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Abstract

This paper argues that, in the presence of nominal wage rigidities, the existence of Rule-of-Thumb agents and price rigidities does not cause a change in the Taylor Principle as suggested by Galí et al. (2004), and that the only rigidity relevant for this result is that faced by Rule-of-Thumb consumers. For doing so, a New-Keynesian model with Rule-of-Thumb agents is proposed. The model discriminates betwen both type of agents when defining wage rigidities, thus allowing to identify and measure the factors that affect the Taylor Principle, this also allows to drop complete markets for Rule-of-Thumb agents, and the simple use of non-separable utility functions in order to determine the incidence of the wealth effect when facing staggered wages.

JEL classifications: E32, E37, C68.

Keywords: Taylor rule, Taylor principle, nominal rigidities, New-Keynesian model, Rule-of-Thumb consumers.

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1 Introduction

The widespread use of simple interest rate rules (henceforth Taylor rules) in order to describe the complex mechanisms behind monetary policy has attracted considerable attention in monetary economics, especially since the work of Taylor (1993, 1999). One of the main concerns generated by the use of a Taylor rule is the possible instability that may arise if the rule is improperly designed or implemented.[1] The conditions that prevent such instability are synthesized by Woodford (2001) in what is called the Taylor principle. The idea is that the monetary authority must be able to increase the real interest rate in response to a surge in inflation, thus affecting the decisions of the agents in the economy, inducing a decrease in aggregate demand and preventing further inflationary pressures. If the monetary authority fails to increase the real interest rate, inflation would generate a rise in aggregate demand and then further inflation. In this context inflation expectations become unbounded, leading to the instability of the equilibrium. In terms of the simple nominal interest rule, this means that the nominal interest rate must respond more than one-for-one to changes in the inflation rate in order to avoid indeterminacy driven by self-fulfilling expectations.

Although the Taylor principle seem to hold in most models applied in New-Keynesian macroeconomics, the mechanism by which it works depends on the degree of response of aggregate demand to changes in the real interest rate, as noted in Rotemberg and Woodford (1999, 109).

The work of Galí, López-Salido, and Vallés (2004) puts this into practice by testing the Taylor principle in a New-Keynesian model with Rule-of-Thumb (or Non-Ricardian) agents.[2] In the model presented in Galí et al. (2004) only a fraction of the households can smooth consumption (by means of capital accumulation), whereas the other fraction (Rule-of-Thumb agents) only have access to contemporaneous labor income for consumption. The model implies a "modified" Taylor principle in which the response of the nominal interest rate to changes in inflation is increasing in the fraction of Rule-of-Thumb agents. This "modified" Taylor principle dictates a stronger response to inflation compared with the traditional Taylor principle in the presence of nominal price rigidities

[1] References to this can be found in Clarida, Galí, and Gertler (2000), Woodford (2001, 2002), Bullard and Mitra (2002), Benhabib, Schmitt-Grohe, and Uribe (2001a,b) and Rotemberg and Woodford (1999). Instability may also arise if there is a strong working capital channel to the marginal costs, as noted by Christiano, Trabandt, and Walentin (2010).

[2] Other examples include Amato and Laubach (2003), Di-Bartolomeo and Rossi (2007) and Bilbiie (2008). The last two use models without capital accumulation and solve analytically for the equilibrium.

and Rule-of-Thumb agents.[3] In this sense Galí et al. (2004) state:

> When the central bank follows a rule that implies an adjustment of the nominal interest rate in response to variations in current inflation and output, the size of the inflation coefficient that is required in order to rule out multiple equilibria is an increasing function of the weight of Rule-of-Thumb consumers in the economy [...] the Taylor principle becomes too weak a criterion for stability when the share of Rule-of-Thumb consumers is large (Galí et al., 2004, 740).

The intuition behind the results is clear. Introducing rule of thumbs agents is equivalent to making part of the aggregate demand independent of the real interest rate (current and expected). In the presence of nominal price rigidities a surge in inflation is associated with countercyclical markups, and thus an increase in the real wage, that allows Rule-of-Thumb agents to consume more, independently of the real interest rate level. In order to guarantee the stability of the system, the effect of the increase in the nominal interest rate over Ricardian aggregate demand must be strong enough to dominate the increase in Rule-of-Thumb agents' consumption. Because of this, the nominal interest rate must be raised more as the share of Rule-of-Thumb agents increases.

Although the results of Galí et al. (2004) ilustrate possible pitfalls and shortcomings that a monetary authority would face when conducting its policy guided by the Taylor principle, I will argue that the "modified" Taylor principle discussed above is just a limiting case, and that in fact the introduction of Rule-of-Thumb agents and nominal rigidities does not imply a significant change in the traditional Taylor principle. The idea is simple, and it rests on the absence of nominal wage rigidities in the model developed in Galí et al. (2004). Although the authors argue that the introduction of wage rigidities (nominal or real) should not affect their findings (Galí et al., 2004, 744), I will use a variation of their model to prove otherwise. In this sense this work is similar to that of Colciago (2011). The difference lies in the way nominal rigidities are introduced, allowing them to affect only Ricardian or Rule-of-Thumb consumers. The model developed here contributes to the literature in two dimensions, one theoretical and the other methodological. First, it allows us to discriminate between the rigidities faced by Ricardian and Rule-of-Thumb agents, and hence to identify the kind and degree of nominal wage rigidity needed to guarantee equilibrium determinacy. These

[3] As will be verified later, Galí et al. (2004) find that the introduction of Non-Ricardian agents is not enough by itself to modify the Taylor principle. Nominal price rigidities are to be included as well.

two features are absent from previous studies. Second, the model makes it possible to drop the complete markets assumption for Rule-of-Thumb agents and also permits a simple use of a wider range of preferences in order to determine the incidence of the wealth effect on Rule-of-Thumb consumers when facing staggered wages.

The intuition behind the main result of this paper is that, in the presence of "sufficient" nominal wage rigidities, Rule-of-Thumb demand faces an automatic stabilizer when inflationary pressures are present. The degree of stickiness needed for this is shown to be near zero and prevents the modification of the Taylor principle, even when the share of Rule-of-Thumb households is near unity. This generalizes the findings of Galí et al. (2004) and Colciago (2011) by presenting both of their models as special cases of the one developed here and finding the minimum degree of wage stickiness needed for restoring the Taylor principle. The results of the articles mentioned are also expanded, for it is shown that the only relevant rigidity for ensuring equilibrium determinacy is the one that affects Rule-of-Thumb consumers.

The rest of this article is organized as follows. Section 2 describes the model. Afterward the parameter values are discussed in Section 3. Section 4 finds conditions of the Taylor rule that guarantee equilibrium determinacy under different configurations of nominal rigidities. Section 5 provides a discussion of alternative assumptions on agents' preferences. Finally, Section 6 concludes.

2 The model

The model presented here is a modified version of the one in Galí, López-Salido, and Vallés (2004, 2007), and it describes an economy with two types of agents. The first is the usual agent known to the literature, who has access to capital accumulation and a risk-free bond in order to smooth consumption over time; it is called Ricardian agent. The second does not have access to any mechanism for consumption smoothing. For that reason it is called Non-Ricardian or Rule-of-Thumb agent. The economy also consists of labor agencies,[4] intermediate goods producers, a final good aggregator, and a monetary authority.

In contrast to the models described in Galí et al. (2004, 2007), the one developed here

[4]It is important to note that the model presented here does not have equilibrium unemployment and that the labor agencies do not stand for those found in the Search and Matching literature. For this see, among others, Christoffel and Kuester (2008), Christoffel and Linzert (2010), Gertler, Sala, and Trigari (2008) and Blanchard and Gali (2010).

assumes that agents (both Ricardian and Rule-of-Thumb) are subject to nominal wage rigidities. For this I follow Erceg, Henderson, and Levin (2000). Introducing staggered wage setting implies that wages cannot respond as quickly to shocks as they do in Galí et al. (2004). This fact, although obvious, has serious consequences for the model dynamics. Nominal wage rigidities act as an automatic stabilizer for the consumption of Rule-of-Thumb agents. The intuition is that, in the presence of wage stickiness, a fraction of the agents will not be able to adjust their nominal wages when facing inflationary pressures. This lowers the average real wage of the Rule-of-Thumb agents and, labor income being their only source of income, their consumption. When facing downward inflationary pressures the opposite happens. The introduction of nominal wage rigidities in this paper differs from the one presented in Colciago (2011) because in this framework Ricardian and Non-Ricardian agents are modeled separately and are allowed to face different levels of wage rigidity. This distinction is relevant because staggered wages have different effects on agents' behavior depending on whether they are or are not Ricardian.

The markets in the model operate as follows. There is a competitive capital market between the Ricardian agents and the intermediate goods firms. The labor supplied by the Ricardian agents is bought in a monopolistically competitive market by a labor agency and packed in an index of Ricardian labor. The same happens to the Rule-of-Thumb agents. Afterward both indexes are bought by a third labor agency that generates a labor index sold to the intermediate goods firms in a competitive market. With capital and labor the firms produce differentiated intermediate goods that are sold, in a market in monopolistic competition to the final good aggregator. Then the aggregator sells the final good to the agents in a competitive market. The final good is used for consumption by both types of agents and for investment in capital. Intermediate goods firms are assumed to be subject to Calvo (1983) rigidities in price setting.

Complete markets that allow consumption insurance against labor market outcomes are assumed for both Ricardian and Rule-of-Thumb agents. This assumption is introduced for comparison with the models of Galí et al. (2004, 2007) and Colciago (2011) and can easily be dropped for Rule-of-Thumb agents, as shown in Section 5.

As for the composition of the representative household it is assumed that a fraction Γ of it is composed of Rule-of-Thumb consumers. This fraction is exogenously determined and held constant over time. In the following the subindex "a" will identify Non-Ricardian agents variables, and the subindex "b" will be used for Ricardian agents.

The rest of this section presents an overview of the model. Appendix A summarizes the linearized equilibrium conditions, and Appendix B summarizes the steady state solution of the model.[5]

Household

The economy is assumed to be populated by a unit measure household with Γ Rule-of-Thumb agents and $1-\Gamma$ Ricardian agents. As in Galí et al. (2007) and Colciago (2011), both agents have the same preferences, characterized by an additively separable utility function of the form:

$$\mathcal{U}_t\left(c_{i,t}, h_{i,t}\right) = \frac{c_{i,t}^{1-\sigma}}{1-\sigma} - \chi\frac{h_{i,t}^{1+\vartheta}}{1+\vartheta} \tag{1}$$

where $i \in (0,1)$ is an agent index, c is consumption, h is labor, σ measures the intertemporal elasticity of substitution, ϑ is the inverse of the Frisch elasticity and χ is a scale parameter.

It is also assumed that both types of agents have access to a set of Arrow-Debreau securities, $a_{i,t}^{(\zeta)}$ (with ζ varying in the event space), which allows them to completely ensure their income across Ricardian or Rule-of-Thumb agents, independently of the labor market outcome.[6]

Rule of Thumb agents

Rule-of-Thumb agent $i \in (0,\Gamma)$ seeks to maximize the expected value of its lifetime utility $\left(E_0\left\{\sum_{t=0}^{\infty}\beta^t\mathcal{U}_t\left(c_{i,t}, h_{i,t}\right)\right\}\right)$ by choosing consumption $(c_{i,t})$, the wage to charge for its labor $(w_{i,t})$ and the acquisition of a set of Arrow-Debreau securities $\left(a_{i,t}^{(\zeta)}\right)$. The agent is subject at all time to a budget constraint:

$$w_{i,t}h_{i,t}^{s} + a_{i,t}^{(\star)} = c_{i,t} + \int q_{i,t+1,t}a_{i,t+1}^{(\zeta)}d\zeta_{t+1,t} \tag{2}$$

where $q_{i,t+1,t}$ is the price of a $t+1$ security at period t, $\star$ indicates the realized event at t, and the integral of the right-hand side is taken over the set of all possible events of period $t+1$ given the information available up to t $(\zeta_{t+1,t})$. The agent is also subject to

[5] A detailed description of the model is available in Ocampo (2012).

[6] This assumption is usually imposed in order to facilitate aggregation and can be easily relaxed for the Rule-of-Thumb agents. Note that one only needs to know the aggregate labor income to aggregate for Non-Ricardian consumption, and that the former can be obtained from the Non-Ricardian labor agency first order conditions.

the labor demand for its type of labor. This will be obtained from the labor agencies problem below. Following Erceg et al. (2000), it is assumed that each Rule-of-Thumb agent is subject to nominal wage rigidity (of the kind introduced in Calvo, 1983). Each period an agent will be allowed to optimally adjust its wage with probability $1-\xi_a$. This probability is the same for all Rule-of-Thumb agents and is time and state independent. If not allowed to optimally adjust its nominal wage an agent must keep it fixed.

After aggregation among Rule-of-Thumb agents the solution to their problem is characterized by their aggregate budget constraint and a wage Phillips curve for the inflation of their average nominal wage, identical to that derived in Erceg et al. (2000) or Galí (2008, Chapter 6).

The introduction of staggered wage setting for the Rule-of-Thumb agents has a great impact on the model's dynamics. This is mainly due to the assumption that nominal wages are kept fixed when there is no optimization. That assumption implies that agents' real wage responds only to inflation when there is no optimization. The response to a surge in inflation is, off course, a decrease in the real wage. The critical point is that, for the Rule-of-Thumb agents, labor income is the only source of income, and thus it is directly linked to consumption. This link is what makes nominal wage rigidities an automatic stabilizer for Rule-of-Thumb consumption (when facing inflationary pressures).

Ricardian agents

The problem of the Ricardian agents is similar to the problem already presented for the Non-Ricardian agents. The difference lies in the extra decisions of the former: they have to optimize their bond holdings ($b_{j,t}$), investment ($x_{j,t}$) and capital accumulation ($k_{j,t}$); and the extra sources of income. It is assumed that firms profits (Pr_t) are evenly distributed among Ricardian agents.

The objective of a Ricardian agent $j \in (\Gamma, 1)$ is to maximize the expected value of its lifetime utility $\left(E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \mathcal{U}_t \left(c_{j,t}, h_{j,t} \right) \right\} \right)$ subject to a budget constraint and the capital accumulation equation. They are given by:

$$r_t^k k_{j,t} + w_{j,t} h_{j,t}^s + b_{j,t-1} \frac{i_{t-1}}{\pi_t} + \frac{\mathrm{Pr}_t}{1-\Gamma} + a_{j,t}^{(\star)} = c_{j,t} + x_{j,t} + b_{j,t} + \int q_{j,t+1,t} a_{j,t+1}^{(\zeta)} d\zeta_{j,t+1,t} \quad (3)$$

$$k_{j,t+1} = \phi \left(\frac{x_{j,t}}{k_{j,t}} \right) k_{j,t} + (1-\delta) k_{j,t} \quad (4)$$

where i_t is the gross nominal interest rate for period t bonds, π_t is the gross inflation rate for the final good price, and $\phi\left(\frac{x_t}{k_t}\right) k_{j,t}$ represents capital adjustment costs. It is assumed that $\phi'(\delta) = 1$ and $\phi(\delta) = \delta$. As with Rule-of-Thumb agents, optimization of the wage is also subject to Calvo rigidities and the labor demand from the Ricardian labor agency. The probability of optimally adjusting the nominal wage for a Ricardian agent is given by $1 - \xi_b$. As before, when there is no optimization nominal wages are kept fixed at their previous values.

The solution of the Ricardian agents problem is characterized by an Euler equation, the two constraints already presented, the definition of a wage Phillips curve for the inflation of their average wage ($w_{b,t}$) and the definition of Tobin's Q (Q_t) as capital's relative price for the agents.

Labor Agencies

There are three types of labor agencies: Ricardian, Non-Ricardian and Aggregate labor agencies. The first two are identical and are designed to buy the differentiated labor from Ricardian and Rule-of-Thumb agents and aggregate it in a Ricardian or Non-Ricardian labor index. The aggregation technology of these agencies is given by:

$$h_{i,t}^{s} = \left[\int_{0}^{1} h_{z,t}^{\frac{\eta-1}{\eta}} dz\right]^{\frac{\eta}{\eta-1}} \tag{5}$$

where $i \in \{a, b\}$ is an agent type index, z is an agent index, $h_{i,t}^{s}$ is the aggregate supply of type i agent labor and η denotes the elasticity of substitution among differentiated labor of type i agent (this parameter is agent invariant). It is important to note that aggregation is done over the space of Ricardian or Non-Ricardian agents; it then holds that, in equilibrium, the demand for Non-Ricardian labor index satisfies $h_{a,t} = \Gamma h_{a,t}^{s}$, and the demand for Ricardian labor satisfies $h_{b,t} = (1 - \Gamma) h_{b,t}^{s}$.

The problem of these first two labor agencies is to maximize their profits subject to the technology described in (5). Since both the Ricardian and Non-Ricardian labor agencies operate in perfect competition when selling the labor indexes to the Aggregate labor agency, their profits are to be zero in equilibrium. The labor indexes are sold at a price $w_{i,t}$, again with $i \in \{a, b\}$. The demand for agent z's differentiated labor is given by:

$$h_{z,t} = \left(\frac{w_{z,t}}{w_{i,t}}\right)^{-\eta} h_{i,t}^{s} \tag{6}$$

and the price ($w_{i,t}$) at which labor index i is sold is given by:

$$w_{i,t} = \left[\int_0^1 w_{z,t}^{1-\eta} dz\right]^{\frac{1}{1-\eta}} \tag{7}$$

The demand for the Ricardian and Non-Ricardian labor indexes comes from the Aggregate labor agency. This agency also operates in perfect competition and produces an aggregate labor index (h_t) that is sold at a price w_t to the intermediate goods producers. Its technology is given by:

$$h_t = h_{a,t}^{\Gamma} h_{b,t}^{1-\Gamma} \tag{8}$$

Firms

The production in the model economy is undertaken by two kind of firms, intermediate goods and final good producers. It is assumed that there is a measure one continuum of intermediate goods producers, each of which uses capital from the Ricardian agents and labor from the Aggregate labor agency to produce differentiated intermediate good ($v_{j,t}$). These goods are later sold to the final good firm that aggregates them into a final good index (Y_t). This index is used for consumption and investment. The final good firm operates in perfect competition and aggregates the intermediate goods using the following technology:

$$Y_t = \left[\int_0^1 v_{j,t}^{\frac{\theta-1}{\theta}} dj\right]^{\frac{\theta}{\theta-1}} \tag{9}$$

The problem of the intermediate goods firms is somewhat more complex since they operate in monopolistic competition and are subject to staggered price setting as in Calvo (1983). A firm has to determine its demand for production factors and its price if allowed to set it. The probability of price adjustment is given at all time by $1-\epsilon$. If not allowed to adjust its price a firm must keep the price charged in the previous period unchanged. The technology of each firm is given by:

$$v_{j,t} = k_{j,t}^{\alpha} h_{j,t}^{1-\alpha} \tag{10}$$

and the solution is characterized by a labor and a capital demand equation, and a New-Keynesian Phillips curve identical to that derived in Galí (2008, Chapter 3).

Monetary Authority

In order to close the model a monetary authority is introduced and is assumed to follow a Taylor rule when setting the value of the nominal interest rate. The linearized condition is given by:

$$i_t = \rho_i i_{t-1} + (1 - \rho_i) (\phi_\pi \pi_t + \phi_y \tilde{y}_t) \tag{11}$$

where $\tilde{y}_t \equiv \ln Y_t - \ln Y$, and Y is the steady state value of Y_t.

3 Parameterization

The parameterization of the model closely follows the one proposed by Galí et al. (2004). This is done in order to facilitate comparison of results. The elasticity of output to capital (α) is set to $1/3$, a value taken for consistency with the US labor income share. The capital depreciation rate (δ) is set to 0.025, implying a 10 percent annual rate of depreciation. The elasticity of the investment-capital ratio with respect to Tobin's Q (ι) is set to unity. The elasticity of substitution among types of labor (η) is set to 6 in order to obtain a steady state markup of 2 on wages. Following Ocampo (2012), the elasticity of substitution between intermediate goods (θ) is chosen to obtain a steady state consumption to output ratio of 80 percent under the baseline parametrization.

As for the utility parameters, the subjective discount factor (β) is set to 0.99, implying a steady state real annual interest rate of 4 percent. The relative risk aversion coefficient (σ) and the Frisch elasticity coefficient (ϑ) are both set to unity. The scaling parameter χ is chosen to obtain a steady state value for aggregate labor equal to $1/2$. Note that this parameter has no direct impact on the linearized equilibrium conditions (Appendix A) and therefore its exact value only affects the models solution by changing the steady state.

The value for the share of Rule-of-Thumb consumers (Γ) is not an easy choice, and the interesting thing to do is to explore the implications of different values for the model dynamics. For the present purpose, however, the baseline value for the parameter will imply that 30 percent of agents are Non-Ricardian.

Lastly, values for the parameters that only affect the dynamic properties of the

Table 1: Baseline Parameters Values

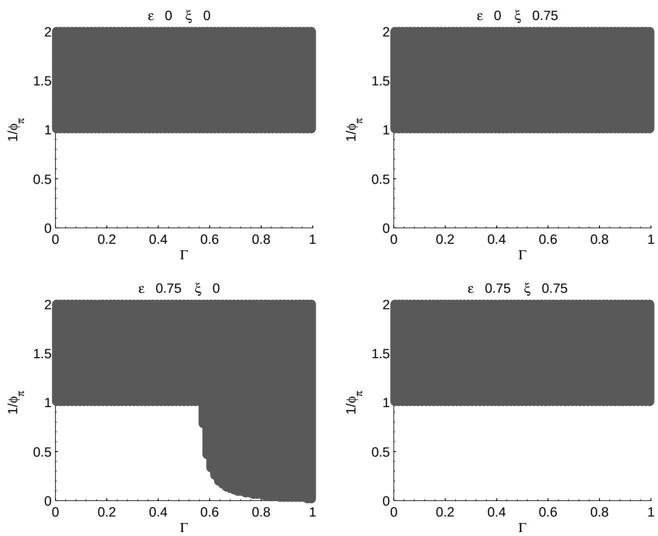
Parameter	Value	Description
α	$1/3$	Output to capital elasticity
δ	0.025	Capital depreciation rate
ι	1	Investment-Capital ratio to Tobin's Q elasticity
η	6	Elasticity of substitution between differentiated labor
θ	-	Elasticity of substitution between intermediate goods
β	0.99	Subjective discount factor
σ	1	Relative risk aversion
ϑ	1	Inverse of the Frisch elasticity
χ	-	Labor disutility scaling parameter
Γ	0.3	Share of Non-Ricardian households
ϵ	$3/4$	Probability of price adjustment
ξ_a	$3/4$	Probability of wage adjustment for Non-Ricardian households
ξ_a	$3/4$	Probability of wage adjustment for Ricardian households
ρ_i	0	Persistence of the Taylor rule
ϕ_π	1.5	Nominal interest response to inflation
ϕ_y	0	Nominal interest response to output

model, and not the steady state value of the variables, are chosen. The probabilities for price and wage adjustment (ϵ, ξ_a, ξ_b) are all set to $3/4$, corresponding to an average price and wage duration of four quarters, as usual in the literature. Note that, although the wage rigidity parameters are set equal under the baseline parameterization, it is possible to have different values for ξ_a and ξ_b. This fact will be exploited in the following sections. The persistence of the nominal interest rate in the Taylor rule (ρ_i) is set to 0 in the baseline parameterization and the other parameters of the Taylor rule are set in a way that the Taylor principle is satisfied (Woodford, 2001). The response to changes in inflation (ϕ_π) is set to 1.5 and the response to output deviations from its steady state value (ϕ_y) to 0. Table 1 summarizes this section.

4 Simulation and equilibrium determinacy

The solution of the log-linear approximation of the model presented in Section 2 can be computed by means of the Klein (2000) algorithm (a generalization of the Blanchard and Kahn, 1980, algorithm). The objective of this section is to establish under which conditions a unique and stable equilibrium exists. Those conditions, given by subsets of the parameter space, will be referred to as determinacy conditions for the equilibrium.

Figure 1: Determinacy Conditions - Composition (Γ)



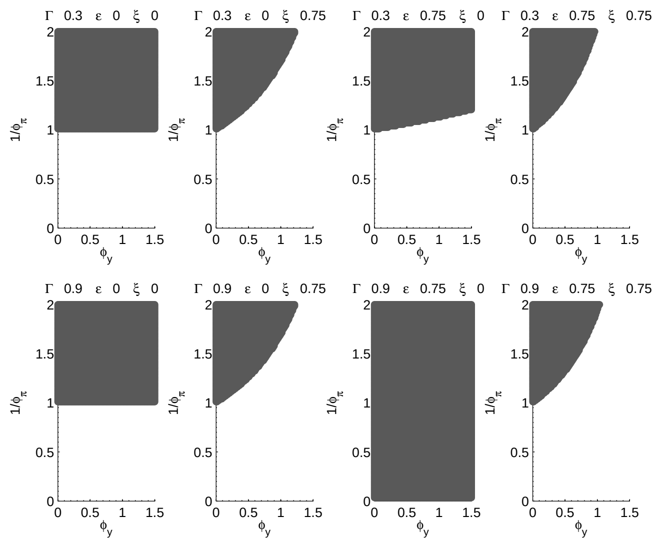
Determinacy conditions for policy response to inflation and Rule-of-Thumb share. Dark areas correspond to non-determinacy regions, and simulations are made under baseline parametrization. It is assumed that $\xi_a = \xi_b = \xi$.

As shown by Galí et al. (2004), the presence of Rule-of-Thumb agents and nominal price rigidities affects the determinacy conditions of the model. In particular, the conditions shift to an area of the parameter space characterized by a stronger response of the monetary authority to inflation. That shift implied a change in the traditional Taylor principle, establishing that the minimum response to inflation required for the equilibrium to be stable and unique is an increasing function of the share of Rule-of-Thumb consumers in the model. In the work of Colciago (2011), the former analysis was complemented by introducing nominal wage rigidities (for Ricardian and Non-Ricardian agents). Assuming a common value for Ricardian and Non-Ricardian wage rigidity ($\xi_a = \xi_b = \xi = 3/4$)), he showed that the Taylor principle was restored.

In order to verify those findings an exercise similar to that of Galí et al. (2004) is conducted. The model is simulated in the (Γ , ϕ_π) space checking the determinacy conditions at each point. In this way a non-determinacy region is constructed for that space. This is done under the model baseline parameterization (Table 1) for four different configurations of nominal rigidities.

The results of the exercise are reported in Figure 1. The left-hand side graphs correspond to the cases presented in Galí et al. (2004). The upper-left graph covers the case with no nominal rigidity (either in prices or wages), as in the cited work the

Figure 2: Determinacy Conditions - Policy Response



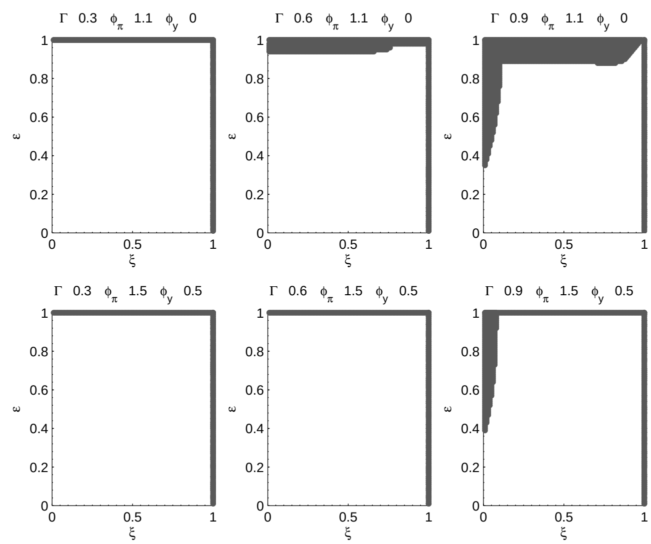
Determinacy conditions over policy response to inflation and output. Dark areas correspond to non-determinacy regions. Simulations are made under baseline parameterization. It is assumed that $\xi_a = \xi_b = \xi$.

determinacy conditions are characterized by the traditional Taylor principle. Note that the non-determinacy region is composed of the points in the $(\Gamma\,,\,\phi_\pi)$ space in which $\phi_\pi < 1$. The lower-left graph corresponds to the case with a high level of price rigidity and absence of nominal wage rigidity. The determinacy conditions are violated when the traditional Taylor principle is not held ($\phi_\pi < 1$) or when the share of Rule-of-Thumb agents increases. In fact, as in Galí et al. (2004), when parameter Γ goes above certain threshold the non-determinacy region begins expanding rapidly up to a point at which there is no response of the monetary authority to inflation sufficiently large as to ensure the determinacy of the equilibrium. The right-hand side graphs correspond to the cases proposed in Colciago (2011), both with the presence of nominal wage rigidity. It is found that in the presence of nominal wage rigidities the determinacy region corresponds to that dictated by the traditional Taylor principle, independently of the presence of Non-Ricardian agents, and whether there are price rigidities or not.

A similar exercise is carried out to check the impact of monetary policy reaction to output on the determinacy conditions (recall that $\phi_y = 0$ under the baseline parameterization). The simulations are done over the $(\phi_y\,,\,\phi_\pi)$ space for different combinations of nominal rigidities and Rule-of-Thumb share (Γ).[7] The results are presented in Figure 2

[7]The original work of Woodford (2001), among others, points out the role of the response to output

Figure 3: Determinacy Conditions - Nominal Rigidities



Determinacy conditions over nominal rigidities parameters. Dark areas correspond to non-determinacy regions. Simulations are made under baseline parameterization. It is assumed that $\xi_a = \xi_b = \xi$.

and are in line with the ones already presented. In short, as shown in Woodford (2001), for a given response to output the necessary response to inflation that ensures determinacy decreases as the former increases. This happens when there is some form of nominal rigidity active in the model. When there is price rigidity in the absence of nominal wage rigidity, the response to output has a smaller effect on the non-determinacy region compared to the case in which the latter is present. The effect is also smaller (or inexistent) when the share of Rule-of-Thumb agents is high. Another exercise is carried out for determining the effects of the Taylor rule smoothing over the determinacy conditions. It is found that the smoothing has no effect over those conditions, as in Schmitt-Grohe and Uribe (2007) and Colciago (2011). For reasons of space the results are not presented here.

Although the previous exercises give information on the effects of Rule-of-Thumb agents and nominal rigidities on the Taylor principle, they do not cover the effects of nominal rigidities on the determinacy conditions. There still remains the question of the extent to which nominal wage rigidities affect the results given in Galí et al. (2004). Under what conditions on nominal rigidities is the Taylor principle valid? To

in the model's determinacy conditions. This response turns out to be crucial when the rule is forward looking as shown by Bernanke and Woodford (1997) and Galí et al. (2004). Colciago (2011) uses a Taylor rule of the form $i_t = \phi_\pi \pi_t$ in his baseline analysis, and his findings do not include the effect of the response to output in the Taylor rule.

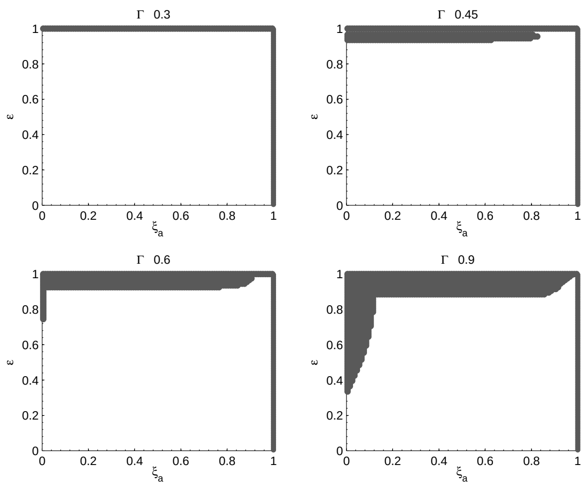
answer that question non-determinacy conditions on the space of nominal rigidities are obtained. This is done for combinations of low, mid and high shares of Rule-of-Thumb agents, and for a weak and a strong response of monetary policy. The weak response of monetary policy is one with no response to output ($\phi_y = 0$) and a response to inflation just above unity ($\phi_\pi = 1.1$). The strong response of monetary policy is one with a stronger response to output ($\phi_y = 0.5$) and inflation ($\phi_\pi = 1.5$). In both cases the traditional Taylor principle is fulfilled.

The results in Figure 3 indicate that the equilibrium is undetermined only when price rigidity is sufficiently high and wage rigidity sufficiently low. The actual values of the minimum and maximum levels of rigidities, for which there is still indeterminacy, are shown to vary depending on the share of Rule-of-Thumb agents (positively) and the strength of the monetary policy reaction to inflation and output (negatively). In either case the level of wage rigidity necessary to ensure the system's stability is no greater than $1/5$, a level below the ones presented in Christiano, Eichenbaum, and Evans (2005) and Smets and Wouters (2007) for the US economy and Smets and Wouters (2003) for the European economy. This finding is in line with the intuition presented in Section 2. The presence of nominal wage rigidity acts as an automatic stabilizer for consumption when there is a surge in inflation, thus making the equilibrium more stable. It is interesting to note, however, the low level of rigidity necessary to obtain this result.

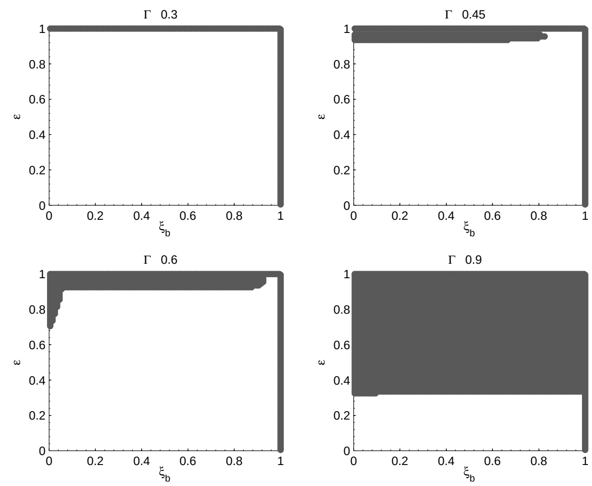
Finally, there is the question of which type of wage rigidity is relevant for equilibrium determinacy. In the past exercises it was assumed that the level of nominal wage rigidity faced by Ricardian and Non-Ricardian agents was equal ($\xi_a = \xi_b = \xi$). Figure 4 presents non-determinacy regions in the (ξ_a, ϵ) and (ξ_b, ϵ) spaces in Panels 4a and 4b respectively. In each case it is assumed that the other type of agent does not face any wage rigidities. It is also assumed that monetary policy response to inflation and output is weak (as in Figure 3) and that the level of price rigidity is high ($\epsilon = 3/4$); this is done because the above conditions make the system more likely to be unstable. The results indicate that the Non-Ricardian wage rigidity is what matters for equilibrium determinacy (note the difference between the lower-right graphs of Panels 4a and 4b). Given enough wage rigidity for Rule-of-Thumb agents, the model's solution exists, and it is unique and stable even when those agents account for 90 percent of the economy, monetary policy has a weak response to inflation and output and there is a significant level of price rigidity.

Figure 4: Determinacy Conditions - Wage Rigidities

(a) Determinacy Conditions - Rule-of-Thumb Wage Rigidity



(b) Determinacy Conditions - Ricardian Wage Rigidity



Panel 4a presents determinacy conditions over price and Non-Ricardian nominal wage rigidities assuming $\xi_b = 0$.

Panel 4b presents determinacy conditions over price and Ricardian nominal wage rigidities assuming $\xi_a = 0$.

Dark areas correspond to non-determinacy regions. Simulations are made under baseline parameterization.

5 Preferences and the wealth effect

The utility function used in Section 2, as well as the one used in Galí et al. (2004), imply that the marginal rate of substitution between labor and consumption (henceforth MRS) depends positively on the agents consumption. This means that there is a direct wealth effect involved in the agents wage decision. As noted, for example, by Jaimovich and Rebelo (2009), the size of the mentioned wealth effect matters for model dynamics. Nevertheless, it is usual in the literature to neglect this fact, and use an additively separable utility function like that in equation (1), because of the aggregation problems that arise when introducing staggered wages as in Erceg et al. (2000) with a non-separable utility function.

The findings of Section 4 indicate that Non-Ricardian wage rigidity is the only relevant rigidity needed to restore the Taylor principle. This fact makes it possible to formulate a model economy that allows for a simple use of a wider range of preferences in order to determine the incidence of the wealth effect on Rule-of-Thumb consumers when facing staggered wages. As will be described below, it is possible to introduce in a simple manner KPR and GHH preferences (King, Plosser, and Rebelo, 1988, and Greenwood, Hercowitz, and Huffman, 1988) into the Rule-of-Thumb framework under nominal wage rigidities. The former preferences imply a "complete" wealth effect, and are in fact the ones used in Galí et al. (2004), while the latter imply no wealth effect on agents' labor market decisions. It is also possible to use a variation of the Jaimovich and Rebelo (2009) preferences that nests KPR and GHH preferences and allows control over the size of the wealth effect.

The use of non-separable utility functions for Non-Ricardian agents is considerably simpler than the use of that function in a Ricardian framework. This follows from two facts. The first one is that, by construction, Non-Ricardian agents face no intertemporal decision besides wage setting (when there are nominal wage rigidities). The second is that their consumption depends only on their labor income, which can be obtained straightforwardly from the labor agency's first order conditions.[8] The first fact implies that agents' marginal utility from consumption has no impact on the equilibrium conditions, other than its effect on the MRS in the labor market decisions. The second

[8] Recall the resource constraint for a Rule-of-Thumb agent i: $c_{i,t} = w_{i,t}h_{i,t}$. Using equation (6) for labor demand: $c_{i,t} = \frac{w_{i,t}^{1-\eta}}{w_{a,t}^{-\eta}} h_{a,t}^{s}$. Aggregating over all Rule-of-Thumb agents: $\Gamma c_{a,t} = \frac{h_{a,t}^{s}}{w_{a,t}^{-\eta}} \int_0^1 w_{i,t}^{1-\eta} di$. With equation 7 and the equilibrium condition for the Non-Ricardian labor index market the aggregation is finished: $c_{a,t} = w_{a,t}h_{a,t}$.

fact makes aggregation feasible even in the absence of Arrow-Debreau securities, hence allowing us to drop the complete markets assumption for Rule-of-Thumb agents.

Equations (12), (13) and (14) present the GHH, KPR and a version of the JR utility functions, as well as their respective MRS. In equation (14) $\Omega_{a,t} = c_{a,t}^{\gamma}\Omega_{a,t-1}^{1-\gamma}$. Note that parameter $\gamma \in [0,1]$ allows us to control the size of the wealth effect by determining the contemporaneous impact of consumption on the MRS, in a way similar to that presented in Jaimovich and Rebelo (2009).[9]

$$\mathcal{U}_t\left(c_{i,t},h_{i,t}\right) = \frac{\left(c_{i,t} - \chi\frac{h_{i,t}^{1+\vartheta}}{1+\vartheta}\right)^{1-\sigma} - 1}{1-\sigma} \qquad \text{MRS}_{i,t} = -\frac{\mathcal{U}_{h,t}\left(c_{i,t},h_{i,t}\right)}{\mathcal{U}_{c,t}\left(c_{i,t},h_{i,t}\right)} = \chi h_{i,t}^{\vartheta} \tag{12}$$

$$\mathcal{U}_t\left(c_{i,t},h_{i,t}\right) = \frac{\left(c_{i,t}\left(1 - \chi\frac{h_{i,t}^{1+\vartheta}}{1+\vartheta}\right)\right)^{1-\sigma} - 1}{1-\sigma} \qquad \text{MRS}_{i,t} = -\frac{\mathcal{U}_{h,t}\left(c_{i,t},h_{i,t}\right)}{\mathcal{U}_{c,t}\left(c_{i,t},h_{i,t}\right)} = \chi c_{i,t} h_{i,t}^{\vartheta} \tag{13}$$

$$\mathcal{U}_t\left(c_{i,t},h_{i,t}\right) = \frac{\left(c_{i,t} - \chi\Omega_{a,t}\frac{h_{i,t}^{1+\vartheta}}{1+\vartheta}\right)^{1-\sigma} - 1}{1-\sigma} \qquad \text{MRS}_{i,t} = -\frac{\mathcal{U}_{h,t}\left(c_{i,t},h_{i,t}\right)}{\mathcal{U}_{c,t}\left(c_{i,t},h_{i,t}\right)} = \chi \Omega_{a,t} h_{i,t}^{\vartheta} \tag{14}$$

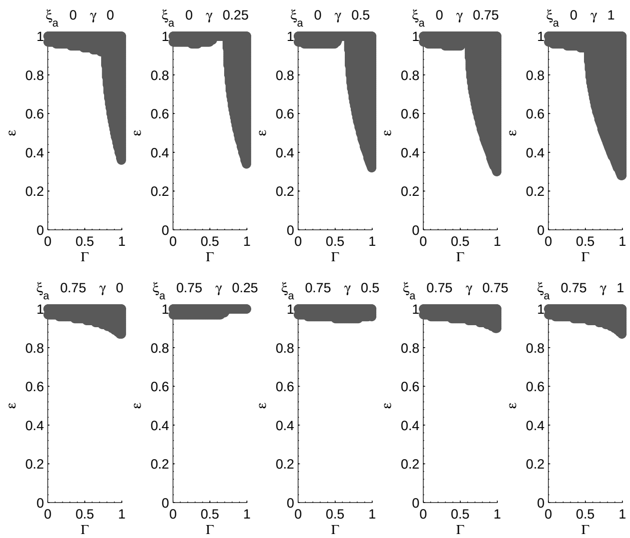
Using any of the preferences mentioned above implies little change to the model presented in Section 2 if they are only used for Rule-of-Thumb consumers or if Ricardian agents are assumed to face no wage rigidities. The only change is in the definition of the wage markup (equation 29 of Appendix A). The wage markup depends on the average MRS of Non-Ricardian agents which is obtained using equation (12), (13) or (14).

In order to determine the effect of the wealth effect on determinacy conditions a variation of the model in Section 2 is simulated. The new model assumes that Non-Ricardian preferences can be represented by the utility function of equation (14) and keeps Ricardian agents unchanged while assuming that they face no wage rigidity. Under the baseline parameterization this implies no changes in the deterministic steady state of the model while $\gamma > 0$ and, as mentioned above, only equation 29 of Appendix A is changed. The equation is now given by:

$$\tilde{\mu}_{w_a,t} = \tilde{w}_{a,t} - \left(\vartheta \tilde{h}_{a,t}^{s} + \tilde{\Omega}_{a,t}\right) \tag{15}$$

[9] In Jaimovich and Rebelo (2009) Ω_t is assumed to be affected by agents' consumption and not aggregate consumption. Agents are assumed to be aware of this when making consumption and labor market decisions. This is not done here for convenience.

Figure 5: Determinacy Conditions - Preferences and Wealth Effect



Determinacy conditions over price rigidity and Rule-of-Thumb share. Dark areas correspond to non-determinacy regions. Simulations are made under baseline parameterization. It is assumed that $\xi_b = 0$ *and different values for parameters* ξ_a *and* γ *are used as stated in the titles.*

with $\tilde{\Omega}_{a,t} = \gamma \tilde{c}_{a,t} + (1-\gamma)\,\tilde{\Omega}_{a,t-1}$. When $\gamma = 0$ the model converges to that of the GHH preferences, equation (12), and in that case the deterministic steady state is changed. The changes are described in Appendix B.[10]

Figure 5 presents the results of the exercise, which consists of characterizing the determinacy conditions of the model in the (Γ, ϵ) space under different values of Non-Ricardian nominal wage rigidity (ξ_a) and wealth effect presence (γ). Recall that when $\gamma = 0$ there is no wealth effect involved in the agent wage decision. There are two main results that arise from this exercise. The first is that changes in the preferences do not alter the effect of nominal wage rigidities on the determinacy conditions. It is still true that the non-determinacy region shrinks as the nominal wage rigidities of Rule-of-Thumb consumers are increased. The second result is that, for $\gamma > 0$, a stronger wealth effect induces indeterminacy in the model. The intuition of this result is the following: when consumption affects agents' labor market decision they are biased towards increasing labor income in response to any shock that decreases consumption. This bias towards a greater labor income, obtained whether by working more hours or

[10]Note that when $\gamma = 1$ the model does not converge to that of the KPR preferences, equation (13). The difference lies in the assumption that $\Omega_{a,t}$ depends on the aggregate Non-Ricardian consumption. Since in the KPR preferences the MRS is a function of agent consumption this changes the aggregation when obtaining the wage Phillips curve. The problem is solved recalling that $\tilde{c}_{i,t} = \tilde{w}_{i,t} + \tilde{h}_{i,t}$ for a Rule-of-Thumb agent i.

increasing the wage, allows Rule-of-Thumb agents to consume more regardless of the nature of the shock and the policy response. This is why there is more instability in the economy as the wealth effect becomes stronger.

When there is no wealth effect, under GHH preferences ($\gamma = 0$), the non-determinacy region is increased compared to a case with a low but positive γ. Nevertheless this result must be taken cautiously for the model has a discrete change when $\gamma = 0$ (for instance in the steady state). However, it is still the case that the non-determinacy region shrinks with respect to the full wealth effect scenario ($\gamma = 1$).

6 Final remarks

When addressing the use of simple policy rules by the monetary authority it is important to keep in mind that they constitute a mere approximation to the complex mechanism behind policymakers' decision making process. In particular, it has been shown that improperly designed policy rules can introduce new sources of instability into the models used to address policy issues. The main result of the literature in this respect is given by Woodford (2001) in what has been called the Taylor principle. Nevertheless, due to its success as a criterion for correctly designing simple interest rate rules, it might be applied in cases in which its validity has not been checked. The work of Galí et al. (2004) is a clear example of the possible pitfalls of the Taylor principle.

In this document the findings of Galí et al. (2004) and Colciago (2011) are revised and complemented by introducing nominal wage rigidities into a New-Keynesian model with Rule-of-Thumb consumers, and establishing the conditions that the policy rule parameters must satisfy in order to ensure equilibrium determinacy. It is found that, as in Colciago (2011), the conditions under which the Taylor principle is modified correspond to the limiting case where agents face no nominal wage rigidity, and that the modification of the Taylor principle (as a function of the share of Rule-of-Thumb consumers) cannot be generalized to a model with staggered wages. Moreover, it is found that the degree of nominal wage rigidity required for this to hold is low enough to be non-binding under the usual parameterization of the model, and that the only relevant rigidity for the results is the one that affects Rule-of-Thumb consumers. This last finding allows us to introduce in a simple manner a wider range of preferences for Rule-of-Thumb consumers, even when facing nominal wage rigidities. Changing the preferences makes it possible to evaluate the impact of the wealth effect on the determinacy conditions of the model. As shown in Section 5, a stronger wealth effect is associated with greater indeterminacy.

As argued in the paper, the intuition behind the Taylor principle is based on in the response of agents to changes in the real interest rate (Rotemberg and Woodford, 1999). This is why the introduction of Rule-of-Thumb consumers is able to modify the Taylor principle (Galí et al., 2004). What is found in this paper is that the introduction of staggered wages into the model acts as an automatic stabilizer for the Rule-of-Thumbers' income (and thus consumption) in the presence of inflationary pressures. This is why it suffices to include nominal wage rigidities for the Non-Ricardian agents in order to re-establish the traditional Taylor principle as a criterion for determinacy in the model.

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A Linear Equilibrium

Variables: $\{i\,,\,\pi\,,\,y\,,\,c\,,\,x\,,\,k\,,\,h\,,\,c_a\,,\,c_b\,,\,h_a\,,\,h_b\,,\,\varphi\,,\,r^k\,,\,w\,,\,w_a\,,\,w_b\,,\,\pi^{w_a}\,,\,\pi^{w_b}\,,\,\mu_{w_a}\,,\,\mu_{w_b}\,,\,q\,,\,\mathrm{pr}\}$

Monetary Policy

$$i_t = \rho_i i_{t-1} + (1-\rho_i)(\phi_\pi \pi_t + \phi_y \tilde{y}_t) + \epsilon_{i,t} \tag{16}$$

Aggregation

$$\tilde{y}_t = \frac{c}{y}\tilde{c}_t + \frac{x}{y}\tilde{x}_t \tag{17}$$

$$\tilde{c}_{a,t} = \tilde{w}_{a,t} + \tilde{h}_{a,t} \tag{18}$$

$$\tilde{c}_t = \frac{c_a}{c}\Gamma\tilde{c}_{a,t} + \frac{c_b}{c}(1-\Gamma)\tilde{c}_{b,t} \tag{19}$$

$$\tilde{\mathrm{pr}}_t = \frac{1}{1-\varphi}\left(\tilde{y}_t - \alpha\varphi\left(\tilde{r}^k_t + \tilde{k}_t\right) - (1-\alpha)\varphi\left(\tilde{w} + \tilde{h}\right)\right) \tag{20}$$

Production

$$\tilde{r}^k_t = \tilde{\varphi}_t - (1-\alpha)\left(\tilde{k}_t - \tilde{h}_t\right) \tag{21}$$

$$\tilde{w}_t = \tilde{\varphi}_t + \alpha\left(\tilde{k}_t - \tilde{h}_t\right) \tag{22}$$

$$\tilde{y}_t = \tilde{z}_t + \alpha\tilde{k}_t + (1-\alpha)\tilde{h}_t \tag{23}$$

Prices

$$\pi_t = \beta E_t\{\pi_{t+1}\} + \frac{(1-\epsilon)(1-\epsilon\beta)}{\epsilon}\tilde{\varphi}_t \tag{24}$$

Labor

$$\tilde{h}_{at} = (\tilde{w}_t - \tilde{w}_{at}) + \tilde{h}_t \tag{25}$$

$$\tilde{h}_{bt} = (\tilde{w}_t - \tilde{w}_{bt}) + \tilde{h}_t \tag{26}$$

$$\tilde{h}_t = \Gamma\tilde{h}_{at} + (1-\Gamma)\tilde{h}_{bt} \tag{27}$$

Non-Ricardian Wage

$$\pi^{w_a}_t = \beta E_t\left\{\pi^{w_a}_{t+1}\right\} - \frac{(1-\xi_a)(1-\beta\xi_a)}{\xi_a(1+\eta\vartheta)}\tilde{\mu}_{w_a,t} \tag{28}$$

$$\tilde{\mu}_{w_a,t} = \tilde{w}_{a,t} - \left(\vartheta\tilde{h}^s_{a,t} + \sigma\tilde{c}_{a,t}\right) \tag{29}$$

$$\tilde{w}_{a,t} = \tilde{w}_{a,t-1} + \pi^{w_a}_t - \pi_t \tag{30}$$

Ricardian Wage

$$\pi^{w_b}_t = \beta E_t\left\{\pi^{w_b}_{t+1}\right\} - \frac{(1-\xi_b)(1-\beta\xi_b)}{\xi_b(1+\eta\vartheta)}\tilde{\mu}_{w_b,t} \tag{31}$$

$$\tilde{\mu}_{w_b,t} = \tilde{w}_{b,t} - \left(\vartheta\tilde{h}^s_{b,t} + \sigma\tilde{c}_{b,t}\right) \tag{32}$$

$$\tilde{w}_{b,t} = \tilde{w}_{b,t-1} + \pi^{w_b}_t - \pi_t \tag{33}$$

Capital

$$\tilde{q}_t = -(i_t - E_t\{\pi_{t+1}\}) + \beta E_t\{\tilde{q}_{t+1}\} + [1-\beta(1-\delta)]E_t\left\{\tilde{r}^k_{t+1}\right\} \tag{34}$$

$$\tilde{x}_t - \tilde{k}_t = \iota\tilde{q}_t \tag{35}$$

$$\tilde{k}_{t+1} = \delta\tilde{x}_t + (1-\delta)\tilde{k}_t \tag{36}$$

Euler Equation

$$i_t - E_t\{\pi_{t+1}\} = \sigma(E_t\{\tilde{c}_{b,t+1}\} - \tilde{c}_{b,t}) \tag{37}$$

B Steady State

Under $\sigma = 1$ and $\vartheta = 1$ the values of the steady state variables is given by the following equations:

$$\pi = 1 \tag{38}$$

$$\varphi = \frac{\theta - 1}{\theta} \tag{39}$$

$$i = \frac{1}{\beta} \tag{40}$$

$$Q = 1 \tag{41}$$

$$r^k = \frac{1}{\beta} + \delta - 1 \tag{42}$$

$$h_a = \left(\frac{\eta}{\eta - 1}\frac{\chi}{\Gamma^2}\right)^{\frac{-1}{2}} \tag{43}$$

$$w = (1-\alpha)\,\varphi z \left(\frac{r^k}{\alpha\varphi z}\right)^{\frac{\alpha}{\alpha-1}} \tag{44}$$

$$h = \left[\frac{(1-\Gamma)^3\,\frac{\eta-1}{\chi\eta} w h_a^{\frac{2\Gamma}{1-\Gamma}}}{z\left(\frac{r^k}{\alpha\varphi z}\right)^{\frac{\alpha}{\alpha-1}} - \delta\left(\frac{r^k}{\alpha\varphi z}\right)^{\frac{1}{\alpha-1}} - \Gamma w}\right]^{\frac{1-\Gamma}{2}} \tag{45}$$

$$h_b = h_a^{\frac{-\Gamma}{1-\Gamma}} h^{\frac{1}{1-\Gamma}} \tag{46}$$

$$k = \left(\frac{r^k}{\alpha\varphi z}\right)^{\frac{1}{\alpha-1}} h \tag{47}$$

$$y = z\left(\frac{r^k}{\alpha\varphi z}\right)^{\frac{\alpha}{\alpha-1}} h \tag{48}$$

$$w_a = \Gamma\frac{w}{h_a} h \tag{49}$$

$$w_b = (1-\Gamma)\, w h_a^{\frac{\Gamma}{1-\Gamma}} h^{\frac{-\Gamma}{1-\Gamma}} \tag{50}$$

$$c_a = \frac{1}{\Gamma} w_a h_a \tag{51}$$

$$c_b = (1-\Gamma)\,\frac{\eta-1}{\chi\eta}\frac{w_b}{h_b} \tag{52}$$

$$c = \Gamma c_a + (1-\Gamma)\, c_b \tag{53}$$

$$x = \delta k \tag{54}$$

$$\Pr = (1-\varphi)\, y \tag{55}$$

Parameter θ can be set as to obtain a consumption output ratio of $\overline{\frac{c}{y}}$ in steady state:

$$\theta = \left[1 - \frac{\frac{1}{\beta} + \delta - 1}{\delta\alpha}\left(1 - \overline{\frac{c}{y}}\right)\right]^{-1} \tag{56}$$

When preferences are of the GHH type, equation (12), conditions (43) and (45) are modified to:

$$h = \left[\frac{(1-\Gamma)^3\,\frac{\eta-1}{\chi\eta} w \left(\frac{\eta-1}{\eta}\frac{\Gamma^2}{\chi} w\right)^{\frac{\Gamma}{1-\Gamma}}}{z\left(\frac{r^k}{\alpha\varphi z}\right)^{\frac{\alpha}{\alpha-1}} - \delta\left(\frac{r^k}{\alpha\varphi z}\right)^{\frac{1}{\alpha-1}} - \Gamma w}\right]^{\frac{1-\Gamma}{2-\Gamma}} \qquad h_a = \left(\frac{\eta-1}{\eta}\frac{\Gamma^2}{\chi} wh\right)^{\frac{1}{2}} \tag{57}$$