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Parental Actions and Siblings' Inequality

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1. Introduction

There is a famous story in the Talmud about two men who having lost their way in the desert find themselves in a precarious situation: if they share their reserves of water, they both will die, but if one keeps the water for himself, he may well survive the ordeal.[1] The morale is clear and appalling: when too much is at stake, fairness considerations become superfluous. Many poor parents, we shall argue, are confronted with a similar dilemma. If they divide equally their scant resources between their children, none of their children will acquire enough human capital to escape poverty. But if they channel most of their resources into a few of their children, the lucky bunch will be put in a much better position to escape a life in poverty. Parents, we shall also argue, often opt for the second option.

We argue that families tend to accentuate (rather than to attenuate) the overall inequality of income and earnings. The same view is shared by some of the previous literature in the economics of the family.[2] This literature has emphasized, in particular, that under some circumstances parents would reinforce the differences in ability of their children by investing disproportionately in the ablest ones (inequality averse parents would also try to countervail the ensuing differences in earrings via financial transfers, but that's another issue).

The theoretical framework presented in this paper is different from the previous literature in three major respects. First, we assume no differences in ability (so reinforcement of innate abilities is not a crucial issue in our model). Second, we assume that in some range the returns to human capital investments increase with the level of human capital. And third, we assume that parental decisions are driven mainly by efficiency considerations (fairness may still be present but it is certainly dwarfed by efficiency in our model).

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[1] See Talmud, Masechet Baba Metzi'a, daff samech-bet. We want to thank Shmuel Shey for finding this reference.

[2] See Becker and Tomes (1976), Behrman, Pollak and Taubman (1982), and Sheshinski and Weiss (1982), and Mulligan (1997).

The model presented in the paper generates important implications concerning how the allocation of resources between children varies across income groups. In the model, poor and middle-income families will tend to channel their resources into a few children whereas rich families (and perhaps very poor families as well) will tend to allocate resources more evenly. As a result, poor and middle income families will tend to generate more inequality relative to richer (and poorer) families.

Testing our model is not simple because we do not observe all forms of parental investments in their children. To be sure, we do not even have good information about the most obvious parental transfers to children (bequests and intervivos transfers). So we have to resort to indirect ways to test our model. Here we use the changes of sibling's earnings inequality across wealth groups to partially test the relevance of our model. The data seems consistent with the model in that we observe greater sibling's earnings inequality among the families at the bottom of the wealth distribution who, arguably, are precisely the ones confronted with dilemma mentioned above.

The rest of this paper is as follows. Section 2 presents the model, Section 3 presents a preliminary test of the model based on differences on earned incomes between siblings. Finally, Section 4 provides some concluding remarks and policy implications.

2. The Model[3]

Consider a small open economy inhabitated by overlapping generations of altruistic individuals. In this economy production of the same (and only) good is performed in either of two sectors. The first sector (S from here on) uses both skilled labor and physical capital. The second sector (U from here on) uses only unskilled labor. We shall assume, for the sake of simplicty, that technology in the S sector is described by a Cobb-Douglas production function, and that technology in the U sector is described by a linear production function. Accordingly,

$$Y_s = AK^a L_s^{1-a}, \qquad (1)$$

[3] This section borrows some of its structure from Galor and Zeira (1993). These authors, however, are more interested in the role of the family in the transmission of inequality while we are more interested in resource allocation within the family. Empirical evidence regarding the transmission of inequality across generations can be found in Becker (1981) and more recently in Gaviria (1998).

$$Y_u = w_u L_u . \tag{2}$$

Firms in this economy operate in perfectly competitive markets and can borrow and lend at the fixed world interest rate, r. We shall assume, in addition, that there is neither depreciation of capital nor adjustments cost of investments. Wages in the S and U sectors can then be written as

$$w_s = A(1-a)\left(\frac{aA}{r}\right)^{\frac{a}{1-a}}, \tag{3}$$

$$w_u = w_u . \tag{4}$$

So here wages are completely determined by a few technological parameters and the world interest rate.

Individuals live for two periods. In the first period they can either work for the U sector (earning w_u) or spend their time acquiring the skills that will later enable them to work in the S sector. We assume that all individuals have the same ability and that they all have to invest in human capital in the first period to be able to work in the S sector during the second period (i.e., there are no natural-born geniuses in the model). Investments in human capital are indivisible and equal to *h* for everybody. Interestingly, this assumption entails the presence of increasing returns to scale in the accumulation of human capital and, as we shall see below, it underlies some of the key results of the model.

In the model, parents have two children and derive utility from their children's average lifetime income.[4] We assume, in particular, that the utility function can be written as

$$U = U(C, \frac{I_1 + I_2}{2}) \tag{5}$$

where C is consumption in the second period and I_i is the lifetime income of child i. Equation (5) implies, among other things, that parents are not averse to inequality in that they will always favor efficiency over fairness when allocating resources to their children. It is worth noting, however, that because the efficiency considerations are so

stark in this setting, the main conclusions below will still hold even if parents do take into account fairness considerations.

Equation (5) is maximized subject to the constraint that consumption plus financial transfers to both children must be equal to lifetime income:

$$I = C + b_1 + b_2, \tag{6}$$

where I is lifetime income in terms of consumption and b_i are financial transfers (bequests for short) to child i. Thus, each individual has to decide not only how much to consume and how much to bequest, but also how to distribute her estate. Here we will focus mainly on the latter decision.

Parents transfer resources to their children who in turn make the decision as to whether or not to invest in human capital. Of course, if parental transfers fall short of education expenses, children will have to borrow in order to invest in human capital. We will assume that individual borrowers pay an interest rate *i* that is higher than the world interest rate, *r*. This may result from either higher monitoring costs or the uncollatarizable nature of human capital investments. We will assume, in addition, that the following two inequalities hold:

$$w_u(2+r) > w_s - h(1+i) \tag{7}$$

$$w_u(2+r) < w_s - h(1+r) \tag{8}$$

The first inequality rules out the trivial case in which investing in human capital is always optimal, and the second rules out the likewise trivial case in which investing in human capital is never optimal.

We can now study the decision as to whether or not to invest in human capital. Consider an individual who inherits *b* from her parents in the first period of her life. A first point follows directly from the assumptions above: if bequests are greater than education expenses (i.e., if b>*h*), investing in human capital will be always optimal and lifetime income will be given by

[4] We abstract from fertility considerations in this paper. For a comprehensive analysis of fertility in this context see Dahan and Tsiddon (1998).

$$I_s = w_s + (b - h)(1 + r) \tag{9}$$

If, on the other hand, parental transfers fall short of education expenses, individuals have to borrow in order to invest in human capital. They will do so only if the S-U wage differential is high enough to compensate the financial expenses and opportunity costs of human capital investments. More precisely, *poor* individuals will get an education if the following inequality holds

$$w_s + (b - h)(1 + i) > w_u(2 + r) + (1 + r)b \tag{10}$$

where the left-hand-side term is the lifetime income of a person who borrows to finance her education and the right-hand-side term is the lifetime income of a person who does not invest in human capital. From (10), we can derive the minimal level of bequests that will compel an individual to invest in human capital, *f:*

$$f = \frac{1}{i - r}(w_u(2 + r) + h(1 + i) - w_s). \tag{11}$$

As shown, *f* increases with both the cost of education and the extent of capital market imperfections, and decreases with the S-U wage differential.

Intra-household Allocation

Two remarks are worthy of mention before studying the allocation of resources within the family. First, it is impossible in this framework to separate the decision of how much to bequest (b_1+b_2) from the decision of how to bequest (b_1 vs. b_2). Second, the main conclusions below should carry over to families with more than two children. More general cases, however, provide little extra insight at the cost of much greater complexity.

We use a backward induction argument to solve the intra-household allocation problem. The idea is simple: first, we substitute total bequests ($B = b_1+b_2$) for average children's income ($\bar{I} = (I_1+I_2)/2$) in the budget constraint, and then we solve the optimization problem in the standard fashion. By doing this, we are able to separate the optimization problem in two stages while preserving the simultaneous nature of the consumption and intra-household allocation decisions.

Figure 1a shows the average children's income (¦) as a function of total bequests (B) for two opposite allocation rules: the first rule depicts the case in which all parental transfers go to one child while the second case depicts the case in which parental transfers are split evenly between the two children. These two rules are important because they encompass all other cases in that no other allocation yields a higher (or a lower) average children's income (see Appendix 1).

We distinguish three different segments in Figure 1. In the first segment, total bequests are less than *f* and hence investments in human capital are not feasible. Here both children enter the U sector and both make w_u in the two periods of their lives regardless of their parents' decisions. In the second segment, B is greater than *f* but less than *h+f*. In this segment all optimal allocations (and, as explained in Appendix 1, it might be several for each value of B) entail an unequal distribution of earnings: one child gets an education and earns w_s, one child enters the U sector and makes w_u. The point here is that parents, realizing that equal splitting will deny both children the opportunity of "making it big", will opt for transferring most resources to a single child. This is, of course, reminiscent of the parable of the desert that we presented in the introduction. In the third segment, total bequests are greater than *h+f*, and optimal allocations (and again it might be several of them for each B) entail both children getting an education and hence making w_s in the second period of their lives.[5]

The previous analysis makes it clear that at least for some levels of total bequests, parental actions may bring about large differences in earned incomes between siblings. This analysis, however, is still incomplete because we are yet to model how parents choose the total level of bequests. Doing this is not difficult because we already know the maximum value of ¦ that can be achieved for each value of B (this is, of course, given by the envelope of the two allocations rules depicted in Figure 1a). The key observation here is that we can use the envelope to rewrite the budget constraint in terms of consumption and ¦ (the very same arguments of the utility function), and hence to solve the optimization problem at hand in the standard fashion (obviously, we don't have to worry

[5] Figure 1b shows the wage of each child as a function of total bequests: as discussed above, wages are the same for both small and large levels of total bequests and quite distinct for intermediate levels.

about allocation problems anymore because they are already incorporated in the envelope curve).

Figure 2 presents the budget constraint along with the optimal choice of consumption and average children's outcomes for various levels of lifetime income. Before looking at the sequence of graphs, there are some points to be made. First, all budget constraints are turned-around versions of Figure 1a (this is, of course, the reason for the kinks). Second, the thin segment of the budget constrain represents the areas where the implied level of total bequests is associated with unequal earnings (i.e., one kid goes to college, the other goes to work). And third, preferences are the same throughout the sequence: income is the only variable that is changing.[6]

We can now study the sequence of graphs of Figure 2. Let us first focus on the upper left corner. Here lifetime income is low; barely enough to cover the education expenses of one kid. In this case moderately altruistic parents will transfer a few resources to their children but these resources will be, in all likelihood, less than *f*, meaning that neither of the kids will invest in human capital. This is exactly the situation depicted in the first segment of Figure 1a. Let us now move to upper right corner. Here income is higher than before–twice the value of education expenses. More importantly, the optimal point lies now on the thin segment of the budget constraint, meaning that parents will choose an unequal distribution of bequests (that in turn will result in highly unequal earnings). This corresponds to the second segment of Figure 1a. Turning to last two graphs, the optimal point lies again on the bold segment of the budget constraint; parents have enough income (given their altruism) to put both kids through school and so earnings are equal for both kids. This is, of course, the situation depicted in the third segment of Figure 1a.

A precise empirical implication concerning how sibling's earnings inequality varies with family wealth can be derived from the previous analysis (as mentioned earlier, this is important because it provides an observable dimension within which the model can be tested). By and large, the model predicts that siblings' earnings inequality should be the greatest among middle-class and moderately poor families: those who can

6 It is important to note that non-altruistic individuals will always consume all their income. Both of their children will then have no choice but entering the U sector. This situation corresponds to the point farthest to the right in the budget constraint.

make it happen for some but not for all children. In these families some children (those hand-picked by the parents) will "make it big" while others will inevitably fall behind. Siblings' inequality, on the other hand, should be the lowest among the rich (and perhaps also among the very poor): those who can make it happen for everybody (or for nobody).

These predictions are different from the predictions of the well-known Becker and Tomes' (1976) model. In this model, parents are inequality averse and children differ in ability with the more able enjoying higher rates of return to human capital investments. In this setting, poor parents will spread human capital investments more evenly among their kids since they both dislike inequality and lack resources to compensate differences in earnings. Wealthy parents, on the other hand, will disproportionately invest in human capital on the ablest kids and will compensate the others via financial transfers. In short, the Becker-Tomes' model predicts, among other things, that siblings' earnings inequality should increase with family wealth.

We can sum up the previous discussion by saying that whereas our model predicts that siblings' inequality in earned incomes should be relatively higher among middle-class (and especially lower-middle class) families, for only these families are confronted with the two-kids-one-life-saver dilemma, the Becker-Tomes model predicts that siblings' inequality in earned incomes should be relatively higher among rich families, for only rich families can reinforce the innate differences of their kids without worry too much about the ensuing inequality.

3. Empirical Analysis

In this section we use a sample of families drawn from the Panel Study of Income Dynamics (PSID) to test the predictions of our model concerning the relation between siblings' earnings inequality and family wealth. Our sample includes the children of the original PSID families that meet the following selection criteria: (1) they must be older than 24, (2) they must have at least three income entries in the period 1985-89, and (3) they must have at least a sibling that meet criteria (1) and (2). The sample comprises 1,722 children distributed in 589 families. The average years of education of the children in the sample is 13.2, and their average age in 1986 was 29.6.

Throughout the analysis we use average earnings to approximate lifetime earnings (the relevant magnitude according to the model). Specifically, we compute earnings as the average annual labor income for the period 1985-89 (all values were converted to 1984 dollars using the consumer price index). For wealth, we use net worth as reported by parents in 1988. It should be noted that questions about wealth in the PSID are sparse. In particular, the questionnaire has never included separate questions for the different forms of wealth, which may explain the lumpy structure of the data (e.g., 15 percent of the families reported a net worth of US$ 50,000).

A definitive test of the model would require detailed information about parental investments on each child. This information is, to the best of our knowledge, not available. Fortunately, the model can still be tested by looking at how siblings' earnings inequality varies across the three segments depicted in Figure 1a. This, however, raises a new difficulty: placing families into these segments is not simple because we don't have precise information about the empirical counterparts of *h* and *f*. To circumvent this problem, we first divide the sample in a few groups according to family wealth, and then we investigate the behavior of siblings' inequality across the wealth groups. This partially solves the problem but creates some uncertainty as to how the wealth groups relate to the theoretical segments (see the discussion below).

Figure 3 summarizes the evidence concerning the relationship between siblings' earnings inequality and family wealth. Two remarks are in order before tackling the evidence. First, we use the coefficient of variation to measure inequality. Second, we divide families in the following six groups: (1) families who reported negative net worth; (2) families who reported positive net worth smaller than US$5,000; (3) families who reported a net worth greater than US$5,000 but less than US$ 50,000; (4) families who reported a net worth of exactly US$50,000; (5) families who reported a net worth greater than US$50,000 but smaller than US$150,000; and (6) families who reported a net worth over US$150,000. All results below are robust to small changes in the groups' definitions.

Figure 3a depicts the changes of siblings' earnings inequality across wealth groups. Inequality is measured here over all siblings: male and females older than 24. As shown, inequality is stable across wealth categories (a slight bump for wealthy families is

noticeable, but it is not statistically significant). This result, however, shouldn't be taken too literally since mixing men and women is known to be problematic in this context (see, for example, Behramn, Pollak, and Taubman, 1995). Indeed, earnings of women are so sensitive to marital and childbearing decisions that they can hardly be interpreted as the reflections of parental investments in human capital. On these grounds, we drop the women of the sample and perform the same analysis. The results are shown in Figure 3b. There is now more variation across wealth groups, and more importantly, within-family inequality is now substantially (and statistically significant) higher for poor families; the small bump for rich families is still noticeable but is again non-significant.

Figure 3c presents perhaps the cleanest evidence concerning the interplay between family wealth and intra-household inequality. Here we focus exclusively on two-son families. Note that by restricting the analysis to these families, we eliminate all random disturbances stemming from family size, birth order and gender.[7] The results are now more in line with the theoretical discussion. First, families at the bottom of the wealth distribution are more unequal, which may be interpreted, at least partially, as a reflection of the make-it-with-one-or-break-with-all dilemma (the differences are now statistically significant). Second, families at the top of the distribution are also relatively more unequal (though the differences are non-significant), which may be interpreted as a reflection of the educate-the-able-and-compensate-the-other phenomenon.

Note that in the previous discussion we implicitly assume a mapping from the wealth groups to the theoretical segments of Figure 1a. We assume, in particular, that the families in the two first wealth groups can be mapped onto the second segment of Figure 1a. This reflects our belief that the presumptions underlying the first segment of Figure 1a (families are so poor that investments in human capital are completely out of question) hardly apply in this case. One reason is that we have imposed several restrictions that may have eliminated the poorest families from our sample.[8] A second reason may be that in the United States the public provision of primary (and even secondary) education is

[7] We don't control for age because we don't expect this variable to change systematically with family wealth (indeed, age barely changes across wealth groups in our sample).
[8] The average annual income of the parents in the whole sample is below US$ 20,000 whereas in our most-preferred sample the same value is US$22,400. For education, the corresponding figures are 10.8 and 11.6, respectively.

almost universal, and hence the cost of investing in basic skills is not more than the forgone wages of menial occupations (i.e., *h* and *f* are low in the United States).

To sum up, we can assert that the empirical results are roughly consistent with the idea that poor families, out of necessity, may channel their investments in human capital into a few of their kids. There have been numerous previous attempts to study allocation of resources within the family. Most of them have looked at financial transfers from parents to children (bequests or intervivos transfers) with the purpose of establishing whether or not parents use these transfers to countervail earnings differences.[9] The evidence in this regard is not only inconclusive but also suffers from the inherent difficulties in measuring parental transfers. Interestingly, these difficulties are circumvented by our approach of using intra-household differences in children's outcomes to test the various models of resource allocation within the family

4. Concluding Remarks

Although the role of the family in the transmission of inequality has long been emphasized in the social mobility literature, the role of the family in the creation of inequality has been somewhat overlooked. In this paper, we present a model in which families play a central role in the creation of inequality. In the model, efficiency considerations lead poor families to channel their scarce resources into a few of their children, which obviously increases the overall level of inequality. The empirical evidence presented here is broadly consistent with the model. A definite test is, however, very difficult since the model deals mainly with unobservable variables (e.g., parental investments).

Interestingly, the model suggests that differences between rich and poor families are not limited to average incomes: the structure of inequality within the family also seems to change across income groups. Although more research is needed to confirm this

[9] Cox and Rank (1992), and McGarry and Schoeni (1995) find that children with smaller earnings received larger gifts and bequests. Menchik (1980) and Wilhelm (1996) find, on the other hand, that equal division is by far the dominant allocation of gifts and bequests. Menchik, in particular, finds that equal sharing occurred in 108 of 173 two-children families drawn from probate records of the Inheritance Tax Division of the Connecticut State Tax Department (the sample includes only net estates above $40,000). Tomes

pattern, the evidence seems to indicate that siblings' differences (and, in particular, those due to parental actions) are an important element in the structure of overall inequality.

Although we limit our empirical analysis to the United States, we have various reasons to believe that the basic intuition underlying our model is even more relevant for developing countries. To start, capital market imperfections are more important in developing countries. In addition, the returns to finishing either high school or college (and finish is the key word here) are much higher in developing countries, which means that efficiency considerations are more likely to override equity considerations in these countries. And last, access to education is much more limited in developing countries: good education, in particular, is severely rationed and funded mainly with family resources in these countries.

At first glance, the policy recommendations of this paper seem feeble. After all, if the family itself is an agent in the generation of inequality, policy options seem rather limited. The main message of the models is, however, positive; namely, policy interventions that ease capital constraints and increase access to education may have a multiplier effect since they not only may close the gap between rich and poor families, but also may close the gap between siblings in poor families. Besides, if increasing returns in the accumulation of human capital are indeed relevant, we may also expect big gains from the same type of policies. In short, family *intermediation* notwithstanding, public policy can still play a fundamental role in the reduction of inequality.

(1981), for his part, finds lower rate of equal sharing -around 50 percent. He uses a sample drawn from the Cleveland probate records that covers all the estate sizes.

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Figure 1a. Average Children's Income Vs. Total Bequests

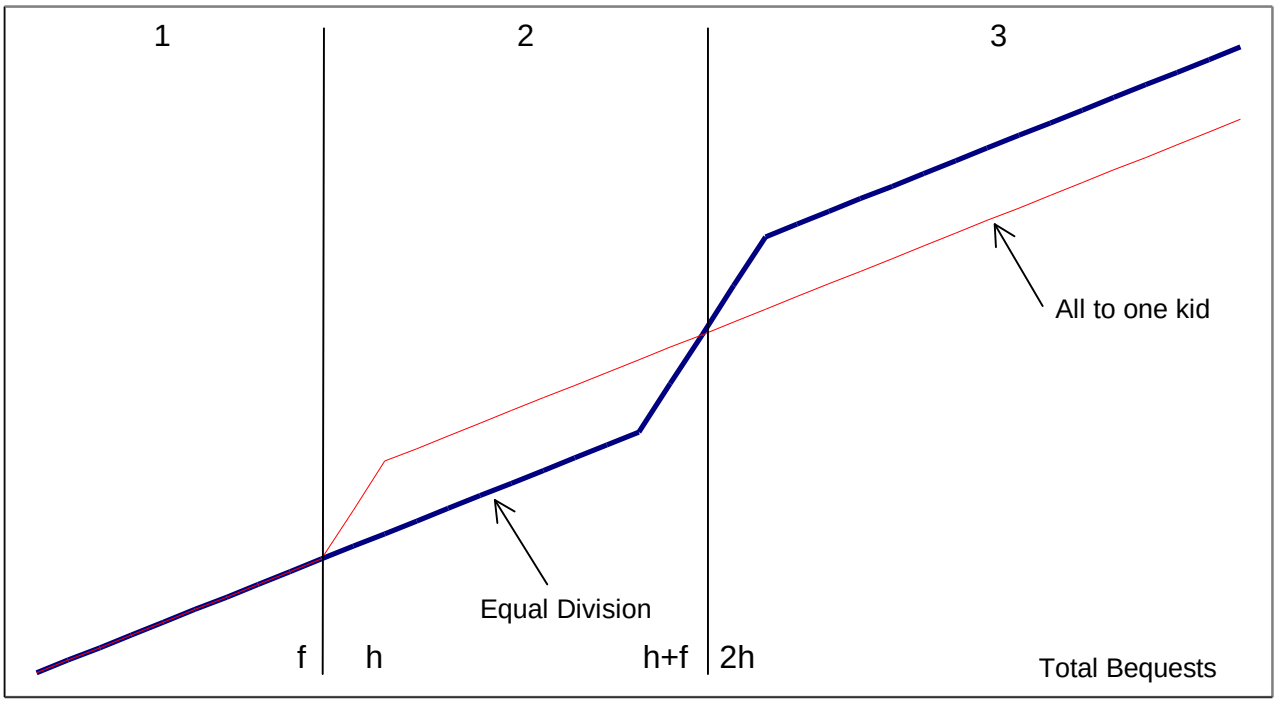


Figure 1b. Wages Vs. Total Bequests

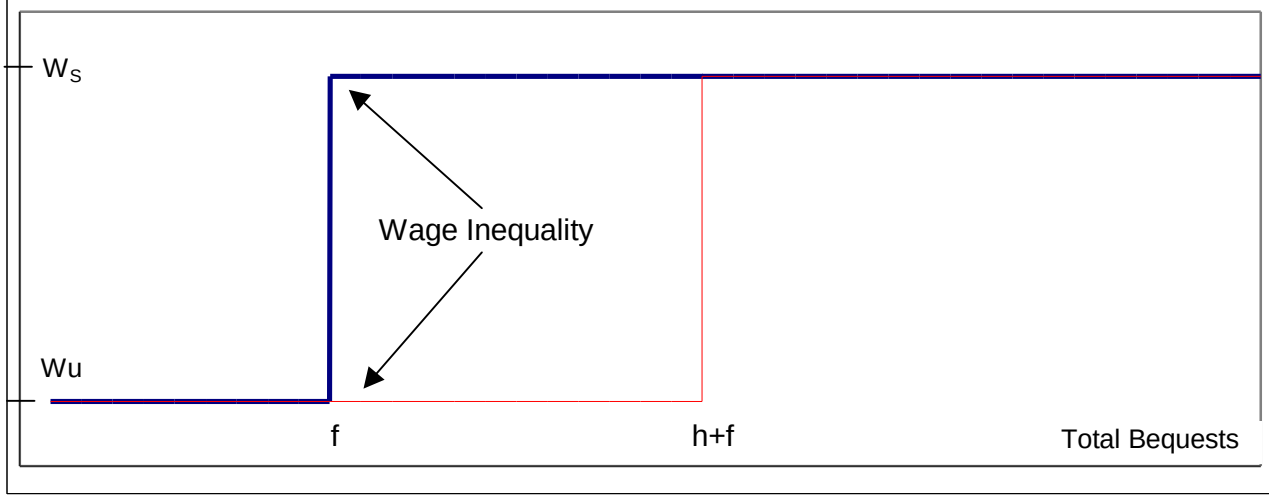
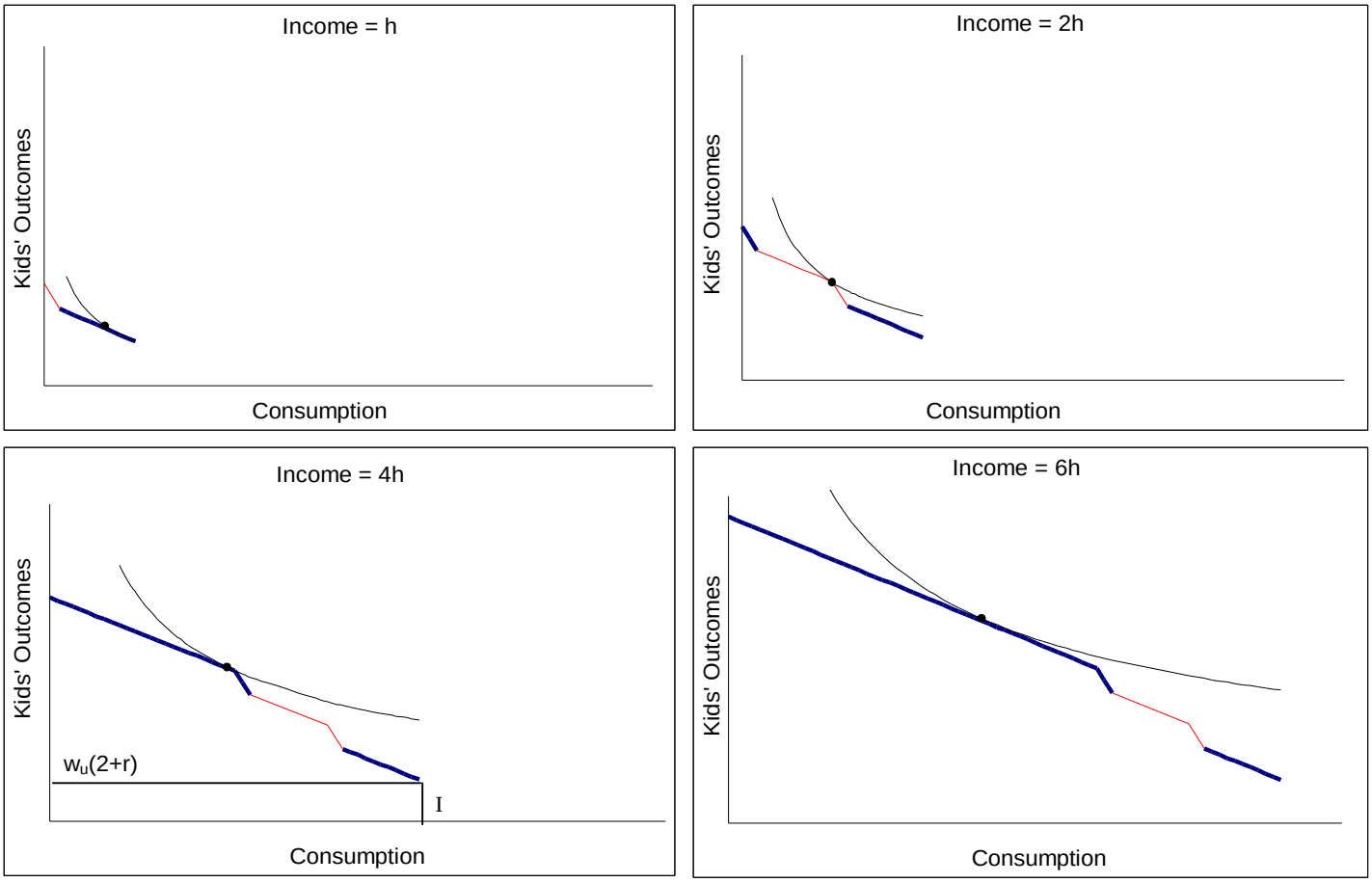


Figure 2. Optimal Choices for Various Levels of Lifetime Income



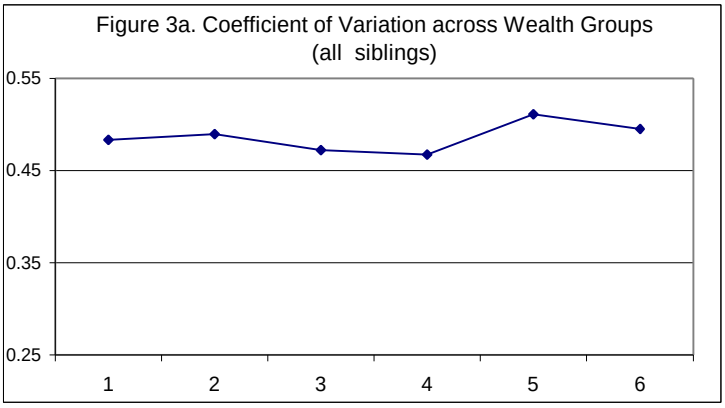


Figure 3a. Coefficient of Variation across Wealth Groups
(all siblings)
0.55
0.45
0.35
0.25
1
2
3
4
5
6

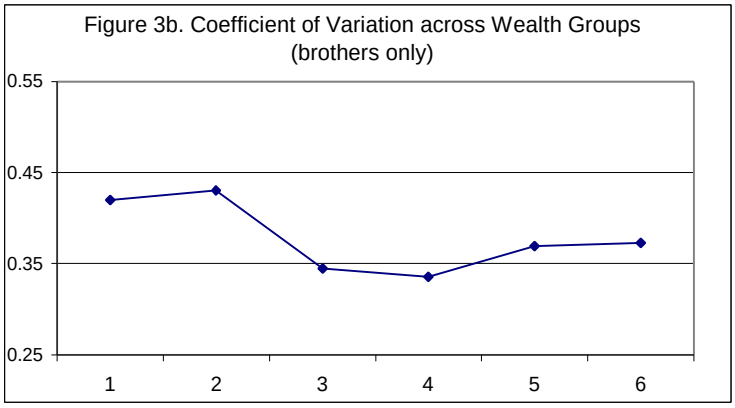


Figure 3b. Coefficient of Variation across Wealth Groups
(brothers only)
0.55
0.45
0.35
0.25
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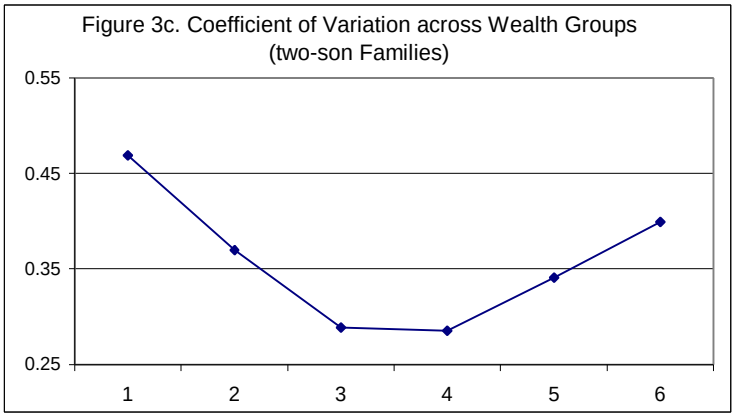
Figure 3c. Coefficient of Variation across Wealth Groups
(two-son Families)
0.55
0.45
0.35
0.25
1
2
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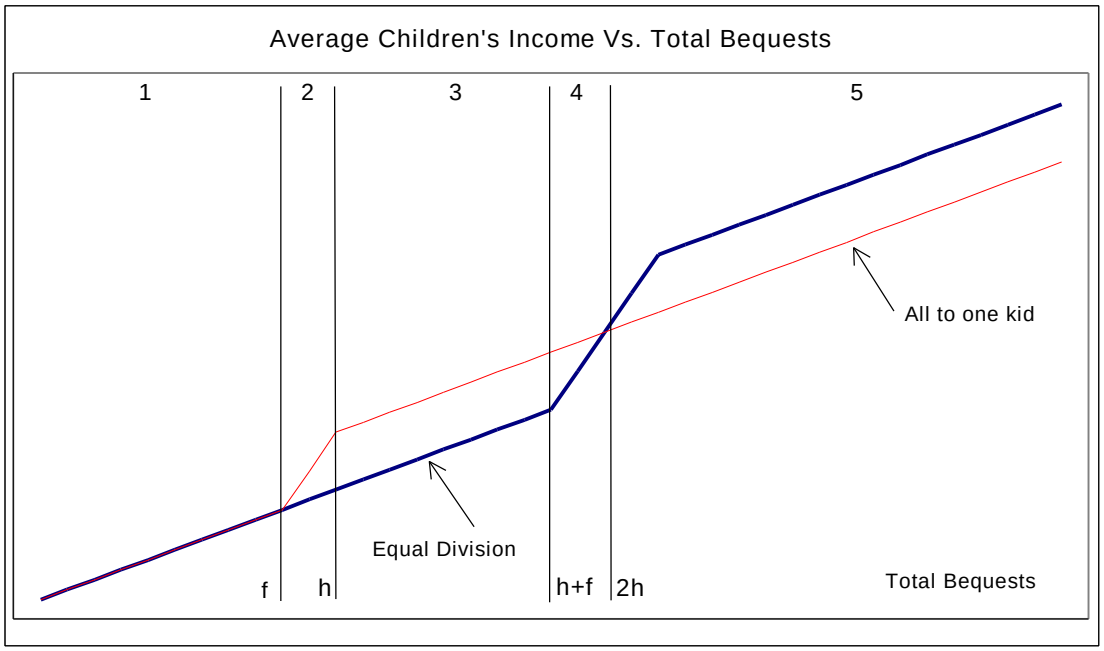
Appendix 1



This graph reproduces Figure 1a in the text. We identify here five different segments (instead of the three segments we identified earlier). Below we comment on the optimal allocations of total bequests (B) in each segment–optimal in that they maximize the average lifetime income of the children.

Segment 1: ($B<f$) all allocations yield the same average children's outcome in this segment. This is a trivial case because total bequests are everywhere below the level necessary to induce one child to invest in human capital.

Segment 2: ($f<B<h$) giving everything to one child is the unique optimal allocation in this segment.

Segment 3: ($h<B<h+f$) giving everything to one child is optimal in this segment, as is giving h to one child and the rest to the other (or vice versa). Equal division is not optimal here. Note that an allocation that would induce both children to get an education (say, by giving f to one child and the rest to the other) may be feasible in this segment but is not optimal. This is a direct consequence of inequalities (7) and (8).

Segment 4: ($h+f<B<2h$) equal division is optimal in this segment, as is giving at least h to one child and the rest to the other (or vice versa)

Segment 5: ($B<2h$) equal division is optimal in this segment, as is any allocation giving at least h to each child.