IDB WORKING PAPER SERIES Nº IDB-WP-785

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April 2017

Cataloging-in-Publication data provided by the
Inter-American Development Bank
Felipe Herrera Library
Robots are us: some economics of human / Seth G. Benzell, Laurence J. Kotlikoff, Guillermo Lagarda, Jeffrey D. Sachs.
p. cm. — (IDB Working Paper Series ; 785)
Includes bibliographic references.
1. Human-machine systems. 2. Labor supply-Effect of automation on. 3. Automation-Economic aspects. I. Benzell, Seth G. II. Kotlikoff, Laurence J. III. Lagarda, Guillermo. IV. Sachs, Jeffrey. V. Inter-American Development Bank. Country Department Central America, Mexico, Panama and the Dominican Republic. VI. Series.
IDB-WP-785

http://www.iadb.org

Robots Are Us: Some Economics of Human Replacement*

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April 2017

Abstract

Will smart machines replace humans like the internal combustion engine replaced horses? If so, can putting people out of work, or at least out of good work, also put the economy out of business? Our model says yes. Under the right conditions, more supply produces, over time, less demand as the smart machines undermine their customer base. Highly tailored skill- and generation-specific redistribution policies can keep smart machines from immiserating our posterity. But blunt policies, such as mandating open-source technology, can make matters worse.

JEL Classification:E20, E24, E27, H42, I30, J11, J20, J30, O30, O38

Keywords:Human Replacement, Labor Supply, Automation, Capital Savings, Welfare, Public Goods, Technological Change

*We would like to thank Dilip Mookherjee, Dietrich Vollrath, John Seater and Kevin Cooke for their useful comments.

1 Introduction

Whether it's bombing our enemies, steering our planes, fielding our calls, rubbing our backs, vacuuming our floors, driving our taxis, or beating us at Jeopardy, it's hard to think of hitherto human tasks that smart machines can't do or won't soon do. Few smart machines look even remotely human. But they all combine brains and brawn, namely sophisticated code and physical capital. And they all have one ultimate creator – us.

Will human replacement - the production by ourselves of ever better substitutes for ourselves - deliver an economic utopia with smart machines satisfying our every material need? Or will our self-induced redundancy leave us earning too little to purchase the products our smart machines can make?

Ironically, smart machines are invaluable for considering what they might do to us and when they might do it. This paper uses the most versatile of smart machines – a run-of-the-mill computer – to simulate one particular vision of human replacement. Our simulated economy – an overlapping generations model – is bare bones. It features two types of workers consuming two goods for two periods. Yet it admits a large range of dynamic outcomes, some of which are quite unpleasant.

The model's two types of agents are called high-tech workers and low-tech workers. The first group has a comparative advantage at analytical tasks, the second in empathetic and interpersonal tasks. Both work full time, but only when young. High-tech workers produce new software code, which adds to the existing stock of code. They are compensated by licensing their newly produced code for immediate use and by selling rights to its future use. The stock of code – new plus old – is combined with the stock of capital to produce automatable goods and services (hereafter referred to as 'goods'). Goods can be consumed or used as capital. Unlike high-tech workers, low-tech workers are right brainers – artists, musicians, priests, astrologers, psychologists, etc. They produce the model's other good, human services (hereafter referred to as 'services'). The service sector does not use capital as an input, just the labor of high and low-tech workers.

Code references not just software but, more generally, rules, instructions, and associated data for generating output from capital. Because of this, code is both created by and is a substitute for the analytical labor provided by high-tech workers in the good (autmomatable) sector. Code is not to be thought of as accumulating in a quantitative way (anyone who has worked on a large software project can testify that fewer lines of code often mean a better program) but rather in efficiency units. Code accumulation may be a result of programmers typing out code directly, of machine learning systems getting better at a task under the supervision of human trainers[1], or of innovation in designing learning

[1] Astro Teller, Google's 'Director of Moonshots', discusses in Madrigal (2014) the importance of this work to Google's current projects:

> *Many of Google's famously computation driven projects –like the creation of Google Maps– employed literally thousands of people to supervise and correct automatic systems. It is one of Google's open secrets that they deploy human*

algorithms themselves. In the United States, more than 5 percent of total wages is paid to those engaged in computer or mathematical occupations[2]; a much larger share of compensation is being paid to those engaged in creating code broadly defined.

Code needs to be maintained, retained, and updated. If the cost of doing so declines via, for example, the invention of the silicon chip, the model delivers a tech boom, which raises the demand for new code. The higher compensation received by high-tech workers to produce this new code engenders more national saving and capital formation, reinforcing the boom. But over time, as the stock of legacy code grows, the demand for new code and, thus for high-tech workers, falls.

The resulting tech bust reflects past humans obsolescing current humans. This process explains the choice of our title, *Robots Are **Us***. The combination of code and capital that produce goods constitutes, in effect, smart machines, aka robots. And these robots contain the stuff of humans – accumulated brain and saving power. Take Junior – 2013's World Computer Chess Champion. Junior can beat every current and, possibly, every future human on the planet. Consequently, his old code has largely put new chess programmers out of business.

Once begun, the boom-bust tech cycle can continue if good producers switch technologies à la Zeira (1998) in response to changes over time in the relative costs of code and capital. But whether or not such Kondratieff waves materialize, tech busts can be tough on high-tech workers. In fact, high-tech workers can start out earning far more than low-tech workers, but end up earning far less.

Furthermore, robots, captured in the model by more code-intensive good production, can leave all future high-tech workers and, potentially, all future low-tech workers worse off. In other words, technological progress can be immiserating. This finding echoes that of Sachs and Kotlikoff (2012). Although our paper includes different features from those in Sachs and Kotlikoff (2012), including two sectors, accumulating code stocks, endogenous technological change, property rights to code, and boom-bust cycle(s), the mechanism by which better technology can undermine the economy is the same. The eventual decline in high-tech worker and, potentially, low-tech worker compensation limits what the young can save and invest. This means less physical capital available for next period's use. It also means that good production can fall over time even though the technological capacity to produce goods expands.

The long run in such cases is no techno-utopia. Yes, code is abundant. But capital is dear. And yes, everyone is fully employed. But no one is earning very

> *intelligence as a catalyst. Instead of programming in that last little bit of reliability, the final 1 or 0.1 or 0.01 percent, they can deploy a bit of cheap human brainpower. And over time, the humans work themselves out of jobs by teaching the machines how to act. "When the human says, 'Here's the right thing to do,' that becomes something we can bake into the system and that will happen slightly less often in the future," Teller said.*

[2]This figure is the share of wages paid to workers in Computer or Mathematical Occupations in the May 2013 NAICS Occupational Employment and Wage Estimates.

much. Consequently, there is too little capacity to buy one of the two things, in addition to current consumption, that today's smart machines (our model's non-human dependent good production process) produce, namely next period's capital stock. In short, when smart machines replace people, they eventually bite the hands of those that finance them.

These findings assume that code is excludable and rival in its use. But we also consider cases in which code is non-excludable, non-rival, or both. Doing so requires additional assumptions but lets us consider the requirement that all code be open source, i.e., non-excludable. Surprisingly, such freeware policies can worsen long-run outcomes.

Our paper proceeds with some economic history – Ned Ludd's quixotic war on machines and the subsequent Luddite movement. As section 2 indicates, Ludd's instinctive fear of technology, ridiculed for over a century, is now the object of a serious economic literature. Section 3 places our model within a broader framework of human competition with robots to indicate what we, for parsimony's sake, exclude. Section 4 presents our model and its solution method. Section 5 illustrates the surprising range of outcomes that even this simple framework can generate. Section 6 considers how the nature of code ownership and rivalry affects outcomes. Section 7 follows Zeira (1998) in letting the choice of production technique respond to relative scarcity of inputs, in our case capital and code. Section 8 presents some potentially supportive evidence. Section 9 concludes.

2 Background and Literature Review

Concern about the downside to new technology dates at least to Ned Ludd's destruction of two stocking frames in 1779 near Leichester, England. Ludd, a weaver, was whipped for indolence before taking revenge on the machines. Popular myth has Ludd escaping to Sherwood Forest to organize secret raids on industrial machinery, albeit with no Maid Marian.

More than three decades later – in 1812, 150 armed workers – self-named Luddites – marched on a textile mill in Huddersfield, England to smash equipment. The British army promptly killed or executed 19 of their number. Later that year the British Parliament passed The Destruction of Stocking Frames, etc. Act, authorizing death for vandalizing machines. Nonetheless, Luddite rioting continued for several years, eventuating in 70 hangings.

Sixty-five years later, Marx (1867) echoed Ned Ludd's warning about machines replacing humans.

> *Within the capitalist system all methods for raising the social productivity of labour are put into effect at the cost of the individual worker; all means for the development of production undergo a dialectical inversion so that they become means of domination and exploitation*

> *of the producers; ... they alienate from him the intellectual potentialities of the labour process in the same proportion as science is incorporated in it as an independent power...*

Keynes (1933) also discussed technology's potential for job destruction writing in the midst of the Great Depression that

> *We are being afflicted with a new disease of which some readers may not yet have heard the name, but of which they will hear a great deal in the years to come – namely, technological unemployment. This means unemployment due to our discovery of means of economizing the use of labor outrunning the pace at which we can find new uses for labor.*

But Keynes goes on to say that "this is only a temporary phase of maladjustment," predicting a future of leisure and plenty one hundred years hence. His contention that short-term pain permits long-term gain reinforced Schumpeter's 1942 encomium to "creative destruction".

In the fifties and sixties, with employment high and rapid real wage growth, Keynes' and Schumpeter's views held sway. Indeed, those raising concerns about technology were derided as Luddites.

Economic times have changed. Luddism is back in favor. Autor, Levy, and Murnane (2003), Acemoglu and Autor (2011), and Autor and Dorn (2013) trace recent declines in employment and wages of middle skilled workers to outsourcing by smart machines. Margo (2013) points to similar *labor polarization* during the early stages of America's industrial revolution. Goos, Manning, and Salomons (2010) offer additional supporting evidence for Europe. However, Mishel, Shierholz, and Schmitt (2013) argue that 'robots' can't be 'blamed' for post-1970's U.S. job polarization given the observed timing of changes in relative wages and employment. A literature inspired by Nelson and Phelps (1966) hypothesizes that inequality may be driven by skilled workers more easily adapting to technological change, but generally predicts only transitory increases in inequality.

Our model supports some of the empirical findings and complements some of the theoretical frameworks in this literature. Its simple elements produce dynamic changes in labor market conditions, the nature and timing of which are highly sensitive to parameterization. But the model consistently features tech booms possibly followed by tech busts, evidence for which is provided in Gordon (2012) and Brynjolfsson and McAfee (2011).

A second prediction of our model is a decline, over time, in labor's share of national income. U.S. national accounts record a stable percent share of national income going to labor during the 1980's and 1990's. But starting in the 2000's labor's share has dropped significantly. Frey and Osborne (2013) try to quantify prospective human redundancy arguing that over 47 percent of current jobs will likely be automated in the next two decades. They also identify the priesthood, psychotherapy and coaching (parts of our service sector) as among the least subject to automation.

While our paper is about smart machines, it's also about endogenous technological change. Schumpeter is clearly the father of this literature. But other classic contributions include Arrow (1962), Lucas (1988), Romer (1990), Zeira (1998), Acemoglu (1998), Zuleta (2008), and Peretto and Seater (2013). Several of these papers endogenize technological change.

Our model accommodates long-run balanced growth arising from population- or labor-augmenting productivity growth.[3] But we abstract from these factors to focus on transitional growth arising from improvements in code retention.

Long term growth may be due to the cumulative impact steady state shifting technologies of this type.[4]

Zeira (1998) considered Leontief technologies and showed that countries with relatively high total factor productivity levels will adopt more capital-intensive techniques in producing intermediate inputs leading to cross-country dispersion in per capita income. But this adoption of new technology benefits workers since the two inputs are perfect complements in production.

Zuleta (2008) considers an economy where the use of more capital-intensive technologies can be optimal, but doing so comes at a cost – a cost that goes beyond simply the price of hiring more capital. Like Zeira (1998), rich economies can get richer while poor economies, which can't afford the capital-intensification process, stagnate.

Peretto and Seater (2013) go considerably beyond Zuleta (2008). They consider monopolistically competitive firms that invest in particular technologies depending on their relative costs. In their model, firms may specialize in the use of one technology or produce with multiple technologies. We investigate this issue here, but in a less robust manner.

Acemoglu also views technology as malleable. In (1998) he models technologies that can be altered to make particular skill groups, including labor, more productive. Hence, a temporary glut of one type of worker can initiate innovations culminating in higher productivity of such workers. It can also alter skill-formation decisions. Acemoglu and Restrepo (2015) endogenize the automation of labor as well as the invention of new labor-intensive products. The former (later) occurs to a greater (lessor) degree when wages are high.

Rourke, et. al. (2013) examines 18th and 19th technological change in England with special focus on the skill premium. His model, which is similar to that of Acemoglu (1998), appears capable of matching the trend in the skill premium over the period.

Following Acemoglu (1998) and Acemoglu and Restrepo (2015), we model labor stocks of both types as exogenous. We make this simplification for three reasons. First, someone predisposed to provide services may not easily switch to producing code. Third, apart from the results on wage inequality, making

[3] A code retention rate below 1, which we assume, ensures balanced growth.

[4] Balanced growth is ruled out Cobb-Douglas utility and an ultimate limit on inputs to the service sector.

all labor perfect substitutes doesn't alter our models' main conclusion. Third, if preferences and production are Cobb-Douglas the skill mix has no impact on the economy's equilibrium transition path.[5]

This literature's generally rather sanguine view of technology, namely as complementing human effort, differs from that presented here. Rather than technology permanently assisting humans, it ultimately largely replaces them. Hemous and Olson (2014) depart somewhat by calibrating a model in which capital can substitute for low-skilled labor while complementing high-skilled labor to explain technology-induced trends in the labor share of income and inequality.

3 A Modeling Framework for Understanding Economic Impacts of Robots

The first ingredient of any model of robot competition is, of course, one or more production processes that can produce particular goods or services with little or no input from humans. The second ingredient is one or more human-based production processes of specific goods and services that do not admit the easy substitution of non-human for human input. The third ingredient is dynamics, since technological change generally doesn't happen over night and since it takes time for new technologies to fully impact the economy. The fourth ingredient is agents that are differentially susceptible to replacement by robots. The fifth and final ingredient is a description of the manner in which robotic technology evolves. This includes the inclination and ability of humans to produce technology that puts themselves out of work.

The first ingredient permits production of particular goods to become less human dependent as robots become more abundant and capable. This process may involve the termination of particular human-intensive production processes. The second ingredient insures that humans have somewhere to go when they are put out of work or out of good work by robots. Taken together the first two ingredients help us consider a basic question surrounding robotic competition: Will the reduction in the cost of goods produced by more advanced robots compensate workers for the lower wages? The third ingredient – dynamics – is essential for determining how physical capital – economic brawn – is impacted through time by robot competition. After all, the counterpart of investment is saving and saving is done by households, not robots. The fourth ingredient, agents that are differentially outmoded by robots, is key for assessing the impact of robots on inequality. And the fifth ingredient, endogenous development of robots, is the driving force of interest.

Our model has each of these ingredients, but not all varieties of them. We don't, for example, include an alternative goods-production technology strictly utilizing labor and capital. Were we to do so, the economy would discretely switch, at some point, from non-robotic to robotic good production. Nor do we assume

[5] As discussed below, this reflects the adjustment of the relative price of the low-tech service good.

that goods production requires any direct human input. Adding this feature would not materially alter the qualitative nature of our findings. Similarly, we do not model code accumulation as contributing to TFP. That type of growth is well understood. Dynamics, the third ingredient, play a central role in our model and admit our central finding that better supply can, over time, mean worse demand. The fourth element – different skill groups – is covered by our inclusion of low-tech as well as high-tech workers. The presence of low-tech workers lets us consider whether technological change can flip the income distribution between people of different skill sets. Finally, our assumption that new software code is purchased provides a realistic means to endogenize development of robots.

4 Our Model

Agents consume the product of both sectors, goods and services. Goods, which can be consumed or invested, are produced using capital and code via a CES production function. The combination of capital and code that makes goods can be viewed as a smart machine or robot. Services, which are consumed when produced, are also created via CES production. New code is written by high-tech workers, and the stock of code is the sum of new and existing code. Old code requires maintenance, retention, and updating. This requirement is modeled as a form of depreciation. High and low-tech workers both live and consume for two periods, but work only when young.

Supply

Time t production of goods, Y_t, and services, S_t, follow (1) and (2),

$$Y_t = D_Y[\alpha(K_t)^{\frac{\varepsilon_y - 1}{\varepsilon_y}} + (1-\alpha)(A_t)^{\frac{\varepsilon_y - 1}{\varepsilon_y}}]^{\frac{\varepsilon_y}{\varepsilon_y - 1}}, \tag{1}$$

$$S_t = D_S[\gamma(H_{S,t})^{\frac{\varepsilon_s - 1}{\varepsilon_s}} + (1-\gamma)(G_t)^{\frac{\varepsilon_s - 1}{\varepsilon_s}}]^{\frac{\varepsilon_s}{\varepsilon_s - 1}}, \tag{2}$$

where $H_{S,t}$ is the amount of high-tech workers in the service sector, and G_t references low-tech workers. D_S and D_Y are total factor productivity terms, γ and α are CES parameters related to factor intensity, and ε_y and ε_s are CES elasticities. The stock of code A_t grows according to,

$$A_t = \delta A_{t-1} + zH_{A,t}, \tag{3}$$

where the "depreciation" factor is $\delta \in [0, 1)$. Higher δ means that legacy code is useful longer.[6] $H_{A,t}$ is the amount of high-tech labor hired by good firms, and z is the productivity of high-tech workers writing code.

[6]The rate of code depreciation in the economy as a whole is unclear. Information technology systems that used to be perfectly adequate are continuously updated and amended to deal with new problems or interface with new complements. A depreciation rate of 30 percent over a 30-40 year generation is assumed in many of our simulations. This corresponds to a typical company needing to replace approximately 1 percent of its code base every year to maintain the same level of output. In calculating depreciation the IRS allows for a 3 year useful lifespan

The good sector's demands for code, high-tech workers, and capital satisfy[7]

$$\max_{K_t, A_t} Y_t(A_t, K_t) - m_t A_t - r_t K_t, \tag{4}$$

where the price of a unit of goods is one, m_t is the rental rate for code, and r_t is the interest rate. Factor demands for services reflect,

$$\max_{H_{S,t}, G_t} q_t S_t(H_{S,t}, G_t) - w_t^G G_t - w_t^H H_{S,t}, \tag{5}$$

where q_t is the price of services, w_t^H is a high-tech worker's wage in the service sector, and w_t^G is a low-tech worker's wage.

Households save in the form of capital and code. Capital accumulation obeys

$$K_{t+1} = \phi I_t - p_t \delta A_t, \tag{6}$$

where I_t is the total resources of those born in t, ϕ is the saving propensity of the young, and $p_t \delta A_t$ is the value of code retained from the current period.

Factor prices satisfy

$$w_t^H = q_t D_S[\gamma(H_{S,t})^{\frac{\varepsilon_s - 1}{\varepsilon_s}} + (1-\gamma)(G_t)^{\frac{\varepsilon_s - 1}{\varepsilon_s}}]^{\frac{1}{\varepsilon_s - 1}}[\gamma(H_{S,t})^{-\frac{1}{\varepsilon_s}}], \tag{7}$$

$$w_t^G = q_t D_S[\gamma(H_{S,t})^{\frac{\varepsilon_s - 1}{\varepsilon_s}} + (1-\gamma)(G_t)^{\frac{\varepsilon_s - 1}{\varepsilon_s}}]^{\frac{1}{\varepsilon_s - 1}}[(1-\gamma)(G_t)^{-\frac{1}{\varepsilon_s}}], \tag{8}$$

$$r_t = D_Y[\alpha(K_t)^{\frac{\varepsilon_y - 1}{\varepsilon_y}} + (1-\alpha)(A_t)^{\frac{\varepsilon_y - 1}{\varepsilon_y}}]^{\frac{1}{\varepsilon_y - 1}}[\alpha(K_t)^{-\frac{1}{\varepsilon_y}}], \tag{9}$$

and

$$m_t = D_Y[\alpha(K_t)^{\frac{\varepsilon_y - 1}{\varepsilon_y}} + (1-\alpha)(A_t)^{\frac{\varepsilon_y - 1}{\varepsilon_y}}]^{\frac{1}{\varepsilon_y - 1}}[(1-\alpha)(A_t)^{-\frac{1}{\varepsilon_y}}]. \tag{10}$$

Households

Whether high-tech or low-tech, households maximize

$$u = (1-\phi)[(1-\kappa)logc_{y,t} + \kappa logs_{y,t}] + \phi[(1-\kappa)logc_{o,t+1} + \kappa logs_{o,t+1}], \tag{11}$$

for licensed software. For software developed in house or purchased bespoke software, costs must be amortized over a 15 year period (as a section 197 intangible). Software which is bundled with hardware depreciates at the rate of the hardware. On the other hand, many programs created over 50 years ago are still in use, such as those written for older nuclear reactors.

[7] To understand this production function consider a firm which provides the service of 'making good chess moves'. Better chess playing smart machines are, in part, distinguished by how many game trees they can investigate and the level of sophistication with which they evaluate board positions and determine which sequences of moves to spend more computational time considering. Therefore, our firm can improve the quality of its output (the chess move it chooses) by increasing either of its inputs. It can either increase the quality of its chess program (increasing its efficiency units of code) or devote more computing time to investigating possible moves and counter-moves (rent more capital). While the logic of decreasing marginal returns to an input seems to hold for production of this type, this does not imply any specific structure on overall returns to scale. Here we restrict our attention to constant returns to scale production.

where $c_{y,t}$, $c_{o,t}$, $s_{y,t}$, $s_{o,t}$, are consumption of goods and services by the young and old, respectively.[8]

Households maximize utility subject to,

$$c_{y,t} + q_t s_{y,t} + \frac{c_{o,t+1} + q_{t+1} s_{o,t+1}}{1 + r_{t+1}} = i_{j,t}, \tag{12}$$

where $i_{j,t}$ is total resources of group j. For low-tech workers,

$$i_{G,t} = w_t^G. \tag{13}$$

For high-tech workers laboring in the service sector,

$$i_{(H,S),t} = w_t^H, \tag{14}$$

and for high-tech workers writing code,

$$i_{(H,A),t} = z(m_t + \delta p_t), \tag{15}$$

where zm_t is revenue from renting out newly produced code and $z\delta p_t$ is revenue from the sale of the intellectual property. Note that like any asset price, p_t is a present value. The second component of the compensation of the code-writing high-tech workers reflect their sale of future rights to their newly written code or their retention and use of this code in their own firms.

High-tech workers are mobile between sectors. Assuming, as we do, no specialization, high-tech workers work in both sectors and receive the same total compensation regardless of where they work.

$$w_t^H = z(m_t + \delta p_t). \tag{16}$$

Household demands satisfy,

$$s_{y,t} = \frac{\kappa(1-\phi) i_{j,t}}{q_t}, \tag{17}$$

$$c_{y,t} = (1-\kappa)(1-\phi) i_{j,t}, \tag{18}$$

$$s_{o,t+1} = \frac{1 + r_{t+1}}{q_{t+1}} [\kappa \phi i_{j,t}], \tag{19}$$

and

$$c_{o,t+1} = [1 + r_{t+1}][(1-\kappa)\phi i_{j,t}]. \tag{20}$$

[8]This selfish OLG framework is, of course, essential for good times to produce bad times. Were agents altruistic they would spread the economic gains from the rise in code retention across all generations via gifts and bequests. But the micro evidence against operational intergenerational altruism is substantial and striking. See, for example, Altonji, Hayashi, and Kotlikoff (1992,1997), Hayashi, Altonji and Kotlikoff (1996), and Abel and Kotlikoff (1994). This is true notwithstanding the popularity of models with infinitely-lived agents. Adding additional periods of life would not impact our qualitative results.

Equilibrium

Equilibrium requires

$$Y_t = C_{y,t} + C_{o,t} + K_{t+1} - K_t, \tag{21}$$

$$H_t = H_{A,t} + H_{S,t}, \tag{22}$$

and

$$S_t = S_{y,t} + S_{o,t}, \tag{23}$$

where C_y, C_o, S_y, S_o, are total consumption demand of goods and services by the young and old respectively.

Asset-market clearing entails equal investment returns on capital and code, i.e.,

$$p_t = \sum_{s=t}^{\infty} R_{s+1,t}^{-1} \delta^{s-t} m_{s+1}, \tag{24}$$

where $R_{s,t}$ is the compound interest factor between t and s, i.e.,

$$R_{s,t} = \prod_{j=t}^{s} (1 + r_j). \tag{25}$$

The Model's Steady State

Despite the model's apparent simplicity, it yields no closed form expression for the steady-state capital stock.[9] However, a unique economically meaningful equilibrium exists in the general case.[10] For the Cobb-Douglas production case, the steady state is implicitly defined by the following two equations in $k = \frac{K}{A}$ and q.

$$\begin{aligned} D_y k^{\alpha} = & [\frac{(1-\phi)(1-\kappa)}{\phi}][k + \frac{(1-\alpha) D_y k^{\alpha} \delta}{1 + \alpha D_y k^{\alpha-1} - \delta}] \\ & + (1-\kappa)[k + \frac{(1-\alpha) D_y k^{\alpha}\delta}{1 + \alpha D_y k^{\alpha-1} - \delta}][1 + \alpha D_y k^{\alpha-1}] \end{aligned} \tag{26}$$

and

$$k + p\delta = \phi[z(m + p\delta)H + (1-\gamma)G(\frac{\gamma}{z(m+p\delta)})^{\frac{\gamma}{1-\gamma}} (qD_s)^{\frac{1}{1-\gamma}}], \tag{27}$$

[9] With population- or labor augmenting technological change, capital per unit of code would be the key steady-state variable.

[10] There are two solutions depending on whether $1 + f_K(k) > \delta$ or not. However, only the former condition permits a positive price of code.

where,

$$m = (1-\alpha)D_y k^{\alpha},$$
$$r = \alpha D_y k^{\alpha-1},$$
$$p = \frac{(1-\alpha)D_y k^{\alpha}}{1+\alpha D_y k^{\alpha-1} - \delta}.$$

Due to the model's analytic intractability, we proceed to a computational approach.[11]

Solving the Model

We calculate the economy's perfect foresight transition path following an immediate and permanent increase in the rate of code retention due, for example, to the development of the silicon chip. The solution is via Gauss-Seidel iteration (see Auerbach and Kotlikoff, 1987). First, we calculate the economy's initial and final steady states. This yields initial and final stocks of capital and code. These steady-state values provide, based on linear interpolation, our initial guesses for the time paths of the two input stocks. Next, we calculate associated guesses of the time paths of factor prices as well as the price paths of code and services. Step three uses these price paths and the model's demand, asset arbitrage, and labor market conditions to derive new paths of the supplies of capital and code. The new paths are weighted with the old paths to form the iteration's next guesses of capital and code paths. The convergence of this iteration, which occurs to a high degree of precision, implies market clearing in each period.

5 Simulating Transition Paths

The models' main novelty is the inclusion of the stock of code in the production of goods. When the code retention rate, δ equals zero, good sector production is conventional – based on contemporaneous amounts of capital and labor (code writers). But when δ rises, good production depends not just on capital and current labor, but, implicitly, on dead high-tech workers as well. We study the effects of this technological change by simulating an immediate and permanent increase in δ.

[11]For the Cobb-Douglas case, we also have

$$\frac{dk}{d\delta} = -\frac{aD_y k^{\alpha-1} + \alpha(1-\alpha)[D_y k^{\alpha-1}]^2 - b}{2\alpha(1-\alpha)cD_y^2 k^{2\alpha-3} + (1-\alpha)eD_y k^{\alpha-2}}$$

where $a = [1 + \frac{(1-\kappa)(1-2\alpha)}{\phi}]$, $b = \frac{(1-\kappa)}{\phi}$, $c = 1-(\alpha+\delta(1-\alpha))$, and $e = (1-\delta)(1-\alpha b) - b(1-(1-\alpha)\delta)$. While this derivative is rather unwieldy, it easy to come up with parameter values such that the derivative is of either sign. This underlies our main point, namely that a higher code retention rate can reduce long-run capital intensity.

The increase in δ initially raises the compensation of code-writing high-tech workers. This draws more high-tech workers into code-writing, thereby raising high-tech worker compensation in both sectors. In most parameterizations, the concomitant reduction in service output raises the price of services. And, depending on the degree to which high-tech workers complement low-tech workers in producing services, the wages of low-tech workers will rise or fall.

Things change over time. As more durable code comes on line, the marginal productivity of code falls, making new code writers increasingly redundant. Eventually the demand for code-writing high-tech workers is limited to those needed to cover the depreciation of legacy code, i.e., to retain, maintain, and update legacy code. The remaining high-tech workers find themselves working in the service sector. The upshot is that high-tech workers can end up potentially earning far less than in the initial steady state.

What about low-tech workers?

The price of services peaks and then declines thanks to the return of high-tech workers to the sector. This puts downward pressure on low-tech workers' wages and, depending on the complementarity of the two inputs in producing services, low-tech workers may also see their wages fall. In this case, the boom-bust in high-tech workers' compensation generates a boom-bust in low-tech compensation. In the extreme, if high and low-tech workers are perfect substitutes, their wages move in lock step.

The economy's dynamic reaction to the higher δ depends on the impact on capital formation. The initial rise in earnings of at least the high-tech workers can engender more aggregate saving and investment. The increased capital makes code and, thus, high-tech workers more productive. But if the compensation of high-tech and, potentially, low-tech workers falls, so too will the saving of the young and the economy's supply of capital. Less capital means lower marginal productivity of code and higher interest rates. This puts additional downward pressure on new code rental rates as well as on the price of future rights to the use of code. A decrease in the depreciation rate of capital would necessarily have an opposite effect, as it raises capital stocks and the marginal product of code.

We next consider four possible transition paths, labeled Immiserizing Growth, Felicitous Growth, The First Will be Last, and Better Tasting Goods. Each simulation features an immediate and permanent rise in the code-retention rate. But the dynamic impact of this technological breakthrough can be good for some and bad for others depending on the size of the shock and other parameters. After presenting these cases, we examine the sensitivity of long-run outcomes to parameter assumptions.

Immiserating Growth

Figure 1 shows that a positive tech shock (the code-preservation rate, δ, rises from 0 to .7) can have negative long-term consequences. The simulation assumes

Cobb-Douglas production of goods and linear production of services; i.e., both types of workers are perfect substitutes in producing services ($\varepsilon_S = \infty$).

As the top left panel indicates, national income quickly rises by 16 percent.[12] But it ultimately declines, ending up 13 percent below its initial steady-state value. Since preferences are logarithmic, expenditures on goods and services change by the same percentage. In the case of services, however, this occurs not only through changes in output levels, but also via changes in relative price.

The relative price of services first rises and then falls steeply, while service output does the opposite. Hence, in the long-run, both young and old agents end up consuming 28 percent less goods. And while their consumption of services is 27 percent larger, it's not worth very much at the margin. In fact, its price is 32 percent lower than before the technological breakthrough.

Both types of workers earn the same under this parameterization. Their compensation initially jumps 11 percent and then starts to fall gradually. In the long run all workers end up earning 32 percent less than was originally the case.

What happens to the welfare of different agents through time? The initial elderly are essentially unaffected by the tech boom. The initial young experience a 14 percent rise in lifetime utility, measured as a compensating differential relative to their initial steady-state utility. But those born in the long run are 17 percent worse off.

The top right chart helps explain why good times presage bad times. The stock of code shoots up and stays high. But the stock of capital immediately starts falling. After six periods there is over 50 percent more code, but 65 percent less capital.

The huge long-run decline in the capital stock and associated rise in its marginal product (the interest rate) has two causes. First, as just stated, wages, which finance the acquisition of capital, are almost cut in half by the implicit competition with dead workers. Second, the advent of a new asset – durable code – crowds out asset accumulation in the form of capital. When δ rises, all workers immediately enjoy an increase in their compensation. This leads to more saving, but not more saving in the form of capital. Instead, their extra saving as well as some of the saving they originally intended to do is used to acquire claims to legacy code. Initially, when the stock of code is small, its price is high. And, later, when the stock of code is large, its price falls to about 40 percent below its initial value. However, the total value of code increases enough to significantly crowd out investment in capital along the entire transition path.

Another way to understand capital's crowding out is to view legacy code, which coders can sell or retain when the code retention rate rises, as a form of future labor income. This higher resource permits more consumption of goods by low-

[12] Throughout, unless otherwise noted, national income, wages, and prices are all reported in real terms. The price index used is a geometric mean of the relative price of goods and services. The weights used are their corresponding shares in consumption. The price index is $\Pi_t = q_t^{\frac{S_{y,t}+S_{o,t}}{C_{y,t}+C_{o,t}}}$

tech workers (and high-tech workers, since they are paid the same) when the shock hits. And this additional good consumption means less goods are saved and invested. But the knock-on effect of having less capital in the economy is lower labor compensation. This reduces the consumption through time of workers, but also their saving.

What happens to labor's share of national income? Initially it rises slightly. But, in the long run, labor's share falls from 76 to 58 percent. This reflects the higher share of output paid to legacy code. The long-run decline in labor's share of national income arises in all our simulations except those in which preferences shift toward the consumption of goods at the same time as the code retention rate rises.

Felicitous Growth

As figure 2 shows, the tech boom need not auger long-term misery. A higher saving preference is the key. In the immiserating growth case above, we assumed a youth saving propensity parameter, ϕ, of .2. This generated a ratio of consumption when young to consumption when old of 1.5 in the initial steady state and .9 in the long run steady state. Here we assume a ϕ of .95 while holding fixed the model's other parameter values. The result is that good times can be good for good. But the road is rocky. Output ends up permanently higher, but only after an intervening depression. Output of both goods peaks in the period after the shock, with national income rising 41 percent. But in the long-run, it is only 18 percent higher – a major decline from its peak. The long-run expansion in output reflects less capital decumulation. In the prior simulation the capital stock immediately declined. Here the capital stock temporarily increases 14 percent above its initial value.

A less rapid decline in the capital stock and higher service prices boosts the common wage in the short term and leaves it at roughly its initial value in the long run. After peaking 47 percent above its initial value, the wage falls, ending up only 1 percent lower. The stock of code ends up more than twice as high. But the capital stock, notwithstanding the high rate of saving, declines by 35 percent.

The respective increase and decrease in the stocks of code and capital produce a significant rise in the economy's interest rate – 77 percent in the long run. Although the labor compensation of high and low-tech workers ends up very close to where it started, this increase in the interest rate permits those living in the future to consume significantly more.

Why does a high enough saving rate keep the δ shock from reducing long-run welfare? The answer is that whatever happens to the stock of code, a higher saving rate entails a higher capital stock and, therefore, higher labor compensation payments to high-tech workers. In the two above examples, we've considered widely varying saving preference parameters. Figure 7 shows how long-run utility varies with ϕ and δ.

The First Will Be Last

If high and low-tech workers are complements in producing services, their wage and utility paths will diverge. Consider, for example, the model with table 2's parameters shown in figure 3. As is always the case, the initial effect for high-tech worker of the δ shock is positive. Indeed, immediately after the shock hits, high-tech workers make 29 percent more than in the previous period. But low-tech workers, who, in this case, need high-tech workers to be productive, see their wages fall one percent as the share of high-tech workers working in services immediately falls from 50 percent to 38 percent.

However, as code accumulates and capital decumulates, high-tech workers start earning less in code-writing and move in great number back to the service sector. Ultimately, 68 percent of high-tech workers work in the service sector. And their return to that sector drives down their wage compared both its initial value and to the long-run wage of low-tech workers. Indeed, in the final steady state, high-tech workers earn 14 percent less than in the initial steady state. Low-tech workers, in contrast, earn 17 percent more. But, interestingly, in period 3 their wage peaks 26 percent above its original value. This rise and fall in the wages of low-tech workers reflects, in part, the rise and fall in the price of services.

Better Tasting Goods

Our assumption above that the share of each type of good in consumption is fixed is an important one. It is reasonable given that there is no strong evidence about whether technological innovations are shifting consumption towards or away from goods that are relatively labor intensive. In this section we reinterpret the utility function of equation 11 as a technology for Cobb-Douglas production of a final consumption good using a combination of goods and services. [13] Figure 4 displays the consequences of having κ fall from .5 to .25 at the same time δ rises. Other parameters are those in the 'First Will Be Last' case.

This additional shock has a dramatic impact on the path of national income. When the shock hits national income increases 7 percent. In the long run it drops 4 percent.

What explains this result? Shouldn't a shift in production functions towards products that have become easier to produce be economically beneficial? As in previous cases, immiseration is caused by capital decumulation. Capital stocks in this case decrease 40 percent in the period after the shock, and 84 percent in the long-run. Capital decumulation is exacerbated by the κ shock in three ways. First, increased immediate consumption demand for goods (i.e., reduced demand for services) increases the share of high-tech workers working as coders. This translates, after one period, into more legacy code and lower labor compensation, the source of saving and capital formation. Second, the increase in immediate good consumption reduces the amount of capital available

[13] It is also unclear how to choose reasonable initial values for κ, but this has little impact on the dynamic effect of code accumulation. Immiseration is still possible for high κ.

to invest. Third, the shift in demand toward goods limits the rise in the price of services. This, too, has a negative impact on wages and capital formation.

The Large Range of Potential Outcomes

As just demonstrated, the model's reaction to the δ shock is highly sensitive to parameter values. We now consider this sensitivity in more detail. Figure 5 jointly displays our previous results. Table 3 shows additional results for several different parameter combinations. The table's baseline simulation (row one) assumes intermediate parameter values. Subsequent rows show the impact of sequentially modifying one parameter. Figure 6 plots the path of national income for each row of the table.

These simulations teach several new things. First, high-tech workers benefit from substitutability in the goods sector. In the perfect substitutability case the productivity of high-tech workers is independent of supplies of code and capital.

Second, with both Cobb-Douglas production and preferences, the path of the capital-to-code ratio in response to a rise in delta, staring from $\delta = 0$, is independent of the absolute and relative numbers of each type of worker.[14]

Third, a positive δ shock always produces a tech boom with increases in both the price of code and the wage of high-tech workers.[15] In most simulations, the boom is short lived, auguring a major tech and saving bust. Fourth, in most simulations capital becomes relatively scarce compared to code leading to a rise in interest rates. Finally, the δ shock generally raises labor share in the short run and lowers it in the long run.

Figure 7 presents a contour graph of the long-run compensating differential. Its top half considers combinations of saving preference parameters ϕ and shocks to δ assuming table 1's values of the other parameters. Because the two types of workers are perfect substitutes, the compensating differential for them is the same. Redder areas denote higher long-run utilities relative to the initial steady state. Bluer areas denote the opposite. Long-run utility increases most when δ

[14] Consider a doubling in H. This will double H_Y in the $\delta = 0$ economy. But if H_Y also doubles along the entire transition path, the path of k will remain unchanged. One can see this by combining the equation for market-clearing in capital with that for market-clearing in code. This, of course, requires the path of H_S be twice as large as well. But this outcome as well as a doubling of path of q_t is implied by equation 16. This k-path invariance to initial levels of H and G is somewhat surprising and suggests that transforming more low-skilled into-high-skilled workers may have less impact on the economy than one might have thought. Still, such a policy, if enacted before the rise in delta, would lower the real wages of skilled workers (their wages valued in capital wouldn't change, but the higher price of S would lower their real wage. It would also improve the relative lot of those who remain unskilled workers since their wage measured in units of capital will rise thanks to the higher marginal revenue of their labor. Additional effects would arise were H or G to vary once delta had risen and the economy was in transition. In this case, the k path would temporarily fall making code and coding less valuable. However, in the long run, the real wages of each type of worker are independent of such transition effects on the path of k.

[15] This, and the previous result, can both be shown analytically.

is large and the saving rate is high. It decreases the most when the δ shock is high and the saving rate is low.

Figure 7's bottom half considers joint shocks to the saving rate and code-writing productivity (z). Higher values of each reinforces their individual positive impacts on long-run utility. As opposed to δ shocks, shocks to code-writing productivity (z) enhance all agents' welfare. The reason is simple – this shock makes living, but not dead high-tech workers more productive. Increasing labor's productivity in other tasks has the same result. As this model has no disutility from labor, reducing labor's productivity is isomorphic to restricting its supply. Policies that attempt to raise wages by reducing labor supply - such as increasing the minimum wage - will therefore backfire.

Figure 8 considers combinations of the saving rate, ϕ, and the good sector's elasticity of substitution, ε_y. It shows the aforementioned sensitivity of long-run utility to the substitutability of code for capital. It also indicates that this sensitivity is greater for low than for high saving rates. Higher substitutability moderates the negative effects of capital's crowding out that occurs with low savings.

6 The Role of Property Rights and Rivalry

To this point we've assumed that code is private and rival. Specifically, we've assumed that when one firm uses code it is unavailable for rent or use by other firms. But unlike physical capital, code represents stored information that may be non-rival in its use. Non-rivalry does not however necessarily imply non-excludability. Patents, copyrights, trade secrets, and other means can be used to limit code's unlicensed distribution. On the other hand, the government can turn code into a public good by mandating it be open source.

This section explores two new scenarios. The first is that code is non rival and non excludable in its use, i.e., it is a public good. The second is that code is non rival, but excludable. To accommodate these possibilities we modify our model in two ways. We assume that each firm faces a fixed cost of entry. And we assume that each firm is endowed with a limited supply of public code. These assumptions ensure a finite number of firms operating with non-trivial quantities of capital. To compare these two new settings with what came above – the case of private (rival and excludable) code, we rewrite our baseline model with the two new assumptions.

Rival, Excludable (Private) Code

With a fixed public code endowment and fixed entry costs, profit maximization satisfies:

$$\pi_{j,t} = F(k_{j,t}, zH_{j,t} + a_{j,t} + \overline{A}) - C - r_t k_{j,t} - m_t a_{j,t}, \tag{28}$$

where $\pi_{j,t}$ are profits for firm j at time t, $F(\bullet)$ is the same CES production function as in the baseline model, $k_{j,t}$ is the amount of capital rented by the firm, $a_{j,t}$ is the amount of code rented by the firm, $H_{j,t}$ is the amount of high-tech labor hired by the firm, $\overline{A}$ is the exogenously set amount of free code in the economy, and C is the cost of creating a new firm. This cost must be paid each period. In equilibrium all firms have zero profits.

$$0 = F(k_{j,t}, zH_{j,t} + a_{j,t} + \overline{A}) - C - r_t k_t - m_t a_{j,t}. \tag{29}$$

Market clearing conditions are,

$$\sum a_{j,t} = \delta A_{t-1}, \tag{30}$$

$$\sum k_{j,t} = K_t, \tag{31}$$

$$\sum H_{j,t} = H_{A,t}, \tag{32}$$

$$Y = c_{o,t} + c_{y,t} - K_t + K_{t+1} + NC, \tag{33}$$

where N is the number of firms. Since all firms are identical, (26) provides an expression for N, the number of firms.

$$0 = NF(\frac{K_t}{N}, zH_t + \frac{1}{N}\delta A_{t-1} + \overline{A}) - NC - r_t K_t - m_t \delta A_{t-1} \tag{34}$$

Firms enter up to the point that the value of the public code they obtain for free, namely $\overline{A}$, equals their fixed cost of production. Thus,

$$\overline{A} F_{a,t} = C. \tag{35}$$

This fixes the marginal product of code at $\frac{C}{\overline{A}}$ in every period. Intuitively, new firms can acquire a perfect substitute for new code, and, thus, new coders at a fixed cost by setting up shop and gaining access to $\overline{A}$ in free code. Given that good production obeys constant returns to scale, fixing code's marginal product means fixing the ratio of capital to code. This, in turn, fixes the interest rate. Hence, the rental rates of coders and capital are invariant to the increase in δ.

Although the increase in δ doesn't raise the current productivity of coders, it does raise the present value of their labor compensation. The reason is that coders can now sell property rights to the future use of their invention. Hence, unlike our initial model, this variant with fixed costs and a free endowment of code does not admit immiserating growth absent some additional assumptions.[16]

Were the number of firms to remain fixed, the jump in δ would entail more code per firm with no higher capital per firm. This would mean a lower marginal productivity of code, which (35) precludes. It would also mean a negative payoff to setting up a new firm. Hence, the number of firms must shrink in order to raise the level of capital per firm as needed to satisfy (35).

[16]If the number of firms is fixed due to oligopilization of the industry, equation 35 would not hold, in which case the marginal productivity of code would again decrease as it accumulates.

To solve the model an additional step is added to the iteration procedure. Given a guess of prices and stocks in a period, (34) is used to calculate N. This guess of N in each period is included in the next iteration to calculate new prices.[17]

Figure 9 shows transition paths for key variables for this excludable, non-rival model based on Table 4's parameter values. Note that high-tech workers earn 14 percent more in the long run and enjoy commensurately higher utility. Low-tech workers are also better off. There is also a modest increase in the economy's capital stock.

Non-Rival, Non-Excludable (Public) Code

Consider next the case that code, in the period after it is produced, is a pure public good used simultaneously by every firm. This possibility could arise by government edict, the wholesale pirating of code, or reverse engineering.

Profits are now

$$\pi_{j,t} = F(k_{j,t}, zH_{j,t} + a_{j,t} + \overline{A}) - C - r_t k_{j,t}, \tag{36}$$

as firms no longer need to rent their stock of code ($a_{j,t}$), where

$$a_{j,t} = \delta A_{t-1} \forall j \tag{37}$$

As before, firm entry and exit imply zero profits,

$$0 = NF(\frac{K_t}{N}, zH_t + \delta A_{t-1} + \overline{A}) - NC - r_t K_t. \tag{38}$$

and

$$(\delta A_{t-1} + \overline{A}) F_{a,t} = C. \tag{39}$$

Finally, with investment in code no longer crowding out investment in capital,

$$K_{t+1} = \phi I_t. \tag{40}$$

Figure 10 shows results for this case again with Table 4's parameter values. The initial steady state is the same as in the prior case of excludable rival code. However, the response to the jump in δ are dramatically different. The jump in δ has no immediate impact on the economy because high-tech workers no longer hold copyright to their code.

In the period after the shock, the economy begins to react. The stock of free public code, which now includes both $\overline{A}$ plus all of the economy's legacy code, is larger. This induces more firm entry. Indeed, the number of firms more than doubles. As indicated in equation 39, with more free code available, new entrants can cover the fixed costs of entry with a lower value per unit of free

[17]In what follows, we consider only equilibria in which high-tech workers work in both sectors. If the public endowment is large enough in a period, goods firms will require no new code.

code, i.e., with a lower marginal product of code. The lower marginal product of code and, thus, of coders leads to an exodus of high-tech workers from coding into services. In the long run, the number of high-tech workers hired for their coding skills falls by 30 percent and their wage falls by 25 percent. National income peaks at 5 percent above its initial level in this period. The interest rate rises by 35 percent and the wage of low-tech workers decreases by 10 percent.

The economy's transition is characterized by a series of damped oscillations as periods of relatively high coder hiring is followed by periods of plentiful free code and relatively low coder hiring. Most importantly, the long-run impact of this change is a net immiseration with long-run national income 8 percent below its initial steady state level.

As in the baseline model, the main mechanism for immiseration is the reduction of the high-tech wage leading to lower capital accumulation. A secondary reason is the inefficiency introduced due to high-tech workers no longer being able to internalize the full value of their creation of new code.

Non-Rival, Excludable (Private) Code

A third possibility is that code is excludable, but non-rival in its use, permitting high-tech workers to license all their code to all firms. The equations for the rival, excludable model hold with the following exceptions. First, profits are given by

$$\pi_{j,t} = F(k_{j,t}, zH_{j,t} + \delta A_{t-1} + \overline{A}) - C - r_t k_{j,t} - m_t \delta A_{t-1} \tag{41}$$

Second, the price of code reflects its use by all firms.

$$p_t = \sum_{s=t}^{\infty} R_{s+1,t}^{-1} \delta^{s-t} m_{s+1} N_{s+1}. \tag{42}$$

As shown in figure 11, the δ shock produces a felicitous transition path, indeed far better than the rival excludable case. As in the rival, excludable case, firms entry satisfies equation 35. Hence, the marginal product of new code is fixed. So is the marginal product of capital, i.e., the interest rate.

7 Endogenous Production Technology

So far we've assumed a single means of producing goods. Here we permit good producers to switch between more and less code-intensive production techniques. To keep matters simple we assume the good sector's production function is Cobb-Douglas and that good producers can choose the parameter on A (and thus on K) such that $\alpha \in [\alpha_1, \alpha_2]$. In the initial steady state, $\alpha_1 = \alpha_2$, but when δ is shocked, the range of possible technologies is expanded as well.

This is simulated via an additional step in the iteration process. After a guess of the path of code and capital is made, an $\alpha \in [\alpha_1, \alpha_2]$ is selected in every period to maximize good output. Subsequently, prices are calculated from marginal products and a new guess of the path of inputs is made.

Given the inputs, and the prevailing stocks of code and capital, output is convex in α. Hence, firms will produce using either the lowest or highest value.[18] This results in the economy flipping back and forth repeatedly, although not necessarily every period, from the most to the least capital-intensive technology. Since our solution method relies on the economy reaching a stable steady state, we set α to a fixed value, namely α_2, far enough in the future such that the transition path for the initial several hundred periods is unaffected.

Figure 12 presents results based on table 5's parameter values. Unlike the previous figures, the absolute amounts of capital and code stocks reflect the dependency of the choice of technology on the ratio of the two stocks. In the initial steady state, the code stock consists just of newly produced code and, naturally, is low. The economy is in a capital-intensive steady state. After the δ shock, code begins to accumulate. In the fourth period, sufficient code is accumulated to lead producers to switch technologies toward more code-intensive production. But the switch to code-intensive production raises wages and, thus, workers' saving. Due to our assumed high saving preference ($\phi = .9$), the increase in saving more than offsets the increase in the value of claims to code and the capital stock increases. If the saving preference were lower, capital stocks would not rise, and the economy would remain permanently in a code-intensive equilibrium. In this case, however, the increase in saving is large enough to drive producers to adopt a capital-intensive technology in the next period. This leads to lower wages, which, over time, means a lower capital-code ratio and a subsequent switch back to code-intensive production.

This ongoing cycle has important welfare implications. High-tech workers who are young when the code-intensive technology is used will earn a high wage when young and high interest rates when old. Those unfortunate enough to be young in a period when a high alpha is chosen will earn low wages while young and low interest rates when old.

Because a period in our model corresponds to roughly 30 years, this cycle of technologically driven booms and busts bears a striking resemblance to the 'long-wave' theories of early economists such as Schumpeter and Kondratieff. While evidence for the existence of such cycles is limited (Mansfield 1983), this model's long-wave cycles reflect a different mechanism from those in Rosenberg and Frischtak (1983).

8 Testable Implications and Supportive Evidence

Each of our model's simulations feature a temporary rise followed by a decline in labor's share of national income as well as a rise in code as a share of total assets. U.S. labor-share data going back four decades provides support for these trends.

[18] $Y''(\alpha) = B\frac{A}{K}^{\alpha}(log(A) - log(K))^2$ must always be positive for K and A greater than zero

There is also recent evidence of a decline in capital per worker, consistent with our model's immiseration scenarios.

Figure 13 displays three measures of labor's share of U.S. income based on three ways to handle a major unknown – labor's share of proprietorship and partnership income. The top two curves use Bureau of Economic Analysis (BEA) data. The first simply displays labor's share of the sum of all non-proprietorship income. The second displays labor's share of corporate income.

The bottom curve displays labor's share of all private businesses output including proprietorships as calculated by the Bureau of Labor Statistics (BLS). The BLS imputes labor's share in proprietorship income by assuming proprietors' and partners' annual labor income equals the annual average wage earning in their industry. Any proprietor income above this amount is considered capital income. This measure is smaller than the others because the BLS's income measure does not net out depreciation.

According to all three measures, labor's share of income is lower in 2015 than in the mid 1970's. In the bottom measure, labor's share peaks in the mid-1970s with the two lowest shares recorded in 2014 and 2015. Between 1975 and 2014, labor's share declines by 5.96 percentage points, 5.88 percentage points, or 4.88 percentage points according to the non-proprietorship labor share, corporate labor share, and BLS measures respectively.

Other authors, including Karabarbounis and Neiman (2013) and Brigdman (2014), report similar findings using related labor-share measures. The consensus view is that labor's share has decreased significantly since peaking in the mid 1970's.

Armenter (2015) considers the possibility that the decrease in the BLS's measure is driven by the assumption that the proprietors pay themselves the average wage in an industry. When he assumes instead that labor's share of proprietor's income remains fixed at 85 percent, labor' share since 1975 still falls, but by less. Karabarbounis and Neiman (2013) attribute the decline to capital accumulation and their finding of gross substitutability between capital and labor. Rather than capital abundance, Rognlie (2015) argues that the decrease in the labor share is due to property scarcity. He attributes the decline in labors' share to an increase in property values and imputed rents.

Code stocks have certainly increased since the invention of the digital computer and the silicon chip. Figure 14 reports stocks of R&D and software as a share of total US fixed assets. According to the BEA, software grew from almost 0 percent of capital in 1960 to over 1.5 percent today. Combined software and R&D stocks have grown as a share of capital by about 3.5 percentage points over the same period. These numbers are likely underestimates of the increasing importance of programmers, scientists and engineers in the economy. Software is decomposed in NIPA table 2.1 into own account, prepackaged, and custom software. The true value of prepackaged software in the economy is likely undercounted because it is often pirated. It is also often free or sold at a discount in order to cross subsidize some other product or subscription

(Parker and Van Alstyne, 2005). BEA estimates of firms' internal creation of their own software are based on very conservative estimates about the share of programmers who are developing new code (rather than maintaining old code) and the rate at which the software stock decays.

Many papers confirm the intuition that the BEA underestimates the stock of organizational capital and code complementary to computers. Brynjolfson, Hitt, and Yang (2002) find that firms with large investments in computer capital have much higher valuations, that computer capital investments lead to disproportionately large increases in firm valuations, and firms that make such investments tend to be more productive in future years. Similarly, Hulten and Hao (2008) find that the book value of R&D- intensive firms in 2006 explains only 31 percent of their valuation. Both these papers argue that only firms who have made large investments in organizational and technological capital are able to implement innovative technologies.

Code and software controlled by firms that aren't counted as assets by the BEA still increase the productivity of firms. Such firms would be more valuable than they should be based on only the assets they are observed to own. Figure 15 shows the value of the US corporate sector less the replacement cost of its physical and financial assets.[19] This measure of the stock of intangible assets is highly cyclical because its numerator is the price of the stock market. Despite this, it shows a clear secular increase starting in the mid 1970s. For firms in the S&P 500, intangible assets increased from 17 percent of market value to in 1975 to 84 percent in 2015 (Ocean Tomo, 2015).

Hall (2001) argues that the increase in the value of economy-wide intangible assets, and therefore Tobin's (average) q, is due to the creation of code and organizational capital within firms, which he calls 'e-capital'. Barkai (2016) also notes that firms' output per unit of observed capital has increased even as the marginal cost of capital (as measured by the real interest rate) has decreased dramatically. Assuming that capital's average product is equal to its marginal product, he interprets this trend as being due to an increase in market power and markups. Of course, an alternative explanation would be the accumulation of unmeasured intangible assets (i.e. the true average product of capital would be much lower if all capital were included). Barkai argues that the stock of intangible assets needed to explain the wedge between the observed average product of capital and its marginal cost is implausibly large. The level of intangible assets in 2014 would need to be 42 Trillion (or 54% of U.S. wealth) in order to explain the discrepancy. However, an extremely rapid increase of the share of intangibles in total assets is a phenomena implied by our model.

Long-run immiseration in our model hinges on a long-run decline in capital per worker. While capital per worker increased at an average rate of 2.5 percent from 1985 to the present, the years 2011 through 2015 have seen a decrease in capital per worker of .5 percent per year on average. This is the longest and most dramatic decrease on record.[20] Further, this measure significantly under-

[19] US Corporate intangible assets are calculated as US corporate equity less corporate net worth from Federal Reserve series Z.1.

[20] Capital-hours ratio; BLS multifactor productivity series, Table PG-2-3. Records date

estimates the extent to which physical capital per person has decreased. Capital services as measured by the BLS include accumulation of intellectual property and capital quality increases (through the deflator) that are attributable in our model not to physical capital per worker but to larger stocks of code.

9 Conclusion

Will smart machines, which are rapidly replacing workers in a wide range of jobs, produce economic misery or prosperity? Our two-period, OLG model admits both outcomes. But it does firmly predict three things - a long-run decline in labor share of income (which appears underway in OECD members), tech-booms followed by tech-busts, and a growing dependency of current output on past software investment.

The obvious policy for producing a win-win from higher code retention is taxing those workers who benefit from this technological breakthrough and saving the proceeds. This will keep the capital stock from falling and provide a fund to pay workers a basic stipend as their wages decline through time. Other policies for managing the rise of smart machines may backfire. For example, restricting labor supply may reduce total labor income. While this may temporarily raise wages, it will also reduce investment and the long-term capital formation on which long-term wages strongly depend. Another example is mandating that all code be open source. This policy removes one mechanism by which capital is crowded out, but it leads firms to free ride on public code rather than hire new coders. This reduces wages, saving, and, in time, the capital stock.

Our simple model illustrates the range of things that smart machines can do for us and to us. Its central message is disturbing. Absent appropriate fiscal policy that redistributes from winners to losers, smart machines can mean long-term misery for all.

back to 1949.

References

Abel, Andrew and Laurence J Kotlikoff, "Intergenerational Altruism and the Effectiveness of Fiscal Policy: New Tests Based on Cohort Data," in "Savings and bequests" 1994, chapter 7, pp. 31–42.

Acemoglu, Daron, "Why Do New Technologies Complement Skills? Directed Technical Change and Wage Inequality," *The Quarterly Journal of Economics*, nov 1998, *113* (4), 1055–1089.

___ and David Autor, "Chapter 12 – Skills, Tasks and Technologies: Implications for Employment and Earnings," in "Handbook of Labor Economics," Vol. 4 2011, pp. 1043–1171.

___ and Pascual Restrepo, "The Race Between Man and Machine: Implications of Technology for Growth, Factor Shares and Employment," 2015.

___, David Autor, David Dorn, Gordon H Hanson, and Brendan Price, "Return of the Solow Paradox? IT, Productivity, and Employment in U.S. Manufacturing," jan 2014.

Altonji, Joseph G., Fumio Hayashi, and Laurence J. Kotlikoff, "Is the Extended Family Altruistically Linked? Direct Tests Using Micro Data," *American Economic Review*, 1992, *82* (5), 1177–98.

Altonji, Joseph G, Fumio Hayashi, and Laurence Kotlikoff, "Parental Altruism and Inter Vivos Transfers: Theory and Evidence," dec 1995.

Armenter, Roc, "A bit of a miracle no more: the decline of the labor share," *Business Review; Federal Reserve Bank of Philadelphia Research Department*, 2015, (Third Quarter), 1–8.

Arrow, Kenneth J., "The Economic Implications of Learning by Doing," *The Review of Economic Studies*, 1962, *3* (June), 155–173.

Autor, David H. and David Dorn, "The growth of low-skill service job and the polarization of the US labor market," *The American Economic Review*, 2013, *103* (5), 1553–1597.

___, Frank Levy, and Richard J. Murnane, "The Skill Content of Recent Technological Change: An Empirical Exploration," *The Quarterly Journal of Economics*, nov 2003, *118* (4), 1279–1333.

Barkai, Simcha, "Declining Labor and Capital Shares," 2016.

Bridgman, Benjamin et al., "Is Labor's Loss Capital's Gain? Gross versus Net Labor Shares," *Washington: US Bureau of Economic Analysis*, 2014.

Frey, Carl Benedikt and Michael A Osborne, "The future of employment: how susceptible are jobs to computerisation," *Retrieved September*, 2013.

Goos, Maarten, Alan Manning, and Anna Salomons, "Explaining Job Polarization in Europe: The Roles of Technology, Globalization and Institutions," *CEP Discussion Papers.*

Hayashi, Fumio, Joseph Altonji, and Laurence Kotlikoff, "Risk-Sharing between and within Families," *Econometrica*, 1996, *64* (2), 261–94.

Hemous, David and Morten Olsen, "The Rise of the Machines: Automation, Horizontal Innovation and Income Inequality," *CEPR Discussion Papers.*

Internal Revenue Service, "Publication 535 (2015), Business Expenses."

Karabarbounis, Loukas and Brent Neiman, "The Global Decline of the Labor Share."

Katz, Lawrence F. and Robert A. Margo, "Technical Change and the Relative Demand for Skilled Labor: The United States in Historical Perspective."

Kelley, Kevin, "The Three Breakthroughs That Have Finally Unleashed AI on the World," *Wired Online Edition*, oct 2014.

Keynes, John Maynard, "Economic Possibilities for our Grandchildren," in "Essays in Persuasion," New York: W.W.Norton & Co, 1963, pp. 358–373.

Kondratieff, Nikolai D, "The Long Waves in Economic Life," *The Review of Economics and Statistics*, 1935, *17* (6), 105–115.

Lucas, Robert E., "On the mechanics of economic development," *Journal of Monetary Economics*, jul 1988, *22* (1), 3–42.

Madrigal, Alexis C., "Inside Google's Secret Drone Delivery Program," *The Atlantic Online Edition*, aug 2014.

Mansfield, Edwin, "Long Waves and Technological Innovation," *American Economic Review*, 1983, *73* (2), 141–45.

Marx, Karl, *Capital: A Critique of Political Economy*, 1992 ed., Penguin Classics, 1887.

Mishel, Lawrence, John Schmitt, and Heidi Shierholz, "Don't blame the robots. Assessing the job polarization explanation of growing wage inequality," 2013.

Nelson, Richard R. and Edmund S. Phelps, "Investment in Humans, Technological Diffusion, and Economic Growth," *American Economic Review*, 1966, *56* (1/2), 69–75.

Ocean Tomo LLC, "Annual Study of Intangible Asset Market Value Ocean Tomo, LLC," 2015.

of Economic Analysis, U.S. Bureau, "National Income and Product Accounts."

of Governors of the Federal Reserve System, Board, "Financial Accounts of the United States - Series Z.1."

O'Rourke, Kevin, Ahmed Rahman, and Alan Taylor, "Luddites, the industrial revolution, and the demographic transition," *Journal of Economic Growth*, 2013, *18* (4), 373–409.

Peretto, Pietro F. and John J. Seater, "Factor-eliminating technical change," *Journal of Monetary Economics*, 2013, *60* (4), 459–473.

Rognlie, Matthew, "Deciphering the fall and rise in the net capital share: accumulation or scarcity?," *Brookings papers on economic activity*, 2016, *2015* (1), 1–69.

Romer, Paul M., "Endogenous Technological Change," *Journal of Political Economy*, 1990, *98* (5), S71–S102.

Rosenberg, Nathan and Claudio R. Frischtak, "Long Waves and Economic Growth: A Critical Appraisal," *American Economic Review*, 1983, *73* (2), 146–151.

Sachs, Jeffrey D and Laurence J Kotlikoff, "Smart Machines and Long-Term Misery," dec 2012.

Schumpeter, Joseph A., *Business Cycles: A Theoretical, Historical and Statistical Analysis of the Capitalist Process*, New York: McGraw Hill Book Co., 1939.

U.S. Bureau of Labor Statistics, "May 2013 OES National Industry-Specific Occupational Employment and Wage Estimates."

——, "Multifactor Productivity and Related Measures."

Website of the National Archives, "Luddites: The Growth of Political Rights in Britain in the 19th Century."

Zeira, Joseph, "Workers, Machines, and Economic Growth," *The Quarterly Journal of Economics*, 1998, *113* (4), 1091–1117.

Table 1
Parameters for Immiserating Growth

Model Parameter	Role	Value
ε_s	Elasticity in Service Sector	∞
ε_y	Elasticity in Good Sector	1
γ	Service High-Tech Input Share Param.	0.5
α	Good Capital Input Share Param.	0.5
δ	Code Retention Rate	0 shocked to 0.7
ϕ	Saving Preference Param.	0.2
H	High-Tech Worker Quantity	1
G	Low-Tech Worker Quantity	1
κ	Service Consumption Share	0.5
z	Code Writing Productivity	1
D_y	TFP in Goods	1
D_s	TFP in Services	1

Table 1: This table gives parameter values for the first illustration of the effects of a one-time, permanent increase in the depreciation rate, δ, from zero to .7. We take the intermediate value of .5 for κ, α, and γ. The productivity terms z, D_Y, and D_S, are set to one. In this and all subsequent simulations invoking an elasticity of 1 (except for the endogenous technology extension) the true elasticity is actually 1.0001

Figure 1
Immiserating Growth

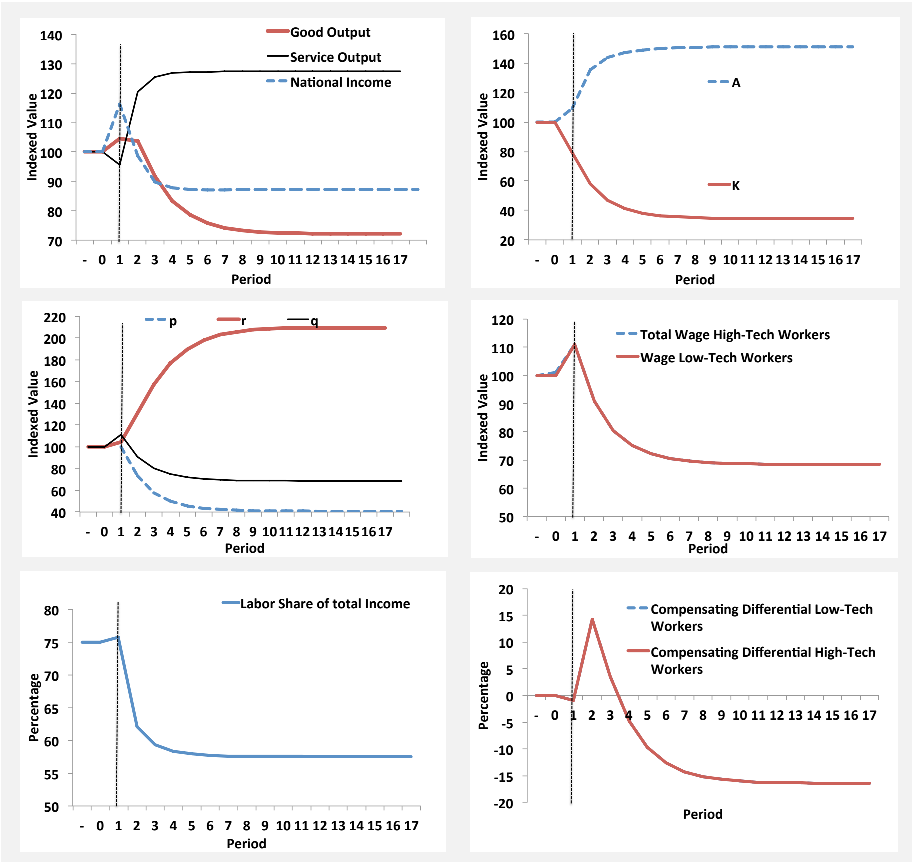

Figure 1: Transition paths based on table 1. "Compensating Differential" references the percentage change in initial steady-state consumption that would be needed for the utility levels of workers to equal their respective transition utility levels. Service and goods output are raw indexed output, not market value. Period 1 non-indexed prices in units of the good are $r = 1.737$, $q = .349$, and $p = .043$.

Figure 2
Felicitous Growth
(higher saving rate, $\phi = .95$)

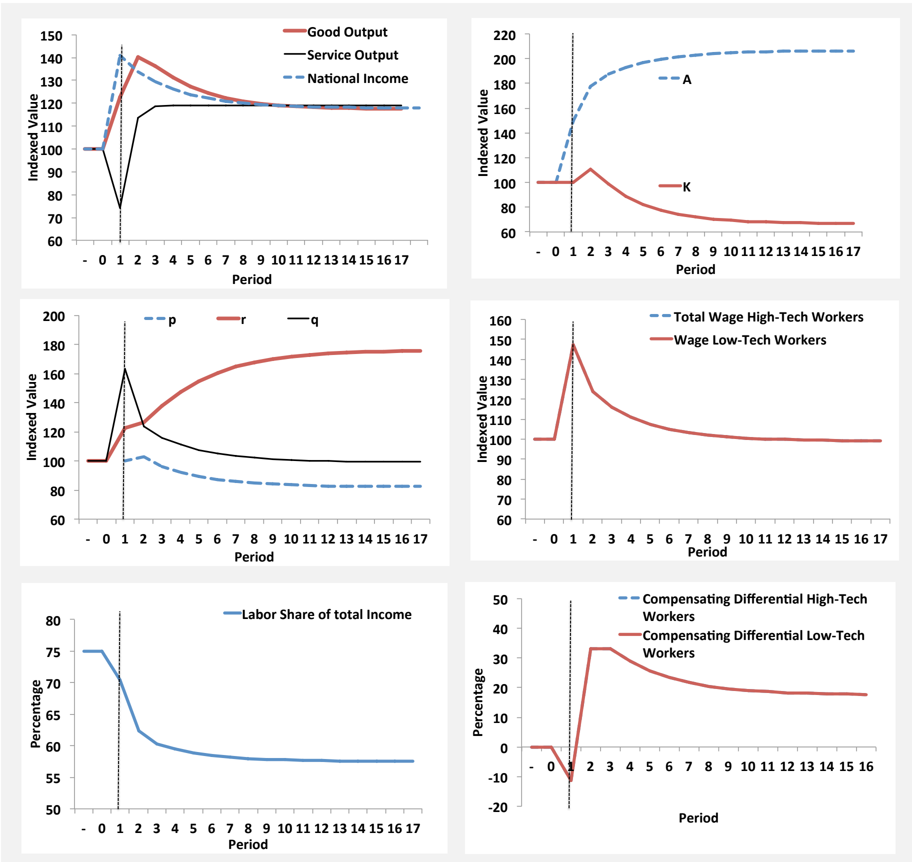

Figure 2: Transition paths based on table 1, with the exception of a higher saving rate ($\phi = .95$). “Compensating Differential” references the percentage change in initial steady-state consumption that would be needed for the utility levels of workers to equal their respective transition utility levels. Service and goods output are raw indexed output, not market value. . Period 1 non-indexed prices in units of the good are $r = .454$, $q = 2.204$, and $p = .631$.

Figure 3
The First Shall Be Last

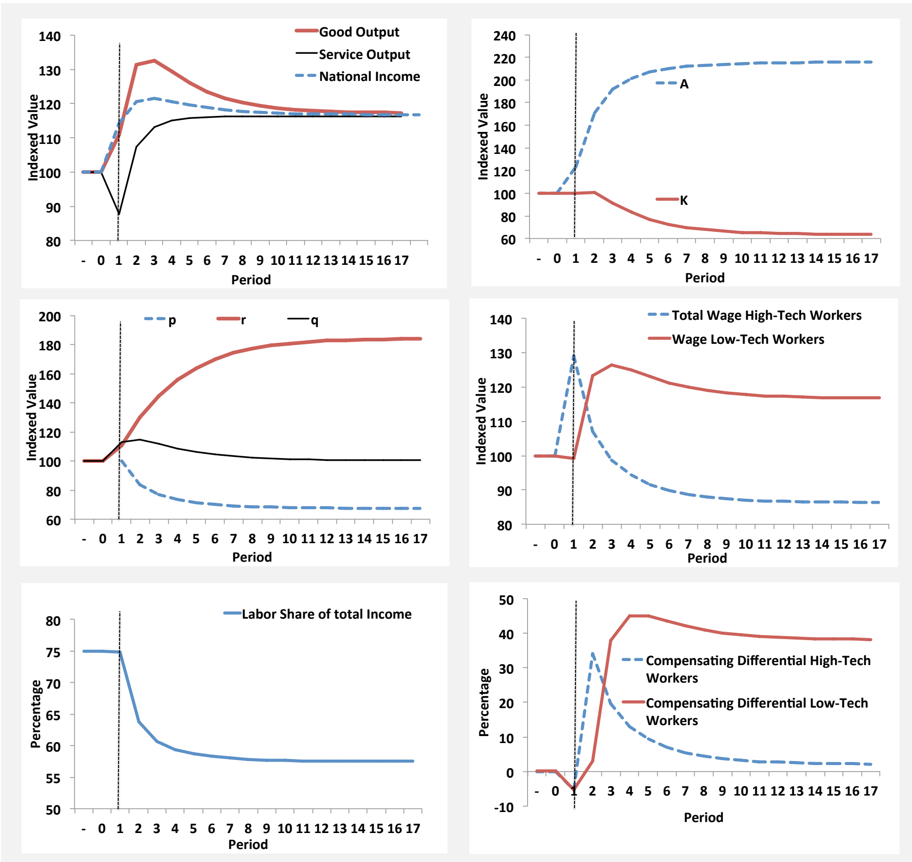

Figure 3: Transition paths based on table 2. “Compensating Differential” references the percentage change in initial steady-state consumption that would be needed for the utility levels of workers to equal their respective transition utility levels. Service and goods output are raw indexed output, not market value. Period 1 non-indexed prices in units of the good are $r = .529$, $q = 1.317$, and $p = .398$.

Figure 4
Better Tasting Goods

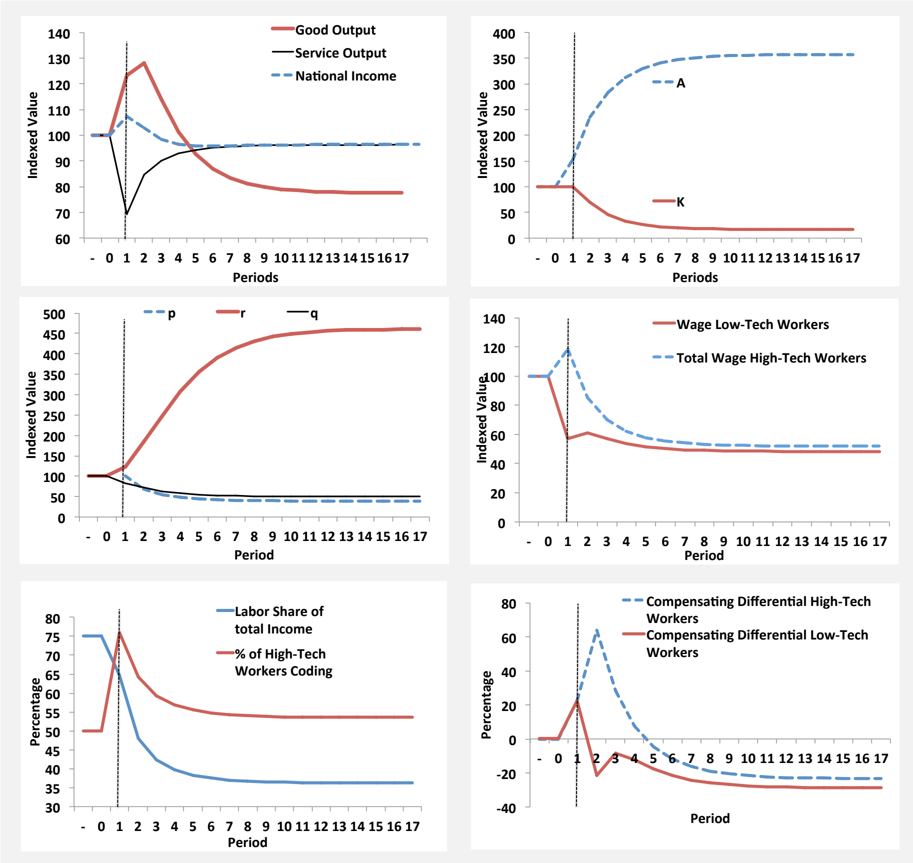

Figure 4: Transtion paths based on table 2, except in addition to the δ shock, κ is simultaneously shocked from .5 to .25. "Compensating Differential" references the percentage change in initial steady-state consumption that would be needed for the utility levels of workers to equal their respective transition utility levels. Service and goods output are raw indexed output, not market value. Period 1 non-indexed prices in units of the good are $r = .587$, $q = .784$, and $p = .200$.

Table 2
Parameters for The First Will Be Last

Model Parameter	Role	Value
ε_s	Elasticity in Service Sector	1
ε_y	Elasticity in Good Sector	1
γ	Service High-Tech Input Share Param.	0.5
α	Good Capital Input Share Param.	0.5
δ	Code-Retention Rate	0 shocked to 0.7
ϕ	Saving Preference Parameter	0.7
H	High-Tech Worker Quantity	2
G	Low-Tech Worker Quantity	1
κ	Service Consumption Share	0.5
z	Code Writing Productivity	1
D_y	TFP in Goods	1
D_s	TFP in Services	1

Table 3: Sensitivity Analysis

Baseline

Period	ε_s	ε_y	δ	ϕ	Labor High-Tech	Labor Low-Tech	κ	γ	z	Nat Income	Corn Output	Spirituality Output	K	A	p	q	r	Wage Hearts	Wage Heads	Labor Share	% Heads Coding	Compensating Differential Heads	Compensating Differential Hearts
Initial Steady State	1.0	1.0	**0.0**	0.5	1.0	1.0	0.5	0.5	1.0	100	100.0	100.0	100.0	100.0	-	100.0	100.0	50.0	100.0	75.0	50.0	0.0	-49.9
1	1.0	1.0	**0.5**	0.5	1.0	1.0	0.5	0.5	1.0	113	117.5	106.4	100.0	112.6	100.0	108.4	106.1	50.7	115.9	66.8	56.3	20.3	-47.4
2	1.0	1.0	0.5	0.5	1.0	1.0	0.5	0.5	1.0	110	116.0	109.1	96.6	143.0	89.7	108.5	121.7	57.7	101.9	65.6	43.4	8.9	-38.3
3	1.0	1.0	0.5	0.5	1.0	1.0	0.5	0.5	1.0	109	113.2	109.6	88.3	152.4	85.7	105.7	131.4	57.6	96.8	65.6	40.5	4.5	-37.7
4	1.0	1.0	0.5	0.5	1.0	1.0	0.5	0.5	1.0	109	111.0	109.7	82.1	156.0	83.6	103.3	137.8	56.7	94.2	64.8	39.9	2.3	-38.5
10	1.0	1.0	0.5	0.5	1.0	1.0	0.5	0.5	1.0	108	107.0	109.6	71.9	159.8	80.5	99.0	149.1	54.3	90.4	62.7	40.0	-1.0	-40.5
Steady State	1.0	1.0	0.5	0.5	1.0	1.0	0.5	0.5	1.0	108	106.7	109.5	71.1	160.0	80.2	98.7	150.0	54.1	90.1	62.5	40.0	-1.2	-40.7

Spirituality Taste Shock κ Jumps from 0.5 to 0.75

Period	ε_s	ε_y	δ	ϕ	Labor High-Tech	Labor Low-Tech	κ	γ	z	Nat Income	Corn Output	Spirituality Output	K	A	p	q	r	Wage Hearts	Wage Heads	Labor Share	% Heads Coding	Compensating Differential Heads	Compensating Differential Hearts
Initial Steady State	1.0	1.0	0.0	0.5	1.0	1.0	**0.5**	0.5	1.0	100	100.0	100.0	100.0	100.0	-	100.0	100.0	50.0	100.0	75.0	50.0	0.0	-49.8
1	1.0	1.0	0.5	0.5	1.0	1.0	**0.75**	0.5	1.0	153	117.2	120.6	100.0	87.1	100.0	135.0	93.4	71.8	127.1	58.4	43.6	20.3	-47.3
2	1.0	1.0	0.5	0.5	1.0	1.0	**0.75**	0.5	1.0	146	129.2	123.6	140.0	98.2	97.7	145.5	83.7	87.8	120.7	67.2	27.3	8.9	-38.2
3	1.0	1.0	0.5	0.5	1.0	1.0	**0.75**	0.5	1.0	143	136.7	124.4	173.0	96.4	99.0	150.9	74.7	93.2	122.1	72.0	23.7	4.5	-37.6
4	1.0	1.0	0.5	0.5	1.0	1.0	**0.75**	0.5	1.0	141	142.0	124.8	200.0	93.4	100.2	154.4	68.4	96.0	124.1	74.6	22.6	2.3	-38.4
10	1.0	1.0	0.5	0.5	1.0	1.0	**0.75**	0.5	1.0	137	154.1	125.6	275.8	85.2	102.8	161.6	55.6	101.5	128.6	80.2	21.2	-1.0	-40.4
Steady State	1.0	1.0	0.5	0.5	1.0	1.0	**0.75**	0.5	1.0	137	156.5	125.8	293.4	83.5	103.2	162.7	53.4	102.3	129.4	81.3	20.9	-1.2	-40.6

Corn Taste Shock κ Jumps from 0.5 to 0.25

Period	ε_s	ε_y	δ	ϕ	Labor High-Tech	Labor Low-Tech	κ	γ	z	Nat Income	Corn Output	Spirituality Output	K	A	p	q	r	Wage Hearts	Wage Heads	Labor Share	Coders Corn Sector	Compensating Differential Heads	Compensating Differential Hearts
Initial Steady State	1.0	1.0	0.0	0.5	1.0	1.0	**0.5**	0.5	1.0	100	100.0	100.0	100.0	100.0	-	100.0	100.0	50.0	100.0	75.0	50.0	0.0	-49.9
1	1.0	1.0	0.5	0.5	1.0	1.0	**0.25**	0.5	1.0	76	117.2	81.6	100.0	149.3	100.0	66.9	122.2	23.8	94.0	70.5	74.7	22.3	-68.9
2	1.0	1.0	0.5	0.5	1.0	1.0	**0.25**	0.5	1.0	69	102.4	84.1	66.0	208.1	71.5	55.7	177.6	22.7	68.3	56.4	66.8	-4.5	-68.2
3	1.0	1.0	0.5	0.5	1.0	1.0	**0.25**	0.5	1.0	65	91.9	84.8	44.9	233.3	58.1	46.9	227.9	19.7	55.8	52.3	64.6	-18.2	-71.0
4	1.0	1.0	0.5	0.5	1.0	1.0	**0.25**	0.5	1.0	62	85.4	85.0	34.4	244.7	51.2	41.7	266.5	17.7	49.1	49.0	64.1	-25.7	-73.2
10	1.0	1.0	0.5	0.5	1.0	1.0	**0.25**	0.5	1.0	59	76.7	84.9	23.1	255.6	42.9	34.8	332.7	14.8	41.0	44.0	64.0	-34.9	-76.5
Steady State	1.0	1.0	0.5	0.5	1.0	1.0	**0.25**	0.5	1.0	59	76.3	84.9	22.7	255.9	42.6	34.6	335.6	14.7	40.7	43.8	64.0	-35.2	-76.6

Low Saving Rate ϕ = 0.1

Period	ε_s	ε_y	δ	ϕ	Labor High-Tech	Labor Low-Tech	κ	γ	z	Nat Income	Corn Output	Spirituality Output	K	A	p	q	r	Wage Hearts	Wage Heads	Labor Share	% Heads Coding	Compensating Differential Heads	Compensating Differential Hearts
Initial Steady State	1.0	1.0	0.0	**0.1**	1.0	1.0	0.5	0.5	1.0	100	100.0	100.0	100.0	100.0	-	100.0	100.0	49.9	100.0	75.1	50.1	0.0	-50.0
1	1.0	1.0	0.5	**0.1**	1.0	1.0	0.5	0.5	1.0	101	100.8	110.5	100.0	102.3	100.0	106.3	101.2	52.5	107.4	77.5	51.2	6.8	-47.9
2	1.0	1.0	0.5	**0.1**	1.0	1.0	0.5	0.5	1.0	98	93.2	113.1	78.8	129.1	82.9	99.0	128.1	54.7	89.6	70.0	39.0	-9.3	-44.6
3	1.0	1.0	0.5	**0.1**	1.0	1.0	0.5	0.5	1.0	97	88.1	113.8	63.7	136.6	75.8	92.7	146.6	52.4	82.0	67.4	36.1	-15.9	-46.2
4	1.0	1.0	0.5	**0.1**	1.0	1.0	0.5	0.5	1.0	97	85.0	113.9	55.8	138.9	72.4	89.1	158.0	50.6	78.4	65.3	35.4	-19.0	-47.6
10	1.0	1.0	0.5	**0.1**	1.0	1.0	0.5	0.5	1.0	96	81.6	114.0	47.8	140.3	68.0	84.6	172.3	48.2	74.2	62.6	35.1	-22.3	-49.6
Steady State	1.0	1.0	0.5	**0.1**	1.0	1.0	0.5	0.5	1.0	96	81.4	114.0	46.9	140.4	68.0	84.6	172.6	48.1	74.1	62.5	35.1	-22.4	-49.6

High Saving Rate ϕ = 0.9

Period	ε_s	ε_y	δ	ϕ	Labor High-Tech	Labor Low-Tech	κ	γ	z	Nat Income	Corn Output	Spirituality Output	K	A	p	q	r	Wage Hearts	Wage Heads	Labor Share	% Heads Coding	Compensating Differential Heads	Compensating Differential Hearts
Initial Steady State	1.0	1.0	0.0	**0.9**	1.0	1.0	0.5	0.5	1.0	100	100.0	100.0	100.0	100.0	-	100.0	100.0	50.0	100.0	75.0	50.0	0.0	-49.9
1	1.0	1.0	0.5	**0.9**	1.0	1.0	0.5	0.5	1.0	121	126.9	103.2	100.0	121.5	100.0	108.5	110.2	48.1	122.5	60.9	60.8	25.0	-50.9
2	1.0	1.0	0.5	**0.9**	1.0	1.0	0.5	0.5	1.0	117	129.4	106.3	104.4	154.4	91.6	112.2	121.6	57.9	108.8	62.3	46.8	16.4	-38.0
3	1.0	1.0	0.5	**0.9**	1.0	1.0	0.5	0.5	1.0	116	128.9	107.0	102.0	164.1	89.2	111.8	126.8	59.5	105.1	63.7	43.5	13.7	-35.7
4	1.0	1.0	0.5	**0.9**	1.0	1.0	0.5	0.5	1.0	116	128.0	107.2	99.2	167.5	88.2	110.9	129.9	59.4	103.6	63.6	42.7	12.4	-35.5
10	1.0	1.0	0.5	**0.9**	1.0	1.0	0.5	0.5	1.0	116	125.7	107.0	92.7	170.9	86.7	108.6	135.8	58.1	101.5	62.7	42.8	10.5	-36.7
Steady State	1.0	1.0	0.5	**0.9**	1.0	1.0	0.5	0.5	1.0	116	125.4	106.9	91.8	171.3	86.5	108.3	136.5	57.9	101.3	62.5	42.8	10.3	-36.9

In all examples, the δ is shocked from 0 in the initial steady state to .5. Simulations subsequent to the baseline have one (highlighted) parameter changed. Output of both products are in units, not at market prices. All endogenous variables are indexed.

Table 3 Continued

Perfect Substitutability in Spiritual Sector $\varepsilon_s = \infty$

Period	ε_s	ε_y	δ	ϕ	Labor High-Tech	Labor Low-Tech	κ	γ	z	Nat Income	Corn Output	Spirituality Output	K	A	p	q	r	Wage Hearts	Wage Heads	Labor Share	% Heads Coding	Compensating Differential Heads	Compensating Differential Hearts
Initial Steady State	**∞**	1.0	0.0	0.5	1.0	1.0	0.5	0.5	1.0	100	100.0	100.0	100.0	100.0	-	100.0	100.0	100.0	100.0	75.0	66.7	0.0	0.0
1	**∞**	1.0	0.5	0.5	1.0	1.0	0.5	0.5	1.0	108	116.1	111.0	100.0	119.5	100.0	111.0	109.3	111.0	111.0	69.1	79.7	17.6	17.6
2	**∞**	1.0	0.5	0.5	1.0	1.0	0.5	0.5	1.0	107	111.3	112.9	97.9	137.7	93.3	106.3	118.6	106.2	106.2	68.0	52.0	14.1	14.1
3	**∞**	1.0	0.5	0.5	1.0	1.0	0.5	0.5	1.0	107	107.3	112.9	86.6	143.0	89.6	102.1	128.5	102.1	102.1	66.7	49.5	10.4	10.4
4	**∞**	1.0	0.5	0.5	1.0	1.0	0.5	0.5	1.0	107	104.7	112.8	79.0	145.7	87.3	99.3	135.8	99.3	99.3	65.3	49.5	7.8	7.8
10	**∞**	1.0	0.5	0.5	1.0	1.0	0.5	0.5	1.0	106	100.4	112.5	67.5	149.8	83.5	94.7	149.0	94.7	94.7	62.7	50.0	3.7	3.7
Steady State	**∞**	1.0	0.5	0.5	1.0	1.0	0.5	0.5	1.0	106	100.1	112.5	66.7	150.1	83.2	94.3	150.0	94.3	94.3	62.5	50.0	3.4	3.4

Strong Complementarity in Spiritual Sector $\varepsilon_s = 0.5$

Period	ε_s	ε_y	δ	ϕ	Labor High-Tech	Labor Low-Tech	κ	γ	z	Nat Income	Corn Output	Spirituality Output	K	A	p	q	r	Wage Hearts	Wage Heads	Labor Share	% Heads Coding	Compensating Differential Heads	Compensating Differential Hearts
Initial Steady State	**0.5**	1.0	0.0	0.5	1.0	1.0	0.5	0.5	1.0	100	100.0	100.0	100.0	100.0	-	100.0	100.0	31.5	100.0	75.0	43.9	0.0	-68.4
1	**0.5**	1.0	0.5	0.5	1.0	1.0	0.5	0.5	1.0	113	117.7	106.0	100.0	112.3	100.0	108.4	106.0	30.0	116.3	66.7	49.2	20.4	-68.9
2	**0.5**	1.0	0.5	0.5	1.0	1.0	0.5	0.5	1.0	110	116.7	108.6	96.5	143.6	89.3	108.8	122.0	38.6	101.6	65.3	38.4	8.3	-58.8
3	**0.5**	1.0	0.5	0.5	1.0	1.0	0.5	0.5	1.0	110	114.1	109.1	88.5	153.8	85.2	106.2	131.8	39.5	96.2	65.4	35.9	3.8	-57.4
4	**0.5**	1.0	0.5	0.5	1.0	1.0	0.5	0.5	1.0	109	112.0	109.2	82.6	157.6	83.1	104.0	138.1	39.1	93.6	64.6	35.4	1.5	-57.6
10	**0.5**	1.0	0.5	0.5	1.0	1.0	0.5	0.5	1.0	108	108.1	109.1	72.6	161.4	80.0	99.7	149.1	37.4	89.8	62.7	35.4	-1.7	-59.0
Steady State	**0.5**	1.0	0.5	0.5	1.0	1.0	0.5	0.5	1.0	108	107.8	109.1	71.9	161.6	79.7	99.4	150.0	37.3	89.5	62.5	35.4	-1.9	-59.1

Strong Complementarity in Corn Sector $\varepsilon_y = 0.8$

Period	ε_s	ε_y	δ	ϕ	Labor High-Tech	Labor Low-Tech	κ	γ	z	Nat Income	Corn Output	Spirituality Output	K	A	p	q	r	Wage Hearts	Wage Heads	Labor Share	% Heads Coding	Compensating Differential Heads	Compensating Differential Hearts
Initial Steady State	1.0	**0.8**	0.0	0.5	1.0	1.0	0.5	0.5	1.0	100	100.0	100.0	100.0	100.0	-	100.0	100.0	52.2	100.0	73.0	47.9	0.0	-47.8
1	1.0	**0.8**	0.5	0.5	1.0	1.0	0.5	0.5	1.0	110	113.1	108.2	100.0	111.3	100.0	107.5	106.3	53.1	113.6	66.7	53.3	18.4	-44.6
2	1.0	**0.8**	0.5	0.5	1.0	1.0	0.5	0.5	1.0	106	109.1	111.1	96.8	137.1	88.7	106.2	121.5	60.0	98.2	65.2	39.0	5.3	-35.7
3	1.0	**0.8**	0.5	0.5	1.0	1.0	0.5	0.5	1.0	105	104.4	111.7	87.9	143.1	83.7	102.4	131.1	59.3	92.2	65.0	35.7	0.0	-35.7
4	1.0	**0.8**	0.5	0.5	1.0	1.0	0.5	0.5	1.0	104	100.7	111.8	80.8	144.6	80.7	99.1	137.8	57.7	88.7	63.9	35.0	-3.1	-36.9
10	1.0	**0.8**	0.5	0.5	1.0	1.0	0.5	0.5	1.0	101	92.3	111.9	65.8	145.0	74.2	91.4	153.6	53.3	81.7	60.8	34.7	-9.3	-40.7
Steady State	1.0	**0.8**	0.5	0.5	1.0	1.0	0.5	0.5	1.0	101	90.9	111.9	63.7	145.0	73.3	90.2	156.2	52.7	80.6	60.2	34.7	-10.3	-41.4

Perfect Substitutability in Corn Sector $\varepsilon_y = \infty$

Period	ε_s	ε_y	δ	ϕ	Labor High-Tech	Labor Low-Tech	κ	γ	z	Nat Income	Corn Output	Spirituality Output	K	A	p	q	r	Wage Hearts	Wage Heads	Labor Share	% Heads Coding	Compensating Differential Heads	Compensating Differential Hearts
Initial Steady State	1.0	**∞**	0.0	0.5	1.0	1.0	0.5	0.5	1.0	100	100.0	100.0	100.0	100.0	-	100.0	100.0	45.5	100.0	80.0	54.6	0.0	-54.5
1	1.0	**∞**	0.5	0.5	1.0	1.0	0.5	0.5	1.0	127	136.0	97.5	100.0	119.0	100.0	114.6	100.0	45.8	130.5	64.1	65.0	27.4	-55.3
2	1.0	**∞**	0.5	0.5	1.0	1.0	0.5	0.5	1.0	123	144.1	99.5	94.6	163.6	95.0	120.9	100.0	53.6	124.0	66.7	56.8	23.5	-46.6
3	1.0	**∞**	0.5	0.5	1.0	1.0	0.5	0.5	1.0	123	147.3	99.9	86.4	182.7	94.1	122.2	100.0	55.3	122.8	69.0	55.0	22.7	-44.7
4	1.0	**∞**	0.5	0.5	1.0	1.0	0.5	0.5	1.0	123	148.7	100.0	81.0	191.5	93.9	122.4	100.0	55.6	122.5	69.6	54.7	22.6	-44.4
10	1.0	**∞**	0.5	0.5	1.0	1.0	0.5	0.5	1.0	122	150.0	100.0	75.1	199.9	93.8	122.5	100.0	55.7	122.5	70.0	54.6	22.5	-44.2
Steady State	1.0	**∞**	0.5	0.5	1.0	1.0	0.5	0.5	1.0	122	150.0	100.0	75.0	200.0	93.8	122.5	100.0	55.7	122.5	70.0	54.6	22.5	-44.2

Code Writing Productivity Shock z Jumps from 1 to 1.5

Period	ε_s	ε_y	δ	ϕ	Labor High-Tech	Labor Low-Tech	κ	γ	**z**	Nat Income	Corn Output	Spirituality Output	K	A	p	q	r	Wage Hearts	Wage Heads	Labor Share	% Heads Coding	Compensating Differential Heads	Compensating Differential Hearts
Initial Steady State	1.0	1.0	0.0	0.5	1.0	1.0	0.5	0.5	**1.0**	100	100.0	100.0	100.0	100.0	-	100.0	100.0	50.0	100.0	75.0	50.0	0.0	-49.9
1	1.0	1.0	0.5	0.5	1.0	1.0	0.5	0.5	**1.5**	137	157.8	105.3	100.0	176.2	100.0	116.4	132.7	52.9	128.1	59.5	58.8	35.2	-44.2
2	1.0	1.0	0.5	0.5	1.0	1.0	0.5	0.5	**1.5**	133	162.3	108.6	112.3	221.7	92.0	122.4	140.5	64.5	116.2	61.2	44.6	25.9	-30.1
3	1.0	1.0	0.5	0.5	1.0	1.0	0.5	0.5	**1.5**	133	162.4	109.4	112.5	234.0	89.8	122.7	144.2	66.7	113.0	62.9	41.1	23.3	-27.3
4	1.0	1.0	0.5	0.5	1.0	1.0	0.5	0.5	**1.5**	133	161.8	109.5	110.9	237.6	89.0	122.3	146.4	66.9	111.8	63.0	40.2	22.3	-26.9
10	1.0	1.0	0.5	0.5	1.0	1.0	0.5	0.5	**1.5**	132	160.2	109.5	107.0	239.9	88.0	121.0	149.7	66.3	110.4	62.6	40.0	21.0	-27.3
Steady State	1.0	1.0	0.5	0.5	1.0	1.0	0.5	0.5	**1.5**	132	160.1	109.5	106.7	240.1	87.9	120.9	150.0	66.2	110.4	62.5	40.0	20.9	-27.4

In all examples δ is shocked from 0 in the initial steady state to .5. Simulations subsequent to the baseline have one (highlighted) parameter changed. Output of both products are in units, not at market prices. All endogenous variables are indexed.

Figure 5
Comparing Four Case Studies

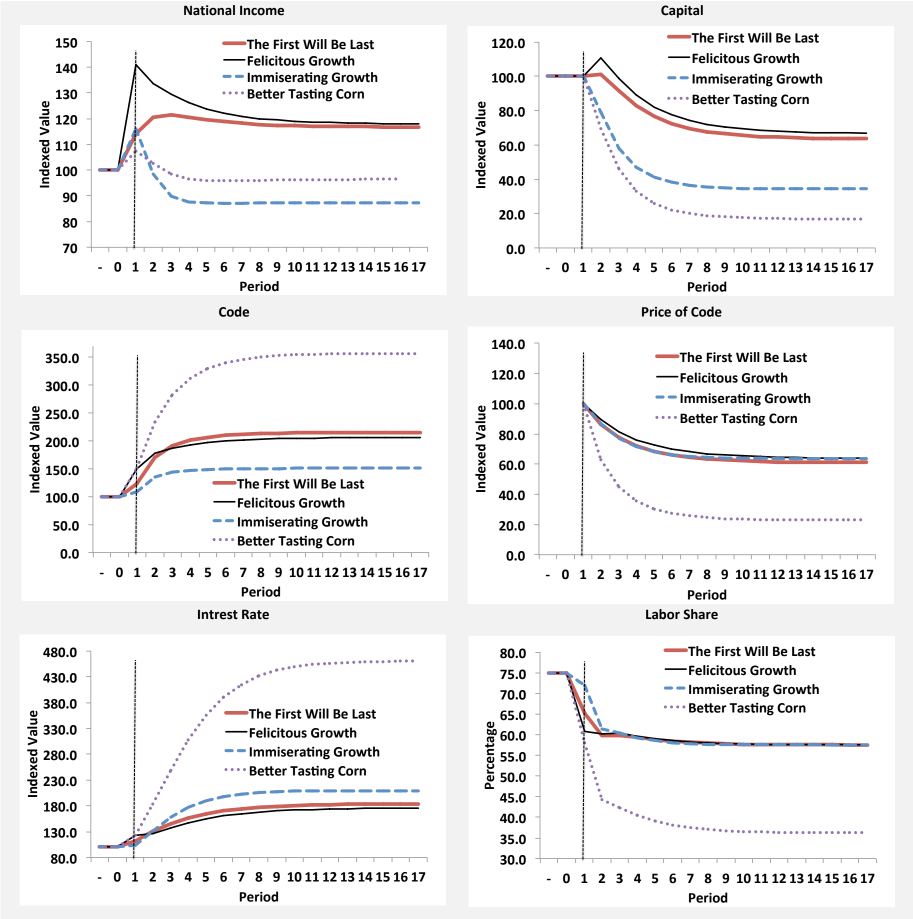

Figure 5: Transition paths from the first 4 cases presented (immiserating growth, etc.) superimposed.

Figure 6
Comparing National Incomes In Sensitivity Analysis

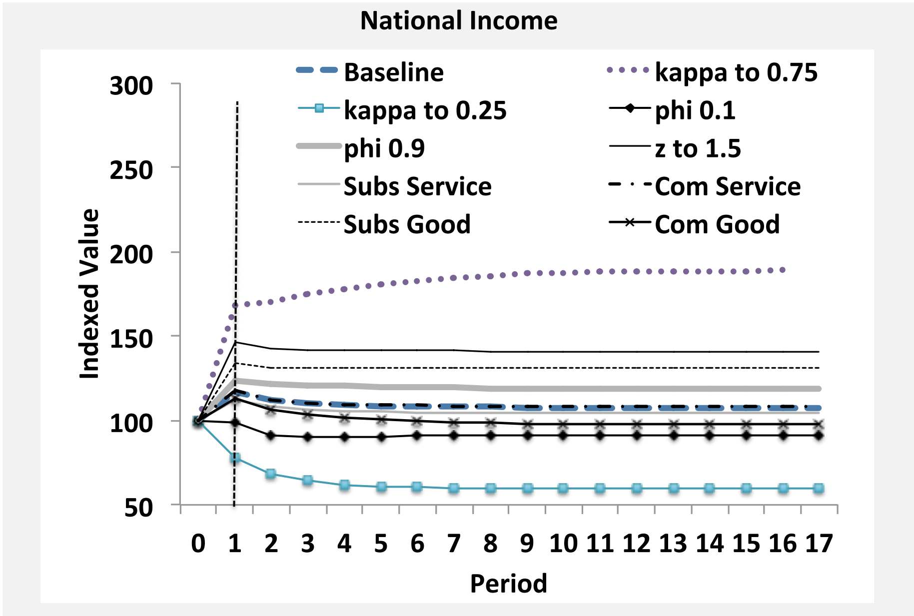

Figure 6: Illustration of the 10 sensitivity analysis cases superimposed. 'Subs' refer to cases in which the production technology of a sector is more substitutable. 'Com' refer to cases in which the production technology is more complementary.

Table 4
Parameters for Institutional Simulations

Model Parameter	Role	Value
ε_s	Elasticity in Service Sector	1
ε_y	Elasticity in Good Sector	1
γ	Service High-Tech Input Share Param.	0.5
α	Good Capital Input Share Param.	0.5
δ	Code Retention Rate	0 shocked to 0.25
ϕ	Saving Rate	0.5
H	High-Tech Worker Quantity	1
G	Low-Tech Worker Quantity	1
κ	Service Consumption Share	0.5
z	Code Writing Productivity	1
D_y	TFP in Good Sector	1
D_s	TFP in Service Sector	1
C	Firm Setup cost	.055
$\overline{A}$	Exogenous Free Code	.25

Table 4: This table gives parameter values for illustrations of the effects of a one-time, permanent increase in the depreciation rate, δ, from zero to .25 given different institutional settings.

Figure 7
Long-Run Compensating Differential for Alternative Saving and Code-Retention and Productivity Shocks

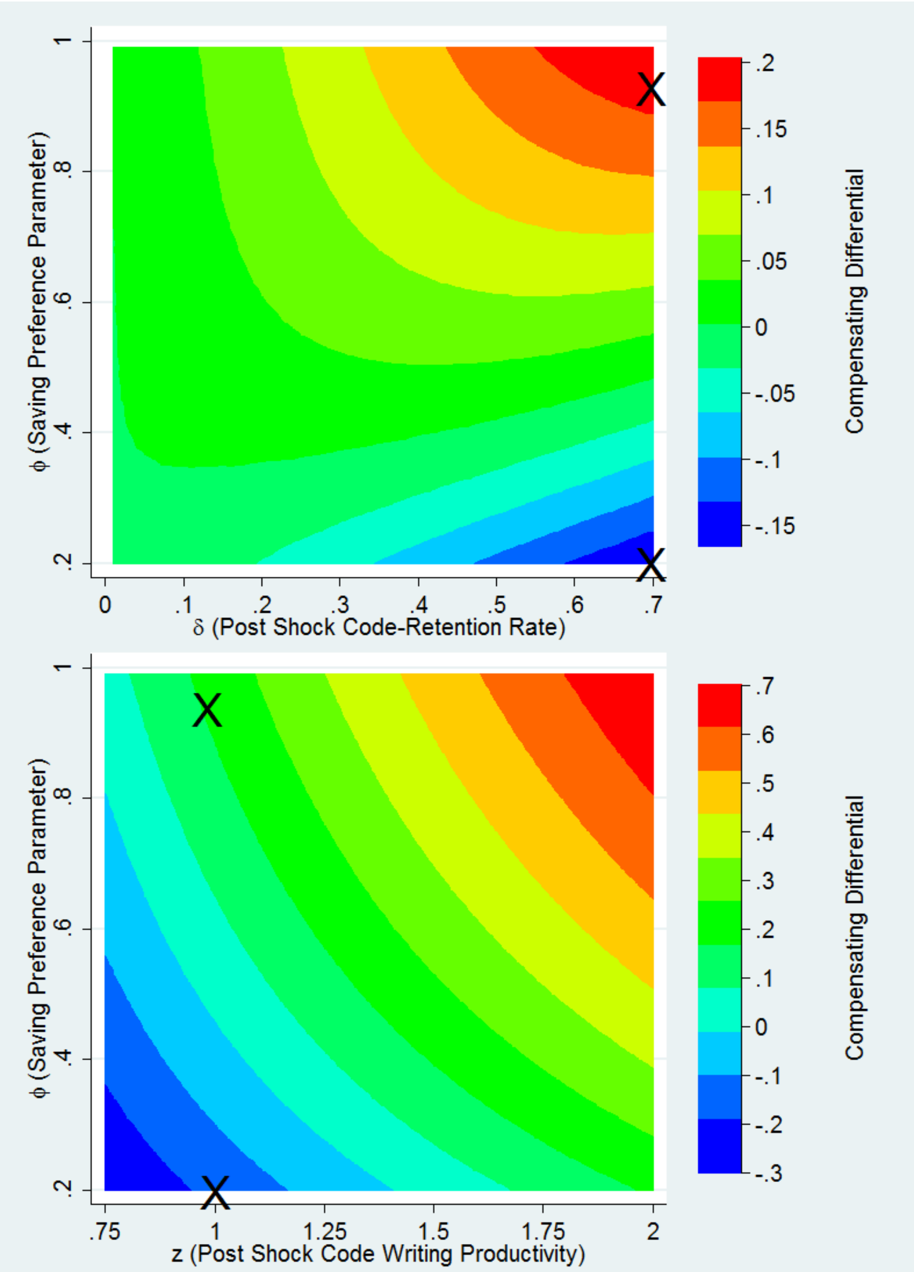

Figure 7: "Compensating Differential" references the percentage change in initial steady-state consumption that would be needed for the utility levels of workers to equal their respective post shock steady-state utility levels. Parameters not on axes are given in table 1. X's denote parameter combinations with transition paths discussed in the text.

Figure 8
Long-Run Compensating Differential for Alternative Saving and Elasticity of Substitution for Low and High-Tech Workers

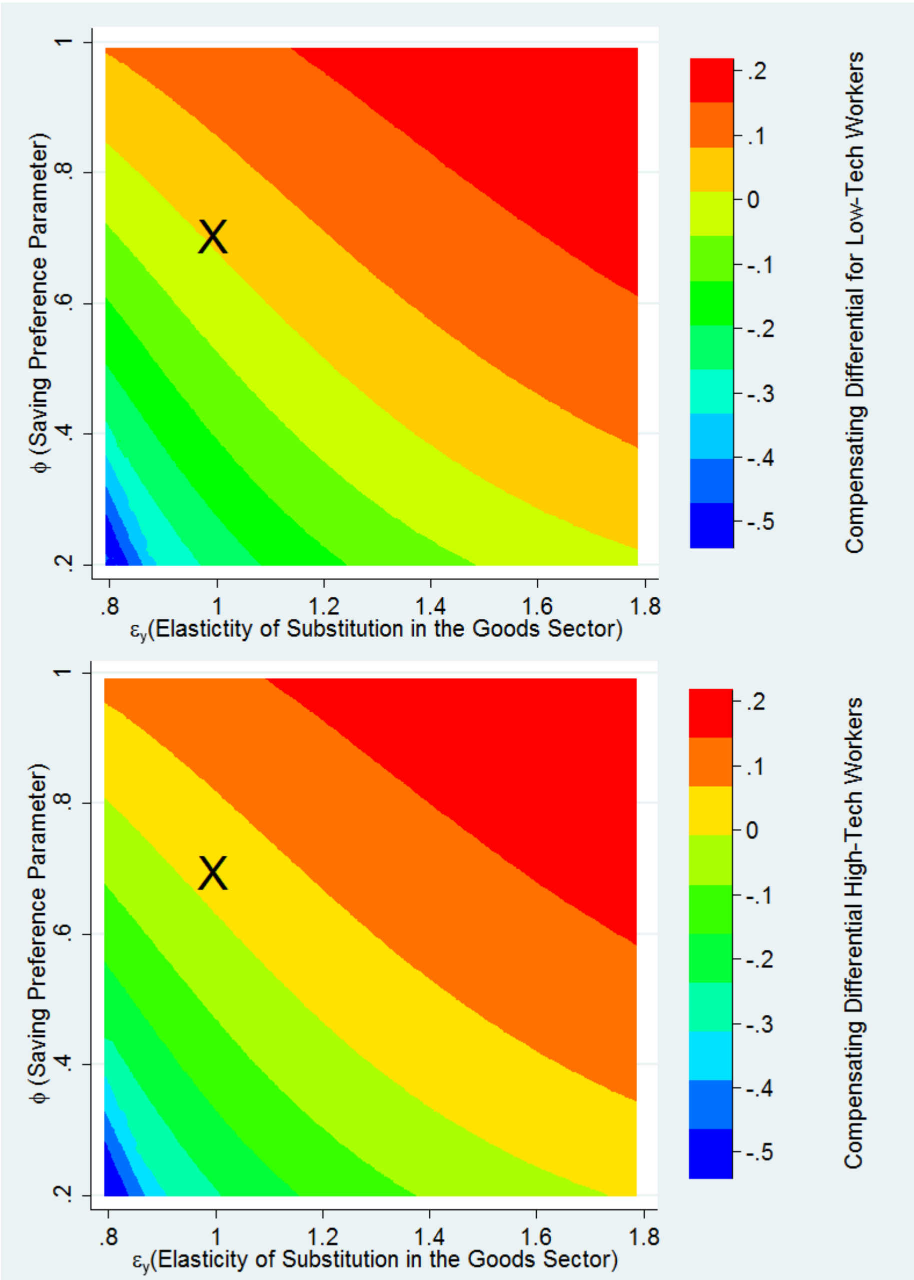

Figure 8: "Compensating Differential" references the percentage change in initial steady-state consumption that would be needed for the utility levels of workers to equal their respective post shock steady-state utility levels. Parameters not on axes are given in table 2. X's denote parameter combinations with transition paths discussed in the text.

Figure 9
Rival, Excludable (Private) Code

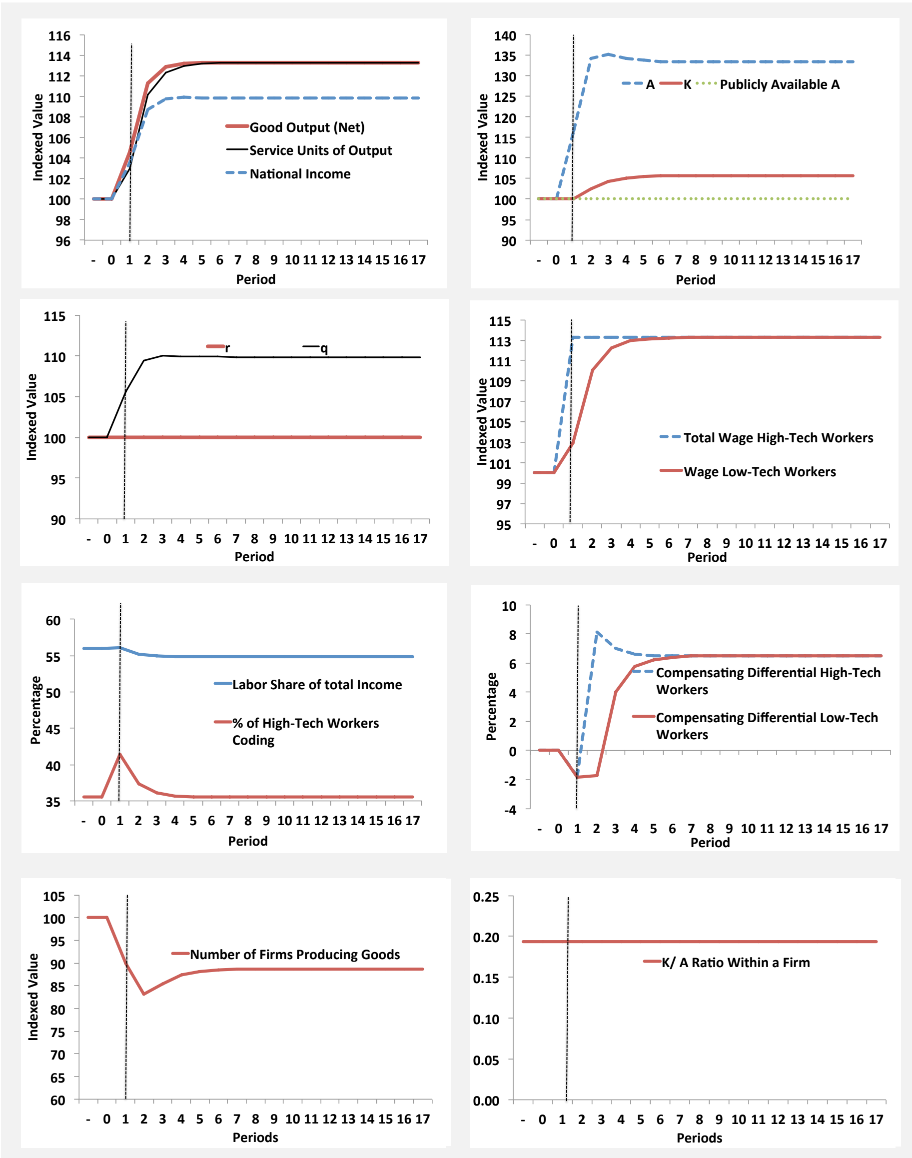

Figure 9: Transition paths based on Table 4. "Compensating Differential" references the percentage change in initial steady-state consumption that would be needed for the utility levels of workers to equal their respective transition utility levels. Service output is raw indexed output, not market value. Period 1 non-indexed prices in terms of the good are $r = 1.136$, $q = .382$, and $p = .117$.

Figure 10
Non-Rival, Non-Excludable (Public) Code

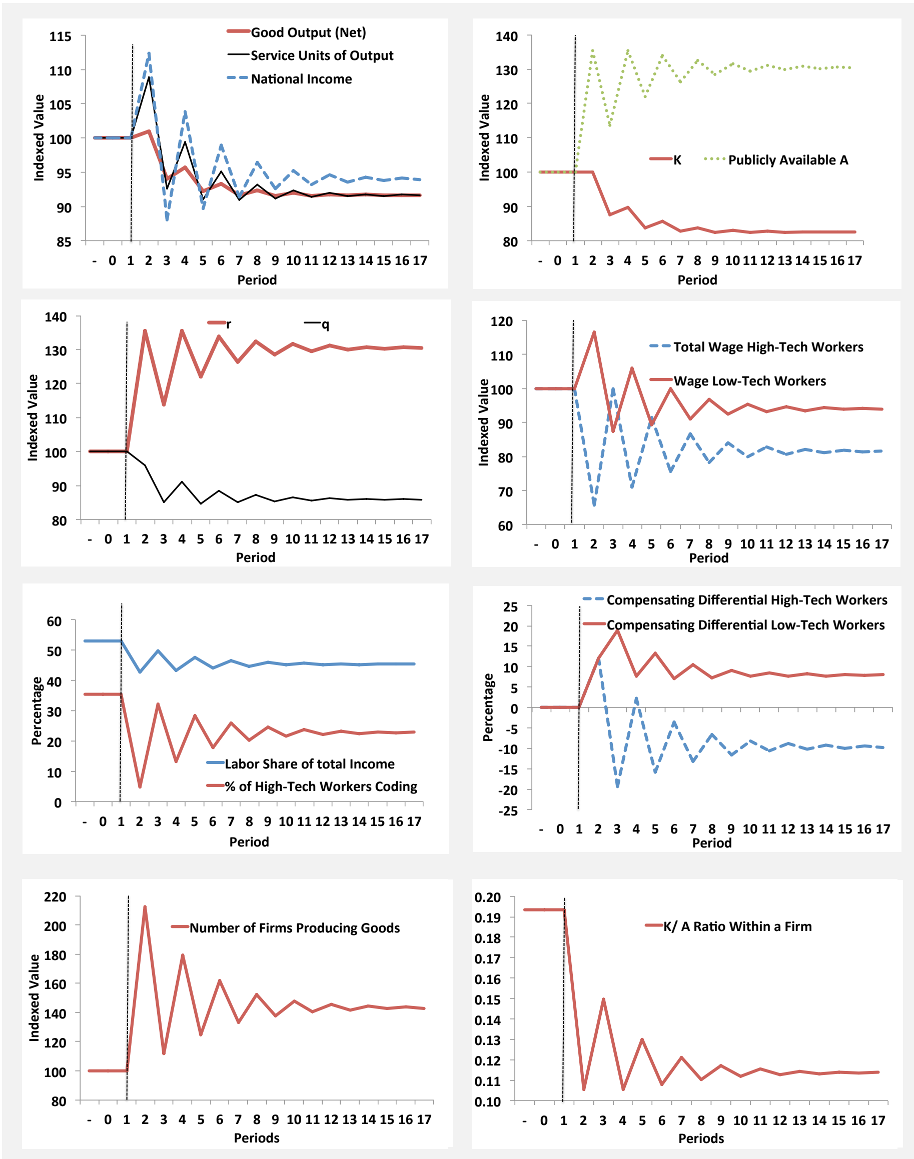

Figure 10: Transition paths based on Table 4. All parameters are identical to Figure 10 except equations are modified as detailed in the text. "Compensating Differential" references the percentage change in initial steady-state consumption that would be needed for the utility levels of workers to equal their respective transition utility levels. Service output is raw indexed output, not market value. Period 1 non-indexed prices in terms of the good are $r = 1.136$, $q = .353$, and $p = NA$.

Figure 11
Non-Rival, Excludable (Private) Code

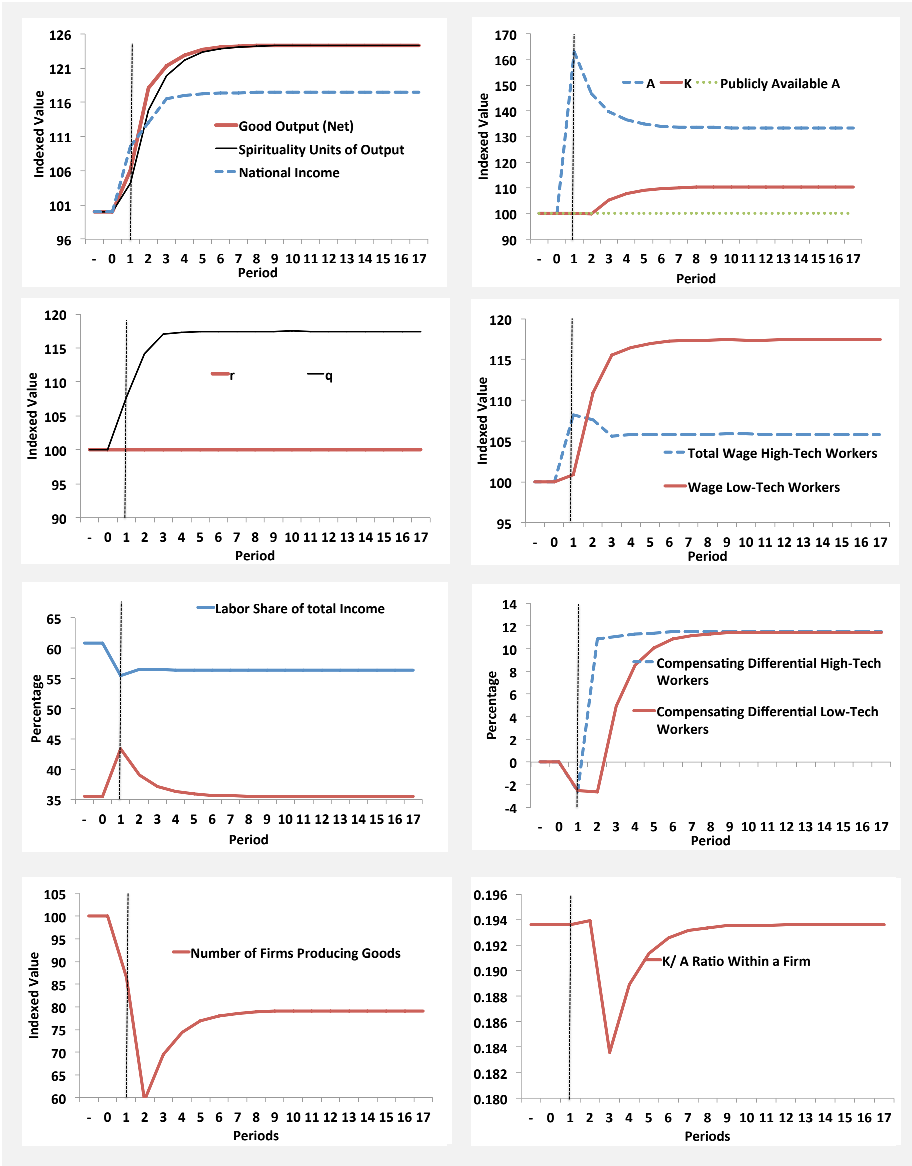

Figure 11: Transition paths based on Table 4. All parameters are identical to Figure 10 except equations are modified as detailed in the text. "Compensating Differential" references the percentage change in initial steady-state consumption that would be needed for the utility levels of workers to equal their respective transition utility levels. Service output is raw indexed output, not market value. Period 1 non-indexed prices in terms of the good are $r = 1.136$, $q = .393$, and $p = .164$.

Table 5
Parameters for the Endogenous Technology Extension

Model Parameter	Role	Value
γ	Service High-Tech Input Share Param.	0.5
α	Good Capital Input Share Param.	[0.3, 0.5]
δ	Code-Retention Rate	0 shocked to 0.6
ϕ	Saving Rate	0.9
H	High-Tech Worker Quantity	1
G	Low-Tech Worker Quantity	10
κ	Service Consumption Share	0.5
z	Code Writing Productivity	1
D_y	TFP in Goods	1
D_s	TFP in Services	1

Figure 12
Endogenous α

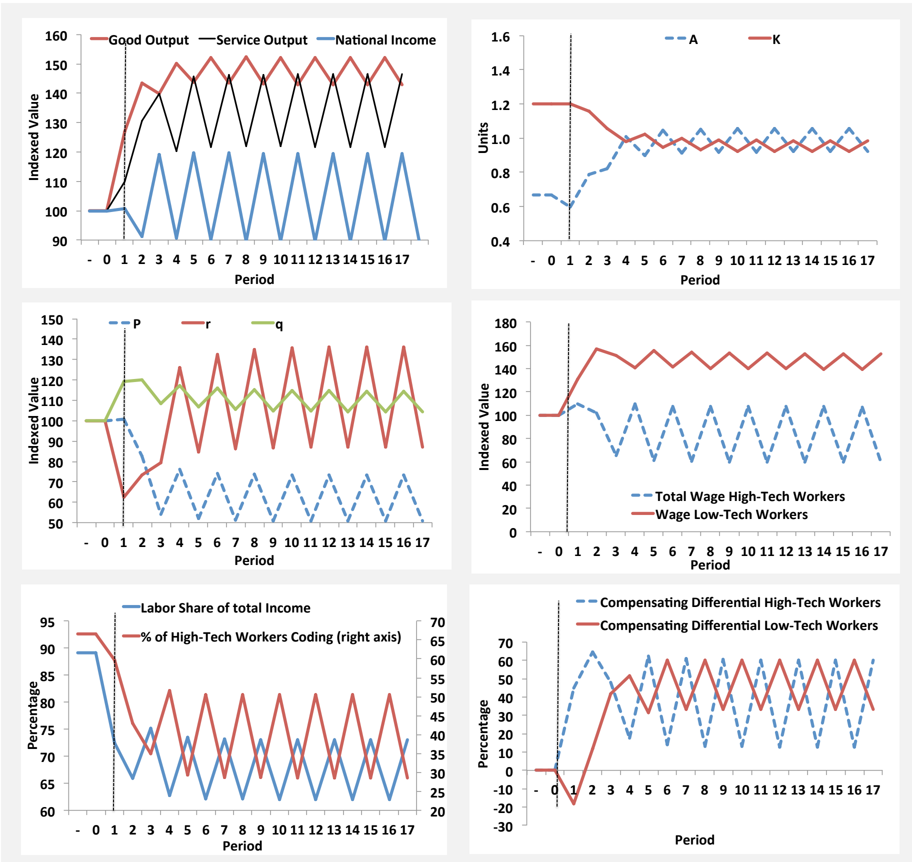

Figure 12: Transition paths based on table 3."Compensating Differential" references the percentage change in initial steady-state consumption that would be needed for the utility levels of workers to equal their respective transition utility levels. Service and goods output are raw indexed output, not market value. Wage of low-tech workers is indexed to the initial steady state wage of high-tech workers.

Figure 13
Three Measures of Labor's Share of Income in the U.S.

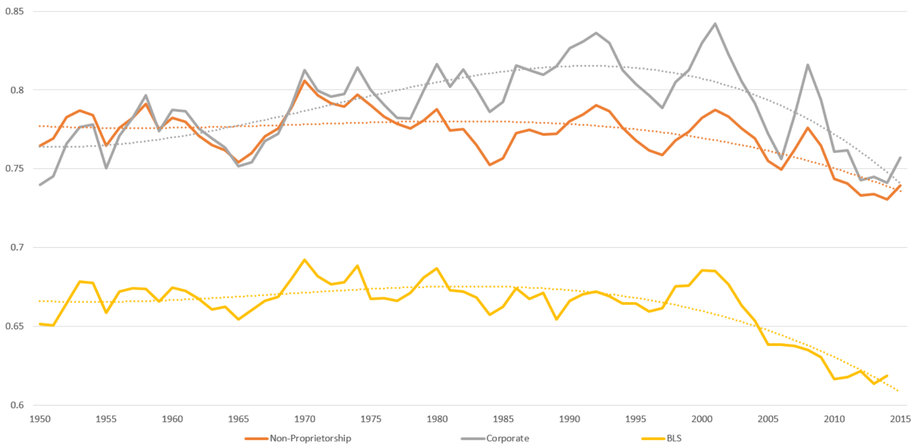

Figure 13: Three measures of the U.S. labor share. The orange curve, labor's share of non-proprietorship income, is calculated as employee compensation divided by national income at producer prices less proprietorship income (precisely, in NIPA table 1.12, lines 2/(1-25+26-18). The gray curve, labor's share of income in the corporate sector, is calculated as corporate employee compensation divided by corporate business income less corporate taxes net of subsidies (NIPA table 1.13 lines 4/(3-9)). The yellow curve is the BLS's measure of labor share in the private business sector (from the BLS multi-factor productivity series). Dashed lines are fitted third degree polynomials.

Figure 14
The Stock of Software and Software and R&D as a Share of U.S. Fixed Assets

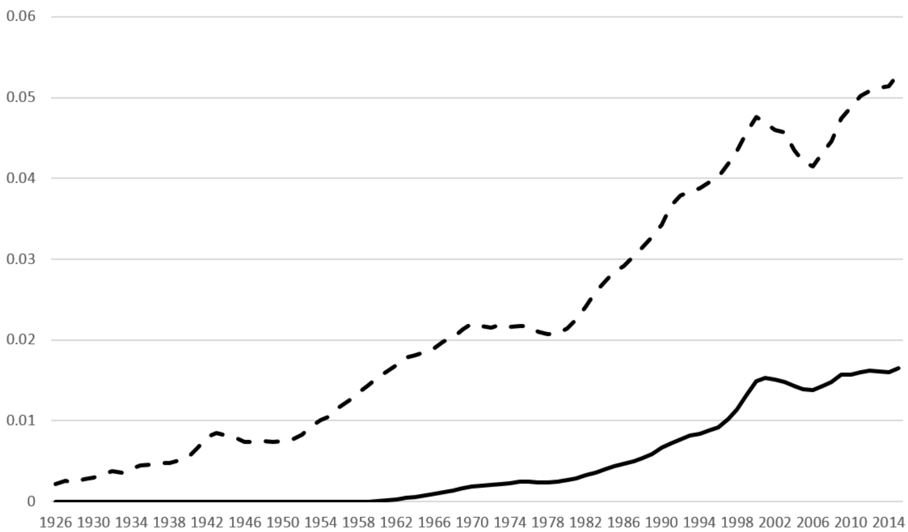

Figure 14: The stock of software (solid line) and software plus R&D assets (dashed line) as a share of total fixed assets (authors' calculation based on NIPA table 2.1).

Figure 15
U.S. Corporate Intangible Assets as a Share of U.S. Wealth

Figure 15: U.S. corporate intangible assets as a share of U.S. wealth is calculated by subtracting the net worth of U.S. corporations from their equity value. Net worth is the replacement cost of fixed assets, plus the market value of other assets, less liabilities apart from owners' equity. This imputed value of intangible corporate assets (goodwill) is divided by total U.S. wealth (authors' calculation based on Federal Reserve financial accounts series Z.1).