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Federico Huneeus
Miguel Del Valle

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Federico Huneeus
Duke University

Miguel Del Valle
Universidad de Chile

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Abstract

To understand the geographic and sectoral impact of productivity, we build the first input-output matrix disaggregated at the geographic level within a country using unique administrative data harmonized to match national accounts. We use this data from Chile to calibrate a state-of-the-art general equilibrium quantitative trade model with production networks, labor mobility, firm selection, international and domestic trade, congestion of fixed factors, and knowledge diffusion. We consider two applications. First, we study the aggregate effects of local productivity shocks. We show that location-sector interactions are crucial: locations and sectors separately account for less than half of the dispersion in GDP elasticities from location-sector-specific productivity shocks. Geography-specific input-output linkages explain 16% of the dispersion, due to the role played by small and influential markets. Second, we analyze the exit of a large steel plant. We show that geographically disaggregated production linkages substantially increase the propagation of the plant exit.

Keywords: Productivity, spatial economics, geographic distribution, input-output linkages, trade, labor mobility.

JEL classification: F11, F17, R13, R15, D57, E01, R12.

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1 Introduction

Understanding how local productivity shocks propagate through the economy is ubiquitous in policy debates. Some of these shocks arise from policy interventions, such as place-based initiatives aimed at boosting productivity in lagging regions or industrial policies targeting specific sectors. Other shocks are due to adverse events, including natural disasters, social conflicts, or the exit of large firms; or positive events such as increases in global demand for commodities that are geographically concentrated. Each of these episodes initially impacts a specific market defined by its sector and location. More importantly, these shocks can then propagate indirectly across locations and sectors through multiple channels, such as upstream and downstream propagation through supply chains, potentially generating uneven impacts, with winners and losers across locations and sectors. Disaggregated quantitative analysis has been constrained by the lack of geographically disaggregated economic accounts, particularly input-output tables, which are crucial for understanding the propagation of such shocks.

We construct novel geographically and sectorally disaggregated economic accounts that characterize economic activity across location and sector pairs in Chile, including geographic and sectoral input-output matrices, using unique administrative data and survey sources. The resulting disaggregated accounts are disciplined to match national accounts and aggregate identities. We combine this new dataset with a state-of-the-art general equilibrium quantitative trade model (Caliendo et al., 2018) to study how local productivity changes are transmitted across all locations and sectors of the economy. These shocks operate through multiple mechanisms, including backward and forward input-output linkages, domestic and international trade, labor migration, congestion in fixed factors such as land and infrastructure, selection and extensive-margin adjustments of firms, and knowledge diffusion.

The disaggregated economic accounts link national accounts aggregates to detailed information disaggregated along both sectoral and location dimensions. Specifically, we build on national accounts built by the Central Bank of Chile (CBC), which provides estimates of regional gross domestic product (GDP), and extend them by geographically disaggregating the input-output matrices. The resulting dataset is internally consistent: aggregating sectoral and spatial flows exactly reproduces national totals and satisfies all accounting identities. Such disaggregated measures of economic activity and interconnections through supply chains remain scarce, both for Chile and the rest of the world.

To implement this measurement, we combine a wide range of administrative data covering the universe of formal firms in Chile between 2018 and 2023. First, we use standard administrative tax records from the Chilean Internal Revenue Service to construct firm-level measures. This data provides direct measures of total, domestic, and international sales; total, domestic, and imported material expenditures; value added; employment; wages; capital stocks and investment. We start from national accounts regional GDP estimates and disaggregate those at the location-sector level using shares from firms' tax records. Second, and more novel, we exploit firm-to-firm value-added tax records to directly measure production linkages across locations and sectors. Existing work typically infers these linkages from surveys or national input-output tables and spatially allocates them using indirect methods, such as gravity equations or proportionality assumptions. In contrast, the Chilean firm-to-firm data report the origin and destination of each transaction for the universe of formal firms, allowing us to observe trade flows across locations and sectors rather than imputing them. This feature enables us to recover the production network at a fine geographic and sectoral resolution without imposing assumptions on the data.

Taken together, this dataset delivers a comprehensive coverage of economic activity across the Chilean territory and provides a direct, measurement-based characterization of geographically and sectorally disaggregated input-output linkages.

Using the model calibrated with the disaggregated economic accounts, we implement two applications to understand how location-sector-specific productivity shocks propagate throughout the economy. The first application studies the aggregate effects of local productivity shocks across all location-sector pairs in the economy. The second application analyzes the exit of a large steel plant in the south of Chile, quantifies its local and aggregate effects, and identifies winners and losers across locations and sectors.

The first application delivers five results. First, when shocks are allowed to vary only by location, or only by sector, we recover the broad qualitative patterns emphasized in the quantitative trade literature. We document the elasticity of GDP to productivity shocks, which we refer to GDP elasticities. We scale these elasticities by the share of GDP that is exposed to the shock, that is, the Domar weight of the shock. We scale by the Domar weight in order to control for mechanical heterogeneity driven by the magnitude of the shock. Regional GDP elasticities are largest in economically central and diversified areas, while sectoral GDP elasticities are strongest in business and social services and comparatively muted in mining. Second, once shocks are disaggregated at the location-sector level, what matters is the dispersion rather than the average effects. Average GDP elasticities remain close to those obtained under one-dimensional shocks, but heterogeneity rises sharply. Furthermore, geography accounts for 12% of the dispersion in aggregate GDP elasticities, sectors account for 16%, and 73% is accounted for by the interaction between the two. Where a shock occurs and which sector is hit are therefore not sufficient statistics for understanding the impact of location-sector-specific shocks. What matters is the interaction between sectors and locations. This underscores the relevance of including geography when measuring economic accounts and the propagation of productivity shocks.

Third, standard sufficient statistics (Baqaee & Farhi, 2020) remain informative but incomplete. If Domar weights were sufficient statistics, all GDP elasticities should be equal to one. The fact that there is significant dispersion means that those sufficient statistics are not sufficient in this framework. As shown by Baqaee and Farhi (2019, 2020, 2024), the difference between these sufficient statistics and the model can be driven by both non-linearities and inefficiencies of the model.

Fourth, trade and input-output linkages shape propagation through distinct channels. Shutting down production networks reduces the dispersion of impacts by 44%. Furthermore, without input-output linkages, sectors become more important. Shutting down trade has an even larger effect, accounting for about 54% of the dispersion of GDP elasticities. Contrary to input-output linkages, without trade, dispersion of GDP elasticities becomes much more geographic and the location-sector interaction falls sharply. These channels are therefore complementary: trade redistributes shocks across sector, while production networks redistribute shocks across space.

Finally, geographically disaggregated input-output tables matter mainly for heterogeneity among smaller and influential markets rather than for average aggregate effects. Replacing the geography-specific input-output matrix with the standard national input-output table leaves average GDP elasticities largely unchanged, but removes about 16% of the overall dispersion of GDP elasticities across location-sectors. This reduction is driven by small markets that have high GDP elasticities in the benchmark model. Furthermore, eliminating the geography aspect of the input-output matrix increases the relevance of sectors in GDP elasticity dispersion by about 19%, highlighting the relevance of appropriately measuring the spatial dimension of input-output tables.

The second application illustrates these insights in a concrete and policy-relevant setting: the exit of a large plant in a location that relies substantially on the economic activity of that plant. Although the plant that exited, *Huachipato*, accounts for only a negligible share of national GDP, its exit generates aggregate losses several times larger than its direct contribution. About three-quarters of the national GDP and employment losses occur outside the location-sector of the plant, showing that localized firm exits can have substantial national consequences once labor mobility, trade and production networks are accounted for.

This case study also highlights the value of geographically disaggregated input-output data. Replacing geography-specific production linkages with national input-output coefficients lowers the GDP elasticity of the shock from 3.41 to 1.51 This comparison shows that spatial detail in production linkages matters quantitatively for the aggregate strength of propagation. This case study reinforces our central message: localized productivity shocks propagate through the economy not because of geography or sector alone, but because of how the two interact within the economy's spatial and production network structure.

Related Literature. We relate to three strands of the literature. First, we contribute to the literature on the construction of disaggregated economic accounts. A prominent set of contributions constructs cross-region and international trade matrices, often by combining shipment data, international trade statistics, input-output tables, and regional labor or output shares as in Leontief and Strout (1963), Caliendo and Parro (2015), and Caliendo et al. (2018, 2019). Because disaggregated and domestic trade data are typically unavailable, these approaches rely on proportionality assumptions or gravity-based allocations to spatially disaggregate producer flows. For instance, Rodríguez-Clare et al. (2025) recover industry-level trade flows across U.S. region pairs using a gravity-based framework, while Andersen et al. (2026) apply a similar approach to Denmark for firm's transactions. On the other hand, Caliendo et al. (2018) measures the geographical aspect of input-output linkages through proportionality assumptions. While these methods have substantially advanced the measurement of spatial trade linkages, they recover cross-location flows indirectly and do not identify the full geography-sector input-output structure of the economy. A closely related strand uses administrative tax records to measure income flows and production linkages at a high level of granularity as in Card et al. (2013), Huneeus (2018), Dhyne et al. (2021), Adão et al. (2022), Bernard et al. (2022), and Atkin et al. (2025). In particular, Atkin et al. (2025) exploits Chilean administrative data to measure detailed bilateral transactions between individuals, firms, and the government, highlighting how distortions influence real income across individuals, but they do not focus on the spatial aspect of the production network let alone its role on propagating productivity shocks.

Our paper contributes to this literature by constructing novel geographically and sectorally disaggregated accounts for Chile that jointly characterize economic activity and production linkages at the location-sector level for an entire country using detailed administrative data and without imposing additional assumptions or structure on the data. Furthermore, all bilateral flows are disciplined to exactly match Chile's national accounts and input-output tables, ensuring national aggregation, internal consistency, and compliance with accounting identities.

Second, we contribute to the literature of propagation through production networks that studies how local productivity shocks impact aggregates and propagate through the economy. A classical starting point is the insight that, first-order aggregate responses can be summarized by Domar weights, as formalized by Hulten (1978). Building on this foundation, a large literature studies shock propagation through input-output linkages and network structure, from early multisector contributions such as Long and Plosser (1983) and Horvath (2000) to modern network-based analyses in Acemoglu et al. (2012, 2016), and nonlinear general-equilibrium frameworks in Baqaee and Farhi (2019, 2020, 2024). This strand of the literature typically abstracts from studying the role of geography. We contribute to this literature by showing the role of geography for the propagation of shocks. In particular we bring to this literature the frameworks from the quantitative spatial literature (Caliendo et al., 2018) which include trade frictions related to geography and we measure the role of geography by building disaggregated economic accounts across space in Chile.

Finally, our paper contributes to the literature of quantitative spatial economics. This literature has developed general equilibrium frameworks to quantify how productivity and trade shocks impact economic activity across locations and affect aggregate welfare through trade costs and geography, starting from the Ricardian structure in Eaton and Kortum (2002) and its counterfactual implementation in Dekle et al. (2008). This approach is extended to multi-sector models with input-output linkages in Caliendo and Parro (2015) and to fully specified spatial equilibria in Caliendo et al. (2018) and Caliendo et al. (2019).[1] Our contribution to this literature is to provide a unified measurement and

[1] Related spatial frameworks, such as Allen and Arkolakis (2014) and Allen et al. (2020), discipline trade costs and the spatial distribution of activity but abstract from explicit production networks.

quantitative framework to study the geographic and sectoral propagation of productivity shocks through empirically identified production networks embedded in a spatial general equilibrium. The closest paper to our approach is Caliendo et al. (2018), which constructs spatial trade linkages by combining data on cross-region shipments, input-output trade between producers, and regional labor or output shares. We depart from this approach by directly measuring bilateral geography-sector input flows using administrative firm-to-firm transaction data, thereby identifying the full origin-destination structure of production without relying on proportional or gravity-based allocations. This richer measurement is especially important for service sectors, whose geographic flows have been largely absent from existing spatial analyses due to their non-physical nature and the lack of survey data. We apply our framework to the Chilean economy, whose pronounced geographic heterogeneity provides a natural setting to study how productivity shocks propagate across sectors and locations.

The remainder of the paper is organized as follows. Section 2 describes the construction of the disaggregated economic accounts, combining state-of-the-art administrative records and survey sources and making them consistent with national accounts. Section 3 presents descriptive evidence on the economic composition of Chile and the geographic structure of the production network. Section 4 introduces the quantitative trade model and defines the sufficient statistics used to measure the aggregate effects of location-sector shocks and the geographic, sectoral, and geography-sector exposures that govern their propagation. Section 5 presents the model calibration. Section 6 reports the results from the quantitative exercises and their applications. Section 7 concludes.

2 Disaggregated Economic Accounts

This section describes the construction of the geographically and sectorally disaggregated economic accounts for Chile.[2] Our analysis uses administrative microdata covering the universe of formal firms in Chile from 2018 to 2023. We link five tax-based data sources from Chile's Internal Revenue Service (SII) using unique and common tax identifiers. The resulting accounts record bilateral flows between location-sector pairs, satisfy accounting identities at the disaggregated and aggregate levels, and aggregate exactly to official national accounts totals. Additional details are provided in Appendix A.[3]

[2] Publicly available at https://dataterritorios.corfo.cl/disaggregated-economic-accounts/

2.1 Data Sources

A key ingredient of disaggregating economic accounts is to measure a disaggregated input-output matrix. Our main source for this is the domestic firm-to-firm transactions in Chile, which cover the universe of formal transactions between firms. All firms report electronic invoices for every domestic transaction, regardless of size or value. Each invoice identifies the supplier and buyer, the transaction date, the nominal value, and the municipalities of both parties, which lets us locate both sides of each transaction. This dataset is unusual by international standards: most countries observe firm-to-firm linkages only partially and must reconstruct them from surveys, input-output tables, shipment records, or proprietary samples. Even if the coverage is good in other countries, geographic information of transactions is also limited. By contrast, the Chilean invoicing system identifies the parties involved and locations for the universe of domestic and formal transactions, allowing us to build a microdata-based geography-sector input-output matrix.

We link the transaction data with tax balance-sheet and labor records. Form 29 provides firm-month level sales, material expenditures, value added, investment, exports, imports, industry classification, and headquarters municipality. Form 22 provides fixed-asset declarations, which we use to initialize firm-level capital stocks. Forms 1879 and 1887 report worker payments at the employee-employer-year level, including wages, bonuses, overtime, and other labor income, allowing us to construct firm-level labor compensation and employment. We measure capital stock using the perpetual inventory method. For this, we combine the fixed asset declaration from form 22 with the investment declaration from form 29. Given that investment from form 29 is mostly related to machinery and equipment, we use LA-KLEMS to rescale investment flows.

We discipline the construction of the disaggregated economic accounts using official values from the Central Bank of Chile (CBC), the National Statistics Institute (INE), and the OECD. The Central Bank provides regional GDP, capital stock, and sectoral input-output tables, which we use to anchor value added, capital, and intermediate input flows. The INE provides employment measured as occupied workers, so it includes both formal

[3] To secure the privacy of workers and firms, the Chilean Internal Revenue Service mandates that the development, extraction, and publication of the results should not allow the identification, directly or indirectly, of natural or legal persons. We implemented all analysis and neither involved nor compromised the SII in doing so. The information contained in the databases of the SII is of a tax nature originating in self-declarations of taxpayers.

and informal employment. The OECD Inter-Country Input-Output (ICIO) tables provide consistent international input-output benchmarks, which we use to discipline Chile's trade flows with the rest of the world.

2.2 Data Cleaning and Consistency with National Accounts

We apply filters to construct the final firm-level sample. A firm is classified as active if it reports positive sales, material expenditures, and labor compensation, and employs at least two workers. For social service organizations, we set sales equal to the maximum of reported sales and observed input costs to align with the national-account treatment of non-market activities. We also winsorize firm-level and transaction-level variables by sector at 1% and 0.1% respectively to limit the influence of outliers. This yields an annual firm-level panel covering 2018 to 2023, with 124 million supplier-buyer-year transactions and 59 million unique firm pairs. The panel includes 2.1 million annual supplier observations and 1.8 million buyer observations, corresponding to 593 thousand unique suppliers, 555 thousand unique buyers, and a total of 677 thousand distinct firms. From this panel, we compute firm-level averages over 2018-2023 for non-zero observations to smooth short-run fluctuations.

We use this panel to allocate official GDP and employment across 56 provinces and 46 economic sectors using a top-down procedure based on microdata shares. Because the CBC GDP series is reported at the 13-sector-by-16-region level and the INE employment series at the 18-sector-by-16-region level, we jointly disaggregate each region-sector cell into our 56-province-by-46-sector partition using firm-level value added and employment shares. Because tax records identify firms' headquarters rather than the exact location of production, we anchor the construction to the CBC regional GDP series, which mitigates headquarters bias across regions by aligning measured activity with production location rather than legal residence. Capital is disciplined separately across province-sector cells using official sectoral targets and province-sector capital shares.

Given the resulting province-sector value added, we reconstruct the remaining province-sector variables from the firm-level administrative data. Using the balance sheet and labor records, we recover sales, material expenditures, labor compensation, and other accounting flows by applying the firm-level relationships between value added and these variables observed in the microdata. We then rescale these reconstructed variables to match the sectoral totals reported in Chile's national input-output tables, ensuring consistency with national accounting identities and avoiding negative values.

We adjust the geographic-sectoral input-output matrix using the MR-GRAS methodology of Temursho et al. (2021), imposing consistency with disaggregated sales and material expenditures as well as with the sectoral aggregation of the national input-output table.[4] This procedure guarantees that the disaggregated accounts are internally coherent and aligned with official national aggregates.

We incorporate international trade using the OECD ICIO tables to add a rest-of-the-world region. We rely on the 2020 ICIO release, which harmonizes input-output data for 76 countries, including Chile. We aggregate all countries other than Chile into a single foreign block and use the ICIO structure to allocate Chile's export and import totals across destination and origin sectors. Specifically, we take province-sector exports and imports from the reconciled Chilean accounts as given and distribute them across foreign sectors using ICIO destination and origin shares, which are assumed to be common across Chilean provinces. This preserves consistency with both Chile's national accounts and the ICIO structure while avoiding additional geographic assumptions.

The resulting data characterize economic activity across location and sector pairs in Chile, including geographic and sectoral input-output matrices, and are disciplined to match national accounts and aggregate identities. This construction yields 2,576 potential province-sector markets. Of these, 67 percent exhibit strictly positive economic activity. At this level of aggregation, the data define 6,635,776 potential province-sector-to-province-sector transaction pairs. Approximately 29 percent of these pairs exhibit strictly positive trade flows.

3 Descriptive Statistics

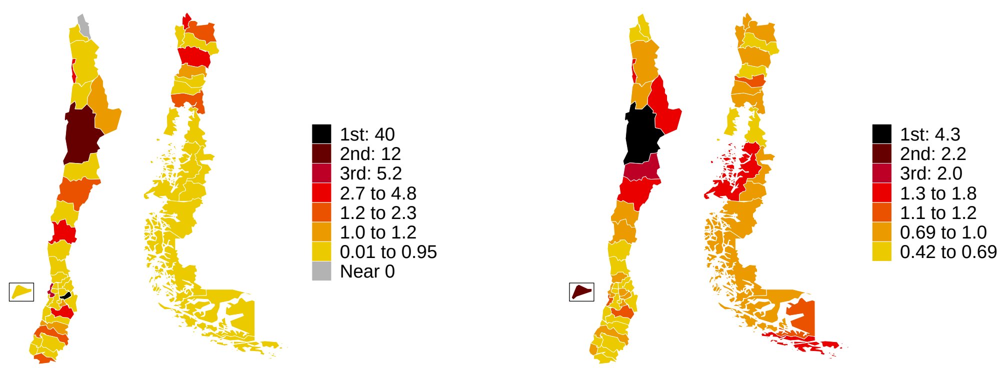
Chile provides a particularly informative setting for studying the spatial propagation of economic activity. The country stretches nearly 4,300 kilometers from north to south while remaining, on average, less than 180 kilometers wide, creating sharp variation in climate, natural resources, transport costs, and economic activity within a single national territory. Mining is concentrated in the north, manufacturing and services in the center, agriculture and forestry in the south, and fishing in the austral regions. Together with Chile's Pacific orientation and distance from major world markets, this geography makes transport

[4] Imposing the first two constraints corresponds to the standard RAS method, widely used for regionalization and temporal updating of input-output matrices. MR-GRAS extends this framework by adding an additional constraint, allowing us to reconcile a sparse multiregional matrix while imposing multiple accounting constraints simultaneously and preserving nonnegativity. This is particularly useful in our setting, as it allows us to match bilateral flows, location-sector totals, and sectoral flows at the same time.

and logistics central to production and trade, and it generates a natural environment for studying how shocks propagate across locations and sectors through geographic-sectoral production linkages. Descriptive evidence on sectoral concentration, province-level specialization, and sectoral differences in GDP and wages is reported in Appendix D.

As Figure 1 shows, economic activity is highly unevenly distributed across provinces. A small number of provinces account for a disproportionate share of national GDP, while many others contribute only marginally to aggregate output. The same figure also reveals substantial dispersion in GDP per worker: provinces in the northern mining belt and parts of the central corridor tend to lie above the national average, whereas many southern and more remote provinces remain below it. This spatial concentration is mirrored in Appendix Figure D.5, which shows that many sectors are concentrated in only a few provinces and that many provinces are specialized across a narrow set of sectors. Together, these patterns suggest that the relevant unit of analysis is not the province or the sector in isolation, but the location-sector pair, since concentration and specialization jointly shape exposure to shocks and the potential for spillovers through the production network.

Figure 1: The Spatial Distribution of Economic Activity

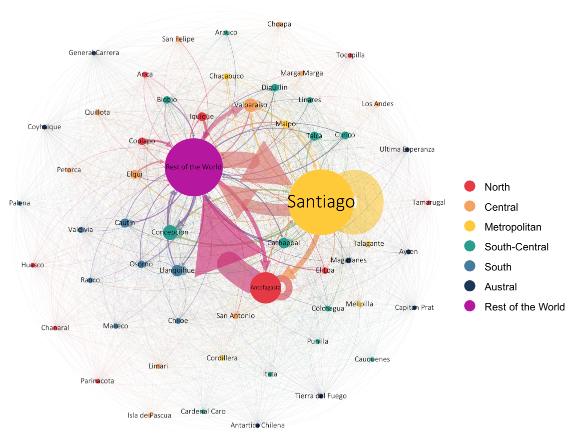


a) Share of National GDP (%) b) GDP per worker (relative to aggregate)

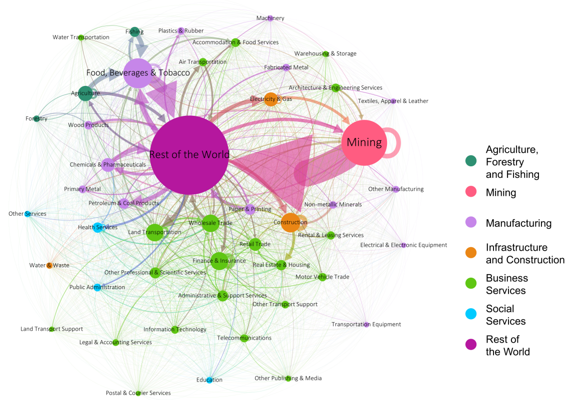
Notes: Panels report provincial shares of national GDP, in percent, and GDP per worker expressed relative to the national average. National annual GDP per worker in 2022 is USD 34,089.

This concentration and specialization are not only visible in the distribution of activity across provinces and sectors, but also in the structure of the production network. Panel A of Figure 2 shows that trade flows are highly centralized across locations, with Santiago emerging as the dominant domestic hub and Antofagasta, where mining activity is concentrated, as a second key node linking the northern macrozone to the rest of the economy and to foreign markets. Geographic distance matters, but only imperfectly: many provinces participate in the network through one or two intermediating hubs, while a small number of locations mediate a large share of domestic and international trade.

Figure 2: The Chilean Trade Flows and Production Network



a) Between Locations

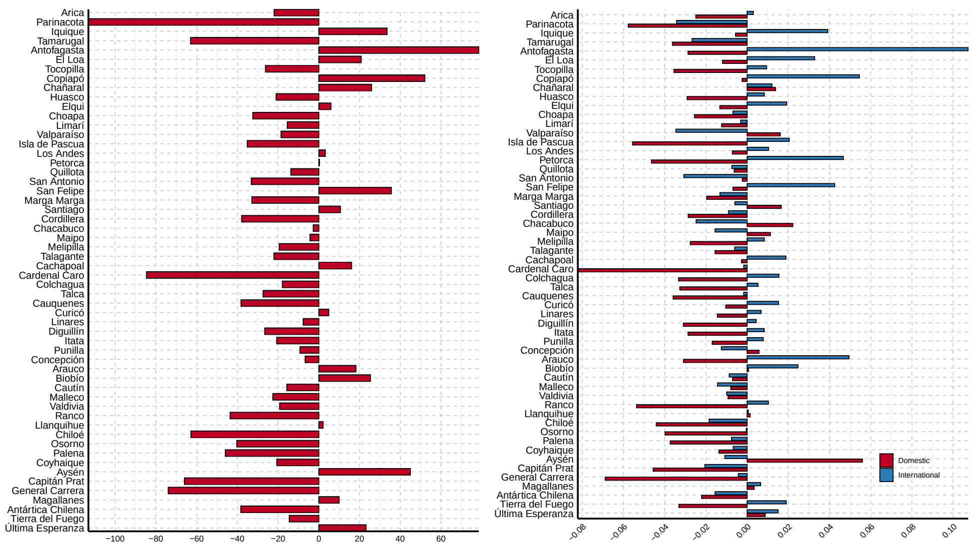


b) Between Sectors

Notes: Panel (a) reports the province-to-province network obtained by aggregating the geographic-sectoral input-output matrix over sectors. Panel (b) applies the same aggregation over provinces to obtain the sector-to-sector network. In both panels, the Rest of the World is kept as a separate node, node size reflects total inflows and outflows, and node positions are determined by the ForceAtlas2 algorithm.

Panel B of Figure 2 shows that sectoral centrality is similarly uneven, but in a different way. Mining is the most central sector in terms of total flows because of its strong links to the Rest of the World, yet it relies relatively little on domestic intermediate inputs. By contrast, manufacturing and several service sectors, especially business services, wholesale and retail trade, construction, and transportation, occupy central positions within the domestic network because they connect many other activities through dense input-output linkages. This asymmetry matters because it implies that sectors differ not only in size, but also in the way they transmit shocks through the economy: some are globally exposed but weakly embedded domestically, while others generate broader internal spillovers.

Figure 3: Provincial Trade Imbalances as a Share of Provincial GDP



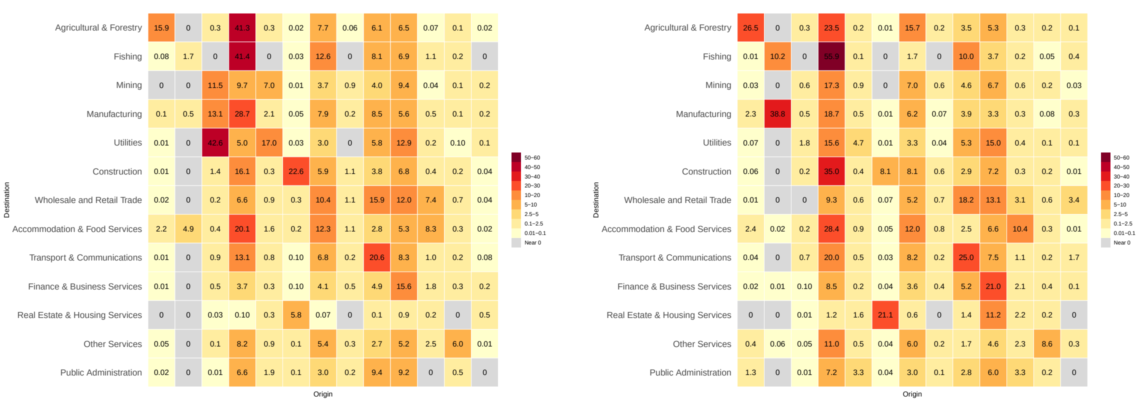
a) Net Trade Imbalance (Exports − Imports)   b) Domestic vs. International Decomposition

Notes: Trade imbalances are expressed as a share of provincial GDP (%) and decomposed into domestic and international components. Provinces are ordered from north to south.

This asymmetric network structure shows up clearly in provincial trade balances. Figures 3 and Appendix Figure D.6 show that trade imbalances are distributed unevenly across space: provinces in the north are systematically net exporters, while provinces in the south and austral regions are predominantly net importers, both relative to provincial GDP and in absolute terms. This pattern is consistent with the concentration of resource-based activities in the northern macrozone and indicates that external surpluses are generated in a limited set of locations rather than being broadly distributed across the territory. At the same time, the domestic component of trade balances follows a different logic: most provinces are net importers within the internal network, and Santiago emerges as the country's principal supplier to the rest of the nation. These figures indicate that Chile's trade geography is shaped by a small set of export-oriented provinces in the north and by Santiago, which functions as the principal domestic supplier and mediates a large share of internal flows.

Finally, to highlight the relevance of geographical input-output tables and trade frictions, we compare the input-output tables of Antofagasta, where mining activity is concentrated, with Puerto Montt, where fishing activity is concentrated. Figures 4 shows, for instance, that manufacturing in Antofagasta mostly sources from mining because it focuses on production related to mining products. On the other hand, manufacturing in Puerto Montt sources mostly from fishing because it produces food. Even though it is the same sector, where it is located changes which is the relevant input-output table and thus how shocks propagate.

Figure 4: Local Input-Output Tables: Illustrative Examples



a) Antofagasta b) Llanquihue

Notes: The figure reports, conditional on location, the share of each input-supplying sector in the total sales of each purchasing sector. For visualization purposes, the 46 model sectors are aggregated into 13 broad sectors. Rows do not sum to 100 because the denominator is total sales, not total intermediate input purchases; adding value added would close each row to 100.

Taken together, these facts point to a production economy in which geography and sectoral specialization jointly shape exposure to shocks and the propagation of shocks across space. Concentration is not only about where output is located, but also about how provinces and sectors are connected through input-output links. This motivates the quantitative analysis in the next section, where we use the disaggregated economic accounts to study how shocks travel through the Chilean production network and economy.

4 Model

Following Caliendo et al. (2018), we present a quantitative multi-regional, multi-sector model to study how local productivity shocks propagate across locations and sectors. Relative to Caliendo et al. (2018), we allow for the following: (1) international trade, and (2) knowledge spillovers as in Liu and Ma (2021).

The model combines trade in intermediate goods, input-output linkages across sectors, knowledge spillovers, labor mobility, and fixed local structures to generate both aggregate and distributional effects. Throughout, we distinguish between domestic locations, indexed by $i, u \in \mathcal{D}$, and foreign countries, indexed by $f \in \mathcal{F}$, with $\mathcal{N} = \mathcal{D} \cup \mathcal{F}$. When referring to the domestic economy, we restrict attention to locations in $\mathcal{D}$; when referring to the full economy, we include foreign countries through $\mathcal{N}$.

The economy is endowed with aggregate labor $\overline{L}$ and location-specific structures $\overline{H}_i$ for $i \in \mathcal{D}$. Labor is freely mobile across regions and sectors, while structures are fixed by location but can be used by all sectors within that location. In each location-sector pair, a continuum of firms draws idiosyncratic productivity from a Fréchet distribution and produces differentiated varieties. These varieties can be used as either final goods or intermediate inputs, and sectors are linked through input-output linkages. Delivering materials across locations entails iceberg trade costs, generating trade across space. Further details are provided in Appendix B.

4.1 Households

The representative household in region $i \in \mathcal{N}$ organizes its consumption bundle according to Cobb-Douglas preferences,

$$u_i = \prod_{k \in \mathcal{K}} (C_{i,k})^{\alpha_k}, \quad \text{where} \quad \sum_{k \in \mathcal{K}} \alpha_k = 1,$$

with expenditure shares α_k over final goods purchased at prices $P_{i,k}$ in all sectors $k \in \mathcal{K}$.

Household expenditure in region i satisfies $\sum_{k \in \mathcal{K}} P_{i,k} C_{i,k} = I_i$, where $P_{i,k}$ is the price of the final good in sector k and region i, and I_i is total household income. Income consists of labor earnings, local returns to structures, and the per-capita return from a global portfolio. Specifically, each region i is endowed with a stock of structures $\overline{H}_i$ that earns rental rate r_i; a share $(1 - \iota_i)$ is retained locally and distributed to residents, while the remaining share ι_i is pooled and redistributed uniformly across all residents. Accordingly,

$I_i = (1-\iota_i) r_i \frac{\overline{H}_i}{L_i} + w_i + \chi$, where $\chi = \sum_{n \in \mathcal{N}} \iota_n r_n \overline{H}_n / \sum_{n \in \mathcal{N}} L_n$ is the per-capita return from the pooled portfolio. The resulting regional imbalance is

$$\Upsilon_i \equiv \iota_i r_i \overline{H}_i - \chi L_i. \tag{1}$$

Households move freely across regions. In equilibrium, they are indifferent across locations, so $v_i = \frac{I_i}{P_i} = \mathcal{U}$, for all $i \in \mathcal{N}$, where $P_i = \prod_{k \in \mathcal{K}} (P_{i,k}/\alpha_k)^{\alpha_k}$ is the ideal price index in region i, and $\mathcal{U}$ is determined in equilibrium.

4.2 Intermediate Goods

In each region i and sector k, a continuum of firms indexed by ω draws an idiosyncratic productivity level $z_{i,k}(\omega)$ from a Fréchet distribution with shape parameter θ_k. Firms in the same location-sector pair (i,k) also share a deterministic productivity component $A_{i,k}$. Intermediate output is produced with labor, structures, and intermediate inputs according to

$$q_{i,k}(z_{i,k}) = z_{i,k} \Big(A_{i,k} \cdot h_{i,k}(z_{i,k})^{\beta_i} \cdot l_{i,k}(z_{i,k})^{1-\beta_i} \Big)^{\gamma_{i,k}} \prod_{h \in \mathcal{K}} M_{i,kh}(z_{i,k})^{\gamma_{i,kh}},$$

where $h_{i,k}(\cdot)$ and $l_{i,k}(\cdot)$ denote demands for structures and labor, respectively, and $M_{i,kh}(\cdot)$ denotes demand for intermediate inputs from sector h. We assume constant returns to scale, so that $\gamma_{i,k} + \sum_{h \in \mathcal{K}} \gamma_{i,kh} = 1$. This technology implies that the unit cost of an intermediate good with productivity $z_{i,k}$ can be written as $x_{i,k}/(z_{i,k} A_{i,k}^{\gamma_{i,k}})$, where $x_{i,k}$ captures the cost of the input bundle used in production:

$$x_{i,k} = B_{i,k} \Big(r_i^{\beta_i} w_i^{1-\beta_i} \Big)^{\gamma_{i,k}} \prod_{h \in \mathcal{K}} P_{i,h}^{\gamma_{i,kh}}. \tag{2}$$

Here, $B_{i,k} = \Big(\gamma_{i,k} (1-\beta_i)^{1-\beta_i} \beta_i^{\beta_i} \Big)^{-\gamma_{i,k}} \prod_{h \in \mathcal{K}} \gamma_{i,kh}^{-\gamma_{i,kh}}$ is a constant.

Deterministic productivity in each location-sector pair (i,k) reflects the effective stock of knowledge available to production. We assume that this stock combines locally accumulated knowledge with spillovers from other location-sector pairs. Formally, $A_{i,k} = \bar{A}_{i,k} \, \Psi_{i,k}^{\varphi}$, where $\bar{A}_{i,k}$ denotes location-sector productivity fundamentals, $\Psi_{i,k}$ is an index of external knowledge accessible to (i,k), and $\varphi \in [0,1)$ governs the elasticity of productivity with respect to spillovers. The knowledge index aggregates knowledge stocks across locations and sectors according to $\Psi_{i,k} = \prod_{u \in \mathcal{N},\, h \in \mathcal{K}} A_{u,h}^{\lambda_{kh} \delta_{iu}}$, where λ_{kh} captures sectoral relatedness and δ_{iu} governs the geographic reach of spillovers. In matrix form, let $\mathbf{W} \equiv$

$\boldsymbol{\Delta} \otimes \boldsymbol{\Lambda}$, with $\boldsymbol{\Delta} = [\delta_{iu}]$ and $\boldsymbol{\Lambda} = [\lambda_{kh}]$. Then $\log \mathbf{A} = \big(\mathbf{I} - \varphi\, \mathbf{W}\big)^{-1} \log \bar{\mathbf{A}}$, so knowledge in any location-sector pair affects productivity both directly and through indirect spillovers. Let $\mathbf{L} \equiv \big(\mathbf{I} - \varphi\, \mathbf{W}\big)^{-1}$, and denote by $\ell_{iu,kh}$ its $(i,k;u,h)$ element. Each $\ell_{iu,kh}$ captures the total contribution of knowledge accumulated in location-sector (u,h) to productivity in (i,k), including both direct and indirect effects. Accordingly, deterministic productivity can be written in levels as

$$A_{i,k} \;=\; \prod_{u\in\mathcal{N},\, h\in\mathcal{K}} \bar{A}_{u,h}^{\,\ell_{iu,kh}}, \tag{3}$$

making explicit that productivity in each location-sector pair emerges from the propagation of knowledge through the economy-wide network of technological complementarities.

In a competitive market for intermediate goods, firms set prices equal to unit costs. Intermediate goods in tradable sectors are costly to trade. Delivering one unit of an intermediate input in sector k to location i from origin u requires $\kappa_{iu,k} \geq 1$ units to be shipped from u, with $\kappa_{ii,k} = 1$. Therefore, the price paid in i for a variety with productivity $z_{i,k}$ is given by, $p_{i,k}(z_{i,k}) = \min_{u\in\mathcal{N}} \left\{\kappa_{iu,k} \cdot x_{u,k} / \big(z_{u,k} A_{u,k}^{\gamma_{u,k}}\big)\right\}$.

4.3 Final goods

Final goods in region i and sector k are produced by combining intermediate goods from the same sector. Let $Q_{i,k}$ denote final output in (i,k), and let $\tilde{q}_{i,k}(z_k)$ denote demand for an intermediate variety with productivity vector z_k. Final-good production is

$$Q_{i,k} = \left[\int \tilde{q}_{i,k}(z_k)^{1-\frac{1}{\eta_{i,k}}}\, d\Phi_k(z_k)\right]^{\eta_{i,k}/(\eta_{i,k}-1)},$$

where $\Phi_k(z_k) = \exp\big\{-\sum_{n\in\mathcal{N}} (z_{n,k})^{-\theta_k}\big\}$ denotes the joint cumulative distribution function of the productivity vector z_k, with marginal distributions given by $\Phi_{i,k}(z_{i,k}) = \exp\big\{-(z_{i,k})^{-\theta_k}\big\}$. Integration is over $\mathbb{R}_+^N$ with respect to the probability measure induced by Φ_k. There is free entry in the production of final goods, and competition implies zero profits.

4.4 Prices and trade shares

Given the distribution of idiosyncratic productivities, we follow Eaton and Kortum (2002) to solve for the distribution of prices. Having solved for the distribution of prices, the price of the final good k in region i is

$$P_{i,k} = \Gamma(\xi_{i,k})^{\frac{1}{1-\eta_{i,k}}} \left[\sum_{u \in \mathcal{N}} (\kappa_{iu,k} \cdot x_{u,k})^{-\theta_k} A_{u,k}^{\theta_k \gamma_{u,k}} \right]^{-\frac{1}{\theta_k}}, \tag{4}$$

where $\Gamma(\xi_{i,k})$ is the gamma function evaluated at $\xi_{i,k} = 1 + (1 - \eta_{i,k})/\theta_k$.

Let $\pi_{iu,k}$ denote the share of total expenditure in region i on intermediate goods of sector k purchased from region u. By properties of the Fréchet distribution,

$$\pi_{iu,k} = \frac{[\kappa_{iu,k} \cdot x_{u,k}]^{-\theta_k} A_{u,k}^{\theta_k \gamma_{u,k}}}{\sum_{m \in \mathcal{N}} [\kappa_{im,k} \cdot x_{m,k}]^{-\theta_k} A_{m,k}^{\theta_k \gamma_{m,k}}}. \tag{5}$$

4.5 Market clearing

Regional labor market clearing requires that total labor used across all sectors equals the available labor force in each region:

$$\sum_{k \in \mathcal{K}} L_{i,k} = \sum_{k \in \mathcal{K}} \int l_{i,k}(z)\, \phi_{i,k}(z)\, dz = L_i, \quad \text{for all } i \in \mathcal{N}, \tag{6}$$

where $L_{i,k}$ denotes employment in region-sector pair (i, k). At the national level, total labor must match total population: $\sum_{i \in \mathcal{N}} L_i = L$.

In regional equilibrium, land and structures are fully allocated across sectors:

$$\sum_{k \in \mathcal{K}} H_{i,k} = \sum_{k \in \mathcal{K}} \int_0^\infty h_{i,k}(z)\, \phi_{i,k}(z)\, dz = \overline{H}_i, \quad \text{for all } i \in \mathcal{N}, \tag{7}$$

where $H_{i,k}$ denotes land and structure usage in (i, k).

Final goods produced in each region-sector are either consumed by households or used as intermediate inputs. Regional market clearing requires:

$$L_i C_{i,k} + \sum_{h \in \mathcal{K}} M_{i,hk} = L_i C_{i,k} + \sum_{h \in \mathcal{K}} \int M_{i,hk}(z)\, \phi_{i,h}(z)\, dz = Q_{i,k}, \quad \text{for all } i \in \mathcal{N},\ k \in \mathcal{K}, \tag{8}$$

where $M_{i,hk}$ is the use of materials for sector k in sector h at location i. Let $X_{i,k}$ denote total expenditure on sector-k final goods in location i (or total revenue). Then, regional market clearing in final goods implies that

$$X_{i,k} = \alpha_k L_i I_i + \sum_{h \in \mathcal{K},\, u \in \mathcal{N}} \gamma_{i,hk}\, \pi_{ui,h}\, X_{u,h}. \tag{9}$$

In equilibrium, each region's total expenditure on intermediate goods, together with its net payment Υ_i, equals its total revenue from supplying intermediate goods to all regions, including itself:

$$\sum_{k \in \mathcal{K},\, u \in \mathcal{N}} \pi_{iu,k} X_{i,k} + \Upsilon_i \;=\; \sum_{k \in \mathcal{K},\, u \in \mathcal{N}} \pi_{ui,k} X_{u,k}. \tag{10}$$

Trade is generally not balanced at the regional level. Equation (10) is a regional accounting identity: a region's net imports are financed by its net factor-income position, Υ_i. Aggregate trade balance holds once regions are added up, since $\sum_{i \in \mathcal{N}} \Upsilon_i = 0$ under the model's ownership structure. As a result, some regions exhibit persistent surpluses and others deficits depending on their ownership of land and structures in other locations. This structure also implies that productivity changes can affect both the magnitude and the direction of regional trade imbalances.

4.6 Labor Mobility

Households are perfectly mobile across locations, so equilibrium requires equal indirect utility across regions. Labor therefore reallocates in response to local real returns, which depend on wages, consumption prices, net payments, and fixed land endowments. The minimization of production costs implies the local factor-price index $\varpi_i \equiv (w_i/(1-\beta_i))^{1-\beta_i} (r_i/\beta_i)^{\beta_i}$. Free mobility then delivers the labor allocation

$$L_i \;=\; \frac{\left(\dfrac{\varpi_i}{\mathcal{U} P_i + \upsilon_i}\right)^{\frac{1}{\beta_i}} \overline{H}_i}{\sum_{u \in \mathcal{N}} \left(\dfrac{\varpi_u}{\mathcal{U} P_u + \upsilon_u}\right)^{\frac{1}{\beta_u}} \overline{H}_u} \overline{L}, \tag{11}$$

where $P_i \equiv \prod_{k \in \mathcal{K}} (P_{i,k}/\alpha_k)^{\alpha_k}$ is the price index, $\upsilon_i \equiv \Upsilon_i / L_i$ denotes per-capita trade imbalances, and $\overline{H}_i$ is the fixed local land endowment. This condition shows labor flows toward locations with higher real returns, while fixed land endowments act as congestion.

4.7 Competitive equilibrium

We define a competitive equilibrium as a set of prices, quantities, and allocations that simultaneously satisfy individual optimality, aggregate consistency, and market-clearing conditions, as described below.

Definition 1. Competitive Equilibrium in Levels. *Given factor endowments* $\{\overline{L}, \overline{H}_i\}$, *region-sector productivities* $\{\bar{A}_{i,k}\}$, *and iceberg trade costs* $\{\kappa_{iu,k}\}$, *a* competitive equilibrium *is characterized by a common level of utility* $\mathcal{U}$*; an allocation of population across regions* $\{L_i\}$ *and across sectors within each region* $\{L_{i,k}\}$*; land allocations* $\{H_{i,k}\}$*; sectoral expenditure, per capita consumption, and intermediate input use* $\{X_{i,k}, C_{i,k}, M_{i,kh}\}$*; final-goods and factor prices* $\{P_{i,k}, r_i, w_i\}$*; regional trade imbalances* $\{\Upsilon_i\}$*; and bilateral expenditure shares on intermediate inputs* $\{\pi_{iu,k}\}$*, and deterministic productivity levels* $\{A_{i,k}\}$*, such that regional trade imbalances satisfy Equation* (1)*, intermediate-goods producers minimize unit costs as in Equation* (2)*, deterministic productivity is consistent with Equation* (3)*, final-goods prices and trade shares are given by Equations* (4) *and* (5)*, market clearing holds as in Equations* (6)*,* (7)*,* (8)*, and* (9)*, aggregate trade balance holds as in Equation* (10)*, and labor mobility is pinned down by Equation* (11)*.*

The equilibrium is solved in relative changes using the *hat-algebra* approach of Dekle et al. (2008), Costinot and Rodríguez-Clare (2014), and Caliendo and Parro (2015), which avoids the need to recover unobserved parameters in levels. This method exploits the fact that many equilibrium conditions are multiplicative, so unobserved fundamentals cancel when the model is written as a ratio between a counterfactual and the baseline equilibrium. Let x' denote the counterfactual value of x and define the relative change of any $x > 0$ as $\hat{x} = x'/x$, with $\hat{x} = 0$ when $x = 0$. Appendix B describes the numerical procedure used to solve the equilibrium in this form.

4.8 Outcomes

We now describe the objects that map productivity shocks into economic outcomes. The first object is deterministic productivity, which captures how shocks propagate across locations and sectors through the knowledge network. We then characterize measured TFP, real GDP, and welfare, each of which incorporates additional mechanisms such as trade-driven selection, labor reallocation, relative-price changes, and interregional income flows.

A key object is deterministic productivity, $\hat{A}_{i,k}$, which transmits shocks through the knowledge network. Using Equation (3), changes in productivity can be written as

$$\log \hat{A}_{i,k} = \sum_{u \in \mathcal{N},\, h \in \mathcal{K}} \ell_{iu,kh} \log \hat{\bar{A}}_{u,h}. \tag{12}$$

This expression shows that a shock in one location-sector pair affects not only local productivity, but also productivity elsewhere through direct and indirect spillovers. In the absence of spillovers, productivity changes remain local.

4.8.1 Measured Total Factor Productivity (TFP)

Measured TFP captures how productivity shocks show up in observed sectoral productivity once trade and selection are taken into account. It is constructed from gross output, deflated by the sectoral price index, and net of observed factor and intermediate-input usage, so it reflects both technological change and the reallocation of varieties induced by trade. The proportional change in measured TFP following a productivity shock is therefore given by $\log \widehat{\text{TFP}}_{i,k} = \log \hat{x}_{i,k} - \log \hat{P}_{i,k}$. Using Equation (5), this can be written as

$$\log \widehat{\text{TFP}}_{i,k} = \gamma_{i,k} \log \hat{A}_{i,k} - \frac{1}{\theta_k} \log \hat{\pi}_{ii,k}. \tag{13}$$

This expression decomposes measured TFP into two mechanisms. The first term captures the direct effect of a productivity shock on value added in location-sector (i, k): the shock raises the productivity of the local production process, but not the productivity of the materials purchased from other sectors, so the effect is attenuated by the value-added share. The second term captures selection in the set of active varieties: as productivity rises, some firms enter or expand while others are displaced, and measured TFP responds through the resulting change in the domestic expenditure share. In the absence of intermediate inputs, trade, and spillovers, measured TFP coincides with fundamental productivity.

4.8.2 Real Gross Domestic Product (GDP)

Real GDP captures how productivity shocks translate into local economic activity once labor reallocation and relative-price changes are taken into account. Using equilibrium factor demands and market-clearing conditions, changes in real GDP satisfy

$$\log \widehat{GDP}_{i,k} = \log \hat{\text{TFP}}_{i,k} + \log \hat{L}_{i,k} + \log \left(\frac{\hat{w}_i}{\hat{x}_{i,k}} \right). \tag{14}$$

This decomposition isolates three channels. The first is measured TFP, which captures the direct productivity and selection effects described above. The second is labor reallocation: when a location-sector becomes more productive, mobile labor shifts toward it, so local GDP rises through a larger employment share. The third is the relative-price channel, which reflects changes in the wage relative to the cost of the full input bundle. This term captures how local factor prices and intermediate-input prices move with the shock, and it is the channel through which fixed structures generate congestion effects. When productivity attracts labor into locations with limited structures, the price of the fixed factor rises and partly offsets the gain in GDP.

In the absence of intermediate inputs, the relative-price term reduces to the standard trade-off between wages and local congestion. With intermediate inputs, the same logic applies to the full input bundle, so GDP effects may be larger or smaller than productivity effects depending on the strength of migration responses and the spatial distribution of fixed factors $\overline{H}_i$. In particular, if productivity induces migration toward locations abundant in fixed structures, the relative-price term tends to reinforce GDP gains. We aggregate GDP using value-added shares at constant prices.

4.8.3 Welfare

We conclude with welfare implications of changes in productivity. Using Equation (11) and equilibrium factor demands, welfare changes measured in consumption-equivalent units are given by $\hat{\mathcal{U}} = \hat{I}_i/\hat{P}_i$. Combining the definition of P_i yields

$$\log \hat{\mathcal{U}} = \sum_{k \in \mathcal{K}} \alpha_k \left(\log \hat{\text{TFP}}_{i,k} + \log \left(\nu_i \frac{\hat{w}_i}{\hat{x}_{i,k}} + (1-\nu_i) \frac{\hat{\chi}}{\hat{x}_{i,k}} \right) \right), \tag{15}$$

where $\nu_i = \frac{(1-\beta_i \iota_i) w_i}{(1-\beta_i \iota_i) w_i + (1-\beta_i)\chi}$. Thus, welfare corresponds to a sector-weighted average of real GDP per capita. If $\iota_i = 0$ for all i, then $\chi = 0$ and $\nu_i = 1$, which yields

$$\log \hat{\mathcal{U}} = \sum_{k \in \mathcal{K}} \alpha_k \left(\log \hat{\text{TFP}}_{i,k} + \log \frac{\hat{w}_i}{\hat{x}_{i,k}} \right).$$

Productivity affects welfare through three channels: (i) changes in measured productivity; (ii) changes in the relative cost of labor versus input bundles; and (iii) changes in returns to global portfolios, weighted by $(1-\nu_i)$.

5 Calibration

We calibrate the model using the disaggregated economic accounts described in Section 2, together with sector-level patent citation data. Table 1 summarizes the baseline data objects and structural parameters used in the calibration, together with their empirical sources and targets. The table distinguishes between objects directly observed in the data, objects constructed from the data, parameters calibrated internally to match model moments, and parameters imported from the literature or imposed as benchmark assumptions. Our goal is to recover the baseline objects $\{I_i, L_i, \Upsilon_i, \pi_{iu,k}\}$ and the structural parameters $\{\theta_k, \alpha_k, \beta_i, \iota_i, \gamma_{i,k}, \gamma_{i,kh}, \lambda_{kh}, \delta_{iu}, \varphi\}$.[5] Further details on how each model object is mapped into the data, together with visual evidence on the calibrated objects and parameter distributions, are provided in Appendix C.

Table 1: Calibrated Baseline Data and Structural Parameters

	Value	Description	Type	Source / Target
Baseline Data				
I_i	Figure C.1	Income per worker	Observed	Computed as $I_i = (VA_i - \Upsilon_i)/L_i$
L_i	Figure C.2	Employment share	Observed	Employment records
Υ_i	Figure C.3	Trade imbalances	Constructed	Provincial trade balances from location-sector accounts
$\pi_{iu,k}$	Figure C.4	Bilateral trade shares	Constructed	Observed firm-to-firm flows with model-based final-demand allocation
Structural Parameters				
α_k	Figure C.5	Final consumption share	Observed	Sectoral final demand measured as sales net of intermediate sales
$1-\beta_i$	Figure C.6	Labor share	Observed	Observed labor income over value added
$\gamma_{i,k}$	Figure C.7	Value-added share	Observed	Observed value added over total sales
$\gamma_{i,kh}$	Figure 4	Local input-output linkages	Observed	Observed intermediate input transactions by location-sector
ι_i	Figure C.10	Global portfolio contribution	Calibrated	Constrained least squares to match provincial trade balances
θ_k	Figure C.11	Productivity dispersion	Literature	Caliendo and Parro (2015) and Caliendo et al. (2018)
λ_{kh}	Figure C.12	Knowledge network	Constructed	Sectoral patent citation shares from LaBelle et al. (2023)
δ_{iu}	$\mathbb{1}\{i = u\}$	Geographic knowledge diffusion	Assumption	Within-location diffusion
φ	0.07	Knowledge-spillover elasticity	Literature	Basso et al. (2025)

Notes: The table summarizes the baseline data targets and structural parameters used to calibrate the model. Entries in the Value column either report the calibrated scalar parameter or point to the figure that displays its empirical distribution across locations or sectors. Source/Target indicates whether the object is directly measured in the data, taken from the literature, or estimated within the calibration.

Per capita income $\{I_i\}$ is constructed from value added, trade balances, and employment as $I_i = (VA_i - \Upsilon_i)/L_i$. This definition allocates to households the portion of value added that remains after net trade payments, and it is consistent with the model's treatment of land and structures as the residual claim on value added. Employment $\{L_i\}$ is taken directly from the data and normalized across locations. Regional trade balances $\{\Upsilon_i\}$ are computed from the location-sector accounts as the difference between the total value

[5] The equilibrium requires all sectors to be defined in every location. We therefore assign location-sector pairs with zero measured value added a small positive activity level, impute employment using the national sectoral value-added-to-employment ratio, and assume no interregional trade. These pairs are treated as unobserved in the final results.

of sales originating in location i and the total value of purchases made by i from other location-sector pairs, including trade with the rest of the world. Finally, bilateral trade shares $\{\pi_{iu,k}\}$ combine observed firm-to-firm intermediate transactions with a model-based allocation of final demand: observed transactions are treated as intermediate flows, while the residual of total sales net of material sales is assigned to local final demand. Aggregating these flows yields $X_{iu,k}$, the value of sector-k goods and services from origin location u absorbed in destination location i, and we define $\pi_{iu,k} = X_{iu,k}/\sum_{u\in\mathcal{N}} X_{iu,k}$.

We next recover the structural parameters disciplined directly by the data. The final consumption shares $\{\alpha_k\}$ are obtained by subtracting material sales from total sales at the sector level and dividing by aggregate consumption. The labor share $\{1-\beta_i\}$ is computed as labor compensation over value added. The value-added share $\{\gamma_{i,k}\}$ is measured as value added over total sales for each province-sector pair. Local input-output coefficients $\{\gamma_{i,kh}\}$ are constructed from observed intermediate input transactions aggregated at the location-sector level, by assigning to each sector the share of sales corresponding to purchases of inputs from sector h.[6] Regional portfolio shares $\{\iota_i\}$ are then internally calibrated by minimizing the distance between observed trade balances and their model-implied counterparts, using the ownership structure implied by the model.

Finally, we discipline the remaining structural parameters using the literature and assumptions. The productivity dispersion parameters $\{\theta_k\}$ are taken from Caliendo and Parro (2015) and Caliendo et al. (2018) and mapped to our sectoral classification. The sectoral knowledge network $\{\lambda_{kh}\}$ is constructed from patent citation shares, following Liu (2019) and LaBelle et al. (2023). Geographic diffusion is restricted to the own location as a benchmark, so that knowledge spillovers operate across sectors within each location, and the knowledge-spillover elasticity is fixed at $\varphi = 0.07$ following Basso et al. (2025).

6 Quantitative Applications

This section presents two quantitative applications of the model that measure how local productivity shocks propagate throughout the economy. The first application measures the aggregate impact of a productivity shock originating in a specific location-sector pair.

[6] Unlike Caliendo et al. (2018), who combine value-added shares with a national input-output matrix to recover intermediate coefficients, we observe intermediate transactions directly in the data and therefore do not need to impute local input-output structure from aggregate tables. Appendix Figure C.9 shows that local and national input-output coefficients are positively related but far from one-to-one, with an R^2 of 0.41. National coefficients therefore capture broad sectoral patterns but miss substantial local geographic heterogeneity.

This exercise applies the core exercise in Caliendo et al. (2018) for the Chilean economy. We compare the multipliers of a local productivity shock on aggregate GDP to standard sufficient statistics, such as Domar weights, and decompose the contribution of each transmission mechanism to the dispersion of aggregate effects across the economy.

The second application evaluates the exit of the country's largest steel plant, *Compañia Siderurgica Huachipato*, located in the southern province of Concepción. We use the model to trace how the shutdown of a geographically concentrated firm transmits through the economy and quantify its effects across locations and sectors.

6.1 Aggregate Effects of Local Productivity Shocks

We consider a 10% increase in the productivity fundamental $\bar{A}_{i,k}$ in each location-sector pair, so that $\hat{\bar{A}}_{i,k} = 1.1$, while keeping trade costs fixed at $\hat{\kappa}_{iu,k} = 1$. We implement this counterfactual for each location-sector or a set of location-sectors, such as all sectors within a specific location. To facilitate comparisons across locations and sectors, we report all aggregate responses as elasticities relative to the weight that the location-sector pair has in the economy. Formally, let $dx \equiv (x' - x)/x$ denote the percentage change in variable x. We define aggregate elasticities for TFP, GDP, and welfare as

$$\text{TFP Elast.} = \frac{d\text{TFP}}{(Y_{i,k}/Y)\, d\bar{A}_{i,k}}, \quad \text{GDP Elast.} = \frac{d\text{GDP}}{(VA_{i,k}/VA)\, d\bar{A}_{i,k}}, \quad \text{Welfare Elast.} = \frac{d\mathcal{U}}{(L_{i,k}/L)\, d\bar{A}_{i,k}}.$$

For TFP and GDP, the aggregate domestic response is normalized by the shock size and the shocked cell's domestic Domar weight. Welfare instead captures the global response: under free mobility, it cannot be separately attributed to the domestic economy and is therefore normalized by the shock size and the cell's global employment share. These normalizations facilitate comparisons of propagation intensity across location-sector cells rather than differences in their economic size.

To understand the mechanisms driving the results, we compute counterfactuals under five scenarios: (i) the benchmark model; (ii) a model without input-output linkages, imposing $\gamma_{i,kh} = 0$ and $\gamma_{i,k} = 1$ for all i, k, h; (iii) a model without regional trade, imposing $\kappa_{iu,k} = \infty$ for all i, u, k; (iv) a model without both input-output linkages and regional trade; and (v) a model that replaces the geographically disaggregated input-output tables and value-added shares with national counterparts that do not vary across locations. The benchmark model is our preferred environment, while the alternatives isolate the role of each channel.

Appendix D.2 reports the full distributions of location, sectoral, and location-sector elasticities, together with additional visualization on the mechanisms behind aggregate impacts.

Location and Sectoral Productivity Shocks. Table 2 summarizes the aggregate elasticities from location shocks. For this, we increase fundamental productivity by 10% in all sectors within a given location, one location at a time; for sectoral shocks, we raise productivity by 10% in all locations within a given sector, one sector at a time. Two patterns stand out. First, measured TFP elasticities remain well below GDP elasticities in both exercises, with mean responses of 0.52 versus 2.04 for location shocks and 0.58 versus 1.97 for sectoral shocks; welfare responses lie in between on average. Second, the dispersion of responses differs across exercises: location shocks generate especially wide variation in welfare, while sectoral shocks generate wider dispersion in GDP. The corresponding distributions are shown in Appendix Figures D.7 and D.8.

For regional shocks, the largest GDP elasticities are concentrated in the Metropolitan and South regions, followed by Central and South-Central, while the North exhibits the weakest GDP amplification despite very high productivity levels. This ordering reflects the fact that central and metropolitan regions are more deeply connected to domestic production networks, so productivity gains there propagate more broadly through trade and factor reallocation. By contrast, the North is highly productive but more specialized, with shorter domestic value chains centered on mining, so shocks there generate weaker GDP amplification. Welfare responses tell a different story: the North exhibits the largest mean welfare gain, indicating that local income and price adjustments need not track GDP one-for-one. The appendix distributions in Figure D.7 show the same ordering visually.

For sectoral shocks, the highest GDP elasticities arise in Business Services and Social Services, followed by Infrastructure and Construction, while Mining exhibits comparatively low GDP amplification despite the highest mean measured TFP and the largest mean welfare response. This ranking reflects the sectoral position of these activities in the domestic network: service sectors occupy central positions and support a wide range of downstream activities, whereas mining is highly productive but less embedded in domestic input chains. As a result, productivity gains in services propagate more strongly across locations and sectors than productivity gains in mining. Figure D.8 in the appendix shows

Table 2: Aggregate Elasticities to Location or Sectoral Productivity Shocks: Benchmark

	Measured TFP		GDP		Welfare	
	Mean (1)	SD (2)	Mean (3)	SD (4)	Mean (5)	SD (6)
Panel A: Regional Shocks						
North	0.57	0.06	1.39	0.28	3.19	1.37
Central	0.53	0.05	2.03	0.25	0.84	0.13
Metropolitan	0.52	0.04	2.37	0.11	1.33	0.11
South-Central	0.46	0.07	1.93	0.19	0.93	0.22
South	0.42	0.06	2.18	0.30	0.89	0.19
Austral	0.54	0.10	1.86	0.24	1.02	0.36
Total	0.52	0.07	2.04	0.42	1.55	1.06
Panel B: Sectoral Shocks						
Agriculture, Forestry, and Fishing	0.42	0.08	1.68	0.14	0.86	0.11
Mining	0.70	—	1.22	—	1.78	—
Manufacturing	0.31	0.09	1.77	0.68	1.09	0.21
Infrastructure and Construction	0.44	0.10	1.98	0.14	1.08	0.05
Business Services	0.59	0.19	2.26	0.28	1.10	0.06
Social Services	0.74	0.08	2.24	0.06	1.15	0.02
Total	0.58	0.19	1.97	0.49	1.22	0.28

Notes: Columns (1), (3), and (5) report weighted averages of TFP, GDP, and welfare elasticities, respectively; columns (2), (4), and (6) report the corresponding weighted standard deviations. Both moments are computed using the relative size of provinces (sectors) within the corresponding macrozone (industry). Sales are used as weights for TFP, value added for GDP, and employment for welfare.

the corresponding sectoral distributions. Overall, the table and appendix figures make the same point: aggregate effects are not driven by sector size alone, but by where activity is located and how it is connected through the economy's spatial production network.

Location-Sector Productivity Shocks. Table 3 summarizes the aggregate effects of shocks that are specific of location-sector pairs. In this exercise, we increase fundamental productivity by 10% in a specific location-sector pair, one pair at a time, using the same normalization as in the location and sectoral counterfactuals.[7] The main takeaway is that average aggregate responses remain broadly similar to the one-dimensional exercises (2.04 in location-sector pair shocks vs 2.04 and 1.97 in location and sectoral shocks, respectively),

[7] Appendix Table D.4 reports the same location and sectoral aggregation levels used in the one-dimensional exercises, now computed from these location-sector shocks, allowing for a direct comparison.

but the dispersion across shocks rises sharply, from 0.42 in location shocks to 0.63 in location-sector pairs. Appendix Figure D.9 shows the full distribution of measured TFP, GDP, and welfare elasticities across location-sector pairs.

Table 3: Aggregate GDP Elasticity of Location-Sector Pair Productivity Shocks

	Mean		Standard Deviation		Variance Decomposition		
	Simple (1)	Weighted (2)	Simple (3)	Weighted (4)	Geography (5)	Sectoral (6)	Interaction (7)
Panel A: Standard Statistics							
Domar Weight ($\times 100$)	0.06	1.78	0.32	2.67	21	4.8	75
Panel B: Aggregate GDP Elasticity							
Benchmark	2.24	2.04	1.09	0.63	12	16	73
No Production Networks	2.55	2.32	1.13	0.82	17	19	64
No Trade Flows	1.83	1.92	0.48	0.43	58	33	8.8
No Trade or Production Networks	1.99	1.97	0.41	0.36	81	18	0.41
No Geography on Input-Output	2.26	2.01	1.04	0.61	13	19	69

Notes: Columns (1) and (2) report means across location-sector pairs. The simple mean is unweighted, while the weighted mean uses Benchmark value-added shares as weights. Columns (3) and (4) report the corresponding standard deviations, unweighted and weighted, respectively. Columns (5)-(7) report a variance decomposition based on an ANOVA with location effects, sector effects, and their interaction, with entries expressed as shares of the total sum of squares. Cells with baseline value added below 0.001% of GDP are excluded, and negative GDP elasticities are set to zero.

The main difference relative to the earlier exercises is not the average response, but the spread. In the benchmark row of Table 3, the mean GDP elasticity is 2.24 in simple terms and 2.04 when weighted, which is close to the averages obtained under regional and sectoral shocks. Appendix Table D.4 shows that this remains true row by row after aggregating the location-sector shocks by region or by sector: the ordering of regions and sectors is broadly preserved, and average TFP, GDP, and welfare responses remain of the same order of magnitude as in the one-dimensional exercises. What changes most is dispersion, which is substantially larger once the shock is localized jointly in geography and sector. One-dimensional shocks therefore average over the omitted dimension and understate how much aggregate effects differ across origins.

Table 3 makes clear where this dispersion comes from. The variance decomposition attributes 12% of the total dispersion in GDP elasticities to geography, 16% to sectors, and 73% to their interaction. This is the key quantitative result in this section, since neither geography nor sector alone is sufficient to summarize the heterogeneity in aggregate effects. The location-sector interaction term dominates, indicating that the same sector can have very different aggregate consequences depending on where it is located, and that the same location can generate very different effects depending on which sector is hit.

The implication is that the relevant unit of analysis is the location-sector pair. Aggregate effects depend on how local factor conditions, labor mobility, production networks, and trade linkages jointly shape the propagation of a shock.

Standard Sufficient Statistics. A large literature uses Domar weights as sufficient statistics for the aggregate effects of microeconomic productivity shocks. In an efficient economy, Hulten's theorem implies that the first-order output effect of a productivity shock equals the shocked producer's Domar weight.[8] Under our normalization, this benchmark implies a unit GDP elasticity. The observed dispersion therefore indicates that Domar weights are informative but not sufficient statistics in our framework. Inefficiencies from how trade imbalances are treated and non-linearities in the model can generate propagation beyond the Hulten benchmark (Baqaee & Farhi, 2019, 2020, 2024).

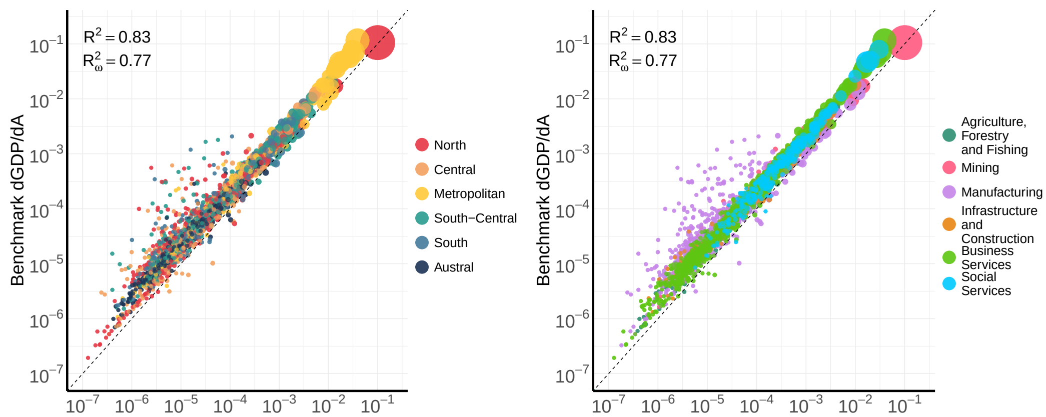
Figure 5 relates Domar weights to counterfactual aggregate GDP responses and GDP elasticities, distinguishing the scale of aggregate effects from the normalized intensity of propagation across location-sector cells. Panels (a) and (b) show that Domar weights account for 83% of the unweighted variation in aggregate GDP effects and 77% when observations are weighted by value added. Thus, they are informative but imperfect predictors of aggregate effects.

The Role of Input-Output and Trade Flows. We assess how trade and production networks shape the cross-sectional dispersion of aggregate GDP elasticities by comparing the Benchmark economy to counterfactuals that shut down input-output linkages, trade flows, or both. Figure 6 compares Benchmark elasticities with their counterfactual counterparts across location-sector pairs, so that the cross-sectional fit captures the contribution of each propagation channel. Table 3 complements this evidence with mean elasticities, standard deviations, and a variance decomposition across geographic, sectoral, and location-sector interaction components.

Panel (a) of Figure 6 shows that removing production networks lowers the fit with the Benchmark to 0.56, so input-output linkages account for 44% of the cross-sectional dispersion in aggregate GDP elasticities. Table 3 shows that this change is accompanied by a shift in the composition of heterogeneity: the geographic share of the variance rises from 12% in the Benchmark to 17%, the sectoral share rises from 16% to 19%, and the interaction

[8] Following Baqaee and Farhi (2020), producer i's revenue-based Domar weight is its sales relative to GDP, $\lambda_i \equiv p_i y_i / \sum_{j=1}^{N} p_j c_j$. Let b denote final expenditure shares, Ω the revenue-based input-output matrix, and $\Psi = (I - \Omega)^{-1}$ the corresponding Leontief inverse. Domar weights then satisfy $\lambda' = b'\Psi$.

Figure 5: Aggregate GDP Effects and Standard Sufficient Statistics



a) Aggregate Effect
(Colors: Locations)

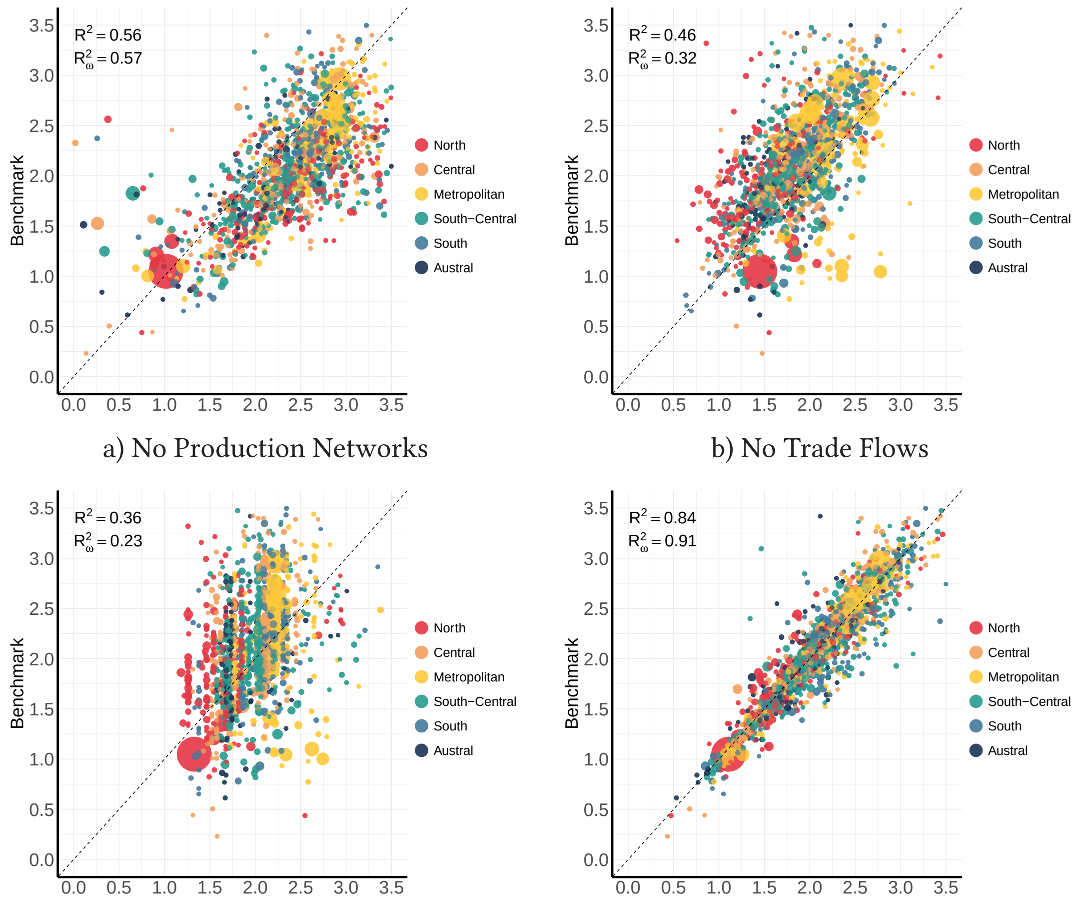
b) Aggregate Effect
(Colors: Sectors)

Notes: This figure compares location-sector aggregate GDP effects with the corresponding Domar-weight normalization. Panel (a) colors observations by macrozone, and panels (b) by industry. Panels (a) and (b) plot aggregate GDP effects. Marker size is proportional to the location-sector value-added share. Axes use a log-log visual scale; regression statistics use untransformed values. Cells with baseline value added below 0.001% of GDP are excluded.

term falls from 73% to 64%. Without production networks, heterogeneity becomes less driven by the location-sector interaction and more by the additive geographic and sectoral components. The economic intuition is that input-output linkages make propagation depend on the specific position of a location-sector pair in the spatial production network, whereas without those linkages shocks are transmitted more through channels that operate separately through geography and sector.

Panel (b) of Figure 6 shows that removing trade flows lowers the fit with the Benchmark to 0.46, and to 0.32 when observations are weighted by value added. Trade therefore accounts for 54% of the cross-sectional dispersion in aggregate GDP elasticities, and for 68% when location-sector pairs are weighted by value added. Table 3 shows that this change is accompanied by a much sharper shift in the composition of heterogeneity than in the no production networks counterfactual: the geographic share of the variance rises from 12% in the Benchmark to 58%, the sectoral share rises from 16% to 33%, and the interaction term falls from 73% to 8.8%. Without trade, heterogeneity becomes much less driven by the location-sector interaction and much more by the additive geographic and sectoral components, especially geography. The economic intuition is that trade transmits

Figure 6: Mechanisms Behind Aggregate GDP Elasticities: Colors by Location



a) No Production Networks

b) No Trade Flows

c) No Trade or Production Networks

d) No Geography on Input-Output

Notes: Panels compare Benchmark aggregate GDP elasticities with those in four counterfactual economies. Each point is a province-sector pair, colored by macrozone. Panel (a) removes production networks, panel (b) removes trade flows, panel (c) removes both, and panel (d) replaces geographically disaggregated input-output linkages with national coefficients. The 45-degree line denotes equality with the Benchmark. Cells with baseline value added below 0.001% of GDP are excluded, and negative GDP elasticities are set to zero.

local productivity shocks across markets through goods demand and sourcing decisions; without that channel, propagation becomes much less tied to the specific location-sector node and much more tied to where activity is located and which sector is hit separately.

Panel (c) of Figure 6 shows that shutting down both trade and production networks lowers the fit with the Benchmark to 0.36, and to 0.23 when observations are weighted by value added. Table 3 shows that the corresponding variance decomposition becomes 81% geography, 18% sector, and only 0.41% interaction. Once both channels are removed, almost all of the heterogeneity in GDP elasticities is accounted for by additive geographic and sectoral components, with very little remaining role for the location-sector interaction. The remaining dispersion reflects channels that continue to operate in this stripped-down environment, in particular labor mobility and congestion in locally fixed factors. Taken together, these counterfactuals show that trade and production networks are complementary propagation channels: both sustain the interaction component of heterogeneity, but trade is the more important channel quantitatively, especially for macroeconomically large location-sector pairs.

The Role of Geographically Disaggregated Input-Output Tables. To isolate the contribution of geographic detail in the input-output structure, we compare the Benchmark to a counterfactual that replaces geographically disaggregated input-output linkages with national sectoral coefficients while keeping the rest of the environment unchanged. Panel (d) of Figure 6 shows a high correspondence between Benchmark elasticities and this counterfactual, with a cross-sectional fit of 0.84 unweighted and 0.91 when observations are weighted by value added. Geographic detail in the input-output matrix therefore accounts for 16% of the unweighted cross-sectional dispersion in Benchmark GDP elasticities, but only 9% of the weighted dispersion. Table 3 shows the same pattern. Average effects remain essentially unchanged, with mean GDP elasticities of 2.24 in the Benchmark and 2.26 in the counterfactual in simple terms, and 2.04 versus 2.01 when weighted by value added. The variance decomposition also changes only modestly: the geographic share rises from 12% to 13%, the sectoral share rises from 16% to 19%, and the interaction term falls from 73% to 69%.

The economic intuition is that national sectoral coefficients are largely shaped by the production structure of the biggest location-sector pairs. As a result, replacing geographically disaggregated linkages with national coefficients remains a reasonable approximation for large markets, which is why the weighted fit remains as high as 0.91. By contrast, smaller location-sector pairs have little influence on national aggregates, so their sourcing and sales patterns change more when geographic detail in the input-output table is removed. One can see in Panel (d) of Figure 6 that GDP elasticities go from more than 3 to 2 when the geography of input-output linkages is ignored.

6.2 Exit of *Huachipato* Steel Plant

We study the exit of Chile's largest steel plant, *Compa n'ıa Sider'urgica Huachipato*, by modeling it as a localized productivity shock to the Concepci'on-Primary Metals node. Specifically, we calibrate a decline in the node's fundamental productivity so that its equilibrium gross output falls by 75%, corresponding to the plant's share of the node's baseline gross sales. Because trade and input substitution absorb part of the disruption, the implied decline in fundamental productivity is considerably larger than the decline in output. This episode is informative because it combines a highly localized disturbance with a plant that was large within its node but small in national exposure, making it a useful case to quantify how a local disruption propagates through the economy. Additional results for this application, including propagation patterns for measured TFP and employment, finer province-sector decompositions, and evidence on the role of distance, are reported in Appendix D.3.

Table 4 shows that the benchmark model implies non-trivial aggregate losses. National GDP falls by 0.14%, national employment by 0.17%, and measured TFP by 0.03%. The corresponding national GDP elasticity is 3.41. Thus, although the shock is highly localized, its aggregate effects are not. The contrast between the small decline in measured TFP and the larger contractions in GDP and employment already points to the role of general equilibrium propagation beyond the directly affected node.

The propagation of the shock is also markedly uneven across sectors and across space. Table 4 shows that GDP falls by 1.3% in Biobío, but also by 0.06% in other regions, while manufacturing contracts by 0.45% compared with 0.09% in the rest of the economy. Figure 7 shows the same point graphically. In the geographic decomposition, Concepción accounts for about 59% of the national GDP decline, so roughly 41% of the aggregate loss occurs outside the province where the shock originates. In the sectoral decomposition, the decline is not confined to the directly affected activity, but propagates broadly through manufacturing and related downstream sectors. At the same time, the figure shows selective offsetting gains in activities such as Wood Products, Agriculture, and Forestry, as well as in some northern and central provinces. The location-sector panels further show that this heterogeneity persists within sectors and within locations. The model therefore implies uneven impacts, with winners and losers distributed across locations and sectors, and can help identify not only the sectors most exposed to the disruption but also the activities most likely to absorb adjustment in its aftermath.

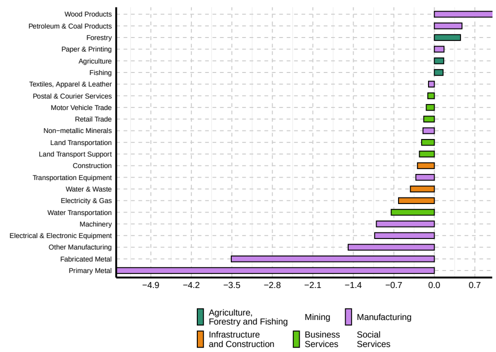
Table 4: Aggregate Effects of the Exit of *Huachipato* Steel Plant

	Measured TFP (1)	Employment (2)	Real GDP (3)
Panel A: National			
Total	−0.03	−0.17	−0.14
Elasticity	0.10	3.49	3.41
Panel B: Across Space			
Biobío			
Total	−0.36	−1.3	−1.3
Elasticity	1.26	26.71	33.28
Other Regions			
Total	−0.002	−0.07	−0.06
Elasticity	0.01	1.54	1.40
Panel C: Across Sectors			
Manufacturing			
Total	−0.10	−0.38	−0.45
Elasticity	0.36	8.01	11.27
Other Sectors			
Total	−0.008	−0.13	−0.09
Elasticity	0.03	2.79	2.34

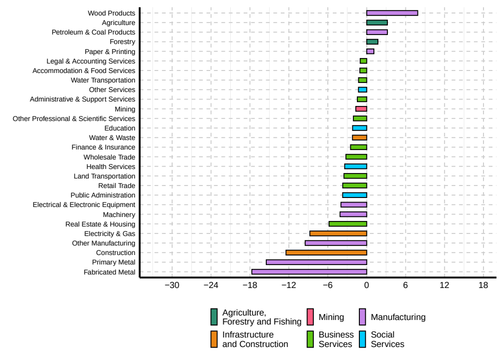
Notes: Columns (1)-(3) report benchmark aggregate effects of Huachipato's exit on measured TFP, employment, and real GDP. Panel A reports national effects. Panel B reports effects aggregated across space, separating Biobío from the rest of the country. Panel C reports effects aggregated across sectors, separating Manufacturing from all other sectors. Within each panel, the rows labeled *Total* report percentage changes, while the rows labeled *Elasticity* normalize each subgroup's total effect by the direct exposure of the shocked node, defined as its baseline share of the relevant aggregate times the size of the fundamental productivity shock, following the convention of Section 6.1. These elasticities are subgroup-specific normalized effects and are therefore not additive across subgroups.

Table 5 shows that production networks are the main source of aggregate amplification in this episode, with knowledge diffusion as a second channel. In the benchmark, the national GDP elasticity is 3.41. Shutting down production networks lowers it to 1.20, so input-output linkages account for about two-thirds of the aggregate amplification. Switching off knowledge spillovers lowers it to 2.83, about a sixth of the benchmark response. By contrast, shutting down trade raises the national GDP elasticity to 5.75, and shutting down both trade and production networks yields 6.87. These comparisons indicate that input-output linkages and knowledge spillovers are the channels that turn a localized

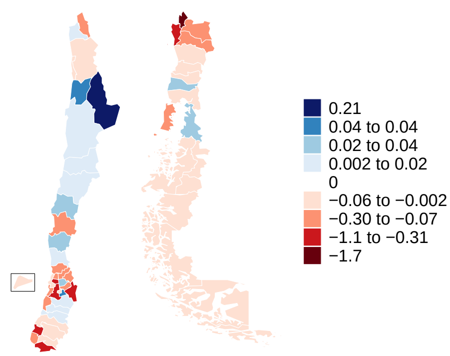
Figure 7: Propagation Across Space and Sector of the Exit of *Huachipato* Steel Plant



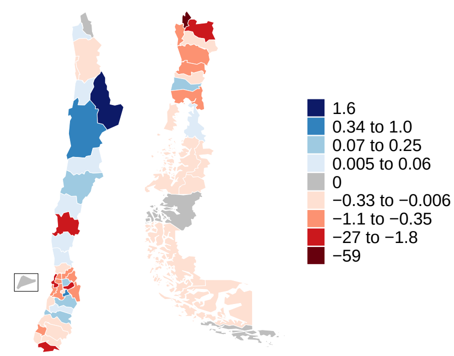
a) Sectoral GDP change (%)



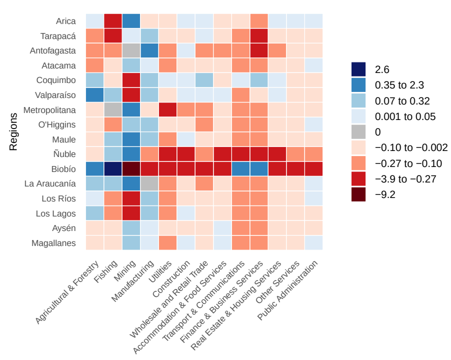
b) Sectoral contribution share (%)



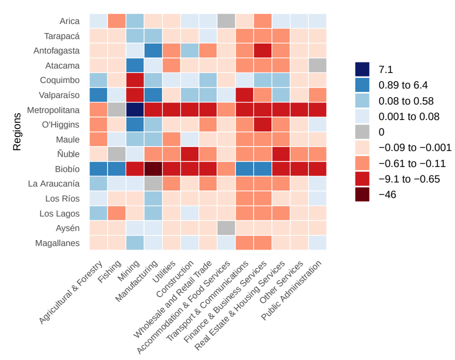
c) Provincial GDP change (%)



d) Provincial contribution share (%)



e) Region-sector GDP change (%)



f) Region-sector contribution share (%)

Notes: Each row reports the same object at a different level of aggregation. The left panels show the percentage change in GDP for each sector, province, or region-sector pair. The right panels show each unit's contribution to the national GDP change, defined as the unit-level GDP change weighted by its national GDP share and expressed relative to the aggregate GDP change. Contributions sum to 100%. Values below the variable-specific plotting threshold are set to zero or omitted, as in the underlying figures.

Table 5: Propagation Mechanisms of *Huachipato*'s Exit

	National Elasticity			Variance Decomposition			Distance
	TFP	Employment	Real GDP	Geography	Sector	Interaction	R^2
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Benchmark	0.10	3.49	3.41	5.5	25	70	41
No Production Networks	0.74	1.14	1.20	0.1	42	58	37
No Trade Flows	0.21	6.27	5.75	72	2.1	26	37
No Trade or Production Networks	2.66	8.65	6.87	81	0.9	18	35
No Geography on Input-Output	0.14	1.59	1.51	1.8	28	71	41
No Knowledge Spillovers	0.07	2.90	2.83	3.7	29	68	42

Notes: Columns (1)-(3) report national elasticities of measured TFP, employment, and GDP to Huachipato's exit across model variants, all evaluated at the same fundamental productivity shock, calibrated in the benchmark economy. The *No Knowledge Spillovers* variant sets $\varphi = 0$, so the shock does not diffuse through the sectoral knowledge network. Columns (4)-(6) report a weighted variance decomposition of GDP responses across geography, sector, and their interaction, with entries expressed as shares of the total sum of squares. Column (7) reports the R^2 from a quadratic fit of GDP responses on distance to Concepción, expressed as a percentage.

disruption into a larger aggregate contraction, whereas trade acts as a spatial adjustment margin that redistributes the shock across markets and therefore dampens its aggregate incidence.

The same table shows that the mechanisms also reshape the nature of heterogeneity in propagation. In the benchmark, the variance decomposition of GDP responses is dominated by the geography-sector interaction, which accounts for 70% of the total variation, compared with 25% for sector and only 5.5% for geography. Without production networks, the sectoral component rises to 42% and the interaction falls to 58%, indicating that shocks depend more on sector identity alone and less on the specific position of a location-sector pair in the production network. Without trade, the pattern reverses: geography rises to 72%, sector falls to 2.6%, and the interaction declines to 26%. When both channels are shut down, heterogeneity is driven predominantly by geography. Knowledge spillovers, by contrast, change the size of the aggregate response without reshaping its composition: the variance decomposition remains nearly identical to the benchmark. Distance captures part of these spatial patterns, but it is not a sufficient statistic for propagation. In the benchmark, a quadratic function of distance to Concepción explains 41% of the cross-sectional variation in GDP responses; the fit remains at 37% without production networks or without trade flows, and at 35% when both are shut down, so the explanatory power of distance comes from neither channel.

Geographic detail in the input-output matrix matters mainly for the magnitude of propagation rather than for its broad cross-sectional structure. Replacing geographically disaggregated linkages with national coefficients lowers the national GDP elasticity from 3.41 to 1.51 and the employment elasticity from 3.49 to 1.59, while raising the measured TFP elasticity from 0.10 to 0.14. By contrast, the variance decomposition and the role of distance remain broadly unchanged. National input-output coefficients therefore preserve the broad geography-sector pattern of propagation, but alter the aggregate strength of the response.

Taken together, the *Huachipato* episode illustrates the paper's central message in a concrete and policy-relevant setting. A localized disruption generates non-trivial aggregate losses, uneven impacts across sectors and places, and substantial adjustment outside the origin market. Understanding these responses requires measuring how trade, production networks, and spatially disaggregated input-output linkages shape propagation across the economy. Measuring these spatial mechanisms is therefore essential for understanding development and adaptation policies.

7 Conclusion

This paper studies how local productivity shocks shape aggregate outcomes and propagate across space and sectors in an economy with rich geographic and sectoral structure. We present and measure a quantitative general-equilibrium model that features input-output linkages along the value chain, international and domestic trade, labor migration, congestion in locally fixed factors, firm selection, and knowledge diffusion. A key contribution of the paper is the construction of geographically and sectorally disaggregated input-output matrices using administrative data, which are, to the best of our knowledge, unique in the world. This dataset allows us to study productivity shocks at a level of spatial and sectoral granularity that has not been feasible in previous work.

Our first main contribution is conceptual. We show that aggregate GDP elasticities to productivity shocks are fundamentally driven by location-sector specific features. Conventional sufficient statistics such as Domar weights capture important aspects of network amplification, but they are imperfect predictors of aggregate effects in such a setting. Both variance decompositions and counterfactual exercises demonstrate that neither geography nor sector alone explains most of the dispersion in aggregate responses. Instead, productivity shocks matter because of how specific sectors are embedded in specific locations within national trade and production networks. This result holds in levels and when elasticities are normalized by size, highlighting that the relevant heterogeneity is economically meaningful rather than mechanical. Trade and input-output networks jointly determine not only the magnitude of aggregate effects, but also how heterogeneity across locations and sectors emerges.

Our second contribution is applied. Using the model, we study the aggregate and spatial consequences of a localized productivity shock, with a detailed application to the exit of the largest steel plant in Chile, *Compañia Siderurgica Huachipato*. This application reinforces the central message of the paper while adding an important distributional dimension. The closure generates large local losses that propagate nationally through production networks and trade, despite the firm's small direct contribution to aggregate GDP. At the same time, the shock produces clear winners and losers across locations and sectors. While steel-intensive activities and the origin location bear the highest costs, other regions and sectors partially benefit through reallocation of labor, demand, and production. These heterogeneous responses imply that aggregate effects conceal substantial redistribution beneath the surface. Identifying where and in which sectors gains and losses materialize is therefore essential for the design of effective public policies, particularly in contexts of industrial restructuring, place-based interventions, or compensation schemes accompanying productivity shocks.

Taken together, the results show that understanding the aggregate and spatial consequences of productivity changes requires modeling the interaction between geography and sectoral structure. Ignoring either dimension leads to systematically incomplete assessments of both aggregate effects and their propagation. More broadly, the paper highlights the central role of trade, production networks, and labor mobility in shaping how local shocks are amplified, redistributed, and ultimately reflected in national outcomes.

Several extensions would further enhance the framework by exploiting the richness of the data. These include relaxing the assumption of uniform consumption shares across locations, allowing for sectoral heterogeneity in the importance of labor and infrastructure within value added, and introducing frictions to labor mobility. Additional extensions include modeling geographically localized agglomeration effects in technology diffusion, allowing workers to transmit knowledge across sectors and regions as they move, and introducing dynamics to capture private capital accumulation and investment responses.

Declaration of generative AI and AI-assisted technologies in the manuscript preparation process

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Appendix

The Geographic and Sectoral Impact of Productivity

Federico Huneeus
Duke University

Miguel Del Valle
Universidad de Chile

A Data Construction

In this appendix, we describe in detail the data sources and procedures used to construct the Disaggregated Economic Accounts. Appendix A.1 provides an overview of the data sources. Appendix A.2 documents the matching procedures, data cleaning, and additional adjustments required to obtain the final datasets used in the model calibration. Finally, Appendix A.3 assesses data robustness, including how each cleaning step affects the number of firms in the sample and the resulting distributions of firm-level variables.

A.1 Description of the Raw Data

This section describes the datasets used in the analysis. We first describe the data used to construct the disaggregated economic accounts and then discuss the additional sources used to calibrate the remaining model parameters.

A.1.1 Disaggregated Economic Accounts

Firm-to-firm transactions. This dataset contains annual information on domestic transactions conducted in Chile over the period 2018-2023. The data are drawn from Electronic Tax Documents (Documentos Tributarios Electrónicos, DTE), collected by the Chilean Internal Revenue Service (Servicio de Impuestos Internos, SII). Since 2018, reporting these documents has been mandatory for all corporate entities, regardless of firm size, revenue, or transaction value. Each document reports the tax identification number (Rol Único Tributario, RUT) of both the seller and the buyer, the transaction date, and the total nominal value of the transaction. In addition, the dataset identifies the municipalities of the establishments where the transaction takes place for both suppliers and purchasers.

We focus on transactions reported through Electronic Invoices (code 33), Electronic Tax-Exempt or Non-Taxed Invoices (code 34), Debit Notes (code 56), and Credit Notes (code 61). Following National Accounts conventions, all transaction values are expressed at basic prices. Specifically, we use the net amount and the tax-exempt amount reported in each document. For documents reporting gross amounts, we deflate values by dividing by $(1 + 0.19)$, reflecting Chile's linear value-added tax (VAT) rate of 19%.

Balance-sheet information. This dataset provides annual financial information for Chilean firms over the period 2018-2023. The data are drawn from the monthly VAT declaration (Formulario 29, F29) and the annual income tax declaration (Formulario 22, F22), both collected by the SII.

From Formulario 29 (F29), we construct the following variables: *total sales*, computed as $538/0.19 + 142 + 587 + 20$; *domestic sales*, $538/0.19 + 142 + 587$; *exports*, 20; *total materials*, $521 + 560 + 562 + 520/0.19 + 528/0.19 + 532/0.19 + 535/0.19$; *domestic materials*, $521+562+520/0.19+528/0.19+532/0.19$; *imports*, $560+535/0.19$; *investment*, $525/0.19+553/0.19$; and *value added*, defined as total sales minus total materials. Because VAT in Chile is levied at a flat rate of 19 percent, we recover pre-tax values by dividing the relevant VAT line items reported in Form F29 by 0.19. We focus on net values, as VAT is ultimately borne by final consumers and refunded to firms, ensuring consistency with National Accounts. From Formulario 22 (F22), we obtain the firm-level *capital stock* using code 647.

The firm registry includes information on each firm's primary industry at the 6-digit level, based on an adapted version of the United Nations' International Standard Industrial Classification (ISIC) used by the SII. We first map these 6-digit activities into the 111-sector classification used by the Central Bank of Chile. We then aggregate those sectors into the 46-sector classification used in our analysis, which provides additional detail within manufacturing while remaining flexible enough to match the sectoral aggregation of other data sources. In particular, the 46 sectors can be consistently re-aggregated to the 13 sectors available in Regional GDP, the 18 sectors in the employment survey, and the 10 sectors in the capital stock data. The sectoral concordance is reported in Table A.5. The dataset also reports the location of firms' headquarters at the municipality, province, and region levels.

Employer-employee relationships. This dataset contains annual labor information for Chilean firms over the period 2018-2023. The data are drawn from the Affidavit on Wages, Salaries, and Pensions (Declaración Jurada 1887, DJ1887) and the Affidavit on Fees for Independent Work (Declaración Jurada 1879, DJ1879), collected annually by the SII. Both declarations report the compensation paid by each firm to each worker, including wages, salaries, bonuses, tips, and other taxable labor income. Because the information is reported at the firm tax identifier (RUT) and worker alias level, we compute the number of workers employed by a firm in a given year as the total number of distinct worker aliases reported by that firm in that year, after merging both declarations to avoid double-counting workers within firms.[9]

[9] Unlike survey data, administrative records may count the same individual multiple times within a year if the worker changes employers during the period.

For DJ1887, we measure labor income as the sum of taxable net pay (code 3), tax-exempt income (code 33), and non-taxable income (code 8). Importantly, code 3 corresponds to net pay and excludes mandatory and voluntary pension and health contributions, which are reported separately in the form under social contributions. For DJ1879, by contrast, the form reports withholding amounts for independent contractors and directors, which we gross up by the applicable withholding rates to recover pre-withholding payments. For directors, the withholding rate was constant between 2018 and 2022 at 10% for code 4 and 35% for code 5. For independent contractors, the withholding rate was 10% in 2018–2019, 10.75% in 2020, 11.5% in 2021, 12.25% in 2022, and 12.5% in 2023. Thus, the two sources are not identical gross-compensation concepts, and we treat them as administrative measures of labor-related income that are subsequently reconciled to national accounts aggregates. For each firm-year, we aggregate compensation and employment across both declarations.

National input-output matrix. We use the national input-output matrix produced by the Central Bank of Chile (BCCh). The purpose of the datasets described above is to disaggregate this national matrix at the local level. We rely on the most recent input-output matrix, corresponding to 2022, constructed for 111 sectors based on an activity classification aligned with ISIC. The matrix includes intermediate consumption, final demand (household consumption, NPISH consumption, government consumption, gross fixed capital formation, inventory changes, and exports at FOB prices), imports (at CIF prices, including product taxes and import duties), value added (compensation of employees and gross operating surplus inclusive of net production taxes), and gross output at basic prices.

National Employment Survey (NENE). The National Employment Survey (Nueva Encuesta Nacional de Empleo, NENE), produced by the National Statistics Institute (INE), provides information on employment in Chile. We use the annualized 2022 survey and its employment measures by economic sector, based on an adaptation of ISIC Rev. 4. We retain the regional level of aggregation because the survey is representative at that level, and use the one-digit sectoral classification, which provides reliable precision in that setting.

Regional GDP. Regional GDP figures in Chile are published by the Central Bank of Chile, which distinguishes between the location of the institutional unit (the firm) and the productive unit (the establishment). Accordingly, regional GDP reflects value added generated in the region where production takes place. These estimates are constructed using multiple indicators that capture production, final use, factor payments, and derived demand at the establishment level, drawing on a combination of administrative records and sectoral surveys. We use the 2022 release, available for each of the 16 regions and 13 economic sectors.

Inter-Country Input-Output Tables. To incorporate international trade, we use the Inter-Country Input-Output Tables (ICIO) database from OECD (2023). This dataset provides sectoral information for the Rest of the World (RoW) in our model, covering the period 1995–2020 and including 76 countries plus the RoW aggregate. Sectors are classified according to ISIC Rev. 4 at the two-digit level, yielding 45 sectors spanning primary, manufacturing, and services. ICIO tables report domestic and cross-border flows by country and sector, as well as final demand, including household final consumption expenditure (HFCE), non-profit institutions serving households (NPISH), government final consumption (GGFC), gross fixed capital formation (GFCF), changes in inventories and valuables (INVNT), and direct purchases abroad by residents (DPABR). We also use the Trade and Employment (TiM) indicators, which provide employment (EMPN) and labor compensation (LABR), and the Trade in Value Added (TiVA) indicators, which report gross output (PROD) and value added (VALU).

Capital measurement. To assist in capital measurement, we use information from the LA-KLEMS database on the disaggregation of capital into equipment and structures across 1-digit sectors in Chile, as well as sector-specific capital depreciation rates.

Construction of the capital stock. We estimate firm-level capital stocks for the period 2018-2023 using the perpetual inventory method. The initial capital stock is obtained from firms' reported fixed assets in Formulario 22 (F22), code 647. Starting from this initial condition, capital stocks are updated using firm-level investment declarations from the first year of reporting onward, with all investment flows expressed in real terms.

Sector-specific depreciation rates and the share of investment allocated to machinery and equipment are taken from the LA-KLEMS database. Because investment reported to the SII primarily reflects expenditures on machinery and equipment, we adjust reported investment flows to account for capital devoted to construction and other structures. This adjustment is implemented using sectoral investment shares from the macro-level LA-KLEMS data, complemented with micro-level estimates based on firms' capital declarations.

We construct two versions of the aggregate capital stock. To compute the user cost of capital (capital payments), we multiply the capital stock by the sum of the depreciation rate and the real interest rate. The real interest rate is defined as the yield on Chilean 10-year sovereign bonds, adjusted for expected inflation (Survey of Economic Expectations) and the country risk premium (EMBI).

A.1.2 Other Sources of Information

Patent data. We use the *International Patent and Citations across Sectors* (INPACT-S) database. Based on PATSTAT, LaBelle et al. (2023) construct a dataset that tracks patenting activity within and across countries and industries over the period 1980-2019. The database features broad coverage of patent offices, sector-level bilateral data, an expanded number of patent applications through imputation, cross-country and cross-industry citation data, and consistent series for domestic and international patents.

Productivity dispersion. Sectoral productivity dispersion parameters are taken from Caliendo and Parro (2015) and Caliendo et al. (2018).

Elasticity of productivity with respect to expenditure. We draw on existing estimates in the literature and adopt the values reported in Basso et al. (2025).

Inter-provincial distance. We construct bilateral travel time and distance measures using the fastest available terrestrial and maritime routes between all pairs of municipalities in Chile, obtained via the Google Maps API. Provincial-level distances are computed as averages across municipalities within each province.

A.2 Data Cleaning

Data matching, sample definition, and outlier treatment. We merge the SII micro-level datasets, Formulario 29 (F29), Formulario 22 (F22), DJ1879, DJ1887, and Electronic Tax Documents (DTE), using firms' tax identifiers (RUT). This yields an annual firm-level panel for 2018 to 2023 that combines inter-firm transactions with balance-sheet, labor, and capital information. At this stage, the panel contains firm-level measures of sales, material expenditures, value added, labor compensation, capital payments, imports, exports, employment, headquarters municipality, and primary economic activity.

We then construct the active-firm sample used throughout the paper. A firm is classified as active if it reports strictly positive sales, material expenditures, and labor compensation, and employs at least two workers. For non-market and social-service activities, we redefine sales as the maximum of reported sales and observed input costs, where input costs are measured as the sum of material expenditures, labor compensation, and capital payments. This adjustment is applied to Administrative and Support Services, Public Administration, Education, Health, and Other Services in order to align the firm-level data with the treatment of non-market production in the national accounts.

Next, we winsorize firm-level and transaction-level variables by sector to reduce the influence of outliers while preserving the sectoral totals used in the top-down reconciliation. Total and domestic sales in the trade sector are winsorized at the 5th and 95th percentiles; in all other sectors, sales are winsorized at the 0.1st and 99.9th percentiles. Total and domestic material expenditures are winsorized at the 5th and 95th percentiles in Mining, Construction, Trade, Hotels and Restaurants, Finance and Insurance, Education, Health, Other Services, and Public Administration; in all remaining sectors, they are winsorized at the 0.1st and 99.9th percentiles. All other firm-level economic variables are winsorized between the 1st and 99th percentiles within each of the 46 sectors used in the analysis, while transaction amounts are winsorized at the 0.01st and 99.99th percentiles. After winsorization, value added, exports, and imports are recomputed, and labor compensation and capital payments are adjusted proportionally whenever their sum exceeds the revised level of value added.

The resulting dataset is an annual firm-level panel for 2018 to 2023 covering 124 million supplier-buyer-year transactions and 59 million unique firm pairs. It contains 2.1 million annual supplier observations and 1.8 million annual buyer observations, corresponding to 593 thousand unique suppliers, 555 thousand unique buyers, and a total of 677 thousand distinct firms.

Averaging, deflation, and confidentiality-preserving aggregation. All monetary variables are deflated to 2018 prices before aggregation. For firm-level economic variables, we compute averages over 2018 to 2023 excluding zero-valued observations, so the resulting panel captures persistent production relationships rather than transitory year-specific fluctuations. For bilateral trade, we start from the annual firm-firm transaction data and construct a firm-firm database using the average annual transaction value in real terms, again excluding zeros. Using non-zero averages allows the construction to recover persistent trading and production relationships from the administrative data, rather than treating temporary inactivity as evidence of permanently absent economic links. This bilateral firm-firm database is then aggregated to the 56-province-by-46-sector level and provides the initial geography-sector transaction matrix used in the balancing step below.

The aggregation must also satisfy SII confidentiality restrictions. In practice, no reported cell can allow for the direct or indirect identification of natural or legal persons, which requires at least 11 firms per published cell. Our main construction therefore works at the 56-province-by-46-sector level, and alternative geographic or sectoral aggregations are generated as re-aggregations of these province-sector accounts rather than from separate raw-data tabulations.

Top-down allocation of official GDP and employment. The purpose of the cleaned microdata is to allocate official macroeconomic totals across province-sector cells. Regional GDP from the CBC is available at the 16-region-by-13-sector level, while employment from the annualized 2022 National Employment Survey (NENE) is available at the 16-region-by-18-sector level. We jointly disaggregate these regional targets into our 56-province-by-46-sector partition using firm-level value-added and employment shares constructed from the cleaned administrative panel.

This top-down step is necessary because tax records identify firms' headquarters rather than the exact location of production. Anchoring the construction to the official CBC regional GDP series mitigates headquarters bias by aligning the spatial allocation with a measure of value added that is designed to reflect production location rather than legal residence. For province-sector cells with no observation in F29, F22, or the DJ labor records, but where the sector is known to exist nationally, we use the SII firm roster, which records each legal entity's 6-digit sector and headquarter commune, to construct provincial shares and allocate the corresponding region-sector total. This rule applies only to province-sector cells with no direct support in the administrative panel but positive sector-level activity in the official aggregates. The same logic is applied to employment whenever the administrative panel does not directly populate a province-sector cell.

Capital allocation. Capital is disciplined separately from GDP and employment. Official capital targets are available at the 10-sector level with no geographic dimension. We therefore estimate firm-level capital stocks using the perpetual inventory method described in the raw-data section and aggregate those estimates to obtain province-sector capital shares. These shares are constructed using the average firm-level capital stock over 2018 to 2023, in the same spirit as the province-sector allocation of value added and employment. We then use these shares to allocate the official 10-sector capital totals across province-sector cells. This procedure preserves consistency with the official sectoral capital aggregates while allowing the spatial distribution of capital to reflect the observed firm-level geography of capital stocks.

Reconstruction of province-sector accounts. Once province-sector value added, employment, and capital have been allocated, we reconstruct the remaining province-sector variables from the firm-level administrative data. Using the observed firm-level relationships between value added and sales, material expenditures, labor compensation, capital payments, and other accounting flows, we recover province-sector totals for the full set of variables required by the model. In practice, for each sector we compute the empirical ratio of each variable to value added in the cleaned microdata and apply those sector-level relationships to the allocated province-sector value added. When a cell has limited direct support in the microdata for a particular variable, we use the corresponding relationship estimated from the full sector. These reconstructed variables are then rescaled to match the sectoral totals reported in Chile's 2022 national input-output matrix, ensuring consistency with national accounting identities while avoiding negative values.

The detailed implementation of this step imposes a sequence of internal-consistency restrictions. If reconstructed value added is negative, material expenditures are scaled proportionally to equal total sales, preserving the relative shares of domestic and imported materials; labor compensation and capital payments are then adjusted proportionally. If value added is zero, employment is set to zero. We also impose that material expenditures do not exceed total sales, scaling both proportionally when necessary. Domestic material expenditures are constrained to match the information recorded in the tax data. Using these adjusted values, sales to final consumers are computed as total sales minus exports and intermediate sales, and domestic sales are defined as the sum of final-consumer sales and intermediate sales. In practice, the reconciliation proceeds in four steps. First, value added, employment, and capital are allocated across province-sector cells. Second, the remaining variables are reconstructed using the corresponding microdata relationships with value added. Third, we impose nonnegativity and accounting consistency restrictions. Fourth, we rescale the reconstructed variables to match official sectoral totals. This procedure is iterative: total sales are initialized from their empirical relationship with value added in the microdata, total materials are backed out as total sales minus value added, and the same logic is applied to labor compensation, capital payments, domestic and imported materials, domestic and international sales, intermediate sales, and final demand until the province-sector accounts are simultaneously consistent with the microdata, the national input-output table, and accounting identities.

Balancing the geography-sector input-output matrix. The initial geography-sector input-output matrix is obtained by aggregating the cleaned firm-firm transaction data to province-sector origin-destination cells. This raw bilateral matrix is informative about the spatial pattern of supplier-buyer linkages, but it does not mechanically satisfy all of the accounting constraints imposed by the reconciled province-sector accounts. We therefore apply the MR-GRAS methodology of Temursho et al. (2021) to reconcile the bilateral matrix with three sets of targets: disaggregated sales, disaggregated material expenditures, and the sectoral totals of the national input-output table.

MR-GRAS is well suited to this setting because the geography-sector matrix is sparse, contains many zero or confidentiality-suppressed cells, and must satisfy multiple accounting constraints simultaneously while preserving nonnegativity. In the matrix that enters MR-GRAS, true zeros and confidentiality-suppressed cells are not distinguished operationally. In our application, MR-GRAS is therefore the final balancing step that transforms the aggregated bilateral transaction data into an internally coherent geography-sector input-output system consistent with the reconciled Chilean accounts.

Incorporation of the Rest of the World. To account for international trade and production linkages, we add a rest-of-the-world block using the 2020 OECD Inter-Country Input-Output Tables. We first harmonize the ICIO sectoral classification with Table A.5. We then aggregate all countries other than Chile into a single foreign block and use the ICIO structure to allocate Chile's reconciled province-sector exports and imports across foreign destination and origin sectors. Province-sector export and import totals from the Chilean accounts are taken as given, and ICIO destination and origin shares are assumed to be common across Chilean provinces. This preserves consistency with both Chile's national accounts and the ICIO structure while avoiding additional geographic assumptions about the foreign sector.

Coverage of the final accounts. The resulting data characterize economic activity across 2,576 potential province-sector markets, of which 67 percent exhibit strictly positive activity. At this level of aggregation, the data define 6,635,776 potential province-sector-to-province-sector transaction pairs. Approximately 29 percent of these pairs exhibit strictly positive trade flows, although most are small (5 percent of pairs exceed USD 1,000), while the remainder correspond either to true zeros or to cells suppressed by confidentiality restrictions in the released data.

A.2.1 Adjustments to Other Datasets

Patent citations across sectors. We harmonize sectoral classifications to Table A.5. Although the original data are based on ISIC Rev. 3, concordance at our level of aggregation is feasible using standard crosswalks. The patent dataset primarily covers manufacturing activities, so some sectors in our model are not directly observed. For those uncovered sectors, we impose a calibration closure rule that they draw knowledge only from within their own sector.

Productivity dispersion. Sectoral measures from Caliendo and Parro (2015) and Caliendo et al. (2018) are classified under ISIC Rev. 3 and NAICS, respectively. At our level of aggregation, these classifications can be mapped directly to Table A.5. These sources report a single dispersion parameter for Agriculture-Forestry and Fishing jointly; because these sectors are separated in our model, we assign the same value to each. For Electrical and Electronic Equipment, four subsectors are reported under ISIC Rev. 3.1, electronic products, communication equipment, computing equipment, and related activities, with values ranging from 7.07 to 12.79. We therefore use their simple average.

A.3 Data Description

This section documents the empirical coverage of the cleaned administrative panel, the distribution of the resulting firm-level and transaction-level variables, and the final geographic and sectoral classifications used to construct the disaggregated accounts.

Coverage and sample selection. Table A.1 summarizes the empirical coverage of the cleaned panel. The raw administrative sources provide broad coverage, especially for sales, materials, value added, and employment, but much thinner coverage for exports, imports, and capital. After merging the five sources, the universe of matched tax identifiers reaches 2.813 million RUTs. The final analytical sample is substantially narrower, falling to 677 thousand firms after applying the cleaning and sample-selection rules described in the previous subsection. The key contrast in the table is therefore between *Total Matching* and *Firms*: the administrative universe is large, but only a subset of firms provides the economically meaningful activity needed to construct the accounts.

Table A.1: Number of Tax Identifiers and Firms (Thousands)

Variables	2018	2019	2020	2021	2022	2023	Total
Total Sales	903	928	935	1011	1066	1073	1644
Domestic Sales	902	926	933	1010	1065	1071	1642
Exports	12.56	13.13	12.74	12.69	14.08	15.95	28.56
Total Materials	1050	1075	1120	1226	1276	1302	1934
Domestic Materials	1050	1075	1120	1226	1276	1302	1934
Imports	33.78	33.98	33.98	40.64	39.50	40.31	80.83
Employment	650	687	675	720	732	718	1235
Capital	845	872	896	927	952	970	999
Value Added	1088	1166	1159	1268	1323	1347	1981
Total Matching	1565	1615	1673	1825	1893	1842	2813
Firms	332	346	344	385	390	375	677

Notes: This table reports the number of distinct tax identifiers (RUTs) with non-missing values for each variable by year. The *Total* column reports the number of distinct RUTs with at least one observation over the period 2018-2023. The row *Total Matching* reports the number of RUTs with at least one observation after merging all data sources. The row *Firms* reports the final number of distinct RUTs used in the aggregation after applying all cleaning, matching, and filtering procedures.

Firm-level distributions after cleaning. Table A.2 reports firm-level descriptive statistics before and after cleaning, using the firm-level averages that underlie the aggregation. Due to SII confidentiality restrictions, only the number of firms, total amounts, means, and standard deviations can be reported, and each released cell must contain at least 11 observations. This restriction has a modest quantitative effect, corresponding to roughly 1 percent of aggregate value added. The main message of the table is that cleaning materially reshapes the upper tail of the firm-level distribution. The effect is not a uniform reduction in all totals. Sales and domestic materials fall sharply, while exports, imports, and value added are affected differently because they are subsequently reconciled within the accounting system. By contrast, the reduction in dispersion is systematic and large across variables, indicating that extreme observations in the raw data are quantitatively important for the construction of the accounts.

Table A.2: Descriptive Statistics of Economic Variables

	Before Cleaning			**After Cleaning**		
	Total	**Mean**	**Std. Dev.**	**Total**	**Mean**	**Std. Dev.**
Total Sales	881.7	717.76	167.415	475.9	612.25	16.528
Domestic Sales	810.6	660.67	167.186	397.9	512.78	11.145
Exports	71.1	2.531	53.780	78.0	649.22	25.140
Total Materials	642.6	493.29	70.292	233.4	300.65	8.808
Domestic Materials	578.3	493.75	69.701	203.0	201.41	6.740
Imports	66.6	881.85	20.163	32.3	250.19	10.119
Employment	15.27	12.99	26.197	13.41	19.81	237.62
Compensation	120.7	89.54	8.270	73.6	94.64	1.821
Investment	22.3	100.28	3.034	17.2	102.67	3.222
Capital	338.5	566.22	41.28	234.1	560.30	36.11
Capital Payments	43.6	72.98	5.820	29.1	69.73	3.700
Value Added	227.1	172.21	171.62	242.6	313.30	11.068

Notes: This table reports descriptive statistics of firm-level economic variables before and after data cleaning, used for aggregation. Statistics correspond to averages over the period 2018-2023, expressed at 2018 prices, which constitute the main analytical dataset. For all variables except employment, the *Total* column reports aggregate amounts in billions of U.S. dollars (USD), obtained by converting from billions of Chilean pesos using an exchange rate of 792 CLP per USD. Mean and standard deviation are reported in millions of Chilean pesos. For employment, totals are reported in millions of workers, while means and standard deviations are expressed in number of workers.

Transaction-level distributions. Table A.3 reports analogous statistics for the firm-to-firm transaction data, both year by year and for the averaged bilateral transaction dataset used as input in the geography-sector matrix. Here the effects of cleaning are even more pronounced. The extensive margin of the network changes little: the numbers of pairs, sellers, and buyers are essentially unchanged before and after cleaning. The intensive margin, however, changes substantially. Total transaction values, means, and especially standard deviations are sharply compressed after cleaning, showing that the bilateral transaction matrix is highly sensitive to large observations. The table also makes clear that the averaged bilateral transaction dataset does not mechanically match domestic materials in the firm-level accounts. This discrepancy is resolved only in the later reconciliation step, where Formulario 29 provides the accounting anchor for domestic intermediate use. The averaged bilateral transaction dataset should therefore be interpreted as a construction input, rather than as a final accounting object.

Table A.3: Descriptive Statistics of Firm-to-Firm Transactions

	# Pairs	# Sellers	# Buyers	Total	Mean	Std. Dev.
Before Cleaning						
2018	187.84	332.63	273.85	2,237.3	121.0	138,080
2019	196.11	345.93	286.02	960.7	49.6	106,900
2020	184.80	343.89	283.69	425.0	23.2	7,420
2021	216.57	385.34	319.34	288.3	13.4	41,260
2022	228.86	390.59	326.35	203.3	9.0	96,970
2023	228.55	375.39	316.88	439.3	19.4	4,680
Average	588.34	593.19	555.27	498.9	22.3	1,180
After Cleaning						
2018	187.84	332.63	273.85	201.0	10.8	210
2019	196.11	345.93	286.02	213.7	11.0	210
2020	184.80	343.89	283.69	214.8	11.7	220
2021	216.57	385.34	319.34	260.3	12.1	220
2022	228.86	390.59	326.35	305.3	13.5	240
2023	228.55	375.39	316.88	288.6	12.7	230
Average	588.34	593.19	555.27	318.9	5.5	120

Notes: This table reports descriptive statistics for firm-to-firm transactions before and after data cleaning, used for aggregation. Statistics are shown for each year over the period 2018-2023, as well as for the firm-level average transaction dataset. The number of pairs is reported in millions of observations, while the number of sellers and buyers is reported in thousands. *Total* reports the aggregate value of transactions in billions of U.S. dollars (USD), obtained by converting from billions of Chilean pesos using an exchange rate of 792 CLP per USD. The mean and standard deviation are reported in millions of Chilean pesos.

Final aggregation objects. Tables A.4 and A.5 define the support of the disaggregated economic accounts used throughout the paper. The location concordance fixes the spatial partition of the economy into 56 provinces, grouped into macro-zones, and adds the rest-of-the-world block used in the open-economy extension. The sector concordance fixes the 46-sector production partition that underlies the empirical analysis and links the working sectoral classification to the official national accounts and to the additional datasets used for calibration. These two concordances are therefore not only descriptive appendices; they are the operational objects used to build the geography-sector network and the final province-sector accounts.

Table A.4: Location Definitions

Macro-zone	Region No.	Region	Provinces
North	15	Arica	1) Arica; 2) Parinacota
	1	Tarapacá	3) Iquique; 4) Tamarugal
	2	Antofagasta	5) Antofagasta; 6) El Loa; 7) Tocopilla
	3	Atacama	8) Copiapó; 9) Chañaral; 10) Huasco
Central	4	Coquimbo	11) Elqui; 12) Choapa; 13) Limarí
	5	Valparaíso	14) Valparaíso; 15) Easter Island; 16) Los Andes; 17) Petorca; 18) Quillota; 19) San Antonio; 20) San Felipe de Aconcagua; 21) Marga Marga
Metropolitan	13	Metropolitan	22) Santiago; 23) Cordillera; 24) Chacabuco; 25) Maipo; 26) Melipilla; 27) Talagante
South-Central	6	O'Higgins	28) Cachapoal; 29) Cardenal Caro; 30) Colchagua
	7	Maule	31) Talca; 32) Cauquenes; 33) Curicó; 34) Linares
	16	Ñuble	35) Diguillín; 36) Itata; 37) Punilla
	8	Biobío	38) Concepción; 39) Arauco; 40) Biobío
South	9	La Araucanía	41) Cautín; 42) Malleco
	14	Los Ríos	43) Valdivia; 44) Ranco
	10	Los Lagos	45) Llanquihue; 46) Chiloé; 47) Osorno; 48) Palena
Austral	11	Aysén	49) Coyhaique; 50) Aysén; 51) Capitán Prat; 52) General Carrera
	12	Magallanes	53) Magallanes; 54) Chilean Antarctica; 55) Tierra del Fuego; 56) Última Esperanza
Rest of the World			57) Rest of the World

Table A.5: Sector Definitions

Macro Industries	Industry Group	Regional Sectors	ISIC Rev. 4 Codes	No.	Sectors
Primary	Agriculture, Forestry and Fishing	Agricultural & Forestry	A01	1	Agriculture
			A02	2	Forestry
		Fishing	A03	3	Fishing
	Mining	Mining	B	4	Mining
Secondary	Manufacturing	Manufacturing	C10T12	5	Food, Beverages & Tobacco
			C13T15	6	Textiles, Apparel & Leather
			C16	7	Wood Products
			C17_18	8	Paper & Printing
			C19	9	Petroleum & Coal Products
			C20−21	10	Chemicals & Pharmaceuticals
			C22	11	Rubber & Plastics
			C23	12	Non-metallic Minerals
			C24	13	Primary Metals
			C25	14	Fabricated Metal Products
			C26−27	15	Electronics & Electrical Equipment
			C28	16	Machinery & Equipment
			C29−30	17	Transport Equipment
			C31−33	18	Other Manufacturing & Repair
	Infrastructure and Construction	Utilities	D	19	Electricity and Gas
			E	20	Water and Waste
		Construction	F	21	Construction
Tertiary	Business Services	Wholesale and Retail Trade	G45	22	Motor Vehicle Trade
			G46	23	Wholesale Trade
			G47	24	Retail Trade
		Accommodation & Food Services	I	25	Accommodation & Food Services
		Transport & Communications	H49	26	Land Transportation
			H50	27	Water Transportation
			H51	28	Air Transportation
			H521	29	Warehousing & Storage
			H5221	30	Land Transport Support
			H5222−229	31	Other Transport Support
			H53	32	Postal & Courier Services
			J61	33	Telecommunications
			J62−63	34	Information Technology
			J58−60	35	Other Publishing & Media
		Real Estate & Housing	L	37	Real Estate & Housing
		Finance & Business Services	K	36	Finance & Insurance
			M69	38	Legal & Accounting Services
			M71	39	Architecture & Engineering Services
			M70 + M72−75	40	Other Professional & Scientific Services
			N77	41	Rental & Leasing Services
			N78−82	42	Administrative & Support Services
	Social Services	Public Administration	O	43	Public Administration
		Other Services	P	44	Education
			Q	45	Health Services
			R−S−T	46	Other Services

B Quantitative Model

This appendix provides the full derivation of the quantitative model described in Section 4. The environment follows Caliendo et al. (2018), extended to allow deterministic productivity to depend on an economy-wide network of knowledge spillovers. We present the model in levels, derive the equilibrium conditions, and then express the system in relative changes for the counterfactual exercises.

Consider an economy composed of a finite set of domestic locations $\mathcal{D} = \{1, \dots, D\}$ and a finite set of foreign countries $\mathcal{F} = \{1, \dots, F\}$. The set of all regions is $\mathcal{N} = \mathcal{D} \cup \mathcal{F}$, with elements indexed by i and u. Each region contains a set of sectors $\mathcal{K} = \{1, \dots, K\}$, indexed by k and h. The economy features two primary factors of production: labor and structures. Aggregate labor, $\overline{L}$, is freely mobile across regions and sectors, supplies one unit of labor per worker, and earns a location-specific wage w_i. Let L_i denote employment in region i. Structures, $\overline{H}_i$, are fixed by location but can be used by all sectors within that location.

In each region-sector pair (i, k), a continuum of firms indexed by $\omega \in \Omega_{i,k}$ draws an idiosyncratic productivity level $z_{i,k}(\omega)$ from a Fréchet distribution. Firms produce differentiated varieties that can be used either as intermediate inputs or in the production of sectoral final goods. Final goods are consumed by households and used as material inputs by firms in other sectors. Sectors are connected through input-output linkages, and shipping materials across regions requires iceberg trade costs. These features generate trade linkages across locations and sectors and allow local productivity shocks to propagate through the economy.

B.1 Households

Households reside in regions indexed by $i \in \mathcal{N}$, where foreign countries are treated as additional regions in the model. Thus, all equilibrium objects below—including labor, structures, factor prices, income, and portfolio positions—are defined for both domestic locations and foreign countries. Households consume sectoral final goods $\{C_{i,k}\}_{k\in\mathcal{K}}$ at local prices $\{P_{i,k}\}_{k\in\mathcal{K}}$. Preferences are Cobb-Douglas,

$$u_i = \prod_{k\in\mathcal{K}} C_{i,k}^{\alpha_k}, \qquad \sum_{k\in\mathcal{K}} \alpha_k = 1. \tag{B.1}$$

The household budget constraint is

$$\sum_{k\in\mathcal{K}} P_{i,k} C_{i,k} = I_i, \tag{B.2}$$

where I_i denotes per-capita income in region i.

Income consists of labor earnings, locally retained rents from structures, and the per-capita return from a global portfolio of structures. Region i is endowed with structures $\overline{H}_i$ that earn rental rate r_i. A share $(1 - \iota_i)$ of these rents is retained locally and distributed to residents of region i, while the remaining share ι_i is pooled in a global portfolio and redistributed uniformly on a per-capita basis. Hence, per-capita income in region i is

$$I_i = w_i + (1 - \iota_i) r_i \frac{\overline{H}_i}{L_i} + \chi,$$

where

$$\chi = \frac{\sum_{n\in\mathcal{N}} \iota_n r_n \overline{H}_n}{\sum_{n\in\mathcal{N}} L_n}$$

is the per-capita return from the pooled portfolio.

This ownership structure generates regional income imbalances. Region i contributes $\iota_i r_i \overline{H}_i$ to the portfolio and receives χL_i in portfolio income. We define the net payment made by region i as

$$\Upsilon_i \equiv \iota_i r_i \overline{H}_i - \chi L_i. \tag{B.3}$$

Thus, $\Upsilon_i > 0$ means that region i pays more into the portfolio than it receives, whereas $\Upsilon_i < 0$ means that it receives net income from the portfolio. By construction,

$$\sum_{i\in\mathcal{N}} \Upsilon_i = 0.$$

Regional trade need not be balanced. Given the sign convention in Equation (B.3), a positive Υ_i corresponds to a regional trade surplus in equilibrium, while a negative Υ_i corresponds to a regional trade deficit. The corresponding regional trade-balance condition is derived below in Section B.6.

The household chooses $\{C_{i,k}\}_{k\in\mathcal{K}}$ to maximize Equation (B.1) subject to Equation (B.2). The first-order conditions imply constant expenditure shares,

$$P_{i,k} C_{i,k} = \alpha_k I_i, \qquad \forall k \in \mathcal{K}.$$

Substituting optimal consumption into preferences yields indirect utility

$$v_i = \frac{I_i}{P_i},$$

where the exact Cobb-Douglas price index is

$$P_i = \prod_{k\in\mathcal{K}} \left(\frac{P_{i,k}}{\alpha_k}\right)^{\alpha_k}.$$

Workers are freely mobile across regions. In equilibrium, all populated regions deliver the same indirect utility,

$$\frac{I_i}{P_i} = \mathcal{U}, \qquad \forall i \in \mathcal{N}.$$

Section B.7 derives the corresponding equilibrium allocation of labor across regions.

B.2 Intermediate-Goods Producers and Knowledge Diffusion

In each region $i \in \mathcal{N}$ and sector $k \in \mathcal{K}$, a continuum of firms produces differentiated intermediate varieties. Firms differ in their idiosyncratic productivity draw $z_{i,k}(\omega)$, where ω indexes varieties. Productivity draws are independently distributed according to a Fréchet distribution,

$$\Pr\left(z_{i,k} \leq z\right) = \exp\left\{-z^{-\theta_k}\right\}, \qquad z > 0,$$

where $\theta_k > 0$ governs productivity dispersion in sector k. In addition to idiosyncratic productivity, all producers in a region-sector pair (i, k) share a deterministic productivity component $A_{i,k}$.

Production technology. For a given deterministic productivity level $A_{i,k}$, a firm with idiosyncratic productivity $z_{i,k}$ produces output using structures, labor, and intermediate inputs. Suppressing the variety index ω, the production function is

$$q_{i,k}(z_{i,k}) = z_{i,k}\left(A_{i,k}\, h_{i,k}(z_{i,k})^{\beta_i} l_{i,k}(z_{i,k})^{1-\beta_i}\right)^{\gamma_{i,k}} \prod_{h\in\mathcal{K}} M_{i,kh}(z_{i,k})^{\gamma_{i,kh}}, \tag{B.4}$$

where $h_{i,k}(z_{i,k})$ and $l_{i,k}(z_{i,k})$ denote the firm's demand for structures and labor, respectively, and $M_{i,kh}(z_{i,k})$ denotes the quantity of sector-h materials used by a producer in sector k located in region i. The parameter β_i governs the share of structures in value-added production, while $1 - \beta_i$ is the corresponding labor share. The parameter $\gamma_{i,k}$ is the value-added share in gross output, and $\gamma_{i,kh}$ is the cost share of materials from sector h. Constant returns to scale require

$$\gamma_{i,k} + \sum_{h\in\mathcal{K}} \gamma_{i,kh} = 1.$$

Unit costs. Given factor prices (w_i, r_i) and material prices $\{P_{i,h}\}_{h\in\mathcal{K}}$, the firm chooses inputs to minimize the cost of producing one unit of output:

$$\min_{l_{i,k},\, h_{i,k},\, \{M_{i,kh}\}_{h\in\mathcal{K}}} r_i h_{i,k} + w_i l_{i,k} + \sum_{h\in\mathcal{K}} P_{i,h} M_{i,kh} \tag{B.5}$$

subject to the production technology in Equation (B.4) delivering one unit of output. The first-order conditions of the cost-minimization problem in Equation (B.5), together with the Cobb-Douglas technology in Equation (B.4), imply that cost shares are equal to the corresponding exponents:

$$\frac{r_i h_{i,k}(z_{i,k})}{c_{i,k}(z_{i,k})} = \gamma_{i,k}\beta_i, \qquad \frac{w_i l_{i,k}(z_{i,k})}{c_{i,k}(z_{i,k})} = \gamma_{i,k}(1-\beta_i), \qquad \frac{P_{i,h} M_{i,kh}(z_{i,k})}{c_{i,k}(z_{i,k})} = \gamma_{i,kh}. \tag{B.6}$$

Combining the cost-share conditions in Equation (B.6) with the technology in Equation (B.4) yields the unit cost of a variety with idiosyncratic productivity $z_{i,k}$:

$$c_{i,k}(z_{i,k}) = \frac{x_{i,k}}{z_{i,k} A_{i,k}^{\gamma_{i,k}}}, \tag{B.7}$$

where $x_{i,k}$ is the cost of the input bundle used by producers in (i,k):

$$x_{i,k} = B_{i,k} \left(r_i^{\beta_i} w_i^{1-\beta_i} \right)^{\gamma_{i,k}} \prod_{h \in \mathcal{K}} P_{i,h}^{\gamma_{i,kh}}. \tag{B.8}$$

The constant $B_{i,k}$ collects Cobb-Douglas share terms:

$$B_{i,k} = \left(\gamma_{i,k} (1-\beta_i)^{1-\beta_i} \beta_i^{\beta_i} \right)^{-\gamma_{i,k}} \prod_{h \in \mathcal{K}} \gamma_{i,kh}^{-\gamma_{i,kh}}. \tag{B.9}$$

Equations (B.7)–(B.9) separate idiosyncratic productivity, deterministic productivity, and the cost of the common input bundle. In the empirical implementation, purchases of durable inputs recorded as intermediate expenditures are included in material demand; the model itself does not feature a separate capital-accumulation decision.

Knowledge diffusion and deterministic productivity. Deterministic productivity in each region-sector pair reflects both local productivity fundamentals and spillovers from knowledge accumulated elsewhere in the economy. We write

$$A_{i,k} = \bar{A}_{i,k} \, \Psi_{i,k}^{\varphi}, \qquad \varphi \in [0,1), \tag{B.10}$$

where $\bar{A}_{i,k}$ denotes region-sector productivity fundamentals, $\Psi_{i,k}$ is the stock of external knowledge accessible to (i,k), and φ governs the elasticity of productivity with respect to knowledge spillovers.

The knowledge index is a Cobb-Douglas aggregator of deterministic productivity across regions and sectors:

$$\Psi_{i,k} = \prod_{u\in\mathcal{N},\, h\in\mathcal{K}} A_{u,h}^{\lambda_{kh}\delta_{iu}}, \tag{B.11}$$

where λ_{kh} captures technological relatedness between sectors k and h, and δ_{iu} captures the geographic reach of knowledge diffusion from region u to region i. We assume

$$\lambda_{kh} \geq 0, \qquad \delta_{iu} \geq 0, \qquad \sum_{h\in\mathcal{K}} \lambda_{kh} = 1, \qquad \sum_{u\in\mathcal{N}} \delta_{iu} = 1. \tag{B.12}$$

Taking logs of the productivity equation in Equation (B.10) and using the knowledge aggregator in Equation (B.11) gives

$$\log A_{i,k} = \log \bar{A}_{i,k} + \varphi \sum_{u\in\mathcal{N},\, h\in\mathcal{K}} \lambda_{kh}\delta_{iu} \log A_{u,h}. \tag{B.13}$$

Let $\log \mathbf{A}$ and $\log \bar{\mathbf{A}}$ collect $\log A_{i,k}$ and $\log \bar{A}_{i,k}$ over all region-sector pairs. Define $\boldsymbol{\Delta} = [\delta_{iu}]$, $\boldsymbol{\Lambda} = [\lambda_{kh}]$, and

$$\mathbf{W} \equiv \boldsymbol{\Delta} \otimes \boldsymbol{\Lambda}, \tag{B.14}$$

so that the element of $\mathbf{W}$ mapping (u, h) into (i, k) is $\delta_{iu}\lambda_{kh}$. Stacking Equation (B.13) over all region-sector pairs and using the definition of $\mathbf{W}$ in Equation (B.14) gives

$$\log \mathbf{A} = \log \bar{\mathbf{A}} + \varphi \mathbf{W} \log \mathbf{A}. \tag{B.15}$$

Using the normalizations in Equation (B.12), $\boldsymbol{\Delta}$ and $\boldsymbol{\Lambda}$ are row-stochastic, and therefore $\mathbf{W}$ is row-stochastic. Hence, $\rho(\mathbf{W}) = 1$, and $\varphi \in [0, 1)$ implies that $\mathbf{I} - \varphi\mathbf{W}$ is invertible. Solving the stacked system in Equation (B.15) gives

$$\log \mathbf{A} = (\mathbf{I} - \varphi\mathbf{W})^{-1} \log \bar{\mathbf{A}}. \tag{B.16}$$

The inverse in Equation (B.16) admits the Neumann-series representation

$$(\mathbf{I} - \varphi\mathbf{W})^{-1} = \sum_{s=0}^{\infty} \varphi^s \mathbf{W}^s,$$

so deterministic productivity incorporates both direct fundamentals and indirect knowledge diffusion through higher-order paths in the economy-wide network.

Let

$$\mathbf{L} \equiv (\mathbf{I} - \varphi \mathbf{W})^{-1}, \tag{B.17}$$

and denote by $\ell_{iu,kh}$ the element of $\mathbf{L}$ mapping fundamentals in region-sector pair (u, h) into deterministic productivity in (i, k). Using the definition of the knowledge multiplier in Equation (B.17) inside Equation (B.16) and exponentiating the resulting log-linear system gives

$$A_{i,k} = \prod_{u \in \mathcal{N},\, h \in \mathcal{K}} \bar{A}_{u,h}^{\ell_{iu,kh}}. \tag{B.18}$$

When $\varphi = 0$, $\mathbf{L} = \mathbf{I}$ and deterministic productivity reduces to local fundamentals, $A_{i,k} = \bar{A}_{i,k}$. For $\varphi > 0$, shocks to fundamentals propagate across regions and sectors according to the knowledge-network multiplier $\mathbf{L}$.

Delivered prices. Intermediate-goods markets are perfectly competitive, so prices equal unit costs. Delivering one unit of a sector-k variety from origin u to destination i requires shipping $\kappa_{iu,k} \geq 1$ units, with $\kappa_{ii,k} = 1$. Combining the unit-cost expression in Equation (B.7) with iceberg trade costs implies that, for a sector-k variety with productivity vector $z_k = \{z_{u,k}\}_{u \in \mathcal{N}}$, the price paid in region i is the minimum trade-cost-adjusted unit cost across all possible origins:

$$p_{i,k}(z_k) = \min_{u \in \mathcal{N}} \left\{ \frac{\kappa_{iu,k} x_{u,k}}{z_{u,k} A_{u,k}^{\gamma_{u,k}}} \right\}. \tag{B.19}$$

Equation (B.19) is the basis for the price-index and bilateral expenditure-share formulas derived in the next sections.

B.3 Final-Goods Aggregation

In each region $i \in \mathcal{N}$ and sector $k \in \mathcal{K}$, a competitive final-good producer aggregates intermediate varieties from the same sector into a sectoral final good. Let $Q_{i,k}$ denote final output in region-sector pair (i, k), and let $\tilde{q}_{i,k}(z_k)$ denote the quantity demanded of a sector-k variety characterized by the productivity vector $z_k = (z_{1,k}, \ldots, z_{N,k})$ across all possible origins. Final-good production is given by

$$Q_{i,k} = \left[\int_{\mathbb{R}_+^N} \tilde{q}_{i,k}(z_k)^{1 - \frac{1}{\eta_{i,k}}} \, d\Phi_k(z_k) \right]^{\frac{\eta_{i,k}}{\eta_{i,k} - 1}}, \tag{B.20}$$

where $\eta_{i,k} > 1$ is the elasticity of substitution across varieties. The joint distribution of productivity draws in sector k is

$$\Phi_k(z_k) = \exp\left\{-\sum_{n\in\mathcal{N}} z_{n,k}^{-\theta_k}\right\},$$

with marginal distribution

$$\Phi_{i,k}(z_{i,k}) = \exp\left\{-z_{i,k}^{-\theta_k}\right\}.$$

The integral in Equation (B.20) is taken with respect to the probability measure induced by Φ_k.

The final-good producer takes as given the delivered price $p_{i,k}(z_k)$ of each variety, where $p_{i,k}(z_k)$ already incorporates the minimum trade-cost-adjusted unit cost across possible origins. Given the final-good price $P_{i,k}$, the producer solves

$$\max_{\{\tilde{q}_{i,k}(z_k)\}} \; P_{i,k}Q_{i,k} - \int_{\mathbb{R}_+^N} p_{i,k}(z_k)\tilde{q}_{i,k}(z_k)\, d\Phi_k(z_k),$$

subject to the final-good technology in Equation (B.20). The first-order condition for each variety implies

$$P_{i,k}Q_{i,k}^{1/\eta_{i,k}}\tilde{q}_{i,k}(z_k)^{-1/\eta_{i,k}} = p_{i,k}(z_k).$$

Hence, demand for an intermediate variety is

$$\tilde{q}_{i,k}(z_k) = Q_{i,k}\left(\frac{p_{i,k}(z_k)}{P_{i,k}}\right)^{-\eta_{i,k}}. \tag{B.21}$$

Free entry and perfect competition in final-good production imply zero profits. Substituting the CES demand in Equation (B.21) into the final-good aggregator in Equation (B.20) gives

$$1 = \left[\int_{\mathbb{R}_+^N}\left(\frac{p_{i,k}(z_k)}{P_{i,k}}\right)^{1-\eta_{i,k}} d\Phi_k(z_k)\right]^{\frac{\eta_{i,k}}{\eta_{i,k}-1}}. \tag{B.22}$$

Solving Equation (B.22) for $P_{i,k}$ yields the CES price index:

$$P_{i,k} = \left[\int_{\mathbb{R}_+^N} p_{i,k}(z_k)^{1-\eta_{i,k}}\, d\Phi_k(z_k)\right]^{\frac{1}{1-\eta_{i,k}}}.$$

The next section evaluates this price index using the Fréchet distribution of productivities and the delivered-price rule derived in Equation (B.19).

B.4 Derivation of the Final-Goods Price Index

This section evaluates the CES price index derived in Equation (B.3) using the Fréchet distribution of productivity draws and the delivered-price rule in Equation (B.19). From Equation (B.3), the final-good price index satisfies

$$P_{i,k}^{1-\eta_{i,k}} = \int_{\mathbb{R}_+^N} p_{i,k}(z_k)^{1-\eta_{i,k}} \, d\Phi_k(z_k). \tag{B.23}$$

We now derive the distribution of the delivered price $p_{i,k}(z_k)$ induced by Equation (B.19).

For a sector-k variety with productivity vector $z_k = \{z_{u,k}\}_{u\in\mathcal{N}}$, Equation (B.19) implies

$$p_{i,k}(z_k) = \min_{u\in\mathcal{N}} \left\{ \frac{\kappa_{iu,k} x_{u,k}}{z_{u,k} A_{u,k}^{\gamma_{u,k}}} \right\}.$$

For each origin $u \in \mathcal{N}$, define the origin-specific component of price competitiveness as

$$\Omega_{iu,k} \equiv \left(\frac{\kappa_{iu,k} x_{u,k}}{A_{u,k}^{\gamma_{u,k}}} \right)^{-\theta_k} = (\kappa_{iu,k} x_{u,k})^{-\theta_k} \, A_{u,k}^{\theta_k \gamma_{u,k}}. \tag{B.24}$$

The aggregate price-competitiveness term for destination i and sector k is

$$\Omega_{i,k} \equiv \sum_{u\in\mathcal{N}} \Omega_{iu,k}. \tag{B.25}$$

For any $p > 0$, the cumulative distribution of the minimum delivered price is

$$\begin{aligned} \Pr(p_{i,k}(z_k) \le p) &= 1 - \Pr(p_{i,k}(z_k) > p) \\ &= 1 - \Pr\left(\frac{\kappa_{iu,k} x_{u,k}}{z_{u,k} A_{u,k}^{\gamma_{u,k}}} > p \quad \forall u \in \mathcal{N} \right) \\ &= 1 - \prod_{u\in\mathcal{N}} \Pr\left(z_{u,k} < \frac{\kappa_{iu,k} x_{u,k}}{p A_{u,k}^{\gamma_{u,k}}} \right), \end{aligned} \tag{B.26}$$

where the last equality uses the independence of productivity draws across origins.

Using the Fréchet distribution in Equation (B.2), each term in the product in Equation (B.26) satisfies

$$\Pr\left(z_{u,k} < \frac{\kappa_{iu,k} x_{u,k}}{p A_{u,k}^{\gamma_{u,k}}}\right) = \exp\left\{-\Omega_{iu,k} p^{\theta_k}\right\}, \tag{B.27}$$

where $\Omega_{iu,k}$ is defined in Equation (B.24). Substituting Equation (B.27) into Equation (B.26) and using the aggregate term in Equation (B.25) gives

$$\Pr(p_{i,k}(z_k) \leq p) = 1 - \exp\left\{-\Omega_{i,k} p^{\theta_k}\right\}. \tag{B.28}$$

Thus, Equation (B.28) implies that the minimum delivered price is Weibull with density

$$f_{i,k}(p) = \theta_k \Omega_{i,k} p^{\theta_k - 1} \exp\left\{-\Omega_{i,k} p^{\theta_k}\right\}. \tag{B.29}$$

Equation (B.23) can therefore be written as the price moment

$$P_{i,k}^{1-\eta_{i,k}} = \int_0^\infty p^{1-\eta_{i,k}} f_{i,k}(p)\, dp. \tag{B.30}$$

Substituting the density in Equation (B.29) into the moment condition in Equation (B.30) yields

$$\begin{aligned} P_{i,k}^{1-\eta_{i,k}} &= \int_0^\infty p^{1-\eta_{i,k}} \theta_k \Omega_{i,k} p^{\theta_k - 1} \exp\left\{-\Omega_{i,k} p^{\theta_k}\right\} dp \\ &= \theta_k \Omega_{i,k} \int_0^\infty p^{\theta_k - \eta_{i,k}} \exp\left\{-\Omega_{i,k} p^{\theta_k}\right\} dp. \end{aligned} \tag{B.31}$$

Using the change of variable

$$y = \Omega_{i,k} p^{\theta_k},$$

Equation (B.31) becomes

$$P_{i,k}^{1-\eta_{i,k}} = \Omega_{i,k}^{\frac{\eta_{i,k}-1}{\theta_k}} \Gamma\left(1 + \frac{1-\eta_{i,k}}{\theta_k}\right). \tag{B.32}$$

The moment in Equation (B.32) is finite provided

$$\xi_{i,k} \equiv 1 + \frac{1-\eta_{i,k}}{\theta_k} > 0,$$

or equivalently $\theta_k > \eta_{i,k} - 1$.

Solving Equation (B.32) for $P_{i,k}$ gives

$$P_{i,k} = \Gamma(\xi_{i,k})^{\frac{1}{1-\eta_{i,k}}} \, \Omega_{i,k}^{-\frac{1}{\theta_k}}. \tag{B.33}$$

Finally, substituting the aggregate price-competitiveness term in Equation (B.25) into Equation (B.33) yields

$$P_{i,k} = \Gamma(\xi_{i,k})^{\frac{1}{1-\eta_{i,k}}} \left[\sum_{u \in \mathcal{N}} (\kappa_{iu,k} x_{u,k})^{-\theta_k} \, A_{u,k}^{\theta_k \gamma_{u,k}} \right]^{-\frac{1}{\theta_k}}. \tag{B.34}$$

Equation (B.34) is the final-good price index used in the counterfactual system.

B.5 Trade Flows and Expenditure Shares

This section derives bilateral expenditure shares using the CES demand system in Equation (B.21), the delivered-price rule in Equation (B.19), and the price-competitiveness terms defined in Equations (B.24) and (B.25). Let

$$X_{i,k} \equiv P_{i,k} Q_{i,k}$$

denote total expenditure in region i on sector-k varieties, and let $X_{iu,k}$ denote the part of this expenditure sourced from origin u. The bilateral expenditure share is

$$\pi_{iu,k} \equiv \frac{X_{iu,k}}{X_{i,k}}.$$

For each origin $u \in \mathcal{N}$, define the delivered price from origin u before minimization across origins as

$$p_{iu,k}(z_{u,k}) \equiv \frac{\kappa_{iu,k} x_{u,k}}{z_{u,k} A_{u,k}^{\gamma_{u,k}}}.$$

Origin u supplies a sector-k variety to destination i whenever $p_{iu,k}(z_{u,k})$ is the lowest delivered price among all origins in Equation (B.19). Let

$$\mathcal{Z}_{iu,k} \equiv \left\{ z_k \in \mathbb{R}_+^N : p_{iu,k}(z_{u,k}) = \min_{n \in \mathcal{N}} p_{in,k}(z_{n,k}) \right\}$$

denote the set of sector-k varieties for which origin u is the lowest-cost supplier to destination i.

Using the Fréchet distribution in Equation (B.2), the delivered-price rule in Equation (B.19), and the price-competitiveness terms in Equations (B.24) and (B.25), the origin-specific contribution to the CES price moment satisfies

$$\int_{\mathcal{Z}_{iu,k}} p_{i,k}(z_k)^{1-\eta_{i,k}}\, d\Phi_k(z_k) = \frac{\Omega_{iu,k}}{\Omega_{i,k}} \int_{\mathbb{R}_+^N} p_{i,k}(z_k)^{1-\eta_{i,k}}\, d\Phi_k(z_k). \tag{B.35}$$

Equation (B.35) follows from the same competing-risk argument used to derive the minimum-price density in Equation (B.29): with independent Fréchet productivity draws, the probability that origin u delivers the lowest price is proportional to its origin-specific price-competitiveness term $\Omega_{iu,k}$.

Using the CES demand in Equation (B.21), expenditure on varieties sourced from origin u is

$$X_{iu,k} = Q_{i,k} P_{i,k}^{\eta_{i,k}} \int_{\mathcal{Z}_{iu,k}} p_{i,k}(z_k)^{1-\eta_{i,k}}\, d\Phi_k(z_k). \tag{B.36}$$

Similarly, total expenditure on sector-k varieties is

$$X_{i,k} = Q_{i,k} P_{i,k}^{\eta_{i,k}} \int_{\mathbb{R}_+^N} p_{i,k}(z_k)^{1-\eta_{i,k}}\, d\Phi_k(z_k). \tag{B.37}$$

Substituting Equation (B.35) into Equation (B.36) and comparing the result with Equation (B.37) yields

$$\pi_{iu,k} = \frac{X_{iu,k}}{X_{i,k}} = \frac{\Omega_{iu,k}}{\Omega_{i,k}}. \tag{B.38}$$

Finally, substituting the origin-specific term in Equation (B.24) and the aggregate term in Equation (B.25) into Equation (B.38) gives

$$\pi_{iu,k} = \frac{\left(\kappa_{iu,k} x_{u,k}\right)^{-\theta_k} A_{u,k}^{\theta_k \gamma_{u,k}}}{\sum_{n \in \mathcal{N}} \left(\kappa_{in,k} x_{n,k}\right)^{-\theta_k} A_{n,k}^{\theta_k \gamma_{n,k}}}. \tag{B.39}$$

Equation (B.39) is the gravity equation for bilateral expenditure shares. It shows that destination i sources a larger share of sector-k varieties from origins with lower trade costs, lower input-bundle costs, or higher deterministic productivity.

B.6 Market Clearing Conditions

Factor markets. Let $L_{i,k}$ and $H_{i,k}$ denote aggregate labor and structures used by producers in region-sector pair (i,k), after aggregating across varieties. Regional labor market clearing requires

$$\sum_{k \in \mathcal{K}} L_{i,k} = L_i, \qquad \forall i \in \mathcal{N}.$$

Aggregate labor is fixed, so population across regions satisfies

$$\sum_{i \in \mathcal{N}} L_i = \overline{L}.$$

Structures are fixed by region and fully allocated across sectors. Hence,

$$\sum_{k \in \mathcal{K}} H_{i,k} = \overline{H}_i, \qquad \forall i \in \mathcal{N}.$$

Final-goods markets. Final goods in each region-sector pair are used either for household consumption or as material inputs by local producers. Let $M_{i,hk}$ denote the aggregate quantity of sector-k final goods used as materials by producers in sector h located in region i. Market clearing for the sector-k final good in region i requires

$$Q_{i,k} = L_i C_{i,k} + \sum_{h \in \mathcal{K}} M_{i,hk}, \qquad \forall i \in \mathcal{N},\ k \in \mathcal{K}. \tag{B.40}$$

Multiplying Equation (B.40) by $P_{i,k}$ and using the definition of sectoral expenditure in Equation (B.5) gives

$$X_{i,k} = P_{i,k} L_i C_{i,k} + \sum_{h \in \mathcal{K}} P_{i,k} M_{i,hk}. \tag{B.41}$$

The first term in Equation (B.41) is household expenditure on sector-k final goods. Using the Cobb-Douglas demand in Equation (B.1), it is equal to

$$P_{i,k} L_i C_{i,k} = \alpha_k L_i I_i. \tag{B.42}$$

The second term in Equation (B.41) is material demand by local producers. Producers in region-sector pair (i, h) spend a share $\gamma_{i,hk}$ of their revenue on sector-k materials. Total revenue of producers in (i, h) is the value of sales to all destinations:

$$R_{i,h} = \sum_{u \in \mathcal{N}} \pi_{ui,h} X_{u,h}, \tag{B.43}$$

where $\pi_{ui,h}$ is the share of destination u's expenditure on sector-h goods sourced from origin i. Therefore,

$$P_{i,k} M_{i,hk} = \gamma_{i,hk} R_{i,h}. \tag{B.44}$$

Substituting Equations (B.42), (B.43), and (B.44) into Equation (B.41) yields the expenditure market clearing condition:

$$X_{i,k} = \alpha_k L_i I_i + \sum_{h \in \mathcal{K}} \gamma_{i,hk} \sum_{u \in \mathcal{N}} \pi_{ui,h} X_{u,h}. \tag{B.45}$$

Equation (B.45) states that expenditure on the sector-k final good in region i equals local household demand plus material demand from all sectors located in region i.

Regional trade balance. Destination i spends $\pi_{iu,k} X_{i,k}$ on sector-k goods sourced from origin u. Hence, aggregate purchases by region i from all origins and sectors are

$$\sum_{k \in \mathcal{K},\, u \in \mathcal{N}} \pi_{iu,k} X_{i,k}.$$

Similarly, aggregate sales by region i to all destinations and sectors are

$$\sum_{k \in \mathcal{K},\, u \in \mathcal{N}} \pi_{ui,k} X_{u,k}.$$

Using the regional imbalance defined in Equation (B.3), regional trade balance requires

$$\sum_{k \in \mathcal{K},\, u \in \mathcal{N}} \pi_{iu,k} X_{i,k} + \Upsilon_i = \sum_{k \in \mathcal{K},\, u \in \mathcal{N}} \pi_{ui,k} X_{u,k}, \qquad \forall i \in \mathcal{N}. \tag{B.46}$$

Given the sign convention in Equation (B.3), a positive Υ_i means that region i makes a net payment to the global portfolio. In equilibrium, this net payment is financed by sales exceeding purchases. Conversely, a negative Υ_i allows purchases to exceed sales.

Equation (B.46) allows trade to be unbalanced at the regional level. Aggregate trade is balanced because regional imbalances sum to zero by Equation (B.1), and because bilateral expenditure shares satisfy $\sum_{u \in \mathcal{N}} \pi_{iu,k} = 1$ for each destination-sector pair (i, k).

B.7 Labor Mobility

Workers are freely mobile across regions. Therefore, in equilibrium, all populated regions deliver the same indirect utility. Using the indirect-utility condition in Equation (B.1), we have

$$\frac{I_i}{P_i} = \mathcal{U}, \qquad \forall i \in \mathcal{N}. \tag{B.47}$$

We now derive the labor allocation implied by Equation (B.47). The derivation uses the cost-share conditions of intermediate-goods producers in Equation (B.6). For a producer in region-sector pair (i, k), the ratio of expenditure on structures to expenditure on labor is

$$r_i h_{i,k}(z_{i,k}) = \frac{\beta_i}{1 - \beta_i} w_i l_{i,k}(z_{i,k}). \tag{B.48}$$

Aggregating Equation (B.48) across varieties and sectors, and using the labor and structures market-clearing conditions in Equations (B.6) and (B.6), gives

$$r_i \overline{H}_i = \frac{\beta_i}{1 - \beta_i} w_i L_i. \tag{B.49}$$

Define the local factor-price index as

$$\varpi_i \equiv \left(\frac{w_i}{1 - \beta_i} \right)^{1 - \beta_i} \left(\frac{r_i}{\beta_i} \right)^{\beta_i}. \tag{B.50}$$

Using Equation (B.49), the rental rate satisfies

$$\frac{r_i}{\beta_i} = \frac{w_i}{1 - \beta_i} \frac{L_i}{\overline{H}_i}. \tag{B.51}$$

Substituting Equation (B.51) into the factor-price index in Equation (B.50) gives

$$\frac{w_i}{1 - \beta_i} = \varpi_i \left(\frac{\overline{H}_i}{L_i} \right)^{\beta_i}. \tag{B.52}$$

Next, use the household income equation in Equation (B.1) and the regional imbalance definition in Equation (B.3). Dividing Equation (B.3) by L_i gives

$$\upsilon_i \equiv \frac{\Upsilon_i}{L_i} = \iota_i r_i \frac{\overline{H}_i}{L_i} - \chi. \tag{B.53}$$

Combining Equation (B.1) with Equation (B.53) gives

$$I_i = w_i + r_i \frac{\overline{H}_i}{L_i} - \upsilon_i. \tag{B.54}$$

Using the aggregate factor-ratio condition in Equation (B.49), Equation (B.54) becomes

$$I_i = \frac{w_i}{1 - \beta_i} - \upsilon_i. \tag{B.55}$$

Substituting the wage-factor-price relation in Equation (B.52) into Equation (B.55) yields

$$I_i = \varpi_i \left(\frac{\overline{H}_i}{L_i} \right)^{\beta_i} - \upsilon_i. \tag{B.56}$$

Substituting Equation (B.56) into the mobility condition in Equation (B.47) gives

$$\mathcal{U} P_i + \upsilon_i = \varpi_i \left(\frac{\overline{H}_i}{L_i} \right)^{\beta_i}. \tag{B.57}$$

Solving Equation (B.57) for L_i gives

$$L_i = \left(\frac{\varpi_i}{\mathcal{U} P_i + \upsilon_i} \right)^{1/\beta_i} \overline{H}_i.$$

This expression assumes $\beta_i \in (0, 1)$, so that structures generate local congestion and the labor allocation is well defined.

Finally, imposing aggregate labor market clearing in Equation (B.6) normalizes the regional labor allocation. The free-mobility condition is therefore

$$L_i = \frac{\left(\frac{\varpi_i}{\mathcal{U} P_i + \upsilon_i} \right)^{1/\beta_i} \overline{H}_i}{\sum_{u \in \mathcal{N}} \left(\frac{\varpi_u}{\mathcal{U} P_u + \upsilon_u} \right)^{1/\beta_u} \overline{H}_u} \overline{L}. \tag{B.58}$$

Equation (B.58) states that labor flows toward regions with higher factor-price indices or larger fixed-factor endowments, and away from regions with higher consumption prices or larger per-capita net payments to the global portfolio.

B.8 General Equilibrium

We first define the competitive equilibrium in levels and then state the system used to solve counterfactual equilibria in relative changes.

Definition 1. Competitive equilibrium in levels. Given factor endowments $\{\overline{L}, \overline{H}_i\}$, region-sector productivities $\{\bar{A}_{i,k}\}$, and iceberg trade costs $\{\kappa_{iu,k}\}$, a *competitive equilibrium* is characterized by a common level of utility $\mathcal{U}$; an allocation of population across regions $\{L_i\}$ and across sectors within each region $\{L_{i,k}\}$; land allocations $\{H_{i,k}\}$; sectoral expenditure, per capita consumption, and intermediate input use $\{X_{i,k}, C_{i,k}, M_{i,kh}\}$; final-goods and factor prices $\{P_{i,k}, r_i, w_i\}$; regional trade imbalances $\{\Upsilon_i\}$; and bilateral expenditure shares on intermediate inputs $\{\pi_{iu,k}\}$, and deterministic productivity levels $\{A_{i,k}\}$, such that:

1. *Regional budget constraints.* Regional trade imbalances are consistent with each region's budget constraint, as implied by Equation (B.3).

2. *Intermediate-goods producers' optimization.* In each region-sector pair, intermediate-goods producers choose input bundles that minimize unit production costs, taking factor and input prices as given, in a manner consistent with Equation (B.8).

3. *Deterministic Productivity.* Deterministic productivity levels are consistent with the knowledge diffusion process described in Equation (B.18).

4. *Final-goods producers' optimization.* Final-goods producers aggregate intermediate varieties sourced from different regions and set competitive prices. Sourcing decisions across origins are governed by the pricing condition and the system of trade shares defined in Equations (B.34) and (B.39).

5. *Market clearing.* All markets clear in equilibrium. In particular, labor markets, land markets, final-goods markets, and sectoral expenditure allocations satisfy Equations (B.6), (B.6), (B.40), and (B.45).

6. *Aggregate trade balance.* The aggregate trade balance holds at the country level, in accordance with Equation (B.46).

7. *Labor mobility and indifference condition.* Households are perfectly mobile across locations, implying equalization of indirect utility across populated regions. Equation (B.58) therefore determines the equilibrium spatial allocation of labor, while sectoral allocation is pinned down by firms' labor demand.

Counterfactual equilibrium in relative changes. We solve counterfactual equilibria using hat algebra. Let x' denote the counterfactual value of variable x, and for strictly positive variables define $\hat{x} \equiv x'/x$. Variables that may be zero or negative, such as regional imbalances, are kept in counterfactual levels rather than expressed as hats.

The counterfactual shocks are changes in productivity fundamentals, trade costs, and fixed factors: $\{\hat{\bar{A}}_{i,k},\ \hat{\kappa}_{iu,k},\ \hat{\bar{H}}_i\}$. The knowledge-network multiplier $\mathbf{L} = (\mathbf{I} - \varphi\mathbf{W})^{-1}$ is held fixed across equilibria.

Productivity. Taking the ratio of the counterfactual and baseline versions of Equation (B.18) gives

$$\hat{A}_{i,k} = \prod_{u\in\mathcal{N},\, h\in\mathcal{K}} \hat{\bar{A}}_{u,h}^{\ell_{iu,kh}}.$$

Input-bundle costs. Taking relative changes in the input-bundle cost in Equation (B.8), and using the factor-price index in Equation (B.50), gives

$$\hat{x}_{i,k} = \hat{\varpi}_i^{\gamma_{i,k}} \prod_{h\in\mathcal{K}} \hat{P}_{i,h}^{\gamma_{i,kh}},$$

where

$$\hat{\varpi}_i = \hat{r}_i^{\beta_i} \hat{w}_i^{1-\beta_i}.$$

Final-good prices. Taking the ratio of the counterfactual and baseline versions of Equation (B.34), using the baseline trade shares in Equation (B.39), gives

$$\hat{P}_{i,k} = \left[\sum_{u\in\mathcal{N}} \pi_{iu,k} \left(\hat{\kappa}_{iu,k} \hat{x}_{u,k} \right)^{-\theta_k} \hat{A}_{u,k}^{\theta_k \gamma_{u,k}} \right]^{-1/\theta_k}. \tag{B.59}$$

The corresponding change in the household price index in Equation (B.1) is

$$\hat{P}_i = \prod_{k\in\mathcal{K}} \hat{P}_{i,k}^{\alpha_k}.$$

Trade shares. Taking the counterfactual version of Equation (B.39) and using Equation (B.59) to normalize across origins gives

$$\pi'_{iu,k} = \pi_{iu,k} \left(\frac{\hat{x}_{u,k}}{\hat{P}_{i,k}} \hat{\kappa}_{iu,k} \right)^{-\theta_k} \hat{A}_{u,k}^{\theta_k \gamma_{u,k}}. \tag{B.60}$$

Equation (B.60) preserves $\sum_{u \in \mathcal{N}} \pi'_{iu,k} = 1$ for every destination-sector pair (i, k).

Labor mobility. Taking relative changes in the unnormalized labor-allocation condition in Equation (B.7) gives

$$\hat{L}_i = \left(\frac{\hat{\varpi}_i}{\frac{P'_i \mathcal{U}' + \upsilon'_i}{P_i \mathcal{U} + \upsilon_i}} \right)^{1/\beta_i} \hat{\overline{H}}_i. \tag{B.61}$$

The denominator in Equation (B.61) can be written as

$$\begin{aligned} \frac{P'_i \mathcal{U}' + \upsilon'_i}{P_i \mathcal{U} + \upsilon_i} &= \hat{P}_i \hat{\mathcal{U}} \frac{P_i \mathcal{U}}{P_i \mathcal{U} + \upsilon_i} + \hat{\upsilon}_i \frac{\upsilon_i}{P_i \mathcal{U} + \upsilon_i} \\ &= \mu_i \hat{P}_i \hat{\mathcal{U}} + (1 - \mu_i) \hat{\upsilon}_i, \end{aligned} \tag{B.62}$$

where

$$\mu_i \equiv \frac{P_i \mathcal{U}}{P_i \mathcal{U} + \upsilon_i} = \frac{1}{1 + \frac{\upsilon_i}{P_i \mathcal{U}}}.$$

Substituting Equation (B.62) into Equation (B.61) yields

$$\hat{L}_i = \left(\frac{\hat{\varpi}_i}{\mu_i \hat{P}_i \hat{\mathcal{U}} + (1 - \mu_i) \hat{\upsilon}_i} \right)^{1/\beta_i} \hat{\overline{H}}_i. \tag{B.63}$$

Imposing aggregate labor-market clearing in Equation (B.6), the normalized regional employment change is

$$\hat{L}_i = \frac{\left[\frac{\hat{\varpi}_i}{\mu_i \hat{P}_i \hat{\mathcal{U}} + (1 - \mu_i) \hat{\upsilon}_i} \right]^{1/\beta_i} \hat{\overline{H}}_i}{\sum_{u \in \mathcal{N}} \left(\frac{L_u}{\overline{L}} \right) \left[\frac{\hat{\varpi}_u}{\mu_u \hat{P}_u \hat{\mathcal{U}} + (1 - \mu_u) \hat{\upsilon}_u} \right]^{1/\beta_u} \hat{\overline{H}}_u}. \tag{B.64}$$

To close the mobility block, use the indirect-utility condition derived in Equation (B.57). Taking relative changes in that equation gives

$$\hat{\mathcal{U}} = \frac{1}{\mu_i}\frac{\hat{\varpi}_i}{\hat{P}_i}\hat{\overline{H}}_i^{\beta_i}\hat{L}_i^{-\beta_i} - \frac{1-\mu_i}{\mu_i}\frac{\hat{\upsilon}_i}{\hat{P}_i}. \tag{B.65}$$

Since $\hat{\mathcal{U}}$ is common across regions and aggregate labor is fixed, Equation (B.65) can equivalently be aggregated using the counterfactual employment weights. Combining Equation (B.65) with aggregate labor-market clearing in Equation (B.6) gives

$$\hat{\mathcal{U}} = \frac{1}{\overline{L}}\sum_{i\in\mathcal{N}} L_i \left[\frac{1}{\mu_i}\frac{\hat{\varpi}_i}{\hat{P}_i}\hat{\overline{H}}_i^{\beta_i}\hat{L}_i^{1-\beta_i} - \frac{1-\mu_i}{\mu_i}\frac{\hat{L}_i\hat{\upsilon}_i}{\hat{P}_i}\right]. \tag{B.66}$$

Portfolio payments and regional imbalances. Using the factor-price representation derived from Equation (B.50), counterfactual factor income in region i can be written as

$$L_i' I_i' + \Upsilon_i' = \hat{\varpi}_i \hat{\overline{H}}_i^{\beta_i}\hat{L}_i^{1-\beta_i}\left(L_i I_i + \Upsilon_i\right). \tag{B.67}$$

Since structures receive a share β_i of factor income, the counterfactual return to the global portfolio is

$$\chi' = \frac{\sum_{n\in\mathcal{N}} \iota_n \beta_n \hat{\varpi}_n \hat{\overline{H}}_n^{\beta_n}\hat{L}_n^{1-\beta_n}\left(L_n I_n + \Upsilon_n\right)}{\overline{L}}. \tag{B.68}$$

The counterfactual regional imbalance is the difference between the contribution of region i to the portfolio and the portfolio income received by its residents:

$$\Upsilon_i' = \iota_i \beta_i \hat{\varpi}_i \hat{\overline{H}}_i^{\beta_i}\hat{L}_i^{1-\beta_i}\left(L_i I_i + \Upsilon_i\right) - \chi' \hat{L}_i L_i. \tag{B.69}$$

Equations (B.68) and (B.69) are the counterfactual counterparts of Equations (B.1) and (B.3).

Final-goods expenditure. The counterfactual version of the expenditure market-clearing condition in Equation (B.45) is

$$X_{i,k}' = \alpha_k L_i' I_i' + \sum_{h\in\mathcal{K}} \gamma_{i,hk} \sum_{u\in\mathcal{N}} \pi_{ui,h}' X_{u,h}'. \tag{B.70}$$

Rearranging Equation (B.67) gives

$$L_i' I_i' = \hat{\varpi}_i \hat{\overline{H}}_i^{\beta_i} \hat{L}_i^{1-\beta_i} \left(L_i I_i + \Upsilon_i \right) - \Upsilon_i'. \tag{B.71}$$

Substituting Equation (B.71) into Equation (B.70) gives

$$X_{i,k}' = \alpha_k \left(\hat{\varpi}_i \hat{\overline{H}}_i^{\beta_i} \hat{L}_i^{1-\beta_i} \left(L_i I_i + \Upsilon_i \right) - \Upsilon_i' \right) + \sum_{h \in \mathcal{K}} \gamma_{i,hk} \sum_{u \in \mathcal{N}} \pi_{ui,h}' X_{u,h}'. \tag{B.72}$$

Regional factor-price index. Counterfactual factor income must equal value added generated by producers located in region i. Using the counterfactual trade shares in Equation (B.60), this condition is

$$L_i' I_i' + \Upsilon_i' = \sum_{k \in \mathcal{K}} \gamma_{i,k} \sum_{u \in \mathcal{N}} \pi_{ui,k}' X_{u,k}'. \tag{B.73}$$

Combining Equation (B.73) with the factor-income representation in Equation (B.67) gives the regional factor-price-index equation:

$$\hat{\varpi}_i = \frac{\sum_{k \in \mathcal{K}} \gamma_{i,k} \sum_{u \in \mathcal{N}} \pi_{ui,k}' X_{u,k}'}{\hat{\overline{H}}_i^{\beta_i} \hat{L}_i^{1-\beta_i} \left(L_i I_i + \Upsilon_i \right)}. \tag{B.74}$$

Definition 2. Competitive equilibrium in relative changes. *Given baseline data* $\{I_i, L_i, \Upsilon_i, \pi_{iu,k}, X_{i,k}\}$ *and the structural parameters for the economy and productivity process* $\{\theta_k, \alpha_k, \beta_i, \iota_i, \gamma_{i,k}, \gamma_{i,kh}, \lambda_{kh}, \delta_{iu}, \varphi\}$, *a* competitive equilibrium in relative changes *is a collection of proportional changes* $\{\hat{A}_{i,k}, \hat{x}_{i,k}, \hat{P}_{i,k}, \hat{P}_i, \hat{L}_i, \hat{\varpi}_i\}$, *together with counterfactual trade shares, expenditure levels, portfolio returns, and regional imbalances* $\{\pi_{iu,k}', X_{i,k}', \chi', \Upsilon_i'\}$, *which, for a given vector of exogenous shocks* $\{\hat{\overline{A}}_{i,k}, \hat{\overline{H}}_i, \hat{\kappa}_{iu,k}\}$, *satisfy the following system:*

1. *Deterministic productivity.* Taking the ratio of the counterfactual and baseline versions of Equation (B.18) gives

$$\hat{A}_{i,k} = \prod_{u \in \mathcal{N},\, h \in \mathcal{K}} \left(\hat{\overline{A}}_{u,h} \right)^{\ell_{iu,kh}}.$$

2. *Input bundle costs.* Taking relative changes in Equation (B.8) and using the factor-price index in Equation (B.50) gives

$$\hat{x}_{i,k} = (\hat{\varpi}_i)^{\gamma_{i,k}} \prod_{h \in \mathcal{K}} \hat{P}_{i,h}^{\gamma_{i,kh}}.$$

3. *Final-goods prices.* Taking the ratio of the counterfactual and baseline versions of Equation (B.34), and using the baseline trade shares in Equation (B.39), gives

$$\hat{P}_{i,k} = \left[\sum_{u \in \mathcal{N}} \pi_{iu,k} (\hat{\kappa}_{iu,k} \hat{x}_{u,k})^{-\theta_k} \left(\hat{A}_{u,k} \right)^{\theta_k \gamma_{u,k}} \right]^{-\frac{1}{\theta_k}}. \tag{B.75}$$

The household price index in Equation (B.1) changes according to

$$\hat{P}_i = \prod_{k \in \mathcal{K}} \hat{P}_{i,k}^{\alpha_k}.$$

4. *Trade shares.* Taking the counterfactual version of Equation (B.39) and using Equation (B.59) to normalize across origins gives

$$\pi'_{iu,k} = \pi_{iu,k} \left(\frac{\hat{x}_{u,k}}{\hat{P}_{i,k}} \hat{\kappa}_{iu,k} \right)^{-\theta_k} \left(\hat{A}_{u,k} \right)^{\theta_k \gamma_{u,k}}. \tag{B.76}$$

5. *Labor mobility.* Taking relative changes in the unnormalized labor-allocation condition in Equation (B.7), and normalizing with aggregate labor clearing in Equation (B.6), gives

$$\hat{L}_i = \frac{\left[\frac{\hat{\varpi}_i}{\mu_i \hat{P}_i \hat{\mathcal{U}} + (1-\mu_i)\hat{\upsilon}_i} \right]^{1/\beta_i} \hat{\bar{H}}_i}{\sum_{u \in \mathcal{N}} \left(\frac{L_u}{\bar{L}} \right) \left[\frac{\hat{\varpi}_u}{\mu_u \hat{P}_u \hat{\mathcal{U}} + (1-\mu_u)\hat{\upsilon}_u} \right]^{1/\beta_u} \hat{\bar{H}}_u}, \tag{B.77}$$

where

$$\mu_i \equiv \frac{P_i \mathcal{U}}{P_i \mathcal{U} + \upsilon_i} = \frac{1}{1 + \frac{\upsilon_i}{P_i \mathcal{U}}}.$$

6. *Portfolio payments and regional imbalances.* Using the counterfactual factor-income representation implied by Equation (B.50), the counterfactual return to the global portfolio is

$$\chi' = \frac{\sum_{n\in\mathcal{N}} \iota_n \beta_n \hat{\varpi}_n \hat{\overline{H}}_n^{\beta_n} \hat{L}_n^{1-\beta_n} (L_n I_n + \Upsilon_n)}{\overline{L}}. \tag{B.78}$$

The counterfactual regional imbalance is

$$\Upsilon_i' = \iota_i \beta_i \hat{\varpi}_i \hat{\overline{H}}_i^{\beta_i} \hat{L}_i^{1-\beta_i} (L_i I_i + \Upsilon_i) - \chi' \hat{L}_i L_i. \tag{B.79}$$

7. *Total expenditure.* The counterfactual version of the expenditure market-clearing condition in Equation (B.45), after substituting the counterfactual expression for household income implied by Equation (B.69), is

$$X_{i,k}' = \alpha_k \left(\hat{\varpi}_i \hat{\overline{H}}_i^{\beta_i} \hat{L}_i^{1-\beta_i} (L_i I_i + \Upsilon_i) - \Upsilon_i' \right) + \sum_{h\in\mathcal{K},\, u\in\mathcal{N}} \gamma_{i,hk}\, \pi_{ui,h}'\, X_{u,h}'. \tag{B.80}$$

8. *Regional factor prices.* Counterfactual factor income must equal value added generated by producers located in region i. Combining this condition with the factor-income representation implied by Equation (B.50) gives

$$\hat{\varpi}_i = \frac{\sum_{k\in\mathcal{K},\, u\in\mathcal{N}} \gamma_{i,k}\, \pi_{ui,k}'\, X_{u,k}'}{\hat{\overline{H}}_i^{\beta_i} \hat{L}_i^{1-\beta_i} (L_i I_i + \Upsilon_i)}. \tag{B.81}$$

An equilibrium in relative changes is thus a fixed point of this system of hat equations, jointly determining counterfactual prices, trade shares, expenditure, portfolio payments, regional imbalances, labor allocations, and factor-price indices without recovering unobserved levels.

B.9 Computation

This section describes the numerical procedure used to solve the counterfactual equilibrium in relative changes. Exogenous shocks are given by any combination of $\{\hat{A}_{i,k}, \hat{\overline{H}}_i, \hat{\kappa}_{iu,k}\}$. The system is solved as a fixed point in the vector of regional factor-price changes $\hat{\boldsymbol{\varpi}} = \{\hat{\varpi}_i\}_{i\in\mathcal{N}}$.

Given a guess for $\hat{\boldsymbol{\varpi}}$, proceed as follows:

Step 1. Productivity. Compute deterministic productivity changes $\hat{A}_{i,k}$ using Equation (B.8). This step maps the exogenous productivity shocks $\{\hat{\bar{A}}_{i,k}\}$ into equilibrium productivity changes through the fixed knowledge-network multiplier.

Step 2. Input-bundle costs and final-good prices. Given the current guess for $\hat{\varpi}$ and the productivity changes computed in Step 1, solve the price-cost subsystem formed by Equations (B.8) and (B.59). This subsystem jointly determines $\{\hat{x}_{i,k}, \hat{P}_{i,k}\}$ because input-bundle costs depend on final-good prices, and final-good prices depend on input-bundle costs. After obtaining $\hat{P}_{i,k}$, compute the household price-index changes $\hat{P}_i$ using Equation (B.8).

Step 3. Trade shares. Given $\{\hat{x}_{i,k}, \hat{P}_{i,k}, \hat{A}_{i,k}\}$ from Steps 1 and 2, compute counterfactual bilateral expenditure shares $\pi'_{iu,k}$ using Equation (B.60). These shares satisfy $\sum_{u\in\mathcal{N}} \pi'_{iu,k} = 1$ for each destination-sector pair (i, k).

Step 4. Labor mobility, portfolio payments, and regional imbalances. Given $\{\hat{\varpi}_i, \hat{P}_i, \hat{\overline{H}}_i\}$, solve the mobility and portfolio block. This block consists of the labor-allocation equation in Equation (B.64), the welfare equation in Equation (B.66), the counterfactual portfolio return in Equation (B.68), and the counterfactual regional imbalance in Equation (B.69). The block jointly determines $\{\hat{L}_i, \hat{\mathcal{U}}, \chi', \Upsilon'_i\}_{i\in\mathcal{N}}$. Per-capita imbalances are updated as $\upsilon'_i = \frac{\Upsilon'_i}{\hat{L}_i L_i}$, $\hat{\upsilon}_i = \frac{\upsilon'_i}{\upsilon_i}$, when $\upsilon_i \neq 0$. When the baseline per-capita imbalance is zero, the implementation uses υ'_i directly in the mobility denominator rather than forming the ratio.

Step 5. Expenditure. Given the counterfactual trade shares from Step 3 and the labor, portfolio, and imbalance objects from Step 4, solve the counterfactual expenditure system in Equation (B.72). This is a linear system in the $N \times K$ unknowns $\{X'_{i,k}\}$.

Step 6. Update regional factor prices. Given $\{\pi'_{iu,k}, X'_{i,k}, \hat{L}_i\}$, update the regional factor-price changes using Equation (B.74):

$$\hat{\varpi}_i^* = \frac{\sum_{k\in\mathcal{K},\, u\in\mathcal{N}} \gamma_{i,k}\, \pi'_{ui,k} X'_{u,k}}{\hat{\overline{H}}_i^{\beta_i}\, \hat{L}_i^{1-\beta_i}\, (L_i I_i + \Upsilon_i)}.$$

Repeat Steps 1–6 until the regional factor-price vector converges:

$$\|\hat{\boldsymbol{\varpi}}^* - \hat{\boldsymbol{\varpi}}\| < \varepsilon.$$

In practice, the fixed-point iteration can be dampened by updating $\hat{\boldsymbol{\varpi}}$ as a weighted average of the previous guess and $\hat{\boldsymbol{\varpi}}^*$. At convergence, the resulting objects satisfy the relative-change equilibrium system in Definition 2.

B.10 Sufficient Statistics

This section describes how the equilibrium objects recovered above map into measured TFP, real GDP, and welfare. These objects are the sufficient statistics used to report the effects of productivity, trade-cost, and fixed-factor shocks.

B.10.1 Measured TFP

Measured TFP in region-sector pair (i, k) is constructed as gross output deflated by the sectoral final-good price index and net of the aggregate input bundle. In relative changes, this model-consistent measure is

$$\log \widehat{\text{TFP}}_{i,k} = \log \hat{x}_{i,k} - \log \hat{P}_{i,k}. \tag{B.82}$$

Equation (B.82) captures the change in measured productivity that remains after accounting for changes in the full input-bundle cost $\hat{x}_{i,k}$ and the output price $\hat{P}_{i,k}$.

To connect Equation (B.82) to trade shares, take the domestic-origin case $u = i$ in the trade-share equation in Equation (B.60). Dividing the counterfactual domestic expenditure share by its baseline value gives

$$\hat{\pi}_{ii,k} = \left(\frac{\hat{x}_{i,k}}{\hat{P}_{i,k}} \hat{\kappa}_{ii,k} \right)^{-\theta_k} \hat{A}_{i,k}^{\theta_k \gamma_{i,k}}. \tag{B.83}$$

Since internal trade costs satisfy $\kappa_{ii,k} = 1$ and are held fixed in the counterfactual, $\hat{\kappa}_{ii,k} = 1$. Combining Equations (B.82) and (B.83) therefore gives

$$\log \widehat{\text{TFP}}_{i,k} = \gamma_{i,k} \log \hat{A}_{i,k} - \frac{1}{\theta_k} \log \hat{\pi}_{ii,k}. \tag{B.84}$$

Equation (B.84) decomposes measured TFP into deterministic productivity and selection. The first term captures the contribution of deterministic productivity to value added in (i,k). The second term captures changes in the set of varieties supplied by local producers, summarized by the domestic expenditure share.

Aggregation. Let

$$Y_{i,k} \equiv \sum_{u \in \mathcal{N}} \pi_{ui,k} X_{u,k}$$

denote baseline gross revenue of producers located in region-sector pair (i,k). Regional, sectoral, and domestic aggregate measured TFP changes are computed using baseline gross-revenue weights:

$$\log \widehat{\text{TFP}}_i = \sum_{k \in \mathcal{K}} \left(\frac{Y_{i,k}}{\sum_{h \in \mathcal{K}} Y_{i,h}} \right) \log \widehat{\text{TFP}}_{i,k},$$

$$\log \widehat{\text{TFP}}_k = \sum_{i \in \mathcal{D}} \left(\frac{Y_{i,k}}{\sum_{u \in \mathcal{D}} Y_{u,k}} \right) \log \widehat{\text{TFP}}_{i,k}, \tag{B.85}$$

and

$$\log \widehat{\text{TFP}} = \sum_{i \in \mathcal{D}} \sum_{k \in \mathcal{K}} \left(\frac{Y_{i,k}}{\sum_{u \in \mathcal{D}} \sum_{h \in \mathcal{K}} Y_{u,h}} \right) \log \widehat{\text{TFP}}_{i,k}. \tag{B.86}$$

The restriction to $i \in \mathcal{D}$ in Equations (B.85) and (B.86) reflects that domestic aggregates are computed only over domestic locations.

B.10.2 Real GDP

Nominal value added in region-sector pair (i,k) is

$$\text{VA}_{i,k} = w_i L_{i,k} + r_i H_{i,k}. \tag{B.87}$$

Using the factor cost-share condition in Equation (B.49), Equation (B.87) can be written as

$$\text{VA}_{i,k} = \frac{w_i L_{i,k}}{1-\beta_i}. \tag{B.88}$$

Real GDP in (i,k) is nominal value added deflated by the sectoral price index $P_{i,k}$. Taking relative changes in Equation (B.88) and deflating by $\hat{P}_{i,k}$ gives

$$\log \widehat{\text{GDP}}_{i,k} = \log \hat{w}_i + \log \hat{L}_{i,k} - \log \hat{P}_{i,k}. \tag{B.89}$$

Using the measured-TFP definition in Equation (B.82), Equation (B.89) becomes

$$\log \widehat{\mathrm{GDP}}_{i,k} = \log \widehat{\mathrm{TFP}}_{i,k} + \log \hat{L}_{i,k} + \log \left(\frac{\hat{w}_i}{\hat{x}_{i,k}} \right). \tag{B.90}$$

Equation (B.90) decomposes real GDP changes into measured TFP, employment, and the wage relative to the full input-bundle cost.

The wage change can be recovered from the factor-price relation in Equation (B.52):

$$\hat{w}_i = \hat{\varpi}_i \hat{\overline{H}}_i^{\beta_i} \hat{L}_i^{-\beta_i}.$$

If sectoral employment changes are needed explicitly, they can be recovered from firms' labor demand. Let $Y_{i,k}$ be gross revenue as defined in Equation (B.10.1). Since labor payments are a fixed share $\gamma_{i,k}(1-\beta_i)$ of revenue,

$$\hat{L}_{i,k} = \frac{\sum_{u \in \mathcal{N}} \pi'_{ui,k} X'_{u,k}}{\sum_{u \in \mathcal{N}} \pi_{ui,k} X_{u,k}} \frac{1}{\hat{w}_i}. \tag{B.91}$$

Aggregation. Let baseline value-added weights be

$$s_{i,k}^{\mathrm{VA}} \equiv \frac{\mathrm{VA}_{i,k}}{\sum_{u \in \mathcal{D}} \sum_{h \in \mathcal{K}} \mathrm{VA}_{u,h}}. \tag{B.92}$$

Regional real GDP changes are aggregated using region-specific value-added weights:

$$\log \widehat{\mathrm{GDP}}_i = \sum_{k \in \mathcal{K}} \left(\frac{\mathrm{VA}_{i,k}}{\sum_{h \in \mathcal{K}} \mathrm{VA}_{i,h}} \right) \log \widehat{\mathrm{GDP}}_{i,k}. \tag{B.93}$$

Domestic sectoral real GDP changes are

$$\log \widehat{\mathrm{GDP}}_k = \sum_{i \in \mathcal{D}} \left(\frac{\mathrm{VA}_{i,k}}{\sum_{u \in \mathcal{D}} \mathrm{VA}_{u,k}} \right) \log \widehat{\mathrm{GDP}}_{i,k}. \tag{B.94}$$

The domestic aggregate real GDP change is

$$\log \widehat{\mathrm{GDP}} = \sum_{i \in \mathcal{D}} \sum_{k \in \mathcal{K}} s_{i,k}^{\mathrm{VA}} \log \widehat{\mathrm{GDP}}_{i,k}. \tag{B.95}$$

B.10.3 Welfare

Welfare is measured by the common indirect utility level. Using the household income equation in Equation (B.1), the factor-price relation in Equation (B.49), and the price index in Equation (B.1), indirect utility in region i can be written as

$$\mathcal{U} = \frac{(1-\beta_i \iota_i) w_i + (1-\beta_i)\chi}{(1-\beta_i) P_i}. \tag{B.96}$$

Taking relative changes in Equation (B.96) gives

$$\hat{\mathcal{U}} = \frac{\nu_i \hat{w}_i + (1-\nu_i)\hat{\chi}}{\hat{P}_i}, \tag{B.97}$$

where

$$\nu_i = \frac{(1-\beta_i \iota_i) w_i}{(1-\beta_i \iota_i) w_i + (1-\beta_i)\chi}. \tag{B.98}$$

Equation (B.97) holds for any populated region in equilibrium, since free mobility equalizes indirect utility.

Using the household price index in Equation (B.8) and the measured-TFP definition in Equation (B.82), Equation (B.97) can be written as

$$\log \hat{\mathcal{U}} = \sum_{k\in\mathcal{K}} \alpha_k \left[\log \widehat{\mathrm{TFP}}_{i,k} + \log\left(\nu_i \frac{\hat{w}_i}{\hat{x}_{i,k}} + (1-\nu_i)\frac{\hat{\chi}}{\hat{x}_{i,k}}\right)\right]. \tag{B.99}$$

Equation (B.99) shows that welfare changes are a sector-weighted average of measured TFP changes and changes in real factor income relative to input-bundle costs.

If $\iota_i = 0$ for all i, then $\chi = 0$ and $\nu_i = 1$. In this case, Equation (B.99) reduces to

$$\log \hat{\mathcal{U}} = \sum_{k\in\mathcal{K}} \alpha_k \left[\log \widehat{\mathrm{TFP}}_{i,k} + \log\left(\frac{\hat{w}_i}{\hat{x}_{i,k}}\right)\right]. \tag{B.100}$$

Using the real-GDP decomposition in Equation (B.90), Equation (B.100) can equivalently be written as

$$\log \hat{\mathcal{U}} = \sum_{k\in\mathcal{K}} \alpha_k \left(\log \widehat{\mathrm{GDP}}_{i,k} - \log \hat{L}_{i,k}\right). \tag{B.101}$$

C Taking the Model to the Data

This appendix complements the calibration discussion in Section 5 by documenting how each baseline object and structural parameter is mapped into the data. The main text summarizes the calibration strategy and reports the main targets in Table 1; here we provide the implementation details, measurement choices, and benchmark assumptions behind each object. We organize the discussion in the same order as Table 1, distinguishing between baseline data objects, parameters directly measured from the data, internally calibrated parameters, parameters imported from the literature, and benchmark assumptions.

The calibration relies on the Disaggregated Economic Accounts described in Section 2 and Appendix A, complemented with sector-level patent citation data and external parameter estimates from the literature. Our objective is to recover the baseline objects $\{I_i, L_i, \Upsilon_i, \pi_{iu,k}\}$ and the structural parameters $\{\alpha_k, \beta_i, \gamma_{i,k}, \gamma_{i,kh}, \iota_i, \theta_k, \lambda_{kh}, \delta_{iu}, \varphi\}$. Table 1 summarizes each object, its empirical counterpart, its type, and the source or target used in the calibration.

The remainder of this appendix proceeds object by object. We first describe the baseline data objects: income per worker, employment, regional trade imbalances, and bilateral expenditure shares. We then describe the structural parameters: final consumption shares, labor shares, value-added shares, local input-output linkages, global portfolio contributions, productivity dispersion, the sectoral knowledge network, geographic knowledge diffusion, and the knowledge-spillover elasticity.

C.1 Baseline Data

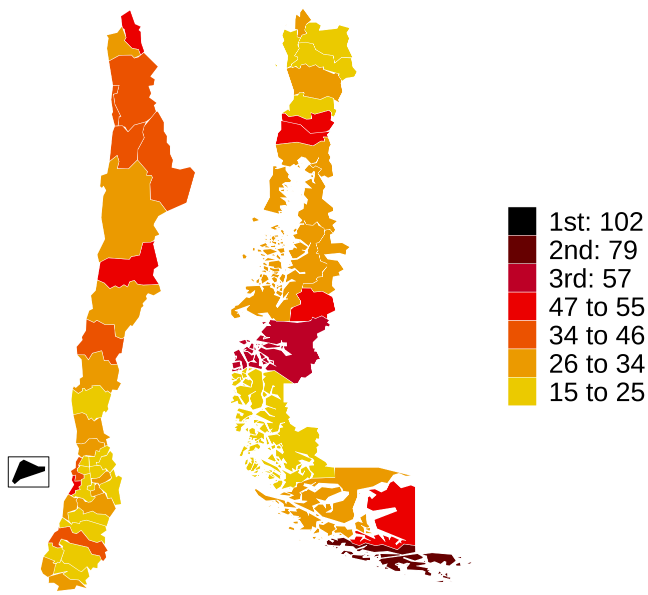
Income per worker, I_i. Income per worker is the empirical counterpart of household income in the model. For each location $i \in \mathcal{N}$, we construct it from regional value added, regional trade imbalances, and employment:

$$I_i = \frac{\mathrm{VA}_i - \Upsilon_i}{L_i}, \qquad \mathrm{VA}_i = \sum_{k \in \mathcal{K}} \mathrm{VA}_{i,k}. \tag{C.1}$$

Equivalently, $L_i I_i + \Upsilon_i = \mathrm{VA}_i$, so value added generated in location i is allocated between income accruing to residents, $L_i I_i$, and the net payment associated with the regional imbalance, Υ_i.

The sign convention follows Equation (B.3). A positive Υ_i means that location i makes a net payment to the global portfolio, so resident income is below value added. A negative Υ_i means that location i receives net income from the global portfolio, so resident income exceeds value added. Figure C.1 reports the resulting distribution of I_i across locations.

Figure C.1: Income per Worker (I_i)

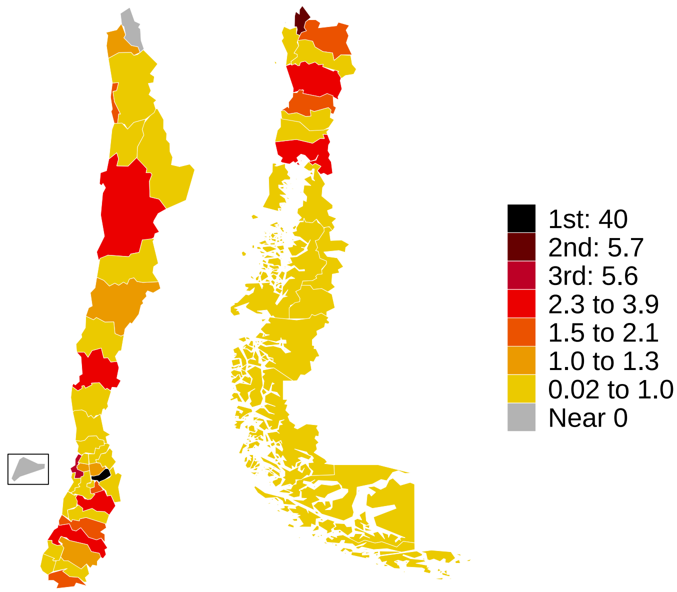


Notes: Income per worker expressed in thousands of 2022 USD. The Rest of the World is calibrated to 36 thousand 2022 USD.

Employment, L_i. Employment is taken directly from the Disaggregated Economic Accounts. For each location $i \in \mathcal{N}$, total employment is obtained by aggregating employment across sectors. This object is used both to construct income per worker in Equation (C.1) and to discipline the baseline spatial allocation of labor in the model.

For the quantitative solution, employment is normalized consistently with the aggregate labor-market clearing condition in Equation (B.6). Figure C.2 reports the resulting employment shares across domestic locations. The rest of the world is treated as an additional location in $\mathcal{N}$ and is included in the model's aggregate labor endowment.

Figure C.2: Employment Shares (L_i)



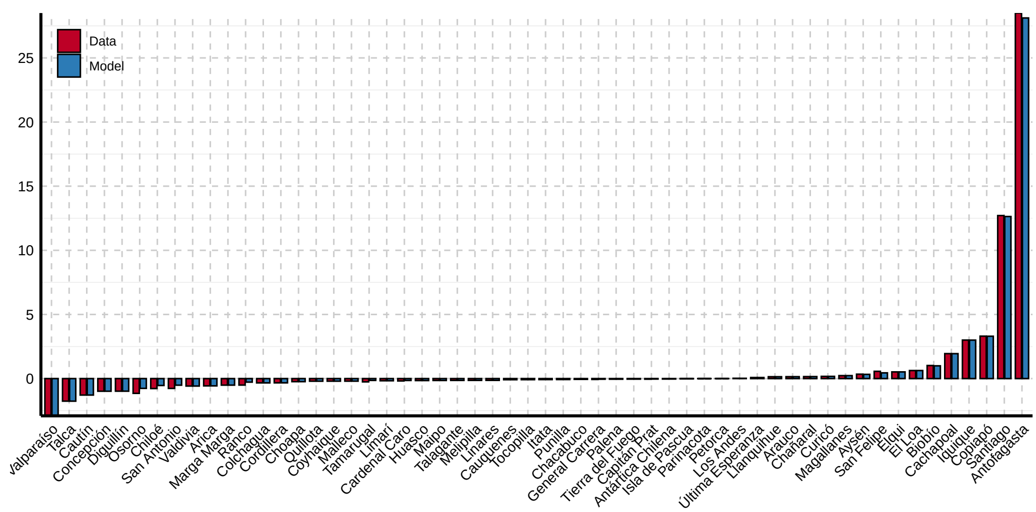
Notes: Shows each province's share of national employment. The Rest of the World is omitted for exposition. The model assumes Chile accounts for 0.36% of world employment (8.89 million workers), while the Rest of the World comprises 2.463 billion workers.

Regional trade imbalances, Υ_i. The empirical counterpart of regional trade imbalances is constructed from the geographic input-output matrix contained in the Disaggregated Economic Accounts, which records origin-destination flows across location-sector pairs. For each location $i \in \mathcal{N}$, we first compute the raw empirical imbalance, Υ_i^{Data}, as the total value of sales originating in location i minus the total value of purchases made by location i, aggregating across sectors and trading partners. This measure includes both domestic trade across locations and international trade with the rest of the world. Thus, a positive Υ_i^{Data} corresponds to a regional trade surplus, while a negative value corresponds to a regional trade deficit.

The model does not take these raw imbalances as unrestricted residuals. Instead, regional imbalances must be rationalized by the ownership structure of fixed factors described in Equation (B.3). We therefore use Υ_i^{Data} as the empirical target when calibrating the global portfolio contribution ι_i. Because the constraint $\iota_i \in [0, 1]$ may bind for some locations, the calibrated portfolio does not necessarily reproduce the raw empirical imbalance exactly. The baseline model uses the model-implied imbalance explained by the calibrated ownership structure, and the remaining unexplained residual is removed from the model baseline.

This adjustment ensures that the regional imbalances used in the quantitative model are consistent with the regional trade-balance condition in Equation (B.46), where purchases by location i plus the net payment Υ_i equal sales by location i. Figure C.3 reports the raw empirical imbalances and their model-implied counterparts after calibrating the global portfolio contribution ι_i.

Figure C.3: Provincial Trade Imbalances in Chile, 2022 (Billion USD)



Notes: Trade imbalances are reported by province in billion USD for 2022. The Rest of the World is omitted for visualization purposes; it records a balance of -37.24 billion USD in the data and -37.90 billion USD in the model calibration.

Bilateral trade shares, $\pi_{iu,k}$. Bilateral trade shares are constructed from the geographic input-output matrix contained in the Disaggregated Economic Accounts. The matrix records flows across origin-destination location-sector pairs, including domestic transactions and trade with the rest of the world. We treat observed firm-to-firm transactions as intermediate flows. Since the administrative data identify transactions between firms but do not observe firm-to-consumer sales with the same bilateral structure, we allocate final demand using the model's assumption that final goods are non-tradable.

Specifically, for each origin-sector pair, we compute final demand as the residual of total sales net of observed intermediate sales, and assign this residual to local final demand. Combining observed intermediate flows with this local final-demand allocation yields $X_{iu,k}$, the value of sector-k goods and services from origin location u absorbed in destination location i. We then define $\pi_{iu,k} = X_{iu,k}/\sum_{n\in\mathcal{N}} X_{in,k}$, so that, for each destination-sector pair (i,k), bilateral expenditure shares sum to one across supplying locations.

This measurement improves on standard implementations that rely on partial shipment surveys or national input-output tables to infer subnational trade flows. For example, Caliendo et al. (2018) use the *Commodity Flow Survey* for a limited set of manufacturing sectors and impose non-tradability in several service sectors because bilateral flow data are not observed. By contrast, the Disaggregated Economic Accounts observe firm-to-firm transactions across locations and sectors, including services and non-manufacturing activities. As a result, we do not need to impose zero tradability on entire industries or recover local input-output structure from aggregate technical coefficients.

This construction preserves the observed geography of intermediate transactions while using the model structure only to close the allocation of final demand. Figure C.4 reports illustrative examples of the resulting bilateral trade shares.

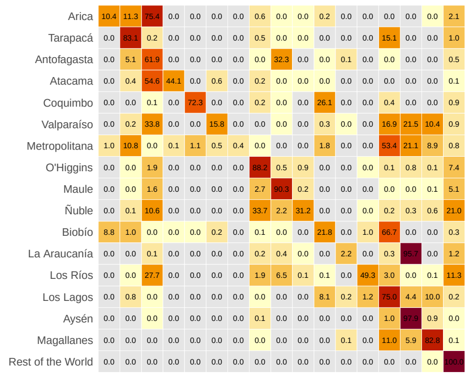
C.2 Structural Parameters

Final consumption shares, α_k. Final consumption shares are measured at the sector level. For each sector k, we compute final demand as total sales net of intermediate sales, aggregating across locations. We then normalize sectoral final demand by aggregate final demand to obtain α_k, so that the consumption shares across sectors sum to one. Because final demand is aggregated across all locations, including the Rest of the World, α_k reflects the composition of aggregate final consumption, which the common-share Cobb–Douglas preferences apply uniformly to every location.

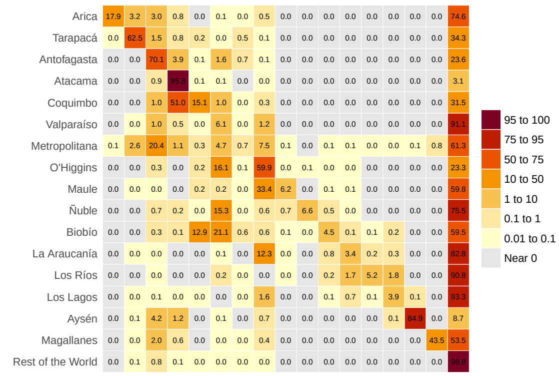
This construction is consistent with the model's treatment of sectoral final goods: final goods enter household consumption, while observed firm-to-firm transactions discipline intermediate demand and bilateral input-output flows. Figure C.5 reports the resulting distribution of final consumption shares across sectors.

Labor shares, $1-\beta_i$. The labor share is measured directly from the Disaggregated Economic Accounts as labor compensation over value added in each location. We aggregate labor compensation and value added across sectors within location i and compute the location-specific labor share $1-\beta_i$. The model parameter β_i, which governs the share of structures in value-added production, is then recovered as the residual.

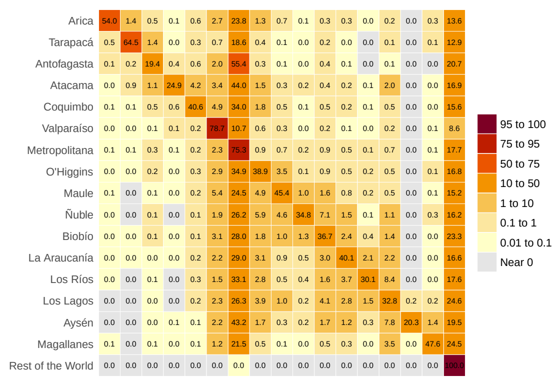
Figure C.4: Bilateral Trade Shares ($\pi_{iu,k}$): Illustrative Examples



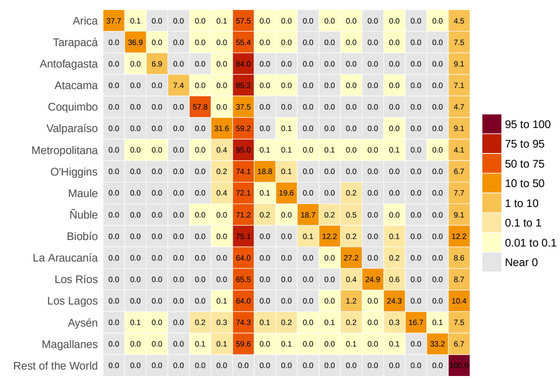
a) Fishing



b) Mining



c) Wholesale Trade

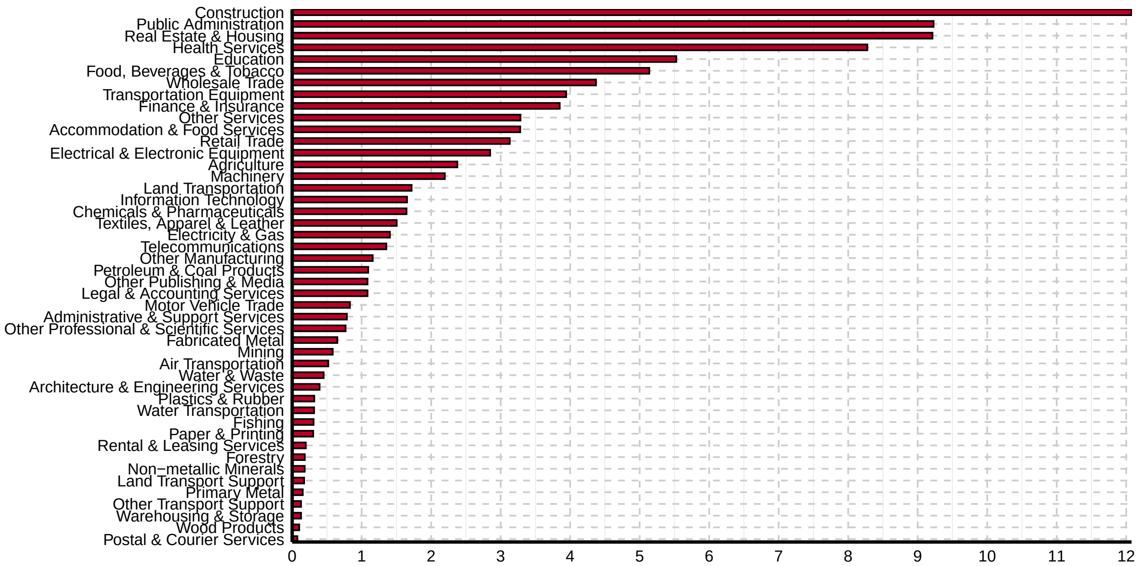


d) Finance & Insurance

Notes: The figure reports, conditional on the supplying sector, each buyer location's expenditure share across supplying locations. Panel (a) shows Fishing and Aquaculture, panel (b) Mining, panel (c) Wholesale Trade, and panel (d) Finance & Insurance. For visualization purposes, locations are aggregated to the regional level, covering Chile's 16 regions and the Rest of the World. The quantitative model, however, uses the same object computed at the province-cell level. Rows sum to 100.

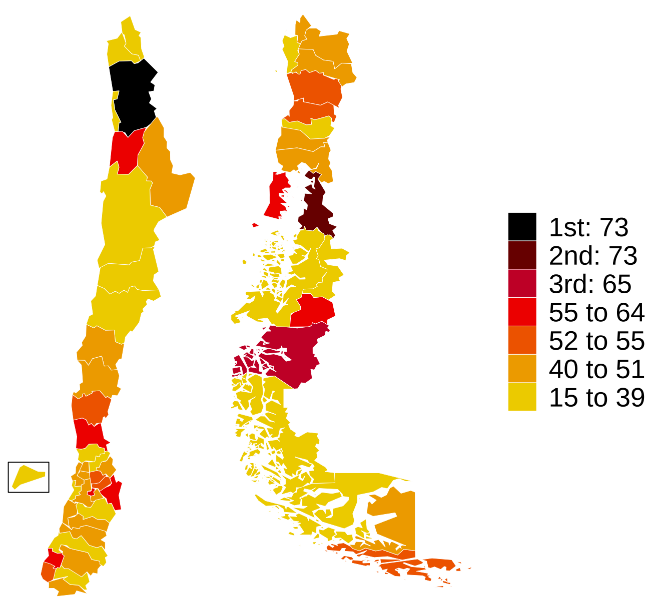
This calibration allows the relative importance of labor and fixed local structures to vary across locations. These differences matter for the strength of local congestion in the mobility block of the model: locations with lower labor shares have a larger fixed-factor component and therefore stronger congestion responses to productivity shocks. Figure C.6 reports the calibrated labor shares across locations.

Figure C.5: Final Consumption Shares by Sector (α_k)



Notes: Shows each sector's share of aggregate final consumption, computed across all locations including the Rest of the World.

Figure C.6: Labor Share of Value Added by Province ($1 - \beta_i$)

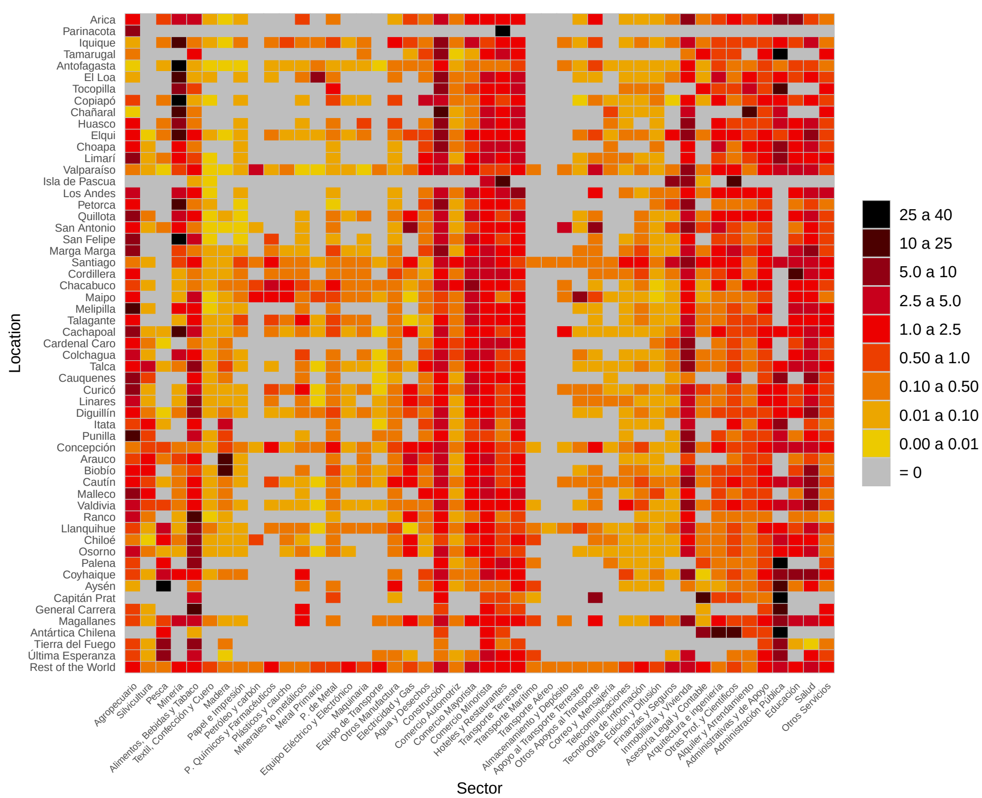


Notes: Labor's share of value added by province. The Rest of the World is set to 51%.

Value-added shares, $\gamma_{i,k}$. Value-added shares are measured directly from the Disaggregated Economic Accounts at the location-sector level. For each pair (i, k), we compute $\gamma_{i,k}$ as value added divided by total sales. This object is the empirical counterpart of the value-added share in the production function and determines the share of gross output paid to primary factors.

Together with the local input-output coefficients $\{\gamma_{i,kh}\}$, the value-added share ensures that production costs are exhausted across primary factors and intermediate inputs. Figure C.7 reports the distribution of $\gamma_{i,k}$ across location-sector pairs.

Figure C.7: Value Added Share in Gross Output by Province-Sector ($\gamma_{i,k}$)



Notes: Value added as a share of total gross output for each province-sector pair.

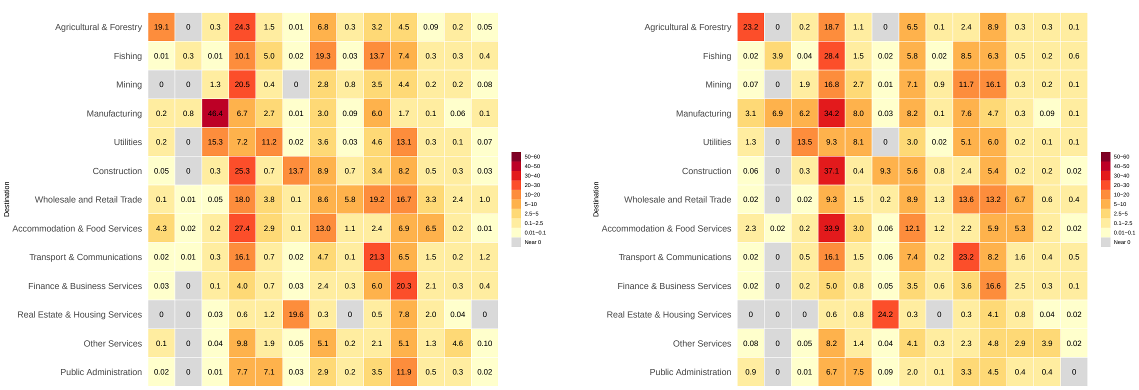
Local input-output linkages, $\gamma_{i,kh}$. Local input-output coefficients are constructed from observed intermediate input transactions in the geographic input-output matrix. For each location-sector buyer (i, k), we aggregate purchases of inputs from supplying sector

h across all origin locations, and divide this amount by total sales of the buyer (i, k). The resulting coefficient $\gamma_{i,kh}$ measures the share of gross output in location-sector (i, k) spent on materials from sector h.

This construction preserves local heterogeneity in production networks. Two locations operating in the same sector can have different input-output coefficients if they purchase different bundles of materials. This is an important difference relative to standard calibrations that recover local input-output coefficients by combining location-sector value-added shares with a national input-output matrix. Because the Disaggregated Economic Accounts observe intermediate transactions directly, we do not need to impose this proportionality rule. Instead, the local cost structure of each location-sector pair is measured from observed purchases of intermediate inputs.

Together with the value-added share $\gamma_{i,k}$, these coefficients discipline the split of gross output between primary factors and intermediate inputs. Figure C.8 reports two additional illustrative examples of the resulting local input-output linkages.

Figure C.8: Material Shares in Sales ($\gamma_{i,kh}$): Illustrative Examples



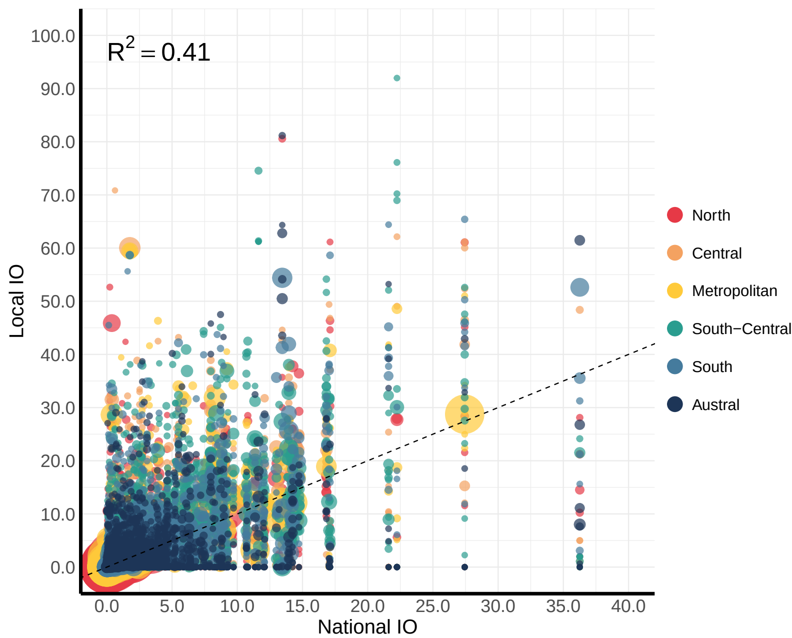
a) Valparaíso b) Concepción

Notes: The figure reports, conditional on location, the share of each input-supplying sector in the total sales of each purchasing sector. For visualization purposes, the 46 model sectors are aggregated into 13 broad sectors. Rows do not sum to 100 because the denominator is total sales, not total intermediate input purchases; adding value added would close each row to 100.

Figure C.9 compares the local input-output coefficients used in our benchmark calibration with the national coefficients used in the alternative model that suppresses spatial heterogeneity in production linkages. The relationship is positive but weak, with an R^2 of only 0.41. National coefficients therefore capture broad average input-output patterns, but they leave most of the cross-province variation in local production linkages unexplained.

For the same sector pair, some provinces exhibit almost no local dependence, while others display much stronger input shares than the national matrix would imply. This is precisely the dimension of heterogeneity that is lost when geographically disaggregated linkages are replaced with national coefficients. The figure thus provides direct evidence that spatially aggregated input-output matrices are an imperfect proxy for the local production structure faced by individual province-sector pairs.

Figure C.9: Local versus National Input-Output Shares



Notes: Each point is a province × origin-sector × destination-sector observation. The x-axis shows the national input-output share for the corresponding sector pair, replicated across provinces; the y-axis shows the province-specific share. Point size is proportional to gross output, and colors denote macrozones. Observations with zero national input-output shares are excluded, while observations with zero local shares are kept when the national share is positive.

Global portfolio contribution, ι_i. The portfolio contribution parameter ι_i is calibrated internally to rationalize regional trade imbalances through the ownership structure of fixed factors. The empirical target is the raw imbalance Υ_i^{Data} constructed from the geographic input-output matrix. The model-implied imbalance is

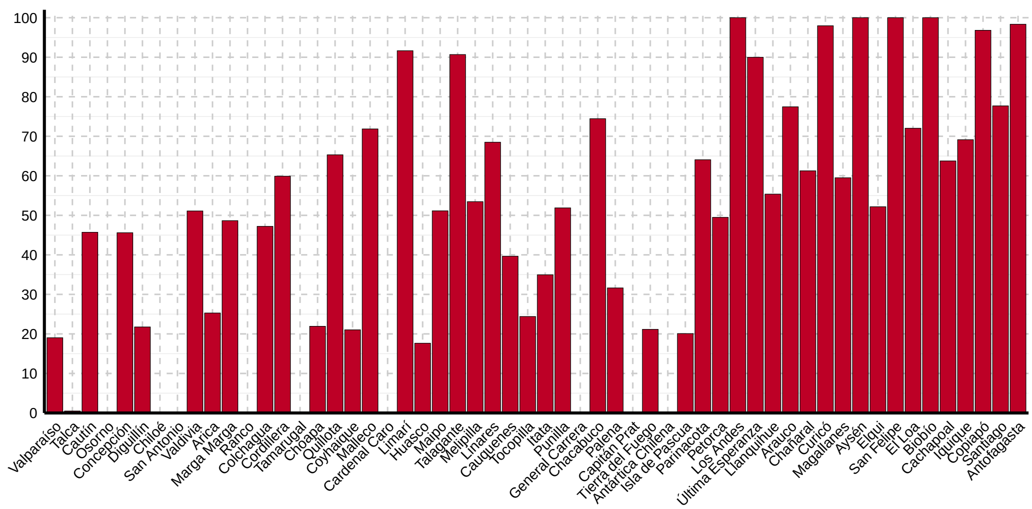
$$\Upsilon_i^{\text{Model}} = \iota_i r_i \overline{H}_i - \chi L_i, \qquad \chi = \frac{\sum_{n \in \mathcal{N}} \iota_n r_n \overline{H}_n}{\sum_{n \in \mathcal{N}} L_n}. \tag{C.2}$$

In the calibration, fixed-factor income is measured using the model's value-added shares, so that $r_i\overline{H}_i$ corresponds to the structures component of regional value added.

We choose $\{\iota_i\}_{i\in\mathcal{N}}$ to minimize the distance between $\Upsilon_i^{\text{Model}}$ and Υ_i^{Data}, subject to $\iota_i \in [0,1]$ for all locations and to $\Upsilon_i^{\text{Model}}$ preserving the sign of Υ_i^{Data} in each location. Because these bounds can bind, the calibrated portfolio does not necessarily match the raw empirical imbalances exactly. The baseline model therefore uses the component of trade imbalances explained by the calibrated ownership structure, while the residual difference between Υ_i^{Data} and $\Upsilon_i^{\text{Model}}$ is removed from the model baseline.

Figure C.10 reports the calibrated values of ι_i across locations, and Figure C.3 compares the raw empirical imbalances with their model-implied counterparts.

Figure C.10: Global Portfolio Contribution Across Locations (ι_i)

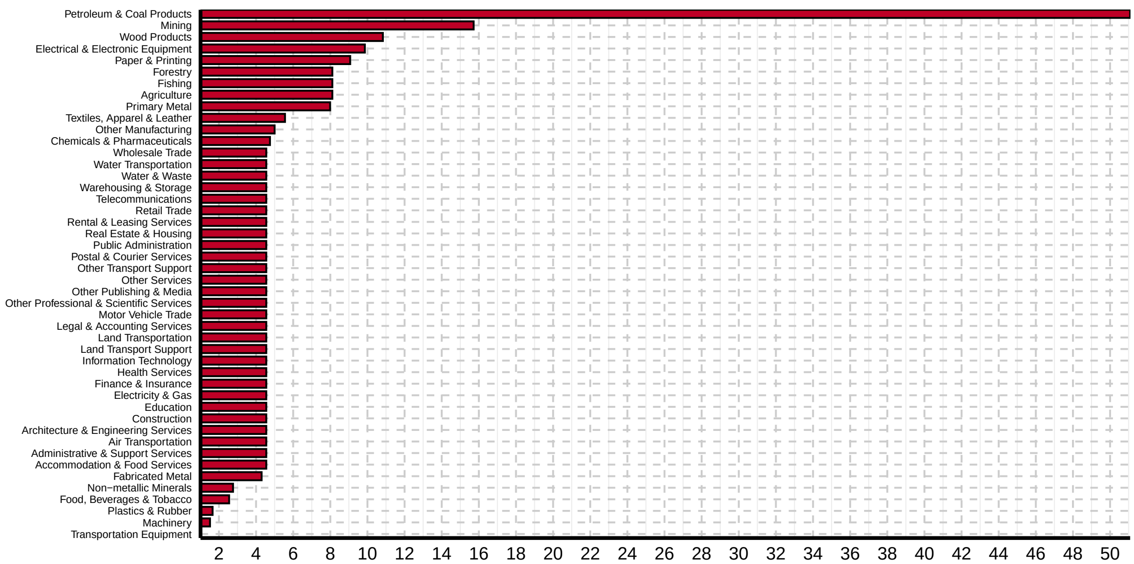


Notes: Calibrated values of ι_i consistent with observed trade imbalances. Values are reported on a 0-100 scale; the Rest of the World equals 50.

Productivity dispersion, θ_k. The productivity dispersion parameters θ_k are imported from Caliendo and Parro (2015) and Caliendo et al. (2018). These parameters govern the dispersion of idiosyncratic productivity draws within each sector and therefore discipline the sensitivity of bilateral expenditure shares to changes in trade costs, input-bundle costs, and deterministic productivity.

We map the estimates from the source classification to our sectoral classification using the sectoral concordance described in Appendix A. When the source estimate is available at a more aggregate level than our model sector, we assign the corresponding estimate to all model sectors within that aggregate category. Thus, θ_k is not chosen to match moments in our data, but is taken from the quantitative trade literature and translated into the sectoral structure of the model. Figure C.11 reports the resulting productivity dispersion parameters by sector.

Figure C.11: Productivity Dispersion by Sector (θ_k)



Notes: Sectoral productivity dispersion parameters are based on estimates from Caliendo and Parro (2015) and Caliendo et al. (2018).

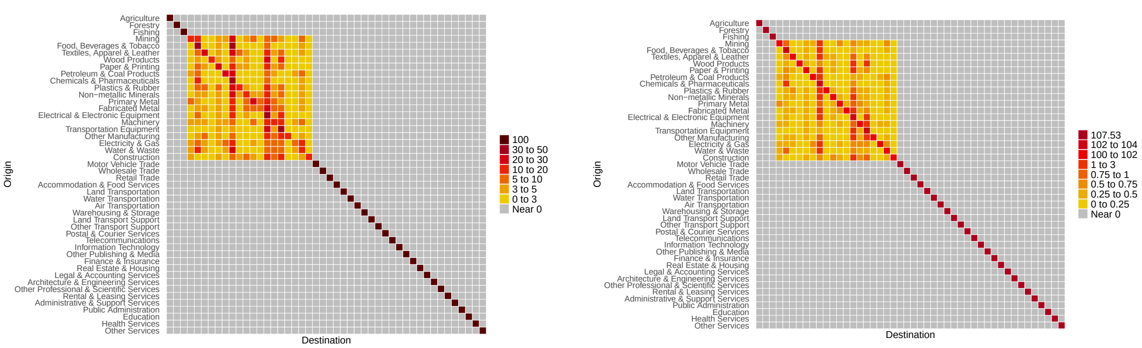
Sectoral knowledge network, λ_{kh}. The sectoral knowledge network is constructed from patent citation data. Following Liu (2019) and LaBelle et al. (2023), we use citation shares to measure technological proximity across sectors. The element λ_{kh} captures the exposure of sector k to knowledge originating in sector h: sectors that cite technologies from sector h more intensively receive a larger weight on that source sector.

We aggregate patent citations to the sectoral classification used in the model and normalize the citation matrix by recipient sector, so that the weights across source sectors sum to one for each sector k. This normalization makes λ_{kh} the empirical counterpart of the sectoral knowledge weights in Equation (B.11). The resulting matrix is constructed outside the model and is not chosen to match trade, productivity, or counterfactual moments.

Geographic knowledge diffusion, δ_{iu}. The geographic component of the knowledge network is imposed as a benchmark assumption. We restrict knowledge diffusion to the own location, $\delta_{iu} = \mathbb{1}\{i = u\}$, so that spillovers operate across sectors within each location. Under this benchmark, the geographic structure of knowledge diffusion introduces no additional cross-location channel beyond those already present through trade costs, input-output linkages, labor mobility, and fixed local structures.

This assumption is deliberately parsimonious. The sectoral knowledge network λ_{kh} disciplines which sectors are technologically connected, while the within-location restriction on δ_{iu} avoids imposing a common cross-location diffusion term that would mechanically inflate the measured elasticities of small units. Spatial heterogeneity in the counterfactual responses is thus driven by the measured geography of production, trade, and input-output linkages rather than by an additional calibrated geography of knowledge diffusion. Figure C.12 reports the benchmark knowledge network and diffusion structure used in the quantitative exercise.

Figure C.12: Knowledge Network and Diffusion



a) Knowledge Network Across Sectors   b) Leontief Inverse Across Sectors, $\times 100$

Notes: Figures displays the calibrated knowledge network across sectors used in the model, and the associated Leontief inverse across sectors for each location, scaled by 100 for visualization purposes, respectively.

Knowledge-spillover elasticity, φ. The knowledge-spillover elasticity is fixed at $\varphi = 0.07$, following Basso et al. (2025). In the model, φ governs the elasticity of deterministic productivity with respect to the external knowledge index in Equation (B.10). Equivalently, it determines the strength of the knowledge-network multiplier in Equation (B.16).

We do not estimate φ internally or choose it to match the counterfactual responses. Instead, we treat it as an externally disciplined benchmark parameter. The value $\varphi = 0.07$ implies economically meaningful but moderate knowledge spillovers, and keeps the calibration on the conservative side of the empirical literature on technological spillovers. Together with the sectoral network λ_{kh} and the geographic diffusion matrix δ_{iu}, this parameter determines how productivity shocks propagate through the knowledge block of the model.

D Additional Results

This appendix reports additional results that complement the empirical and quantitative findings presented in the main text. Section D.1 provides a more detailed description of the geographic and sectoral structure of the chilean disaggregated economic accounts. Section D.2 expands the analysis of aggregate effects from local productivity shocks, documenting additional heterogeneity across locations and sectors. Section D.3 provides further details on the *Huachipato* counterfactual, including the spatial and sectoral propagation of the plant-level shock. Together, these results provide supporting evidence for the main quantitative findings without altering the baseline model or calibration.

D.1 Descriptive Statistics

This subsection reports additional descriptive evidence on the spatial and sectoral organization of economic activity in Chile. The goal is not to introduce new measurement objects, but to document more fully the empirical patterns summarized in Section 3. We organize the evidence in two parts. The first documents the spatial, sectoral, and joint spatial-sectoral distribution of economic activity, productivity, employment, and wages. The second describes the geographic-sectoral production network and the trade imbalances that arise from it.

D.1.1 Spatial and Sectoral Distribution of Economic Activity

The disaggregated economic accounts reveal substantial heterogeneity across locations, sectors, and location-sector pairs. This heterogeneity matters for the quantitative analysis because exposure to shocks depends not only on where output is located, but also on which sectors operate in each location and how those location-sector pairs are connected through input-output linkages.

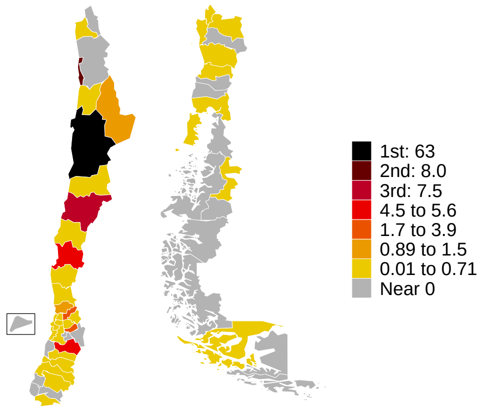
Fact 1. Activity, productivity, employment, and wages are unevenly distributed across space and sectors. The main text shows that aggregate GDP and GDP per worker are highly uneven across provinces. Figure D.1 complements that evidence by showing that the geography of GDP differs sharply across sectors. Mining is concentrated in a small set of northern provinces, fishing is concentrated in coastal and austral locations, agriculture is more prominent in central and southern provinces, and manufacturing is more concentrated around the central macrozone. These patterns show that spatial concentration is sector-specific: provinces that are central for one sector need not be central for another.

Figure D.2 takes the opposite perspective and shows the sectoral composition of GDP within each province. Each cell reports the importance of a sector in a province's GDP, with colors normalized within province: darker cells identify the sectors that account for the largest shares of local GDP. The figure shows that sectoral specialization is highly heterogeneous across space. Mining is dominant in several northern provinces, such as Antofagasta and El Loa; Santiago displays a more service-oriented structure, with large contributions from trade, finance, business services, and other urban activities; and several southern and austral provinces show larger local roles for agriculture, forestry, fishing, or public and social services. This reinforces the main text's point that the relevant unit is the location-sector pair rather than the province or the sector in isolation.

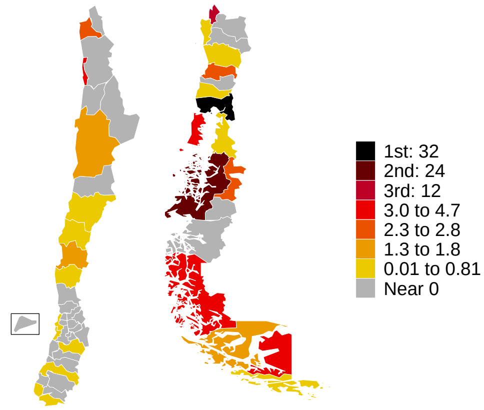
Employment and wages display related but less extreme patterns. Figure D.3 shows that employment is less spatially concentrated than GDP, while wage differences across provinces are smaller than differences in GDP per worker. This suggests that spatial productivity differences reflect not only labor income, but also variation in capital intensity, natural-resource rents, and local fixed factors.

The same distinction appears across sectors. Figure D.4 shows that sectoral size and sectoral productivity are not the same object. Mining, real estate and housing, construction, and large service sectors account for large shares of national GDP, but GDP per worker is especially high in asset- and capital-intensive activities such as real estate and housing, petroleum and coal products, legal and accounting services, and mining. Employment and wage payments are instead concentrated in large labor-intensive services such as retail trade, construction, education, health, public administration, and wholesale trade.

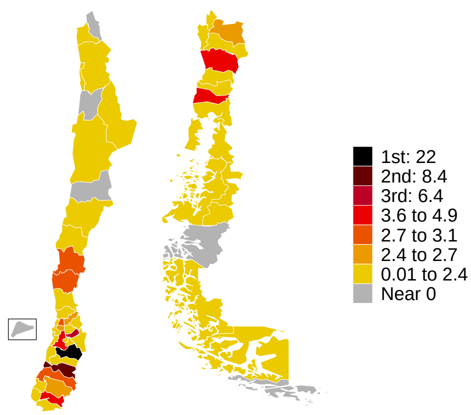
Figure D.1: Spatial Distribution of GDP: Sectoral Examples (% of National Sectoral GDP)



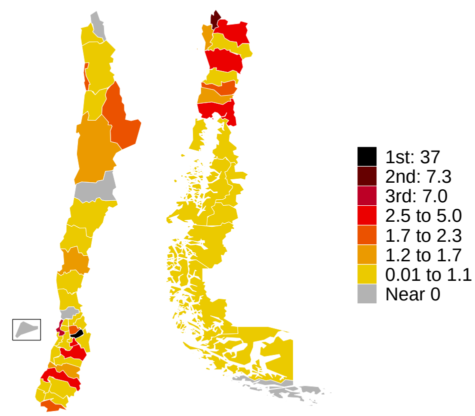
a) Mining



b) Fishing



c) Agriculture

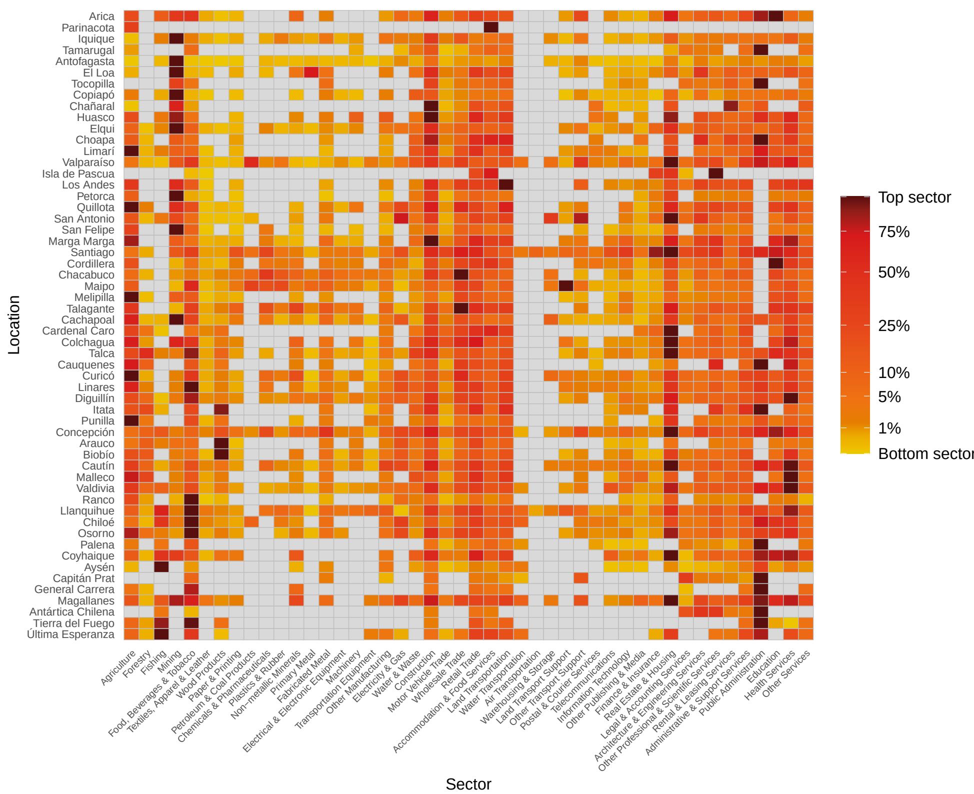


d) Manufacturing

Notes: Figures report each province's contribution to national GDP within the selected sector (%). Shares sum to 100 across provinces separately for each sector. The figure shows Mining, Fishing, Agriculture, and Manufacturing.

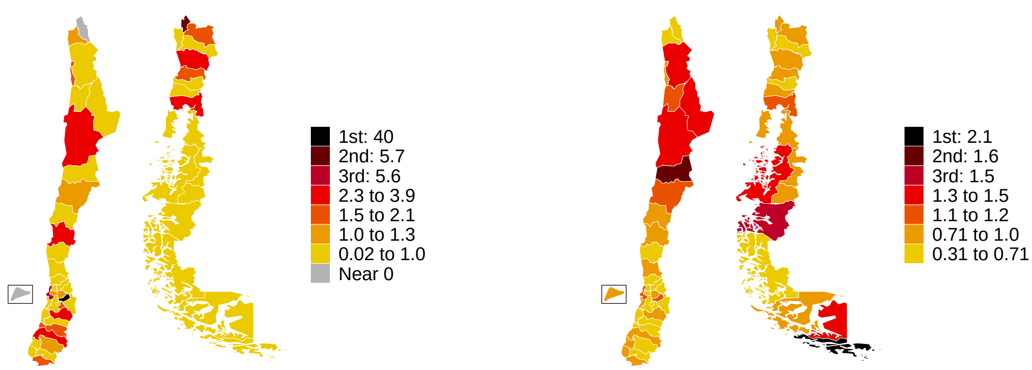
Fact 2. Extreme regions are more specialized, while central provinces are more diversified. Figure D.5 summarizes concentration using the Herfindahl-Hirschman Index (HHI). From the sectoral perspective, some industries are highly concentrated in a small number of provinces, especially activities tied to natural resources, ports, or specialized infrastructure. Other sectors are more spatially diffuse, particularly services that follow local demand.

Figure D.2: Geographic-Sector Distribution of Economic Activity



Notes: Figure reports each sector's share of total provincial GDP (%). Colors are normalized within province: darker red indicates the most important sector in that province, while lighter yellow indicates sectors with smaller GDP shares.

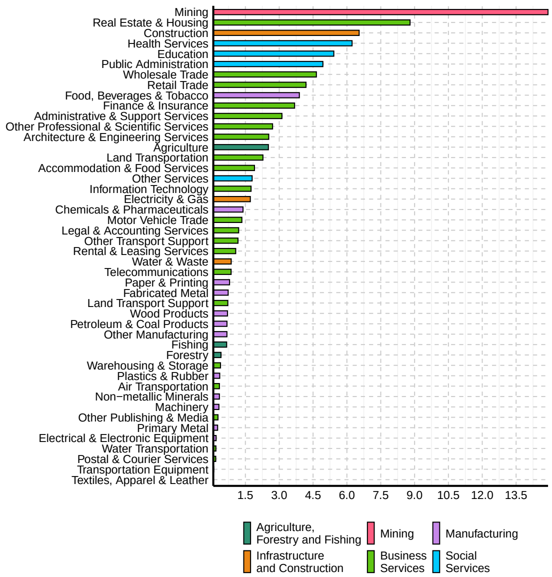
Figure D.3: Spatial Distribution of Economic Activity in Chile: Employment and Wages



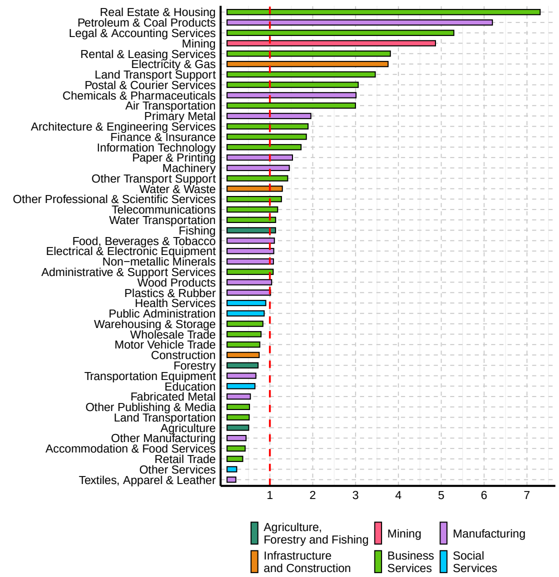
a) Share of National Employment (%) b) Wages per worker (relative to aggregate)

Notes: Figures show provincial shares of national employment (%), and wages per worker expressed relative to national aggregate values. National annual wages per worker are USD$14,870 for 2022.

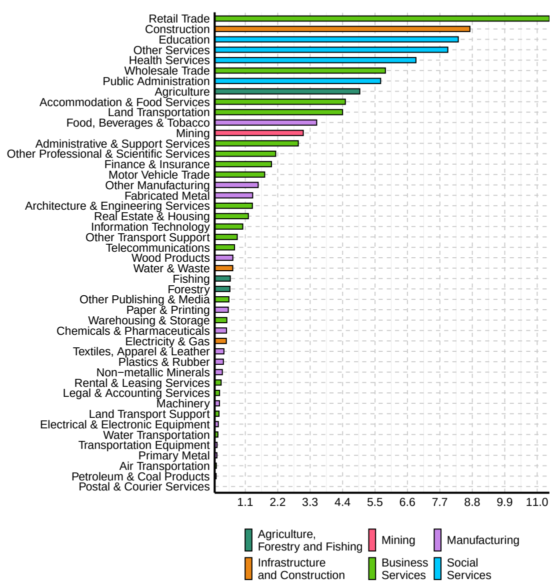
Figure D.4: Sectoral Distribution of Economic Activity in Chile



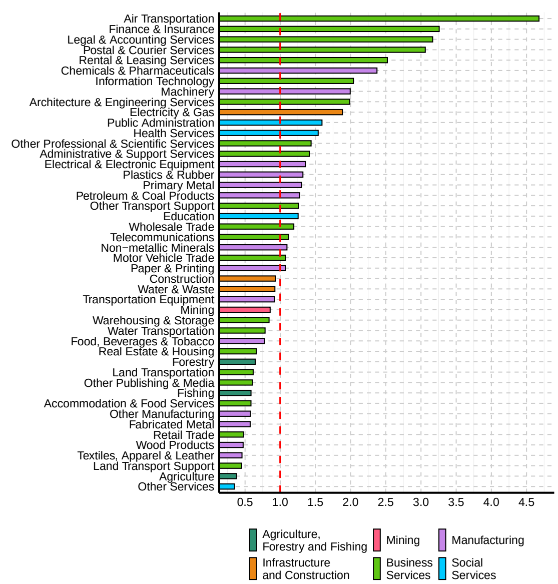
a) Share of National GDP (%)



b) GDP per worker (relative to aggregate)



c) Share of National Employment (%)

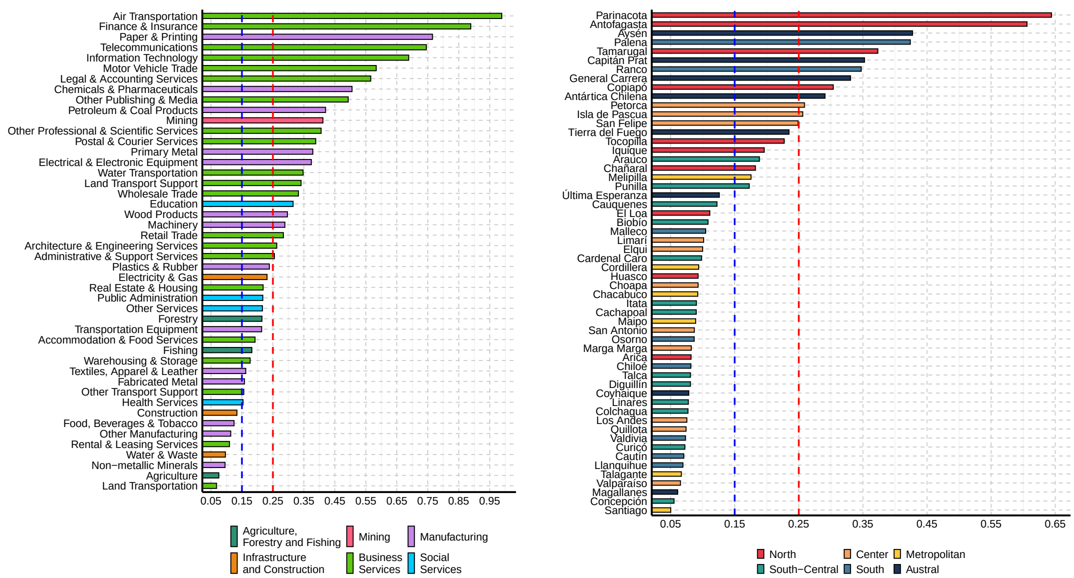


d) Wages per worker (relative to aggregate)

Notes: Panel A reports each sector's share of national GDP (%). Panel B reports GDP per worker by sector, expressed relative to the national average; the red vertical line marks parity with national GDP per worker. Panel C reports each sector's share of national wage payments (%). Panel D reports wages per worker by sector, expressed relative to the national average; the red vertical line marks parity with national wages per worker. National annual GDP per worker in 2022 is USD 34,089, and national annual wages per worker are USD 14,870.

From the provincial perspective, concentration is highest in remote and extreme regions, where a small number of sectors account for a large share of local GDP. Central and metropolitan provinces are more diversified, reflecting a broader mix of services, manufacturing, logistics, and administrative activities. These patterns are consistent with comparative advantage and agglomeration forces, and they imply that local exposure to shocks depends on the interaction between geography and sectoral specialization.

Figure D.5: Spatial and Sectoral Concentration of Economic Activity in Chile



a) Spatial concentration by sector (HHI) b) Sectoral concentration by province (HHI)

Notes: The left panel reports, for each sector, the Herfindahl-Hirschman Index (HHI) computed across provinces using sectoral GDP shares. The right panel reports, for each province, the HHI computed across sectors using provincial GDP shares. Vertical lines at 0.15 and 0.25 indicate the thresholds commonly used to classify distributions as unconcentrated and moderately concentrated, respectively.

Fact 3. The most relevant heterogeneity is at the location-sector level. Table D.1 quantifies the concentration patterns shown in the figures. Looking only across provinces or only across sectors understates the degree of heterogeneity in the economy. Across location-sector cells, the top 10% accounts for 79% of national GDP, while the bottom half accounts for only 1.8%. The same table shows large dispersion in GDP per worker at the location-sector level: the coefficient of variation is 2.6, and the 90th-to-50th percentile ratio is 5.3. Employment and wages exhibit similar two-dimensional heterogeneity.

Table D.1: The Spatial Sectorial Distribution of Economic Activity in Chile

	(1)	(2)	(3)	(4)	(5)	(6)
Panel A: Relative Size Distribution						
	Shares of National GDP			Shares of National Employment		
	Spatial	Sectoral	Interaction	Spatial	Sectoral	Interaction
HHI	0.184	0.056	0.018	0.179	0.057	0.011
Bottom 10%	0.13	0.75	0.03	0.14	0.57	0.07
Bottom 25%	1.1	3.1	0.25	1.2	2.2	0.46
Bottom 50%	5.0	11	1.8	7.4	7.9	2.6
Top 50%	95	89	98	93	92	97
Top 25%	85	69	92	79	75	89
Top 10%	69	42	79	62	43	72
Top 10% / Bottom 50%	14	3.7	44	8.4	5.5	27
Panel B: Productivity Distribution						
	GDP per Worker			Wages per Worker		
	Spatial	Sectoral	Interaction	Spatial	Sectoral	Interaction
Coefficient of Variation	0.65	0.95	2.6	0.39	0.69	1.1
10th percentile	17	16	5.5	7.9	7.1	2.7
25th percentile	21	25	11	8.7	9.3	5.1
50th percentile	27	37	21	12	16	9.8
75th percentile	34	64	48	15	23	20
90th percentile	51	129	113	19	36	35
Ratio P90/P50	1.9	3.4	5.3	1.6	2.3	3.6
Panel C: Correlation Between Both Measures						
	GDP Share vs Employment Share			GDP per Worker vs Wages per Worker		
Spearman	0.97	0.83	0.83	0.77	0.61	0.82
R^2 weighted	0.97	0.00	0.05	0.48	0.00	0.04

Notes: Panel A reports concentration and tail measures for the distributions of national GDP and employment shares across locations, sectors, and location-sector pairs. Panel B reports analogous distributional statistics for GDP per worker and wages per worker. Panel C reports rank and weighted linear correlations between size-based and productivity-based measures. Shares are expressed in percent.

The variance decomposition in Table D.2 reinforces this point. Sectoral components explain a large share of dispersion in GDP per worker and wages per worker, but the interaction between space and sectors is also quantitatively important. In the two-way decomposition, the interaction accounts for 36.2% of the variance in log GDP per worker and 43.2% of the variance in log wages per worker. These shares imply that productivity and wage differences cannot be summarized by province effects and sector effects alone.

Table D.2: Variance Decomposition of Productivity in Chile

	log(GDP/Worker)	log(Wages/Worker)
Panel A: Between versus Within Locations		
Between	0.10	0.12
Within	0.90	0.88
Panel B: Between versus Within Sectors		
Between	0.62	0.54
Within	0.38	0.46
Panel C: Two-Way Across Locations and Sectors		
Spatial	0.08	0.10
Sectoral	0.56	0.47
Spatial-Sectoral	0.36	0.43

Notes: Entries report variance shares from ANOVA decompositions of log GDP per worker and log wages per worker across location-sector observations. Panels A and B decompose variation into between and within components for locations and sectors separately. Panel C reports the two-way decomposition into spatial, sectoral, and interaction components. All components sum to one within each column.

Taken together, these facts show that the distribution of economic activity in Chile is not only spatially uneven or sectorally uneven. It is jointly spatial-sectoral. This motivates the model's use of location-sector productivity, local input-output coefficients, and bilateral trade shares across locations and sectors.

D.1.2 The Chilean Production Network

The concentration patterns documented above matter because production is connected through intermediate input flows. A province can be highly exposed to shocks outside its own borders if it relies on inputs produced elsewhere, and a sector can transmit shocks widely if it supplies many downstream activities. The geographic-sectoral input-output matrix allows us to measure these linkages directly.

Fact 4. The location network is centralized around Santiago and Antofagasta. Panel A of Figure 2 shows that interregional production flows are highly centralized. Santiago is the dominant domestic hub, concentrating large inflows and outflows and maintaining direct links with many provinces. Several central provinces are tightly connected to Santiago, reflecting dense production, logistics, and service linkages around the metropolitan area.

Antofagasta is a second key node. Its centrality reflects the concentration of mining activity and its role as a gateway between the northern macrozone, the rest of the domestic economy, and foreign markets. Other northern provinces tend to cluster around Antofagasta, while many austral provinces remain more peripheral and connect to the rest of the economy through a small number of hubs. Distance matters for this core-periphery structure, but imperfectly: some remote provinces remain integrated because of resource-based activities or logistics infrastructure.

Fact 5. The sectoral network is asymmetric: mining is globally central, while manufacturing and services sustain domestic linkages. Panel B of Figure 2 shows that sectoral centrality is also uneven. Mining is central in total flows because of its strong connection to the Rest of the World, but it is less connected to domestic upstream suppliers than its size would suggest. By contrast, manufacturing and several service sectors are deeply embedded in domestic input-output linkages. Food, beverages, and tobacco are closely connected to agriculture and fishing. Business services, wholesale and retail trade, construction, and transportation support production across many sectors even though they are less directly export-oriented.

Table D.3 quantifies this network structure. Most trade flows occur across, rather than within, locations, sectors, and location-sector pairs. This is especially stark at the joint location-sector level, where between-cell flows account for 92.6% of total trade. The table also shows that domestic trade accounts for 55.1% of aggregate flows, while international trade accounts for 44.9%. These magnitudes confirm that both domestic production linkages and foreign exposure are central for understanding propagation.

Table D.3: The Chilean Production Network Flows

	Billion USD	% of Total Trade	% of GDP
Panel A: Domestic Flows Across Locations			
Within	114.7	26.5	38.1
Between	123.8	28.6	41.1
Panel B: Domestic Flows Across Sectors			
Within	49.2	11.4	16.3
Between	189.3	43.8	62.8
Panel C: Domestic Flows Across Locations and Sectors			
Within	32.0	7.4	10.6
Between	206.5	47.7	68.5
Panel D: Aggregate Flows			
International	194.0	44.9	64.4
Domestic	238.5	55.1	79.2
Total Trade	432.5	100.0	143.6

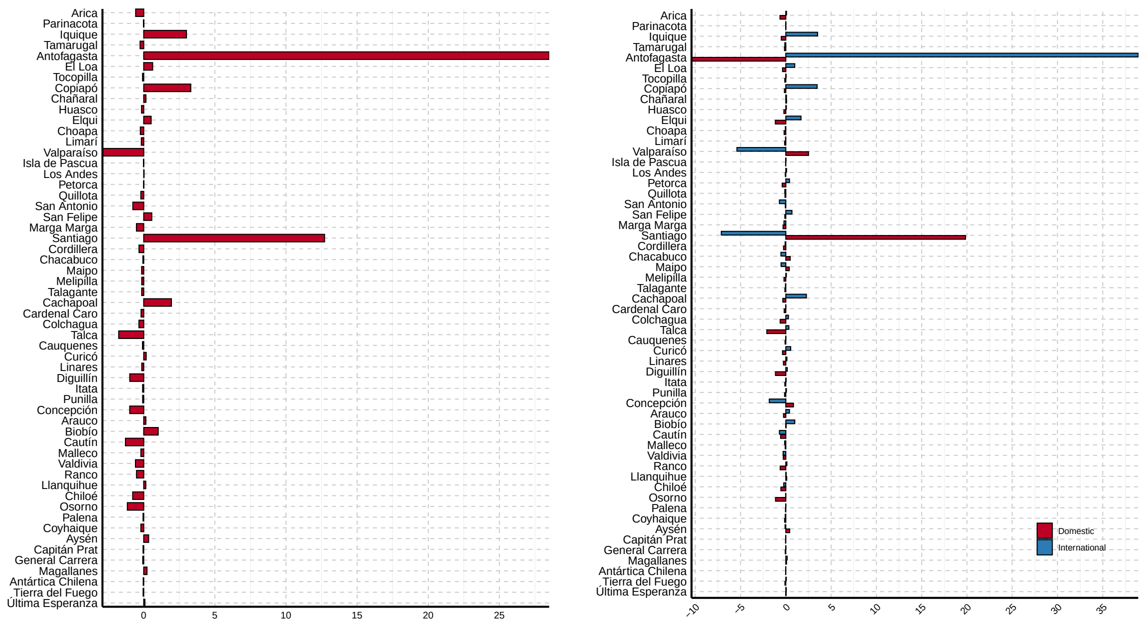
Notes: Entries report the value of domestic and international trade flows in 2022. Within and between refer, respectively, to flows occurring within and across locations, sectors, or location-sector pairs. Percent-of-total-trade columns are computed relative to aggregate trade flows, and percent-of-GDP columns are computed relative to national GDP.

Fact 6. Trade balances display a north-south gradient and a distinct domestic-trade structure. Figure 3 in the main text shows that provincial trade balances, measured relative to provincial GDP, are sharply polarized across space. Figure D.6 complements that evidence by reporting the same imbalances in levels. Northern provinces tend to run positive balances, driven by international exports from resource-intensive activities. Southern and austral provinces are more often net importers, both from abroad and from other domestic locations.

The domestic component follows a different pattern. Most provinces are net importers within the domestic network, while Santiago stands out as the principal net domestic supplier. This distinction matters for propagation: the provinces that generate external surpluses are not necessarily the same locations that mediate domestic input flows.

The Chilean production network is therefore shaped by both export-oriented northern provinces and a central domestic hub that supplies and intermediates activity across the rest of the country.

Figure D.6: Provincial Trade Imbalances in Billion USD



a) Net Trade Imbalance (Exports − Imports) b) Domestic vs. International Decomposition

Notes: Trade imbalances are expressed as billion USD and decomposed into domestic and international components. Provinces are ordered from north to south.

Overall, the additional evidence in this appendix reinforces the central empirical message of the paper: Chile's production structure is organized jointly by geography and sectoral specialization. Output, productivity, employment, wages, input-output flows, and trade balances all vary substantially across location-sector pairs. This two-dimensional heterogeneity is precisely the object disciplined by the geographic input-output accounts and used in the quantitative exercises in the main text.

D.2 Aggregate Effects of Local Productivity Shocks

This subsection reports additional results for the first quantitative application in Section 6.1. The main text reports the central moments, variance decompositions, sufficient-statistics comparisons, and mechanism comparisons. Here we provide the underlying distributions and complementary visual evidence for regional, sectoral, and location-sector productivity shocks. Throughout this subsection, we follow the normalization used in the main text:

aggregate responses are reported as elasticities after scaling by the size of the directly affected unit. These elasticities therefore measure amplification relative to the direct exposure of the shocked province, sector, or province-sector pair. The goal is to document the heterogeneity behind the aggregate elasticities and to show how the main findings vary across outcomes, locations, sectors, and model mechanisms.

Regional productivity shocks. Figure D.7 reports the full distribution of aggregate elasticities from regional productivity shocks. In each counterfactual, we increase fundamental productivity by 10% in all sectors within one province and solve for the new equilibrium. Each point therefore corresponds to one province-level counterfactual. As in the main text, aggregate responses are reported as elasticities after scaling by the size of the directly affected province, so the figure captures amplification relative to direct regional exposure rather than mechanical differences in province size.

The figure complements the regional-shock moments in Table 2. Measured TFP elasticities are relatively compressed across provinces, indicating that regional productivity gains are partly absorbed through input-output linkages, price changes, and trade-induced selection. GDP elasticities display substantially more spatial heterogeneity, because they also reflect labor reallocation, changes in factor prices, fixed-factor congestion, and propagation through domestic trade. Welfare elasticities follow a distinct spatial pattern, reflecting the additional role of consumption prices and portfolio income. Thus, the same regional productivity shock can generate different rankings depending on whether the outcome is measured TFP, aggregate GDP, or welfare.

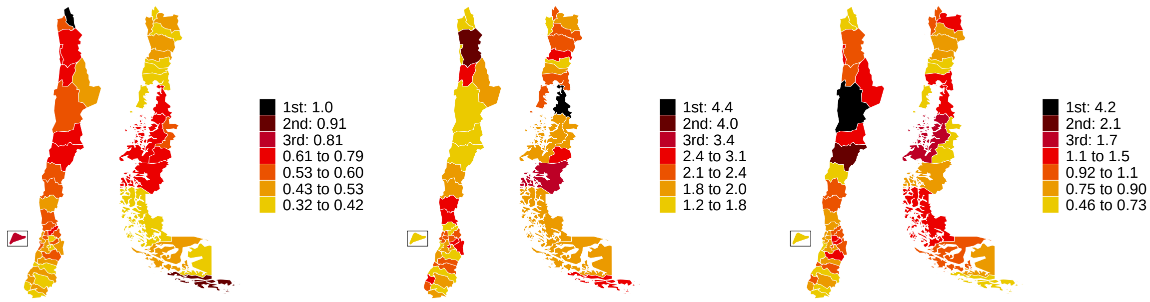
Three patterns stand out. First, GDP elasticities are systematically larger than measured TFP elasticities, with welfare responses lying in between. This ordering is informative about the mechanisms at work. Measured TFP captures value-added efficiency net of intermediate input costs. GDP additionally captures general-equilibrium reallocation across locations and sectors. Welfare further incorporates changes in consumer prices and income from the global portfolio. Second, dispersion differs across outcomes: measured TFP responses are tightly clustered, whereas GDP and welfare responses vary substantially across provinces. This indicates that most of the spatial heterogeneity in aggregate responses comes from general-equilibrium reallocation and propagation, rather than from local value-added productivity alone. Third, the ranking of provinces is not driven only by local productivity levels. Central and metropolitan provinces generate large GDP elasticities because they are more deeply embedded in domestic production networks and supply a broad set of downstream locations and sectors.

The Northern region is especially informative. Northern provinces have high GDP per worker and account for a large share of national output, but their GDP elasticities are comparatively muted. This reflects the structure of mining-based local economies: mining is highly productive and strongly connected to foreign markets, but it is less embedded in domestic input-output chains than many services and manufacturing activities. As a result, productivity gains in northern provinces propagate less through the domestic economy than productivity gains in more diversified and network-central regions. This does not imply small welfare effects, because welfare also reflects price changes, portfolio income, and the redistribution of gains under labor mobility.

These patterns are closely related to the mechanisms emphasized by Caliendo et al. (2018), but their quantitative expression differs in the Chilean economy. As in their benchmark analysis, measured TFP elasticities are smaller and more compressed than GDP and welfare elasticities, consistent with input-output linkages transmitting part of the productivity gains through lower intermediate input prices. The key difference is that Chile's sharper spatial concentration and stronger resource specialization make the location of the shock especially important. In the U.S. benchmark, regional propagation is shaped by a more diversified spatial production structure. In Chile, by contrast, the contrast between mining-oriented northern provinces, central domestic hubs, and more peripheral southern and austral locations generates stronger spatial differences in GDP amplification. The comparison therefore reinforces the paper's central message: the same class of mechanisms can produce different aggregate elasticities depending on the geography of production and the domestic embeddedness of the shocked region.

Sectoral productivity shocks. Figure D.8 reports the analogous distributions for sectoral productivity shocks. In each counterfactual, we increase fundamental productivity by 10% in all locations within one sector. This exercise isolates how the aggregate effect of a sectoral productivity improvement depends on the sector's position in the production network and on the spatial distribution of the activity being shocked. As in the regional exercise, elasticities are normalized by the size of the directly affected sector, so the cross-sectoral variation captures amplification rather than mechanical differences in sector size.

Figure D.7: Aggregate Elasticities of Regional Productivity Shocks



a) Measured TFP b) GDP c) Welfare

Notes: Panels report aggregate elasticities of measured TFP, GDP, and welfare to a 10% increase in fundamental productivity applied uniformly across all sectors within a province. Each point corresponds to one province-level counterfactual equilibrium. Outcomes are normalized by the corresponding province share in national sales, value added, or employment.

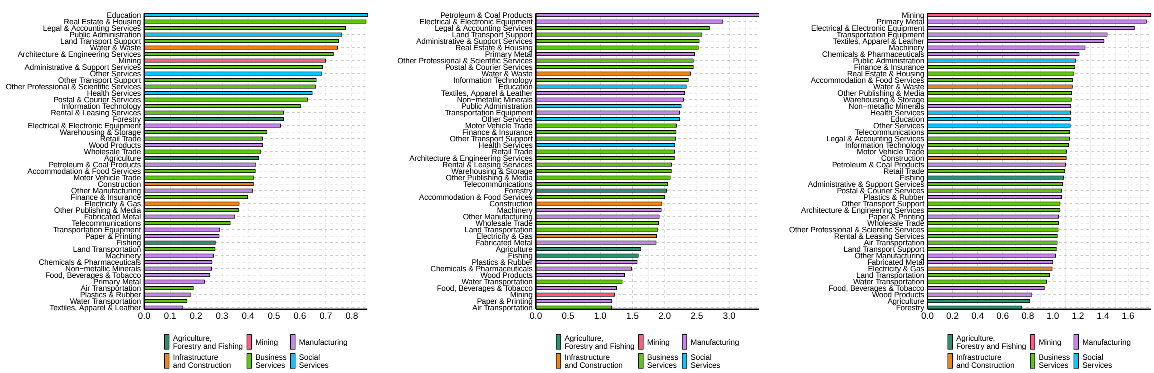
The figure shows that sectoral rankings differ sharply across outcomes. Measured TFP elasticities are high in sectors where productivity gains translate relatively directly into value-added efficiency. GDP elasticities are largest in sectors that are broadly used across the economy and occupy central positions in domestic input-output linkages, especially business services, logistics-related activities, and other intermediate service sectors. Welfare elasticities are more compressed across sectors, because consumption prices, portfolio income, and labor mobility redistribute gains beyond the directly affected sector. These differences show that sectoral productivity effects depend not only on the sector's size, but also on how the sector is connected to other activities and where it operates geographically.

The contrast between mining and services is central. Mining has high measured TFP effects and is strongly connected to foreign markets, but it generates more limited GDP amplification because its domestic input-output linkages are narrower. By contrast, several business and logistics services generate larger GDP elasticities because they are used by many downstream sectors and operate across a broad set of locations. Productivity gains in these sectors reduce input costs for many firms, raise the marginal productivity of factors elsewhere in the economy, and propagate through domestic trade and production networks. This is why some service sectors generate larger aggregate GDP responses than sectors that are larger or more productive in direct terms.

The comparison with Caliendo et al. (2018) is useful because it highlights which results are general and which are specific to Chile's production structure. As in their analysis, input-output linkages attenuate measured TFP responses and amplify aggregate

GDP responses for sectors that are central suppliers to the rest of the economy. However, the ranking of sectors differs. In the U.S. benchmark, large aggregate responses arise from a more balanced mix of manufacturing and service sectors, consistent with deeper domestic manufacturing supply chains. In Chile, the largest GDP elasticities are concentrated in business and social services, while mining-related productivity gains propagate less through domestic value chains despite the sector's macroeconomic importance. This difference reflects both the resource orientation of the Chilean economy and our measurement of intersectoral and interregional flows across all sectors, including services. The result underscores that sectoral productivity shocks cannot be evaluated only from national sector size or direct productivity; their aggregate effects depend on the sector's domestic network embeddedness and spatial footprint.

Figure D.8: Aggregate Elasticities of Sectoral Productivity Shocks



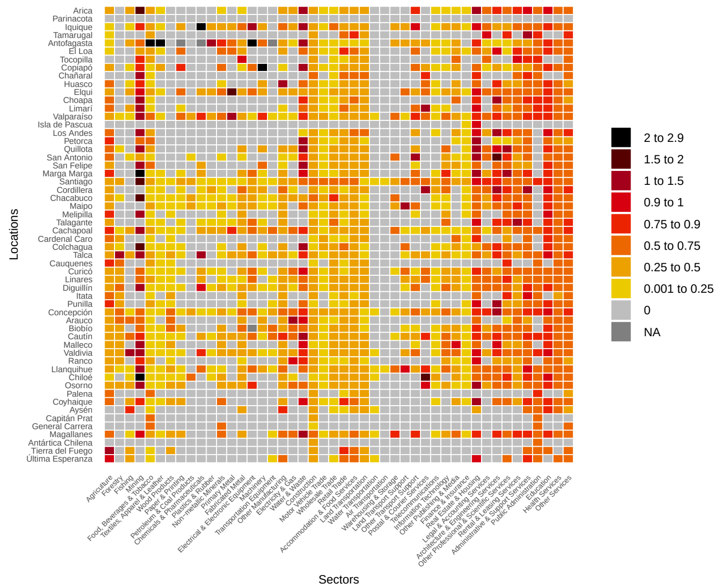
a) Measured TFP   b) GDP   c) Welfare

Notes: Panels report aggregate elasticities of measured TFP, GDP, and welfare to a 10% increase in fundamental productivity applied uniformly across all locations within a sector. Each point corresponds to one sector-level counterfactual equilibrium. Outcomes are normalized by the corresponding sector share in national sales, value added, or employment.

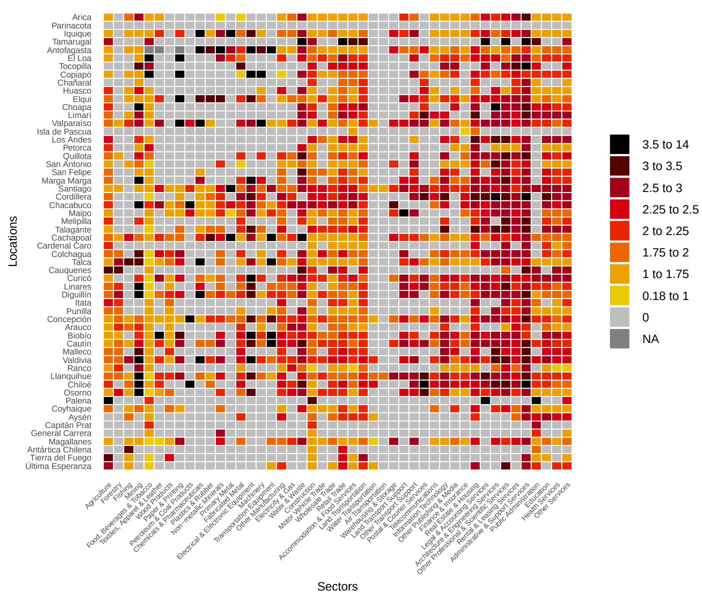
Location-sector productivity shocks. Figure D.9 reports the full set of counterfactuals in which productivity increases in one province-sector pair at a time. This exercise is the most disaggregated version of the first application and is the relevant benchmark for studying how geography and sectoral specialization interact in determining aggregate propagation.

The main text shows that average responses under location-sector shocks are similar to the averages obtained under regional or sectoral shocks, but that dispersion rises substantially once the shock is localized jointly in geography and sector. The figure

Figure D.9: Aggregate Elasticities of Location-Sector Productivity Shocks



a) Measured TFP

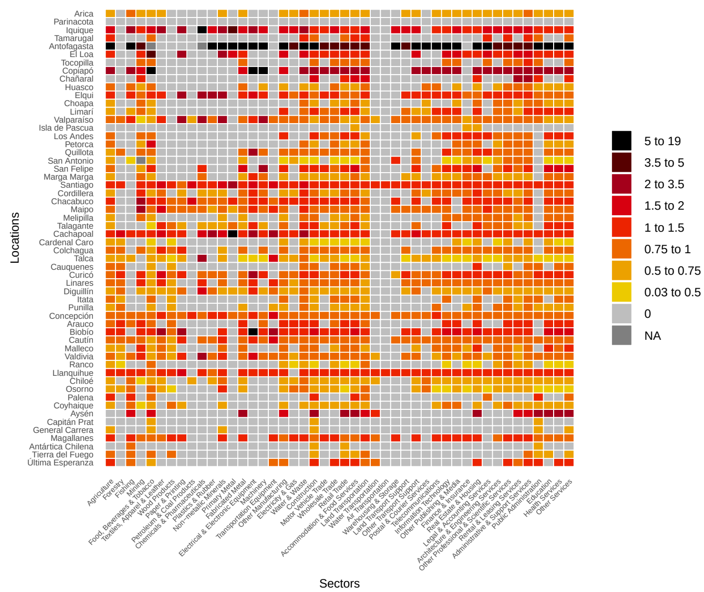


b) GDP

Notes: Panels report aggregate elasticities of measured TFP and GDP to a 10% productivity increase in one province-sector pair. Each point is one counterfactual equilibrium. Outcomes are normalized by the corresponding national share.

documents this dispersion visually for measured TFP, GDP, and welfare. Table D.4 then

Figure D.9: Aggregate Elasticities of Location-Sector Productivity Shocks (continued)



c) Welfare

Notes: Panel reports aggregate welfare elasticities to a 10% productivity increase in one province-sector pair. Each point is one counterfactual equilibrium. The outcome is normalized by the corresponding national employment share.

reports the corresponding aggregation by macrozone and industry. Together, these objects show that one-dimensional shocks average over an important source of heterogeneity: the same sector can generate different aggregate effects depending on where it is located, and the same province can generate different effects depending on which sector is shocked.

Mechanisms. Figure D.10 complements the mechanism comparison in the main text by coloring each province-sector pair by industry rather than by macrozone. We focus the mechanism comparison on GDP elasticities because GDP is the main aggregate object used to quantify propagation and because the variance-decomposition result in Table 3 is stated for GDP. Each panel compares Benchmark aggregate GDP elasticities with the corresponding elasticities in a counterfactual economy that removes one or more propagation channels. The comparison shows whether the same location-sector pairs that are important in the Benchmark remain important when a mechanism is shut down.

Panel (a) removes input-output linkages. The positive fit with the Benchmark indicates that some of the cross-sectional ordering remains, but the dispersion around the 45-degree line shows that production networks account for a sizable part of the hetero-

Table D.4: Aggregate Elasticities to Location-Sector Productivity Shocks: Benchmark

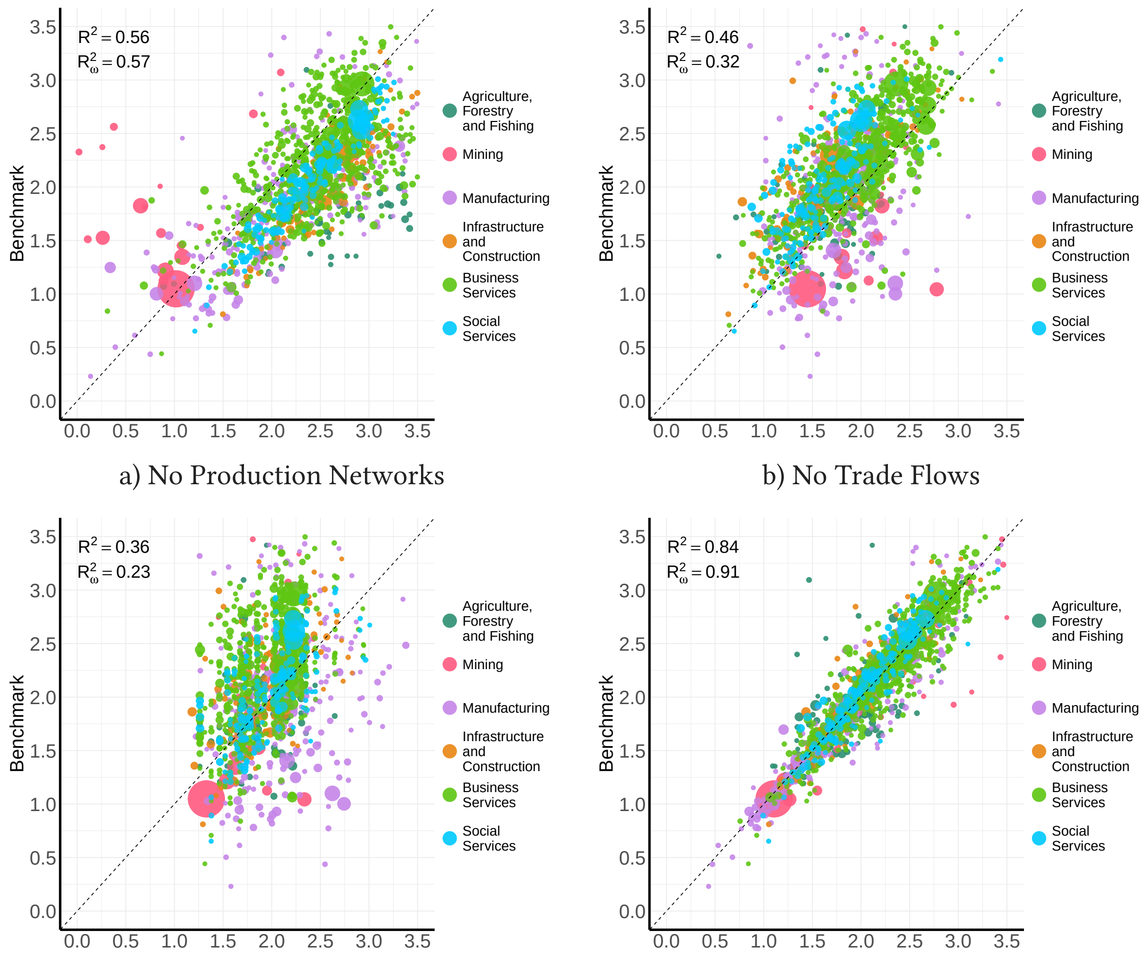
	Measured TFP		GDP		Welfare	
	Mean (1)	SD (2)	Mean (3)	SD (4)	Mean (5)	SD (6)
Panel A: Regional Aggregation						
North	0.60	0.22	1.45	0.62	3.15	1.84
Central	0.65	0.29	2.10	0.50	0.84	0.20
Metropolitan	0.65	0.28	2.33	0.51	1.28	0.16
South-Central	0.59	0.26	1.95	0.49	0.93	0.28
South	0.55	0.27	2.09	0.59	0.87	0.18
Austral	0.63	0.23	1.82	0.46	1.01	0.39
Total	0.62	0.27	2.04	0.63	1.50	1.18
Panel B: Sectoral Aggregation						
Agriculture, Forestry, and Fishing	0.54	0.27	1.79	0.35	0.99	0.41
Mining	0.74	0.31	1.22	0.35	2.85	1.27
Manufacturing	0.33	0.23	1.56	0.78	1.21	1.86
Infrastructure and Construction	0.47	0.17	1.99	0.37	1.44	1.25
Business Services	0.63	0.25	2.36	0.43	1.26	0.71
Social Services	0.76	0.11	2.30	0.40	1.26	0.73
Total	0.62	0.27	2.04	0.63	1.50	1.18

Notes: Averages are computed using the relative size of province-sector pairs within the corresponding macrozone or industry. Sales are used as weights for TFP, value added for GDP, and employment for welfare.

geneity in GDP elasticities. Panel (b) removes regional trade. The fit falls more sharply, indicating that trade is a central propagation channel for localized productivity shocks. Panel (c) removes both regional trade and input-output linkages, leaving little of the Benchmark cross-sectional structure. Panel (d) replaces the geographically disaggregated input-output structure with national sectoral coefficients. In that case, the fit remains high, especially for larger province-sector pairs, indicating that geographic input-output detail matters most for heterogeneity among smaller and more locally specific markets.

The sector-colored version of the figure also clarifies that the mechanism results are not driven by a single industry. Business services and social services remain prominent among high-elasticity observations in the Benchmark, while manufacturing and resource-based sectors display more heterogeneous responses depending on the channel being removed. This reinforces the main conclusion of the application: aggregate propagation depends on the joint position of a province-sector pair in trade, production networks, and factor markets.

Figure D.10: Mechanisms Behind Aggregate GDP Elasticities: Colors by Sector



a) No Production Networks

b) No Trade Flows

c) No Trade, No Production Networks

d) No Geography on Input-Output

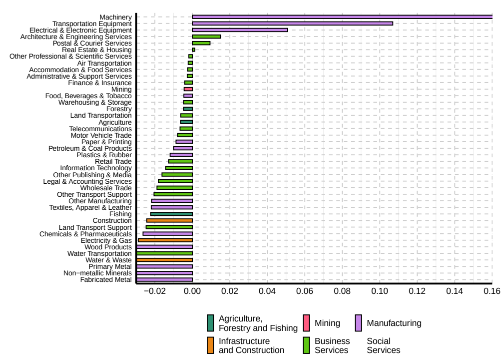
Notes: Panels compare Benchmark aggregate GDP elasticities for province-sector productivity shocks with the corresponding elasticities from four counterfactual economies. Each point is a province-sector pair, colored by industry. Panel (a) removes production networks, panel (b) removes trade flows, panel (c) removes both channels, and panel (d) replaces geographically disaggregated input-output linkages with national coefficients. The 45-degree line indicates equality with the Benchmark.

D.3 Exit of *Huachipato* Steel Plant

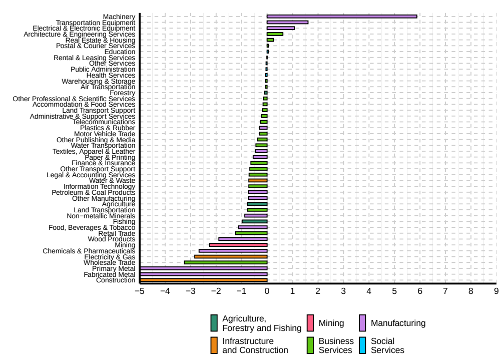
This subsection reports additional results for the *Huachipato* counterfactual studied in Section 6.2. The main text reports the aggregate effects, the benchmark propagation patterns for GDP, and the mechanism comparison across model variants. Here we complement those results by showing how the shock propagates across additional outcomes and at a finer province-sector level of disaggregation. The goal is to document the incidence of the shock beyond the directly affected Concepción–Primary Metals node and to clarify how the spatial and sectoral patterns differ across measured TFP, employment, and GDP.

Propagation across outcomes. Figure D.11 reports the propagation of the *Huachipato* shock for measured TFP. Compared with the GDP results in Figure 7, measured TFP responses are more localized and more composition-driven. The largest negative contribution is concentrated in the directly affected Primary Metals activity and in the Biobío/Concepción area, while several other sectors and locations show small positive or negative contributions. This pattern is consistent with the national results in Table 4: the shock generates a much larger contraction in GDP and employment than in measured TFP, because measured TFP captures only part of the general-equilibrium adjustment induced by changes in sourcing, prices, factor allocation, and local congestion.

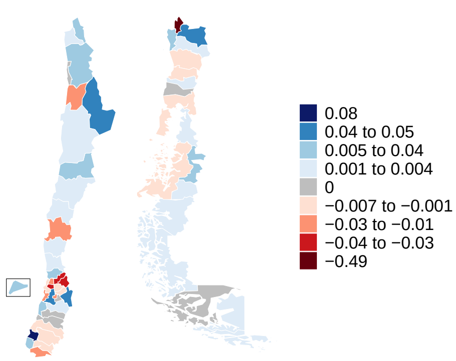
Figure D.11: Propagation Across Space and Sector of the Exit of *Huachipato* Steel Plant: Measured TFP



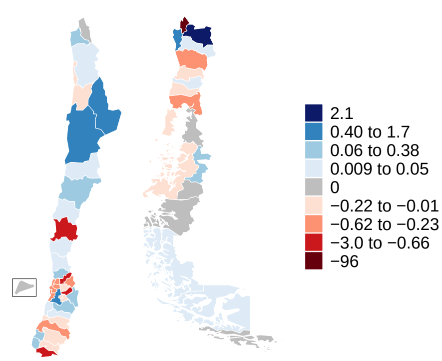
a) Sectoral measured TFP change (%)



b) Sectoral contribution share (%)

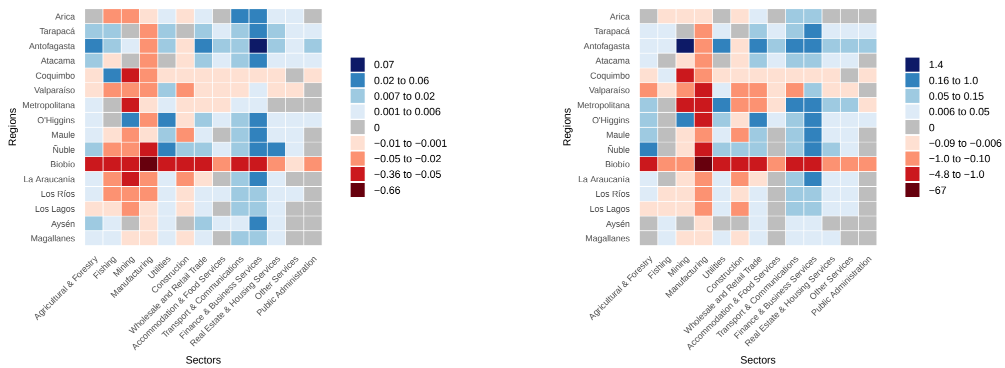


c) Provincial measured TFP change (%)



d) Provincial contribution share (%)

Figure D.11: Propagation Across Space and Sector of the Exit of *Huachipato* Steel Plant: Measured TFP (continued)



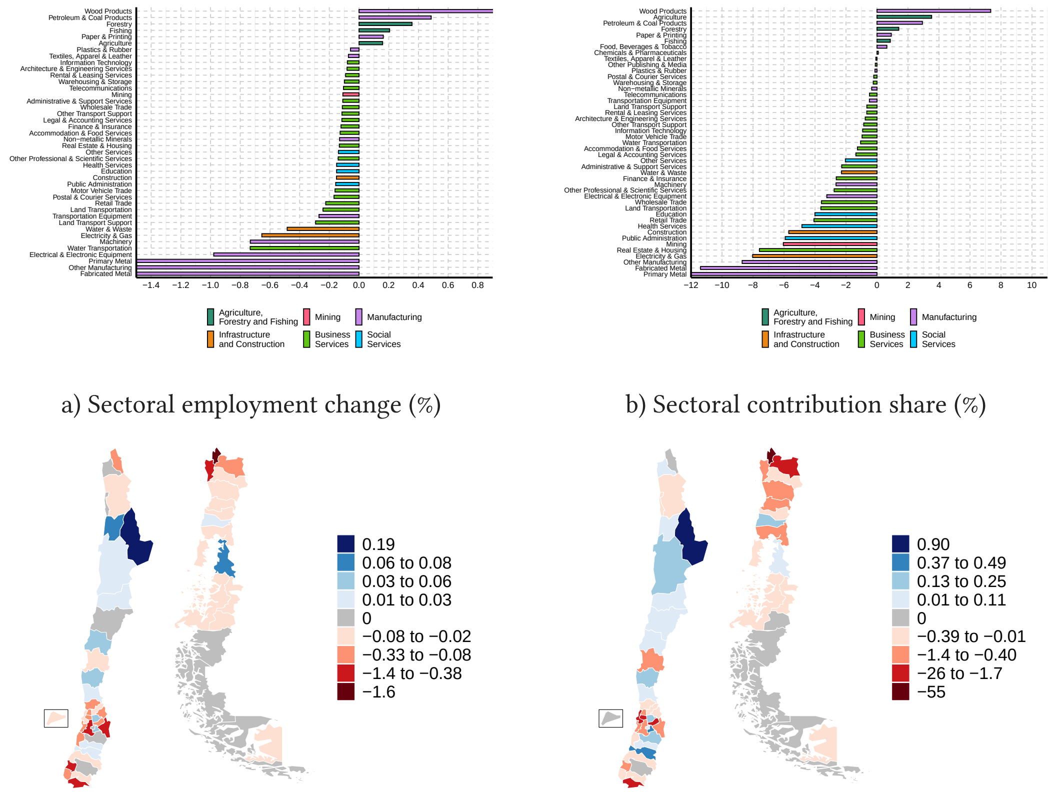
e) Region-sector measured TFP change (%) f) Region-sector contribution share (%)

Notes: Each row reports the same measured TFP object at a different level of aggregation. The left panels show the percentage change in measured TFP for each sector, province, or region-sector pair. The right panels show each unit's contribution to the national measured TFP change, defined as the unit-level change weighted by its national share and expressed relative to the aggregate measured TFP change. Contributions sum to 100%. To make the remaining sectoral distribution legible, the Primary Metals bar is visually truncated in panels (a) and (b). Its actual sectoral measured TFP change is -1.70%, and its actual contribution to the aggregate measured TFP change is -91.6%. Values below the variable-specific plotting threshold are set to zero or omitted, as in the underlying figures.

Figure D.12 reports the corresponding propagation patterns for employment. Employment responses are more dispersed than measured TFP responses, and they are closer to the GDP patterns reported in the main text. The largest losses are concentrated in Primary Metals and downstream manufacturing activities, but the adjustment also extends to services and to locations outside Biobío. This reflects the fact that employment adjusts through labor mobility and changes in local factor demand, so the incidence of the shock is not limited to the province-sector pair where productivity falls.

Taken together, Figures D.11 and D.12 show that the same shock has different incidence depending on the outcome. Measured TFP is more concentrated around the directly affected activity, while employment and GDP propagate more broadly through the economy. This distinction is useful for interpreting the aggregate results in Table 4: the small measured TFP loss does not imply that the shock is economically small, because the main adjustment operates through quantities, factor reallocation, and input-output propagation.

Figure D.12: Propagation Across Space and Sector of the Exit of *Huachipato* Steel Plant: Employment



a) Sectoral employment change (%)

b) Sectoral contribution share (%)

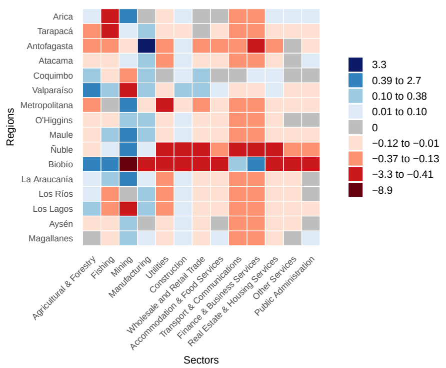
c) Provincial employment change (%)

d) Provincial contribution share (%)

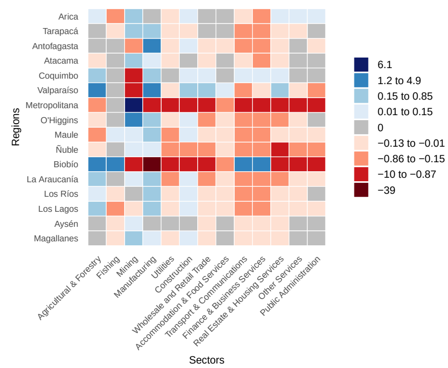
Fine province-sector incidence. The main text shows the region-sector incidence of the *Huachipato* shock for GDP. Figure D.13 reports the same type of decomposition at the finer province-sector level and compares measured TFP, employment, and GDP. Each cell reports the contribution of a province-sector pair to the corresponding national aggregate effect. This finer decomposition makes clear that the shock is not summarized by either geography or sector alone. Instead, the largest contributions come from specific combinations of locations and sectors.

The directly affected Concepción–Primary Metals cell is the most visible negative contributor, but the figure also shows substantial spillovers to other province-sector pairs. In GDP and employment, negative contributions extend to downstream manufacturing and related service activities both inside and outside Biobío. At the same time, several province-sector pairs display positive contributions, reflecting reallocation, substitution,

Figure D.12: Propagation Across Space and Sector of the Exit of *Huachipato* Steel Plant: Employment (continued)



e) Region-sector employment change (%)



f) Region-sector contribution share (%)

Notes: Each row reports the same employment object at a different level of aggregation. The left panels show the percentage change in employment for each sector, province, or region-sector pair. The right panels show each unit's contribution to the national employment change, defined as the unit-level change weighted by its national employment share and expressed relative to the aggregate employment change. Contributions sum to 100%. To make the remaining sectoral distribution legible, the Primary Metals bar is visually truncated in panels (a) and (b). Its actual sectoral employment change is -7.20%, and its actual contribution to the aggregate employment change is -19.3%. Values below the variable-specific plotting threshold are set to zero or omitted, as in the underlying figures.

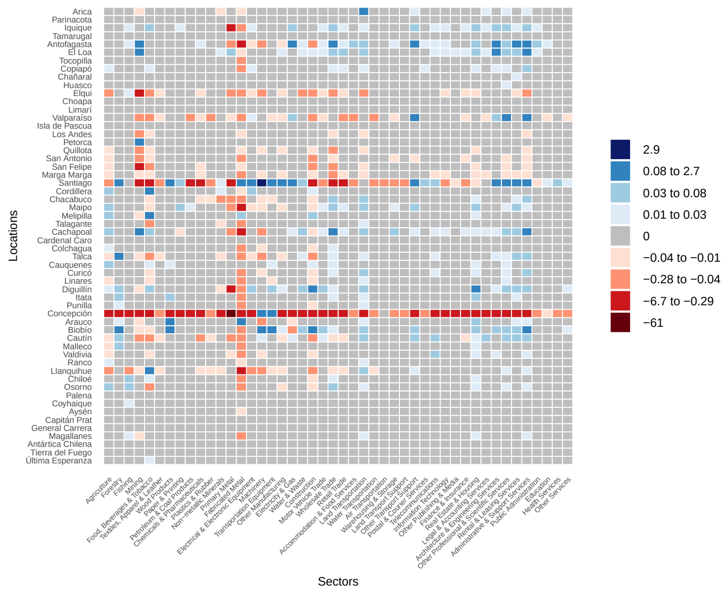
and changes in relative prices. These offsetting gains mitigate the aggregate contraction but do not overturn it. The fine decomposition therefore provides a visual counterpart to the variance decomposition in Table 5: propagation is inherently geographic-sectoral.

Distance and spatial propagation. Figure D.14 relates provincial GDP responses to distance from Concepción. The figure shows a clear spatial gradient: provinces closer to the origin of the shock tend to experience larger GDP losses. Distance alone explains a nontrivial share of the spatial variation, with a quadratic fit accounting for about 42% of the cross-sectional dispersion in provincial GDP responses. This pattern is consistent with the role of trade costs in the model.

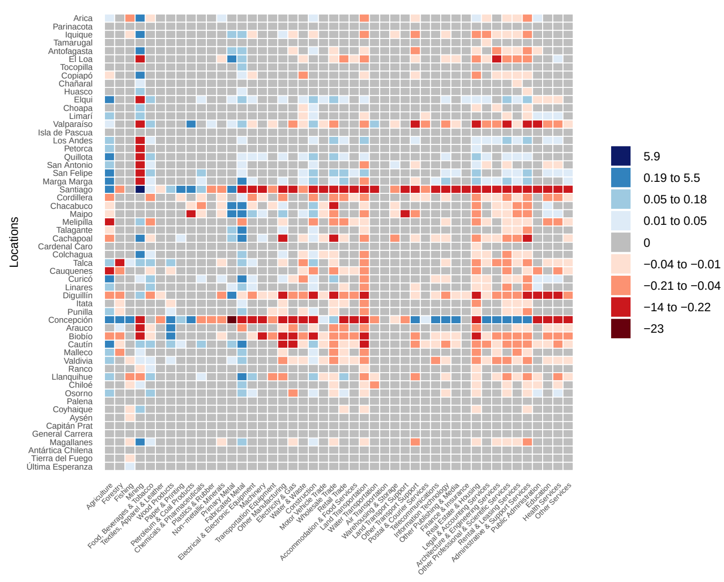
Distance, however, is not a sufficient statistic for propagation. Provinces at similar distances from Concepción can display different GDP responses because they differ in sectoral composition, input-output exposure, and network centrality. Some provinces are more exposed because they use steel-intensive inputs or are connected to affected downstream sectors, while others are less exposed or experience offsetting gains through reallocation. This is why the mechanism decomposition in Table 5 assigns a large role to the geography-sector interaction: distance shapes the spatial decay of the shock, but sectoral linkages determine which locations are most exposed.

Overall, the additional evidence in this appendix reinforces the main conclusion of the *Huachipato* application. The plant exit is local in origin, but its incidence is not local in equilibrium. Measured TFP effects are relatively concentrated, while GDP and employment effects propagate across locations and sectors. The fine province-sector decompositions and the distance gradient show why both geography and sectoral linkages are necessary to understand the shock: distance matters, but the strongest propagation patterns arise from the interaction between where production takes place and how sectors are connected through the input-output network.

Figure D.13: Province-Sector Contributions to National Aggregate Effects



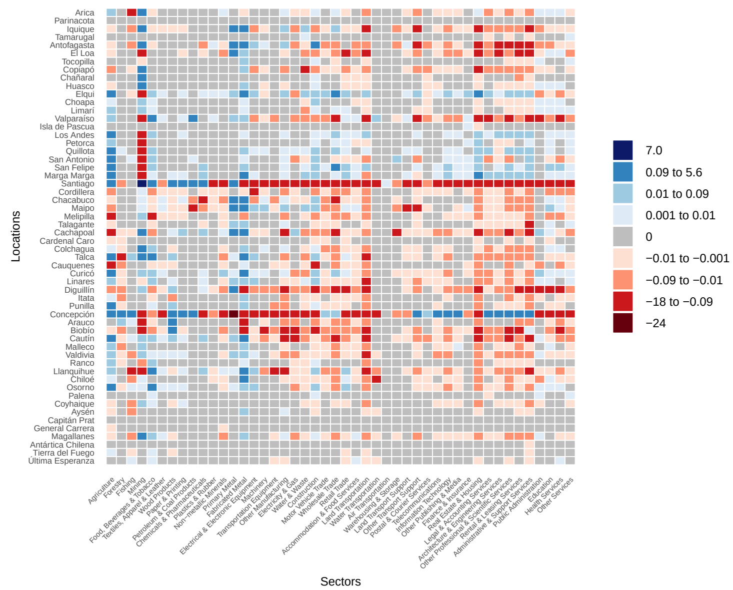
a) Measured TFP



b) Employment

Notes: Each panel reports the contribution of province-sector pairs to the corresponding national aggregate effect of the *Huachipato* shock. Contributions are defined as the province-sector percentage change weighted by its national share in the relevant outcome and expressed relative to the corresponding national aggregate change. Positive values indicate offsetting gains, while negative values indicate contributions to the aggregate loss. Values below the variable-specific plotting threshold are set to zero or omitted.

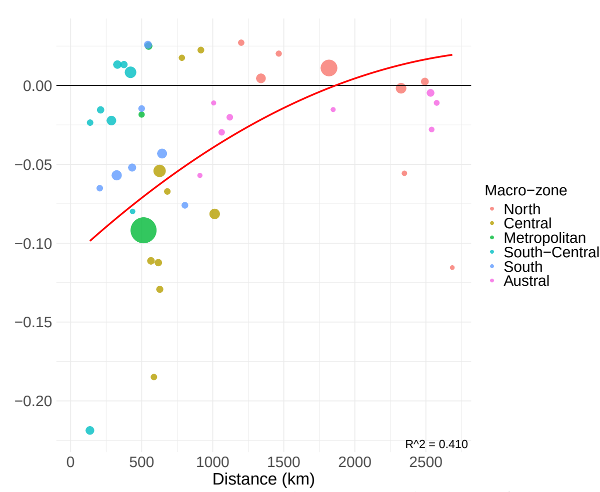
Figure D.13: Province-Sector Contributions to National Aggregate Effects (continued)



c) GDP

Notes: Each panel reports the contribution of province-sector pairs to the corresponding national aggregate effect of the *Huachipato* shock. Contributions are defined as the province-sector percentage change weighted by its national share in the relevant outcome and expressed relative to the corresponding national aggregate change. Positive values indicate offsetting gains, while negative values indicate contributions to the aggregate loss. Values below the variable-specific plotting threshold are set to zero or omitted.

Figure D.14: Propagation of the Exit of *Huachipato* by Distance: GDP



Notes: Figure relates provincial GDP percentage changes to distance from Concepción. The reported R^2 corresponds to a quadratic fit. For visualization purposes, the vertical axis is truncated to focus on the central part of the distribution; the main text reports the corresponding distance fit across model variants.