

# The Firm Dynamics of Business Cycles

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# The Firm Dynamics of Business Cycles\*

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## Abstract

We use firm dynamics statistics on employment by age, entry, exit, and job flows to identify sources of business cycle fluctuations in the U.S. economy since 1980. We extend the [Hopenhayn \(1992\)](#) firm dynamics model by incorporating capital and debt accumulation to the firm's problem and savings to the consumer's problem. Analyzing the implications of unexpected productivity, credit, labor wedge, and investment wedge shocks for firm dynamics statistics, we show that (a) productivity shock accounts for the 1990-91 and 2001 recessions, and (b) productivity and credit shocks jointly account for the 1980-82 and 2007-09 recessions.

*Keywords:* firm dynamics, business cycles.

*JEL classification:* D21, D22, E24, E32.

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# 1 Introduction

In this paper, we ask the following question: how can one use firm dynamics statistics to identify business cycle shocks? To answer this question, we use a model of heterogeneous firms to analyze how aggregate shocks account for patterns of firm employment by age, entry and exit, and job creation and job destruction over the business cycle. Our study complements the empirical literature on firm dynamics led by [Davis, Haltiwanger, and Schuh \(1998\)](#), which documents patterns of firm dynamics statistics over the business cycle, but abstracts from analyzing the data through the lenses of a model with heterogeneous firms. Our study also complements the business cycle accounting literature pioneered by [Chari, Kehoe, and McGrattan \(2007\)](#), which estimates productivity, labor, and investment wedges using a representative agent model, but abstracts from firm heterogeneity.

We use administrative firm dynamics data from the Business Dynamics Statistics of the U.S. Census Bureau, which is publicly available and contains annual information on private businesses in the United States.<sup>1</sup> We analyze four U.S. recessions (1980-82, 1990-91, 2001, and 2007-09) and document the following patterns. During the 1990-91 and 2001 recessions, the (cross-sectional) average employment size decreased across all age groups of firms (intensive margin), firm entry and firm exit did not vary much (extensive margin), job creation decreased, and job destruction increased. The 1980-82 and 2007-09 recessions differ from the 1990-91 and 2001 recessions with respect to their implications for the extensive margin: firm entry decreased and firm exit increased. In our quantitative exercise, we consider a set of aggregate shocks that are often used in the business cycle literature to identify which ones can replicate these patterns.

Our general equilibrium model economy builds on [Hopenhayn \(1992\)](#)'s model of firm entry and exit. We enrich its environment by adding savings and labor choices to the representative consumer's problem, and capital and debt decisions, together with borrowing and non-negative dividend payments constraints, to the firm's problem. The savings and labor choices are important to generate procyclical fluctuations in employment and investment, as in the standard real business cycle literature. The inclusion of debt and capital accumulation, together with borrowing and non-negative dividend payments constraints, generates lifecycle dynamics of firms, in which they grow by accumulating capital and debt. Furthermore, this environment allows us to study the implications of credit-type shocks.

We calibrate the initial stationary equilibrium of our model to match key moments in the U.S. economy such as non-finance business debt to GDP and the risk-free rate. Given that we emphasize the firm dynamics statistics discussed above, we validate the

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<sup>1</sup>The main benefit of using this data set is coverage. It includes almost every private employer firm in the U.S. economy, and accounts for 72-82 percent of total employment from 1977 to 2015.

model by comparing it to data with non-targeted firm life cycle moments such as firm survival probability and firm employment. For the main exercise, we compute separate transitions with shocks to productivity, the borrowing constraint (credit shock), the labor wedge, and the investment wedge—the last two refer to shocks (wedges) from the business cycle accounting literature. The shock is unexpected in period one and is chosen such that it leads to a 5 percent drop in output—the average drop for the U.S. economy in the last four recessions. After that, the shock is assumed to persist at a rate such that it takes 5 years for the economy to complete more than half of the recovery since the beginning of the recession—the average for the U.S. economy in the last four recessions. All of these shocks lead to a drop in output, investment, and employment. However, we show that they have different implications for firm employment by age, entry and exit, and job creation and job destruction. Subsequently, we use those implications to identify the shocks that were most important during the last four U.S. recessions.

As mentioned above, during the 1990-91 and 2001 recessions, the (cross-sectional) average employment size decreased across all age groups of firms (intensive margin), firm entry and firm exit did not vary much (extensive margin), job creation decreased, and job destruction increased. These patterns are consistent with the implications from a productivity shock. It decreases the marginal products of both capital and labor for every firm in the economy leading to a decrease in the (cross-sectional) average employment size across all age groups of firms. This happens even though both interest and wage rates decrease in equilibrium. The extensive margin of firm entry and exit do not vary much because of two offsetting effects. While lower aggregate productivity by itself decreases the value of operation, the resulting decrease in wages reduces both the entry and operating costs, which are assumed to be in units of labor. This assumption plays an important role in our analysis, and is also a key difference from previous studies on firm entry and exit over the business cycle, as we discuss below. Furthermore, this assumption is supported by the fact that it implies a stationary average employment size of firms in a balance growth path, which is consistent with the data for the U.S.<sup>2</sup> Finally, job creation decreases because growing firms hire fewer workers, and job destruction increases because shrinking firms fire more workers. The shocks to credit and to labor and investment wedges are not individually or jointly consistent with the firm dynamics patterns observed during the 1990-91 and 2001 recessions, because all of them decrease firm entry.

The 1980-82 and 2007-09 recessions differ from the 1990-91 and 2001 recessions with respect to their implications for the extensive margin: firm entry decreased and firm exit increased. As discussed in the previous paragraph, the productivity shock accounts for the 1990-91 and 2001 recessions. Furthermore, the remaining three shocks (credit, labor

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<sup>2</sup>Klenow, Li, and Bollard (2013) provide direct empirical evidence for the assumption of entry costs in units of labor.

wedge, and investment wedge) decrease firm entry. Hence, we focus on firm exit. We find that only the credit shock increases firm exit by making it harder for an incumbent firm to satisfy the non-negative dividend constraint. The labor wedge shock leads to a large drop in firm entry, but does not affect firm exit; the investment wedge shock decreases firm exit. We also show that the conclusions from a tighter borrowing constraint are robust to alternative ways of modeling a credit shock, such as increasing the cost of borrowing (captured by an exogenous spread on the risk-free rate) or increasing the fixed cost of operating.<sup>3</sup> Therefore, we conclude that a productivity shock and a credit shock were most important during the 1980-82 and 2007-09 recessions.

Our paper contributes to three strands of the literature: the empirical literature on firm dynamics, the business cycle accounting literature, and the literature on models of firm dynamics and aggregate fluctuations. The empirical literature includes, but is not limited to [Davis, Haltiwanger, and Schuh \(1998\)](#), [Davis, Faberman, and Haltiwanger \(2012\)](#), [Fort, Haltiwanger, Jarmin, and Miranda \(2013\)](#), [Decker, Haltiwanger, Jarmin, and Miranda \(2014a,b\)](#), and [Siemer \(2018\)](#). In particular, [Fort, Haltiwanger, Jarmin, and Miranda \(2013\)](#) and [Decker, Haltiwanger, Jarmin, and Miranda \(2014b\)](#) emphasize the importance of firm age in addition to firm size in the data. The analysis of our model also emphasizes the importance of firm age in addition to firm size to understand the heterogeneous responses of firms to business cycle shocks. This is because in the presence of financial frictions and firm entry and exit, age is one of the determinants of a firm's level of productivity, capital, and debt. In our model, these variables determine a firm's response to an aggregate shock. [Siemer \(2018\)](#), using confidential data on the universe of firms in the U.S., argues that financial constraints affected firm employment growth primarily through firm entry and exit during the Great Recession. This empirical finding complements the implications from our model economy, which shows that the credit shock decreases firm entry and increases firm exit.

The business cycle accounting literature estimates productivity, investment, labor, and government wedges to exactly match aggregates such as output, investment, consumption, and labor ([Chari, Kehoe, and McGrattan, 2007](#)). Upon estimation, the researcher could use these wedges to identify potential shocks. For example, [Chari, Kehoe, and McGrattan \(2007\)](#) and [Brinca, Chari, Kehoe, and McGrattan \(2016\)](#) emphasize the role of productivity and labor wedges during the 1980-82 recession and the role of the labor wedge during the 2007-09 recession, respectively. In our approach, instead of exactly matching aggregates (output, investment, consumption, and labor), we analyze the implications of shocks for the firm dynamics statistics, discussed above. The advantage of our approach is that the shocks are qualitatively consistent with both the aggregates and the firm dynamics statistics observed in the data. This approach leads us to the conclu-

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<sup>3</sup>The increase in the fixed cost of operating hurts small firms more than big firms, as modeled in [Clementi, Khan, Palazzo, and Thomas \(2015\)](#) and [Khan, Senga, and Thomas \(2016\)](#).

sion that productivity and credit shocks were most important during both the 1980-82 and 2007-09 recessions.

Finally, our paper contributes to the literature that uses firm dynamics models to study aggregate fluctuations. [Khan and Thomas \(2013\)](#) study productivity and credit shocks in a firm dynamics model with collateral constraints and partial irreversibility of investment. [Khan, Senga, and Thomas \(2016\)](#) study productivity and credit shocks in a firm dynamics model with endogenous default. Both papers argue that credit shocks are important to account for the behavior of aggregates and employment among small and large firms during the Great Recession. [Clementi, Khan, Palazzo, and Thomas \(2015\)](#), [Sedláček \(2015\)](#), and [Siemer \(2016\)](#) study how the decrease in firm entry during the Great Recession affected the slow recovery, which they term the “missing generation effect”. [Buera, Fattal-Jaef, and Shin \(2015\)](#) analyze productivity shocks and credit shocks in a model with frictions in both the labor and the financial markets. In their model, a productivity shock has no effect on employment. A credit shock decreases employment among both young firms or small firms and increases employment among old and large firms. [Mehrotra and Sergeyev \(2017\)](#) study the effects of productivity, credit, and discount rate shocks on jobs flows during the Great Recession. They argue that the Great recession was primarily driven by a discount rate shock and a credit shock. [Schott \(2015\)](#) studies the effects of productivity shocks and house price shocks in a model with collateral constraints and a frictional labor market. He argues the slow recovery of house prices, which tightens the collateral constraint, accounts for the jobless recovery since the Great Recession. These papers analyze implications for some, but not all of the firm dynamics statistics, discussed above. Furthermore, they have emphasized the role of credit shocks and discount rate shocks during the Great Recession. We add to this literature, first by analyzing the implications of a larger set of shocks (productivity, credit, labor wedge, investment wedge) for all the firm dynamics statistics, discussed above. Second, we compare the implications from the model economy with data for every U.S. recession since 1980. In Section 6, we discuss the firm dynamics implications from these papers in detail.

Our paper is also related to the literature that studies firm/establishment/plant entry and exit over the business cycle such as [Clementi and Palazzo \(2016\)](#), [Lee and Mukoyama \(2016\)](#), [Osotimehin and Pappadà \(2017\)](#), and [Woo \(2015\)](#). In particular, [Clementi and Palazzo \(2016\)](#) extend [Hopenhayn \(1992\)](#) to analyze how a shock to aggregate productivity propagates to changes in output in a model with and without firm entry and exit. In contrast, our analysis uses firm dynamics statistics to learn about the sources of aggregate fluctuations. In addition, our modeling choices differ from [Clementi and Palazzo \(2016\)](#), as we assume a general equilibrium model with endogenous labor supply, unbounded mass of potential entrants, and entry and operating costs in units of labor.<sup>4</sup>

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<sup>4</sup>Our paper also complements the literature that focuses on how firm dynamics affect investment rather than employment: [Ottonello and Winberry \(2018\)](#), [Winberry \(2018\)](#), and [Xiao \(2018\)](#). Other



## 2 Data

In this section, we discuss the business cycle patterns of the following set of firm dynamics statistics: (cross-sectional) average employment size by age group, entry and exit rates, and job creation and job destruction rates. The data are from the Business Dynamics Statistics (BDS), published by the Center of Economic Studies in the U.S. Census Bureau. This publicly available data set contains annual (mid-March) information on private businesses in the United States from 1977 to 2015. It is based on administrative records and covers most of the private non-agricultural sector of the economy. Furthermore, only employer firms are included (firms with at least one payroll employee). The main exclusions are self-employed individuals, employees of private households, agricultural production employees, and most government employees. Although information is available both at firm and establishment levels, we choose the firm as the main economic unit of our analysis because the firm makes the relevant decisions regarding the economic activities of its establishments.<sup>5</sup>

We begin by analyzing the time series of the (cross-sectional) average employment size of two age groups of firms: firms that are one or two years old (Figure 1a) and firms that are three years or older (Figure 1b). The average employment size is defined as the total number of employees in a group divided by the total number of firms in the same group. We distinguish between age groups because in Section 6, we show our model economy has different implications for each age group, depending on which shock hits the economy. In the data, however, both age groups show the same pattern — the average employment size decreases during recessions. The shaded areas in the figures correspond to recession periods according to the NBER classification of U.S. business cycles. From 1977 to 2015, NBER identifies four recessions.<sup>6</sup>

Next, we analyze the time series of firm entry and firm exit rates (Figures 1c and 1d). Figure 1c illustrates the series of entry both in levels (blue line, left axis) and in deviations from a linear trend (red line, right axis). The entry rate is defined as the number of entrants, firms of age 0, divided by the total number of firms. The black line corresponds to the linear trend and it shows the entry rate has decreased over time, as has been documented by Haltiwanger, Jarmin, and Miranda (2012). Our interest is on high frequency movements in the data. Hence, we focus our analysis on entry in terms of deviations from the linear trend (red line, right axis). Figure 1c shows the entry rate

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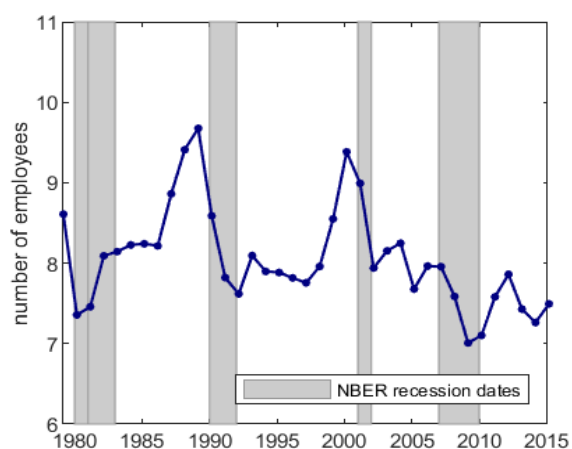
papers related to our study are Sedláček and Sterk (2017), Bloom et al. (2018), Arellano et al. (2018), Gilchrist et al. (2014), Dyrda (2016), Jermann and Quadrini (2012), Moscarini and Postel-Vinay (2012), Gomes and Schmid (2010), Gavazza et al. (2016), Lee and Mukoyama (2015), and Adrian et al. (2012).

<sup>5</sup>An establishment is defined as a single physical location where production takes place, whereas a firm corresponds to a group of establishments linked to each other by ownership. We refer to Haltiwanger, Jarmin, and Miranda (2002) and Jarmin and Miranda (2002) for a description of the data.

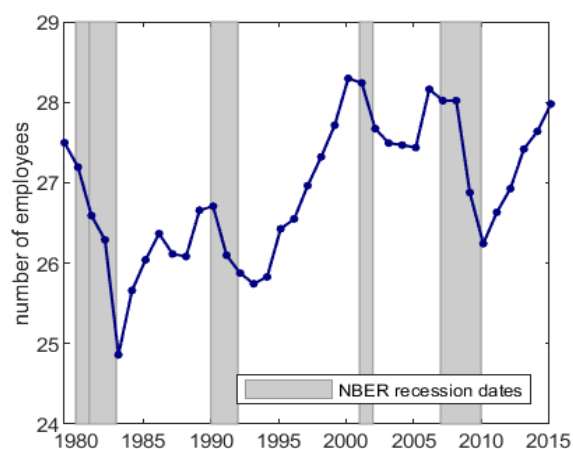
<sup>6</sup>NBER indentifies two recessions in the early 1980s, one from January 1980 to July 1980, and another from July 1981 to November 1982. Given the proximity of the two recessions, we treat them as one.

Figure 1: Firm employment size, entry/exit, and job flows (1979-2015)

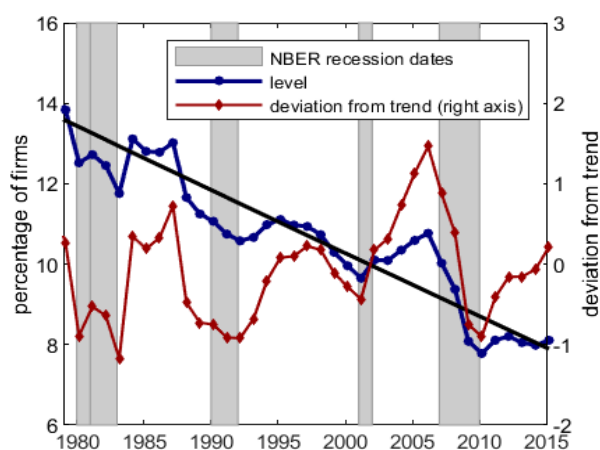
(a) Employment size: age: 1-2 years



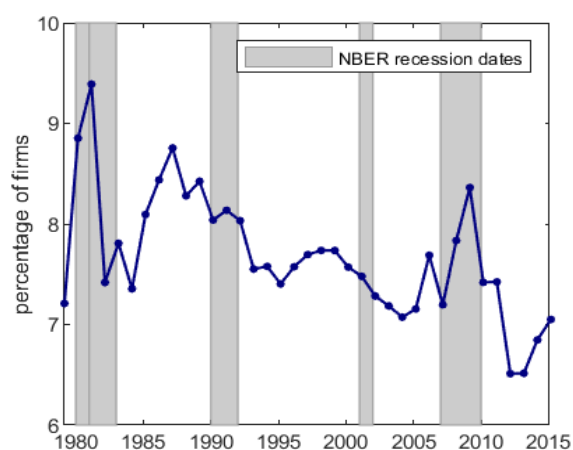
(b) Employment size: age 3+ years



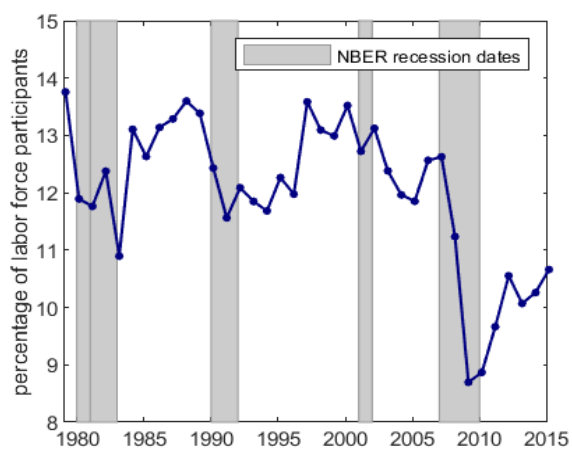
(c) Firm entry rate



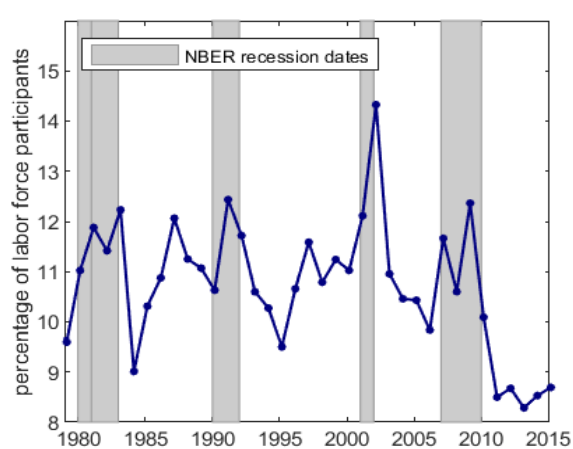
(d) Firm exit rate: age 3+ years



(e) Job creation



(f) Job destruction



Sources: U.S. Census and NBER.

decreased sharply during the 1980-82 and 2007-09 recessions, but did not vary much during the 1990-91 and 2001 recessions. Turning attention to the series of the exit rate in Figure 1d, we observe a similar pattern. The exit rate increased significantly during the 1980-82 and 2007-09 recessions, but did not vary much during the 1990-91 and 2001 recessions. The exit rate in Figure 1d corresponds to the exit rate of the group of firms that are three years or older, and it is defined as the total number of firms that exited the economy in that age group divided by the total number of firms in the same age group. We focus on the exit rate of firms that are three years or older for the following reason: total firm exit lags firm entry, because younger firms are more likely to exit than older firms. By focusing on the exit rate of firms that are three years or older, we control for the lagged effect of firm entry.

Finally, Figures 1e and 1f show the time series of job creation and job destruction rates, respectively. Job creation/destruction rates are defined as the number of jobs created/destroyed divided by the total number of labor force participants. The figures show the job creation rate decreased and the job destruction rate increased in every recession.

The creation and destruction of jobs are due to variations in employment in both the intensive margin (changes in the average employment size of firms) and the extensive margin (changes in the number of firms due to entry and exit), and they add up to variations in total employment. In Section 6, we use the business cycle patterns of the firm dynamics statistics observed in Figure 1, to identify which aggregate shocks generate the same pattern in our model economy as in the data. More specifically, we consider a set of aggregate shocks that is often used in the business cycle literature to identify which shocks can replicate the following patterns: decreasing average employment size of firms that are one or two years old and three years and older, decreasing job creation rates, and increasing job destruction rates — in every recession; no significant change in firm entry and firm exit during the 1990-91 and 2001 recessions; and decreasing firm entry and increasing firm exit during the 1980-82 and 2007-09 recessions. In the next section, we present the model economy.

### 3 Model

Our model economy consists of a representative household, heterogeneous incumbent firms, and ex-ante identical potential entrants. The representative household receives dividend payments from the firms and makes decisions regarding consumption, labor, and savings to maximize utility. Incumbent firms are heterogeneous with respect to their capital, debt, and productivity. They make decisions on whether to operate (or exit), as well as decisions on labor, investment, and borrowing to maximize profits. Ex-ante identical potential entrants make decisions on whether to enter or not. Despite

the idiosyncratic risk, the aggregate state of the economy evolves deterministically, and agents have perfect foresight regarding prices. We describe the problem of each agent in detail below.

**Households:** the representative household chooses a sequence of consumption  $\{C_t\}$ , labor  $\{H_t\}$ , and asset holdings  $\{A_{t+1}\}$  to maximize her lifetime utility subject to a sequence of budget, feasibility, and no-Ponzi-scheme constraints:

$$\begin{aligned} \max_{\{C_t, H_t, A_{t+1}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t U(C_t, H_t), \quad \text{s.t.} \\ C_t + A_{t+1} \leq (1 - \tau_t^h) w_t H_t + (1 + r_t) A_t + \Pi_t + T_t, \\ C_t \geq 0, \quad H_t \in [0, 1], \quad A_t \geq -\bar{A}, \quad A_0 \text{ given}, \end{aligned} \quad (1)$$

where  $U(C_t, H_t; \psi, \phi) = \log C_t - \psi \frac{H_t^{1+\phi}}{1+\phi}$ .  $\beta \in (0, 1)$  denotes the discount factor,  $w_t$  the wage rate,  $\tau_t^h$  the labor wedge,  $r_t$  the interest rate, and  $T_t$  the lump-sum transfers. Furthermore, we assume the representative household owns the firms and receives dividend payments  $\Pi_t$  in each period.

**Incumbent firms:** let  $V_t^{inc}(k, b, \epsilon)$  denote the value of an incumbent firm in period  $t$ . The idiosyncratic state is given by the stock of physical capital  $k$ , the stock of debt  $b$ , and the idiosyncratic productivity  $\epsilon$ . In the beginning of each period, after observing the idiosyncratic productivity, the firm chooses whether to exit ( $d = 0$ ) or to operate ( $d = 1$ ) to maximize its value:

$$V_t^{inc}(k, b, \epsilon) = \max_{d \in \{0, 1\}} d \times V_t^{op}(k, b, \epsilon) + (1 - d) \times V_t^{exit}(k, b), \quad (2)$$

in which  $V_t^{op}(k, b, \epsilon)$  denotes the value of operating, and  $V_t^{exit}(k, b)$  denotes the value of exiting. If the firm continues to operate, it chooses labor  $l$ , next period capital  $k'$ , and next period debt  $b'$  to maximize its value subject to non-negative dividend payments and borrowing constraints. The dividend payment of a firm is equal to its revenues net of operational costs, investment in physical capital, interest payments on its current debt, plus revenue from new debt issuance. It is expressed as

$$\begin{aligned} D_t(k', b', l, k, b, \epsilon) = & f(k, l, \epsilon; Z_t) - w_t(l + f_t^o) - (1 + \tau_t^k)(k' - (1 - \delta^k)k) \\ & - (1 + r_t + \tau_t^b)b + b', \end{aligned}$$

where  $f(k, l, \epsilon; Z_t)$  denotes the production function,  $\tau_t^k$  the investment wedge,  $\delta^k$  the depreciation rate of physical capital,  $f_t^o$  the fixed operating cost (in units of labor), and  $\tau_t^b$  the exogenously given spread on the debt interest rate (borrowing wedge).<sup>7</sup> As in

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<sup>7</sup>Both operating and entry costs are in units of labor, such that the model delivers stationary entry/exit

Hopenhayn (1992), idiosyncratic productivity follows a first order Markov process, in which  $F(\epsilon'|\epsilon)$  denotes the distribution of  $\epsilon'$  conditional on the realization of  $\epsilon$ . The value of the incumbent firm that decides to operate is

$$\begin{aligned} V_t^{op}(k, b, \epsilon) &= \max_{l, k', b'} D_t(k', b', l, k, b, \epsilon) + \frac{1}{1 + r_{t+1}} \int V_{t+1}^{inc}(k', b', \epsilon') F(d\epsilon'|\epsilon), \\ \text{s.t.} \quad &D_t(k', b', l, k, b, \epsilon) \geq 0, \quad b' \in [0, \bar{b}_t]. \end{aligned} \quad (3)$$

We assume firms cannot save ( $b' < 0$ ) and that the amount of next period debt (borrowing) is limited by  $\bar{b}_t$ .<sup>8</sup> Upon exit, firms suffer a reduction in their stock of capital given by  $\chi$ , where  $0 < \chi < 1$ . Exiting firms must sell their capital and pay any debt entirely:<sup>9</sup>

$$V_t^{exit}(k, b) = (1 + \tau_t^k)\chi k - (1 + r_t + \tau_t^b)b. \quad (4)$$

**Entrants:** we assume an unbounded mass of ex-ante identical potential entrants. They enter with zero physical capital and zero debt. Upon entry, they pay a fixed cost  $f_t^e$  (in units of labor) and draw an idiosyncratic productivity shock from the initial distribution  $G$ . Hence they have to borrow in the period they enter to invest and start producing in the subsequent period. The value of entry  $V_t^{entry}$  is given by

$$V_t^{entry} = -w_t f_t^e + \int V_t^{inc}(0, 0, \epsilon) G(d\epsilon). \quad (5)$$

**Dividend payments:** let  $\Omega_t(k, b, \epsilon)$  denote the distribution of incumbent firms over the idiosyncratic states. Total dividend payments,  $\Pi_t$ , will be equal to the sum of the total dividend payments from incumbent firms that choose to operate, the total dividend payments from incumbent firms that choose to exit, and the total dividend payments from entrants:

$$\begin{aligned} \Pi_t &= \int D_t(k'_t(k, b, \epsilon), b'_t(k, b, \epsilon), l_t(k, b, \epsilon), k, b, \epsilon) d_t(k, b, \epsilon) \Omega_t(dk \times db \times d\epsilon) \\ &\quad + \int [(1 + \tau_t^k)\chi k - (1 + r_t + \tau_t^b)b] (1 - d_t(k, b, \epsilon)) \Omega_t(dk \times db \times d\epsilon) \\ &\quad + m_t \left[ \int D_t(k'_t(0, 0, \epsilon), b'_t(0, 0, \epsilon), l_t(0, 0, \epsilon), 0, 0, \epsilon) d_t(0, 0, \epsilon) G(d\epsilon) - w_t f_t^e \right]. \end{aligned} \quad (6)$$

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rates and average employment size of firms in a balanced growth path. See Klenow, Li, and Bollard (2013).

<sup>8</sup>Having a borrowing constraint instead of a collateral constraint allows us to have firms that enter the economy with zero capital.

<sup>9</sup>Note we are allowing for negative dividend payments upon exit. That is, if the stock of physical capital of an exiting firm is not sufficient to cover debt repayments, the household must transfer to the firm the amount of final goods necessary to cover the repayment. That implies firms are not allowed to default on their debt.

**Production technology:** each firm produces the homogeneous consumption good using the following production technology:

$$f(k, l, \epsilon; Z) = (Z\epsilon)^{(1-\alpha\nu)} (k^\alpha l^{1-\alpha})^\nu,$$

where  $\epsilon$  denotes the firm's idiosyncratic productivity,  $Z$  the aggregate productivity,  $k$  the physical capital input, and  $l$  the labor input. The capital-labor share is denoted by  $\alpha \in (0, 1)$ , and  $\nu \in (0, 1)$  denotes the span of control parameter, as in Lucas (1978).

**Lump-sum transfers:** following the business cycle accounting literature, investment, borrowing, and labor wedges work like taxes, and they are rebated to the household as lump-sum transfers. Total transfers are given by

$$\begin{aligned} T_t = & \tau_t^k \left[ \int d_t(k, b, \epsilon) k'_t(k, b, \epsilon) \Omega_t(dk \times db \times d\epsilon) + m_t \int k'_t(0, 0, \epsilon) d_t(0, 0, \epsilon) G^\epsilon(d\epsilon) \right] \\ & - \tau_t^k \left[ \int [d_t(k, b, \epsilon) (1 - \delta^k) k + (1 - d_t(k, b, \epsilon)) \chi k] \Omega_t(dk \times db \times d\epsilon) \right] \\ & + \tau_t^b \int b \Omega_t(dk \times db \times d\epsilon) + \tau_t^h w_t H_t. \end{aligned}$$

### 3.1 Equilibrium

A *competitive equilibrium* consists of an initial level of asset holdings  $A_0$ , initial distribution over idiosyncratic states  $\Omega_0(k, b, \epsilon)$ , sequences of wage rate  $\{w_t\}$ , interest rate  $\{r_t\}$ , aggregate productivity  $\{Z_t\}$ , investment wedge  $\{\tau_t^k\}$ , labor wedge  $\{\tau_t^h\}$ , interest rate spread (borrowing wedge)  $\{\tau_t^b\}$ , borrowing constraint  $\{\bar{b}_t\}$ , consumption  $\{C_t\}$ , labor supply  $\{H_t\}$ , asset holdings  $\{A_{t+1}\}$ , dividend payments  $\{\Pi_t\}$ , lump-sum transfers  $\{T_t\}$ , mass of entrants  $\{m_t\}$ , distribution of incumbent firms over the idiosyncratic state  $\{\Omega_t(k, b, \epsilon)\}$ , operate/exit decision function  $\{d_t(k, b, \epsilon)\}$ , labor demand function  $\{l_t(k, b, \epsilon)\}$ , capital decision function  $\{k'_t(k, b, \epsilon)\}$ , debt decision function  $\{b'_t(k, b, \epsilon)\}$ , value of entry  $\{V_t^{entry}\}$ , value of incumbent function  $\{V_t^{inc}(k, b, \epsilon)\}$ , value of operating function  $\{V_t^{op}(k, b, \epsilon)\}$ , and value of exiting function  $\{V_t^{exit}(k, b)\}$ , such that:

(i) given  $A_0$ ,  $\{w_t\}$ ,  $\{r_t\}$ ,  $\{\Pi_t\}$ ,  $\{T_t\}$ ,  $\{\tau_t^h\}$ , the allocations  $\{C_t\}$ ,  $\{H_t\}$ , and  $\{A_{t+1}\}$  solve the household problem in (1);

(ii) for each  $t$  and for each idiosyncratic state  $(k, b, \epsilon)$ , given  $r_t$ ,  $w_t$ ,  $Z_t$ ,  $\tau_t^k$ ,  $\tau_t^b$ , and  $V_{t+1}^{inc}(k, b, \epsilon)$ , the allocations  $l_t(k, b, \epsilon)$ ,  $k'_t(k, b, \epsilon)$ ,  $b'_t(k, b, \epsilon)$ , and  $d_t(k, b, \epsilon)$  solve the incumbent firm problem in (2), (3), and (4), with the respective maximum values equal to  $V_t^{inc}(k, b, \epsilon)$ ,  $V_t^{op}(k, b, \epsilon)$ , and  $V_t^{exit}(k, b)$ ;

(iii) for each  $t$ , the distribution of firms  $\Omega_t(k, b, \epsilon)$  evolves according to:

$$\begin{aligned} \Omega_{t+1}(k', b', \epsilon') = & \\ & \int \mathbb{I}\{k'_t(k, b, \epsilon) \leq k', b'_t(k, b, \epsilon) \leq b'\} d_t(k, b, \epsilon) F(\epsilon'|d\epsilon) \Omega_t(dk \times db \times d\epsilon) \\ & + m_t \int \mathbb{I}\{k'_t(0, 0, \epsilon) \leq k', b'_t(0, 0, \epsilon) \leq b'\} d_t(0, 0, \epsilon) F(\epsilon'|d\epsilon) G(d\epsilon); \end{aligned}$$

(iv) for each  $t$ , the value of entry is equal to zero:  $V_t^{entry} = 0$ ;

(v) for each  $t$ , the wedges are lump sum rebated to the household;

(vi) for each  $t$ , the labor market clears:

$$\begin{aligned} H_t = & \int d_t(k, b, \epsilon) [l_t(k, b, \epsilon) + f^o] \Omega_t(dk \times db \times d\epsilon) \\ & + m_t f^e + m_t \int [l_t(0, 0, \epsilon) + f^o] d_t(0, 0, \epsilon) G^\epsilon(d\epsilon). \end{aligned}$$

(vii) for each  $t$ , the asset market clears:

$$\begin{aligned} A_{t+1} = & \int d_t(k, b, \epsilon) b'_t(k, b, \epsilon) \Omega_t(dk \times db \times d\epsilon) \\ & + m_t \int b'_t(0, 0, \epsilon) d_t(0, 0, \epsilon) G^\epsilon(d\epsilon). \end{aligned}$$

(viii) for each  $t$ , the goods market clears:

$$\begin{aligned} C_t + & \int d_t(k, b, \epsilon) k'_t(k, b, \epsilon) \Omega_t(dk \times db \times d\epsilon) + m_t \int k'_t(0, 0, \epsilon) d_t(0, 0, \epsilon) G^\epsilon(d\epsilon) = \\ & \int d_t(k, b, \epsilon) f(k, l_t(k, b, \epsilon), \epsilon; Z) \Omega_t(dk \times db \times d\epsilon) + \\ & \int d_t(k, b, \epsilon) (1 - \delta^k) k \Omega_t(dk \times db \times d\epsilon) + \int (1 - d_t(k, b, \epsilon)) \chi k \Omega_t(dk \times db \times d\epsilon). \end{aligned}$$

Given the definition of a competitive equilibrium, the definition of a stationary competitive equilibrium is straightforward. A *stationary competitive equilibrium* is a competitive equilibrium in which the sequences of prices, allocations, and distributions, are constant across time (so time subscripts  $t$  can be dropped).

## 4 Calibration

Some parameters are calibrated outside the stationary equilibrium and the remaining parameters are calibrated jointly to target moments in the stationary equilibrium. [Table 1](#) shows the parameters calibrated outside the stationary equilibrium. A period is assumed to be one year, because the BDS data are annual. The discount rate  $\beta$  is set such that the

stationary equilibrium interest rate is 0.04 - an estimate from [McGrattan and Prescott \(2003\)](#).  $\phi$  is set to 0.5, which leads to a Frisch elasticity of 2. The borrowing spread  $\tau^b$  is set to the average of the Moody's Seasoned Aaa Corporate spread (1983-present). For all the statistics, we use data for all available years.

The decreasing returns parameter  $\nu$  is set to 0.836, which is the average of the estimates used in [Khan and Thomas \(2008, 2013\)](#), [Bloom et al. \(2018\)](#), and [Clementi et al. \(2015\)](#). The depreciation rate  $\delta^k$  is set to 0.04. The recovery rate of capital upon exit  $\chi$  is set such that it equals  $1 - \delta^k$ . This parameterization ensures the operate/exit decision is weakly monotonic in current capital. Both the fixed entry cost  $f^e$  and aggregate labor productivity  $Z$  are set to one. Both the labor and investment wedges ( $\tau^h$  and  $\tau^k$ ) are set to zero.

Table 1: Parameters determined outside of the stationary equilibrium

	Parameter	Description	Value
$\beta$	Discount rate	$(1 + r)\beta = 1$	0.960
$\phi$	Labor elasticity parameter	Frisch elasticity=2	0.500
$\tau^b$	Borrowing spread	Average Aaa corporate bond spread	0.013
$\nu$	Decreasing returns	<a href="#">Khan and Thomas (2008, 2013)</a> , <a href="#">Bloom et al. (2018)</a>	0.836
$\delta^k$	Depreciation rate		0.040
$\chi$	Recovery rate	$\chi = 1 - \delta^k$	0.960
$\rho$	Persistence	<a href="#">Khan and Thomas (2008)</a>	0.859
$f^e$	Fixed entry cost		1
$Z$	Labor productivity		1
$\tau^h$	Labor wedge		0
$\tau^k$	Investment wedge		0

The idiosyncratic productivity  $\epsilon$  is assumed to follow a log  $AR(1)$  process given by  $\log \epsilon' = \rho_\epsilon \log \epsilon + \eta'$ , where the innovation  $\eta'$  is iid normal with mean zero and variance  $\sigma_\eta^2$ . The persistence parameter  $\rho$  is set to 0.859, following [Khan and Thomas \(2008\)](#). The variance parameter  $\sigma_\eta^2$  will be calibrated jointly in equilibrium, discussed below. The productivity process is discretized to a Markov chain with 40 grid points, using the [Tauchen \(1986\)](#) method. Entrants draw their initial productivities from the stationary distribution of the discretized Markov chain.

The five remaining parameters are the borrowing constraint  $\bar{b}$ , leisure share  $\psi$ , capital share  $\alpha$ , variance of innovations  $\sigma_\eta^2$ , and fixed operating cost  $f^o$ , which are presented in [Table 2](#). They are calibrated jointly to match the following moments: average non-finance business debt to GDP (1948-2015), total employment to working age population of 0.6, average labor share (1970-2015), firm size 21-25 years/firm size 1 year (BDS, 1977-2015), and entrants' 5 year survival rate (BDS, 1977-2015).



Table 2: Parameters determined jointly in equilibrium

Parameter		Target	Value
$\bar{b}$	Borrowing limit	Non-finance business debt to GDP = 50.15 percent	3.02
$\psi$	Leisure share	Total labor = 0.60	1.67
$\alpha$	Capital share	Labor share = 64.53 percent	0.37
$\sigma_\eta^2$	Variance of innovations	Firm size 21-25 years/firm size 1 year = 3.24	0.15
$f^o$	Fixed operating cost	Entrants' 5 year survival rate = 46.23 percent	0.16

## 5 Properties of the initial stationary equilibrium

This section analyzes the policy functions of the firm's problem and compares the non-targeted life-cycle moments of the model to the data.

### 5.1 Policy functions

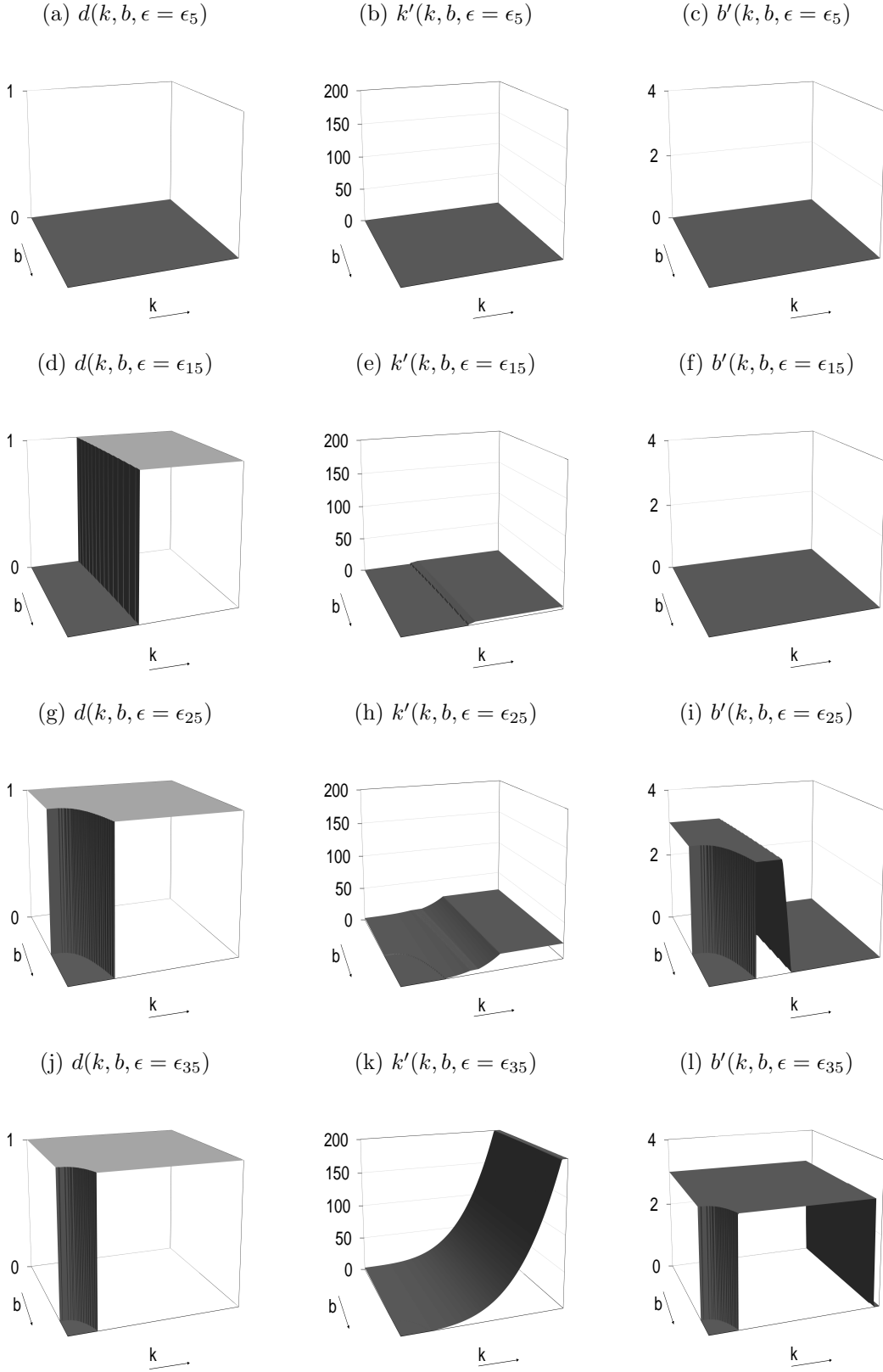
The policy functions of the firm are the operating/exit decision  $d(k, b, \epsilon) \in \{0, 1\}$ , next period capital choice  $k'(k, b, \epsilon)$ , and next period debt choice  $b'(k, b, \epsilon)$ . They are illustrated in Figure 2 as a function of  $k$  and  $b$  for different values of idiosyncratic productivity ( $\epsilon_5, \epsilon_{15}, \epsilon_{25}, \epsilon_{35}$ ). Subscripts denote their position in the grid, and  $i < j$  implies  $\epsilon_i < \epsilon_j$ . The operating decision is weakly increasing in idiosyncratic productivity and in capital, and weakly decreasing in debt (Figures 2a, 2d, 2g, and 2j). That is, the firm is more likely to operate for higher levels of idiosyncratic productivity given capital and debt; for higher levels of capital given idiosyncratic productivity and debt; and for lower levels of debt given idiosyncratic productivity and capital. Moreover, when idiosyncratic productivity increases, the firm is more likely to operate with lower levels of capital and higher levels of debt (Figures 2d, 2g, and 2j).

For a firm that chooses to operate, the next period capital choice is strictly increasing in idiosyncratic productivity (Figures 2e, 2h, and 2k). If the non-negative constraint does not bind, the firm will choose a  $k'$  that is only a function of the idiosyncratic productivity  $\epsilon$ . That is,  $k'$  will not be a function of the current level of capital  $k$  and debt  $b$ . Furthermore, the firm will choose  $b' = 0$  if borrowing is costly ( $1 + r + \tau^b > \beta$ ). In this case,  $k'$  is characterized by the following equation:

$$\beta E_{\epsilon'|\epsilon}[d(k', 0, \epsilon')(f_k(k', l'(k', \epsilon'), \epsilon') + 1 - \delta^k) + (1 - d(k', 0, \epsilon'))\chi] = 1$$

This unconstrained choice of  $k'$  can be seen in Figures 2e, 2h, and 2k for higher levels of capital  $k$ . If the firm chooses to operate and the non-negative dividend constraint does bind for the unconstrained choice of  $k'$ , the firm will borrow. An example is when the firm has a low level of capital, but wants grow by choosing a higher  $k'$ , as observed in Figures 2h, 2k, 2i, and 2l.

Figure 2: Policy functions (initial stationary equilibrium)



## 5.2 Non-targeted life cycle moments

Figures 3 and 4 compare the conditional survival probability by age and the average employment size by age in the model to the data. For survival probabilities, we compare the model with data until age 4 because the survival probability for subsequent years cannot be computed in the BDS public data. For the average employment size by age, we compare the model with data for a longer horizon from age 0 to 25. The targets were firm size 21-25 years/firm size 1 year and entrants' 5-year survival rate. The model does well in matching the non-targeted life cycle profiles.

Figure 3: Survival probability by age

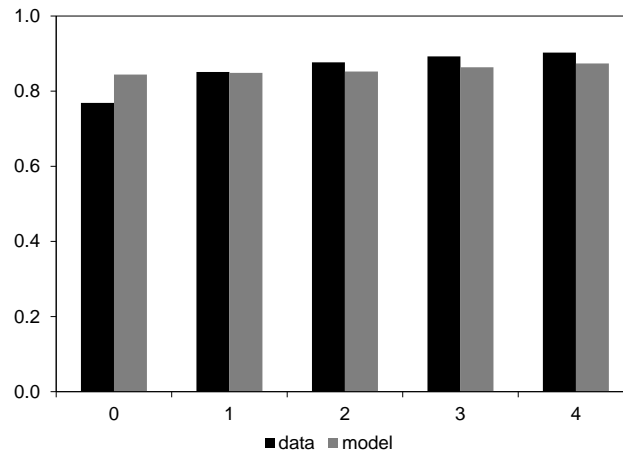
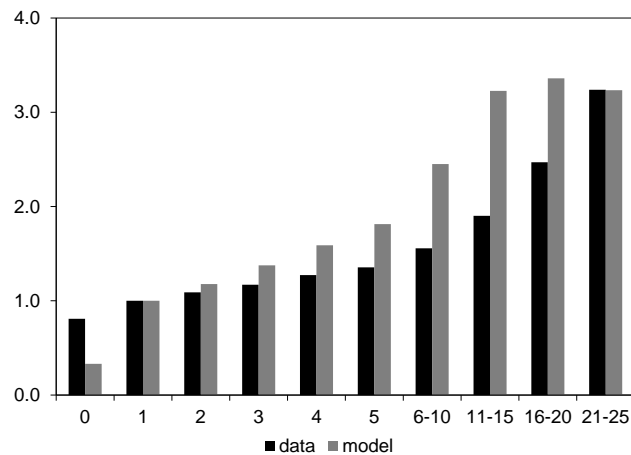
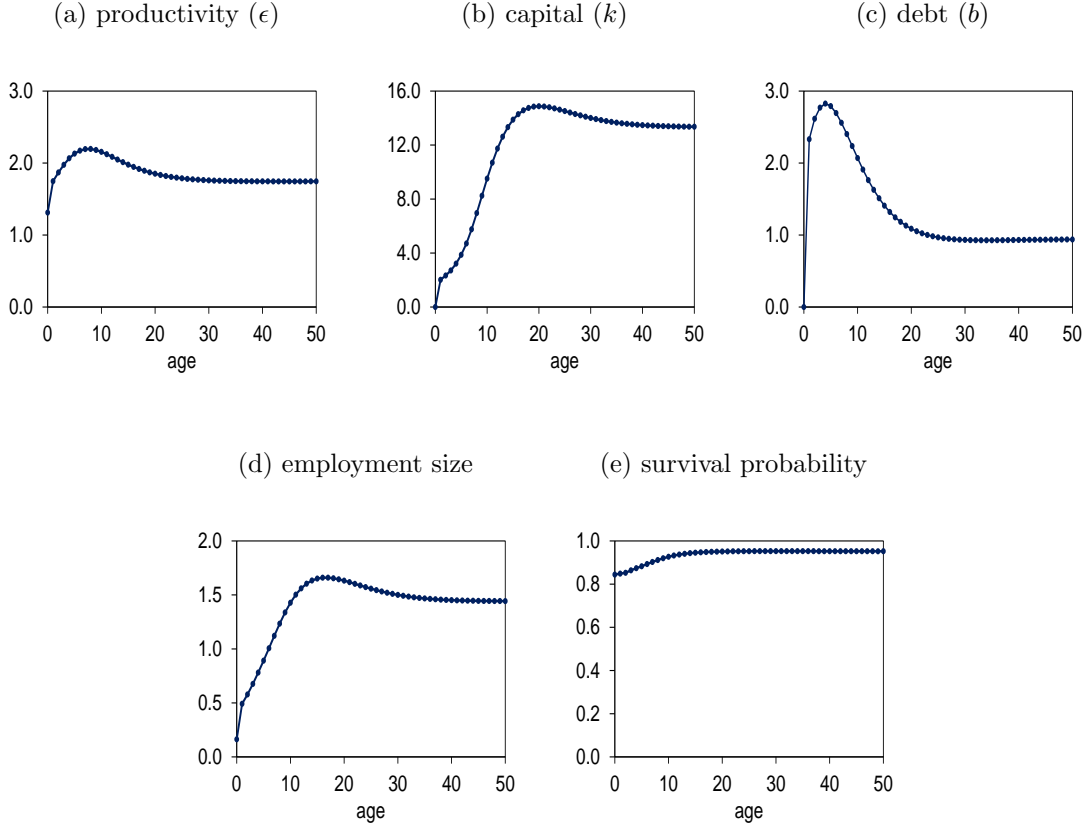


Figure 4: Employment size by age (index, age 1 = 1)



In the model, we see that both the survival probability and the average employment size are non-monotonic in age. Figure 5 plots the life cycle dynamics of the average firm in the benchmark economy for idiosyncratic productivity, capital, debt, employment size, and conditional survival probability for the first 50 years. We assume a firm enters the economy with zero capital and zero debt. Hence, the average firm accumulates debt in the early part of its life cycle to invest and grow (Figure 5c). As the firm accumulates debt, it gets closer to the non-negative dividend constraint. At this point, a firm that experiences a relatively low idiosyncratic productivity shock is more likely to exit because either the firm cannot satisfy the non-negative dividend constraint (even by choosing  $k' = 0$ ) or a binding non-negative dividend constraint limits the firm's ability to invest in capital. This mechanism of firm exit leads to overshooting of the average productivity, capital, and employment size over the life cycle (Figures 5a, 5b, and 5d). Later in the life cycle, the average firm reduces its debt as it grows closer to its optimal size of capital (5b and 5d).

Figure 5: Life cycle properties



## 6 Results

We compute separate transitions with shocks to productivity, the borrowing constraint (referred to as a credit shock), labor wedge, and investment wedge. The size of the shock is calibrated to target the average drop in GDP per labor force participant (from peak to trough) in the U.S. economy in the last four recessions (5.24 percent). Each shock is unexpected in period one. After that, we assume perfect foresight and an annual persistence of 0.75. With the assumed persistence, from the beginning of the recession, it takes the economy 4-5 years to complete more than half the recovery, which is close to the average for the U.S. economy in the last four recessions (4.82 years). The results from the four shocks are presented in Figures 6, 7, 8, and 9.<sup>10</sup> All four shocks lead to a drop in output, investment, and labor (Appendix A.3). However, they have different implications for firm employment size by age group, entry and exit, and job flows. We first discuss the firm dynamics implications of each shock, and subsequently identify the important shocks during the last four U.S. recessions.

A negative productivity shock (Figure 6) reduces the marginal products of both capital and labor — for every firm in the economy. Hence, the (cross-sectional) average employment size decreases for both age groups of firms: 1-2 years and 3 years and older (Figures 6c and 6d). This happens although the interest rate and the wage rate decrease in equilibrium (Figures 6a and 6b). Firm entry and firm exit (extensive margin) do not vary much (Figures 6e and 6f), because of the following opposing mechanisms. Lower aggregate productivity decreases the value of operation. However, the lower wage rate decreases the cost of labor including fixed entry costs and fixed operating costs; the lower interest rate decreases the cost of borrowing. Due to opposing mechanisms, firm entry and firm exit do not vary much. Finally, job creation decreases because growing firms (with respect to both capital and productivity) hire fewer workers. Analogously, job destruction increases because shrinking firms fire more workers (Figures 6g and 6h).

In this model, firms borrow when the non-negative dividend constraint binds for the unconstrained choice of capital, as discussed in Section 5.1. When the borrowing limit is tightened (Figure 7), and if the non-negative dividend constraint binds, firms invest less in productive capital and subsequently employ fewer workers. However, a tighter borrowing constraint also leads to both a lower interest rate and a lower wage rate (Figures 7a and 7b). A lower wage rate leads firms with unconstrained dividends to employ more workers. Young firms (1-2 years) are more likely to be dividend constrained, and therefore their (cross-sectional) average employment size decreases (Figure 7c). Older firms (3 years and older) are less likely to be dividend constrained, and therefore their (cross-sectional) average employment size increases (Figure 7d). Firm entry decreases because a tighter

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<sup>10</sup>In Appendix A.1, we also analyze alternative ways of modeling a credit shock, such as shocks to the borrowing wedge or the fixed operating cost. That analysis does not alter our conclusions.

borrowing constraint decreases the entrant's value of operation (Figure 7e). Firm exit increases because a tighter borrowing constraint makes it harder for a firm to satisfy the non-negative dividend constraint (Figure 7f). Job creation decreases due to lower firm entry and lower investment in capital (and therefore lower employment) by constrained firms (Figure 7g). Job destruction decreases because shrinking firms fire fewer workers, due to lower wages (Figure 7h).

The increase in the labor wedge (Figure 8) increases the marginal cost of working for the household. In equilibrium, the interest rate decreases and the wage rate increases (Figures 8a and 8b). Note that the fundamentals of an incumbent firm are not affected. Hence, firm exit does not vary much (Figure 8f). However, firm entry decreases significantly (Figure 8h). The higher wage rate decreases the average employment size of firms that are 1-2 years in the first period. In the subsequent period, the average size increases because of the lagged effect of lower firm entry (Figure 8c). That is, young firms are small, and therefore, significantly lower firm entry increases the average size of firms that are 1-2 years in the subsequent period. The (cross-sectional) average employment size of firms that are 3 years and older decreases because of higher wages (Figure 8d). Job creation decreases because of lower firm entry and higher wages (Figure 8g). Job destruction initially increases due to higher wages, and subsequently decreases due to the lagged effect of lower firm entry (Figure 8h).

The increase in the investment wedge (Figure 9) increases the marginal cost of investing in productive capital, which leads to lower investment. In equilibrium, the interest rate decreases significantly and the wage rate increases (Figures 9a and 9b). Firm size decreases for both age groups due to higher wages (Figures 9c and 9d). Both firm entry and firm exit decrease (Figures 9e and 9f). Firm entry decreases because the increase in the investment wedge decreases the value of operation for an entrant. Firm exit decreases because the significantly lower interest rate decreases the cost of debt. Job creation decreases due to lower firm entry, lower investment, and higher wages (Figure 9g). Job destruction initially increases due to higher wages, and subsequently decreases due to the lagged effect of lower firm entry (Figure 9h).

Now we turn to an analysis of the last four U.S. recessions. Recall that during the 1990-91 and 2001 recessions, the (cross-sectional) average employment size decreased for all age groups (intensive margin), firm entry and firm exit were hardly affected (extensive margin), job creation decreased, and job destruction increased (Figure 1). These patterns are consistent with the implications from a productivity shock (Figure 6). The credit shock, labor wedge shock, and investment wedge shock are not individually or jointly consistent with their implications for the firm dynamics patterns observed during the 1990-91 and 2001 recessions because they decrease firm entry (Figures 7e, 8e, and 9e). Hence, we conclude that the productivity shock was important during the 1990-91 and 2001 recessions.

The 1980-82 and 2007-09 recessions differ from the 1990-91 and 2001 recessions with respect to their implications for the extensive margin — firm entry decreased and firm exit increased (Figures 1c and 1d). As discussed in the previous paragraph, the productivity shock accounts for the 1990-91 and 2001 recessions. Furthermore, the remaining three shocks decrease firm entry. Hence, we focus on implications for firm exit. In the remaining three shocks, only the credit shock increases firm exit (Figures 7f, 8f, and 9f). Therefore, we conclude that a productivity shock and a credit shock were important during the 1980-82 and 2007-09 recessions.

## 6.1 Related literature on firm dynamics implications

In Table 3, we summarize the firm dynamics implications from our paper and related papers in the literature for negative output shocks (if available). With regard to a negative productivity shock, [Mehrotra and Sergeyev \(2017\)](#) find no effect on the employment size of young firms, but old firms decrease in size. We find that all age groups decrease in size. The key differences are that we incorporate dynamics to both the consumer’s problem and the firm’s problem, endogenize firm entry and firm exit, and analyze implications in a closed economy (endogenous interest rate) instead of a small open economy (exogenous interest rate). Implications for firm entry and firm exit are mixed. [Khan et al. \(2016\)](#) and [Siemer \(2016\)](#) find that a negative productivity shock has no effect on firm entry. [Clementi et al. \(2015\)](#) find that a negative productivity shock decreases firm entry. With regard to firm exit, both [Clementi et al. \(2015\)](#) and [Khan et al. \(2016\)](#) find that a negative productivity shock increases firm exit. In our paper, neither firm entry nor exit are affected. The key difference is that we assume fixed costs in units of labor, which dampens the effect of a productivity shock through lower wages. Finally, the implications for job flows from a negative productivity shock are consistent with [Mehrotra and Sergeyev \(2017\)](#) and [Schott \(2015\)](#). With regard to a credit shock, the firm dynamics implications from a shock to the borrowing constraint in our model are consistent with the papers listed in Table 3.

## 7 Conclusion

We extend the [Hopenhayn \(1992\)](#) firm dynamics model with entry and exit to include capital and debt accumulation to the firm’s problem and savings to the consumer’s problem. Using our model, we discuss the implications of a set of aggregate shocks (productivity, credit, labor wedge, and investment wedge) — for firm employment by age, entry and exit, and job flows. We find that a productivity shock was important during the 1990-91 and 2001 recessions, and that both productivity and credit shocks were important during the 1980-82 and 2007-09 recessions.

Figure 6: Productivity shock (unit = p.p. deviation)

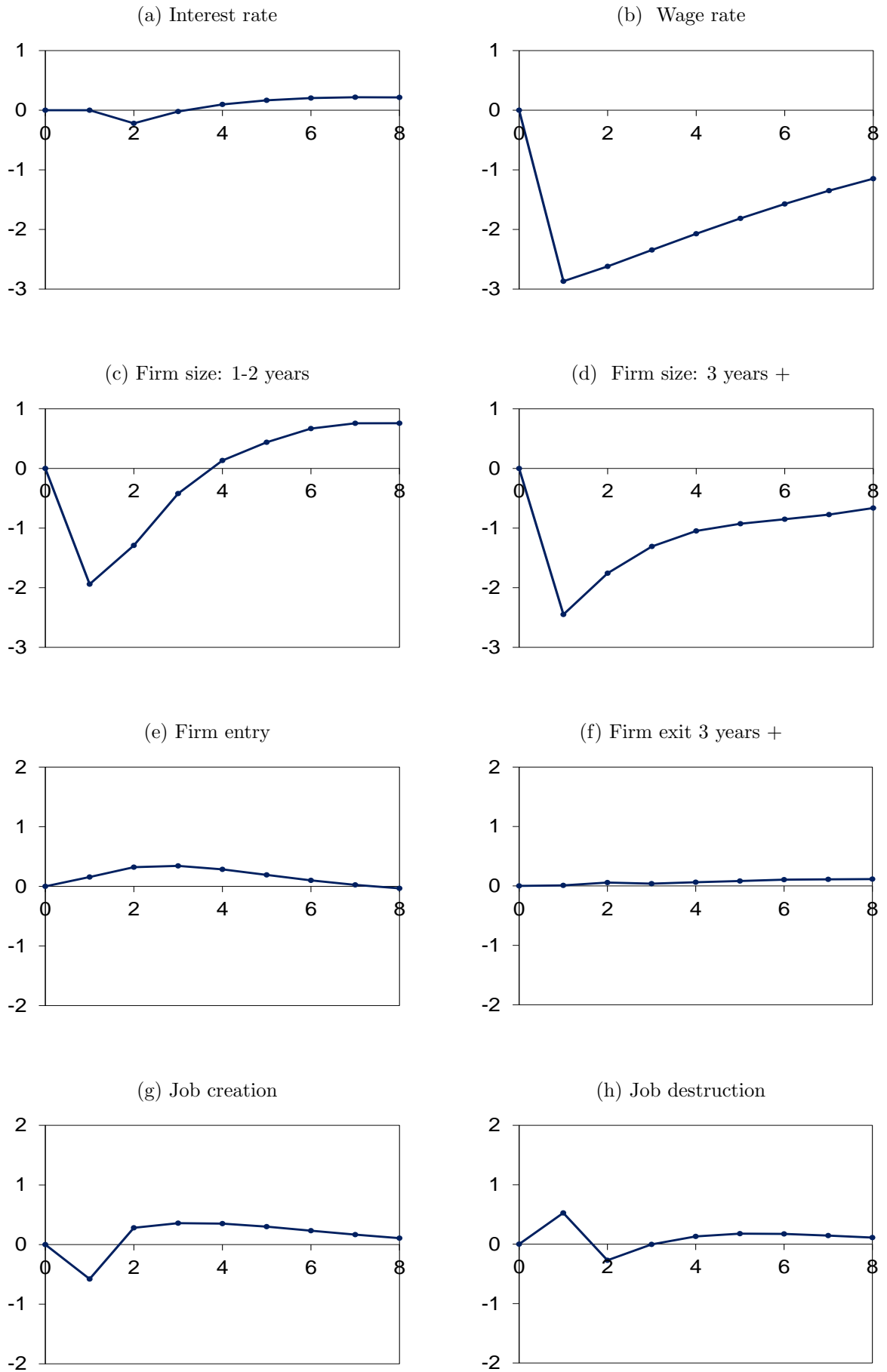




Figure 7: Credit shock (unit = p.p. deviation)

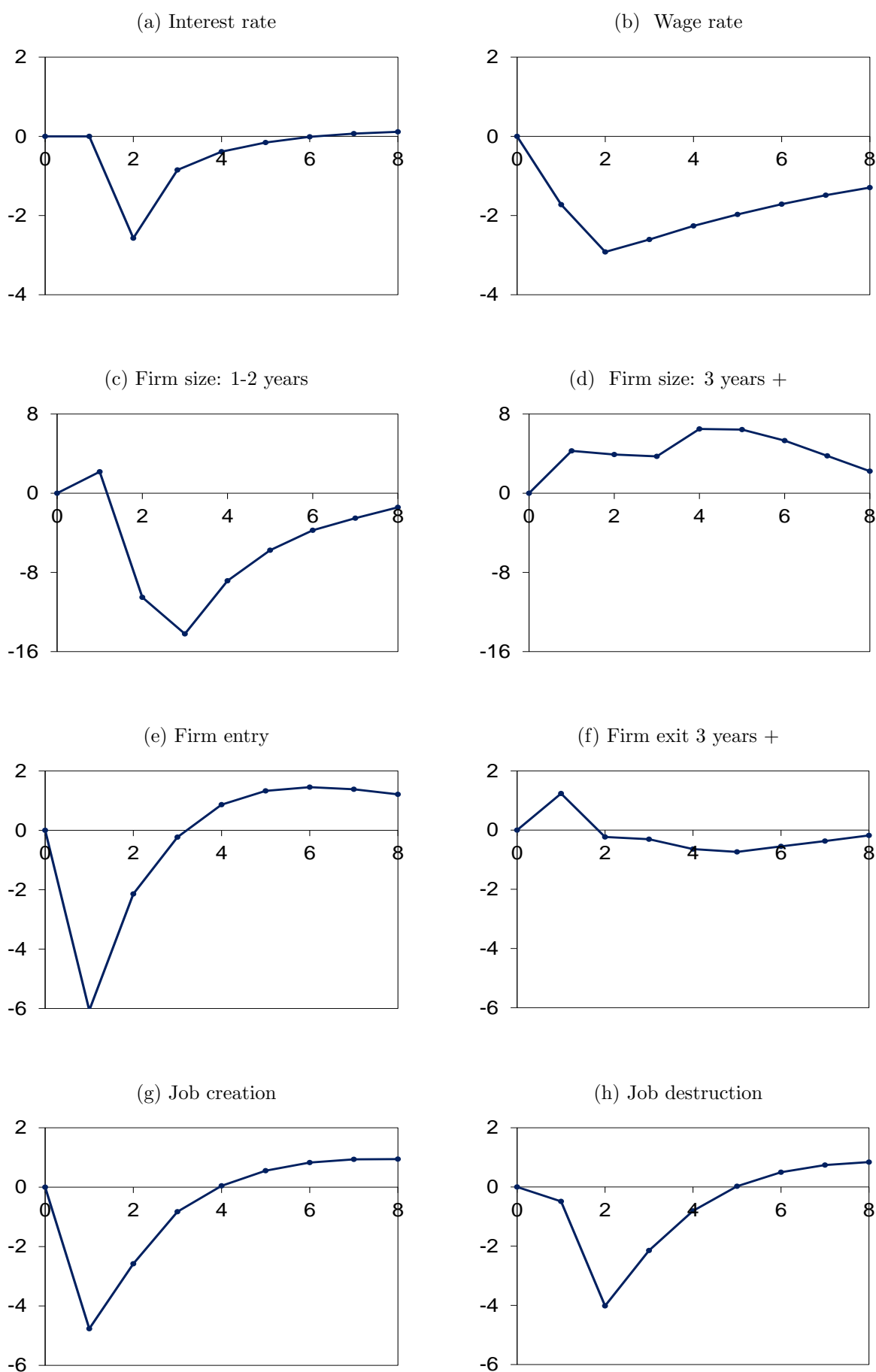


Figure 8: Labor wedge shock (unit = p.p. deviation)

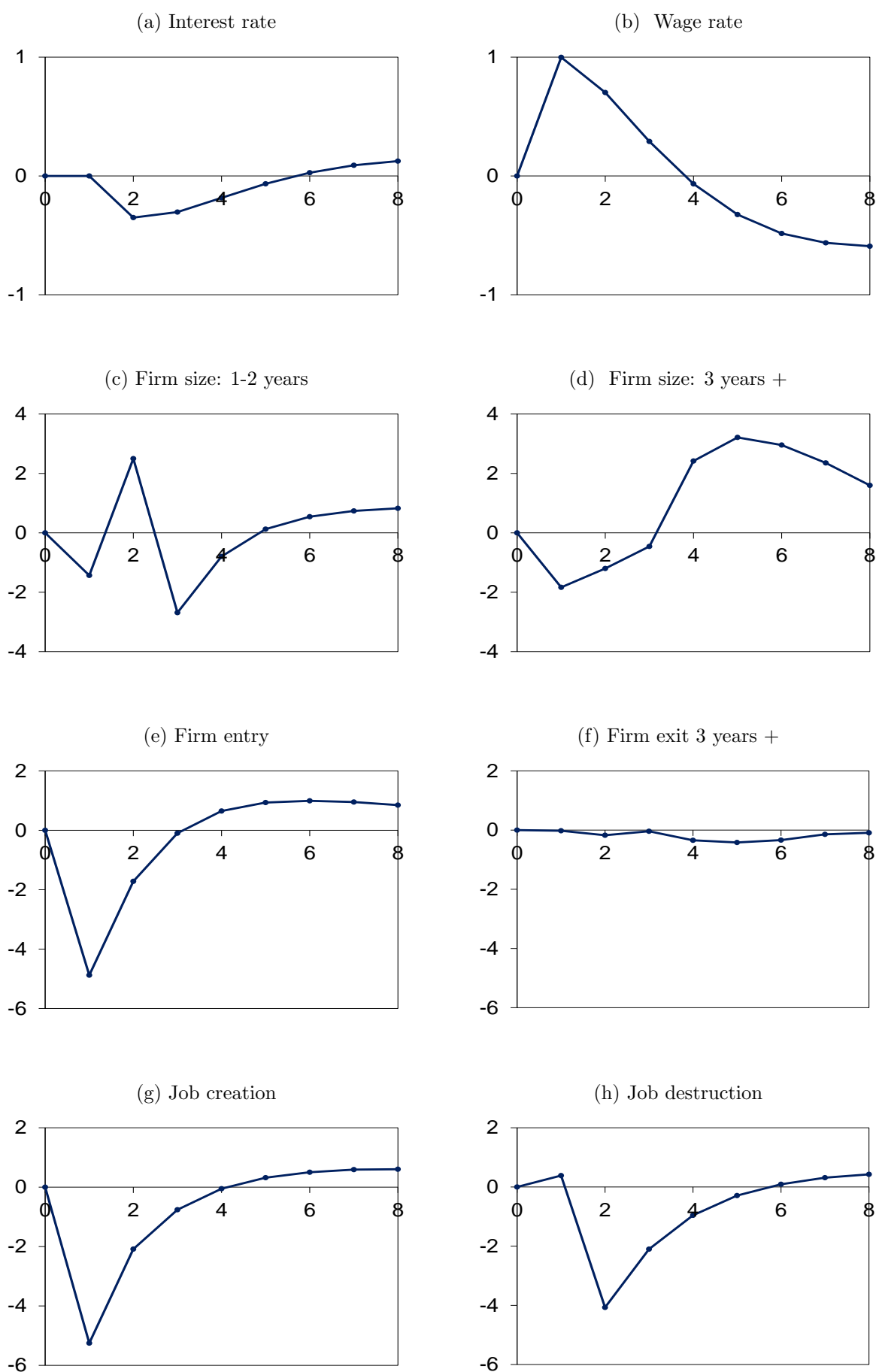


Figure 9: Investment wedge shock (unit = p.p. deviation)

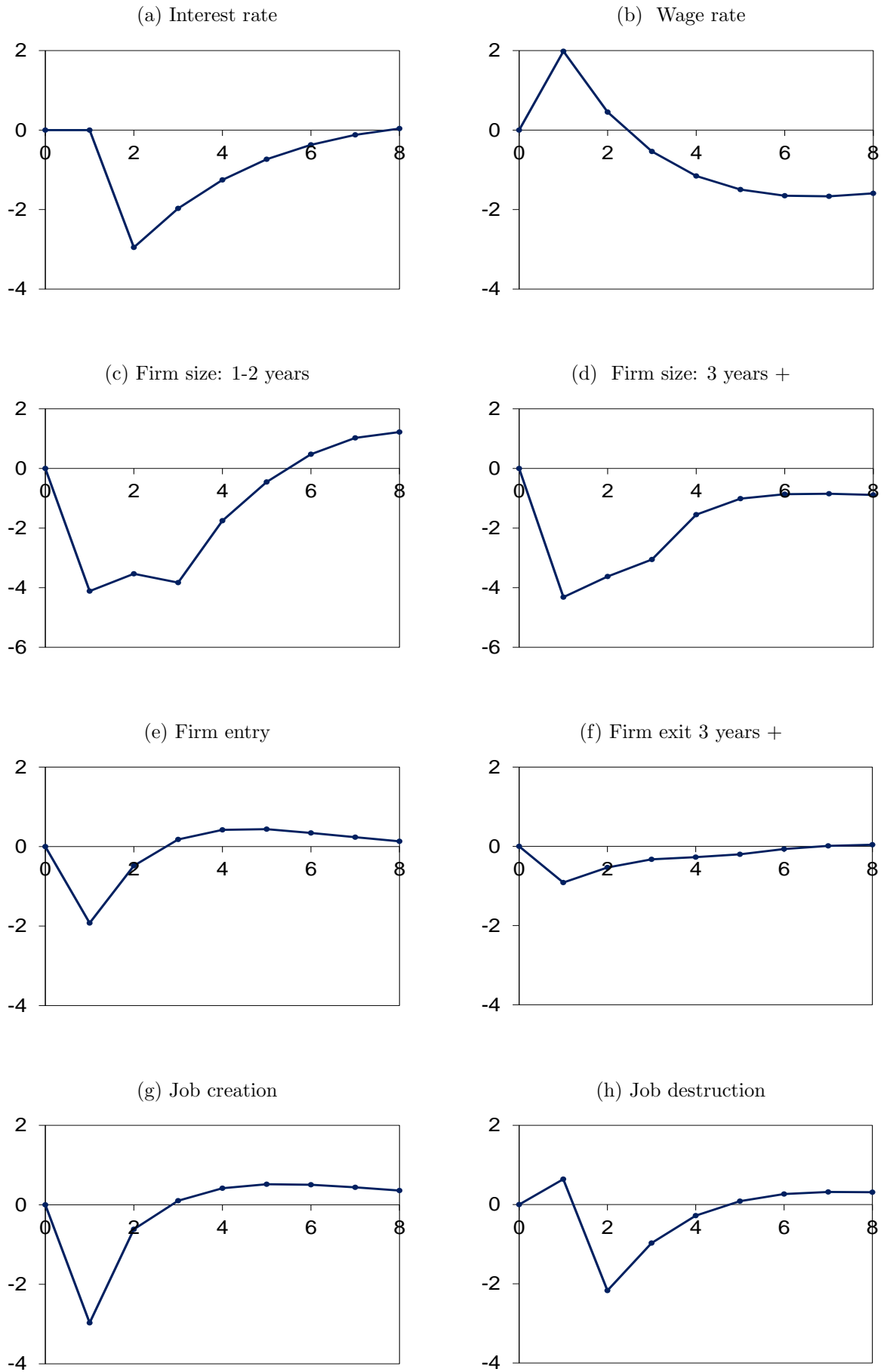


Table 3: Summary of firm dynamics implications

Paper	Period	Model key features	Shocks	Employment by age <sup>a</sup>	Entry	Exit	Job flows <sup>d</sup>
This paper	Recessions post 1980	Endogenous entry and exit, borrowing limit	Productivity Credit 1 (borrowing limit) Labor wedge Investment wedge Credit 2 (spread) Credit 3 (higher operating cost)	young decreases; old decreases young decreases; old increases young decreases <sup>c</sup> ; old decreases young decreases; old decreases young increases; old increases young increases; old increases	no change decreases decreases decreases decreases decreases	no change increases no change decreases increases increases	JC decreases; JD increases JC decreases; JD decreases JC decreases; JD increases <sup>d</sup> JC decreases; JD increases JC decreases; JD decreases JC decreases; JD decreases
Buera et al. (2015)	Great recession	Collateral constraint, search friction in the labor market	Productivity Credit (collateral constraint)	- small/young decrease; old-large increase	- -	- -	- -
Clementi et al. (2015)	Great recession	Endogenous entry and exit, capital adjustment cost	Productivity Credit (higher operating cost)	- -	decreases decreases	increases increases	- -
Khan et al. (2016)	Great recession	Default	Productivity Credit (higher operating cost)	- -	no change decreases	increases increases	- -
Mehrotra and Sergeyev (2017) <sup>e</sup>	Great recession	Collateral constraint	Productivity Credit (collateral constraint) Discount factor	young no change; old decreases young decreases; old increases -	- - -	- - -	JC decreases; JD increases JC decreases; JD decreases JC increases; JD increases
Schott (2015)	Great recession	Search friction in the labor market	Productivity House price	- -	decreases decreases	- -	startup JC decreases; incumbent JD increases startup JC decreases; incumbent JC increases
Sedláček (2015)	Great recession	Search friction in the labor market	Productivity Entry cost	- -	decreases decreases	- -	- -
Stemer (2016)	Great recession	Default	Productivity Credit (lower recovery rate)	- -	no change decreases	- -	- -

<sup>a</sup> young (old) refers to employment size of firms less than (greater than) 5 years old in Buera et al. (2015) and Mehrotra and Sergeyev (2017); 3 years old in our paper. 3 years allows us to analyze dynamics during the 1980 recession.

<sup>b</sup> JC = job creation; JD = job destruction.

<sup>c</sup> Subsequently increases.

<sup>d</sup> Subsequently decreases.

<sup>e</sup> We report implications for the case of their preferred calibration with a Frisch elasticity of 1.

## References

- Adrian, T., P. Colla, and H. S. Shin (2012). Which financial frictions? Parsing the evidence from the financial crisis of 2007 to 2009. *NBER Macroeconomics Annual* 2012 27(3), 159–214.
- Arellano, C., Y. Bai, and P. J. Kehoe (2018). Financial frictions and fluctuations in volatility. *Journal of Political Economy*, Forthcoming.
- Bloom, N., M. Floetotto, N. Jaimovich, I. Saporta-Eksten, and S. J. Terry (2018). Really uncertain business cycles. *Econometrica*, Forthcoming.
- Brinca, P., V. V. Chari, P. J. Kehoe, and E. McGrattan (2016). Accounting for business cycles. *Handbook of Macroeconomics* 2A, 1013–1063.
- Buera, F. J., R. N. Fattal-Jaef, and Y. Shin (2015). Anatomy of a credit crunch: From capital to labor markets. *Review of Economic Dynamics* 18, 101–117.
- Chari, V. V., P. J. Kehoe, and E. McGrattan (2007). Accounting for business cycles. *Econometrica* 75(3), 781–836.
- Clementi, G. L., A. Khan, B. Palazzo, and J. K. Thomas (2015). Entry, exit and the shape of aggregate fluctuations in a general equilibrium model with capital heterogeneity. *Working paper*.
- Clementi, G. L. and B. Palazzo (2016). Entry, exit, firm dynamics, and aggregate fluctuations. *American Economic Journal: Macroeconomics* 8(3), 1–41.
- Davis, S. J., R. J. Faberman, and J. Haltiwanger (2012). Labor market flows in the cross section and over time. *Journal of Monetary Economics* 59, 1–18.
- Davis, S. J., J. Haltiwanger, and S. Schuh (1998). Job creation and destruction. *MIT Press Books* 1.
- Decker, R., J. Haltiwanger, R. S. Jarmin, and J. Miranda (2014a). The role of entrepreneurship in u.s. job creation and economic dynamism. *Journal of Economic Perspectives* 28(3), 3–24.
- Decker, R., J. Haltiwanger, R. S. Jarmin, and J. Miranda (2014b). The secular decline in business dynamism in the u.s. *Working paper*.
- Dyrda, S. (2016). Fluctuations in uncertainty, efficient borrowing constraints, and firm dynamics. *Working paper*.
- Fort, T. C., J. Haltiwanger, R. S. Jarmin, and J. Miranda (2013). How firms respond to business cycles: the role of firm age and firm size. *IMF Economic Review*.

- Gavazza, A., S. Mongey, and G. L. Violante (2016). Aggregate recruiting intensity. *Working paper*.
- Gilchrist, S., J. W. Sim, and E. Zakrajšek (2014). Uncertainty, financial frictions, and investment dynamics. *Working paper*.
- Gomes, J. F. and L. Schmid (2010). Equilibrium credit spreads and the macroeconomy. *Working paper*.
- Haltiwanger, J., R. Jarmin, and J. Miranda (2002). Business dynamics statistics: An overview. Kauffman Foundation.
- Haltiwanger, J., R. Jarmin, and J. Miranda (2012). Where have all the young firms gone? Kauffman Foundation.
- Hopenhayn, H. (1992). Entry, exit, and firm dynamics in long run equilibrium. *Econometrica* 60(5), 1127–1150.
- Hopenhayn, H. and R. Rogerson (1993). Job turnover and policy evaluation: a general equilibrium analysis. *Journal of Political Economy* 101(5), 915–938.
- Jarmin, R. and J. Miranda (2002). The longitudinal business database. Center for Economic Studies WP 02-17.
- Jermann, U. and V. Quadrini (2012). Macroeconomic effects of financial shocks. *American Economic Review* 102(1), 238–271.
- Khan, A., T. Senga, and J. K. Thomas (2016). Default risk and aggregate fluctuations in an economy with production heterogeneity. *Working paper*.
- Khan, A. and J. K. Thomas (2008). Idiosyncratic shocks and the role of nonconvexities in plant and aggregate investment dynamics. *Econometrica* 76(2), 395–436.
- Khan, A. and J. K. Thomas (2013). Credit shocks and aggregate fluctuations in an economy with production heterogeneity. *Journal of Political Economy* 121(6), 1055–1107.
- Klenow, P., H. Li, and A. Bollard (2013). Entry costs rise with development. 2013 Meeting Papers 471 Society for Economic Dynamics.
- Lee, Y. and T. Mukoyama (2015). Entry and exit of manufacturing plants over the business cycle. *European Economic Review* 77, 20–27.
- Lee, Y. and T. Mukoyama (2016). A model of entry, exit, and plant-level dynamics over the business cycle. *Working paper*.

- Lucas, R. (1978). On the size distribution of business firms. *The Bell Journal of Economics* 9(2), 508–523.
- McGrattan, E. R. and E. C. Prescott (2003). Average debt and equity returns: puzzling? *American Economic Review, Papers and Proceedings* 93(2), 392–397.
- Mehrotra, N. and D. Sergeyev (2017). Financial shocks, firm credit and the Great Recession. *Working paper*.
- Moscarini, G. and F. Postel-Vinay (2012). The contribution of large and small employers to job creation in times of high and low unemployment. *American Economic Review* 102(6), 2509–2539.
- Osootimehin, S. and F. Pappadà (2017). Credit frictions and the cleansing effect of recessions. *Economic Journal* 127(602), 1153–1187.
- Ottonello, P. and T. Winberry (2018). Financial heterogeneity and the investment channel of monetary policy. *Working paper*.
- Schott, I. (2015). Start-ups, house prices, and the U.S. recovery. *Working paper*.
- Sedláček, P. (2015). Lost generations of firms and aggregate labor market dynamics. *Working paper*.
- Sedláček, P. and V. Sterk (2017). The growth potential of startups over the business cycle. *American Economic Review* 107(10), 3182–3210.
- Siemer, M. (2016). Firm entry and employment dynamics during the great recession. *Working paper*.
- Siemer, M. (2018). Employment effects of financial constraints during the great recession. *Review of Economic Statistics, forthcoming*.
- Tauchen, G. (1986). Finite state markov-chain approximations to univariate and vector autoregressions. *Economics Letters* 20(2), 177–181.
- Winberry, T. (2018). Lumpy investment, business cycles, and stimulus policy. *Working paper*.
- Woo, J. (2015). The cyclicalities of entry and exit: a general equilibrium analysis with imperfect information. *Working paper*.
- Xiao, J. (2018). Capital allocation and investment dynamics in credit crises. *Working paper*.

# A Appendix

## A.1 Other credit shocks

Section 6 analyzes a credit shock as a tightening of the borrowing constraint. Alternative ways of modeling a credit shock are to increase the cost of borrowing (captured by the borrowing wedge  $\tau^b$ ) or increase the fixed cost of operation  $f^o$ . As discussed in Clementi et al. (2015) and Khan et al. (2016), the latter captures a credit shock because it hurts small firms more than big firms. Again, we calibrate the size and the persistence of the shocks to target the moments discussed in Section 6 — average drop in GDP per labor force participant and average time for half the recovery in the last four recessions. The results from these shocks are presented in Figures 10 and 11, respectively. We first discuss the firm dynamics implications of these two shocks.

When the borrowing wedge (Figure 10) increases, firms borrow less and decrease investment in capital. Subsequently, these firms employ fewer workers. However, the wage rate and the interest rate decrease in equilibrium (Figures 10a and 10b). We see that the second mechanism dominates for both young and old incumbent firms: their average employment size increases (Figures 10c and 10d). In the extensive margin, firm entry decreases because a higher cost of borrowing decreases the entrant's value of operation (Figure 10e). Firm exit increases because a higher cost of borrowing decreases the value of operation for incumbents who have higher levels of debt (Figure 10f). Job creation decreases due to lower firm entry and lower investment in productive capital (and subsequently less employment) (Figure 10g). Job destruction decreases due to both lower wages and the lagged effect of lower firm entry (Figure 10h).

An increase in the fixed operating cost (Figure 11) increases firm exit (Figure 11f). The (cross-sectional) average employment size of incumbent firms increases across all age groups because of higher fixed operating costs (in units of labor), exit of low productivity firms, and lower wages (Figures 11c and 11d). Firm entry decreases due to a lower value of operation (Figure 11e). Job creation decreases due to lower firm entry (Figure 11g). Job destruction decreases because of the higher fixed operating cost, lower wages, and the lagged effect of lower firm entry (Figure 11h).

To summarize, both shocks decrease total employment through the extensive margin (lower firm entry and higher firm exit) rather than the intensive margin (lower firm employment size). Furthermore, the implications for the extensive margin are similar to those from a shock to the borrowing constraint. Hence, our conclusion that a credit shock was important during the 1980-82 and 2007-09 recessions is robust to these two alternative ways of modeling the shock.



Figure 10: Borrowing wedge shock (unit = p.p. deviation)

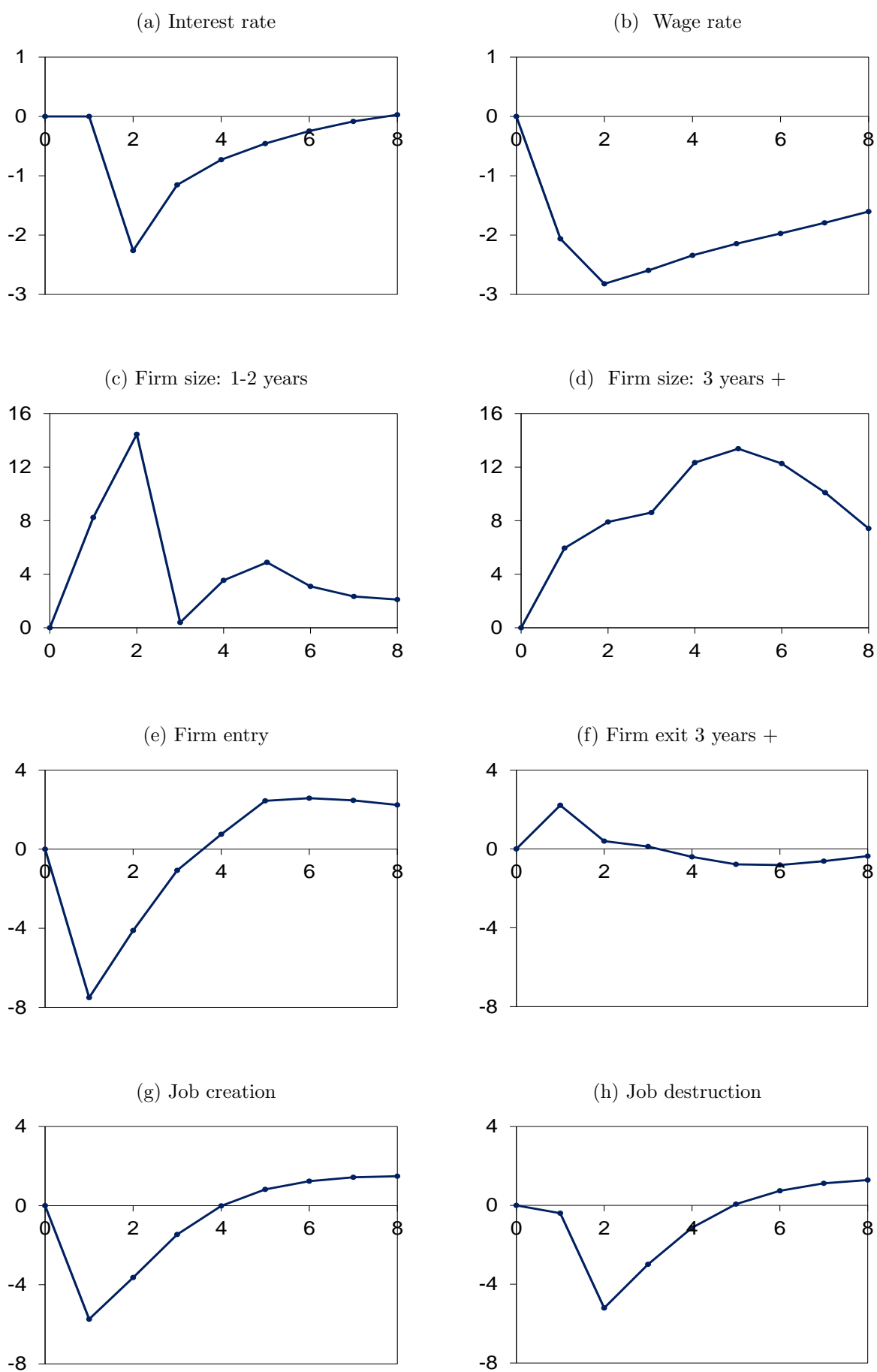
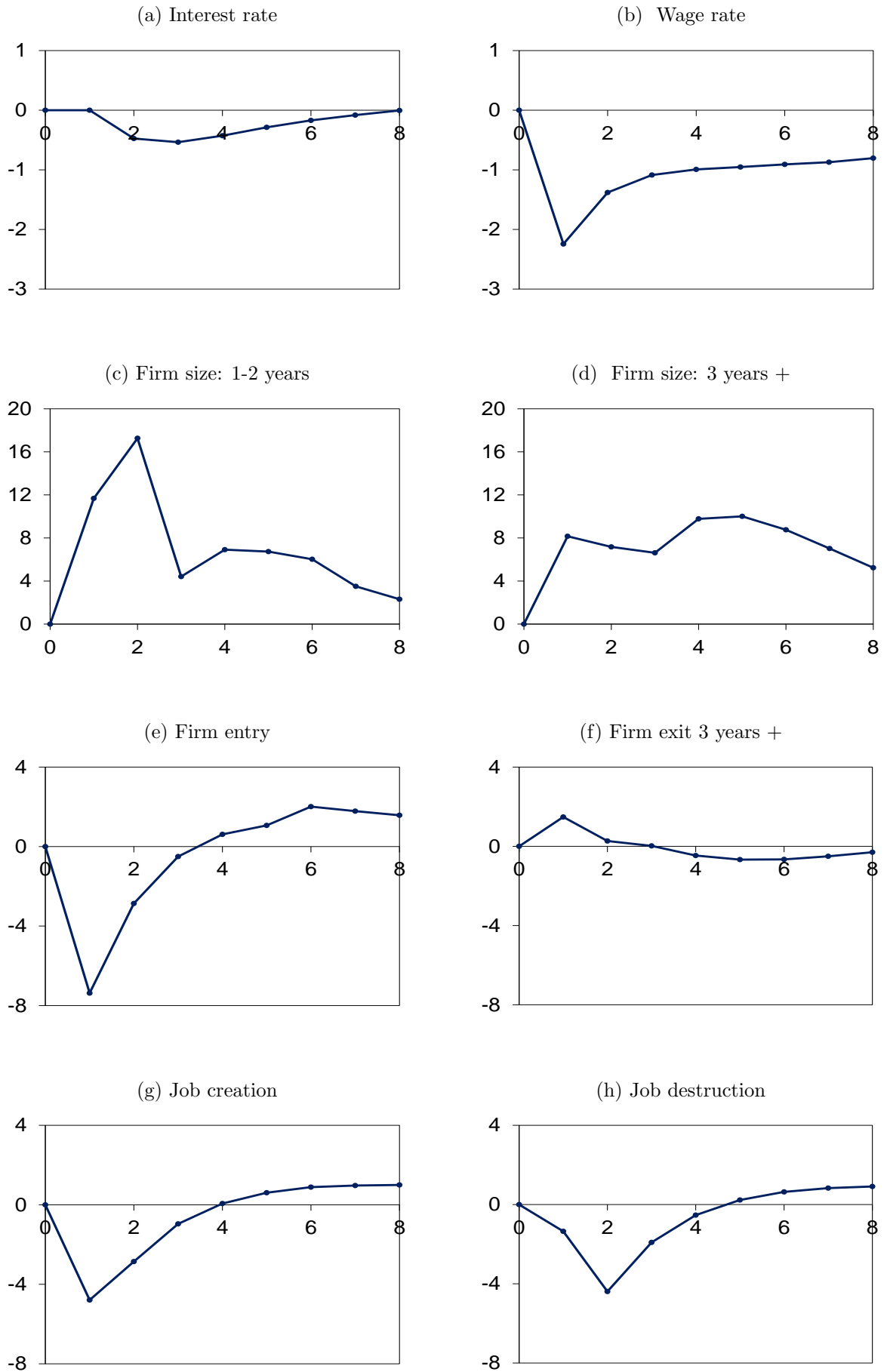


Figure 11: Fixed operating cost shock (unit = p.p. deviation)



## A.2 Algorithm to compute transitions

- Guess sequence of  $r_t$
- Back out sequence of  $C_t$  given that  $C_t = C_{t+1}/(\beta(1 + r_{t+1}))$
- Guess  $w_t$  and solve for incumbent firm's value and policy functions; solve for sequence of  $w_t$  through backward induction such that  $V_t^{entry} = 0$
- Back out sequence of  $H_t$  from household intra-temporal condition
- Solve for sequence of  $m_t$  such that labor market clears
- Back out sequence for  $\Pi_t$ ,  $A_{t+1}$ , and  $C_t$
- Update  $r_t$  until convergence

## A.3 Aggregates

Figures 12, 13, and 14 plot the shocks and the corresponding aggregates such as output, employment, and investment for the results discussed in Section 6 and Appendix A.1. Output in the model is computed as the sum of production and fixed entry costs (in units of labor),  $\int d(k_t, b_t, \epsilon_t) f(k_t, l_t, \epsilon_t; Z_t) \Omega_t(dk \times db \times d\epsilon) + m_t w_0 f^e$ , where  $w_0$  is the initial stationary equilibrium wage rate. The figures show that all of these shocks decrease output, employment, and investment.

Figure 12: Aggregates: productivity and credit shocks (unit = p.p. deviation)

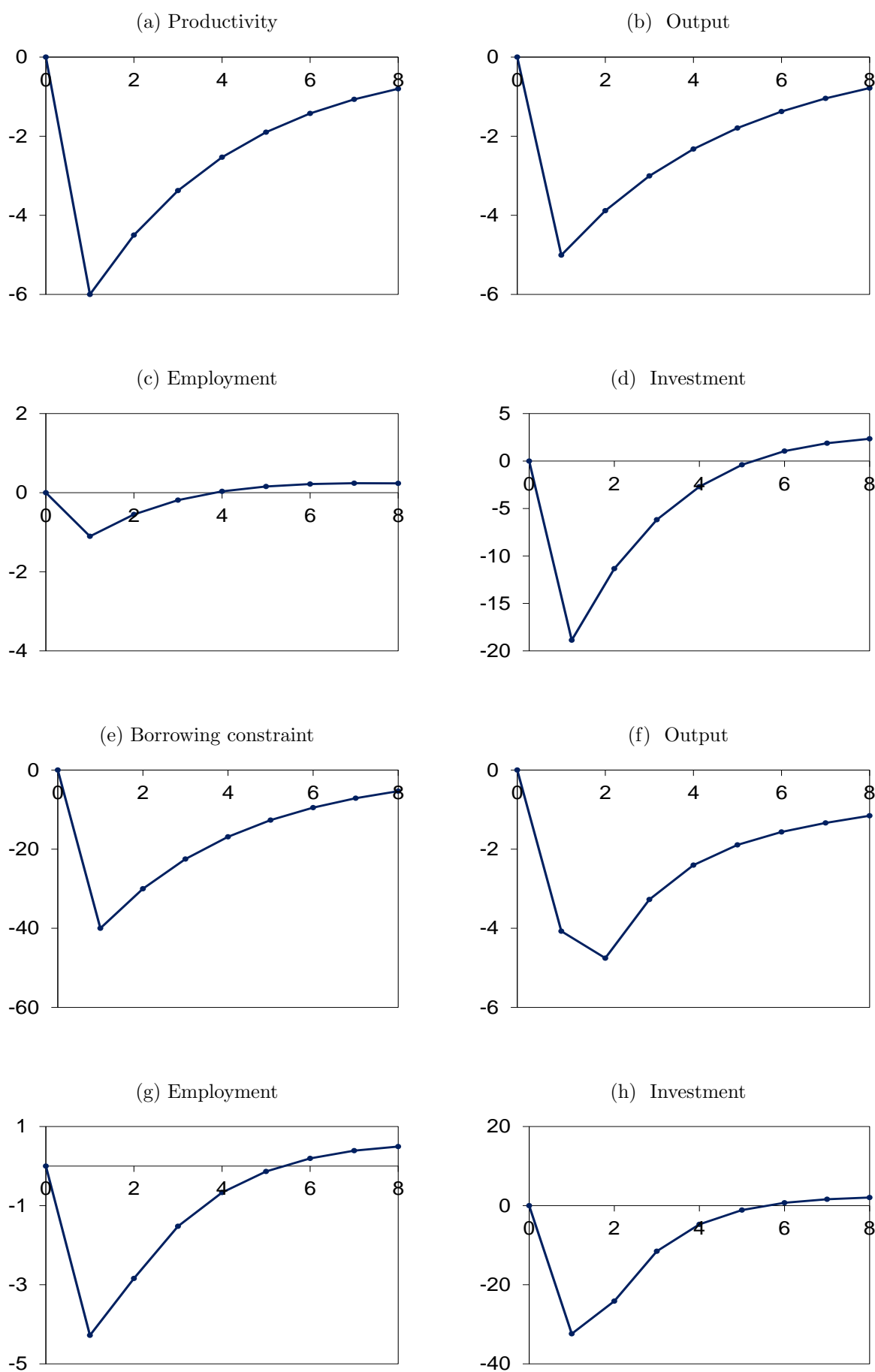


Figure 13: Aggregates: labor wedge and investment wedge (unit = p.p. deviation)

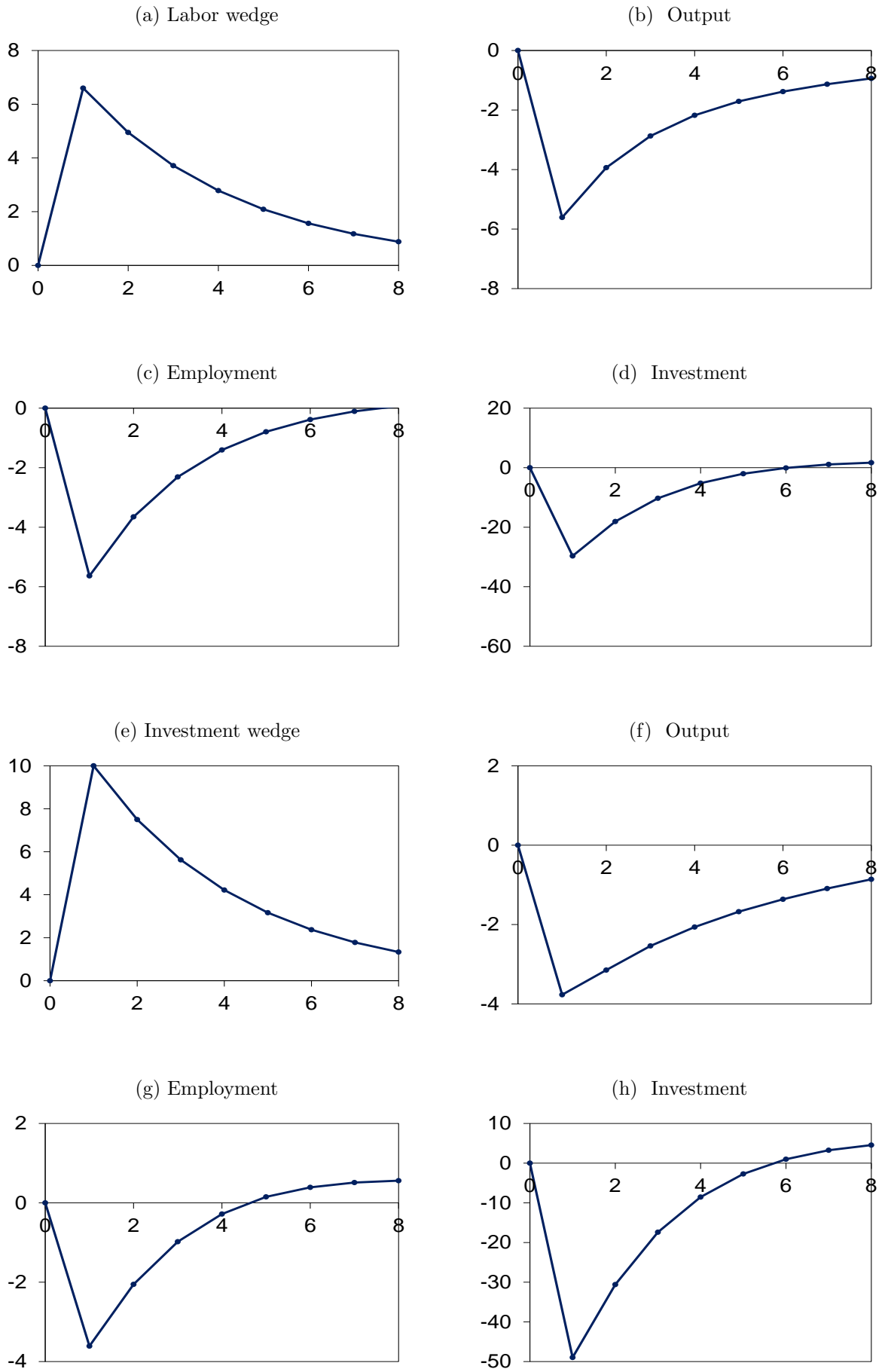


Figure 14: Aggregates: borrowing wedge and fixed operating cost (unit = p.p. deviation)

