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Abstract

Singapore's unique monetary policy consists of a managed exchange rate framework that can be characterized as a Taylor-like reaction function with the nominal devaluation rate instead of the nominal interest rate as the main policy instrument. We build a small open economy New Keynesian model to estimate and characterize such a monetary rule from a welfare perspective. Welfare gains under an exchange rate rule (ERR) relative to the more standard interest rate-based Taylor rule (IRR) are unambiguously increasing in the degree of trade openness (defined as exports plus imports as a share of GDP). For Singapore, where trade openness is 280% of GDP, we estimate welfare gains of 1.48% of permanent consumption under an ERR. In a counterfactual thought experiment, we find that Chile, an established inflation-targeting economy using an IRR, would be better off under an ERR for any degree of openness above 100% (currently at 70%).

JEL classifications: E52, E58, F41
Keywords: Monetary policy, Exchange rate management, Open economy macroeconomics

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1 Introduction

Over the last couple of decades, a bipolar view has dominated research about monetary and exchange rate policy. On the one hand, most modern inflation-targeting economies conduct monetary policy using the interest rate (or a monetary aggregate) as their primary policy instrument, leaving the exchange rate (ER) floating freely in response to macroeconomic shocks. Letting the exchange rate act as the "shock absorber," the argument goes, helps accommodate foreign shocks while providing independence to the monetary authority to set interest rates and achieve internal balance. On the other hand, a large body of research has focused on the costs and benefits of fixed exchange rate regimes, in which the monetary authority commits to keeping the nominal exchange rate at a given value, effectively giving up on conducting an autonomous monetary policy.

In practice, however, the vast majority of countries worldwide operate some intermediate regime, such as dirty floating (with occasional interventions to reduce ER volatility), crawling bands (that allow exchange rate fluctuations within a periodically adjusted band), or a crawling peg (in which the band collapses to a potentially adjustable fixed value). Table 1 shows that 70 of 112 analyzed countries use some intermediate exchange rate framework (or soft peg), 36 classify as free floaters, and only 6 use a hard peg. The table also reveals that, on average, economies that actively manage their exchange rates exhibit much lower exchange rate volatility and are significantly more open to trade.

Table 1: Exchange Rate (ER) Regime, Degree of Openness and Exchange Rate Volatility

	No. Countries	Imports/GDP, %	Exports/GDP, %	ER volatility, %
Hard Peg	6	94	93	1.0
Soft Peg	70	47	40	2.7
Free Floating	36	37	33	5.0

Notes: Based on data from the IMF's AREAER and IFS. The GDP ratios are in nominal terms. The exchange rate (ER) volatility is the standard deviation of the quarterly nominal devaluation rate. All statistics are obtained as simple averages across countries in the period 2009Q1-2019Q4. We exclude countries in the European Monetary Union and those whose exchange rate regime changed between 2009 and 2011.

In small and open economies, the exchange rate constitutes a crucial transmission channel of monetary policy (Svensson (2000); Parrado and Velasco (2002); Galí and Monacelli (2005)). On the monetary side, there is an exchange rate pass-through, as a nominal devaluation directly increases inflation through its effect on the domestic currency price of imported goods. On the real side of the economy, the real exchange rate affects the terms of trade, affecting domestic demand for foreign goods and aggregate demand (both domestic and foreign) for home-produced goods. In such a context, should small open economies fear floating their exchange rates? Is there a case for a managed exchange rate regime? What is the optimal degree of exchange rate volatility in an open economy buffeted with a wide variety of foreign shocks? How open to trade does an economy have to be to justify exchange rate management?

In this paper, we quantitatively evaluate the macroeconomic and welfare implications of Singapore's unique managed exchange rate system and its potential applicability to other small and open economies. The Monetary Authority of Singapore (MAS) operates a basket, band, and crawl (BBC) exchange rate-based monetary policy framework in which the country's nominal effective exchange rate is managed against a trade-weighted basket of currencies. While the MAS does not follow an official inflation-targeting regime, it declares that a core inflation rate of about 2 percent is consistent with overall price stability. Parrado (2004) and McCallum (2007) show this policy framework can be characterized as a *de facto* Taylor rule-like reaction function, with the nominal exchange rate instead of the more standard nominal interest rate as the main policy instrument. According to the IMF (2022 Article IV Consultation), Singapore's monetary policy framework has served the country well as a robust anchor of price stability. Notably, since its inception in 1981, the country has enjoyed low inflation and high economic growth, becoming one of the most developed economies in the world based on its outstanding trade integration.

Is Singapore better off under its exchange-rate-based monetary policy rule (ERR, henceforth) than a more standard interest rate-based Taylor rule (IRR, henceforth)? Should other small and open developing economies follow Singapore's lead and allow for higher degrees of exchange rate management? To answer these questions, we build a New Keynesian model of a small open economy, in which the monetary authority follows an unconventional Taylor rule with the nominal devaluation rate as its main policy instrument.

We run counterfactual analyses asking whether Singapore could improve welfare by using a standard interest rate-based Taylor rule (IRR) instead of its estimated benchmark exchange rate rule (ERR). Conversely, we run an analogous exercise for the case of Chile, asking whether an inflation targeter using an IRR could improve welfare under a counterfactual ERR à la Singapore. We chose Chile for several reasons. On the one hand, the country is an established inflation-targeter using a credible IRR with very few interventions in the foreign exchange markets (almost free floater). On the other hand, the paper focuses on the openness dimension, so we wanted a free floater economy that is very open to trade and financial transactions. Of course, besides the degree of openness, there are other reasons to fear floating the exchange rate, e.g., lack of credibility in the monetary authority or liability dollarization (see Calvo and Reinhart, 2002). Since Chile is well-known for its highly credible Central Bank and low liability dollarization, we can indeed focus the analysis on the relationship between trade openness and the desirability of exchange rate management.

We start by estimating the models for Singapore and Chile, respectively, using a large set of observable macroeconomic time series and applying state-of-the-art Bayesian methods. We provide estimates of the benchmark monetary rules in both countries as well as counterfactual rules used later for welfare comparisons. Before proceeding with simulations and the welfare analysis, we validate our estimated models by comparing a large set of second moments generated by the model against the data. To understand the main transmission channels at work, we characterize the model dynamics to a wide array of key domestic and foreign shocks, including a monetary policy shock under the unconventional ERR.

From a welfare perspective, we find both countries are better off under the benchmark monetary policy rules they have used over the last several decades. For Singapore, adopting an IRR over an ERR would result in a significant welfare reduction: the average household is willing to give up 1.48% of permanent consumption under the benchmark ERR to avoid living in an economy in which the Central Bank uses an IRR. In the case of Chile, instead, we find that households are willing to sacrifice 0.49% of the consumption profile under the benchmark IRR to avoid living under the counterfactual Singaporean-style ERR. We find that the ERR in Singapore effectively reduces inflation volatility by about 5% relative to a counterfactual IRR, while in Chile, the ERR significantly increases inflation volatility by 68% (from 0.63% to 1.06%).

We illustrate how these welfare implications depend heavily on the degree of trade openness: while exports plus imports represent around 280% of Singapore's GDP, the analogous figure is around 70% in Chile. Motivated by this evidence, we generalize the welfare analysis by means of a thought experiment asking how open a country has to be to justify an inflation-targeting framework centered on the nominal exchange rate as the main policy instrument. We confirm that welfare gains under the ERR are unambiguously increasing in both countries' degree of trade openness. In our baseline results, the threshold degree of openness from which the ERR begins outperforming the IRR is $(x + m)/y$ =32% in Singapore and $(x + m)/y$ =100% in Chile.

Under an ERR, the monetary authority keeps the exchange rate under tight control, changing the rate of nominal devaluations smoothly to obtain low and stable inflation. To give the IRR the best chance to outperform the ERR in Singapore, in a complementary exercise, we augment it with an additional feedback parameter aimed at smoothing excessive exchange rate volatility, following Lubik and Schorfheide (2007). Loosely speaking, we allow for "dirty floating" under the IRR. Notably, the generalized IRR (GIRR) significantly improves the performance of standard interest rate rules, although it is still far from beating the ERR in very open economies.

Related literature. This study is related to the literature that uses general equilibrium models to evaluate the welfare implications of Taylor (1993)-type monetary policy rules in small open economies (Parrado and Velasco, 2002; Galí and Monacelli, 2005; De Paoli, 2009; Corsetti et al., 2010; García-Cicco, 2022). More specifically, we focus on a particular type of Taylor rule in which the monetary authority uses the exchange rate as its policy instrument, leaving the interest rate to be determined by market forces, as in Chow et al. (2014) and Heipertz et al. (2022).

Parrado (2004) followed by McCallum (2006) were the first authors formalizing Singapore's unique monetary policy as a Taylor-type reaction function, but using the exchange rate rather than the interest rate as the policy instrument. They both use GMM techniques to estimate the reaction function. Closer to us, Chow et al. (2014) estimate Singapore's ERR using Bayesian techniques, although they do not allow for feedback on the output gap, and more importantly, they do not report the model's ability to match first and second moments in the data, nor do they provide a clean and systematic welfare comparison between the ERR and the IRR as we do in the present article.

More recently, Heipertz et al. (2022) employ a calibrated version of Galí and Monacelli (2005) to evaluate the conditions under which an ERR à la Singapore may outperform the standard IRR. Unlike Heipertz et al. (2022), we use a small open economy framework with incomplete markets instead of the two-country model with perfect risk sharing initially proposed by Galí and Monacelli (2005). More importantly, we extend the model with two additional dimensions of openness besides the standard foreign goods in the consumption basket of the representative household. First, we introduce investment and capital, part of which also needs to be imported. Second, we allow for an imported input in the production function, which is essential to match Singapore's extreme degree of openness while capturing additional channels by which exchange rate movements affect the macroeconomy. Similar to us, Lombardo and Ravenna (2014) study optimal monetary policy in open economies where production requires an imported input. Unlike us, they quantify the welfare loss from an exchange rate peg relative to the Ramsey policy, while we focus on the welfare implications of a managed exchange rate framework.

The rest of the paper is organized as follows. Section 2 outlines the model. Section 3 delineates the parameterization strategy with an emphasis on the estimated monetary policy rules. Section 4 describes the model dynamics, while Section 5 discusses the welfare analysis. Section 6 concludes.

2 The Model

We present a small open economy model à la Parrado and Velasco (2002) and Galí and Monacelli (2005) extended with incomplete financial markets and additional dimensions of trade openness in investment goods and production. Consumption and investment goods require a share of imported goods, and domestic production requires an imported input besides labor and capital. There is habit formation in consumption and adjustment costs in investment, and firms face a Calvo-pricing problem with partial backward indexation. The economy is subject to shocks to preferences, technology, monetary policy, foreign GDP, foreign inflation, foreign interest rates and country spreads. In the case of the Chilean economy, we add a commodity endowment that is fully exported at exogenously given international prices.

2.1 Households

A unit mass of infinitely-lived households populates the economy. Consumption and hours worked are identical across family members. Household preferences are defined over per capita consumption and per capita hours. We use uppercase (Latin and Greek) letters for variables containing a unit root (either because of steady-state growth or positive steady-state inflation). Expected lifetime utility is given by:

$$E_0 \sum_{t=0}^{\infty} \beta^t \xi_t^\beta \left\{ \frac{\widehat{C}_t^{1-\sigma}}{1-\sigma} - \Xi_t \frac{h_t^{1+\psi}}{1+\psi} \right\} \tag{1}$$

where $\widehat{C}_t = C_t - \phi_c \breve{C}_{t-1}$, C_t is consumption, $\breve{C}_t$ is average consumption,[1] h_t is hours worked, ξ_t^β is an intertemporal preference shock, Ξ_t is a term that affects the disutility of work (explained below), and parameters β, σ, ϕ_c, and ψ govern time discount, risk aversion, habit formation in consumption, and the elasticity of labor supply, respectively. The disutility of work term is given by $\Xi_t = \eta \xi_t^h A_{t-1}^{1-\sigma} \Theta_t$, where η is a constant,[2] ξ_t^h is an intratemporal preference shock, A_t is the growth trend of the economy (with balanced growth $a_t = \frac{A_t}{A_{t-1}}$), and the term $\Theta_t = A_{t-1}^{\sigma} (\breve{C}_t - \phi_c \breve{C}_{t-1})^{-\sigma}$ is engineered to eliminate the wealth effect of labor supply as in Galí et al. (2012).

The budget constraint, expressed in home currency units, is given by

$$P_t C_t + P_t^I I_t + B_t + S_t B_t^* = W_t h_t + P_t^H r_t^K K_{t-1} + r_{t-1} B_{t-1} + S_t r_{t-1}^* B_{t-1}^* + \hat{\Sigma}_t, \tag{2}$$

where P_t is the price of the consumption basket, P_t^I is the price of investment, P_t^H is the price of the home-produced good, S_t is the nominal exchange rate, defined as the price of one unit of foreign currency in terms of domestic currency (a positive value of $\pi_t^S \equiv \frac{S_t}{S_{t-1}}$ means a devaluation of the domestic currency), I_t denotes investment, K_t is the stock of capital, r_t^K is the gross rental rate of capital, B_t and B_t^* are the stock of domestic and foreign bonds acquired in period t that pay non-state contingent gross returns r_t and r_t^*, respectively. The term $\hat{\Sigma}_t$ collects profits from firms and foreign rents.[3]

[1] In equilibrium, $\breve{C}_t = C_t$, but when optimizing, the household takes $\breve{C}_t$ as given (external habits).

[2] Constant parameter used to target a normalized steady state value for hours worked.

[3] See the Technical Appendix G for further details.

Physical capital evolves according to:

$$K_t = (1-\delta)K_{t-1} + \left[1 - \Gamma\left(\frac{I_t}{I_{t-1}}\right)\right] I_t \xi_t^i \tag{3}$$

where δ is the capital depreciation rate, ξ_t^i is an exogenous AR(1) process affecting the marginal efficiency of investment, and the function $\Gamma(.)$ represents convex adjustment costs of the form:

$$\Gamma\left(\frac{I_t}{I_{t-1}}\right) = \frac{\phi_k}{2}\left(\frac{I_t}{I_{t-1}} - a\right)^2$$

where ϕ_k governs the degree real rigidity and a is the economy's long-run growth rate.

The interest rate paid on foreign bonds is given by $r_t^* = r_t^{W*} \cdot spr_t$, where r_t^{W*} is the risk-free world interest rate, and spr_t is a country-specific spread, composed by an endogenous component that depends on the economy-wide net foreign asset position, and an exogenous component, (ξ_t^{S*}):

$$spr_t = \overline{spr} \cdot \exp\left[-\phi_b\left(\frac{S_t B_t^*}{P_t^Y Y_t} - \bar{b}\right) + \frac{\xi_t^{S*} - \xi^{S*}}{\xi^{S*}} + \frac{\xi_t^{U*} - \xi^{U*}}{\xi^{U*}}\right] \tag{4}$$

where $\overline{spr}$ is the steady state spread, $\left(\frac{S_t B_t^*}{P_t^Y Y_t}\right)$ is the domestic-currency debt-to-output ratio with steady state value $\bar{b}$, ϕ_b govern the spread elasticity to deviations of the debt-to-output ratio, ξ_t^{S*} is an observable Uncovered Interest Rate Parity (UIP) shock, and ξ_t^{U*} represents an unobservable (risk premium) shocks. The debt-elastic spread is the closing device to avoid a unit root in the net foreign asset position and induce stationarity in the small-open economy, as in Schmitt-Grohe and Uribe (2003). Variables r_t^{W*}, ξ_t^{S*} and ξ_t^{U*} follow exogenous AR(1) processes.

2.2 Consumption Goods

We distinguish between core and non-core consumption goods to improve the model's fit. In Singapore, for instance, the final consumption good, C_t, is a CES basket combining core consumption, $C_{Z,t}$, and two non-core consumption bundles, $C_{V,t}$ (volatiles) and $C_{E,t}$ (energy-related), as follows:

$$C_t = \left[(\gamma_Z)^{\frac{1}{\varrho_C}}\left(C_{Z,t}\right)^{\frac{\varrho_C - 1}{\varrho_C}} + (\gamma_V)^{\frac{1}{\varrho_C}}\left(C_{V,t}\right)^{\frac{\varrho_C - 1}{\varrho_C}} + (\gamma_E)^{\frac{1}{\varrho_C}}\left(C_{E,t}\right)^{\frac{\varrho_C - 1}{\varrho_C}}\right]^{\frac{\varrho_C}{\varrho_C - 1}} \tag{5}$$

where $\gamma_Z = 1 - \gamma_V - \gamma_E$, γ_V and γ_E are the respective shares and ϱ_C is the elasticity of substitution across consumption goods.[4]

The three consumption goods are, in turn, CES baskets combining home-produced and foreign goods. Then, for each $\Psi \in \{Z, V, E\}$:

$$C_{\Psi,t} = z_{\Psi,t}\left[(1-\gamma_C)^{\frac{1}{\varrho_C}}\left(C_{\Psi,t}^H\right)^{\frac{\varrho_C - 1}{\varrho_C}} + (\gamma_C)^{\frac{1}{\varrho_C}}\left(C_{\Psi,t}^F\right)^{\frac{\varrho_C - 1}{\varrho_C}}\right]^{\frac{\varrho_C}{\varrho_C - 1}} \tag{6}$$

[4] We adapt the non-core CPI components to the specific practices of the Central Banks of Chile and Singapore. For Chile, the basket $C_{V,t}$ is mapped to food consumption, while $C_{E,t}$ is mapped to energy consumption. For Singapore, they are mapped to housing-related and transport-related consumption.

where $z_{\Psi,t}$ sector-specific productivity shocks, and γ_C is the share of foreign consumption goods, the openness dimension used by Galí and Monacelli (2005) and Heipertz et al. (2022). For simplicity, we assume the same elasticity of substitution (ϱ_C) between core and non-core goods and between home and foreign consumption goods.

Each consumption bundle, $C^{\Omega}_{\Psi,t}$ for $\Psi \in \{Z, V, E\}$ and $\Omega \in \{H, F\}$, is a Dixit-Stiglitz style aggregate of monopolistically competitive varieties indexed by $i \in [0, 1]$, as follows:

$$C^{\Omega}_{\Psi,t} = \left[\int_0^1 C^{\Omega}_{\Psi,t}(i)^{\frac{\epsilon-1}{\epsilon}} di\right]^{\frac{\epsilon}{\epsilon-1}} \tag{7}$$

where ϵ is the elasticity of substitution across varieties. These unique varieties $i \in [0, 1]$ are produced by monopolistically competitive firms, using labor, capital, and intermediate inputs, subject to Calvo-type nominal price rigidities (more details below).

2.3 Investment Goods

Investment goods are produced by a set of competitive firms operating a CES technology combining home-produced and foreign investment goods, as follows:

$$I_t = \left[(1-\gamma_I)^{\frac{1}{\varrho_I}} \left(I^H_t\right)^{\frac{\varrho_I-1}{\varrho_I}} + (\gamma_I)^{\frac{1}{\varrho_I}} \left(I^F_t\right)^{\frac{\varrho_I-1}{\varrho_I}}\right]^{\frac{\varrho_I}{\varrho_I-1}} \tag{8}$$

where γ_I is the share of foreign investment and ϱ_I is the elasticity of substitution across home and foreign investment goods. γ_I represents the second dimension of openness in the model, in contrast to Galí and Monacelli (2005) and Heipertz et al. (2022) who use only γ_C.

2.4 Home Good Production

There is a continuum of firms indexed by $i \in [0, 1]$ with a CRS Cobb-Douglas technology to produce output $Y^H_t(i)$ combining capital $K_{t-1}(i)$, labor $h_t(i)$, and an imported intermediate input $M^F_t(i)$ as follows:

$$Y^H_t(i) = z_t \left[K_{t-1}(i)\right]^{\alpha_K} \left[M^F_t(i)\right]^{\alpha_M} \left[A^H_t h_t(i)\right]^{1-\alpha_K-\alpha_M} \tag{9}$$

where α_K is the capital share and α_M is the foreign input share, constituting the model's third dimension of openness. The productivity shock z_t follows a stationary AR(1) process, A^H_t is a (labor-augmenting) non-stationary stochastic trend in productivity, with growth rate given by $a^H_t \equiv \frac{A^H_t}{A^H_{t-1}}$. To maintain a balanced growth path, we assume home productivity A^H_t cointegrates with the global productivity trend A_t so that $A^H_t = \left(aA^H_{t-1}\right)^{1-\Gamma} (A_t)^{\Gamma}$, where parameter Γ governs the speed of adjustment to the global growth trend.

In the case of Chile, we assume there is also an endowment of commodity goods (namely, copper) that follows an exogenous and stationary AR(1) process, z^{Co}_t. To maintain balanced growth, we assume the endowment of copper grows at the same productivity trend as the rest of the economy so that $Y^{Co}_t = A_t z^{Co}_t$. Copper output is fully exported at the foreign-currency price P^{Co*}_t.

2.5 Foreign Good Intermediation

There is a continuum of firms indexed by $i \in [0,1]$ with a simple technology to transform an *homogeneous* imported input $M_t(i)$ into a *differentiated* variety $Y_t^F(i)$ as follows:

$$Y_t^F(i) = M_t(i) \tag{10}$$

The price of the homogeneous imported input is given by P_t^M. By the law of one price $P_t^M = S_t P_t^{F*}$, where P_t^{F*} is the foreign-currency price of imported goods and follows an AR(1) process. Cost minimization implies that the input price equals the firms' marginal cost $P_t^M = MC_t^F$. Note the difference between the price of the imported input P_t^M and the average price of the foreign basket P_t^F.

2.6 Price Setting

Firms have monopolistic power over their respective variety $i \in [0,1]$ and set prices à la Calvo (1983). Each period, firms face a probability $(1-\theta^J)$, $J \in \{H, F, H*\}$, of re-optimizing its nominal price $\tilde{P}_t^J(i)$ to maximize expected profits, taking the demand for its variety and marginal costs as given. With probability θ^J firms cannot choose prices optimally and use a passive price updater which depends on a weighted average of lagged sectoral inflation (π_{t-1}^J) and the Central Bank's inflation target (π), with weights ζ^J: $\left[\left(\pi_{t-1}^J\right)^{\zeta^J}(\pi)^{1-\zeta^J}\right]$. See Technical Appendices D, E and F, for further details and derivations.

2.7 Monetary Policy

We evaluate two alternative monetary policy rules. First, a standard interest rate rule (IRR), in which the Central Bank manages the nominal interest rate in order to smooth deviations of inflation and output growth:

$$\frac{r_t}{r} = \left(\frac{r_{t-1}}{r}\right)^{\rho}\left[\left(\frac{\pi_t^Z}{\pi}\right)^{\alpha_\pi}\left(\frac{Y_t}{aY_{t-1}}\right)^{\alpha_y}\right]^{(1-\rho)}\xi_t^m \tag{11}$$

with $\rho \in (0,1)$, $\alpha_y \geq 0$, $\alpha_\pi > 1$, and where $\pi_t^Z = \frac{P_t^Z}{P_{t-1}^Z}$ is inflation (with positive steady state value π), and $\frac{Y_t}{Y_{t-1}}$ is the growth rate of real GDP (defined below), with long-run steady state growth a, and ξ_t^m is a random monetary policy shock following a stationary AR(1) process.

Alternatively, under an exchange rate-based monetary policy rule (ERR), the monetary authority manages the nominal devaluation rate:

$$\frac{\pi_t^S}{\pi^S} = \left(\frac{\pi_{t-1}^S}{\pi^S}\right)^{\rho}\left[\left(\frac{\pi_t^Z}{\pi}\right)^{-\alpha_\pi}\left(\frac{Y_t}{aY_{t-1}}\right)^{-\alpha_y}\right]^{(1-\rho)}\xi_t^m. \tag{12}$$

2.8 Rest of the World

The rest of the world buys the continuum of exportable varieties produced by the small open economy $C_t^{H*}(i)$ which have standard Dixit-Stiglitz form, $C_t^{H*}(i) = \left(\frac{P_t^{H*}(i)}{P_t^{H*}}\right)^{-\epsilon} C_t^{H*}$, where $P_t^{H*}(i)$

is the price of exportable variety i, P_t^{H*} is the average price,[5] and C_t^{H*} is foreign demand which depends on the relative price set by domestic producers, the global economic cycle Y_t^*, and an i.i.d. demand shock for local manufacturing goods ξ_t^{H*}, as follows:

$$C_t^{H*} = \left[a_{t-1} C_{t-1}^{H*}\right]^{\rho^{H*}} \left[\left(\frac{P_t^{H*}}{P_t^*} \right)^{-\epsilon^*} Y_t^* \right]^{1-\rho^{H*}} \xi_t^{H*} \tag{13}$$

where ϵ^* is the price elasticity. Foreign output evolves according to $Y_t^* = A_t z_t^*$, where A_t is the global productivity trend, $a_t = \frac{A_t}{A_{t-1}}$ is the growth of the trend (following an AR(1) process), and z_t^* is a productivity shock following an AR(1) process.

The domestic currency prices of foreign goods obey the law of one price as follows:

$$P_t^M = S_t P_t^{F*} \tag{14}$$

$$P_t^{Co} = S_t P_t^{Co*} \tag{15}$$

where $p_t^{F*} = \frac{P_t^{F*}}{P_t^*}$ and $p_t^{Co*} = \frac{P_t^{Co*}}{P_t^*}$ follow stationary AR(1) processes.[6] Foreign inflation $\pi_t^* = \frac{P_t^*}{P_{t-1}^*}$ also follows an exogenous AR(1). Finally, we define the real exchange rate as $rer_t = \frac{S_t P_t^*}{P_t}$ (increase means depreciation), where the nominal devaluation rate $\pi_t^S = \frac{S_t}{S_{t-1}}$ satisfies

$$\frac{rer_t}{rer_{t-1}} = \frac{\pi_t^S \pi_t^*}{\pi_t}. \tag{16}$$

2.9 Aggregation and Market clearing

The model is closed by a series of aggregating equations and market-clearing conditions. In particular, the home and foreign goods markets as well as the labor market clear in equilibrium. Technical Appendix G shows that the balance of payments can be written as:

$$S_t B_t^* = S_t r_{t-1}^* B_{t-1}^* + TB_t + REN_t \tag{17}$$

where the following definitions for the trade balance TB_t and the income payments (rents) balance REN_t, in domestic currency terms, apply:

$$TB_t = P_t^H C_t^{H*} + P_t^{Co} Y_t^{Co} - P_t^M M_t \tag{18}$$

$$REN_t = S_t \xi^{R*} A_{t-1} - (1-\chi) P_t^{Co} Y_t^{Co} \tag{19}$$

where ξ^{R*} is a steady state constant used to match the observed current account to GDP ratio in the steady state calibration algorithm. Technical Appendices A, B and C provide the full set of equilibrium conditions in non-stationary, stationary, and static (steady state) form.

[5] Note we assume the same elasticity of substitution ϵ across varieties as in the local market.

[6] The commodity price channel P_t^{Co*} is only active in the Chilean model.

3 Parametrization Strategy

We calibrate a set of parameters from previous literature and to match key macroeconomic moments. The remaining parameters, including elasticities, nominal and real rigidities, exogenous AR(1) processes, and the monetary policy rule parameters (α_π, α_y, ρ, σ_{ξ^m}), are estimated using Bayesian techniques as in An and Schorfheide (2007). Appendix A provides a detailed description of the calibrated and estimated parameters for both countries, along with the observable data used in the estimation. We focus here on highlighting the estimation results regarding the monetary policy rules introduced in the previous section (IRR and ERR described in equations (11) and (12)).

We estimate two models for each country: one model using the benchmark rule (ERR for Singapore and IRR for Chile) and another using the counterfactual rule (IRR for Singapore and ERR for Chile). The counterfactual estimation is useful as a reasonable and internally consistent rule to be compared against the benchmark. However, later in the welfare analysis, we generalize this counterfactual rule to take many possible values over a grid of potential feedback coefficients to test if there is any circumstance in which the counterfactual may beat the benchmark policy rule.

A couple of technical points regarding the counterfactual estimation are worth mentioning. First, we just reestimate the monetary policy rule parameters (α_π, α_y, ρ, σ_{ξ^m}), keeping all remaining parameters at their benchmark values. Second, we drop the interest and devaluation rates as observables to avoid forcing the model to accommodate data on the policy instrument and shock absorber recorded under an alternative monetary rule. Of course, the same argument would be valid for the rest of the observable variables, but the main second moments that change under IRR vs ERR are precisely the nominal exchange rate and interest rate. So, by dropping them, we leave the model free to find the parameters of the counterfactual rule that would have generated the business cycle moments observed for all remaining observables.

Table 2 presents the estimations for Singapore's ERR as the benchmark (Panel (a)), and the IRR as the counterfactual scenario (Panel (b)). The estimates indicate that Singapore's monetary authority has conducted its benchmark ERR by assigning greater importance to inflation than output. For instance, a one-percent positive inflation gap triggers a $(1-\rho)\alpha_\pi = (1-0.35)1.37 = 0.89\%$ policy-driven appreciation in the nominal exchange rate, whereas a one-percent positive output growth gap prescribes only a $(1-\rho)\alpha_y = (1-0.35)0.16 = 0.10\%$ nominal appreciation: that is, a relative feedback ratio of $0.89/0.10 = 8.6$.

Our estimation of Singapore's ERR is within the estimates of previous research under alternative methods and data. For instance, using monthly data for the 1999-2002 period and GMM econometric techniques, Parrado (2004) obtained an inflation gap coefficient of $(1-\rho)\alpha_\pi = (1-0.84)0.98 = 0.16$ and an output gap coefficient of $(1-\rho)\alpha_y = (1-0.84)0.14 = 0.02$: that is, a relative feedback ratio of $0.16/0.02 = 8$.

Regarding Singapore's counterfactual IRR, a one-percent positive inflation gap induces the monetary authority to raise the interest rate by $(1-\rho)\alpha_\pi = (1-0.67)2.45 = 0.81\%$, while a one-percent positive output growth gap results in an interest rate increase of $(1-\rho)\alpha_y = (1-0.67)0.13 = 0.04\%$ only.

Table 2: Singapore: Estimated Monetary Policy Rules

	(a) Benchmark (ERR)			**(b) Counterfactual** (IRR)		
	mean	5^{th} pct.	95^{th} pct.	mean	5^{th} pct.	95^{th} pct.
α_π	**1.37**	0.92	1.80	**2.45**	2.07	2.81
α_y	**0.16**	0.07	0.24	**0.13**	0.03	0.22
ρ	**0.35**	0.21	0.51	**0.67**	0.53	0.80
$100\sigma_{\xi^m}$	**1.10**	0.93	1.27	**0.27**	0.11	0.48

Notes: Singapore's monetary policy operates under the Exchange Rate Taylor rule (ERR) as its established benchmark, while the Interest Rate Taylor rule (IRR) serves as the counterfactual scenario. The benchmark results, derived from the baseline model estimation using the Metropolis-Hastings algorithm with 100,000 replications, are compared with the counterfactual estimation. In the counterfactual estimation, we kept all the parameters at their benchmark model parameterization values, except for the monetary policy rule, which was re-estimated using the same set of observable variables but the interest rate and the nominal devaluation rate.

Table 3: Chile: Estimated Monetary Policy Rules

	(a) Benchmark (IRR)			**(b) Counterfactual** (ERR)		
	mean	5^{th} pct.	95^{th} pct.	mean	5^{th} pct.	95^{th} pct.
α_π	**1.61**	1.44	1.79	**1.74**	1.29	2.16
α_y	**0.17**	0.07	0.26	**0.17**	0.07	0.27
ρ	**0.78**	0.73	0.83	**0.36**	0.21	0.52
$100\sigma_{\xi^m}$	**0.18**	0.14	0.22	**3.13**	2.37	3.88

Notes: The notes for Table 2 apply with the sole distinction that Chile's monetary policy operates under the IRR as its established benchmark, while the ERR serves as the counterfactual scenario.

Table 3 reports the analogous estimates for Chile. Under the benchmark IRR reported in Panel (a), a one-percent positive inflation gap induces the Central Bank to hike the interest rate by $(1-\rho)\alpha_\pi = (1-0.78)1.61 = 0.35\%$, while a one-percent positive output growth gap results in a $(1-\rho)\alpha_y = 0.04\%$ increase. These estimates for Chile align closely with those reported by (García et al., 2019) for the period 2001-2017, which obtain feedbacks of $(1-\rho)\alpha_\pi = (1-0.74)1.95 = 0.51\%$ for inflation and $(1-\rho)\alpha_y = (1-0.74)0.13 = 0.03\%$ for output growth.

The counterfactual ERR for Chile presented in Panel (b) indicates that a one-percent increase in the inflation gap induces the monetary authority to appreciate the exchange rate by $(1-\rho)\alpha_\pi = (1-0.36)1.74 = 1.11\%$, while a one-percent increase in the output growth gap results in a policy appreciation of $(1-\rho)\alpha_y = 0.11\%$.

4 Model Dynamics

This section characterizes the dynamic properties of the model. First, we establish the good performance of our estimated models in matching a wide range of second moments observed in the data for both countries. Second, we document the type of shocks to which each of these economies with different degrees of openness is subject. Finally, we compare model dynamics by means of impulse-response functions under the two alternative monetary policy rules.

4.1 Model Fit

Table 4 compares model-based second moments against their data counterparts for a subset of key domestic variables. The model statistics are reported for Singapore (Panel (a)) and Chile (Panel (b)) under their respective benchmark monetary policy rules.

Table 4: Second Moments

		100*s.d.(x_t)		corr$(x_t, \Delta \log y_t)$		corr(x_t, x_{t-1})	
x_t	Description	data	model	data	model	data	model
	(a) Singapore						
$\Delta \log y_t$	GDP growth	1.73	1.71	1.00	1.00	0.22	0.10
$\Delta \log c_t$	Total consumption growth	2.08	2.37	0.23	0.08	-0.22	0.06
$\Delta \log i_t$	Total investment growth	4.65	7.07	0.31	0.10	-0.18	0.62
tb_t/y_t	Trade balance/GDP	6.92	11.20	-0.04	-0.17	0.80	0.94
$\Delta \log h_t$	Hours growth	0.79	0.98	0.22	0.05	0.71	0.10
π_t^C	Headline inflation	0.58	0.58	0.18	-0.26	0.52	0.24
π_t^Z	Core inflation	0.43	0.50	0.05	-0.12	0.26	0.29
π_t^V	Housing inflation	1.37	1.31	-0.07	-0.18	0.62	0.04
π_t^T	Transport inflation	1.92	1.86	0.37	-0.17	0.10	-0.01
r_t	Nominal Interest Rate	0.30	1.70	0.10	-0.12	0.90	0.84
spr_t	Spread	0.25	0.45	-0.17	-0.03	0.84	0.95
rer_t	Real exchange rate	6.41	6.90	0.24	0.08	0.98	0.96
π_t^S	Nominal devaluation rate	1.12	1.21	-0.03	-0.05	0.11	0.35
	(b) Chile						
$\Delta \log y_t$	GDP growth	1.01	0.87	1.00	1.00	0.25	0.19
$\Delta \log c_t$	Total consumption growth	1.01	0.98	0.63	0.39	0.35	0.45
$\Delta \log i$	Total investment growth	3.75	3.71	0.52	0.41	0.33	0.55
tb_t/y_t	Trade balance/GDP	5.41	3.81	0.31	0.04	0.79	0.89
$\Delta \log h_t$	Hours growth	0.84	1.11	0.38	0.22	0.21	0.11
π_t^C	Headline inflation	0.62	0.63	0.10	-0.23	0.56	0.46
π_t^Z	Core inflation	0.53	0.48	-0.18	-0.08	0.80	0.74
π_t^V	Food inflation	2.12	1.94	0.16	-0.23	0.37	0.04
π_t^T	Energy inflation	3.44	3.18	0.19	-0.14	0.14	-0.00
r_t	Nominal Interest Rate	0.48	0.53	-0.11	-0.07	0.92	0.91
spr_t	Spread	0.19	0.24	-0.35	-0.08	0.82	0.88
rer_t	Real exchange rate	7.71	7.81	-0.12	-0.05	0.90	0.91
π_t^S	Nominal devaluation rate	4.59	4.50	-0.13	0.02	0.23	-0.01

Notes: The table presents the standard deviation (percent), the correlation with GDP growth, and the first-order autocorrelation for a subset of endogenous variables under the benchmark model in each country (ERR in Singapore, IRR in Chile). The data sample period is 1991Q1-2019Q3 for Singapore and 1996Q2-2019Q3 for Chile. See Appendix A for further details on data construction.

For both countries, the model does a good job of matching the unconditional volatility of most variables, although overestimating the volatility of investment and the trade balance in Singapore while underestimating the standard deviation of the trade balance in Chile. Notably, the model also overestimates the volatility of spreads in both countries due to the debt-elastic spread mechanism used as the small open economy closing device (see equation (4) and Schmitt-Grohe and Uribe (2003)). Consequently, the model also tends to overestimate the volatility of interest rates, especially in Singapore. Overall, the model performs remarkably well in matching the volatilities of GDP and consumption growth, inflation rates, the real exchange rate, and the nominal devaluation rate.

The model also captures most correlations of key variables with GDP growth and autocorrelations. Most cases in which the model fails to match the sign of these business cycle moments are for relatively small (close to zero and therefore statistically insignificant) correlations. We are not aware of other articles studying the quantitative properties of ERR reporting the goodness of fit of the model.

4.2 What Accounts for the Business Cycle?

Table 5 documents the sources of business cycle fluctuations by means of a variance decomposition of shocks under the benchmark monetary policy rule in each country (ERR in Singapore, Panel (a); and IRR in Chile, Panel (b)). Productivity shocks, grouped under supply shocks, explain most business cycle fluctuations in real output (86% in SGP and 72% in CHL), while demand shocks, including preference and investment shocks, drive consumption growth in both countries (86% in SGP and 83% in CHL).

Table 5: Variance Decomposition

	(a) Singapore, ERR				**(b) Chile, IRR**			
x_t	Supply	Demand	Policy	Foreign	Supply	Demand	Policy	Foreign
$\Delta \log y_t$	86.4	7.1	0.6	5.9	71.6	23.2	0.9	4.2
$\Delta \log c_t$	2.0	85.8	0.1	12.0	11.3	83.0	1.7	4.0
$\Delta \log i_t$	1.1	52.7	2.0	44.2	4.3	81.6	0.7	13.3
tb_t/y_t	2.1	15.0	10.8	72.1	8.2	8.5	0.8	82.6
π_t^C	74.9	0.1	7.4	17.6	52.6	24.2	4.1	19.1
r_t	0.5	0.4	3.8	95.3	11.7	44.3	14.9	29.0
π_t^S	9.2	0.1	82.7	8.0	3.7	0.8	2.7	92.8
rer_t	14.0	0.0	27.2	58.7	39.4	9.6	1.3	49.7

Notes: The table presents the variance decomposition for a set of macroeconomic variables under the benchmark rule in each country. The shocks are grouped into four groups: supply shocks (growth trend and productivity), demand shocks (discount factor, labor wedge, and investment), monetary policy shocks, and foreign shocks (foreign productivity, imported prices, foreign inflation, world interest rates, and UIP shocks).

The two economies differ more significantly regarding investment dynamics. In Singapore, investment growth is driven nearly equally by (investment) demand (53%) and (UIP) foreign (44%) shocks, whereas in Chile, the investment shock itself explains 82%, while foreign shocks explain only 13% of the overall volatility in investment. Regarding the trade balance, both countries share a high incidence of foreign shocks (72% in Singapore vs 83% in Chile), although Singapore stands out because of a non-trivial 11% incidence of (exchange rate-based) monetary policy shocks compared to just 1% in Chile.

Turning to monetary variables, it is noteworthy that inflation in Singapore is mostly supply-driven, with 75% of inflation volatility being explained by supply shocks and 0% by demand shocks. In contrast, the decomposition in Chile is much more balanced between supply-side (53%) and demand-side (24%) inflationary shocks.

As is well known, under an IRR, the monetary authority controls the interest rate to attain internal balance, while the exchange rate is left floating to absorb foreign shocks and secure external balance. Panel (b) of Table 5 precisely illustrates this pattern in Chile: 93% of nominal devaluations are explained by foreign shocks. As expected, the monetary policy interest rate tool responds much more strongly to demand (44%) rather than supply (12%) shocks, while discretionary (unexpected) monetary policy shocks represent 15% of the overall variation in the interest rate.

In stark contrast, under an ERR, the monetary authority manages the nominal devaluation rate to achieve price stability and sustainable economic growth, letting the interest rate be determined by market forces (in essence, to satisfy the UIP condition). Panel (a) of Table 5 shows that in Singapore, indeed, the interest rate becomes the shock absorber, with foreign shocks (mainly UIP shocks and foreign interest rate) accounting for 95% of the overall variance in the domestic interest rate. Perhaps the most notable difference between both rules is the share of policy-induced shocks embedded in the monetary instrument: while monetary policy shocks explain 15% of interest rates under IRR, they account for 83% of the variance in the nominal exchange rate under ERR.

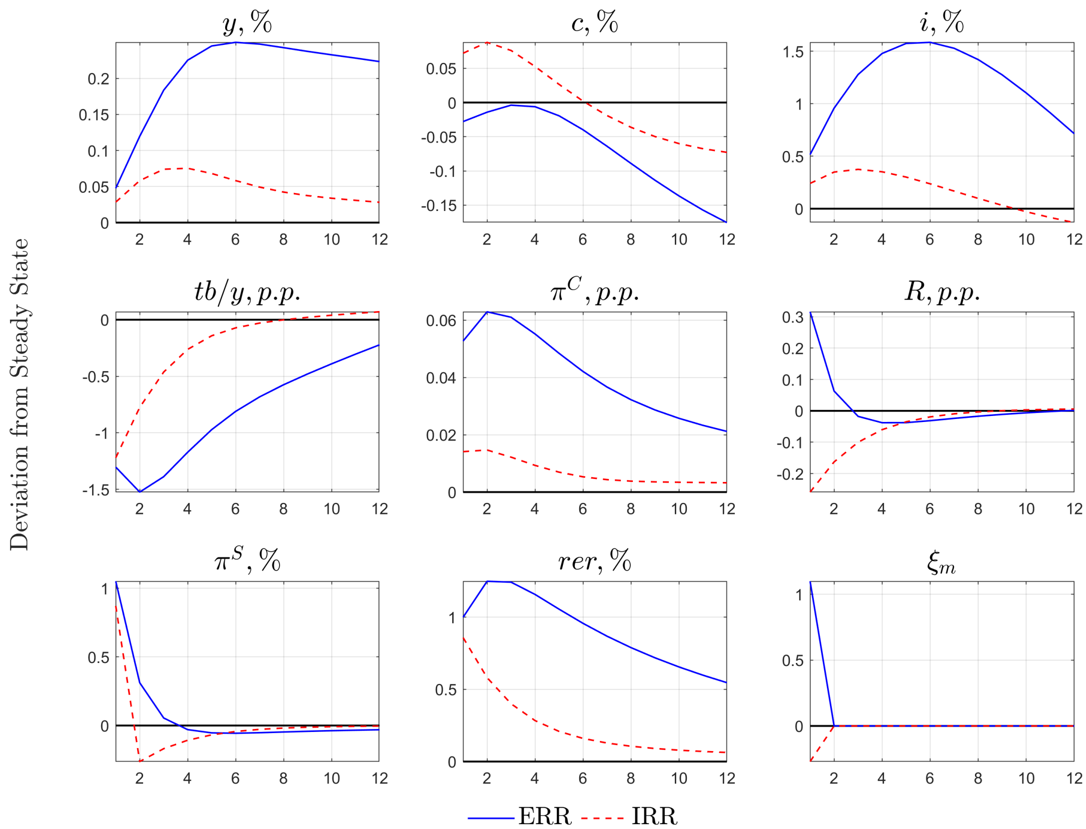
4.3 The Effects of a Monetary Policy Shock under an ERR

Figure 1 characterizes the dynamics of a monetary policy shock under the estimated benchmark ERR and counterfactual IRR for Singapore. Appendix B provides the analogous for the case of Chile. To simplify the comparison across rules, we turned off the feedback on output growth ($\alpha_y = 0$).

A one standard deviation monetary policy shock under the IRR corresponds to a 27-basis points decline in the nominal interest rate. The surprise fall in the interest rate boosts consumption and investment, while higher domestic absorption, a large share of which is imported, deteriorates the trade balance. Capital outflows induce an impact devaluation of the nominal exchange rate, followed by several periods of expected revaluations consistent with the UIP condition. In the short run, prices are sticky, so the nominal devaluation translates into real exchange rate depreciation.

In contrast, a one standard deviation monetary shock under the ERR paradigm corresponds to a 1.1% nominal devaluation at impact, with estimated persistence of the monetary instrument of 0.35. This implies that the initial surprise devaluation is followed by two additional devaluations of 0.31% and 0.05%, respectively, followed by a period of expected revaluations. To satisfy the UIP ($\hat{r}_t = \hat{r}_t^* + E_t\hat{\pi}_{t+1}^S$), the interest rate must increase for a couple of quarters (e.g., $\hat{r}_1 = E_1\hat{\pi}_2^S$), followed by a prolonged period of below-trend interest rates. Low nominal interest rates and higher expected inflation lead to protractedly low real interest rates, fueling a significant investment boom that persistently increases output.

Figure 1: Monetary Policy Shock under Both Monetary Policy Rules



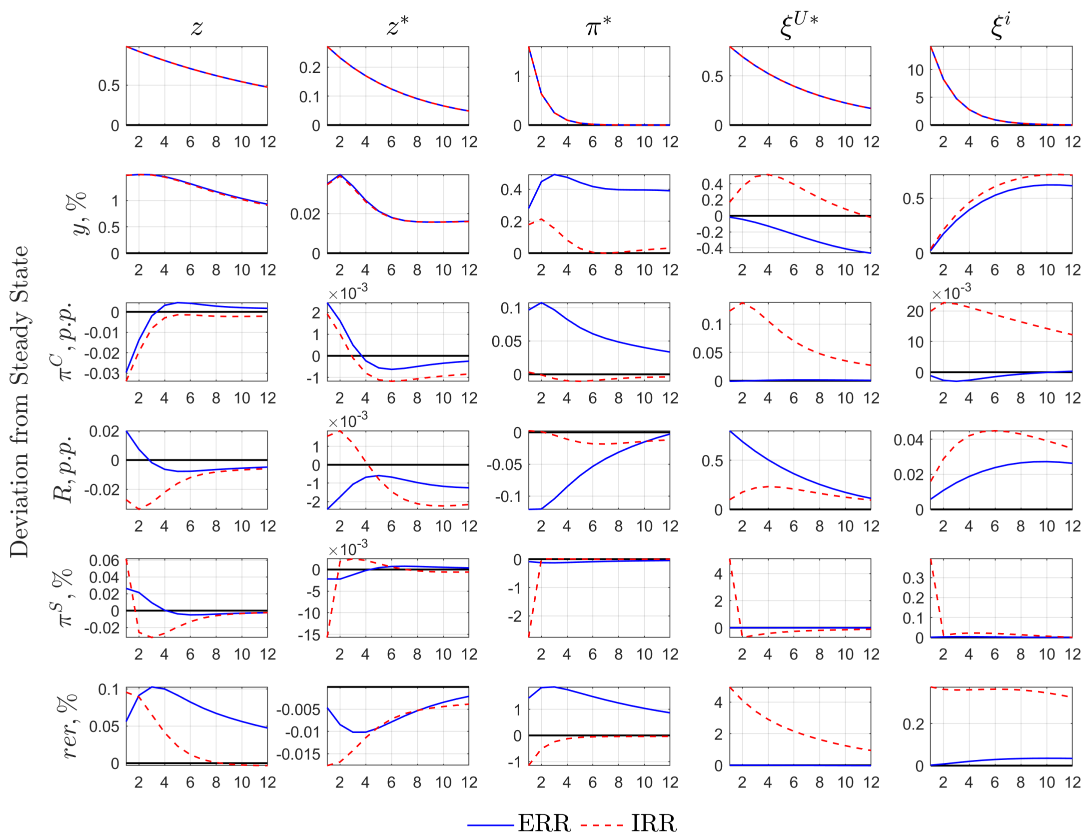
Notes: Responses to an expansionary monetary policy shock under the benchmark ERR versus the counterfactual IRR estimated for Singapore. The shock size is one standard deviation estimated for each rule: a 1.1% nominal devaluation in the exchange rate under the ERR and a fall of 27 basis points in the nominal interest rate under the IRR. For simplicity, both monetary rules are assumed to target inflation only ($\alpha_y = 0$). All variables are expressed in percent deviations from the steady state, except for the trade balance-to-output ratio which is expressed as percentage points of GDP relative to the steady state, and inflation and interest rates which are expressed in percentage points. The model is approximated to the first order.

4.4 The Transmission Mechanisms of Supply, Demand, and Foreign Shocks

Figure 2 summarizes the model's dynamics of key macroeconomic variables in response to standard shocks studied in the New Keynesian literature: a domestic productivity shock (z_t), foreign productivity (z_t^*) and foreign inflation (π_t^*) shocks, a UIP or spread shock (ξ^{U*}), and an innovation to the marginal efficiency of investment (ξ_i). All these shocks are of one standard deviation and reported in the first row of the Figure (See Tables A.4 and A.8 to find the persistences and volatilities estimated for each shock.) A domestic productivity (z_t) shock generates an economic boom led by investment, consumption, and a trade surplus. Ceteris paribus, higher productivity generates an excess supply of domestic goods, reducing inflation pressures and inducing the monetary authority to reduce the interest rate under an IRR. To satisfy the UIP condition, the nominal exchange rate overshoots, depreciating on impact, followed by a three-year period of expected nominal revaluations.

Instead, under an ERR, the Central Bank responds to lower inflation by announcing a smooth path of nominal devaluations of the domestic currency. Expected devaluations reduce the demand for domestic bonds. In equilibrium, interest rates must rise to close the excess supply of domestic bonds and satisfy the UIP.

Figure 2: Impulse Responses to Selected Driving Forces



Notes: Responses to alternative shocks under the benchmark ERR versus the counterfactual IRR estimated for Singapore. All shocks are of one standard deviation. For simplicity, both monetary rules are assumed to target inflation only ($\alpha_y = 0$). All variables are expressed in percent deviations from the steady state, except for inflation and interest rates, which are expressed in percentage points. The model is approximated to the first order.

The dynamics induced by a foreign productivity (z_t^*) shock are quite similar to domestic productivity, inducing an economic boom via higher exports. The only difference is foreign productivity acts as a demand shock for domestically produced goods, which raises inflation and interest rates under an IRR in the short run. The higher monetary policy interest rates increase the demand for domestic bonds, attracting capital inflows that appreciate the nominal exchange rate on impact, followed by a period of expected devaluations.

Under an ERR, the Central Bank responds to inflationary pressures by announcing a smooth revaluation path lasting one year. Expected currency appreciation increases the demand for domestic bonds, inducing a fall in the (market-determined) interest rate consistent with UIP. Overall, as shown in Heipertz et al. (2022), conditional on a productivity shock, either domestic or foreign, the nominal exchange rate is much more volatile under an IRR than an ERR. Next, we illustrate this is also true for most driving forces in the model.

We now turn to the key foreign inflation (π_t^*) shock. Under a standard IRR, the nominal exchange rate appreciates 2.8% at impact, fully absorbing/counteracting imported inflation and even slightly reducing headline inflation. The Central Bank reduces the interest rate accordingly, lifting consumption, investment, and overall output. Notably, despite the one standard deviation increase in foreign inflation, the large exchange rate appreciation induces the RER also to appreciate 1% on impact and stay below trend for several quarters.

Under an ERR, instead of letting the exchange rate jump at impact, the monetary authority announces a smooth appreciation path to be distributed over the following three years, gradually reverting inflation back to the target. To satisfy the UIP, the interest rate needs to jump down on impact and stay below trend for three years, fueling an investment boom and inflation pressures that persist much longer than the original shock. Unlike under IRR, in this case, the RER depreciates more than 1% at impact and stays below trend for several years, as foreign inflation increases much more than domestic inflation, while the nominal exchange rate is quasi-fixed under the managed float ERR. Notably, regarding inflation volatility, this is probably the only shock for which IRR is preferred: at the cost of much larger π_t^S volatility.

On the other hand, a surprise hike in the "risk premium" (ξ_t^{U*}) shock, illustrated in the fourth column of Figure 2, increases the incentive to save, reducing consumption and investment. Since a large share of consumption and investment is imported, real imports decline, and the trade balance improves. The net effect on output depends on which effect dominates: In Singapore, the improvement in the trade balance dominates the fall in consumption and investment, so real output rises; in Chile, the opposite happens, and the UIP shock reduces output. The fall in domestic demand causes a real depreciation, which, due to sticky prices, is obtained via a corresponding nominal exchange rate depreciation at impact, followed by a period of expected revaluations. The initial depreciation pass-through to headline inflation and the Central Bank is forced to raise the monetary policy interest rate (recall we set $\alpha_y = 0$).

Under an ERR, the monetary authority lets the domestic interest rates fully absorb the shock, increasing one-to-one with the increase in foreign interest rates. Higher interest rates induce an even larger decline in consumption and investment than under the IRR, inducing an overall recession with no inflation whatsoever and, hence, no required adjustments in the policy devaluation rate or the real exchange rate.

We conclude this section by studying a positive shock to the marginal efficiency of investment (ξ_t^i). Under an IRR, the shock is expansionary, as it reduces firms' real marginal costs through a decline in the rental rate of capital. Real investment increases 4% on impact and stays above trend for several years, generating a protracted increase in the stock of capital and the demand for labor, which raises real output and wages. The trade balance deteriorates as a significant share of the investment basket requires foreign imported goods. Accordingly, the investment shock leads to a real exchange rate depreciation. In the short run, prices are sticky, so a real depreciation is obtained by a corresponding nominal depreciation. Over time, the exchange rate depreciation is inflationary, to which the Central Bank reacts by increasing the policy interest rate.

Under an ERR, the Central Bank does not allow the exchange rate to depreciate at impact, while the interest rate does rise to satisfy the UIP. In this case, there is no pass-through from depreciation to inflation. In fact, the higher efficiency of investment boosts domestic output, while higher interest rates reduce private consumption. In equilibrium, inflation actually falls slightly, and the Central Bank announces a path of small devaluations to lift inflation back to the target.

5 Welfare Evaluation of Alternative Monetary Instruments

What are the welfare implications of managing the exchange rate, instead of the interest rate, in conducting monetary policy? Under what conditions would inflation-targeter countries in Latin America be better off under an exchange rate-based (ERR) versus an interest rate-based (IRR) monetary policy rule? In this section, we evaluate whether the ERR prevailing in Singapore over the last four decades has been optimal from a welfare perspective relative to a more standard IRR. Analogously, we compute the welfare implications for the Chilean economy of a counterfactual switch from its current IRR to the Singaporean-style ERR.

Let $\hat{\theta} = (\alpha_\pi, \alpha_y, \rho, \sigma_{\xi^m})$ represent the parameter vector of the **benchmark** monetary policy rule estimated for each country: the ERR for Singapore (Table 1, Panel (a)) and the IRR for Chile (Table 2, Panel (a)). Similarly, let $\overline{\theta}$ denote the monetary policy parameters estimated under the **counterfactual** rules: the IRR for Singapore (Table 1, Panel (b)) and the ERR for Chile (Table 2, Panel (b)). We compute consumption equivalent units, λ, as the fraction of lifetime consumption a household is willing to give up to be indifferent between the benchmark $\hat{\theta}$ and the counterfactual monetary policy rule $\overline{\theta}$:

$$E_0 \sum_{t=0}^{\infty} \beta^t U(c_t(\overline{\theta}), h_t(\overline{\theta})) = E_0 \sum_{t=0}^{\infty} \beta^t U((1+\lambda)c_t(\hat{\theta}), h_t(\hat{\theta})).$$

A negative value of λ means the household strictly prefers the benchmark monetary policy rule ($\hat{\theta}$) over the counterfactual ($\overline{\theta}$), in the sense that households are willing to sacrifice $\lambda\%$ of permanent consumption under the benchmark to avoid living under the counterfactual monetary rule. We numerically approximate these value functions using a second-order approximation of the model around the non-stochastic steady state.

5.1 Should Singapore Fear Floating Its Exchange Rate? Would Chile Be Better Off under an ERR?

We start by comparing the welfare and macroeconomic implications of a counterfactual switch from the estimated benchmark to each country's (also estimated) counterfactual monetary policy rule. Later, we generalize the analysis, allowing the counterfactual to operate under a wide range of possible feedback coefficients, thereby giving the counterfactual the best chance to outperform the already operating benchmark rule.

Table 6 summarizes our main results for Singapore (Panel (a)) and Chile (Panel (b)). The table reports the feedback parameters of each monetary rule, the welfare gain/loss of living under the counterfactual rule, and the model-implied volatilities for selected macroeconomic variables. Panel (a) shows that adopting an IRR over an ERR would result in a significant welfare reduction in Singapore: the average household is willing to give up 1.48% of consumption every period to avoid living in an economy in which the Central Bank uses an IRR. However, this does not imply the ERR is necessarily welfare-improving for any small open economy. In fact, our analogous results for Chile in Panel (b) reveal that households are willing to sacrifice 0.49% of the consumption profile under the benchmark IRR to avoid living under the counterfactual Singaporean-style ERR.

Table 6: Welfare Gains under the Counterfactual Rule

	Monetary rule		Welfare gain, %	Macroeconomic Volatility s.d., %								
(a) Singapore	α_π	α_y	λ	y	c	i	tb/y	π^C	R	π^S	rer	b/y
Benchmark ERR	1.37	0.16	-	7.4	5.5	31.2	10.8	0.58	1.7	1.2	6.6	140.3
Counterfactual IRR	2.45	0.13	**-1.48**	7.2	5.5	21.7	10.3	0.61	0.9	6.2	10.4	78.2
ERR to IRR Ratio				1.03	1.01	1.44	1.05	0.95	1.79	0.20	0.64	1.79
(b) Chile	α_π	α_y	λ	y	c	i	tb/y	π^C	R	π^S	rer	b/y
Benchmark IRR	1.61	0.17	-	3.9	3.5	14.3	3.7	0.63	0.5	4.6	8.1	32.8
Counterfactual ERR	1.74	0.17	**-0.49**	4.2	4.3	15.5	4.4	1.06	1.6	3.1	8.5	43.2
ERR to IRR Ratio				1.08	1.22	1.09	1.19	1.68	3.03	0.68	1.06	1.32

Notes: The monetary rule parameters (α_π, α_y) are the estimated values reported in Tables 2 and 3. Welfare gains, λ, are expressed in consumption equivalent units (percent): e.g., $\lambda = -1$ means a welfare loss under the counterfactual (IRR for SGP; ERR for Chile), as the household is willing to sacrifice 1% of permanent consumption under the benchmark to avoid living in the counterfactual. Macroeconomic volatilities are reported as unconditional standard deviations (percent). The model is approximated to the second order.

Notably, in both countries, the ERR induces higher volatility in real variables, such as output, consumption, and investment, than the more standard IRR. However, such excess volatility is much milder in Singapore than in Chile when focusing on real output or consumption. For instance, the unconditional volatility of real consumption in Singapore is 1% higher under the ERR, while in Chile, such volatility increases by 22% (from 3.5% to 4.3%) when moving from an IRR environment to an ERR. In contrast, in Singapore, real investment is 44% more volatile under the ERR relative to the IRR, while in Chile, such volatility is only 9% higher.

The differences between both countries are even starker when looking at the volatilities of monetary variables. The ERR in Singapore reduces inflation volatility by 5%, while in Chile, the ERR significantly increases inflation volatility by 68% (from 0.63% to 1.06%). Under the ERR, the interest rate acts as the econom-wide shock absorber, so its volatility naturally increases, although it does so much more in Chile (from 0.5% to 1.6%) than in Singapore (from 0.9% to 1.7%).

As is expected from a managed float regime, the ERR effectively reduces exchange rate volatility, but there is significant heterogeneity across countries. For instance, in Singapore, the nominal devaluation rate has a volatility of 1.2% under the ERR, which would increase more than five times to 6.2% under a counterfactual IRR. In contrast, Chile under the ERR would have a volatility of 3.1% in the nominal devaluation rate (almost three times larger than in Singapore under the same policy rule), and the volatility reduction relative to the IRR is not as large (from 4.6% under IRR to the 3.1% under ERR).

Lower volatility in inflation combined with much lower fluctuations in the nominal devaluation rate leads to lower volatility in Singapore's real exchange rate (6.6% under ERR versus 10.4% under IRR). Instead, in Chile, the RER is (slightly) more volatile under the ERR, as the fall in the volatility of the nominal devaluation is not enough to counteract larger fluctuations in the inflation rate.

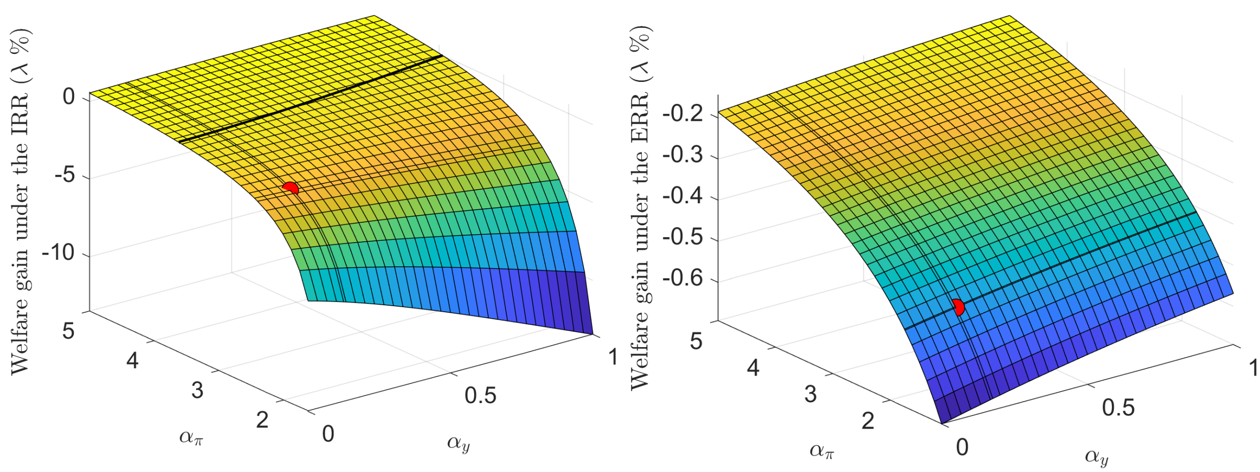
Next, we test the robustness of our results by challenging the estimated benchmark rule against a continuum of possible counterfactual rules. Specifically, we repeat the welfare evaluation reported in Table 6 for many possible combinations of the counterfactual monetary policy feedback parameters ($\alpha_\pi \in [1.1, 5]$ and $\alpha_y \in [0, 1]$). The persistence of the rule and the size of monetary policy shocks are kept fixed at their respective estimated values. Figure 3 illustrates the results. The z-axis shows the welfare gains obtained by comparing the benchmark against the counterfactual rule determined by α_π (in the x-axis) and α_y (in the y-axis). The thick black line on the surface represents all counterfactual rules delivering the same welfare as the benchmark rule, while the red dot denotes the estimated counterfactual rule reported in Table 6.

The results for Singapore in Panel (a) reveal that the benchmark ERR can only be outperformed by an extremely hawkish IRR with $\alpha_\pi > 3.5$, with little sensitivity over the α_y dimension. Adopting a less hawkish IRR raises welfare losses, driven by higher volatility in consumption, inflation, and exchange rates. In the case of Chile, instead, there is no feasible ERR that can outperform the traditional benchmark IRR. Although the counterfactual ERR rule is more attractive as it becomes increasingly hawkish, we find that the country still faces welfare losses using an unrealistic value of $\alpha_\pi = 10$ (not reported).

To summarize, from a welfare perspective, both countries are better off under the benchmark monetary policy rules they have been using over the last decades: the ERR for Singapore and the IRR for Chile. In the next section, we illustrate how this result depends heavily on the degree of openness of the countries under analysis.

Figure 3: Welfare Gains over a Continuum of Counterfactual Rules

(a) Singapore (b) Chile



Notes: Welfare gains (if $\lambda > 0$) and losses (if $\lambda < 0$) are reported in the z-axis over a continuum of possible counterfactual rules relative to the benchmark. For Singapore in Panel (a), we explore counterfactual IRR over $(\alpha_\pi, \alpha_y) \in [1.5, 5] \times [0, 1]$. For Chile in Panel (b), we explore counterfactual ERR over $(\alpha_\pi, \alpha_y) \in [1.1, 5] \times [0, 1]$. The thick black line on the surface represents all counterfactual rules delivering the same welfare as the benchmark rule, while the red dot denotes the estimated counterfactual reported in Table 6.

5.2 Trade Openness and the Desirability of Exchange Rate Management

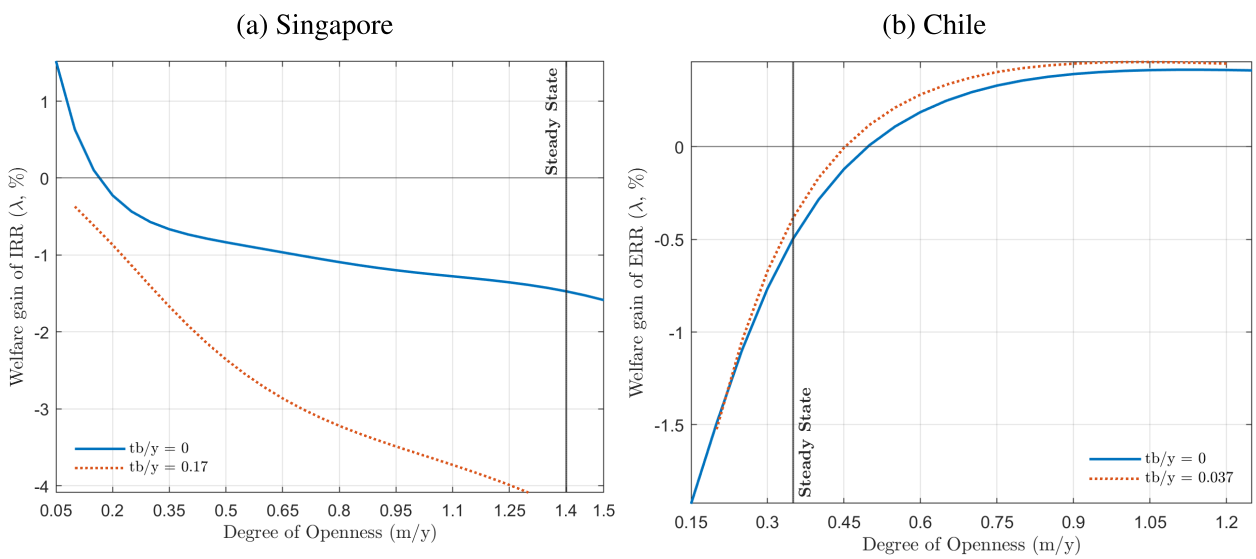
What makes some countries like Singapore better off under an ERR, while others like Chile prefer an IRR? One salient feature of Singapore is its extremely high degree of trade integration with the rest of the world. Between 1986 and 2019, average exports (imports) accounted for 149%

(132%) of GDP, an average trade surplus of 17% of GDP. Accordingly, the exchange rate channel of monetary policy transmission is much more important than in other open economies like Chile, whose (1996-2019) average exports (imports) accounted for 37% (33%) of GDP.

In this section, we generalize the welfare analysis for a continuum of possible degrees of trade openness. In this thought experiment, we ask whether Singapore would still be better off under the ERR if its imports plus exports to GDP ratio is, say 100% instead of the 280% used in the baseline calibration ($m/y = x/y$ =140%).[7] Analogously, we ask how open Chile has to be to eventually justify a monetary framework centered on the nominal exchange rate as the main policy instrument?

Figure 4 shows that welfare gains under the ERR are unambiguously increasing in the degree of trade openness for both countries. The threshold openness from which the ERR begins outperforming the IRR differs, which can be attributed to other country-specific parameters estimated from the data. Under the baseline calibration with zero trade balance ($tb/y = 0$, solid blue lines), the threshold is $m/y = 16\%$ (implying $(x+m)/y$ = 32%) in Singapore (Panel (a)) and $m/y = 50\%$ ($(x + m)/y$ = 100%) in Chile (Panel (b)).

Figure 4: Trade Openness and the Choice of a Monetary Framework



Notes: Welfare gains (if $\lambda > 0$) and losses (if $\lambda < 0$) of the counterfactual monetary rule in the y-axis over a continuum of possible degrees of trade openness in the x-axis. Welfare gains are expressed in consumption equivalent units (percent). The degree of openness is measured as the imports to GDP ratio. Vertical lines correspond to each country's baseline calibration, aligning with the welfare results reported in Table 6. The monetary rule parameters are kept constant at their estimated values reported in Tables 2 and 3. Solid blue lines depict the baseline calibration while dotted orange lines display an alternative calibration using the actual trade surplus observed in each country.

In Singapore, the welfare losses of the counterfactual IRR are even larger when calibrating the steady state to the observed trade surplus of $tb/y = 0.17$ (dotted orange line). Analogously, in Chile, the welfare gains of the ERR are larger when the model is calibrated to the observed trade

[7] We opted for a clean calibration with a zero trade balance in steady-state and thus zero net foreign asset position in the long run. To operationalize such an assumption, in the model, we target export to GDP and imports to GDP to equal the average values observed in the data. See Appendix A for a detailed discussion of the calibration and estimation results in each country. We also provide results using the actual observed trade surplus in the calibration.

surplus, although the distance between both lines is much smaller than in Singapore, given the also much smaller observed trade surplus of $tb/y = 0.034$.

5.3 Generalizing the IRR with Exchange Rate Smoothing

An IRR generates too much exchange rate volatility for economies heavily dependent on international trade. In this section, we test whether an IRR can improve its performance by allowing a feedback coefficient intended to smooth excessive fluctuations in the nominal exchange rate, as in Lubik and Schorfheide (2007). The generalized interest rate rate rule (GIRR) is given by:

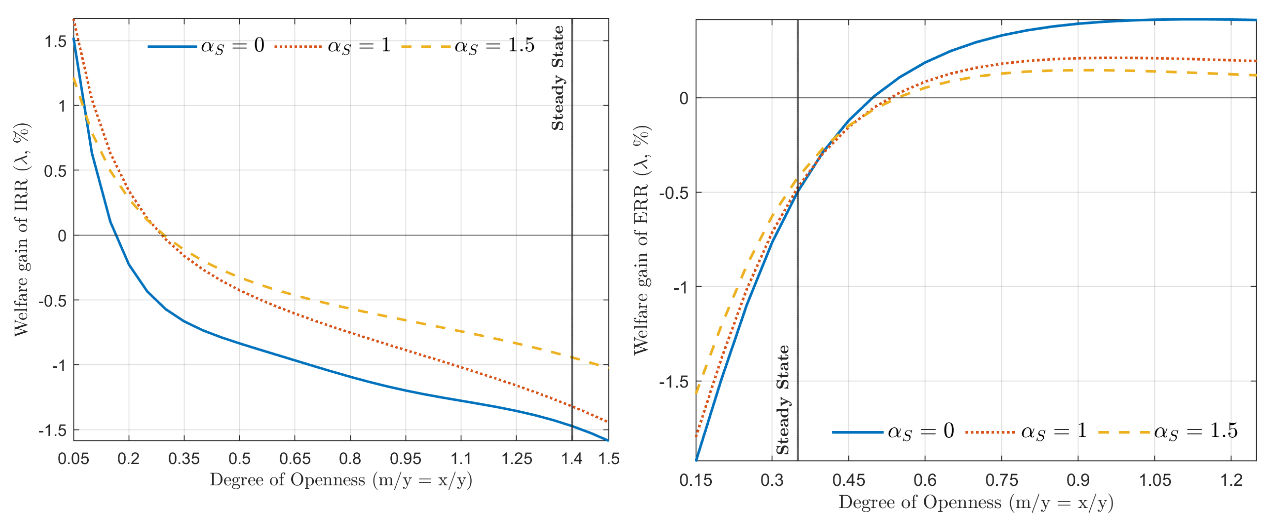
$$\frac{r_t}{r} = \left(\frac{r_{t-1}}{r}\right)^{\rho} \left[\left(\frac{\pi_t^Z}{\pi}\right)^{\alpha_\pi} \left(\frac{Y_t}{aY_{t-1}}\right)^{\alpha_y} \left(\frac{\pi_t^S}{\pi^S}\right)^{\alpha_S} \right]^{(1-\rho)} \xi_t^m$$

where α_S controls the degree of exchange rate smoothness.

Figure 5 redoes the welfare analysis using a GIRR with $\alpha_S \in \{0, 1, 1.5\}$, where the case $\alpha_S = 0$ replicates the baseline result presented in Figure 4. The results show setting $\alpha_S > 0$ would significantly improve the performance of an IRR in Singapore. If for instance $\alpha_S = 1$, the threshold openness from which the ERR beats the IRR is $m/y > 29\%$ (or equivalently, $(x + m)/y > 58\%$), almost double the value with $\alpha_S = 0$. That said, it is important to remember that the ERR still clearly beats the IRR for the quantitatively relevant range of trade openness.

Figure 5: Welfare Gains Using a GIRR

(a) Singapore (b) Chile



Notes: See notes from Figure 4. Solid blue lines depict the baseline calibration without exchange rate smoothing ($\alpha_S = 0$), dotted orange lines display the case with $\alpha_S = 1$, and dashed yellow the case with $\alpha_S = 1.5$.

6 Conclusion

We characterize Singapore's unique monetary and exchange rate policy framework in a state-of-the-art general equilibrium model with trade openness in consumption, investment, and production.

We show that Singapore's managed exchange rate regime can be characterized as a Taylor-like reaction function but using the nominal exchange rate instead of the nominal interest rate as the main policy instrument. This framework allows the monetary authority to massively smooth exchange rate volatility while letting interest rates be determined by market forces to satisfy uncovered interest rate parity.

Quantitatively, we find the exchange rate rule (ERR) has delivered **significant** welfare gains of 1.48% of permanent consumption for Singapore relative to a more standard interest rate-based Taylor rule (IRR). Moreover, the IRR could only beat the ERR by being implausibly hawkish, and even in that case, the welfare gains would be relatively **small**. Importantly, we explore how results are affected by the degree of trade openness. In our baseline results, the threshold openness from which the ERR begins outperforming the IRR is $(x+m)/y = 32\%$ in Singapore and $(x+m)/y = 100\%$ in Chile.

We have abstracted from several dimensions that can enrich future research. On the one hand, we assume rational expectations and full credibility about the Central Bank's actions. To be consistent with our assumptions, we have focused the analysis on two very open economies (Singapore and Chile) with highly credible monetary authorities and established policy frameworks: the ERR in Singapore and the IRR in Chile. As is well-known, however, for the average small open economy, managing the exchange rate may be prone to speculative attacks against the domestic currency, so additional research is required to generalize prescriptions to countries with low degrees of credibility. On the other hand, we have abstracted from pure nontradable goods, which are shown to be an important determinant of optimal monetary policy by Lombardo and Ravenna (2014).

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Appendix

A Calibration Strategy and Estimated Parameters

A.1 Singapore

We calibrate a set of parameters from previous literature and to match key macroeconomic moments. The remaining parameters, including elasticities, nominal and real rigidities, autoregressive processes, and the monetary policy rule (α_π, α_y, ρ, σ_{ξ^m}), are estimated using Bayesian techniques as in An and Schorfheide (2007).

Table A.1 presents the values of parameters fixed a priori, based on previous literature, or to match sample averages in the data. We set the long-run productivity growth of the economy at $a = 2\%$ (annual, per capita), consistent with an average GDP growth of 5.5% and an average labor force growth of 3.3%. The long-run inflation rate is fixed at $\pi = 2\%$ (annual), the Singapore Monetary Authority's inflation target. The foreign long-run inflation rate $\pi^* = 2.0\%$ (annual) is set to normalize the relative price of the foreing basket of goods to $p^F/p^I = 1$. The risk-free interest rate is set to $r^W = 3.4\%$ (annual) and the steady-state spread $spr = 2.6\%$ (annual), the former to normalize $\pi^S = 1$ and the latter the sample average for the Singapore's EMBI.

The scale parameters governing the disutility of work for households are set to normalize total hours to $h = 1$.

We set the risk aversion parameter to $\sigma = 2.0$, a value typically used in the literature, implying an intertemporal elasticity of substitution equal to $IES = 1/\sigma = 0.5$. We follow Medina and Soto (2007) and García et al. (2019) in calibrating the elasticities of substitution across varieties $\epsilon = 11$, implying a markup of $10\% = \epsilon/(\epsilon - 1)$), and capital depreciation rates ($\delta = 0.015$ quarterly).

The volatiles ($\gamma_V = 0.25$) and transport ($\gamma_T = 0.25$) shares in the consumption bundle are taken directly from the CPI basket weights in the data.

Table A.1: Calibrated Deep Parameters

Parameter	Value	Description	Source
$a^4 - 1$	2.0	Trend growth rate (annual. %)	Data: Per Capita Growth
$\pi^4 - 1$	2.0	Inflation rate (annual. %)	Data: Average Inflation
$(\pi^*)^4 - 1$	2.0	Foreign inflation rate (annual, %)	Normalize $p^F/p^I = 1$
$(R^{W*})^4 - 1$	3.4	Foreign risk-free interest rate (annual, %)	Normalize $\pi^S = 1$
$spr^4 - 1$	2.6	Country spread (annual. %)	Data: EMBI spread
η	0.49	Scale parameter disutility of work	Normalize $h = 1$
σ	2.0	Inverse elasticity of intertemporal substitution	Literature
ϵ	11	Elasticity of subst. across varieties	Literature
δ	0.015	Depreciation rate of capital (quarterly)	Literature
γ_V	0.25	Share of Volatiles in CPI basket	Data: CPI basket weights
γ_T	0.17	Share of Transport in CPI basket	Data: CPI basket weights

Table A.2 presents a set of parameters endogenously determined in the steady-state algorithm to match key macroeconomic ratios. The subjective discount factor is set to $\beta = 0.99997$ to hit a nominal interest of 3.8%, the sample average for Singapore's prime interest rate, consistent with a neutral real interest rate of $R - \pi = 3.8\%$. The share of foreigns goods in the consumption basket (γ_C), the share of foreign investment basket (γ_I) and the share of foreign intermediate input used in production (α_M) is calibrated in order to match an imports-to-GDP ratio of 140.5%, under the assumption that $\gamma_I = \gamma_C$ and $\alpha_M = 0.5\gamma_C$. The capital share in production α_K is calibrated to match the investment-to-GDP ratio of 28%. Finally, foreign productivity z^* and foreign rents ξ^{R*} are calibrated to match a zero trade balance and zero current account to GDP ratio, respectively.

Table A.2: Parameters Calibrated to Match Macroeconomic Targets

Parameter	Value	Description	Target	Data	Model
β	0.99997	Subjective time discount factor (quarterly)	Real Interest Rate	3.8	4.1
γ_C	0.92	Degree of openness in consumption basket	Imports-to-GDP ratio	140.5	140.5
γ_I	0.92	Degree of openness in investment basket	$\gamma_I = \gamma_C$	-	-
α_M	0.46	Degree of openness in production	$\alpha_M = 0.5\gamma_C$	-	-
α_K	0.26	Capital share in production	Investment-to-GDP ratio	28.0	28.0
z^*	2.1	SS foreign productivity	Trade balance-to-GDP ratio	0	0
ξ^{R*}	0	SS foreign rents	Current Account-to-GDP ratio	0	0

The remaining parameters are estimated using Bayesian methods following An and Schorfheide (2007). Let z_t be the vector of exogenous processes in the model:

$$z_t = \{a_t, z_t, z_t^{Co}, z_{Z,t}, z_{V,t}, z_{T,t}, \xi_t^i, \xi_t^\beta, \xi_t^h, \xi_t^m, z_t^*, \pi_t^*, p_t^{F*}, p_t^{Co*}, \xi_t^{H*}, r_t^{W*}, \xi_t^{S*}, \xi_t^{U*}\}.$$

Each element of z_t follows an independent AR(1) process given by:

$$z_t = (1 - \rho_z)z + \rho_z z_{t-1} + \sigma_z \varepsilon_{z,t}$$

with $\rho_z \in (0, 1)$, $\sigma_z > 0$, $\varepsilon_{z,t} \sim N(0, 1)$. The set of observables used to inform the model consists of 15 macroeconomic variables at quarterly frequency covering 1991Q1-2019Q3.[8] These variables include:

- **GDP supply side:** (1) real growth of GDP.
- **GDP demand side:** (2) real growth rate of non-durable consumption and services, (3) total investment, and the (4) ratio of the nominal trade balance to GDP.
- **Labor market:** (5) real growth rate of hours worked.
- **Macro prices**: (6) inflation rate of core CPI, (7) volatile CPI and (8) transport CPI; as well as (9) the country premium (EMBI spread), (10) the nominal devaluation rate and (11) the real exchange rate.
- **External variables:** (12) foreign (trade partners) GDP growth rate, (13) foreign (risk-free) interest rate, (14) foreign (trade partners) inflation rate, and (15) import prices inflation rates.

[8]The source for all variables is the Singapore Department of Statistics. Variables are seasonally adjusted and demeaned. All growth rates are changes from two consecutive quarters.

The estimation procedure includes i.i.d. measurement errors for all observables. The variance of the measurement errors is calibrated to 10% of the variance of the corresponding observable. We follow García et al. (2019) in setting the shapes, means, and standard deviations for the priors. Posterior distributions are obtained from a random walk Metropolis Hasting chain with 100,000 draws after a burn-in of 50,000 draws. We also follow García et al. (2019) in scaling the elasticity of the spread with respect to the country's net foreign asset position and the AR(1) processes' standard deviations to have similar parameter magnitudes, thereby improving the efficiency of the joint optimization. Tables A.3 and A.4 report prior and posterior distributions for structural parameters and AR(1) processes, respectively.

Table A.3: Prior and Posterior Distributions: Structural Parameters

Parameters	Description	Initial Prior			Posterior		
		distr.	mean	s.d.	mean	5^{th} pct.	95^{th} pct.
φ	Inverse Frisch elasticity	Gamma	1.50	0.50	**4.49**	3.17	5.94
ϱ^C	Elast. of subst. in home-foreign cons.	Gamma	0.90	0.25	**0.93**	0.47	1.46
ϱ^I	Elast. of subst. in home-foreign inv.	Gamma	0.90	0.25	**0.90**	0.43	1.36
ϕ_c	Habit formation	Beta	0.75	0.10	**0.73**	0.57	0.88
ϕ_b	Country premium debt elast.	Inv-Gamma	1.00	Inf.	**0.29**	0.19	0.40
ϕ_k	Inv. adjustment cost elast.	Gamma	5.00	1.50	**5.28**	2.33	8.30
θ^H	Calvo probability H	Beta	0.75	0.07	**0.64**	0.47	0.82
θ^F	Calvo probability F	Beta	0.75	0.07	**0.95**	0.93	0.97
θ^{H*}	Calvo probability H^*	Beta	0.75	0.07	**0.67**	0.55	0.79
ζ^H	Lagged inflation price adj. weight H	Beta	0.50	0.20	**0.13**	0.01	0.29
ζ^F	Lagged inflation price adj. weight F	Beta	0.50	0.20	**0.28**	0.03	0.55
ζ^{H*}	Lagged inflation price adj. weight H^*	Beta	0.50	0.20	**0.47**	0.10	0.81
Γ	Global growth trend pass-through	Beta	0.50	0.20	**0.62**	0.27	0.95
ϵ^*	Price elasticity of foreign demand	Inv-Gamma	0.20	0.05	**1.54**	0.93	2.23
α_y	Monetary rule response to GDP growth	Normal	0.15	0.05	**0.16**	0.07	0.24
α_π	Monetary rule response to total inflation	Normal	1.50	0.25	**1.37**	0.92	1.80
ρ	Monetary rule smoothing parameter	Beta	0.50	0.15	**0.35**	0.21	0.51

Notes: The table shows posterior distributions obtained from a random walk Metropolis Hasting chain with 100,000 draws after a burn-in of 50,000 draws for the benchmark ERR model. The estimation sample is 1991Q1-2019Q3.

Table A.4: Prior and Posterior Distributions: Exogenous AR(1) processes

Parameters	Description	Initial Prior			Posterior		
		distr.	mean	s.d.	mean	5^{th} pct.	95^{th} pct.
AR(1) coefficient							
$\rho_{c^{H*}}$	Foreign demand	Beta	0.50	0.20	**0.48**	0.24	0.70
ρ_a	Global unit root tech. shock	Beta	0.50	0.20	**0.63**	0.44	0.81
ρ_z	Productivity shock	Beta	0.85	0.07	**0.94**	0.89	0.98
ρ_{z^Z}	Productivity shock, Z	Beta	0.50	0.20	**0.97**	0.94	0.99
ρ_{z^V}	Productivity shock, V	Beta	0.50	0.20	**0.97**	0.95	0.99
ρ_{z^T}	Productivity shock, T	Beta	0.50	0.20	**0.94**	0.90	0.98
ρ_{ξ^β}	Preference shock	Beta	0.50	0.20	**0.10**	0.01	0.21
ρ_{ξ^h}	Labor supply shock	Beta	0.50	0.20	**0.98**	0.95	1.00
ρ_{ξ^i}	Inv. prod. shock	Beta	0.75	0.07	**0.58**	0.38	0.85
ρ_{z^*}	Foreign productivity shock	Beta	0.85	0.07	**0.86**	0.71	0.97
ρ_{π^*}	Foreign inflation shock	Beta	0.50	0.20	**0.40**	0.24	0.56
$\rho_{p^{F*}}$	Imported input price shock	Beta	0.50	0.20	**0.69**	0.46	0.89
$\rho_{R^{W*}}$	Foreign interest rate shock	Beta	0.50	0.20	**0.93**	0.89	0.97
$\rho_{\xi^{S*}}$	Spread shock (observed)	Beta	0.75	0.07	**0.81**	0.71	0.90
$\rho_{\xi^{U*}}$	Spread shock (unobserved)	Beta	0.75	0.07	**0.87**	0.78	0.95
Innovations s.d.							
σ_a	Global unit root tech. shock	Inv-Gamma	0.50	Inf.	**0.35**	0.23	0.47
σ_z	Productivity shock	Inv-Gamma	0.50	Inf.	**0.99**	0.86	1.14
σ_{z^Z}	Productivity shock, Z	Inv-Gamma	0.50	Inf.	**0.41**	0.35	0.48
σ_{z^V}	Productivity shock, V	Inv-Gamma	0.50	Inf.	**1.25**	1.08	1.42
σ_{z^T}	Productivity shock, T	Inv-Gamma	0.50	Inf.	**1.83**	1.56	2.08
σ_{ξ^β}	Preference shock	Inv-Gamma	0.50	Inf.	**15.29**	6.74	25.87
σ_{ξ^h}	Labor supply shock	Inv-Gamma	0.50	Inf.	**4.79**	3.49	5.98
σ_{ξ^i}	Inv. prod. shock	Inv-Gamma	0.50	Inf.	**14.14**	0.11	26.64
$\sigma_{\xi^{H*}}$	Foreign demand	Inv-Gamma	0.50	Inf.	**0.27**	0.12	0.45
σ_{z^*}	Foreign productivity shock	Inv-Gamma	0.50	Inf.	**0.27**	0.15	0.38
σ_{π^*}	Foreign inflation shock	Inv-Gamma	0.50	Inf.	**1.61**	1.37	1.86
$\sigma_{p^{F*}}$	Imported input price shock	Inv-Gamma	0.50	Inf.	**1.11**	0.89	1.33
$\sigma_{R^{W*}}$	Foreign interest rate shock	Inv-Gamma	0.50	Inf.	**0.14**	0.11	0.17
$\sigma_{\xi^{S*}}$	Spread shock (observed)	Inv-Gamma	0.50	Inf.	**0.13**	0.11	0.15
$\sigma_{\xi^{U*}}$	Spread shock (unobserved)	Inv-Gamma	0.50	Inf.	**0.80**	0.20	1.64
σ_{ξ^m}	Monetary policy shock	Inv-Gamma	0.50	Inf.	**1.10**	0.93	1.27

Notes: The table shows posterior distributions obtained from a random walk Metropolis Hasting chain with 100,000 draws after a burn-in of 50,000 draws for the benchmark ERR model. The estimation sample is 1991Q1-2019Q3.

A.2 Chile

Table A.5 presents the values of parameters fixed a priori, based on previous literature, or to match sample averages in the data. We set the long-run productivity growth of the economy at $a = 1\%$ (annual, per capita), consistent with an average GDP growth of 3.5% and an average labor force growth of 2.5%. The long-run inflation rate is fixed at $\pi = 3\%$ (annual), the Chilean Central Bank's inflation target. The foreign long-run inflation rate $\pi^* = 3.0\%$ (annual) is set to normalize the relative price of the foreing basket of goods to $p^F/p^I = 1$. The risk-free interest rate is set to $r^W = 3.5\%$ (annual) and the steady-state spread $spr = 1.5\%$ (annual), the former to normalize $\pi^S = 1$ and the latter the sample average for the Chilean EMBI.

The scale parameters governing the disutility of work for households are set to normalize total hours to $h = 1$.

We set the risk aversion parameter to $\sigma = 2.0$, a value typically used in the literature, implying an intertemporal elasticity of substitution equal to $IES = 1/\sigma = 0.5$. We follow Medina and Soto (2007) and García et al. (2019) in calibrating the elasticities of substitution across varieties $\epsilon = 11$, implying a markup of $10\% = \epsilon/(\epsilon - 1))$, the capital depreciation rates ($\delta = 0.015$ quarterly) and the share of households income from Co production $\chi = 0.33$.

The food ($\gamma_V = 0.19$) and energy ($\gamma_T = 0.06$) shares in the consumption bundle are taken directly from the CPI basket weights in the data.

Table A.5: Calibrated Deep Parameters

Parameter	Value	Description	Source
$a^4 - 1$	1.0	Trend growth rate (annual. %)	Data: Per Capita Growth
$\pi^4 - 1$	3.0	Inflation rate (annual. %)	Data: Average Inflation
$(\pi^*)^4 - 1$	3.0	Foreign inflation rate (annual, %)	Normalize $p^F/p^I = 1$
$(R^{W*})^4 - 1$	3.5	Foreign risk-free interest rate (annual, %)	Normalize $\pi^S = 1$
$spr^4 - 1$	1.5	Country spread (annual. %)	Data: EMBI spread
η	1.62	Scale parameter disutility of work	Normalize $h = 1$
σ	2.0	Inverse elasticity of intertemporal substitution	Literature
ϵ	11	Elasticity of subst. across varieties	Literature
δ	0.015	Depreciation rate of capital (quarterly)	Literature
γ_V	0.19	Share of Volatiles in CPI basket	Data: CPI basket weights
γ_T	0.06	Share of Transport in CPI basket	Data: CPI basket weights
χ	0.33	Share of household income from Co production	García et al. (2019)
p^{Co*}	1	SS exported commodity price (foreign currency)	Normalized

Table A.5 presents a set of parameters endogenously determined in the steady-state algorithm to match key macroeconomic ratios. The subjective discount factor is set to $\beta = 0.99997$ to hit a nominal interest of $R = 4.5\%$, consistent with recent estimates for the Chilean neutral real interest rate of $R - \pi = 1.5\%$ (see Ceballos et al. (2017)). The share of foreigns goods in the consumption basket (γ_C), the share of foreign investment basket (γ_I) and the share of foreign intermediate input used in production (α_M) is calibrated in order to match an imports-to-GDP ratio of 35.3%, under the assumption that $\gamma_I = \gamma_C$ and $\alpha_M = 0.5\gamma_C$. The capital share in production α_K is calibrated to match the investment-to-GDP ratio of 25.3%. The foreign productivity z^* and foreign rents ξ^{R*} are calibrated to match a zero trade balance and zero current account to GDP ratio, respectively.

Finally, the commodity sector productivity is calibrated to match the commodity output share of 0.14.

Table A.6: Parameters Calibrated to Match Macroeconomic Targets

Parameter	Value	Description	Target	Data	Model
β	0.99997	Subjective time discount factor (quarterly)	Real Interest Rate	1.4	2.0
γ_C	0.27	Degree of openness in consumption basket	Imports-to-GDP ratio	35.3	35.3
γ_I	0.27	Degree of openness in investment basket	$\gamma_I = \gamma_C$	-	-
α_M	0.14	Degree of openness in production	$\alpha_M = 0.5\gamma_C$	-	-
α_K	0.34	Capital share in production	Investment-to-GDP ratio	25.3	25.3
z^*	-0.25	SS foreign productivity	Trade balance-to-GDP ratio	0	0
ξ^{R*}	0.37	SS foreign rents	Current Account-to-GDP ratio	0	0
z^{Co}	0.55	SS productivity Co sector	Commodity Output Share	0.14	0.14

The remaining parameters are estimated using Bayesian methods following An and Schorfheide (2007). Let z_t be the vector of exogenous AR(1) processes in the model:

$$z_t = \{a_t, z_t, z_t^{Co}, z_{Z,t}, z_{V,t}, z_{T,t}, \xi_t^i, \xi_t^\beta, \xi_t^h, \xi_t^m, z_t^*, \pi_t^*, p_t^{F*}, p_t^{Co*}, \xi_t^{H*}, r_t^{W*}, \xi_t^{S*}, \xi_t^{U*}\}.$$

Each element of z_t follows an independent AR(1) process given by:

$$z_t = (1-\rho_z)z + \rho_z z_{t-1} + \sigma_z \varepsilon_{z,t}$$

with $\rho_z \in (0,1)$, $\sigma_z > 0$, $\varepsilon_{z,t} \sim N(0,1)$. The set of observables used to inform the model consists of 17 macroeconomic variables at quarterly frequency covering 1996Q2-2019Q3.[9] These variables include:

- **GDP supply side:** (1) real growth of GDP and (2) real growth of commodity GDP.
- **GDP demand side:** (3) real growth rate of non-durable consumption and services, (4) total investment, and the (5) ratio of the nominal trade balance to GDP.
- **Labor market:** (6) real growth rate of hours worked.
- **Macro prices**: (7) inflation rate of core CPI, (8) food CPI and (9) energy CPI; as well as (10) the monetary policy interest rate, (11) the country premium (EMBI spread), and (12) the real exchange rate.
- **External variables:** (13) foreign (trade partners) GDP growth rate, (14) foreign (risk-free) interest rate, (15) foreign (trade partners) inflation rate, (16) import prices inflation rates, and (17) the dollar-denominated inflation of the commodity price.

The estimation procedure includes i.i.d. measurement errors for all observables. The variance of the measurement errors is calibrated to 10% of the variance of the corresponding observable. We follow García et al. (2019) in setting the shapes, means, and standard deviations for the priors. Posterior distributions are obtained from a random walk Metropolis Hasting chain with 100,000

[9]The source for all variables is the Central Bank of Chile. Variables are seasonally adjusted and demeaned. All growth rates are changes from two consecutive quarters.

draws after a burn-in of 50,000 draws. We also follow García et al. (2019) in scaling the elasticity of the spread with respect to the country's net foreign asset position and the AR(1) processes' standard deviations to have similar parameter magnitudes, thereby improving the efficiency of the joint optimization. Tables A.7 and A.8 report prior and posterior distributions for structural parameters and AR(1) processes, respectively.

Table A.7: Prior and Posterior Distributions: Structural Parameters

Parameters	Description	Initial Prior			Posterior		
		distr.	mean	s.d.	mean	5^{th} pct.	95^{th} pct.
φ	Inverse Frisch elasticity	Gamma	1.50	0.50	**2.68**	1.65	3.71
ϱ^C	Elast. of subst. in home-foreign cons.	Gamma	0.90	0.25	**0.79**	0.44	1.13
ϱ^I	Elast. of subst. in home-foreign inv.	Gamma	0.90	0.25	**0.88**	0.43	1.36
ϕ_c	Habit formation	Beta	0.75	0.10	**0.62**	0.50	0.71
ϕ_b	Country premium debt elast.	Inv-Gamma	1.00	Inf.	**0.35**	0.19	0.53
ϕ_k	Inv. adjustment cost elast.	Gamma	5.00	1.50	**3.79**	1.98	6.02
θ^H	Calvo probability H	Beta	0.75	0.07	**0.87**	0.83	0.91
θ^F	Calvo probability F	Beta	0.75	0.07	**0.74**	0.69	0.80
θ^{H*}	Calvo probability H^*	Beta	0.75	0.07	**0.75**	0.61	0.89
ζ^H	Lagged inflation price adj. weight H	Beta	0.50	0.20	**0.87**	0.75	0.98
ζ^F	Lagged inflation price adj. weight F	Beta	0.50	0.20	**0.42**	0.14	0.70
ζ^{H*}	Lagged inflation price adj. weight H^*	Beta	0.50	0.20	**0.46**	0.11	0.82
Γ	Global growth trend pass-through	Beta	0.50	0.20	**0.48**	0.20	0.81
ϵ^*	Price elasticity of foreign demand	Inv-Gamma	0.20	0.05	**0.18**	0.12	0.26
α_y	Monetary rule response to GDP growth	Normal	0.15	0.05	**0.17**	0.07	0.26
α_π	Monetary rule response to total inflation	Normal	1.70	0.10	**1.61**	1.44	1.79
ρ	Monetary rule smoothing parameter	Beta	0.75	0.05	**0.78**	0.73	0.83

Notes: The table shows posterior distributions obtained from a random walk Metropolis Hasting chain with 100,000 draws after a burn-in of 50,000 draws for the benchmark IRR model. The estimation sample is 1996Q2-2019Q3.

Table A.8: Prior and Posterior Distributions: Exogenous AR(1) processes

Parameters	Description	Initial Prior			Posterior		
		distr.	mean	s.d.	mean	5^{th} pct.	95^{th} pct.
AR(1) coefficient							
$\rho_{c^{H*}}$	Foreign demand	Beta	0.50	0.20	**0.55**	0.19	0.94
ρ_a	Global unit root tech. shock	Beta	0.50	0.20	**0.65**	0.61	0.70
ρ_z	Productivity shock	Beta	0.85	0.07	**0.95**	0.91	0.99
ρ_{z^Z}	Productivity shock, Z	Beta	0.50	0.20	**0.79**	0.47	1.00
ρ_{z^V}	Productivity shock, V	Beta	0.50	0.20	**0.96**	0.93	0.99
ρ_{z^T}	Productivity shock, T	Beta	0.50	0.20	**0.92**	0.85	0.98
$\rho_{z^{Co}}$	Productivity shock, Co	Beta	0.85	0.07	**0.90**	0.82	0.98
ρ_{ξ^β}	Preference shock	Beta	0.50	0.20	**0.78**	0.61	0.93
ρ_{ξ^h}	Labor supply shock	Beta	0.50	0.20	**0.62**	0.18	0.96
ρ_{ξ^i}	Inv. prod. shock	Beta	0.75	0.07	**0.62**	0.51	0.74
ρ_{z^*}	Foreign productivity shock	Beta	0.85	0.07	**0.87**	0.73	0.97
ρ_{ξ^*}	Foreign inflation shock	Beta	0.50	0.20	**0.33**	0.22	0.45
$\rho_{p^{F*}}$	Imported input price shock	Beta	0.50	0.20	**0.75**	0.51	0.96
$\rho_{\xi^{Co*}}$	Commodity price shock	Beta	0.50	0.20	**0.91**	0.86	0.96
$\rho_{R^{W*}}$	Foreign interest rate shock	Beta	0.50	0.20	**0.92**	0.88	0.96
$\rho_{\xi^{S*}}$	Spread shock (observed)	Beta	0.75	0.07	**0.81**	0.71	0.90
$\rho_{\xi^{U*}}$	Spread shock (unobserved)	Beta	0.75	0.07	**0.89**	0.83	0.95
Innovations s.d.							
σ_a	Global unit root tech. shock	Inv-Gamma	0.50	Inf.	**0.30**	0.23	0.37
σ_z	Productivity shock	Inv-Gamma	0.50	Inf.	**0.73**	0.61	0.86
σ_{z^Z}	Productivity shock, Z	Inv-Gamma	0.50	Inf.	**0.17**	0.11	0.24
σ_{z^V}	Productivity shock, V	Inv-Gamma	0.50	Inf.	**1.85**	1.54	2.17
σ_{z^T}	Productivity shock, T	Inv-Gamma	0.50	Inf.	**3.20**	2.74	3.70
$\sigma_{z^{Co}}$	Productivity shock, Co	Inv-Gamma	0.50	Inf.	**3.14**	2.63	3.75
σ_{ξ^β}	Preference shock	Inv-Gamma	0.50	Inf.	**4.40**	3.37	5.48
σ_{ξ^h}	Labor supply shock	Inv-Gamma	0.50	Inf.	**2.45**	0.13	4.28
σ_{ξ^i}	Inv. prod. shock	Inv-Gamma	0.50	Inf.	**6.69**	3.30	10.21
$\sigma_{\xi^{H*}}$	Foreign demand	Inv-Gamma	0.50	Inf.	**1.19**	0.12	2.31
σ_{z^*}	Foreign productivity shock	Inv-Gamma	0.50	Inf.	**0.19**	0.12	0.26
σ_{ξ^*}	Foreign inflation shock	Inv-Gamma	0.50	Inf.	**2.13**	1.80	2.46
$\sigma_{p^{F*}}$	Imported input price shock	Inv-Gamma	0.50	Inf.	**1.31**	1.00	1.65
$\sigma_{\xi^{Co*}}$	Commodity price shock	Inv-Gamma	0.50	Inf.	**10.17**	8.66	11.68
$\sigma_{R^{W*}}$	Foreign interest rate shock	Inv-Gamma	0.50	Inf.	**0.14**	0.11	0.18
$\sigma_{\xi^{S*}}$	Spread shock (observed)	Inv-Gamma	0.50	Inf.	**0.11**	0.09	0.13
$\sigma_{\xi^{U*}}$	Spread shock (unobserved)	Inv-Gamma	0.50	Inf.	**0.38**	0.20	0.57
σ_{ξ^m}	Monetary policy shock	Inv-Gamma	0.50	Inf.	**0.18**	0.14	0.22

Notes: The table shows posterior distributions obtained from a random walk Metropolis Hasting chain with 100,000 draws after a burn-in of 50,000 draws for the benchmark IRR model. The estimation sample is 1996Q2-2019Q3.

A.3 Steady State Fit

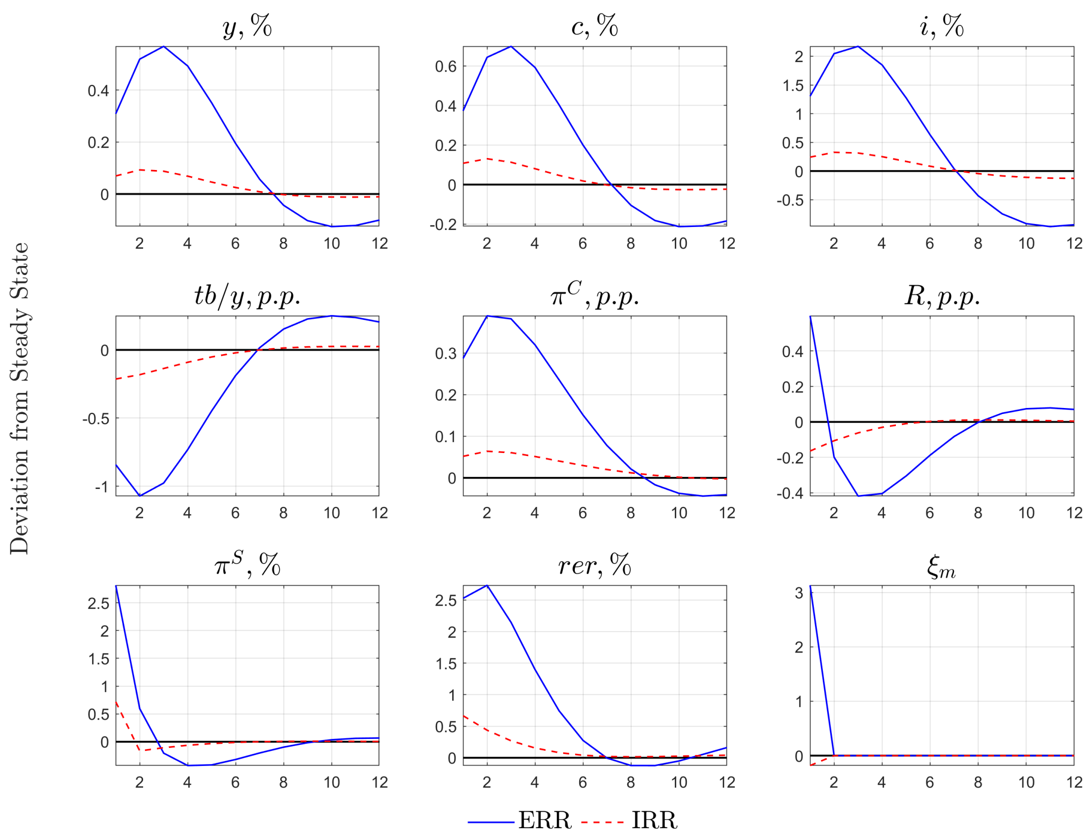
Table A.9: Steady State Fit

		(a) Singapore		**(b) Chile**	
Variable	Description	data	model	data	model
c/y	Consumption/GDP	72.0	72.0	74.7	74.7
i/y	Investment/GDP	28.0	28.0	25.3	25.3
x/y	Exports/GDP	140.5	140.5	35.3	35.3
m/y	Imports/GDP	140.5	140.5	35.3	35.3
tb/y	Trade balance/GDP	0.0	0.0	0.0	0.0
$rents/y$	Rents/GDP	0.0	0.0	0.0	0.0
ca/y	Current account/GDP	0.0	0.0	0.0	0.0
m^C/m	Consumption imports/imports	25.0	43.0	26.0	52.3
m^M/m	Intermediate imports/imports	50.0	40.2	53.0	30.0
m^I/m	Investment imports/imports	25.0	16.7	21.0	17.7
$100((R/\pi)^4 - 1)$	Real int. rate	3.8	4.1	1.4	2.0
$100((R)^4 - 1)$	Nominal int. rate	5.8	6.1	4.4	5.1
$100((\pi)^4 - 1)$	Domestic inflation	2.0	2.0	3.0	3.0
$100((R*)^4 - 1)$	Foreign int. rate	5.3	6.1	3.6	5.1
$100((R^W)^4 - 1)$	Risk-free int. rate	2.7	3.4	2.1	3.5
$100((SPR)^4 - 1)$	Spread	2.6	2.6	1.5	1.5
$100((\pi*)^4 - 1)$	Foreign inflation	2.0	2.0	2.5	3.0

Notes: The table presents the data and model steady states. For Singapore, the variables are calculated for the period 1991Q1-2019Q3, while for Chile we use the period 1996Q2-2019Q3.

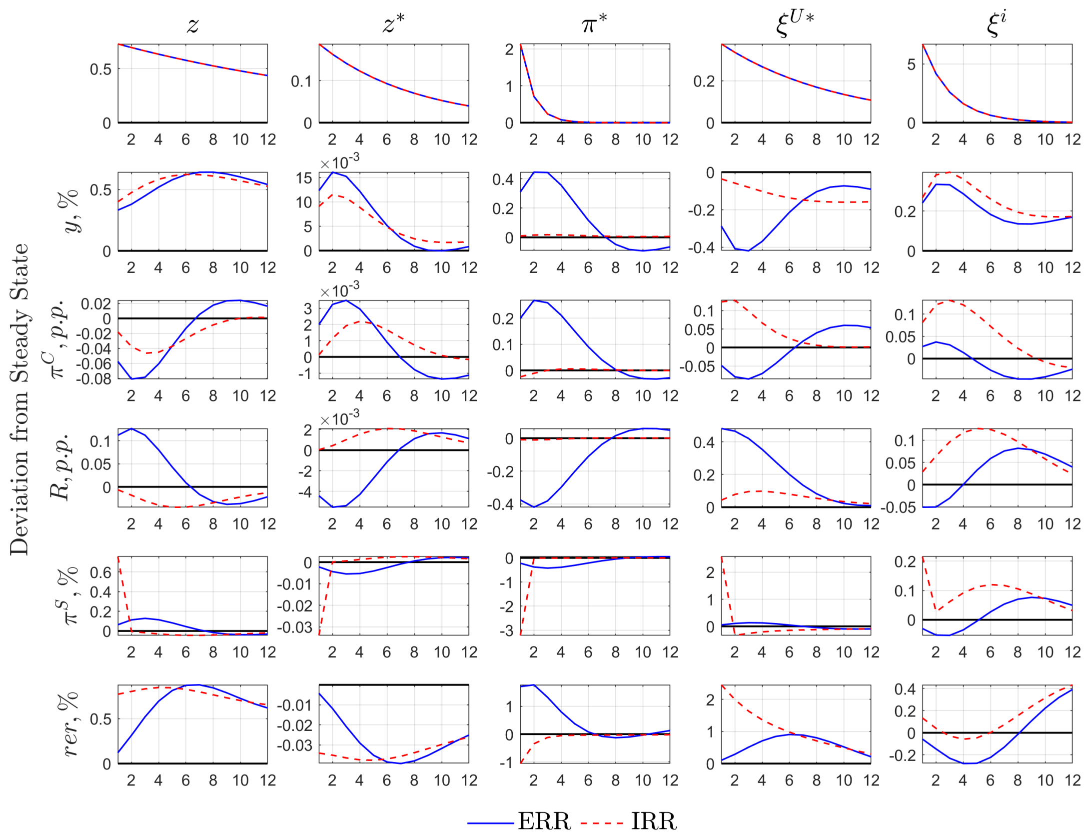
B Impulse Response Functions for Chile

Figure B.1: Monetary Policy Shock under the Interest Rate Rule



Notes: Responses to an expansionary monetary policy shock under the benchmark IRR versus the counterfactual ERR estimated for Chile. The shock size is one standard deviation estimated for each rule: a 3.13% nominal devaluation in the exchange rate under the ERR and a fall of 18 basis points in the nominal interest rate under the IRR. For simplicity, both monetary rules are assumed to target inflation only ($\alpha_y = 0$). All variables are expressed in percent deviations from the steady state, except for the trade balance-to-output ratio which is expressed as percentage points of GDP relative to the steady state, and inflation and interest rates which are expressed in percentage points. The model is approximated to the first order.

Figure B.2: Impulse responses to selected driving forces



Notes: Responses to alternative shocks under the benchmark IRR versus the counterfactual ERR estimated for Chile. All shocks are of one standard deviation. For simplicity, both monetary rules are assumed to target inflation only (α_y = 0). All variables are expressed in percent deviations from the steady state, except for inflation and interest rates, which are expressed in percentage points. The model is approximated to the first order.

Technical Appendix

A Equilibrium Conditions

1. **Endogenous Variables: 71**

   - Flows: $\{\widehat{C}_t, C_t, C_{Z,t}, C_{V,t}, C_{T,t}, C^H_{Z,t}, C^H_{V,t}, C^H_{T,t}, C^F_{Z,t}, C^F_{V,t}, C^F_{T,t}, C^{H*}_t, I_t, I^H_t, I^F_t\}$ = 15
   - Output: $\{Y_t, Y^{Co}_t, \tilde{Y}^H_t, Y^H_t, Y^F_t, M_t, M^F_t\}$ = 7
   - Stocks: $\{B^*_t, K_t, h_t\}$ = 3
   - Definitions: $\{TB_t, XN_t, MN_t, CA_t, REN_t, TBR_t, XR_t, MR_t, U_t, V_t, \Theta_t\}$ = 11
   - Prices: $\{W_t, P_t, P_{Z,t}, P_{V,t}, P_{T,t}, P^H_t, P^{H*}_t, P^F_t, P^I_t, P^Y_t, P^M_t, P^*_t, \tilde{P}^H_t, \tilde{P}^F_t, \tilde{P}^{H*}_t, P^{Co}_t\}$ = 16
   - Rates: $\{r_t, r^*_t, spr_t, r^K_t, \pi_t, S_t, rer_t, \Lambda_t, Q_t, MC^H_t, MC^F_t, A^H_t, Y^*_t\}$ = 13
   - Calvo: $\{F^H_t, F^F_t, F^{H*}_t, \Delta^H_t, \Delta^F_t, \Delta^{H*}_t\}$ = 6

2. **Exogenous Variables: 18**

   - *Domestic shocks*: 10
     - Supply: $\{a_t, z_t, z^{Co}_t, z_{Z,t}, z_{V,t}, z_{T,t}\}$ = 6
     - Demand: $\{\xi^\beta_t, \xi^h_t, \xi^i_t\}$ = 3
     - Policy: $\{\xi^m_t\}$ = 1
   - *Foreign shocks*: 8
     - Foreign demand: $\{z^*_t, \xi^{H*}_t\}$ = 2
     - Foreign prices: $\{\pi^*_t, p^{F*}_t, p^{Co*}_t\}$ = 3
     - Interest rates: $\{r^{W*}_t, \xi^{S*}_t, \xi^{U*}_t\}$ = 3

A.1 Households

$$U_t = \xi_t^{\beta} \left\{ \frac{\widehat{C}_t^{1-\sigma}}{1-\sigma} - \eta \xi_t^h A_{t-1}^{1-\sigma} \Theta_t \frac{h_t^{1+\psi}}{1+\psi} \right\} \tag{1}$$

$$\widehat{C}_t = C_t - \phi_c C_{t-1} \tag{2}$$

$$\Theta_t = A_{t-1}^{\sigma} \widehat{C}_t^{-\sigma} \tag{3}$$

$$V_t = U_t + \beta V_{t+1} \tag{4}$$

$$\Lambda_t P_t = \widehat{C}_t^{-\sigma} \tag{5}$$

$$\Lambda_t W_t = \eta \xi_t^h A_{t-1}^{1-\sigma} \left(h_t\right)^{\psi} \Theta_t \tag{6}$$

$$1 = \beta E_t \left\{ \frac{\xi_{t+1}^{\beta}}{\xi_t^{\beta}} \frac{\Lambda_{t+1}}{\Lambda_t} r_t \right\} \tag{7}$$

$$1 = \beta E_t \left\{ \frac{\xi_{t+1}^{\beta}}{\xi_t^{\beta}} \frac{\Lambda_{t+1}}{\Lambda_t} \frac{S_{t+1}}{S_t} r_t^* \right\} \tag{8}$$

A.2 Consumption baskets

$$C_{Z,t} = \gamma_Z \left(\frac{P_{Z,t}}{P_t} \right)^{-\varrho_C} C_t \tag{9}$$

$$C_{V,t} = \gamma_V \left(\frac{P_{V,t}}{P_t} \right)^{-\varrho_C} C_t \tag{10}$$

$$C_{T,t} = \gamma_T \left(\frac{P_{T,t}}{P_t} \right)^{-\varrho_C} C_t \tag{11}$$

$$P_t = \left[\gamma_Z \left(P_{Z,t}\right)^{1-\varrho_C} + \gamma_V \left(P_{V,t}\right)^{1-\varrho_C} + \gamma_T \left(P_{T,t}\right)^{1-\varrho_C} \right]^{\frac{1}{1-\varrho_C}} . \tag{12}$$

$$C_{Z,t}^H = z_{Z,t}^{\varrho_C - 1} (1-\gamma_C) \left(\frac{P_t^H}{P_{Z,t}} \right)^{-\varrho_C} C_{Z,t} \tag{13}$$

$$C_{Z,t}^F = z_{Z,t}^{\varrho_C - 1} \gamma_C \left(\frac{P_t^F}{P_{Z,t}} \right)^{-\varrho_C} C_{Z,t} \tag{14}$$

$$P_{Z,t} = \frac{1}{z_{Z,t}} \left[(1-\gamma_C) \left(P_t^H \right)^{1-\varrho_C} + \gamma_C \left(P_t^F \right)^{1-\varrho_C} \right]^{\frac{1}{1-\varrho_C}} \tag{15}$$

$$C_{V,t}^H = z_{V,t}^{\varrho_C - 1} (1-\gamma_C) \left(\frac{P_t^H}{P_{V,t}} \right)^{-\varrho_C} C_{V,t} \tag{16}$$

$$C_{V,t}^F = z_{V,t}^{\varrho_C - 1} \gamma_C \left(\frac{P_t^F}{P_{V,t}} \right)^{-\varrho_C} C_{V,t} \tag{17}$$

$$P_{V,t} = \frac{1}{z_{V,t}} \left[(1-\gamma_C) \left(P_t^H \right)^{1-\varrho_C} + \gamma_C \left(P_t^F \right)^{1-\varrho_C} \right]^{\frac{1}{1-\varrho_C}} \tag{18}$$

$$C_{T,t}^H = z_{T,t}^{\varrho_C - 1} (1-\gamma_C) \left(\frac{P_t^H}{P_{T,t}} \right)^{-\varrho_C} C_{T,t} \tag{19}$$

$$C_{T,t}^F = z_{T,t}^{\varrho_C - 1} \gamma_C \left(\frac{P_t^F}{P_{T,t}} \right)^{-\varrho_C} C_{T,t} \tag{20}$$

$$P_{T,t} = \frac{1}{z_{T,t}} \left[(1-\gamma_C) \left(P_t^H \right)^{1-\varrho_C} + \gamma_C \left(P_t^F \right)^{1-\varrho_C} \right]^{\frac{1}{1-\varrho_C}} \tag{21}$$

A.3 Investment Baskets

$$I_t^H = (1-\gamma_I) \left(\frac{P_t^H}{P_t^I} \right)^{-\varrho_I} I_t \tag{22}$$

$$I_t^F = \gamma_I \left(\frac{P_t^F}{P_t^I} \right)^{-\varrho_I} I_t \tag{23}$$

$$P_t^I = \left[(1-\gamma_I) \left(P_t^H \right)^{1-\varrho_I} + \gamma_I \left(P_t^F \right)^{1-\varrho_I} \right]^{\frac{1}{1-\varrho_I}} \tag{24}$$

A.4 Capital

$$K_t = (1-\delta) K_{t-1} + \left[1 - \Gamma \left(\frac{I_t}{I_{t-1}} \right) \right] I_t \xi_t^i \tag{25}$$

$$Q_t = \beta E_t \left\{ \frac{\xi_{t+1}^\beta}{\xi_t^\beta} \frac{\Lambda_{t+1}}{\Lambda_t} \left[P_{t+1}^H r_{t+1}^K + (1-\delta) Q_{t+1} \right] \right\} \tag{26}$$

$$
\begin{aligned}
P^I_t =& Q_t \left[\left(1 - \Gamma \left(\frac{I_t}{I_{t-1}} \right) \right) + \left(-\Gamma' \left(\frac{I_t}{I_{t-1}} \right) \right) \cdot \left(\frac{I_t}{I_{t-1}} \right) \right] \xi^i_t \\
&+ \beta E_t \left\{ \frac{\xi^\beta_{t+1}}{\xi^\beta_t} \frac{\Lambda_{t+1}}{\Lambda_t} Q_{t+1} \left(-\Gamma' \left(\frac{I_{t+1}}{I_t} \right) \right) \cdot (-1) \left(\frac{I_{t+1}}{I_t} \right)^2 \xi^i_{t+1} \right\}
\end{aligned} \tag{27}
$$

A.5 Production

$$
P^H_t r^K_t = MC^H_t \cdot \alpha_K \frac{Y^H_t}{K_{t-1}} \tag{28}
$$

$$
P^F_t = MC^H_t \cdot \alpha_M \frac{Y^H_t}{M^F_t} \tag{29}
$$

$$
W_t = MC^H_t \cdot (1 - \alpha_K - \alpha_M) \frac{Y^H_t}{h_t} \tag{30}
$$

$$
MC^H_t = \frac{1}{z_t} \left(\frac{P^H_t r^K_t}{\alpha_K} \right)^{\alpha_K} \left(\frac{P^F_t}{\alpha_M} \right)^{\alpha_M} \left(\frac{W_t / A^H_t}{1 - \alpha_K - \alpha_M} \right)^{1 - \alpha_K - \alpha_M} \tag{31}
$$

$$
Y^F_t = M_t \tag{32}
$$

$$
MC^F_t = P^M_t \tag{33}
$$

$$
Y^{Co}_t = A_t z^{Co}_t \tag{34}
$$

A.6 Price setting

$$
F^H_t = \left(\frac{\tilde{P}^H_t}{P^H_t} \right)^{-\epsilon} \tilde{Y}^H_t mc^H_t + (\theta^H) E_t \left\{ \Psi_{t,t+1} \left(\Gamma^{(1)}_t \frac{\tilde{P}^H_t}{\tilde{P}^H_{t+1}} \right)^{-\epsilon} \left(\frac{P^H_{t+1}}{P^H_t} \right) F^H_{t+1} \right\} \tag{35}
$$

$$
F^H_t = \left(\frac{\tilde{P}^H_t}{P^H_t} \right)^{1-\epsilon} \tilde{Y}^H_t \left(\frac{\epsilon - 1}{\epsilon} \right) + (\theta^H) E_t \left\{ \Psi_{t,t+1} \left(\Gamma^{(1)}_t \frac{\tilde{P}^H_t}{\tilde{P}^H_{t+1}} \right)^{1-\epsilon} \left(\frac{P^H_{t+1}}{P^H_t} \right) F^H_{t+1} \right\} \tag{36}
$$

$$
(P^H_t)^{1-\epsilon} = (1 - \theta^H)(\tilde{P}^H_t)^{1-\epsilon} + \theta^H \left[\left(\pi^H_{t-1} \right)^{\zeta^H} (\pi)^{1-\zeta^H} P^H_{t-1} \right]^{1-\epsilon} \tag{37}
$$

$$F_t^F = \left(\frac{\tilde{P}_t^F}{P_t^F}\right)^{-\epsilon} Y_t^F mc_t^F + (\theta^F) E_t \left\{ \Psi_{t,t+1} \left(\Gamma_t^{(1)} \frac{\tilde{P}_t^F}{\tilde{P}_{t+1}^F}\right)^{-\epsilon} \left(\frac{P_{t+1}^F}{P_t^F}\right) F_{t+1}^F \right\} \tag{38}$$

$$F_t^F = \left(\frac{\tilde{P}_t^F}{P_t^F}\right)^{1-\epsilon} Y_t^F \left(\frac{\epsilon-1}{\epsilon}\right) + (\theta^F) E_t \left\{ \Psi_{t,t+1} \left(\Gamma_t^{(1)} \frac{\tilde{P}_t^F}{\tilde{P}_{t+1}^F}\right)^{1-\epsilon} \left(\frac{P_{t+1}^F}{P_t^F}\right) F_{t+1}^F \right\} \tag{39}$$

$$(P_t^F)^{1-\epsilon} = (1-\theta^F)(\tilde{P}_t^F)^{1-\epsilon} + \theta^F \left[\left(\pi_{t-1}^F\right)^{\zeta^F} (\pi)^{1-\zeta^F} P_{t-1}^F \right]^{1-\epsilon} \tag{40}$$

$$F_t^{H*} = \left(\frac{\tilde{P}_t^{H*}}{P_t^{H*}}\right)^{-\epsilon} \left(\frac{P_t^{H*}}{P_t^*}\right)^{-1} \frac{P_t^H}{P_t} \frac{mc_t^H}{rer_t} C_t^{H*} + \theta^{H*} E_t \left\{ \Psi_{t,t+1} \left(\Gamma_t^{(1)} \frac{\tilde{P}_t^{H*}}{\tilde{P}_{t+1}^{H*}}\right)^{-\epsilon} \left(\frac{P_{t+1}^{H*}}{P_t^{H*}}\right) F_{t+1}^{H*} \right\} \tag{41}$$

$$F_t^{H*} = \left(\frac{\tilde{P}_t^{H*}}{P_t^{H*}}\right)^{1-\epsilon} C_t^{H*} \left(\frac{\epsilon-1}{\epsilon}\right) + \theta^{H*} E_t \left\{ \Psi_{t,t+1} \left(\Gamma_t^{(1)} \frac{\tilde{P}_t^{H*}}{\tilde{P}_{t+1}^{H*}}\right)^{1-\epsilon} \left(\frac{P_{t+1}^{H*}}{P_t^{H*}}\right) F_{t+1}^{H*} \right\} \tag{42}$$

$$(P_t^{H*})^{1-\epsilon} = (1-\theta^{H*})(\tilde{P}_t^{H*})^{1-\epsilon} + \theta^{H*} \left[\left(\pi_{t-1}^{H*}\right)^{\zeta^{H*}} (\pi^*)^{1-\zeta^{H*}} P_{t-1}^{H*} \right]^{1-\epsilon} \tag{43}$$

A.7 Rest of the world

$$C_t^{H*} = \left[a_{t-1} C_{t-1}^{H*}\right]^{\rho^{H*}} \left[\left(\frac{P_t^{H*}}{P_t^*}\right)^{-\epsilon^*} Y_t^* \right]^{1-\rho^{H*}} \xi_t^{H*} \tag{44}$$

$$Y_t^* = A_t z_t^* \tag{45}$$

$$A_t^H = (a A_{t-1}^H)^{1-\Gamma} (A_t)^\Gamma \tag{46}$$

$$P_t^M = S_t P_t^{F*} \tag{47}$$

$$P_t^{Co} = S_t P_t^{Co*} \tag{48}$$

$$rer_t = \frac{S_t P_t^*}{P_t} \tag{49}$$

$$r_t^* = r_t^{W*} \cdot spr_t \tag{50}$$

$$spr_t = \overline{spr} \cdot \exp\left[-\phi_b\left(\frac{S_t B_t^*}{P_t^Y Y_t} - \bar{b}\right) + \frac{\xi_t^{S*} - \xi^{S*}}{\xi^{S*}} + \frac{\xi_t^{U*} - \xi^{U*}}{\xi^{U*}}\right] \tag{51}$$

A.8 Monetary policy

$$\frac{r_t}{r} = \left(\frac{r_{t-1}}{r}\right)^{\rho}\left[\left(\frac{\pi_t^Z}{\pi}\right)^{\alpha_\pi}\left(\frac{Y_t}{aY_{t-1}}\right)^{\alpha_y}\right]^{(1-\rho)} \xi_t^m \tag{52}$$

$$\frac{\pi_t^S}{\pi^S} = \left(\frac{\pi_{t-1}^S}{\pi^S}\right)^{\rho}\left[\left(\frac{\pi_t^Z}{\pi}\right)^{-\alpha_\pi}\left(\frac{Y_t}{aY_{t-1}}\right)^{-\alpha_y}\right]^{(1-\rho)} \xi_t^m$$

A.9 Aggregation and Market clearing

$$\tilde{Y}_t^H \equiv C_{Z,t}^H + C_{V,t}^H + C_{T,t}^H + I_t^H \tag{53}$$

$$Y_t^H = \Delta_t^H \tilde{Y}_t^H + \Delta_t^{H*} C_t^{H*} \tag{54}$$

$$Y_t^F = \Delta_t^F \left(C_{Z,t}^F + C_{V,t}^F + C_{T,t}^F + I_t^F + M_t^F\right) \tag{55}$$

$$\Delta_t^H = (1-\theta^H)\left(\frac{\tilde{P}_t^H}{P_t^H}\right)^{-\epsilon} + \theta^H\left(\frac{P_{t-1}^H}{P_t^H}\right)^{-\epsilon}\left[\left(\pi_{t-1}^H\right)^{\zeta^H}(\pi)^{1-\zeta^H}\right]^{-\epsilon}\Delta_{t-1}^H \tag{56}$$

$$\Delta_t^F = (1-\theta^F)\left(\frac{\tilde{P}_t^F}{P_t^F}\right)^{-\epsilon} + \theta^F\left(\frac{P_{t-1}^F}{P_t^F}\right)^{-\epsilon}\left[\left(\pi_{t-1}^F\right)^{\zeta^F}(\pi)^{1-\zeta^F}\right]^{-\epsilon}\Delta_{t-1}^F \tag{57}$$

$$\Delta_t^{H*} = (1-\theta^{H*})\left(\frac{\tilde{P}_t^{H*}}{P_t^{H*}}\right)^{-\epsilon} + \theta^{H*}\left(\frac{P_{t-1}^{H*}}{P_t^{H*}}\right)^{-\epsilon}\left[\left(\pi_{t-1}^{H*}\right)^{\zeta^{H*}}(\pi^*)^{1-\zeta^{H*}}\right]^{-\epsilon}\Delta_{t-1}^{H*} \tag{58}$$

A.10 Balance of payments

$$S_t B_t^* = S_t r_{t-1}^* B_{t-1}^* + TB_t + REN_t \tag{59}$$
$$TB_t = XN_t - MN_t \tag{60}$$
$$REN_t = S_t \Xi_t^{R*} - (1-\chi) P_t^{Co} Y_t^{Co} \tag{61}$$
$$XN_t = P_t^H C_t^{H*} + P_t^{Co} Y_t^{Co} \tag{62}$$
$$MN_t = P_t^M M_t \tag{63}$$
$$CA_t = S_t \left(B_t^* - B_{t-1}^* \right) \tag{64}$$

A.11 Definitions

$$P_t^Y Y_t = P_t C_t + P_t^I I_t + TB_t \tag{65}$$
$$Y_t = C_t + I_t + TBR_t \tag{66}$$
$$TBR_t = XR_t - MR_t \tag{67}$$
$$XR_t = C_t^{H*} + Y_t^{Co} \tag{68}$$
$$MR_t = C_{Z,t}^F + C_{V,t}^F + C_{T,t}^F + I_t^F + M_t^F \tag{69}$$

$$\pi_t = \frac{P_t}{P_{t-1}} \tag{70}$$
$$\pi_t^* = \frac{P_t^*}{P_{t-1}^*} \tag{71}$$

B Stationary Equilibrium Conditions

1. **Endogenous Variables: 69**
   - Flows: $\{\widehat{c}_t, c_t, c_{Z,t}, c_{V,t}, c_{T,t}, c^H_{Z,t}, c^H_{V,t}, c^H_{T,t}, c^F_{Z,t}, c^F_{V,t}, c^F_{T,t}, c^{H*}_t, i_t, i^H_t, i^F_t\}$ = 15
   - Output: $\{y_t, y^{Co}_t, \tilde{y}^H_t, y^H_t, y^F_t, m_t, m^F_t\}$ = 7
   - Stocks: $\{b^*_t, k_t, h_t\}$ = 3
   - Definitions: $\{tb_t, xn_t, mn_t, ca_t, ren_t, tbr_t, xr_t, mr_t, u_t, v_t, \Theta_t\}$ = 11
   - Prices: $\{w_t, p_{Z,t}, p_{V,t}, p_{T,t}, p^H_t, p^F_t, p^{H*}_t, p^I_t, p^Y_t, p^M_t, \tilde{p}^H_t, \tilde{p}^F_t, \tilde{p}^{H*}_t, p^{Co}_t\}$ = 14
   - Rates: $\{r_t, r^*_t, spr_t, r^K_t, \pi_t, \pi^S_t, rer_t, \lambda_t, q_t, mc^H_t, mc^F_t, a^H_t, y^*_t\}$ = 13
   - Calvo: $\{f^H_t, f^F_t, f^{H*}_t, \Delta^H_t, \Delta^F_t, \Delta^{H*}_t\}$ = 6
2. **Exogenous Variables: 18**
   - Domestic shocks: $\{a_t, z_t, z^{Co}_t, z_{Z,t}, z_{V,t}, z_{T,t}, \xi^\beta_t, \xi^h_t, \xi^i_t, \xi^m_t\}$ = 10
   - Foreign shocks: $\{z^*_t, \xi^{H*}_t, \pi^*_t, p^{F*}_t, p^{Co*}_t, r^{W*}_t, \xi^{S*}_t, \xi^{U*}_t\}$ = 8

Detrended Real Quantities:
$x_t = \dfrac{X_t}{A_{t-1}}$ for $X_t = \{C_t, C^H_t, C^F_t, C^{H*}_t, I_t, I^H_t, I^F_t, Y_t, Y^H_t, Y^F_t, M_t, M^F_t, K_t, TBR_t, XR_t, MR_t, F^H_t, F^F_t, F^{H*}_t\}$

Detrended Nominal Quantities:
$x_t = \dfrac{X_t}{P_t A_{t-1}}$ for $X_t = \{TB_t, XN_t, MN_t, CA_t, REN_t\}$

$x_t = \dfrac{X_t}{P^*_t A_{t-1}}$ for $X_t = \{B^*_t\}$

Relative Prices:
$p^J_t = \dfrac{P^J_t}{P_t}$ for $P^J_t = \{P_t, P^H_t, P^F_t, P^I_t, P^Y_t, P^M_t, Q_t\}$

$p^{J*}_t = \dfrac{P^{J*}_t}{P^*_t}$ for $P^{J*}_t = \{P^*_t, P^{F*}_t, P^{H*}_t\}$

$\tilde{p}^J_t = \dfrac{\tilde{P}^J_t}{P^J_t}$ for $\tilde{P}^J_t = \{\tilde{P}^H_t, \tilde{P}^F_t, \tilde{P}^{H*}_t\}$

$mc^J_t = \dfrac{MC^J_t}{P^J_t}$ for $MC^J_t = \{MC^H_t, MC^F_t\}$

Special Variables:

$w_t = \dfrac{W_t}{P_t A_{t-1}}$

$\lambda_t = \dfrac{\Lambda_t P_t}{A^{-\sigma}_{t-1}}$

$u_t = \dfrac{U_t}{A^{1-\sigma}_{t-1}}$

$v_t = \dfrac{V_t}{A^{1-\sigma}_{t-1}}$

B.1 Households

$$u_t = \xi_t^{\beta} \left\{ \frac{\widehat{c}_t^{1-\sigma}}{1-\sigma} - \eta \xi_t^h \Theta_t \frac{h_t^{1+\psi}}{1+\psi} \right\} \tag{1}$$

$$\widehat{c}_t = c_t - \phi_c \frac{c_{t-1}}{a_{t-1}} \tag{2}$$

$$\Theta_t = \widehat{c}_t^{-\sigma} \tag{3}$$

$$v_t = u_t + \frac{\beta}{a_t^{\sigma-1}} v_{t+1} \tag{4}$$

$$\lambda_t = \widehat{c}_t^{-\sigma} \tag{5}$$

$$\lambda_t w_t = \eta \xi_t^h \left(h_t\right)^{\psi} \Theta_t \tag{6}$$

$$1 = \frac{\beta}{a_t^{\sigma}} E_t \left\{ \frac{\xi_{t+1}^{\beta}}{\xi_t^{\beta}} \frac{\lambda_{t+1}}{\lambda_t} \frac{r_t}{\pi_{t+1}} \right\} \tag{7}$$

$$1 = \frac{\beta}{a_t^{\sigma}} E_t \left\{ \frac{\xi_{t+1}^{\beta}}{\xi_t^{\beta}} \frac{\lambda_{t+1}}{\lambda_t} \frac{r_t^* \pi_{t+1}^S}{\pi_{t+1}} \right\} \tag{8}$$

B.2 Consumption baskets

$$c_{Z,t} = \gamma_Z \left(\frac{p_{Z,t}}{1} \right)^{-\varrho_C} c_t \tag{9}$$

$$c_{V,t} = \gamma_V \left(\frac{p_{V,t}}{1} \right)^{-\varrho_C} c_t \tag{10}$$

$$c_{T,t} = \gamma_T \left(\frac{p_{T,t}}{1} \right)^{-\varrho_C} c_t \tag{11}$$

$$p_t = \left[\gamma_Z \left(p_{Z,t}\right)^{1-\varrho_C} + \gamma_V \left(p_{V,t}\right)^{1-\varrho_C} + \gamma_T \left(p_{T,t}\right)^{1-\varrho_C} \right]^{\frac{1}{1-\varrho_C}} . \tag{12}$$

$$c_{Z,t}^H = z_{Z,t}^{\varrho_C - 1} (1-\gamma_C) \left(\frac{p_t^H}{p_{Z,t}} \right)^{-\varrho_C} c_{Z,t} \tag{13}$$

$$c_{Z,t}^F = z_{Z,t}^{\varrho_C - 1} \gamma_C \left(\frac{p_t^F}{p_{Z,t}} \right)^{-\varrho_C} c_{Z,t} \tag{14}$$

$$p_{Z,t} = \frac{1}{z_{Z,t}} \left[(1-\gamma_C) \left(p_t^H\right)^{1-\varrho_C} + \gamma_C \left(p_t^F\right)^{1-\varrho_C} \right]^{\frac{1}{1-\varrho_C}} \tag{15}$$

$$c_{V,t}^H = z_{V,t}^{\varrho_C - 1} (1-\gamma_C) \left(\frac{p_t^H}{p_{V,t}} \right)^{-\varrho_C} c_{V,t} \tag{16}$$

$$c_{V,t}^F = z_{V,t}^{\varrho_C - 1} \gamma_C \left(\frac{p_t^F}{p_{V,t}} \right)^{-\varrho_C} c_{V,t} \tag{17}$$

$$p_{V,t} = \frac{1}{z_{V,t}} \left[(1-\gamma_C) \left(p_t^H\right)^{1-\varrho_C} + \gamma_C \left(p_t^F\right)^{1-\varrho_C} \right]^{\frac{1}{1-\varrho_C}} \tag{18}$$

$$c_{T,t}^H = z_{T,t}^{\varrho_C - 1} (1-\gamma_C) \left(\frac{p_t^H}{p_{T,t}} \right)^{-\varrho_C} c_{T,t} \tag{19}$$

$$c_{T,t}^F = z_{T,t}^{\varrho_C - 1} \gamma_C \left(\frac{p_t^F}{p_{T,t}} \right)^{-\varrho_C} c_{T,t} \tag{20}$$

$$p_{T,t} = \frac{1}{z_{T,t}} \left[(1-\gamma_C) \left(p_t^H\right)^{1-\varrho_C} + \gamma_C \left(p_t^F\right)^{1-\varrho_C} \right]^{\frac{1}{1-\varrho_C}} \tag{21}$$

B.3 Investment Baskets

$$i_t^H = (1-\gamma_I) \left(\frac{p_t^H}{p_t^I} \right)^{-\varrho_I} i_t \tag{22}$$

$$i_t^F = \gamma_I \left(\frac{p_t^F}{p_t^I} \right)^{-\varrho_I} i_t \tag{23}$$

$$p_t^I = \left[(1-\gamma_I) \left(p_t^H\right)^{1-\varrho_I} + \gamma_I \left(p_t^F\right)^{1-\varrho_I} \right]^{\frac{1}{1-\varrho_I}} \tag{24}$$

B.4 Capital

$$k_t = (1-\delta) \frac{k_{t-1}}{a_{t-1}} + \left[1 - \frac{\phi_k}{2} \left(\frac{i_t}{i_{t-1}} a_{t-1} - a \right)^2 \right] i_t \xi_t^i \tag{25}$$

$$q_t = \frac{\beta}{a_t^\sigma} E_t \left\{ \frac{\lambda_{t+1}}{\lambda_t} \frac{\xi_{t+1}^\beta}{\xi_t^\beta} \left[p_{t+1}^H r_{t+1}^K + (1-\delta) q_{t+1} \right] \right\} \tag{26}$$

$$p_t^I = q_t \left[1 - \frac{\phi_k}{2} \left(\frac{i_t}{i_{t-1}} a_{t-1} - a \right)^2 - \phi_k \left(\frac{i_t}{i_{t-1}} a_{t-1} - a \right) \left(\frac{i_t}{i_{t-1}} a_{t-1} \right) \right] \xi_t^i$$
$$+ \frac{\beta}{a_t^\sigma} E_t \left\{ \frac{\lambda_{t+1}}{\lambda_t} \frac{\xi_{t+1}^\beta}{\xi_t^\beta} q_{t+1} \phi_k \left(\frac{i_{t+1}}{i_t} a_t - a \right) \left(\frac{i_{t+1}}{i_t} a_t \right)^2 \xi_{t+1}^i \right\} \tag{27}$$

B.5 Production

$$r_t^K = mc_t^H \cdot \alpha_K \frac{y_t^H}{k_{t-1}/a_{t-1}} \tag{28}$$
$$p_t^F = mc_t^H \cdot \alpha_M \frac{p_t^H y_t^H}{m_t^F} \tag{29}$$
$$w_t = mc_t^H \cdot (1 - \alpha_K - \alpha_M) \frac{p_t^H y_t^H}{h_t} \tag{30}$$
$$p_t^H mc_t^H = \frac{1}{z_t} \left(\frac{p_t^H r_t^K}{\alpha_K} \right)^{\alpha_K} \left(\frac{p_t^F}{\alpha_M} \right)^{\alpha_M} \left(\frac{w_t/(\nabla_{t-1}^H a_t^H)}{1 - \alpha_K - \alpha_M} \right)^{1-\alpha_K-\alpha_M} \tag{31}$$

$$y_t^F = m_t \tag{32}$$
$$mc_t^F = \frac{p_t^M}{p_t^F} \tag{33}$$

$$y_t^{Co} = a_t z_t^{Co} \tag{34}$$

B.6 Price setting

$$f_t^H = \left(\tilde{p}_t^H\right)^{-\epsilon} \tilde{y}_t^H mc_t^H + \frac{\beta \theta^H}{a_t^{\sigma-1}} E_t \left\{ \frac{\xi_{t+1}^\beta}{\xi_t^\beta} \frac{\lambda_{t+1}}{\lambda_t} \left(\frac{p_t^H}{p_{t+1}^H} \right)^{-1-\epsilon} \left(\frac{(\pi_t^H)^{\zeta^H} (\pi)^{1-\zeta^H}}{\pi_{t+1}} \cdot \frac{\tilde{p}_t^H}{\tilde{p}_{t+1}^H} \right)^{-\epsilon} f_{t+1}^H \right\} \tag{35}$$

$$f_t^H = \left(\tilde{p}_t^H\right)^{1-\epsilon} \tilde{y}_t^H \left(\frac{\epsilon - 1}{\epsilon} \right) + \frac{\beta \theta^H}{a_t^{\sigma-1}} E_t \left\{ \frac{\xi_{t+1}^\beta}{\xi_t^\beta} \frac{\lambda_{t+1}}{\lambda_t} \left(\frac{p_t^H}{p_{t+1}^H} \right)^{-\epsilon} \left(\frac{(\pi_t^H)^{\zeta^H} (\pi)^{1-\zeta^H}}{\pi_{t+1}} \cdot \frac{\tilde{p}_t^H}{\tilde{p}_{t+1}^H} \right)^{1-\epsilon} f_{t+1}^H \right\} \tag{36}$$

$$1 = (1 - \theta^H)(\tilde{p}_t^H)^{1-\epsilon} + \theta^H \left[\frac{\left(\pi_{t-1}^H\right)^{\zeta^H} (\pi)^{1-\zeta^H}}{\pi_t} \frac{p_{t-1}^H}{p_t^H} \right]^{1-\epsilon} \tag{37}$$

$$f_t^F = \left(\tilde{p}_t^F\right)^{-\epsilon} y_t^F mc_t^F + \frac{\beta\theta^F}{a_t^{\sigma-1}} E_t \left\{ \frac{\xi_{t+1}^\beta}{\xi_t^\beta} \frac{\lambda_{t+1}}{\lambda_t} \left(\frac{p_t^F}{p_{t+1}^F}\right)^{-1-\epsilon} \left(\frac{(\pi_t^F)^{\zeta^F} (\pi)^{1-\zeta^F}}{\pi_{t+1}} \cdot \frac{\tilde{p}_t^F}{\tilde{p}_{t+1}^F} \right)^{-\epsilon} f_{t+1}^F \right\} \tag{38}$$

$$f_t^F = \left(\tilde{p}_t^F\right)^{1-\epsilon} y_t^F \left(\frac{\epsilon-1}{\epsilon}\right) + \frac{\beta\theta^F}{a_t^{\sigma-1}} E_t \left\{ \frac{\xi_{t+1}^\beta}{\xi_t^\beta} \frac{\lambda_{t+1}}{\lambda_t} \left(\frac{p_t^F}{p_{t+1}^F}\right)^{-\epsilon} \left(\frac{(\pi_t^F)^{\zeta^F} (\pi)^{1-\zeta^F}}{\pi_{t+1}} \cdot \frac{\tilde{p}_t^F}{\tilde{p}_{t+1}^F} \right)^{1-\epsilon} f_{t+1}^F \right\} \tag{39}$$

$$1 = (1-\theta^F)(\tilde{p}_t^F)^{1-\epsilon} + \theta^F \left[\frac{\left(\pi_{t-1}^F\right)^{\zeta^F} (\pi)^{1-\zeta^F}}{\pi_t} \frac{p_{t-1}^F}{p_t^F} \right]^{1-\epsilon} \tag{40}$$

$$f_t^{H*} = \left(\tilde{p}_t^{H*}\right)^{-\epsilon} \frac{p_t^H}{p_t^{H*}} \frac{mc_t^H}{rer_t} c_t^{H*} + \frac{\beta\theta^{H*}}{a_t^{\sigma-1}} E_t \left\{ \frac{\xi_{t+1}^\beta}{\xi_t^\beta} \frac{\lambda_{t+1}}{\lambda_t} \frac{rer_{t+1}}{rer_t} \left(\frac{p_t^{H*}}{p_{t+1}^{H*}}\right)^{-1-\epsilon} \left(\frac{(\pi_t^{H*})^{\zeta^{H*}} (\pi^*)^{1-\zeta^{H*}}}{\pi_{t+1}^*} \cdot \frac{\tilde{p}_t^{H*}}{\tilde{p}_{t+1}^{H*}} \right)^{-\epsilon} f_{t+1}^{H*} \right\} \tag{41}$$

$$f_t^{H*} = \left(\tilde{p}_t^{H*}\right)^{1-\epsilon} \left(\frac{\epsilon-1}{\epsilon}\right) c_t^{H*} + \frac{\beta\theta^{H*}}{a_t^{\sigma-1}} E_t \left\{ \frac{\xi_{t+1}^\beta}{\xi_t^\beta} \frac{\lambda_{t+1}}{\lambda_t} \frac{rer_{t+1}}{rer_t} \left(\frac{p_t^{H*}}{p_{t+1}^{H*}}\right)^{-\epsilon} \left(\frac{(\pi_t^{H*})^{\zeta^{H*}} (\pi^*)^{1-\zeta^{H*}}}{\pi_{t+1}^*} \cdot \frac{\tilde{p}_t^{H*}}{\tilde{p}_{t+1}^{H*}} \right)^{1-\epsilon} f_{t+1}^{H*} \right\} \tag{42}$$

$$1 = (1-\theta^{H*})(\tilde{p}_t^{H*})^{1-\epsilon} + \theta^{H*} \left[\frac{\left(\pi_{t-1}^{H*}\right)^{\zeta^{H*}} (\pi^*)^{1-\zeta^{H*}}}{\pi_t^*} \frac{p_{t-1}^{H*}}{p_t^{H*}} \right]^{1-\epsilon} \tag{43}$$

B.7 Rest of the world

$$c_t^{H*} = \left[c_{t-1}^{H*}\right]^{\rho^{H*}} \left[\left(p_t^{H*}\right)^{-\epsilon^*} y_t^*\right]^{1-\rho^{H*}} \xi_t^{H*} \tag{44}$$

$$y_t^* = a_t z_t^* \tag{45}$$

$$a_t^H = \frac{\nabla_t^H}{\nabla_{t-1}^H} a_t \quad \text{with} \quad \nabla_t^H = \left(\frac{a}{a_t} \nabla_{t-1}^H\right)^{1-\Gamma} \tag{46}$$

$$p_t^M = rer_t p_t^{F*} \tag{47}$$
$$p_t^{Co} = rer_t p_t^{Co*} \tag{48}$$

$$\frac{rer_t}{rer_{t-1}} = \frac{\pi_t^S \pi_t^*}{\pi_t} \tag{49}$$

$$r_t^* = r_t^{W*} \cdot spr_t \tag{50}$$

$$spr_t = \overline{spr} \cdot \exp\left[-\phi_b \left(\frac{rer_t b_t^*}{p_t^Y y_t} - \overline{b}\right) + \frac{\xi_t^{S*} - \xi^{S*}}{\xi^{S*}} + \frac{\xi_t^{U*} - \xi^{U*}}{\xi^{U*}}\right] \tag{51}$$

B.8 Monetary policy

$$\frac{r_t}{r} = \left(\frac{r_{t-1}}{r}\right)^{\rho} \left[\left(\frac{\pi_t^Z}{\pi}\right)^{\alpha_\pi} \left(\frac{a_{t-1}}{a}\frac{y_t}{y_{t-1}}\right)^{\alpha_y}\right]^{(1-\rho)} \xi_t^m \tag{52}$$

$$\frac{\pi_t^S}{\pi^S} = \left(\frac{\pi_{t-1}^S}{\pi^S}\right)^{\rho} \left[\left(\frac{\pi_t^Z}{\pi}\right)^{-\alpha_\pi} \left(\frac{a_{t-1}}{a}\frac{y_t}{y_{t-1}}\right)^{-\alpha_y}\right]^{(1-\rho)} \xi_t^m$$

B.9 Aggregation and Market clearing

$$\tilde{y}_t^H = c_{Z,t}^H + c_{V,t}^H + c_{T,t}^H + i_t^H \tag{53}$$
$$y_t^H = \Delta_t^H \tilde{y}_t^H + \Delta_t^{H*} c_t^{H*} \tag{54}$$
$$y_t^F = \Delta_t^F \left(c_{Z,t}^F + c_{V,t}^F + c_{T,t}^F + i_t^F + m_t^F\right) \tag{55}$$

$$\Delta_t^H = \left(1-\theta^H\right)\left(\tilde{p}_t^H\right)^{-\epsilon} + \theta^H \left(\frac{p_{t-1}^H}{p_t^H}\frac{\left(\pi_{t-1}^H\right)^{\zeta^H}\left(\pi\right)^{1-\zeta^H}}{\pi_t}\right)^{-\epsilon} \Delta_{t-1}^H \tag{56}$$

$$\Delta_t^F = \left(1-\theta^F\right)\left(\tilde{p}_t^F\right)^{-\epsilon} + \theta^F \left(\frac{p_{t-1}^F}{p_t^F}\frac{\left(\pi_{t-1}^F\right)^{\zeta^F}\left(\pi\right)^{1-\zeta^F}}{\pi_t}\right)^{-\epsilon} \Delta_{t-1}^F \tag{57}$$

$$\Delta_t^{H*} = \left(1-\theta^{H*}\right)\left(\tilde{p}_t^{H*}\right)^{-\epsilon} + \theta^{H*} \left(\frac{p_{t-1}^{H*}}{p_t^{H*}}\frac{\left(\pi_{t-1}^{H*}\right)^{\zeta^{H*}}\left(\pi^*\right)^{1-\zeta^{H*}}}{\pi_t^*}\right)^{-\epsilon} \Delta_{t-1}^{H*} \tag{58}$$

B.10 Balance of payments

$$rer_t b_t^* = \frac{rer_t r_{t-1}^* b_{t-1}^*}{\pi_t^* a_{t-1}} + tb_t + ren_t \tag{59}$$

$$tb_t = xn_t - mn_t \tag{60}$$

$$ren_t = rer_t \xi_t^{R*} - (1-\chi) p_t^{Co} y_t^{Co} \tag{61}$$

$$xn_t = p_t^H c_t^{H*} + p_t^{Co} y_t^{Co} \tag{62}$$

$$mn_t = p_t^M m_t \tag{63}$$

$$ca_t = rer_t \left(b_t^* - \frac{b_{t-1}^*}{\pi_t^* a_{t-1}} \right) \tag{64}$$

B.11 Definitions

$$p_t^Y y_t = c_t + p_t^I i_t + tb_t \tag{65}$$

$$y_t = c_t + i_t + tbr_t \tag{66}$$

$$tbr_t = xr_t - mr_t \tag{67}$$

$$xr_t = c_t^{H*} + y_t^{Co} \tag{68}$$

$$mr_t = c_{Z,t}^F + c_{V,t}^F + c_{T,t}^F + i_t^F + m_t^F \tag{69}$$

C Steady State

1. **Endogenous Variables: 74**
   - Flows: $\{\widehat{c}, c, c_Z, c_V, c_T, c_Z^H, c_V^H, c_T^H, c_Z^F, c_V^F, c_T^F, c^{H*}, i, i^H, i^F\}$ = 15
   - Output: $\{y, y^{Co}, \tilde{y}^H, y^H, y^F, m, m^F\}$ = 7
   - Stocks: $\{b^*, k, h\}$ = 3
   - Definitions: $\{tb, xn, mn, ca, ren, tbr, xr, mr, u, v, \Theta, COY, CAY, TBY, MY, IY\}$ = 16
   - Prices: $\{w, p_Z, p_V, p_T, p^H, p^{H*}, p^F, p^I, p^Y, p^M, \tilde{p}^H, \tilde{p}^F, \tilde{p}^{H*}, p^{Co}\}$ = 14
   - Rates: $\{r, r^*, spr, r^K, \pi, \pi^S, rer, \lambda, q, mc^H, mc^F, a^H, y^*\}$ = 13
   - Calvo: $\{f^H, f^F, f^{H*}, \Delta^H, \Delta^F, \Delta^{H*}\}$ = 6
2. **Exogenous Variables: 18**
   - Domestic shocks: $\{a, z, z^{Co}, z_Z, z_V, z_T, \xi^\beta, \xi^h, \xi^i, \xi^m\}$ = 10
   - Foreign shocks: $\{z^*, \xi^{H*}, \pi^*, p^{F*}, p^{Co*}, r^{W*}, \xi^{S*}, \xi^{U*}\}$ = 8

C.1 Households

$$u = \xi^{\beta} \left\{ \frac{\widehat{c}^{1-\sigma}}{1-\sigma} - \eta \xi^{h} \Theta \frac{h^{1+\psi}}{1+\psi} \right\} \tag{1}$$

$$\widehat{c} = c - \phi_c \frac{c}{a} \tag{2}$$

$$\Theta = \widehat{c}^{-\sigma} \tag{3}$$

$$v = u + \frac{\beta}{a^{\sigma-1}} v \tag{4}$$

$$\lambda = \widehat{c}^{-\sigma} \tag{5}$$

$$\lambda w = \eta \xi^{h} \left(h\right)^{\psi} \Theta \tag{6}$$

$$r\beta = a^{\sigma} \pi \tag{7}$$

$$r = r^{*} \pi^{S} \tag{8}$$

C.2 Consumption baskets

$$c_Z = \gamma_Z \left(\frac{p_Z}{1}\right)^{-\varrho_C} c \tag{9}$$

$$c_V = \gamma_V \left(\frac{p_V}{1}\right)^{-\varrho_C} c \tag{10}$$

$$c_T = \gamma_T \left(\frac{p_T}{1}\right)^{-\varrho_C} c \tag{11}$$

$$1 = \left[\gamma_Z \left(p_Z\right)^{1-\varrho_C} + \gamma_V \left(p_V\right)^{1-\varrho_C} + \gamma_T \left(p_T\right)^{1-\varrho_C}\right]^{\frac{1}{1-\varrho_C}}. \tag{12}$$

$$c_Z^H = z_Z^{\varrho_C - 1} (1-\gamma_C) \left(\frac{p^H}{p_Z}\right)^{-\varrho_C} c_Z \tag{13}$$

$$c_Z^F = z_Z^{\varrho_C - 1} \gamma_C \left(\frac{p^F}{p_Z}\right)^{-\varrho_C} c_Z \tag{14}$$

$$p_Z = \frac{1}{z_Z} \left[(1-\gamma_C) \left(p^H\right)^{1-\varrho_C} + \gamma_C \left(p^F\right)^{1-\varrho_C}\right]^{\frac{1}{1-\varrho_C}} \tag{15}$$

$$c_V^H = z_V^{\varrho_C - 1}(1-\gamma_C)\left(\frac{p^H}{p_V}\right)^{-\varrho_C} c_V \tag{16}$$

$$c_V^F = z_V^{\varrho_C - 1}\gamma_C\left(\frac{p^F}{p_V}\right)^{-\varrho_C} c_V \tag{17}$$

$$p_V = \frac{1}{z_V}\left[(1-\gamma_C)\left(p^H\right)^{1-\varrho_C} + \gamma_C\left(p^F\right)^{1-\varrho_C}\right]^{\frac{1}{1-\varrho_C}} \tag{18}$$

$$c_T^H = z_T^{\varrho_C - 1}(1-\gamma_C)\left(\frac{p^H}{p_T}\right)^{-\varrho_C} c_T \tag{19}$$

$$c_T^F = z_T^{\varrho_C - 1}\gamma_C\left(\frac{p^F}{p_T}\right)^{-\varrho_C} c_T \tag{20}$$

$$p_T = \frac{1}{z_T}\left[(1-\gamma_C)\left(p^H\right)^{1-\varrho_C} + \gamma_C\left(p^F\right)^{1-\varrho_C}\right]^{\frac{1}{1-\varrho_C}} \tag{21}$$

C.3 Investment Baskets

$$i^H = (1-\gamma_I)\left(\frac{p^H}{p^I}\right)^{-\varrho_I} i \tag{22}$$

$$i^F = \gamma_I\left(\frac{p^F}{p^I}\right)^{-\varrho_I} i \tag{23}$$

$$p^I = \left[(1-\gamma_I)\left(p^H\right)^{1-\varrho_I} + \gamma_I\left(p^F\right)^{1-\varrho_I}\right]^{\frac{1}{1-\varrho_I}} \tag{24}$$

C.4 Capital

$$i = \frac{(a-1+\delta)}{a}k \tag{25}$$

$$r^K = \left[\frac{a^\sigma}{\beta} - (1-\delta)\right]\frac{p^I}{p^H} \tag{26}$$

$$p^I = q \tag{27}$$

C.5 Production

$$r^K = mc^H \cdot \alpha_K \frac{y^H}{k/a} \tag{28}$$

$$p^F = mc^H \cdot \alpha_M \frac{p^H y^H}{m^F} \tag{29}$$

$$w = mc^H \cdot (1 - \alpha_K - \alpha_M) \frac{p^H y^H}{h} \tag{30}$$

$$p^H mc^H = \frac{1}{z} \left(\frac{p^H r^K}{\alpha_K} \right)^{\alpha_K} \left(\frac{p^F}{\alpha_M} \right)^{\alpha_M} \left(\frac{w/(\nabla^H a^H)}{1 - \alpha_K - \alpha_M} \right)^{1-\alpha_K-\alpha_M} \tag{31}$$

$$y^F = m \tag{32}$$

$$mc^F = \frac{p^M}{p^F} \tag{33}$$

$$y^{Co} = az^{Co} \tag{34}$$

C.6 Price setting

$$mc^H = \left(\frac{\epsilon - 1}{\epsilon} \right) \tag{35}$$

$$f^H = \frac{\tilde{y}^H mc^H}{1 - \frac{\beta\theta^H}{a^{\sigma-1}}} \tag{36}$$

$$mc^F = \left(\frac{\epsilon - 1}{\epsilon} \right) \tag{37}$$

$$f^F = \frac{y^F mc^F}{1 - \frac{\beta\theta^F}{a^{\sigma-1}}} \tag{38}$$

$$p^{H*} = \frac{p^H}{rer} \tag{39}$$

$$f^{H*} = \frac{c^{H*} mc^H}{1 - \frac{\beta\theta^{H*}}{a^{\sigma-1}}} \tag{40}$$

$$\tilde{p}^H = 1 \tag{41}$$
$$\tilde{p}^F = 1 \tag{42}$$
$$\tilde{p}^{H*} = 1 \tag{43}$$

C.7 Rest of the world

$$c^{H*} = \left(p^{H*}\right)^{-\epsilon^*} y^* \tag{44}$$
$$y^* = az^* \tag{45}$$

$$a^H = a \quad \text{with} \quad \nabla^H = 1 \tag{46}$$

$$p^M = rer \cdot p^{F*} \tag{47}$$
$$p^{Co} = rer \cdot p^{Co*} \tag{48}$$

$$\pi^* = \frac{\pi}{\pi^S} \tag{49}$$

$$r^* = r^{W*} \cdot spr \tag{50}$$
$$spr = \overline{spr} \tag{51}$$
$$\pi = \pi \tag{52}$$

C.8 Aggregation and Market clearing

$$\tilde{y}^H = c_Z^H + c_V^H + c_T^H + i^H \tag{53}$$
$$y^H = \Delta^H \tilde{y}^H + \Delta^{H*} c^{H*} \tag{54}$$
$$y^F = \Delta^F \left(c_Z^F + c_V^F + c_T^F + i^F + m^F\right) \tag{55}$$

$$\Delta^H = 1 \tag{56}$$
$$\Delta^F = 1 \tag{57}$$
$$\Delta^{H*} = 1 \tag{58}$$

C.9 Balance of payments

$$rer \cdot b^* = \frac{tb + ren}{1 - \frac{r^*}{a\pi^*}} \tag{59}$$

$$tb = xn - mn \tag{60}$$

$$ren = rer\xi^{R*} - (1-\chi)p^{Co}y^{Co} \tag{61}$$

$$xn = p^H c^{H*} + p^{Co}y^{Co} \tag{62}$$

$$mn = p^M m \tag{63}$$

$$ca = rer \cdot b^* \left(1 - \frac{1}{a\pi^*}\right) \tag{64}$$

C.10 Definitions

$$p^Y y = c + p^I i + tb \tag{65}$$

$$y = c + i + tbr \tag{66}$$

$$tbr = xr - mr \tag{67}$$

$$xr = c^{H*} + y^{Co} \tag{68}$$

$$mr = c_Z^F + c_V^F + c_T^F + i^F + m^F \tag{69}$$

$$COY = \frac{p^{Co}y^{Co}}{p^Y y} \tag{70}$$

$$CAY = \frac{ca}{p^Y y} \tag{71}$$

$$TBY = \frac{tb}{p^Y y} \tag{72}$$

$$MY = \frac{p^M mr}{p^Y y} \tag{73}$$

$$IY = \frac{p^I i}{p^Y y} \tag{74}$$

D Calvo Price Setting $J = H$

Let $\tilde{Y}_t^J = C_t^J + I_t^J$. Any firm $i \in [0,1]$ that receives the Calvo signal (with prob. $(1-\theta^J)$) solves:

$$\begin{aligned}
\max_{\tilde{P}_t^J(i)} \quad & E_t \sum_{k=0}^{\infty} (\theta^J)^k \Psi_{t,t+k} \left(P_{t+k}^J(i) - MC_{t+k}^J \right) \tilde{Y}_{t+k}^J(i) \\
\text{s.t.} \quad & \tilde{Y}_t^J(i) = C_t^J(i) + I_t^J(i) = \left(\frac{P_t^J(i)}{P_t^J} \right)^{-\epsilon} \tilde{Y}_t^J \\
& P_{t+k}^J(i) = \Gamma_t^{(k)} \tilde{P}_t^J(i)
\end{aligned}$$

where the "passive" updater $\Gamma_t^{(k)}$ for those that cannot reoptimize (with prob. θ^J) is given by:

$$\Gamma_t^{(k)} \equiv \Gamma_t^{(k-1)} \cdot \left[(\pi_{t+k-1}^J)^{\zeta^J} (\pi)^{1-\zeta^J} \right], \quad \Gamma_t^{(0)} \equiv 1$$

the stochastic discount factor for any period $k \geq 0$ is:

$$\Psi_{t,t+k} = \beta^k \left(\frac{\xi_{t+k}^\beta}{\xi_t^\beta} \right) \left(\frac{\Lambda_{t+k}}{\Lambda_t} \right)$$

and recall

$$(P_t^J)^{1-\epsilon} = \int_0^1 \left(P_t^J(i) \right)^{1-\epsilon} di = (1-\theta^J)(\tilde{P}_t^J)^{1-\epsilon} + \theta^J \left[\left(\pi_{t-1}^J \right)^{\zeta^J} (\pi)^{1-\zeta^J} P_{t-1}^J \right]^{1-\epsilon}.$$

Plugging the constraints in the objective, dropping index i because all solve same problem, we obtain the first-order condition :

$$E_t \sum_{k=0}^{\infty} (\theta^J)^k \Psi_{t,t+k} \left[(1-\epsilon)(\Gamma_t^{(k)})^{1-\epsilon} (\tilde{P}_t^J)^{-\epsilon} - (-\epsilon) MC_{t+k}^J (\Gamma_t^{(k)})^{-\epsilon} (\tilde{P}_t^J)^{-\epsilon-1} \right] \left(P_{t+k}^J \right)^{\epsilon} \tilde{Y}_{t+k}^J = 0$$

Multiplying by $\frac{\tilde{P}_t^J}{P_t^J}$, dividing by $(-\epsilon)$, and using $mc_t^J = \frac{MC_t^J}{P_t^J}$, the FOC can be expressed as three equations in three unknowns $(F_{1t}^J, F_{2t}^J, \tilde{P}_t^J)$:

$$F_{1t}^J = F_{2t}^J$$

$$\begin{aligned}
F_{1t}^J &= E_t \sum_{k=0}^{\infty} (\theta^J)^k \Psi_{t,t+k} (\Gamma_t^{(k)})^{-\epsilon} (\tilde{P}_t^J)^{-\epsilon} (P_t^J)^{-1} (P_{t+k}^J)^{\epsilon+1} \tilde{Y}_{t+k}^J mc_{t+k}^J \\
F_{2t}^J &= E_t \sum_{k=0}^{\infty} (\theta^J)^k \Psi_{t,t+k} (\Gamma_t^{(k)})^{1-\epsilon} (\tilde{P}_t^J)^{1-\epsilon} (P_t^J)^{-1} (P_{t+k}^J)^{\epsilon} \tilde{Y}_{t+k}^J \left(\frac{\epsilon-1}{\epsilon} \right)
\end{aligned}$$

Rewriting the infinite sums recursively and letting $F_{1t}^J = F_{2t}^J = F_t^J$, the FOC can also be expressed as two equations in $(F_t^J, \tilde{P}_t^J)$:

$$\begin{aligned}
F_t^J &= \left(\frac{\tilde{P}_t^J}{P_t^J} \right)^{-\epsilon} \tilde{Y}_t^J mc_t^J + (\theta^J) E_t \left\{ \Psi_{t,t+1} \left(\Gamma_t^{(1)} \frac{\tilde{P}_t^J}{\tilde{P}_{t+1}^J} \right)^{-\epsilon} \left(\frac{P_{t+1}^J}{P_t^J} \right) F_{t+1}^J \right\} \\
F_t^J &= \left(\frac{\tilde{P}_t^J}{P_t^J} \right)^{1-\epsilon} \tilde{Y}_t^J \left(\frac{\epsilon-1}{\epsilon} \right) + (\theta^J) E_t \left\{ \Psi_{t,t+1} \left(\Gamma_t^{(1)} \frac{\tilde{P}_t^J}{\tilde{P}_{t+1}^J} \right)^{1-\epsilon} \left(\frac{P_{t+1}^J}{P_t^J} \right) F_{t+1}^J \right\}.
\end{aligned}$$

E Calvo Price Setting $J = F$

Any firm $i \in [0,1]$ that receives the Calvo signal (with prob. $(1-\theta^J)$) solves:

$$\max_{\tilde{P}_t^J(i)} \quad E_t \sum_{k=0}^{\infty} (\theta^J)^k \Psi_{t,t+k} \left(P_{t+k}^J(i) - MC_{t+k}^J \right) Y_{t+k}^J(i)$$

$$\text{s.t.} \quad Y_t^J(i) = C_t^J(i) + I_t^J(i) + M_t^J(i) = \left(\frac{P_t^J(i)}{P_t^J} \right)^{-\epsilon} Y_t^J$$

$$P_{t+k}^J(i) = \Gamma_t^{(k)} \tilde{P}_t^J(i)$$

where the "passive" updater $\Gamma_t^{(k)}$ for those that cannot reoptimize (with prob. θ^J) is given by:

$$\Gamma_t^{(k)} \equiv \Gamma_t^{(k-1)} \cdot \left[(\pi_{t+k-1}^J)^{\zeta^J} (\pi)^{1-\zeta^J} \right], \quad \Gamma_t^{(0)} \equiv 1$$

the stochastic discount factor for any period $k \geq 0$ is:

$$\Psi_{t,t+k} = \beta^k \left(\frac{\xi_{t+k}^\beta}{\xi_t^\beta} \right) \left(\frac{\Lambda_{t+k}}{\Lambda_t} \right)$$

and recall

$$(P_t^J)^{1-\epsilon} = \int_0^1 \left(P_t^J(i) \right)^{1-\epsilon} di = (1-\theta^J)(\tilde{P}_t^J)^{1-\epsilon} + \theta^J \left[\left(\pi_{t-1}^J \right)^{\zeta^J} (\pi)^{1-\zeta^J} P_{t-1}^J \right]^{1-\epsilon}.$$

Plugging the constraints in the objective, dropping index i because all solve same problem, the first-order condition is:

$$E_t \sum_{k=0}^{\infty} (\theta^J)^k \Psi_{t,t+k} \left[(1-\epsilon)(\Gamma_t^{(k)})^{1-\epsilon} (\tilde{P}_t^J)^{-\epsilon} - (-\epsilon) MC_{t+k}^J (\Gamma_t^{(k)})^{-\epsilon} (\tilde{P}_t^J)^{-\epsilon-1} \right] \left(P_{t+k}^J \right)^{\epsilon} Y_{t+k}^J = 0$$

Multiplying by $\frac{\tilde{P}_t^J}{P_t^J}$, dividing by $(-\epsilon)$, and using $mc_t^J = \frac{MC_t^J}{P_t^J}$, the FOC can be expressed as three equations in three unknowns $(F_{1t}^J, F_{2t}^J, \tilde{P}_t^J)$:

$$F_{1t}^J = F_{2t}^J$$

$$F_{1t}^J = E_t \sum_{k=0}^{\infty} (\theta^J)^k \Psi_{t,t+k} (\Gamma_t^{(k)})^{-\epsilon} (\tilde{P}_t^J)^{-\epsilon} (P_t^J)^{-1} (P_{t+k}^J)^{\epsilon+1} Y_{t+k}^J mc_{t+k}^J$$

$$F_{2t}^J = E_t \sum_{k=0}^{\infty} (\theta^J)^k \Psi_{t,t+k} (\Gamma_t^{(k)})^{1-\epsilon} (\tilde{P}_t^J)^{1-\epsilon} (P_t^J)^{-1} (P_{t+k}^J)^{\epsilon} Y_{t+k}^J \left(\frac{\epsilon - 1}{\epsilon} \right)$$

Rewriting the infinite sums recursively and letting $F_{1t}^J = F_{2t}^J = F_t^J$, the FOC can also be expressed as:

$$F_t^J = \left(\frac{\tilde{P}_t^J}{P_t^J} \right)^{-\epsilon} Y_t^J mc_t^J + (\theta^J) E_t \left\{ \Psi_{t,t+1} \left(\Gamma_t^{(1)} \frac{\tilde{P}_t^J}{\tilde{P}_{t+1}^J} \right)^{-\epsilon} \left(\frac{P_{t+1}^J}{P_t^J} \right) F_{t+1}^J \right\}$$

$$F_t^J = \left(\frac{\tilde{P}_t^J}{P_t^J} \right)^{1-\epsilon} Y_t^J \left(\frac{\epsilon - 1}{\epsilon} \right) + (\theta^J) E_t \left\{ \Psi_{t,t+1} \left(\Gamma_t^{(1)} \frac{\tilde{P}_t^J}{\tilde{P}_{t+1}^J} \right)^{1-\epsilon} \left(\frac{P_{t+1}^J}{P_t^J} \right) F_{t+1}^J \right\}.$$

F Calvo Price Setting $J = H*$

Any firm $i \in [0,1]$ that receives the Calvo signal (with probability $(1-\theta^{H*})$) solves:

$$\max_{\tilde{P}_t^{H*}(i)} E_t \sum_{k=0}^{\infty} (\theta^{H*})^k \Psi_{t,t+k} \left(P_{t+k}^{H*}(i) - \frac{MC_{t+k}^H}{S_{t+k}} \right) C_{t+k}^{H*}(i)$$

$$\text{s.t. } C_t^{H*}(i) = \left(\frac{P_t^{H*}(i)}{P_t^{H*}} \right)^{-\epsilon} C_t^{H*}$$

$$P_{t+k}^{H*}(i) = \Gamma_t^{(k)} \tilde{P}_t^{H*}(i)$$

where the "passive" updater $\Gamma_t^{(k)}$ for those that cannot reoptimize (with prob. θ^{H*}) is given by:

$$\Gamma_t^{(k)} \equiv \Gamma_t^{(k-1)} \cdot \left[(\pi_{t+k-1}^{H*})^{\zeta^J} (\pi^*)^{1-\zeta^J} \right], \quad \Gamma_t^{(0)} \equiv 1$$

and the stochastic discount factor for any period $k \geq 0$ is:

$$\Psi_{t,t+k} = \beta^k \left(\frac{\xi_{t+k}^\beta}{\xi_t^\beta} \right) \left(\frac{\Lambda_{t+k}}{\Lambda_t} \right) \left(\frac{S_{t+k}}{S_t} \right)$$

and recall

$$(P_t^{H*})^{1-\epsilon} = \int_0^1 \left(P_t^{H*}(i) \right)^{1-\epsilon} di = (1-\theta^{H*})(\tilde{P}_t^{H*})^{1-\epsilon} + \theta^{H*} \left[\left(\pi_{t-1}^{H*} \right)^{\zeta^{H*}} (\pi^*)^{1-\zeta^{H*}} P_{t-1}^{H*} \right]^{1-\epsilon}.$$

Plugging the constraints in the objective and dropping index i, the first-order condition is:

$$E_t \sum_{k=0}^{\infty} (\theta^{H*})^k \Psi_{t,t+k} \left[(1-\epsilon)(\Gamma_t^{(k)})^{1-\epsilon} (\tilde{P}_t^{H*})^{-\epsilon} - (-\epsilon) \frac{MC_{t+k}^H}{S_{t+k}} (\Gamma_t^{(k)})^{-\epsilon} (\tilde{P}_t^{H*})^{-\epsilon-1} \right] \left(P_{t+k}^{H*} \right)^{\epsilon} C_{t+k}^{H*} = 0$$

Multiplying by $\frac{\tilde{P}_t^{H*}}{P_t^{H*}}$, dividing by $(-\epsilon)$,using $mc_t^H = \frac{MC_t^H}{P_t^H}$ and $rer_t = \frac{S_t P_t^*}{P_t}$. Rearranging, the FOC can be expressed as three equations in three unknowns: $(F_{1t}^{H*}, F_{2t}^{H*}, \tilde{P}_t^{H*})$:

$$F_{1t}^{H*} = F_{2t}^{H*}$$

$$F_{1t}^{H*} = E_t \sum_{k=0}^{\infty} (\theta^{H*})^k \Psi_{t,t+k} (\Gamma_t^{(k)})^{-\epsilon} (\tilde{P}_t^{H*})^{-\epsilon} (P_t^{H*})^{-1} (P_{t+k}^{H*})^{\epsilon} P_{t+k}^H C_{t+k}^{H*} \frac{mc_{t+k}^H}{rer_{t+k}} \frac{P_{t+k}^*}{P_{t+k}}$$

$$F_{2t}^{H*} = E_t \sum_{k=0}^{\infty} (\theta^{H*})^k \Psi_{t,t+k} (\Gamma_t^{(k)})^{1-\epsilon} (\tilde{P}_t^{H*})^{1-\epsilon} (P_t^{H*})^{-1} (P_{t+k}^{H*})^{\epsilon} C_{t+k}^{H*} \left(\frac{\epsilon-1}{\epsilon} \right)$$

Rewriting the infinite sums recursively and letting $F_{1t}^{H*} = F_{2t}^{H*} = F_t^{H*}$, the FOC can also be expressed as two equations in $(F_t^{H*}, \tilde{P}_t^{H*})$:

$$F_t^{H*} = \left(\frac{\tilde{P}_t^{H*}}{P_t^{H*}} \right)^{-\epsilon} \left(\frac{P_t^{H*}}{P_t^*} \right)^{-1} \frac{P_t^H}{P_t} \frac{mc_t^H}{rer_t} C_t^{H*} + \theta^{H*} E_t \left\{ \Psi_{t,t+1} \left(\Gamma_t^{(1)} \frac{\tilde{P}_t^{H*}}{\tilde{P}_{t+1}^{H*}} \right)^{-\epsilon} \left(\frac{P_{t+1}^{H*}}{P_t^{H*}} \right) F_{t+1}^{H*} \right\}$$

$$F_t^{H*} = \left(\frac{\tilde{P}_t^{H*}}{P_t^{H*}} \right)^{1-\epsilon} C_t^{H*} \left(\frac{\epsilon-1}{\epsilon} \right) + \theta^{H*} E_t \left\{ \Psi_{t,t+1} \left(\Gamma_t^{(1)} \frac{\tilde{P}_t^{H*}}{\tilde{P}_{t+1}^{H*}} \right)^{1-\epsilon} \left(\frac{P_{t+1}^{H*}}{P_t^{H*}} \right) F_{t+1}^{H*} \right\}.$$

G Derivation of the Balance of Payments

Recall the household budget constraint:

$$P_tC_t + P_t^I I_t + B_t + S_tB_t^* = W_th_t + P_t^H r_t^K K_{t-1} + r_{t-1}B_{t-1} + S_t r_{t-1}^* B_{t-1}^* + \hat{\Sigma}_t$$

with

$$\hat{\Sigma}_t = P_t^H Y_t^H - W_th_t - P_t^H r_t^K K_{t-1} - P_t^M M_t^F + (P_t^F - P_t^M)(M_t - M_t^F) + \chi P_t^{Co} Y_t^{Co} + S_t \Xi^{R*}$$

The baskets for consumption and investment can be written as:

$$\begin{aligned} P_tC_t &= P_t^H(C_{Z,t}^H + C_{V,t}^H + C_{T,t}^H) + P_t^F(C_{Z,t}^F + C_{V,t}^F + C_{T,t}^F) \\ P_t^I I_t &= P_t^H I_t^H + P_t^F I_t^F \end{aligned}$$

Market clearing conditions imply:

$$\begin{aligned} Y_t^H &= \Delta_t^H\left(C_{Z,t}^H + C_{V,t}^H + C_{T,t}^H + I_t^H\right) + \Delta_t^{H*}\left(C_t^{H*}\right) \\ Y_t^F &= \Delta_t^F\left(C_{Z,t}^F + C_{V,t}^F + C_{T,t}^F + I_t^F + M_t^F\right) = M_t \end{aligned}$$

After some algebra, and assuming $\Delta_t^H = \Delta_t^F = \Delta_t^{H*} = 1$, we have:

$$S_tB_t^* = S_t r_{t-1}^* B_{t-1}^* + P_t^H C_t^{H*} + P_t^{Co} Y_t^{Co} - P_t^M M_t + S_t \Xi^{R*} - (1-\chi) P_t^{Co} Y_t^{Co}$$

Defining the trade balance and the (non-interest) rents account:

$$\begin{aligned} S_tB_t^* &= S_t r_{t-1}^* B_{t-1}^* + TB_t + REN_t \\ TB_t &= XN_t - MN_t \\ REN_t &= S_t \Xi^{R*} - (1-\chi) P_t^{Co} Y_t^{Co} \\ XN_t &= P_t^H C_t^{H*} + P_t^{Co} Y_t^{Co} \\ MN_t &= P_t^M M_t \end{aligned}$$

The current account (in domestic currency) is obtained as the change in the net foreign asset position:

$$CA_t = S_t\left(B_t^* - B_{t-1}^*\right) = TB_t + RENT_t$$

where we have defined the total rents account, $RENT_t$:

$$RENT_t \equiv REN_t + S_t(r_{t-1}^* - 1)B_{t-1}^*.$$