on this score unless a bid level low enough to produce  $F_j = 0$  is offered. And, similarly, there will be uncertainty at the upper end unless a bid level high enough to produce  $F_j = 1$  is offered.<sup>7</sup> At a simple intuitive level this second kind of uncertainty can lead to the sorts of difficulties involving negative mean willingness to pay sketched in Figure 1.

Consider three different set of yes/no responses to three bid levels, \$2.50, \$5.00, and \$7.50. The data are then used to infer linear cumulative distribution functions over bids, as shown in the figure.<sup>8</sup> For function (1), for example, 30 percent of those offered \$2.50 say yes; 20 percent of those offered \$5.00 say yes, and 10 percent of those offered \$7.50 say yes. With the hypothetical sample points indicated in the figure the equations for the three cumulative probability functions for positive responses ( $1-F(B_i)$ ) are:

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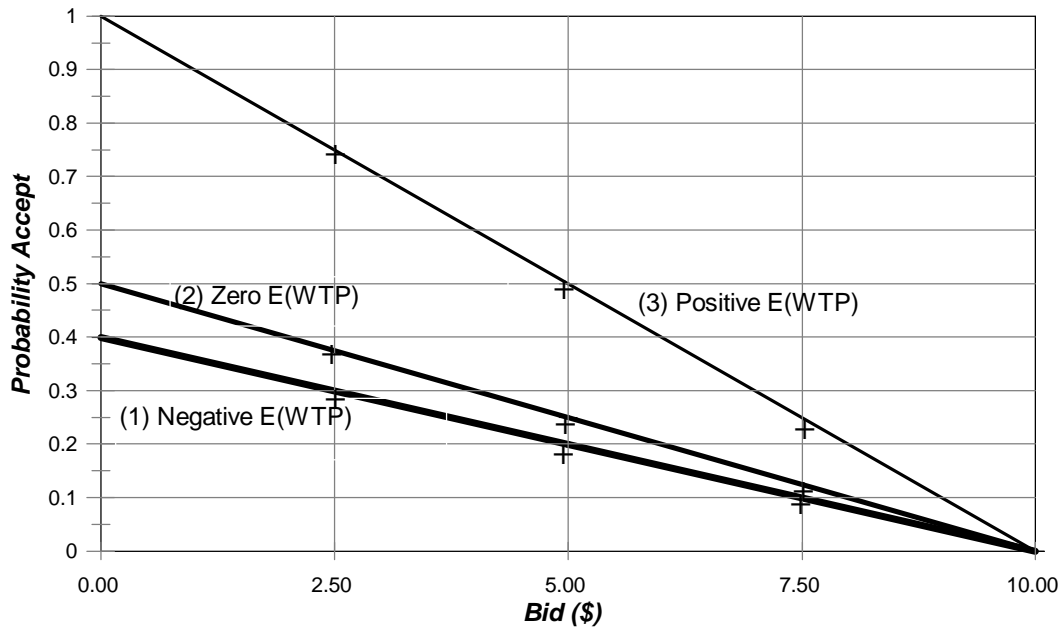
<sup>5</sup> There is an entire literature on the issue of bid design. For a brief and useful review of the implications of different designs see Creel (1998).

<sup>6</sup> Probably for every project there are some people who will see themselves as losing something because of it. Thus, the prospective neighbors of a wastewater treatment plant might prefer the status quo because of the plant's local negative externalities, especially if they are upstream of the current outfall. In an actual survey setting some negative WTPs were stated for an improvement in drinking water quality in a study undertaken by Kwak and Russell (1994). These people turned out to be vendors in the vicinity of springs that were heavily visited because of the existing perception of a potable water quality problem.

<sup>7</sup> Even with a survey design including very low and very high bid levels there will be uncertainty because of sampling, but at least a simple plot of the approximate cumulative probability of acceptance or rejection will be defined. See the nonparametric methods discussion.

<sup>8</sup> Strictly speaking, the relationship between bid and the probability of rejection is the familiar upward sloping cumulative distribution function  $F(\text{bid})$  and the downward sloping relation between bid and the probability of acceptance shown in the figure is the inverse cumulative distribution function (also called the survivor function), or  $1-F(\text{bid})$ .

**Figure 1. Three Possible Linear Inverse Cumulative Distribution Functions for WTP**



- (1) Prob(Yes) =  $(1-F_1) = 0.40 - 0.04$  (Bid)
- (2) Prob(Yes) =  $(1-F_2) = 0.50 - 0.05$  (Bid)
- (3) Prob(Yes) =  $(1-F_3) = 1.00 - 0.10$  (Bid)

Note in the figure that only function (3) covers the allowable probability range of zero to one for bids greater than or equal to zero. The other two functions have to be extended into the negative bid quadrant (not shown) to yield acceptance probabilities greater than 0.5 and 0.4, respectively. The specifications of  $(1-F_2)$  and  $(1-F_3)$  above imply acceptance probabilities of 1.0 only at negative bid levels of -\$10.00 and -\$15.00 respectively. The linearity of the three inverse cumulative distributions means that they represent uniform probability density functions over the intervals they cover such that:<sup>9</sup>

- (1)  $f_1(x) = 0.04$  from  $x = -\$15.00$  to  $\$10.00$
- (2)  $f_2(x) = 0.05$  from  $x = -\$10.00$  to  $\$10.00$
- (3)  $f_3(x) = 0.10$  from  $x = \$0.00$  to  $\$10.00$

<sup>9</sup> The probability density over  $x$ ,  $f(x)$ , is obtained by taking the derivative of the cumulative distribution function  $F(x)$  with respect to  $x$ . In this case  $x$  is the bid and the slopes of the three linear cumulative density functions yield the probability density.

The corresponding means or expected values of WTP are equal to the integral of over the bid range of the product of the density and  $x$ . In the linear case the bid range is between the maximum bid driving  $F(x)$  to zero and the minimum bid that sets  $F(x)$  to 1.0.<sup>10</sup>

$$E(\text{WTP}) = E(\text{Bid}) = \int_{\text{Min}}^{\text{Max}} x f(x) dx \quad \text{where Bid} = x$$

Calculation for the  $f_1(x)$  case gives:

$$E_1(\text{WTP}) = \int_{-15}^{10} x (0.04) dx = (0.04 x^2)' \Big|_{-15}^{10} = -\$2.50$$

Similarly for the other two inferred bid distributions,  $E_2(\text{WTP}) = \$0$  and  $E_3(\text{WTP}) = \$5$ .

Summing up, in the example the bid level at which all of the respondents would say “yes” was an uncertain number that was not provided directly by the sample data, but had to be inferred by fitting a (linear) inverse cumulative distribution. The sample information available at positive bid levels implied, for two of the exercises, that the bid at the 100 percent acceptance level was negative—in one case -\$15 and -\$10 in the other. The available information, used in the simplest way,<sup>11</sup> consequently produced mean WTP estimates ranging from a negative \$2.50 to a positive \$5.00. The first general lesson of this section (which carries over to more sophisticated nonlinear functional forms discussed below) is that if, at a zero bid, the predicted acceptance rate is less than 50 percent and the model is not confined to only positive bids in estimation or function evaluation, the predicted mean willingness to pay will be negative.

Even if the odd “objector” might show up, it seems highly unlikely that a negative mean WTP would be a correct inference. The target population could reject a proposed project because the investment is not producing a “good” on net for the average person, perhaps because negative externalities generated by the investment (the treatment plant) are so severe and widespread that the current without-project situation is preferred. Or, those surveyed could exhibit such a strong case of “status quo bias” that they require a subsidy as well as an environmental improvement to voluntarily move away from the current situation (Adamowicz *et al.* 1998), although this behavior would seem to be irrational and at odds with the utility-theoretic basis of CV. Finally, a very plausible cause of this result could be that those surveyed do not believe the scenario because they are cynical about the possibility that an actual investment in environmental quality will be made. This is a questionnaire design problem involving an unpopular choice of payment vehicle and project executor that usually can be discovered and addressed through focus groups and pre-testing before the final survey is administered.

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<sup>10</sup>More generally with nonlinear cumulative densities that are asymptotic to the upper and lower probability bounds of 0 and 1 the bid range runs from minus to plus infinity. See the discussion of analysis issues in a subsequent section.

<sup>11</sup>In practice, no analysts actually use the uniform density/linear cumulative density assumption anymore. While it is easy to fit a linear equation to binary 0,1 choice data with ordinary least squares (OLS), that approach has several undesirable econometric properties (e.g. heteroskedasticity). The advent of accessible computer software for qualitative dependent variable analysis in the 1980s (Logit, Probit) made the OLS shortcut unnecessary and irrelevant.

Besides the causes and cures for negative mean WTP, a second lesson from the example is that there is something to be said for “spreading out” the range of bids presented to respondents so that the chance of discovering the low bid that drives the acceptance rate close to 100 percent is higher, as is the chance of finding the high bid for which acceptance is near zero (assumed to be \$10 in the example). Knowledge about the tails of the cumulative distribution of acceptance as a function of bid level can be very useful in constructing a nonparametric estimate of the mean to serve as a check against more complicated econometric density function estimation approaches that impose a pre-specified shape and range of support (see below). One way to identify the bid levels determining the upper and lower tails of the density is to do an open-ended CV survey in a pre-test, and design the bid groups and sub-sample sizes accordingly (Cooper 1993).

The inference problem, however, is larger but more subtle than that of occasionally producing a negative mean WTP. Simply stated it is that there are many ways of attacking referendum CV data; that each will in general produce a different mean WTP; and that none of them is so obviously superior that all the others can be rejected. “True” benefits are then unknowable, even if we believe the referendum CV method to be in principle capable of eliciting them. The extent of the resulting uncertainty is illustrated using data from the project described in the next section.

# THE PROJECT – CLEANING UP THE TIETÊ RIVER

The parts of the Tietê River and its tributaries that flow through the São Paulo, Brazil, Metropolitan Area (SPMA), are the most polluted bodies of water in the State. The Tietê enters the metropolitan area with acceptable water characteristics but in Guarulhos, at the confluence of the Jacu, it becomes anaerobic or close to it (see Map 1). From the Jacu downstream the large volume of domestic and industrial waste dumped into the relatively small volume of river flow has made the river an open sewer that supports no aquatic life, and smells most of the year over a stretch of more than 80 kilometers. The reason for this is that the tributaries of the Tietê in the metropolitan area and the Tietê itself receive waste well beyond the river's natural processing capacity. At present the organic load is predominantly from households (360 tons per day, 80 percent of the total) with surface runoff accounting for another 62 tons per day (14 percent of the total) and industry contributing another 30 tons per day (7 percent). The problem is severe all year long and becomes critical in the dry season.

## The Project

The proposed project for cleaning up the Tietê involves extension of sewers to currently unsewered households (and businesses) and the provision of wastewater treatment plants at the discharge ends of those sewers. The major concern is the removal of oxygen-demanding organic materials (measured as biochemical oxygen demand, BOD) and disposal or recycling of these materials in ways and places that do not tax the dissolved oxygen (DO) carrying capacity of the river. The overall project is divided into three stages. The first stage has already been completed and the beginning of operation of the next two stages is contemplated for 2003 and 2008 respectively. The predicted effects of the project's three stages on water quality, as measured by BOD, DO, and the sewage-related health threat proxy of fecal coliform bacteria counts, are summarized in Table 1.

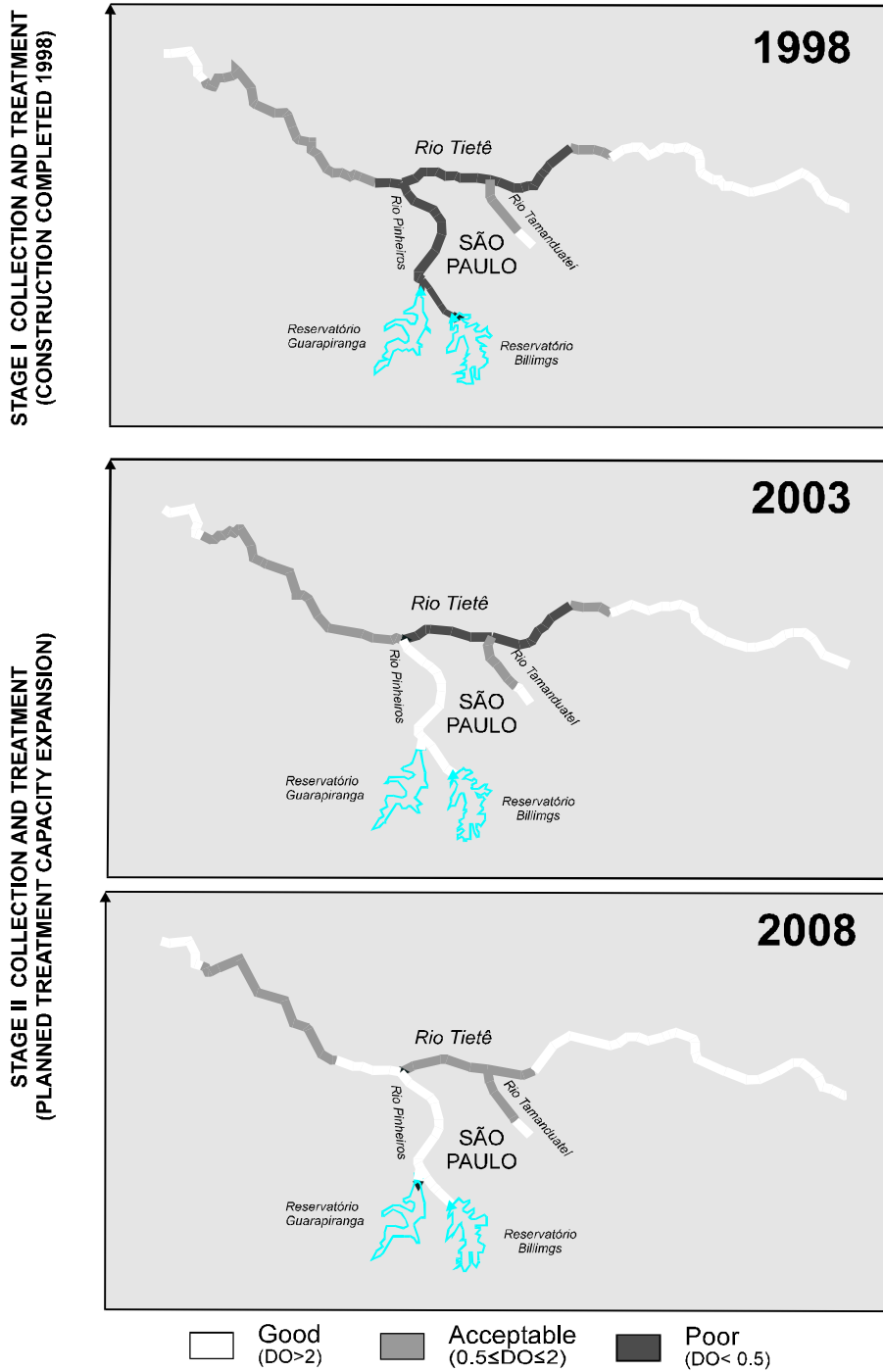
The project has local and general water quality benefits. The local benefits of the overall project involve the "neighborhood" effects created by sewerage – that is, the cleaning up of locally offensive and dangerous conditions. But the benefits component of interest here is that arising from general improvements in the mainstream and tributaries as summarized in Table 1 and Map 2. This was approached in the project analysis via a dichotomous CV survey asking WTP for the described dissolved oxygen improvements, which were from 0 to at least 1.5 mg/l in the segments asked about. To reflect the quality that would actually result from the proposed works, the questionnaire used the maps displayed in map set 2 to show what parts of the river would improve and when.

There are also benefits from resuming the use of Tietê water for hydroelectric generation after transfer to a different sub-basin. This use had been stopped because the low quality of the Tietê water was damaging the much cleaner reservoir into which the Tietê was diverted. There is a possibility that after completion of Stage III, water from the Tietê will once again be suitable for power generation.



# PRESENT AND PREDICTED WATER QUALITY IN THE TIETÊ RIVER AND MAIN TRIBUTARIES

MAP 2.



This map, prepared by the Inter-American Development Bank, has not been approved by any competent authority and its inclusion in the loan document has the exclusive objective of indicating the area of influence of the project proposed for financing.

Sustainable Development Department  
Geographic Information System (SDS/gis) 4/20/99



<b>Measured At:</b>	<b>1998 Stage I</b>			<b>2003 Stage II</b>			<b>2008 Stage III</b>		
	<b>DO mg/l</b>	<b>BOD mg/l</b>	<b>FC (10<sup>6</sup>/ 100ml)</b>	<b>DO mg/l</b>	<b>BOD mg/l</b>	<b>FC (10<sup>6</sup>/ 100ml)</b>	<b>DO mg/l</b>	<b>BOD mg/l</b>	<b>FC (10<sup>6</sup>/ 100ml)</b>
Tietê Confluence Tamanduatei	0.00	33.5	.85	0.00	23.2	.55	1.46	13.6	.23
Tietê Confluence Pinheiros	0.00	28.9	.78	0.43	22.3	.56	1.98	12.6	.25
Pinheiros Pumping Station	0.00	32.2	.59	0.55	16.7	.29	2.18	11.6	.15
Edgar Souza	0.98	32.0	.65	2.95	26.1	.46	4.01	13.1	.17
Pirapora	4.35	14.3	.002	4.50	13.4	.002	4.27	8.4	.0008
Note: DO = Dissolved Oxygen; BOD = Biochemical Oxygen Demand; FC = Fecal Coliforms. Low flow is taken to be that flow exceeded 90 percent of the time.									

## **The Survey**

The valuation survey indicated that the greatest improvement that could be expected was that the water quality would permit boating and the existence of fish in some segments. It emphasized that it would not be safe to swim in any of the rivers. To take the benefit estimates seriously, regardless of inference method, one would like to be confident that respondents correctly understood the implications of the low flow assumption and the implications of the DO levels for their lives. But for the purposes of this paper, any difficulties along those lines are irrelevant, being equally present in all method results.

The survey was conducted with 600 households divided among five sub-regions of the relevant region. These reflect households both close to and far from the river. This sample was also split another way – into five groups offered different “bid” levels to respond to. The levels, chosen on the basis of focus group comments, were, in Reals (\$R), 0.50, 2.00, 5.00, 12.00, and 20.00, and were presented as monthly payments that would be made over the 10 years of construction of stages two and three. A translation of the key valuation question follows directly.

After the survey was completed, it turned out that at the lowest bid level, the proportion of “yes” responses from the far-from river sample amounted to fewer than 50 percent of the sub-sample total. This is a warning flag indicating that without restrictions on functional form or the use of limits in calculating the mean, a negative mean WTP will result.

## The Valuation Question

Look at **Map 1**. The triangles and circles depict SABESP's five water treatment plants. The larger the size of the symbol the larger the quantity of wastewater treated. The two plants represented by the triangle have been operational for some time, treating 20% of SPMA wastewater.

In 1993, SABESP initiated works for Stage I of the River Tietê decontamination program. Three new plants (depicted by the circles) are planned to be operational by the year 1998. With these new stations, 40% of the industrial and domestic load will be treated. Consequently, water quality of the Tietê River and its tributaries will improve. Still, 60% of the domestic and industrial load will reach the rivers untreated.

Even with three new treatment plants operational by 1998 water quality of the Rio Pinheiros will continue to be poor. The sections of the rivers in grey depict an acceptable level of water quality mainly due to the elimination of odors; still, no aquatic life is supported. On the other hand, the river sections delineated in white support some aquatic life and boating is permitted.

SABESP has a project to continue the decontamination of the River Tietê. Under the new project, more treatment plants will be built and an expansion of the existing treatment plants is foreseen. If the project is pursued, in 10 years 95% of pollutants will be treated, improving water quality of the rivers. **Map 2** depicts the improvement in water quality during the next 10 years.

As shown in the map, in the next five years, the Rio Pinheiros will show a considerable improvement in water quality. On the other hand, water quality in the River Tietê and Tamanduateí will not improve. By 2008, at the conclusion of the proposed project, all of the rivers will have an acceptable or good water quality level.

The costs involved in such a project are high and there are not enough financial resources. What would you prefer:

***Pay R\$ (bid amounts: 0.5, 2, 5, 12, or 20) rendered as an increase in you monthly water utility bill for the next 10 years for an improvement in water quality as depicted in Map 2 or not pay and the project will not be executed leaving water quality of the rivers of Sao Paulo at the current levels?***

# ANALYSIS ISSUES

The potential for a negative estimate of the expected value of willingness to pay is only a special example of a more general issue with referendum CV, which is that the willingness to pay value extracted from the data can be heavily influenced by the methodological approach taken.

There are basically two routes to analyzing referendum data. The one most frequently pursued by project economists involves several steps, beginning with the specification and statistical estimation of one or more probability models of individual choice, employing prior assumptions about the form of the inverse distribution, and the covariates belonging in the distribution which serve to change its location and shape across respondents. This is followed by the evaluation of conditional mean or median formulas derived from the choice model, which depend on its estimated parameters. After calculating individual-specific means or medians, averages are taken over the entire sample to produce global central tendency measures. A less frequently traveled but much easier route ignores covariates and does not specify any particular inverse distribution. Instead it uses all the data in pooled form (i.e. the marginal distribution) to produce nonparametric measures of central tendency. Given its prominence, the next two sections concentrate on the parametric route, followed by a discussion of nonparametric options.

The parametric route can quickly become quite complex, producing a wide array of central tendency estimates. It is not uncommon to find instances where predicted WTP can vary from low to high by a factor of two, five or ten with the same data, depending on the analyst's choice of density function, the specification of the functional form of the indirect utility index and its arguments, and whether a mean, a truncated mean, or a median is used. In short, with referendum data there are a host of possible measures of central tendency of willingness to pay.<sup>12</sup> Gauged by their frequency of use by practitioners, all of them might seem equally legitimate, but this is not a useful criterion. For instance, the untruncated mean extracted from Logit estimation of a random utility model (see Table 2 below) has been one of the most popular measures used in IDB project analysis and in the literature more generally, even though it is potentially vulnerable to the problem of negative WTP.

## Some Basic Mechanics With Referendum Data and Rum Models

Consider an individual who must decide whether to answer yes or no to the following: *Would you vote for a program to increase environmental quality from  $q^0$  to  $q^1$  if it would decrease your annual income by \$B ?* Let the indirect utility function be  $u(Y,q,X)$  where  $\mathbf{X}$  is a vector of individual characteristics and the vector of market prices  $\mathbf{P}$  is omitted since prices are assumed to be constant.

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<sup>12</sup> Benefits uncertainty and the influence of analyst choices in econometric estimation is not unique to referendum CV. Similar issues arise with fitting econometric demand or participation models to revealed preference data. Striking examples appear in Smith (1990), Vaughan and Russell (1982) and Ziemer *et al.* (1980) for recreation demand and in Bachrach and Vaughan (1994) for potable water.

The individual responds yes if:

$$(1) \quad u(Y-B, q^1, X) - u(Y, q^0, X) \geq 0$$

and no otherwise.

Let  $h(\bullet)$  be the observable component of utility. Here,  $h$  represents an indirect utility function which in statistical estimation is often called the index function or utility index, denoted as the summed product of the parameter estimates and the explanatory variables,  $X\beta$  (Greene 1990, p. 673). The probability of a “yes” response is given by:

$$(2) \quad P_1 = P[h(Y - B, q^1, X) + e_1 > h(Y, q^0, X) + e_0]$$

Where  $e_i$  ( $i=0,1$ ) are independent, identically distributed random variables with zero means and the error term represents influences on utility not observed by the analyst, or just random error in the choice process itself. Assuming the error difference follows a Logistic distribution,<sup>13</sup> the probability of a “yes” response can be expressed as an estimable random utility (difference) model, or RUM:

$$(3) \quad P_1 = e^{\beta h} / (1 + e^{\beta h}) = (1 + e^{-\beta h})^{-1}$$

Where  $\beta h = h^1 - h^0$ . The linear utility difference index  $\beta h$  in the “no income effects” RUM is usually specified as a function of the bid level,  $B$ , and a set of socioeconomic variables,  $S$ , including a constant term but not including income as an argument (i.e.  $\beta h = (a_1 - a_0) + \beta B + \gamma S$ ). This most basic of specifications imposes the assumption of a constant marginal utility of income, which simplifies recovery of an expected value for WTP.

By reversing the sign on the probability difference, we get the expression for the probability of rejecting the offer:

$$(4) \quad P_0 = (1 + e^{\beta h})^{-1}$$

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<sup>13</sup> In the literature, only the Logit and the Probit modeling approaches appear with any frequency, although Hazilla (forthcoming) demonstrates a number of other possibilities. The Logit discrete choice model follows from the assumption that the errors  $e_0$  and  $e_1$  each are independently and identically distributed with Weibull density functions. The difference between any two random variables with (log) Weibull distributions has a Logistic distribution  $L(\bullet)$ , whose cumulative density  $F(\bullet)$  equals  $e^{\bullet} / (1 + e^{\bullet})$ , where the number “e” is the base of the natural system of logarithms and the  $(\bullet)$  represents the index function whose parameters are found by estimating the model (Formby, Hill and Johnson 1984, p. 353). The probability density of the Logit,  $f(\bullet)$ , is just the product of the cumulative density and one minus the cumulative, or  $F(\bullet)(1 - F(\bullet))$ . Its closed form analytical expressions make the Logit more tractable mathematically than the alternative assumption of using a normal error distribution,  $F(\bullet)$  to fit a Probit model. Although the Logistic distribution is thicker in the tails than the Normal, in most cases the Logit and Probit approaches to binary choice estimation produce similar prediction probabilities and elasticity responses, so the choice between them is largely a matter of convenience (Maddala, 1983, p. 23, Green 1990, p. 666).

We define the willingness-to-pay (WTP) for  $q^1$  by the amount of money that must be taken away from the individual enjoying an improved amenity level,  $q^1$ , that leaves s/he as well off as the initial amenity and income situation.

$$(5) \quad u(Y - \text{WTP}, q^1) = u(Y, q^0)$$

and

$$(6) \quad h(Y - \text{WTP}, q^1) + e_1 - e_0 = h(Y, q^0)$$

Because of the term  $e_1 - e_0$ , WTP is a random variable. Then, the probability of accepting the offer is also the probability that  $\text{WTP} \leq B$ , and the probability of rejecting the offer is also the probability that  $\text{WTP} > B$ . This is a cumulative distribution function and can be denoted as  $F(\text{WTP})$ . As pointed out by Hanemann (1984), the truncated expected value of the random variable (WTP) can be found from the cumulative distribution function as follows:

$$(7) \quad E[\text{WTP}] = \int_0^{\infty} [1 - F(\text{WTP})] d\text{WTP}$$

Here, the integration is only over positive values of WTP, because if there is utility improvement, WTP theoretically cannot be negative (although it can depend on who you ask and how the question is phrased, as noted in the preceding section and footnote 3 above). Similarly, the untruncated expected value of the random variable (WTP) can be found from the cumulative density function:

$$(8) \quad E[\text{WTP}] = \int_{-\infty}^{\infty} [1 - F(\text{WTP})] d\text{WTP} + \int_{-\infty}^0 F(\text{WTP}) d\text{WTP}$$

The latter, treating the negative domain of WTP as admissible, will generally be less than or equal to the truncated WTP represented by the first term in the above expression (Johansson *et al.* 1989) because of the inference described intuitively in the section introducing the problem.

For the Logit probability model, Hanemann (1984, 1989) and Ardila (1993) provide the WTP formulas shown in Table 2 for the unrestricted expected value, the median, and the truncated expected value that restricts WTP to be positive.

The  $a$  term in the table is shorthand for an augmented intercept absorbing the estimated constant and the socioeconomic variable influences on  $h$  ( $a$  below equals  $(a_1 - a_0) + \beta S$ ). The letter  $C$  in the table is shorthand for the central tendency measure of WTP, following the notation of Hanemann (1984, 1989), the original source. In models with several explanatory variables, the parameter  $a$  can be replaced by an augmented intercept, using the coefficient estimates evaluated at the means of the independent variables, except of course, the bid price,

$\beta$ .<sup>14</sup> For reference, function evaluation formulas are provided in Ardila (1993), Haab and McConnell (1998), and Hazilla (forthcoming), among others.

<b>Table 2. Formulae for Central Tendencies from the Probability Model</b>		
<b>Description</b>	<b>Symbol</b>	<b>Equation</b>
Mean, $E(WTP)$ , $-4 < WTP < 4$	$C+$	$a/\beta$
Median WTP	$C^*$	$a/\beta$
Truncated Mean, $E(WTP)$ , $0 < WTP < 4$	$C'$	$\ln(1+\exp(a))/\beta$
Truncated Mean, $E(WTP)$ , $0 < WTP < B_{max}$ where $B_{max}$ is the maximum bid	$C\sim$	$1/\beta \ln[(1+\exp(a))/(1+\exp(a-\beta B_{max}))]$
Truncated Mean, Log Transform, $E(\exp^{\ln(WTP)})$ , $-4 < \ln WTP < 4$ (utility difference logit, log of bid, 0 Lower Limit, No Upper Limit)	$C_{\ln}^+$	$\exp(-a/\beta) [(p/\beta)/(\sin(p/\beta))]$ (Only applies if $0 < 1/\beta < 1$ , otherwise numerical approximation required)
Truncated Mean, Log Transform, $E(\exp^{\ln(WTP)})$ , $-4 < \ln WTP < \ln \text{Income}$ (utility difference logit, log of bid, 0 Lower Limit, Income Upper Limit)	$C_{\ln}\sim$	No Analytic Expression–Requires Numerical Approximation
Truncated Median, Log Transform	$C_{\ln}^*$	$\exp(-a/\beta)$

### Model Assumptions

Given the theoretical underpinnings of the conventional random utility model (RUM) sketched above, it is necessary to recognize that when the RUM is specified as a Logit model with a linear utility difference index specification, a fundamental contradiction arises because the Logit potentially allows predicted willingness to pay to fall between minus and plus infinity, admitting the possibility of negative values. Negative WTP should be ruled out for well conceived environmental improvements, as should expected payments exceeding actual income.<sup>15</sup> The expedients for guaranteeing satisfaction of one or both of these limits by evaluating the linear

<sup>14</sup> The augmented intercept,  $a$ , referred to in Tables 2 and 4 is simply the original intercept (for purposes of this note call it  $\beta_0$ ) plus the rest of the  $i=1\dots n-1$  parameter estimates other than the bid parameter estimate multiplied by the respective sample means of the explanatory variables  $\mathbf{X}_i$ . The  $\beta$  attached to bid in Table 1 is, in this notation, equivalent to  $\beta_n$ .

<sup>15</sup> This is strictly true only if the answer supplied reflects an understanding that payments for the good offered are to be taken out of current income without drawing down savings or liquidating other forms of wealth. It is unlikely that low income survey respondents (who usually dominate CV surveys taken in developing countries), would either have assets to pledge or be willing

utility index model estimated with Logit or Probit from zero bid to either plus infinity or income (truncated means), or by forcing the estimated density to lie in the positive region by using the logarithm of bid rather than the untransformed bid in estimation, leave a great deal to be desired. They are just ad-hoc fixes to the conventional random utility model's fundamental specification error of an unrestricted error term.<sup>16</sup>

Although it was originally discussed in the late 1980s (Johansson, Kriström and Mäler, 1989; Hanemann 1989) the issue has recently been brought more fully to light by Haab and McConnell (May 1998). The latter suggest employing a beta distribution for the density of willingness to pay to consistently hold WTP between zero and some upper bound such as income. In an unpublished study Haab and McConnell (August 1997, January 1999) have proposed an alternative way to achieve a similar restriction by bounded Probit (or Logit) estimation. Because this method is much simpler to implement than the beta, it is applied to the Tietê project referendum survey data, where it produces reasonable estimates for the median, but curious estimates for the mean.

## Central Tendency Measures

A second, related issue is which measure of central tendency to use, once having estimated some probability-of-bid-acceptance model from referendum data. Again, the debate goes back at least ten years. Hanemann (1989) and Haab and McConnell (1997, August 1997, July 1998, January 1999) argue for the median of individual WTP because in probability models it is less sensitive to distributional misspecification and estimation method. Hanemann (1989) also points out that the median is a more equitable social choice rule for aggregation of willingness to pay across the population for a cost-benefit test, even though it violates the Kaldor-Hicks potential compensation criterion.<sup>17</sup>

Sometimes the discrepancies among the alternative central tendency measures can be large enough to confound a project acceptance or rejection decision using CB criteria — the project passes the test using some subset of central tendency measures and fails it using others. Put simply, the unbounded expected value measure obtained

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to pledge them in excess of current income when valuing a non-unique environmental good like water quality improvement. However, the preservation of unique natural assets or irreplaceable historical sites may evoke contributions in excess of income, especially among the upper strata, and especially if the question is posed as a one-time payment rather than a series of payments strung out over several years.

<sup>16</sup> Creel (1998) sounds a more optimistic note by demonstrating that the marginal expected value of willingness to pay, truncated from below at zero and from above at a maximum that drives the probability of acceptance to zero can be consistently estimated from the simplest possible logit model (intercept and price parameters only) providing the bids are spread uniformly between the upper and lower bounds, the upper bound is known a-priori, and the acceptance probability is integrated only up to the upper bound in calculating the mean.

<sup>17</sup> The use of a global mean to get an estimate of gross project benefits, which is the focus of this paper, should not be confused with designing a tariff structure to recover project costs. For rate determination, a global fee based on average WTP would be inappropriate because in aggregate it potentially could induce actual welfare losses among low income rate payers with WTP below the mean that offset the net welfare gains accruing to upper income households whose WTP exceeds the global mean charge. For rate setting, progressive, income-differentiated charges would avoid the equity problem, and calculating them on the basis of referendum WTP data would require a utility index specification that includes income as a regressor.

by using a linear utility index in estimation of a probability model is not generally satisfactory and may understate benefits. But, when distributional asymmetry is introduced to correct for this by either truncating the range of expected value function evaluation or by introducing non-linearity in the utility index, the mean individual WTP extracted from referendum models no longer equals the median and will usually exceed it. In this case using the median as a benefit measure means that project acceptance will not be as strongly influenced by a few extreme observations lying in the tails of the (asymmetric) WTP distribution as it would be using the mean. Experienced analysts know that to get the highest benefits possible and unabashedly seek project acceptance under an NPV or EIRR criterion, the mean of an asymmetric distribution can be used, but its median will provide a more cautious, conservative lower bound on project payoff. It seems reasonable to recommend at least taking a look at the latter, or reporting both mean and median.



# PARAMETRIC CHOICE MODEL ANALYSIS

To demonstrate, the standard central tendency measures described above were obtained by applying a Logit choice model to the 600 survey sample observations, coding the dependent variable as 1 if the offer was accepted, and 0 if not. Simple linear and log bid specifications of the utility index were used.<sup>18</sup>

## Probability Model Estimation

The independent variables in the statistical Logit model included the bid value, the age of the respondent, and a household wealth/social status indicator. A dummy variable was included to distinguish between residents who live close to the river (184 households), and are significantly more affected by its pollution, than households not residing in close proximity. The estimation results appear in Table 3.

All parameter estimates are significant at better than the 5% level, and most have signs that are consistent with prior expectations. Households close to the river are more likely to be willing to pay than more distant households, as are wealthier households.

Predictions of the acceptance rates across bid levels for both models, evaluated at their respective sub-sample means, are displayed in Figure 2. Notice that the logarithmic specification confines all of the distribution function to the positive bid quadrant, while the linear specification potentially extends to the left of zero, even though this region is omitted from the figure. The thicker tails of the log bid models suggest arithmetic means that should exceed the arithmetic means of the linear bid models. However, we used geometric means for the log bid models, which explains why they fall below the arithmetic means of the linear bid models in Table 4 below.

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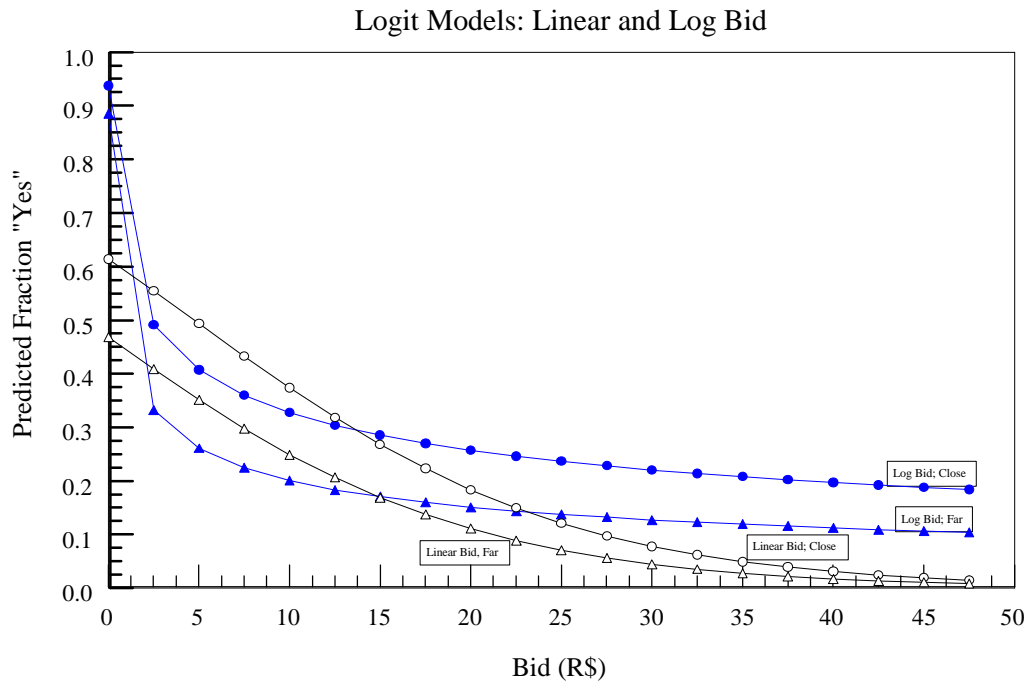
<sup>18</sup> Note that the dummy variable specification shifts the function but imposes the restriction that households living near or far from the river share the same regime with respect to the other parameters. The log bid model's expected value could not be evaluated using an analytic formula because its parameters fell outside the limits of the formula's applicability (Hanemann 1984, p. 337). Numerical approximation was used to compute the means of the log bid model (see Annex 1).

**Table 3. Logit Model Parameter Estimates and Variable Means**

Variable	Linear Bid Model Coefficient ( <i>t stat.</i> )	Log Bid Model Coefficient ( <i>t stat.</i> )	Means of Variables		
			Full Sample	Close Sub-Sample	Far Sub-Sample
<b>Constant</b>	0.7769 (2.38)	0.7608 (2.30)	...	...	...
<b>Close to River (1 if Yes, 0 Else)</b>	0.6551 (3.29)	0.6629 (3.33)	0.3066	1	0
<b>Status (1 if Upper, 0 Else)</b>	0.8357 (2.92)	0.7968 (2.78)	0.11	0.1467	0.0938
<b>Age of Household Head (Years)</b>	-0.0221 (-3.20)	-0.0227 (-3.27)	45.88	49.38	44.34
<b>Bid (R\$/Household/Month)</b>	-0.0978 (-6.78)	...	7.9	7.99	7.86
<b>Log of Bid (ln R\$/Household/Month)</b>	...	-0.4954 (-6.99)	1.42	1.43	1.41

Note: For the linear bid index model, Unrestricted Log Likelihood=-350.00, Restricted Log Likelihood (intercept only) = -389.08, Chi-squared statistic = 78.15, significant at >1% level, and Pseudo R<sup>2</sup> = 0.10. For the log bid index model, Unrestricted Log Likelihood=-350.65, Restricted Log Likelihood (intercept only) =-389.08, Chi-squared statistic = 76.79, significant at >1% level, and Pseudo R<sup>2</sup> = 0.098.

Figure 2



## Central Tendency Measures

Applying the expected value and median formulas produces the WTP estimates in Table 4 for the untruncated mean, the mean truncated at zero but untruncated from above, the truncated mean confined between zero and the maximum bid (20 reais), and the median.<sup>19</sup>

<b>Table 4. Parametric Central Tendency Estimates</b>			
<b>Central Tendency Measure</b>		<b>Household Willingness to Pay per Month (1998 Reals)</b>	
		<b>Close to River</b>	<b>Far from River</b>
Median = Untruncated Mean, $E(WTP)$ , $-4 < WTP < 4$ (utility difference logit, linear in bid)	C+ C*	4.74 (SE=1.66)	-1.27 (SE=1.56)
Truncated Mean, $E(WTP)$ , $0 < WTP < 4$ (utility difference logit, linear in bid)	C'	9.73 (SE=1.29)	6.16 (SE=0.75)
Truncated Mean, $E(WTP)$ , $0 < WTP < B_{max}$ (utility difference logit, linear in bid)	C~	7.66 (SE=0.71)	5.03 (SE=0.44)
Truncated Mean, Log Transform, $E(\exp^{\ln(WTP)})$ , $-4 < \ln WTP < 4$	C <sub>ln</sub> <sup>+</sup>	4.66	1.46
Truncated Mean, Log Transform, $E(\exp^{\ln(B)})$ , $-4 < \ln WTP < \ln \text{Income}$	C <sub>ln</sub> ~	3.49	1.23
Truncated Median, Log Transform	C <sub>ln</sub> <sup>*</sup>	2.34	0.61
Note: The augmented intercepts are 0.4634 for Close and -0.1246 for Far in the linear model. For both cases, $\beta$ , the marginal utility of income estimate, is 0.09776 (after multiplying by -1 to make it positive). In the log of bid model the augmented intercepts are 0.4201 for Close and -0.2427 for Far. For both cases, $\beta$ on the natural log of bid is 0.49454 (after multiplying by -1 to make it positive). Geometric means were calculated for the log transform models by taking the antilog of the mean log bid found by numerical approximation. Approximate standard errors are reported in parentheses (SE=) in cases where an analytical formula for expected value enabled them to be estimated via a Taylor's series approximation (the "delta" method) using LIMDEP's WALD procedure (See Hazilla, forthcoming).			

<sup>19</sup> The unit of currency used throughout is the Brazilian real (reais), denoted as R\$. The rate of exchange in March 1998 was 1.14 reais per U.S. dollar. All estimates presented were produced by evaluating the relevant formulas at the means of the explanatory variables rather than calculating individual-specific values and averaging them over the sample to obtain a grand mean. Since the Logit is a nonlinear density function the two routes will not, in general, produce exactly the same estimate of mean WTP. The two routes would yield the same sample mean if the arguments in the indirect utility difference model were confined to an intercept and the bid, omitting all individual-specific variables involving income and personal characteristics, because then the individual-specific means would all be the same. This reduced model is likely to suffer from biased coefficients caused by omitted variables, so the mean, which is a function of these parameter estimates, will be biased as well to an unknown extent. Creel (1998) shows that the impact of misspecification bias on  $E(WTP)$  can be controlled by arraying bids uniformly over the span from zero to an upper limit (like income), although he does not recommend fitting a simplistic and deliberately misspecified model.

These results pose two dilemmas. First, the unrestricted mean WTP for households living far from the river is negative. Second, there is a large disparity between the several alternative truncated means using either a linear or log bid specification.

If project justification (rather than analysis) is the goal, it might be tempting to use the truncated mean that gives the highest benefit and ignore the subtleties. Few would ever detect this sleight of hand. However, an honest project appraisal would admit that things are not quite so simple. Hanneman (1989) indicates that the measure  $C'$  unambiguously overstates the true mean in situations where the augmented intercept is greater than zero (i.e. when the probability of acceptance at a zero bid is greater than 0.5).

Also, it is inconsistent to use an untruncated distributional assumption for estimation and a truncated rule like  $C'$  for function evaluation. In other words, an inconsistency arises because in estimation of a Logit model with a linear utility index difference the domain of the fitted cumulative density is theoretically allowed to include all the real numbers even though the random variable is known a-priori to exclude negative values. Then, in function evaluation, a "correction" like  $C'$  or  $C\sim$  is made ex-post by using only that portion of the fitted distribution lying in the positive probability/bid quadrant to compute the expected value integral. For instance, return to the didactic linear probability function  $f_1(x)$  in the introduction. Evaluating the integral between the limits \$0 to \$ 10 produces a positive truncated mean of +\$2.00 instead of the original unrestricted negative mean of -\$2.50 obtained by evaluating the integral from -\$15 to +\$10.<sup>20</sup>

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<sup>20</sup> The assumption behind this truncated mean calculation is that the negative domain of the CDF now piles up at the zero bid level, which is like assigning a zero to every observation in the sample whose  $E(WTP)$  is negative, a technique used by Jorge Ducci in the IDB's very first CV experiment (see Ardila *et al.* 1998). To do even more violence to the estimation results, in function evaluation it could be assumed that instead of clustering at zero, the negative WTP part of the CDF should be reallocated to the positive part. In our example, the probability of a non-negative WTP at zero bid is only 40%, but were it 100% then  $E(WTP)$  would be \$5.00 (i.e.  $\$2.00 \div 0.4$ ). This calculation, which cannot be recommended, re-normalizes the positive domain of the estimated inverse CDF to include all the probability mass by rescaling the estimated inverse CDF of function #1 in Figure 1 to instead look like function #3 (i.e.  $1 - F_1 = (0.40/0.40) - (0.04/0.40)$  (Bid)). The negative  $E(WTP)$  problem has been solved by simply ignoring the negative domain of the estimated inverse cumulative distribution, even though that domain was not ruled out in the estimation step.

# ANALYSIS OPTIONS

Haab and McConnell offer two simple yet effective alternatives for estimating WTP that overcome the necessity of arbitrarily truncating WTP at zero or some upper bound (or both) in discrete choice referendum models, taking it as given that the unrestricted mean explained at the beginning is undesirable. The first route is a “distribution-free” nonparametric technique for getting lower-bound estimates of the mean and median (McConnell 1995; Haab and McConnell 1997). The other involves a reformulation of the Probit or Logit model that automatically guarantees that median WTP will be greater than a lower bound of zero but never be greater than income (Haab and McConnell, August 1998, January 1999). At a minimum, it is probably a good idea to calculate a nonparametric<sup>21</sup> estimate of the mean and median before getting too deeply involved in estimation of WTP, just to have a benchmark.

## The Turnbull Nonparametric Technique

Consider a stylized contingent valuation question. Respondents are asked: “Would you be willing to pay an amount  $b_j$ ?” The  $b_j$  are indexed  $j = 0, 1 \dots M+1$  and  $b_j > b_k$  for  $j > k$ , and  $b_0 = 0$ . Let  $p_j$  be the probability that the respondent’s WTP is in the bid interval  $b_{j-1}$  to  $b_j$ . This can be written:<sup>22</sup>

$$(9) \quad p_j = P(b_{j-1} < w \leq b_j) \text{ for } j = 1, \dots, M+1.$$

Alternatively, the cumulative distribution function (CDF) is written:

$$(10) \quad F_j = P(w \leq b_j) \text{ for } j = 1, \dots, M+1, \text{ where } F_{M+1} = 1.$$

For reasons already discussed, one aims to have  $b_{M+1}$  high enough that  $F_{M+1} = 1$ . That is,  $b_{M+1}$  is effectively infinite in the problem setting. Then

$$(11) \quad p_j = F_j - F_{j-1}$$

and  $F_0 = 0$ . The Turnbull can be estimated by treating either the  $F_j, j = 1 \dots M$  or  $p_j, j = 1 \dots M$  as parameters.

The  $p$ ’s can be estimated quite simply. Let  $N_j$  represent the number of “no” responses registered in each bid group  $j$ . If  $[N_j / (N_j + Y_j)] > [N_{j-1} / (N_{j-1} + Y_{j-1})]$  for all  $j$  between one and  $M$ , then  $p_j = [N_j / (N_j + Y_j)] - [N_{j-1} / (N_{j-1} + Y_{j-1})]$ . The probability  $N_j / (Y_j + N_j)$  represents the proportion of respondents who say ‘no’ to  $b_j$ . As such, it

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<sup>21</sup> In the context of this paper, nonparametric means “distribution-free”; that is, the distribution function of the random variable producing the data need not be specified.

<sup>22</sup> This section is an abridged version of the presentation in McConnell (1995). A complete treatment is available in Haab and McConnell (1997).

is a natural estimator of  $F_j$ .<sup>23</sup>

Hence, the estimator of  $p_j$  could be written:

$$(12) \quad p_j = F_j \text{ \& } F_{j\&l}, \text{ where } F_j = \frac{Y_j}{N + Y_j}$$

Expected willingness to pay can be written as:

$$(13) \quad E(WTP) = \int_0^4 WTP \, dF(WTP) = \sum_{j=1}^{M\&l} \int_{b_{j\&l}}^{b_j} WTP \, dF(WTP)$$

Replacing willingness to pay by the lower bound of each interval produces a lower bound estimate of the expected value of willingness to pay:

$$(14) \quad E(LB_{WTP}) = 0 \cdot P(0 \leq w < b_1) + b_1 P(b_1 \leq w < b_2) + \dots + b_M P(b_M \leq w < b_{M\&l}) = \sum_{j=1}^{M\&l} b_{j\&l} p_j$$

where  $p_{M+1} = 1 - F_M$ . The variance of the lower bound mean is:

$$(15) \quad V\left(\sum_{j=1}^{M\&l} p_j b_{j\&l}\right) = \sum_{j\&l} b_{j\&l}^2 (V(F_j) + V(F_{j\&l})) + 2 \sum_{j=1}^M b_j b_{j\&l} V(F_j)$$

where the variance of each proportion  $V(F_j)$  is equal to  $F_j(1-F_j) / (N_j + Y_j)$ .

This too can be calculated rather easily from a simple table of proportions of yes's or no's and the total number of respondents in each grouping. The results of applying these formulas are displayed in Tables 5 and 6, which also provide a linear interpolation for the median.

Notice in the tables that  $b_M$  is the highest bid actually offered respondents and is the lower bound of the final interval running from  $b_M$  to infinity. In the expected value formula in Eq. (14),  $b_M$  is used with no attempt to guess at an appropriate value to apply to the portions of the two sub-samples who had WTPs greater than R\$20 (24 and 11 percent respectively). This is what produces the lower bound label and distinguishes the Turnbull approach from Kriström's method discussed next.

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<sup>23</sup> The estimate of  $F_j$  assumes the proportion of no responses increases as the bid increases across all bid classes. If not, McConnell and Haab (1997) show how to join bid groups to achieve monotonically increasing proportions. This was not necessary with the Tietê survey data, except for the first two bid groups in the far- from- river sub-sample.

Bid Group j	Bid (\$/month)	Bid Range	Total # of "No" Answers N <sub>j</sub>	Total # of Obs. TOTAL <sub>j</sub>	CDF=F <sub>j</sub> = N <sub>j</sub> /TOTAL <sub>j</sub>	PDF=P <sub>j</sub> = F(j)-F(j-1)	Lower Bound Estimate of E(WTP)	
0	0.50	0-0.50	10	37	0.270	0.270	0.00	
1	2.00	0.50 - 2.00	13	33	0.394	0.124	0.06	
2	5.00	2.00 - 5.00	26	41	0.634	0.240	0.48	
3	12.00	5.0 - 12.00	26	35	0.743	0.109	0.54	
4	20.00	12.00 -20.00	29	38	0.763	0.020	0.24	
5	>20.0				1.000	0.237	4.74	
		Totals :	104	184		1		
Note:							E(WTP):	R\$6.07
The median bid was found by linear interpolation between the bids attached to the cumulative frequencies (CDF values) above and below 50%. That is, Med=B <sub>i</sub> + k(i) where B <sub>i</sub> is the lower (left) boundary of the class containing the median (\$2.00), i is the class interval (\$3.00) and k approximates where the 50% point lies inside the CDF values at the lower and upper boundaries ((0.5-0.394)/(0.634-0.394)). So, \$3.33=\$2.00 + 0.44*\$3.00.							Variance E(WTP)	R\$2.99
							Median WTP	R\$3.33

Bid Group j	Bid (\$/month)	Bid Range	Total # of "No" Answers N <sub>j</sub>	Total # of Obs. TOTAL <sub>j</sub>	CDF=F <sub>j</sub> = N <sub>j</sub> /TOTAL <sub>j</sub>	PDF=P <sub>j</sub> = F(j)-F(j-1)	Lower Bound Estimate of E(WTP)	
0	2.00	0.00 - 2.00	93	170	0.547	0.547	0	
1	5.00	2.00 - 5.00	57	79	0.722	0.174	0.35	
2	12.00	5.0 - 12.00	62	85	0.729	0.008	0.04	
3	20.00	12.00 -20.00	73	85	0.89	0.161	1.93	
4	>20.0				1	0.11	2.2	
		Totals :	285	416		1		
Note:							E(WTP):	R\$4.51
The median bid was found by linear interpolation between the bids attached to the cumulative frequencies (CDF values) above and below 50%. That is, Med=B <sub>i</sub> + k(i) where B <sub>i</sub> is the lower (left) boundary of the class containing the median (\$0.00), i is the class interval (\$2.00) and k approximates where the 50% point lies inside the CDF values at the lower and upper boundaries (0.5/0.547). So, \$1.83=\$0.00 + 0.914*\$2.00.							Variance E(WTP)	R\$1.31
							Median WTP	R\$1.83



## Kriström's Nonparametric Mean

Kriström's (1990) nonparametric method is even easier to calculate and understand than the Turnbull. In words, all one does is array the frequency of affirmative responses in each bid class in monotonically descending order with ascending bids, connect the points by linear interpolation, and approximate the integral under the resultant empirical cumulative density to get the mean (see Annex 1). The figures below show the approximate empirical distributions. Average income in the close-to-river sample is 30 percent higher than the far-from-river average, which probably causes the corresponding density to be more stretched out toward the higher bid levels.

**Figure 3. Nonparametric Inverse Cumulative Distributions**



Unlike the Turnbull, the bid that drives the probability of acceptance to zero must be specified by the analyst if the survey does not reveal it, so Kriström's mean depends in part on this arbitrary value. To construct the empirical cumulative densities pictured above, a conservative upper limit of R\$40 for  $b_{M+1}$  was assumed, which is approximately three percent of average household income (see Ardila *et al.* 1998). Tables 7 and 8 show the calculation steps.

The influence of the final interval between the last posited bid and the assumed bid driving acceptance to zero is evident from the entries in the penultimate row and last column of the tables, just above their shaded "Average WTP" cells. In the close-to-river case, this value accounts for nearly seventy-five percent of the overall mean value, and in the far-from-river-case, forty-five percent of the mean value is due to the last interval. If the upper limit driving the acceptance rate to zero were set to R\$30 rather than R\$40, the close and far means would fall by about 50¢ and 75¢, respectively, illustrating their sensitivity to this assumption. The nonparametric estimates of location would probably be better had the sample included more bid intervals spanning a wider bid range.

**Table 7. Kriström Nonparametric Mean: Close to River Sample**

Bid Group j	Bid (R\$/month)	Bid Range	Bid Mid-Point	Total # of "Yes" Answers (Y <sub>j</sub> )	Total # of Obs. Total j	1-F <sub>j</sub> = Y <sub>j</sub> /Total j	P <sub>j</sub> = [1-F <sub>j-1</sub> ]-[1-F <sub>j</sub> ]	Kriström Estimate of WTP
na	0.00	0	0	na	na	1	na	0.00
0	0.50	0-0.5	0.25	27	37	0.7297	0.2703	0.07
1	2.00	0.5-2.0	1.25	20	33	0.6061	0.1237	0.15
2	5.00	2.0-5.0	3.5	15	41	0.3659	0.2402	0.84
3	12.00	5.0-12.0	8.5	9	35	0.2571	0.1087	0.92
4	20.00	12.0-20.0	16	9	38	0.2368	0.0203	0.32
5	40.00	20-40	30	0	0	0.0000	0.2368	7.11
Note: The median bid was found by linear interpolation between the actually offered bids (not mid-points) attached to the cumulative frequencies (CDF values) above and below 50% acceptance. That is, $Med=B_u - k*i$ where $B_u$ is the bid in the first class containing more than 50% of "yes" observations (\$5.00), $i$ is the interval between adjacent bids bordering the median (\$3.00) and $k$ approximates where the 50% point lies $((0.6061-0.50)/(0.6061-0.3659))$ . So, $\$3.67 = \$5.00 - 0.44*\$3.00$ .								<b>Average WTP:</b> R\$9.42  <b>Median WTP:</b> R\$3.67

**Table 8. Kriström Nonparametric Mean: Far from River Sample**

Bid Group j	Bid (R\$/month)	Bid Range	Bid Mid-Point	Total # of "Yes" Answers (Y <sub>j</sub> )	Total # of Obs. Total j	1-F <sub>j</sub> = Y <sub>j</sub> /Total j	P <sub>j</sub> = [1-F <sub>j-1</sub> ]-[1-F <sub>j</sub> ]	Kriström Estimate of WTP
na	0.00	0	0	na	na	1	na	0.00
0	2.00	0.0-2.0	1.25	77	170	0.4529	0.5471	0.55
1	5.00	2.0-5.0	3.5	22	79	0.2785	0.1745	0.61
2	12.00	5.0-12.0	8.5	23	85	0.2706	0.0079	0.07
3	20.00	12.0-20.0	16	9	82	0.1098	0.1608	2.57
4	40.00	20-40	30	0	0	0.0000	0.1098	3.29
Note: The median bid was found by linear interpolation between the actually offered bids (not mid-points) attached to the cumulative frequencies (CDF values) above and below 50% acceptance. That is, $Med=B_u - k*i$ where $B_u$ is the bid in the first class containing more than 50% of "yes" observations (\$2.00), $i$ is the interval between adjacent bids bordering the median (\$2.00) and $k$ approximates where the 50% point lies in the interval $(([1-.4529]-.50)/(1.00-0.4529))$ . So, $\$1.83 = \$2.00 - 0.086*\$2.00$ .								<b>Average WTP:</b> R\$7.09  <b>Median WTP:</b> R\$1.83

## The Bounded Probit or Logit of Haab and McConnell

Rather than starting from a RUM model specification as we did in Equations 1 through 6 above and then backing out the expression it implies for the median or mean WTP, Haab and McConnell (August 1997, July 1998, January 1999) start at the other end with an expression for WTP that represents the amount of income the individual is willing to pay, expressed as the product of income and a proportion of income lying between zero and one. Somewhat analogous to the conventional RUM, the proportion is estimated as a function of the bid amount and other socioeconomic variables (see Eqs. 17 and 18) but the bid-related variable disappears when predicting the median proportion (see Eq. 19).<sup>24</sup>

While this approach makes no claim to being consistent with any theoretical indirect utility function, it solves the practical problem of finding a non-zero WTP that at the same time will not exceed income. Haab and McConnell suppose that WTP lies between zero and some upper bound,  $A_i$ , such that:

$$(16) \quad \text{Median}(WTP_i) = \frac{A_i}{1 + e^{-X_i \beta - e_i}} = p(e_i) A_i$$

where  $p(e_i) = 1/(1+e^{-X_i \beta - e_i})$  falls in the (0,1) interval,  $e_i \sim N(0, s^2)$ ,  $X_i \beta$  is the inner product of the  $J$  covariates ( $X_i = X_{i1} \dots X_{iJ}$ ) and a vector of coefficients  $\beta$  and  $A_i$  is a known constant for individual  $i$ , such as income, which is assumed to be a reasonable upper bound on willingness to pay. When  $A_i$  is interpreted as income, equation (16) shows that WTP goes to zero for very large negative errors or  $X_i \beta$  and to income with very large positive errors or  $X_i \beta$ .

If the  $i$ th respondent is asked “Would you pay ‘ $B_i$ ’ for a proposed water quality improvement?” the probability of a no response is the probability that willingness to pay would be less than  $B_i$ . Haab and McConnell write this as:

$$(17) \quad P(WTP_i < B_i) = P\left(\frac{A_i}{1 + e^{-X_i \beta - e_i}} < B_i\right) = P\left(\frac{e_i}{s} < \frac{-\ln\left(\frac{A_i - B_i}{B_i}\right) - X_i \beta}{s}\right)$$

When  $e_i$  is distributed  $N(0,1)$ , the last expression on the right hand side is the contribution to the likelihood function for a standard probit model, where the probability of a ‘no’ response is modeled with the covariates  $X_i$  and  $\ln [(A_i - B_i)/B_i]$ . Similarly, the probability of a ‘yes’ response becomes:

$$(18) \quad P(WTP < B_i) = P\left(\frac{e_i}{s} < \frac{\ln\left(\frac{A_i - B_i}{B_i}\right) + X_i \beta}{s}\right)$$

Combining (17) and (18) results in a standard probit model with  $X_i$  (including a constant) and  $\ln [(A_i - B_i)/B_i]$  as covariates. The estimated coefficient on  $X_i$  will be an estimate of  $\beta/s$  and the estimated coefficient for  $\ln [(A_i - B_i)/B_i]$  will be an estimate of  $1/s$ . The unscaled  $\beta$ s can be recovered by dividing the estimates of  $\beta/s$  by the estimated parameter  $1/s$  attached to the constructed variable  $\ln [(A_i - B_i)/B_i]$ . The median WTP for each individual is then obtained by setting  $e_i$  in (16) to zero because that is the value that splits the symmetric error distribution in half.

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<sup>24</sup> The balance of this section is drawn directly from parts of Haab and McConnell’s papers.

$$(19) \quad Median(WTP_i) = \frac{A_i}{1 + e^{-X_i \beta}} = p(\mathbf{e}_i) A_i$$

Application of the Bounded Probit estimator to the Tietê data leads to the median calculations demonstrated in Tables 9 and 10, using individual household income for the upper limit.<sup>25</sup>

The first two columns of each table refer to estimation of a Probit probability model for each of the two subsamples (Close, Far) where the dependent variable is 0 if the respondent rejected the survey offer (a “no”) and 1 if it was accepted (a “yes”). The Probit parameter estimates are reported in the third column. In general (Maddala 1983, p. 23) they are measurable and estimable only up to a scalar (1/s) but the model specification in this particular case provides an independent estimate of that scalar (see the Btrans variable row in the tables) that allows unscaled parameter estimates to be recovered. They are reported in the fourth column. The summed product of the untransformed parameters and the explanatory variables gives an estimate of the average value of the index function  $X\beta$ . Inserting that index function value in Eq. 19's expression  $1/(1+e^{-(X\beta)})$  produces a median estimate of the fraction of income that would be offered to get the water quality improvements provided by the project (0.0021 for beneficiaries close to the river and 0.0005 for those living farther away). Multiplying the fraction by average income (“A” in Eq. 19) produces a Bounded Probit estimate of Median (WTP). The results of this exercise are reported in the summary table in the next section where all the WTP estimates are collected.

There is no closed form analytical solution for the expected value of WTP in the bounded probit or logit formulation, so it must be found by numerical integration (Haab and McConnell, August 1997). The general form of expected willingness to pay is given by:

$$(20) \quad E(WTP_i) = \int_{-4}^4 WTP(X_i \beta, e) f(e) de$$

The integral in (20) can be approximated by:

$$(21) \quad E(WTP) \approx \sum_{k=1}^n (1/s) f\left(\frac{e_k}{s}\right) \cdot WTP(X_i \beta, e_k) (e_k - e_{k-1})$$

where  $f(\bullet)$  is the standard normal pdf,  $e_k$  are points on the distributional support of  $e$  and  $n$  is large enough so that the approximation is smooth. We used 5000 points to apply (21), approximating  $f(\bullet)$  by successive differences in the standard normal CDF,  $F(\bullet)$ , a technique explained in Annex 1.<sup>26</sup> Table 11 immediately following the median calculations illustrates selected portions of the 5000 evaluation points used to get a numerical approximation to the Bounded Probit mean for the close-to-river group. Similar calculations (not shown) were done for the far-from-river group.

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<sup>25</sup> At the request of a referee, similar mean and median calculations were done based on estimation of a Bounded Probit model imposing an upper limit on WTP at 20% of household income. Bounds much less than 20% could not be imposed using the full sample since in some cases the bid offered was around 18% of income, so going below that would involve a negative sign on the variable  $(A-B)/B$ , which has no logarithm. For the medians, not much was gained or lost by imposing the limit. The bounded median under a 20% of income constraint was \$3.25 for the close-to-river group and \$0.60 for those far away.

<sup>26</sup> Haab and McConnell (1997) provide a quick numerical approximation technique based on a few point estimates of the pdf that dispenses with setting up a large number of points,  $n$ , but is less smooth and hence less accurate than (21). Although we applied it to test whether our more exact approximation worked, it is not discussed here because the shortcut can be fairly imprecise if the range in the standard normal deviate,  $e/s$ , and the number of evaluation points are not properly chosen.

<b>Table 9. Bounded Probit Median: Close to River Sub-Sample</b>					
Limit=100% of Income (Mean Income = \$1,524.39 Reals/Household/Month)					
Variable	Variable Definition	Original Probit Parameter Estimates <sup>2</sup> (β/s)	Unscaled Parameter Estimates <sup>3</sup> (β)	Variable Means (X)	Variable Means *Unscaled Parameters (XB)
Constant		-1.3089	-5.5886 *	1	-5.5886
Status	1 if Upper;0 Else	0.2715	1.1592	0.147	0.1704
Age	Age of Household Head, Years	-0.0108	-0.0459 *	49.38	-2.2677
Btrans <sup>1</sup>	ln ((Income-Bid)/Bid)	0.2342	0.2342 *	5.324	n.a
Barrio	1 if Close to River; 0 Else	0.3569	1.5237 *	1	1.0000
Notes: <sup>1</sup> This is the bounding variable whose parameter estimate, 1/s, is used to unscale the rest of the βs. <sup>2</sup> A * denotes significance at the 1% level or better. <sup>3</sup> Original parameter estimates divided by 1/s, the parameter attached to Btrans.				Xβ=Column Sum	-6.1622
				Fraction of Income =1/(1+exp(-Xβ))	0.0021
				Median =Share*Income	R\$3.21

<b>Table 10. Bounded Probit Median: Far from River Sub-Sample</b>					
Limit=100% of Income (Mean Income = \$1,148.97 Reals/Household/Month)					
Variable	Variable Definition	Original Probit Parameter Estimates <sup>2</sup> (β/s)	Unscaled Parameter Estimates <sup>3</sup> (β)	Variable Means (X)	Variable Means *Unscaled Parameters (XB)
Constant		-1.3089 *	-5.5886	1	-5.5886
Status	1 if Upper;0 Else	0.2715	1.1592	0.094	0.1090
Age	Age of Household Head, Years	-0.0108 *	-0.0459	44.34	-2.0363
Btrans <sup>1</sup>	ln ((Income-Bid)/Bid)	0.2342 *	n.a	5.324	n.a
Barrio	1 if Close to River; 0 Else	0.3569 *	1.5237	0	0.0000
Notes: <sup>1</sup> This is the bounding variable whose parameter estimate, 1/s, is used to unscale the rest of the βs. <sup>2</sup> A * denotes significance at the 1% level or better. <sup>3</sup> Original parameter estimates divided by 1/s, the parameter attached to Btrans.				Xβ=Column Sum	-7.5159
				Fraction of Income =1/(1+exp(-Xβ))	0.0005
				Median =Share*Income	R\$0.63

**Table 11. Numerical Approximation of Bounded Probit Mean WTP: Close to River Sub-Sample**

Location of Median	Step #	Standard Normal Deviate (e/s)	Error (e)	Cumulative Normal Density, CDF F (e/s)	Approximate pdf f(e/s) . ? F (e/s)	-Xβ-e	WTP Ratio, R 1/(1+e(-XB-e))	Product of WTP Ratio*pdf ? F (e/s)*R
	1	-6.0000	-25.618	9.8659e-10				
	2	-5.9976	-25.608	1.0013e-09	1.4691e-11	31.7702	0.0000e+00	2.3412e-25
	•	•	•	•	•	•	•	•
	417	-5.0014	-21.354	2.8458e-07	3.5227e-09	27.5167	1.1211e-12	3.9495e-21
	418	-4.9990	-21.344	2.8814e-07	3.5653e-09	27.5064	1.1327e-12	4.0384e-21
	•	•	•	•	•	•	•	•
	834	-4.0004	-17.081	3.1618e-05	3.1921e-07	23.2427	8.0504e-11	2.5698e-17
	835	-3.9980	-17.070	3.1940e-05	3.2229e-07	23.2325	8.1334e-11	2.6213e-17
	•	•	•	•	•	•	•	•
	1250	-3.0018	-12.825	1.3419e-03	1.0543e-05	18.9868	5.6770e-09	5.9854e-14
	1251	-2.9994	-12.807	1.3526e-03	1.0619e-05	18.9687	5.7807e-09	6.1388e-14
	•	•	•	•	•	•	•	•
	1667	-2.0008	-8.543	0.022707	1.2909e-04	14.7050	4.1086e-07	5.3036e-11
	1668	-1.9984	-8.533	0.022837	1.2971e-04	14.6948	4.1509e-07	5.3841e-11
	•	•	•	•	•	•	•	•
	2083	-1.0022	-4.279	0.158123	5.7887e-04	10.4413	2.9201e-05	1.6903e-08
	2084	-0.9998	-4.269	0.158704	5.8026e-04	10.4310	2.9501e-05	1.7119e-08
	•	•	•	•	•	•	•	•
Median R ->	2500	-0.0012	-0.005	0.499521	9.5765e-04	6.1673	0.00209	2.0038e-06
	2501	0.0012	0.005	0.500479	9.5765e-04	6.1571	0.00211	2.0244e-06
	•	•	•	•	•	•	•	•
	2917	0.9998	4.269	0.841296	5.8166e-04	1.8934	0.13086	7.6117e-05
	2918	1.0022	4.279	0.841877	5.8026e-04	1.8831	0.13203	7.6614e-05
	•	•	•	•	•	•	•	•
	3333	1.9984	8.533	0.977163	1.3033e-04	-2.3704	0.91454	1.1919e-04
	3334	2.0008	8.543	0.977293	1.2971e-04	-2.3806	0.91534	1.1873e-04
	•	•	•	•	•	•	•	•
	3750	2.9994	12.807	0.998647	1.0696e-05	-6.6443	0.99870	5.5230e-05
	3751	3.0018	12.817	0.998658	1.0619e-05	-6.6546	0.99871	1.0606e-05
	•	•	•	•	•	•	•	•
	4166	3.9980	17.070	9.9997e-01	3.2540e-07	-10.9081	9.9998e-01	3.2539e-07
	4167	4.0004	17.081	9.9997e-01	3.2229e-07	-10.9183	9.9998e-01	3.2228e-07
	•	•	•	•	•	•	•	•
	4583	4.9990	21.344	1.0000e+00	3.6083e-09	-15.1820	1.0000e+00	3.6083e-09
	4584	5.0014	21.354	1.0000e+00	3.5653e-09	-15.1923	1.0000e+00	3.5653e-09
	•	•	•	•	•	•	•	•
	4999	5.9976	25.608	1.0000e+00	1.4904e-11	-19.4458	1.0000e+00	1.4904e-11
	5000	6.0000	25.618	1.0000e+00	1.4691e-11	-19.4560	1.0000e+00	1.4691e-11
				<b>Grand Totals:</b>	1.0000e+00			0.0919
				<b>E(WTP) = 3 ? F (e/s) * R * Income = 0.0919 * R\$1524:</b>				<b>R\$140.10</b>

Note: A • indicates intervening calculations that are not shown. Givens for the approximation are 5000 evaluation points; an e/s range from -6 to +6; a step size (? (e/s) of 0.0024, a standard deviation (s) of 4.2697 (Table 9, reciprocal of the Btrans parameter), and an index value (-Xβ) of 6.1622 (Table 9).

The Bounded Probit mean results (R\$140.10 and R\$60.46 for households close to and far from the river, respectively) are completely inconsistent with all that has come before, being more than a factor of ten greater than the highest of all of the preceding estimates, and 45 and 100 times larger than their respective close and far-from-river sub-sample Bounded Probit medians.<sup>27</sup>

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<sup>27</sup> While this phenomenon might be an artifact of one or more mistakes in setting up the approximation, we were able to replicate all of the examples given in Haab and McConnell (1997) successfully. In addition, in the example in Table 7 of their paper, the Bounded Probit mean exceeds the median by a factor of 38, which is similar to what happens with the Tietê data. Reference to Table 11 shows that the median ratio is properly located, but the distribution is heavily skewed. Imposing a bound on median WTP at 20% of income brought the near and far means down to \$50.05 and \$25.54 which are only slightly more plausible. Some doubt about the usefulness of the Bounded Probit mean (but not the median) in CB analysis is probably warranted.

## **UNCERTAINTY IN COST-BENEFIT ANALYSIS: A COMPARISON OF RESULTS**

Uncertainty about WTP need not translate into uncertainty about a project's net benefits. If the analysis decision is unambiguous because a project's net present value (NPV) is either consistently positive or consistently negative across the plausible range of possible WTP estimates, then any one of them will suffice. But it is impossible to establish in advance, without actually doing the exercise, that a given project analysis decision will be impervious to variations in the central tendency measure of WTP, making the choice of measure a matter of indifference. On the contrary, uncertainty about benefits is a project-specific issue, and has to be handled on a case-by-case basis. Robustness is likely to be the exception rather than the rule.

The second and third columns in Table 12 collect all of the central tendency measures calculated from the Tietê project's WTP survey data. Sorting them from high to low in the near-to-river category confirms rather dramatically the introductory warning that a wide range of plausible estimates can be extracted from referendum data. Even disregarding the bounded Probit mean, the highest near-to-river WTP exceeds the lowest by a factor of four, and the factor is ten for the far-from river estimates.

The next three columns of the table show, in deterministic sensitivity fashion, the effect that using each of the alternative WTP measures would have on the economic feasibility of the project at issue, expressed in terms of net present value (NPV) using a twelve percent interest rate. In general, under optimistic assumptions about execution timing and the earliest possible manifestation of energy benefits<sup>28</sup> (the "best case" scenario), the project decision is not severely affected by the wide variety of per household benefit measures available to appraise it in this particular case. Under the most optimistic of assumptions the project as a whole (Stages I, II and III) is not viable except under the Bounded Probit mean benefit measure, while the incremental project (Stages II and III) that treats Stage I costs as sunk is economically justified for all but the lowest WTP measure. Said otherwise, if the initial conditions were set optimistically and the problem posed to different analysts each using a different WTP measure, the final conclusion would be near unanimous and unaffected by the measure chosen.

The apparent absence of a grey area or zone of ambiguity in the incremental project appraisal decision vanishes when the initial conditions are set less favorably (the "worst case" scenario in the table). While the project as a whole gets even worse and is consistently rejected, the once favorable decision on the configuration of Stages II and III becomes cloudier if the execution period is extended over fifteen years rather than completed in ten and if energy benefits do not materialize at all. Then, the final column of the table shows that the incremental project only looks economically feasible for six of the measures, mostly means, and is infeasible (negative NPV) for the other five, which are mainly medians of one sort or another. This result demonstrates another remark made early-on about the implications of using the mean rather than the median — the former will generally produce a more favorable outcome with WTP distributions that are skewed to the right.

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<sup>28</sup> These benefits arise from resuming the use of water from the Tietê for hydro-electric generation after transfer to a different sub-basin. This use had been suspended because the low quality of the Tietê water was degrading the reservoir into which the Tietê was diverted.



**Table 12. Cost-Benefit Comparisons**

Central Tendency Measure	WTP per Household per Month (1998 Reals)		Net Present Value (Million Reals)		
	Close	Far	Scenario & Project Stages		
			Best Case I, II & II	Best Case II&III	Worst Case II & III
Bounded Probit Mean, Limit =100% of Income	140.10	60.46	10722	11456	6665
Truncated Mean, E(C), $0 < C < 4$ (utility difference logit, linear in bid, C)	9.73	6.16	-40	684	310
Kriström's Nonparametric Mean	9.42	7.09	-20	704	322
Truncated Mean, E(C), $0 < C < B_{max}$ (utility difference logit, linear in bid)	7.65	5.03	-233	501	202
Turnbull Nonparametric Lower Bound Mean	6.07	4.51	-348	376	128
Untruncated Mean, E(C)= Median, $-4 < C < 4$ (utility difference logit, linear in bid, C)	4.74	-1.27	-628	96 <sup>b</sup>	-38 <sup>b</sup>
Truncated Mean, Log Transform, 4UL (utility difference logit, log of bid)	4.66	1.46	-570	153	-4
Truncated Mean, Log Transform, Income UL (utility difference logit, log of bid)	3.49	1.23	-656	67	-55
Nonparametric Median (Linear Interpolation)	3.33	1.83	-641	83	-46
Bounded Probit Median, Limit =100% of Income	3.21	0.63	-700	24	-81
Truncated Median, Log Transform (utility difference logit, log of bid)	2.34	0.61	-758	-34	-115

Notes:

- The "Best Case" sets the construction period to 5 years each for Stages II and III, and has energy benefits on line in the first year after Stage II is built. The "Worst Case" sets the execution period to 10 years for Stage II and 5 years for Stage III, and assumes no energy benefits come on line over a 30-year horizon.
- Far-from-river WTP arbitrarily set to zero to compute NPV

However, the median measure only indicates the price at which a project proposal would be accepted by a majority vote under a one-person, one-vote rule. If the project's NPV is negative using the median, that does not necessarily imply it is not worth doing from a social welfare standpoint. Aggregating up using the mean to get total benefits is more consistent with standard cost-benefit practice where the "votes" are in monetary units, and outliers with high willingness to pay count in the calculation of the ability of the winners to compensate the losers and still come out ahead (McFadden and Leonard 1993, p. 193).

There is no golden rule for resolving ambiguities about project approval brought on by uncertainty about the central tendency measure of willingness to pay except, perhaps, to be aware of this source of uncertainty and to explicitly acknowledge it rather than ignore or conceal it. At a minimum, a search for the existence of a grey area should be conducted. If the project is either economically unjustified using the highest of all legitimate benefit measures or justified using the lowest among the candidates, all the better because benefits uncertainty is demonstrably not an issue.

If, on the other hand, the project acceptance decision is reversed somewhere along the spectrum of possible measures, there are several simple decision rules that could be applied, including picking the greatest WTP to push the project ahead and avoid controversy, choosing a measure somewhere in the middle of the range to impart some balance to the final recommendation, or taking a conservative posture by selecting a measure at the low end. A more sophisticated approach would be to fold all of the empirical distributions of the expected value measures together, either with equal probability of drawing from each (akin to picking something in the middle) or with unequal weights reflecting the analyst's judgement or confidence.<sup>29</sup> Finally, one could try to argue for a specific choice on theoretical or econometric grounds, although abstruse technical explanations are unlikely to be popular with decision makers who are ultimately responsible for financing multi-million dollar projects.<sup>30</sup>

Looking at the preceding table, it would be prudent to discard the Bounded Probit and the Untruncated Rum means—the former is ridiculously high and the latter is theoretically inconsistent and ridiculously negative. The choice between means and medians is philosophical; choosing a mean is consistent with standard aggregation practice in CBA. Eschewing the medians and moving on to the remaining means, the Kriström nonparametric mean is too heavily influenced by tail value assumptions to be reliable in this case. After this process of elimination, the remaining means are all legitimate contenders. If one had to choose a single measure, a reasonable choice would be the Turnbull expected value because it is a conservative lower bound measure that in this case falls in the middle of the pack.

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<sup>29</sup> Ardila (1993) and Hazilla (forthcoming) show how empirical distributions of mean WTP can be generated, given knowledge of the variances and covariances of the statistically estimated parameter estimates that appear in the E(WTP) formulas.

<sup>30</sup> For example, one reviewer suggested that the analyst could legitimately argue for the E(WTP) from the parametric log bid model if in fact the presence of a fat right tail in the distribution is caused by a high percentage of positive responses at high bid levels. A statistical test of the parametric linear versus log bid models, suggested by both reviewers, would be even more rigorous, but more difficult, since these are non-nested hypotheses (see Ozuna *et al.* 1993; McFadden 1994; McFadden and Leonard 1993 for possible tests). As stated at the onset, the specification issue and associated statistical tests which might help narrow the field of candidate benefit measures in parametric approaches that condition average WTP on covariates is beyond the scope of this paper and has not been pursued. Our point is that, while the issue may be worth further thought, simple nonparametric approaches (like the Turnbull and Kriström methods) obviate the need for such testing because they directly produce an estimate of population E(WTP) from the marginal rather than the conditional distribution of WTP. Moreover, they are at least as precise as conditional mean WTP parametric approaches under most circumstances, do not require any prior assumptions about the distribution of preferences, and yield a consistent measure of E(WTP) which is not susceptible to the misspecification errors that at least potentially can plague the parametric distribution-fitting techniques, rendering their E(WTP) estimates statistically inconsistent (McFadden 1994).

## CONCLUDING OBSERVATIONS

No mysterious code of silence has been broken here by revealing the uncertainty inherent in referendum CV estimates of WTP — the academic literature, particularly of late, has covered the issue in some depth and many experienced project analysts are probably well aware of it. Yet that literature is at times inaccessible and hard to understand, and no synthesis exists emphasizing the implications of using these several CV measures in investment project appraisal. Therefore, the main purpose of this paper has been to explain, in simple terms using worked examples, the nature of the problem and the solutions available to everyday practitioners. That having been done, what practical recommendations can be made? The most obvious would seem to be:

- ! Do an open-ended survey at the pre-test stage to get an idea of the bid range to use in a full-blown referendum survey and produce a tentative benchmark WTP from the open-ended data to compare against.
- ! Design the referendum to cover the bid range so nonparametric means and medians can be computed reliably. Monitor the survey results, perhaps executing it in phases, so adjustments in the bid range can be made if coverage deficiencies become apparent.
- ! Run a battery of central tendency measures, definitely including a nonparametric measure and perhaps including the bounded Probit median, rather than arbitrarily picking one or two of the more familiar parametric measures.
- ! Explore the influence of the several WTP measures on the cost-benefit analysis outcome, looking for the existence or absence of the uncertain grey area.
- ! Reach a reasoned final recommendation about project feasibility based on the above, and be able to explain it.

In sum, before becoming completely and inextricably caught up in the fine points of econometric estimation of parametric choice models it is worth pausing to consider the options available and the point of the exercise. If the primary goal is to explain and understand respondent behavior, verify whether CV survey responses are consistent with economic theory, or estimate WTP for a population other than the one sampled, parametric choice models must be estimated. If all one needs is a benefit measure for CB analysis, on the other hand, nonparametric estimates of WTP may have the edge. McFadden and Leonard (1993, pp. 167-168) summarize the advantages and disadvantages of each route:

*...direct approaches to valuing a resource do not require any parameterization of preferences or the distribution of tastes, and do not require that WTP be related to any consumer characteristics such as age or income, because the final impact of these variations is taken care of by random sampling from the population...The advantages of parametric methods are that they make it relatively easy to impose preference axioms, pool data across experiments, and extrapolate the calculations of value to different populations than the sampled population. Their primary limitation is that, if the parameterization is not flexible enough to describe behavior, then the misspecification will usually cause the mean WTP calculated from the estimated model to be a biased estimate of true WTP.*

## NUMERICAL INTEGRATION FORMULAS FOR THE MEAN

The mean  $E(x)$  of a continuous random variable  $x$  with a cumulative distribution function  $F(x)$  and probability density function  $f(x)$  –which is the first derivative of  $F(x)$  w.r.t.  $x$ – is given by:

$$(1) E(x) = \int_{-4}^{+4} x f(x) dx$$

The problem is to use a discrete approximation to (1) above to compute:

$$2) E(x) \approx \sum x f(x)$$

where the range of  $x$  is approximately minus to plus infinity for the untruncated mean and zero to some upper limit  $x_{max}$  for the truncated mean.

The fundamental theorem of the calculus tells us that the area under a curve  $f(x)$  between the limits  $x_1$  and  $x_2$  is (i) the sum of a number of infinitesimally small subdivisions in  $x$  of length  $n$ ; (ii) the definite integral of  $f(x)$  between the limits; or the difference between the integral  $F(x)$  evaluated at  $x_1$  and  $x_2$ :

$$(3) \lim_{n \rightarrow \infty} \sum_{i=1}^n f(x_i) \Delta x_i = \int_{x_1}^{x_2} f(x) dx = F(x_2) - F(x_1)$$

We know the value of  $F(x)$  for any bid  $x$  from the bid group proportions. Therefore, we can split the  $x$  range into “small” intervals and sum the means from each small interval to get the grand mean. That is, the contribution to the overall mean from the approximate mean *within* any bid group interval is the product of some  $x$  within the interval (i.e. the lower limit,  $x_1$ , the upper limit,  $x_2$ , or some arbitrary value of value of  $x$  in between which Kriström’s method sets at the group mid-point) times the probability that  $x$  lies between  $x_1$  and  $x_2$ :

$$(4) E(x) \text{ in interval } x_2 - x_1 = \int_{x_1}^{x_2} x f(x) dx = x[F(x_2) - F(x_1)] \text{ for } (x_1 \sim x \sim x_2):$$

Generalizing, then, the grand mean is the sum of the interval sub-means. That is, symbolically, using the lower limit of each interval for each  $x_i$  and repeatedly applying (4) above:

$$(5) E(x) \approx x_1[F(x_2) - F(x_1)] + x_2[F(x_3) - F(x_2)] + x_3[F(x_4) - F(x_3)] \dots + x_{n-1}[F(x_n) - F(x_{n-1})]$$

where  $x_1$  = a large negative number for the unrestricted mean or 0 for the truncated mean and  $x_n$  equals a large positive number for the unrestricted mean and the truncated mean bounded at zero but unbounded from above, or  $x_{max}$  when bounding from above at average income or some fraction thereof.

In addition, the density (a.k.a. pdf and  $f(x)$ ), at some point in any interval given ascending values for  $x$  (i.e.  $x_1 < x_2 < x_3 < \dots < x_n$ ) is approximated by — and proportional to — the difference between adjacent CDF values (Freund and Walpole, Theorem 3.3, p. 80), where the factor of proportionality is the sum of  $f(x)$  over the sampled points to normalize to one (Pollard 1977):

$$(6) f(x_i) \approx \frac{1}{3} \frac{[F(x_i) - F(x_{i-1})]}{\sum f(x)}$$

The above relationships can be used to compute the mean by numerical integration for any of the formulas in Table 1, even without access to specialized software. While admittedly crude, with a sufficient number of points it is possible to come very close to the analytical results in a simple spreadsheet setup by computing the sum of the products of the interval mid-points (or lower bounds) times the difference in adjacent CDF values,  $\sum x_i [F(x_i) - F(x_{i-1})]$ . Equivalently,  $f(x)$  values can be multiplied by the successive values of  $x$  and summed, but the result has to be divided by the normalizing factor  $\sum f(x)$  to get the mean.<sup>31</sup>

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<sup>31</sup> McFadden and Leonard (1993, p. 195) also provide a numerical approximation formula for the mean based on the integral under the cumulative distribution function (Eq. 8 in the main text) rather than our approximation based on the summed product of the density function and  $x$  (Eq. 5 in this Annex). The integral under the distribution function can be obtained easily using mathematical software; for example, LIMDEP's FINTEGRATE command. But if the variable of integration is allowed to take negative values, and FINTEGRATE or an equivalent route is taken, the user should be careful to take the integrals under the positive and negative domains of  $x$  in two separate steps and subsequently sum them. We checked our spreadsheet-based integrals with FINTEGRATE and obtained comparable answers.

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