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Labor Markets Division

Daniel Mantilla-García

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Daniel Mantilla-García

Universidad de Los Andes (Colombia), School of Management
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PLAC Network

NETWORK FOR PENSIONS IN LATIN AMERICA AND THE CARIBBEAN

Best Practices Series 3

TARGET-INCOME DESIGN OF INCENTIVES, BENCHMARK PORTFOLIOS AND PERFORMANCE METRICS FOR PENSION FUNDS

Daniel Mantilla-García
Universidad de Los Andes (Colombia), School of Management
EDHEC-Risk Institute (France)

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Foreword

The Labor Markets Division of the Inter-American Development Bank (IDB) supports countries in Latin America and the Caribbean to build stronger pension systems by seeking to increase their coverage (support in old age to the vast majority of the population), sufficiency (pension benefits that allows for a dignified life in old age) and sustainability (pension benefits financed in the present and in the future). To advance these objectives, the IDB created the Network for Pensions in Latin America and the Caribbean (PLAC Network) in 2015. The PLAC Network is a regional public good that serves as a platform for dialogue and learning among pension institutions and experts. It is one of the mechanisms through which the IDB supports the efforts of countries in the region to improve the institutional and technical capacity of their pension entities.

In this context, we are delighted to launch the third in a series of PLAC Network Best Practices Documents. These documents address main topics of interest and concern to pension policymakers in the region, chosen through consultation among all PLAC Network members. The work is led by an expert who receives insights from the PLAC Network team and is subject to several rounds of comments and contributions from members of the Network. We also invite you to review the first two documents in the series, *Pensions Supervision and Guidelines for the Design and Implementation of the Payout Phase.*

This third document is entitled *Target-Income Design of Incentives, Benchmark Portfolios, and Performance Metrics for Pension Funds*. The document presents a set of best practices and delivers a practical set of tools to assist regulators and supervisors in designing and implementing a regulatory and supervisory framework for funded systems that improves security and sufficiency of retirement income and provides relevant and timely information to pension fund affiliates. The framework achieves these goals by fostering integration of the accumulation and payout phases and the alignment of the regulatory incentives for pension fund management companies with the objectives of pension fund affiliates.

In this document, Professor Daniel Mantilla-García, from Universidad de Los Andes (Colombia), draws lessons from the literature on the emerging paradigm of Target Retirement Income (TRI) designs for pensions systems, to provide a framework, a set of tools, and an illustration of the benefits of applying the TRI principles, in the context of Latin America and the Caribbean region. The author received support throughout this project from IDB members Waldo Tapia, Carolina Cabrita Felix, Laura Karina Gutierrez, Carolina Gonzalez Velosa, and Mariano Bosch, as well as from Luis Felipe Jimenez Salazar, Manuel García Huitrón, Diego Andres Castro Bayona, Gloria Janet Sarmiento Martinez, Javier Serrano and Miguel Martinez-Carrasco. Edgar Robles also made comments and suggestions on the document as an external reviewer. Additionally, the results of this work were presented to and discussed with all PLAC Network member institutions on May 26th, 2021. The author would also like to thank as well Daniel Romero Correa and Omar Youssef Nassar Dorrego for their excellent research assistance in data collection and treatment tasks.

Please direct **any comments or inquiries about this publication** to the PLAC Network team

redplac@iadb.org

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"We can't manage risk right if we measure it wrong."
— Robert C. Merton, *Global Pensions Programme, 2020*

Executive summary and Best Practices

Arguably, a pension system should aim to:[1]

1. Provide security and sufficiency of retirement income to its affiliates.
2. Generate realistic expectations and provide relevant and timely information to its affiliates to support their consumption and saving decisions during the accumulation phase.
3. Be financially sustainable (i.e., avoid generating deficits and debt).
4. Have simple and progressive cross-subsidies (if any).

On the one hand, pay-as-you-go defined-benefits (DB) plans have the advantages of fulfilling objectives 1 and 2 above, because *i.* the benefits are guaranteed by the sponsoring institution, *ii.* the benefits are defined in terms of retirement income (e.g., replacement rate), and *iii.* the conditions to obtain a sufficient level of benefits are usually very clear for the affiliates (e.g., a minimum number of contribution years). However, contribution time has a loose relation with the cost of financing a given level of predefined retirement income. Hence, the difference between the cost of the defined benefits and the total contributed assets must be covered by the pension provider. This has an advantage and a disadvantage. The advantage is that the pension provider, which is responsible for managing the contributions, has all the incentives to minimize the chances of any potential shortfall between the assets and the costs of the benefits because the latter constitute a liability for the institution. The disadvantage is that the way in which benefits are defined implies unhedgeable liabilities, which means the system almost inevitably creates deficits for the sponsoring institution. When the latter is a public institution, deficits translate into public debt, to be paid with interest in the future, which might not be desirable in terms of intergenerational justice. Furthermore, if benefits are defined in terms of replacement rate, then any shortfall covered by the State represents a subsidy that would be proportional to the individual's level of income. Hence, other factors being equal, it would embed a regressive subsidy, in contrast with objective 4 above.

In a context of aging populations, many countries have migrated to systems that increasingly rely on funded defined contributions (DC) plans, to avoid growing and persistent deficits (addressing objective 3). Most DC

1. Arguably, another desirable property for a pension system, is maximizing the probability of having a viable planet at the time when its affiliates expect to enjoy their retirement income. This objective is not addressed in this document, but the proposals hereafter do not conflict with it in principle

plans simply consist of fixing a level of periodic contributions to pension funds (by the affiliates, and/or sponsoring institutions) that is independent of the funding situation of the plan (i.e., the level of assets versus the cost of financing retirement income over time). In fact, in most DC plans, the benefits remain undefined. As a consequence, most current DC plans do not fulfill objectives 1 and 2 above.[2] This implies a transfer of the risk from institutions to individuals, and the latter have no control over the management of their contributed assets. Furthermore, as discussed in this document, current regulations in the region do not provide any incentives for the fund management institutions to control potential losses measured in terms of retirement income for their affiliates, which is the primary risk that the latter face. As illustrated hereafter, this induces ineffective risk management practices and a misalignment between the investment strategies of the managers and the objectives of its affiliates.

To address these central flaws in the design of DC systems, a new paradigm of funded pension plans is recently emerging, called "target retirement income" (TRI).[3] The TRI paradigm aims to:[4]

- Put the retirement income *objective* of pension fund affiliates explicitly at the core of the plan design.
- Include explicit incentives for pension fund managers to *hedge* against a primary risk in retirement investing: the variations in the conversion or 'exchange rate' that determine the resulting level of retirement income for affiliates for a given level of cumulated assets.
- Foster the use of investment strategies, and risk and performance metrics that shift the current *short-term* focus of DC plans towards the *long-term* nature of the retirement problem.
- Provide retirement income security and clear and early signals for affiliates to make timely adjustments in their consumption and savings decisions.

Based on the aforementioned TRI principles, in what follows, we provide a list of **good practices**. Then, Section 2 presents a series of specific tools and regulatory designs that constitute possible alternatives for implementing those practices. Section 3 describe in more detail the implementation of those alternatives and illustrates how they comply with the aforementioned TRI objectives.

1. A key aspect of the architecture of retirement systems is the allocation of risk between the plan affiliates and the managing institution. In that sense, if the affiliates bear all investment risk (as is the case in most DC plans), it is considered **good practice** to design regulations or contracts that explicitly align the incentives of the pension fund management institutions with the outcome of the portfolios that they manage, in terms of the *retirement income* of their fund affiliates. Those incentives can be introduced by:

2. In terms of the justice of cross-subsidies (objective 4 above), DC plans do not tend to bear much criticisms, as their design allows them to integrate progressive components easily.

3. See for instance the IOPS report by Stańko (2015), as well as Martellini et al. (2019), and Mantilla-Garcia et al. (2019a). Another term used by some pension providers in Canada (such as Aon Corporation) with similar objectives is "Target Benefits Plans". A previous call for DC reform with similar principles can be found in Impavido et al. (2010), chapter 4, who proposed "target annuitization funds". See also Blake (2008) for a theoretical discussion.

4. Another way to describe the Target Retirement Income paradigm presented above, is to design retirement plans with financially sustainable defined benefits, that do not generate deficits and debt.

- Defining performance metrics that take into account the conversion risk from asset to retirement income. Section 2.3 presents these type of performance metrics, such as funding ratio and surplus adapted for funded DC plans.
- Designing payment contracts that align the payoff of the fund management company with the retirement income objective (and risk exposure) of their affiliates. Section 2.5 presents several alternatives to define this type of contract, which creates incentives not only to increase the expected retirement income levels of their affiliates but also to limit possible losses in their pension levels. This type of contracts implies a linear payoff in terms of the funding ratio of the pension plan, unlike common AUM-based or contribution-based payment structures, which induce a short-term asset-only focus.[5]

2. Defining regulatory incentives that are aligned with the retirement income objective of fund affiliates requires defining explicit metrics for that objective. Hence, it is considered a **good practice** to define affordable levels of target benefits, in terms of *retirement income,* so that pension fund managers explicitly incorporate that objective into their investment strategies.

   Sections 2.1 and 2.2 provide possible ways to define affordable levels of target benefits. In order to implement them, it is necessary to have a measure of the cost of a retirement unit during the accumulation phase. This can be achieved in at least one of two ways: *i.* using the yield curve derived from the prices of regular government bonds to estimate the value of a retirement bond, and an investable replication benchmark portfolio, as discussed and illustrated in Section 3.1, or *ii.* issuing at least small quantities of retirement bonds for each of the cohorts in the accumulation phase. For a detailed definition and description of retirement units and retirement bonds, see Section 2.1.

3. Investment strategies that manage risk efficiently require the separation of *i.* performance-seeking, *ii.* risk-hedging, and *iii.* overall allocation objectives in long-term investment strategies. Hence, it is considered **good practice** that regulations and supervision tools encourage separate management for these three distinct 'defense,' 'attack,' and 'allocation' functions, within pension fund management companies. This can be achieved by defining different and separate benchmark portfolios and risk and performance metrics that are adequate for the (pension) liability-hedging portfolio (LHP), the performance-seeking portfolio (PSP), and the overall allocation strategy between the LHP and the PSP. In particular,

   - Section 2.4 provides four types of benchmark strategies for the overall asset-allocation portfolio that insure four different possible risk-management objectives, all of which are defined in terms of securing increasing target levels of retirement income. A possible and simple implementation of these benchmark strategies is presented in Section 3.2, which illustrates its advantages in a historical simulation (Section 4 summarizes the conclusions of that empirical exercise).
   - Section 3.1 discusses and illustrates the implementation of liability-hedging strategies that can be used as benchmark indices for LHPs in pension funds. Such indices would also serve as a means of estimating the affordable number of retirement units for each affiliate (discussed in Sections 2.1 and 2.2) and for

5. Furthermore, as discussed in Section 2.5, the proposed payment structures avoid the drawback of the typical asymmetric performance fee contracts, which create incentives to increase risk, but no incentive to control losses.

determining the level of management fee payments for the pension fund management company, as proposed in Section 2.5.

- The PSP should be managed using diversification strategies, in order to access risk premia through the available investment vehicles. Current benchmarks and practices in the region focus on this type of objective already. Hence, we do not provide proposals related to PSP management in this document as this is not the aspect that requires most attention at the moment.

4. As illustrated in Section 3.2, the conversion risk from assets to retirement income is the largest market risk borne by pension fund affiliates, and it can be hedged using a LHP. The constitution of such a LHP crucially depends on the time to retirement. Hence, it is considered **good practice** to define a LHP benchmark per cohort of individuals with (approximately) the same expected retirement date. On the other hand, the same PSP benchmark can be used for all cohorts.[6]

5. As Section 3.1 shows, in some cases, in the absence of retirement bonds, it can be necessary for pension funds to use some level of leverage with the objective to have a good match of the duration of their target retirement income. Hence, it is considered **good practice** that regulations that define limits on leverage to pension funds (if any), account for the fact that using leverage for performance purposes in the PSP can increase risk, while using leverage for reducing interest rate risk exposure within the LHP, has the opposite effect of reducing risk (when measured in the right units, i.e., retirement units).

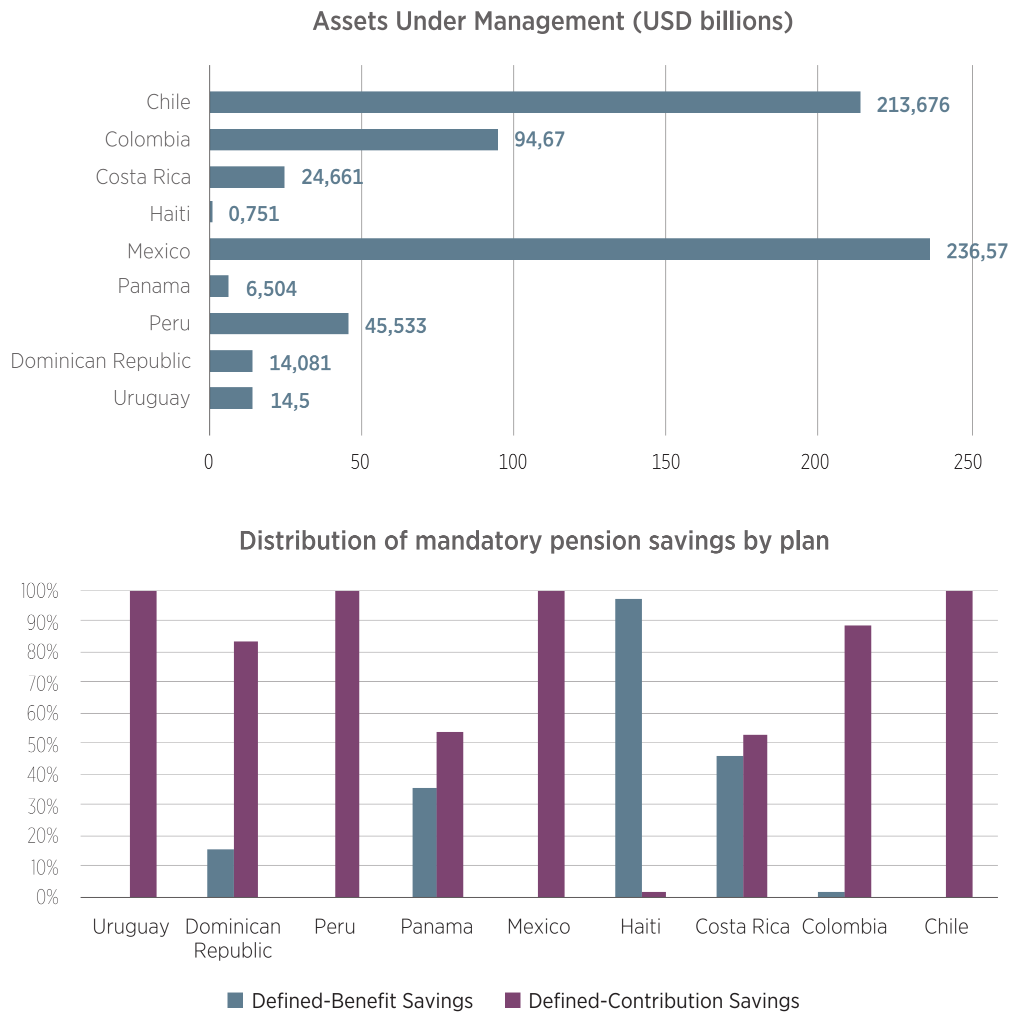
6. As illustrated in Section 3.2, dynamic-allocation based portfolio-insurance strategies allow investment in a risky PSP while at the same time securing an increasing level of retirement income for the affiliates. However, the same level of minimum retirement income can be secured with different allocations to the PSP and the LHP. As discussed in Section 2.4, a given ‘floor’ value for the portfolio can be secured with different values for the multiplier parameter of this kind of strategies. Hence, it is considered **good practice** that the supervisor or regulator defines floor limits in terms of the affordable number of retirement units instead of defining limits to the weights assigned to the different asset classes. Currently, the opposite approach is observed in the regulation of several countries in the region; that is, there are constant regulatory limits on the allocation to asset classes, while there are no minimum target levels of retirement income to be secured.

6. According to portfolio theory, the PSP should maximize short-term risk-adjusted expected return, and its construction is in principle independent of the investment horizon. Hence, it is often referred in the literature as the myopic portfolio, or myopic demand. However, the optimal allocation of the overall strategy to the PSP, does depend on the investment horizon.

1 Introduction

The speed of population aging is increasing around the world, and the region of Latin America and the Caribbean, which used to have an important demographic bonus, is showing one of the fastest aging patterns in their populations (see Cavallo et al., 2016). This persistent trend in aging populations around the world has spawned a global trend from defined-benefit (DB) pension systems towards defined-contribution (DC) systems, and the latter type of system is already prevalent in the region (see Figure 1).

FIGURE 1 ▪ **SURVEY ANSWERS ON ASSETS UNDER MANAGEMENT, AND DB/DC DISTRIBUTION**



Several authors have pointed out that this is a serious concern since it implies that retirement risks are increasingly transferred from the sponsor institutions to individuals (e.g., Cobb, 2015; Martellini and Milhau, 2020). This concern is aggravated by accumulated evidence that pension and investment risks are not properly managed within DC funds (see Merton, 2014; van Bilsen et al., 2020), and is reflected in the significantly lower replacement rates for individuals that have relied on DC plans. For instance, Altamirano et al. (2018) report replacement rates for DC systems of less than half of those of the DB systems in Latin America and the Caribbean.[7] In other words, while financial sustainability of DB systems is at risk in many cases, the retirement income security provided by the current DC systems is quivery.

In the absence of a formal definition of pension liabilities, it is perhaps not surprising that investment risks are not managed to the best interests of the beneficiaries within individual or collective DC funds. After all, the emergence of sound liability-driven investment (LDI) and risk management strategies in the DB pension space is intimately linked to the presence of well-identified, explicit pension liabilities, which are used as references for risk management purposes by setting the assets-to-liability ratio (or pension surplus, which is assets minus liabilities) as the main performance metric. However, according to our survey, the regulation of pension funds in many countries in the region lacks any definition of pension liability and tends to focus on (only) assets' returns.[8]

Indeed, the absence of a pension liability definition in DC plans has several undesirable consequences. First, it leads to the use of performance metrics that are ill-adapted for pension funds, such as asset-only returns (nominal or inflation adjusted), which induces a disconnect between the strategic asset-allocation strategies followed by pension fund managers and the retirement income objective of affiliates. Furthermore, communication with pension fund affiliates is misleading and ineffective as, in most cases, they will not receive the total amount of assets accumulated by the retirement date but a stream of cashflows spread over many years. As discussed hereafter, the exchange rate of assets to cashflow significantly changes over time and should be captured in the pension liability definition. To address this misalignment, Mantilla-Garcia et al. (2019a) introduce a financially sustainable definition of pension liability for funded DC plans and propose incentives definition for pension fund managers that foster efficient risk management strategies (the latter are also discussed is Sections 2.2 and 2.5).[9] The latter apply key lessons from the literature on optimal portfolio selection in long-horizon settings, which includes combining a LHP with a diversified PSP, using asset-allocation based portfolio insurance strategies adapted to the asset-liability management context.

The importance of hedging portfolios to protect investors from changes in key variables, such as variations in interest rates, is stressed in the literature on portfolio optimization, particularly when the investment horizon is relatively long. Since the classic works of Merton (1969, 1971, 1973), we know that optimal investment strategies are composed of a "speculative term" (i.e., the risky block) and a term that prescribes how investors should

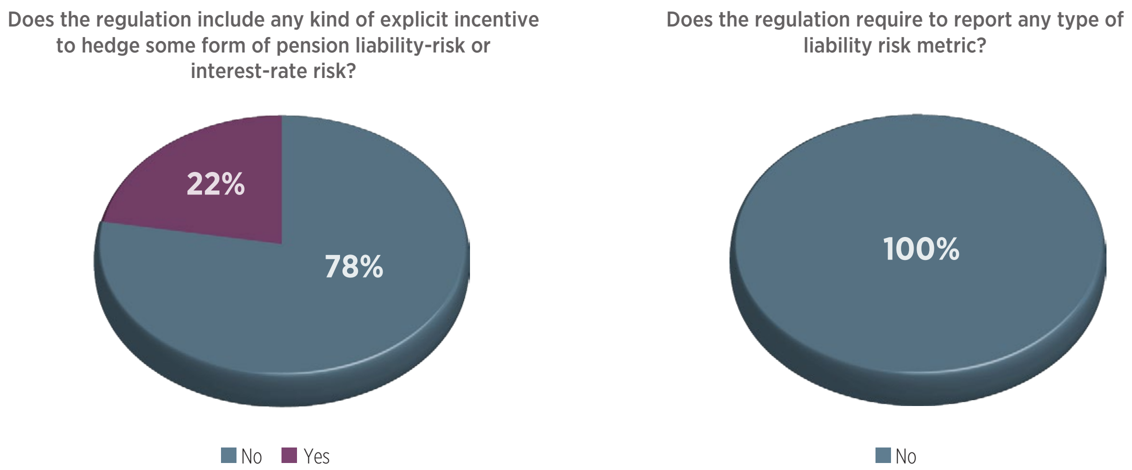
7. They perform this comparison for the same period of time, and in some cases even for the same country, as some of them have both systems currently running.

8. As Figure 2 shows, among the 9 countries that responded the survey, none of their regulation requires the reporting of liability- related risk metrics. By contrast, in the EDHEC's European survey (Badaoui et al., 2014) there was 46% positive answer by pension funds to the question of "Do you hedge your liabilities?".

9. The pension liability definition proposed in Mantilla-Garcia et al. (2019a), builds in the concept of retirement bonds, introduced and discussed in articles such as Muralidhar (2015), Muralidhar et al. (2016), Merton and Muralidhar (2017) and Kobor and Muralidhar (2018).

hedge changes in state variables that impact their utility. In accordance with that general prescription, Omberg (1999), Sørensen (1999), Brennan and Xia (2000), and Viceira and Campbell (2001), among others, find that optimal portfolios should hedge variations in interest rates by investing in the zero-coupon bonds expiring at the investment horizon of the investor (or a portfolio of regular coupon-paying bonds that replicates the zero-coupon bond). The problem solved in those papers concerns an investor with a consumption objective at a single date in the future. When generalized to a problem for which the objective is financing consumption at a series of future dates (as in the retirement period), the hedging asset would naturally be a bond that pays an indexed cash flow on those dates (or equivalently, a set of zero coupon bonds with maturities matching the target consumption dates).[10] This central piece of advice is widely disregarded by pension fund managers of DC plans. For instance, target-date funds and pension fund benchmarks in DC plans commonly use as the "low risk" portfolio a bond index with relatively short and constant duration.[11] Indeed, the sensible bond allocation for a pension fund is a portfolio that matches at best, the cash flow needs of its investors in retirement. Hence, the duration of such bond portfolio should be higher than the values observed, and decrease as the retirement date approaches, instead of remaining constant. In this sense, the target-income strategies described hereafter are based primarily on the use of a truly safe asset relative to the objective of financing consumption after retirement. The correct choice of the safe block in the overall strategy is crucial for risk-management and, as a consequence, for long-term performance.[12]

FIGURE 2 ▪ **SURVEY ANSWERS ON PENSION LIABILITIES-DRIVEN REGULATORY INCENTIVES**



10. This is precisely the motivation behind the concept of retirement bonds (see Merton and Muralidhar, 2017). Whenever such bonds are not available, the hedging asset is a liability hedging portfolio that aims to replicate the theoretical value of the retirement bond, using dynamic interest-rate hedging strategies, as illustrated in Section 3.1.

11. For instance, the Duration of the Bloomberg Barclays US Aggregate Bond Index during the period 1978-01-01 to 2019-09-30 presented a mean-reverting behavior around an average value of 4.9 years. This bond index is used as the bond benchmark by Target-Date funds, such as the LifePath Index 2055 Fund of BlackRock (they report $200 billion invested in "LifePath Target Date fund's assets").

12. For instance, from a theoretical standpoint, all the portfolios in Markowitz's efficient frontier are dominated by the portfolios in Tobin (1958)'s Capital Market Line that result from adding the risk-free asset in Markowitz (1952)'s one-period model.

Another prevailing feature in the literature of long-term optimal strategies, which is present in professional investment management practice, is the horizon effect in asset allocation. This effect was first referred as a puzzle by Samuelson (1963) but later reconciled with rational portfolio theory in works by authors such as Samuelson (1989, 1994), Kim and Omberg (1996), Campbell and Viceira (1999), Barberis (2000), Wachter (2002), and Munk et al. (2004). It consists of a higher allocation to equities for younger investors and a lower allocation to equities as the investment horizon approaches. This feature is clearly reflected in the dynamic allocation rule of target date funds.

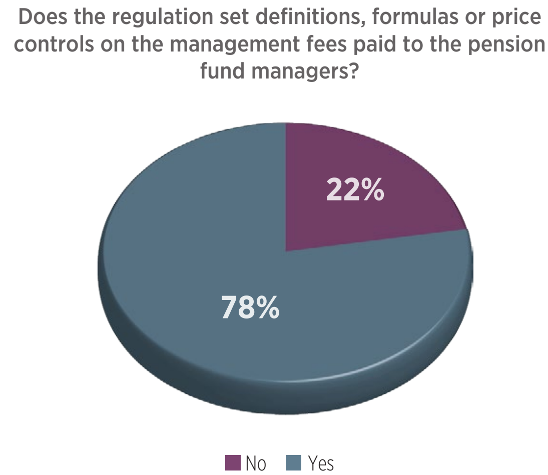
A third feature of asset-allocation advice documented by Canner et al. (1997) is that professional investment advisors systematically recommend increasingly higher bonds-to-stocks ratios as risk aversion increases. Brennan and Xia (2000) and Munk et al. (2004) find that taking into account interest rate uncertainty introduces this asset allocation feature into the optimal strategy of a rational investor with constant relative risk aversion. The target-income strategies in Mantilla-Garcia et al. (2019a), which are discussed and illustrated in this document, integrate all the aforementioned recommendations from the long-term investing literature. They have the additional advantage of expressing the riskiness of each multi-fund in terms of the maximum tolerable loss of retirement income pesos or as the minimum level of retirement income that the investor could secure, which are metrics that are much easier to grasp for investors, and hence simplify their fund choice.

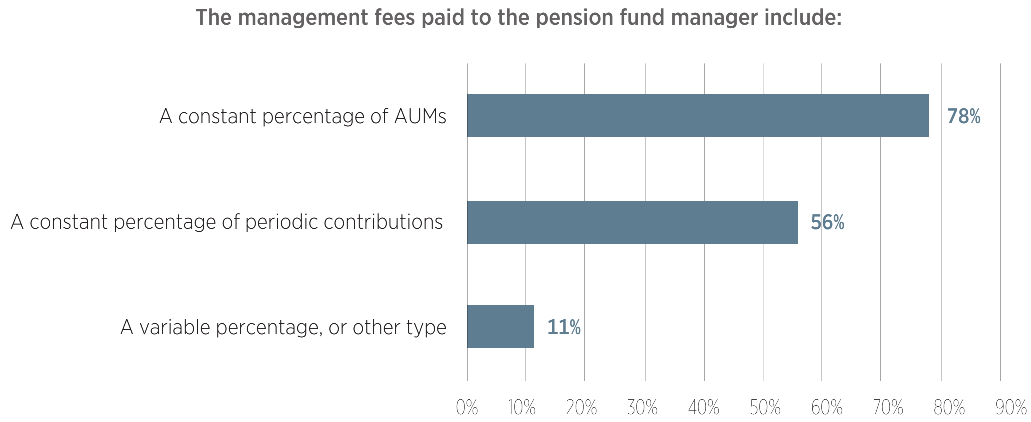
One last feature present in the optimal portfolio theory literature is the constraint of securing a minimum level of replacement income. For instance, Martellini and Milhau (2012) solve for an optimal asset allocation strategy that maximizes retirement income under a constraint on the minimum level of replacement income. They find that "in the presence of funding ratio constraints, the optimal policy is shown to involve dynamic allocation strategies that are reminiscent of portfolio insurance strategies, extended to an asset-liability management (ALM) context." The asset-allocation rules discussed hereafter have very similar traits but have the additional advantage that they avoid relying on parameter estimates or model assumptions in order to insure the minimum target retirement income.

A crucial tool that regulators have used in some countries is the design of incentives for pension fund management companies.[13] However, according to our survey, regulatory management fees for pension funds in the region are functions of variables that do not incentivize efforts to improve risk-adjusted performance, such as a fixed percentage of mandatory contributions, or induce a short-term asset-only focus on pension fund strategies (see right panel of Figure 3). Such fee types contrast with findings in the literature that "incentive contracts in which the agent's payoff is not based on the principal's objective do not in general provide first-best incentives" (Baker, 1992). In Section 2.5, we propose a series of management fee structures that align the incentives of pension fund managers with the retirement income objective of their affiliates. Motivated by recommendations from contract theory, the compensation design suggested has a simpler form than the current nonlinear payoff structure of typical performance fees observed in the hedge fund industry. The latter has been found to create a gap between the stated rates and the effective fee rates (Ben-David et al., 2020) and to generate pervasive risk-taking incentives (Dai et al., 2020).

13. For example, 78% of the respondent countries in our survey regulate management fees, as shown in Figure 3.

FIGURE 3 ▪ SURVEY ANSWERS ON MANAGEMENT FEES REGULATION





2 Methodology

2.1 Retirement Units

A pension fund's main objective is to provide a steady stream of cash flows after one's retirement date, by investing the contributions in an efficient way that maximizes the resulting replacement income during retirement while having a reasonable level of uncertainty around the possible outcomes.

Muralidhar (2015), Muralidhar et al. (2016), Merton and Muralidhar (2017), and Kobor and Muralidhar (2018) postulate that bond-issuer institutions, like governments, easily can provide an ideal retirement- income-hedging instrument that could greatly simplify a central aspect of risk management for the retirement problem. This risk-free asset is simply a bond with indexed coupons to be paid on terms that match the investor's cash-flow needs in retirement. These retirement bonds would be truly safe assets, from the standpoint of an agent investing with the goal of financing a replacement income during retirement. To see this, first consider what usually happens when an individual who has contributed part of their income to their retirement account during their working years finally reaches their retirement date. At that point, they usually cannot access the lump sum, but those retirement savings are exchanged for a series of cash flows that will cover their consumption after their retirement date. Now, what amount of pesos per period during retirement should the individual accept in exchange for that given lump sum in their retirement account?

To answer that question, let's assume that the individual expects that the level of those future cash flows should be certain (or as certain as possible) and also assume that consumption in retirement will happen only during a fixed deterministic time period (this latter assumption is relaxed later). Suppose that the fixed period is set as the life expectancy after retirement of the cohort of people who are the same age as the individual in our example.

Denoting T as the retirement date, the cash flows are needed at dates $T+h$, where h is $1, 2, ..., K$. Since the cash flows should be as certain as possible, the cash flows should be discounted at the zero-coupon rates $R_{T,h}$ which corresponds to the yield of a risk-free zero bond paying \$1, h years ahead from current time T. Thus, for every peso of income per year during the K retirement years, the amount that the individual should be willing to pay at retirement date T is:

$$B_T = \sum_{h=1}^{K} \$1 e^{-hR_{T,h}} \tag{1}$$

Hence, if the total value of the retirement account at retirement date is A_T, then the number of pesos per year that the individual would receive in exchange for that lump sum is

$$\tilde{N}_T = \frac{A_T}{B_T} \tag{2}$$

Notice that the cash flow amount per period in equation (1) was assumed to be constant over time, equal to $1. If such cash flows represent nominal pesos, then the individual would be losing purchasing power during their retirement years, which is impractical if consumption is the objective. Thus, cash flows should include cost-of-living adjustments (COLAs). One way to do so is to let the pension's nominal cash flows grow at a fixed growth rate g, which may be set as the expected level of inflation. In that case, the conversion factor of future consumption for pesos of T would be:

$$B_T = \sum_{h=1}^{K} (1+g)^{T+h} e^{-hR_{T,h}} \tag{3}$$

If a perfect hedge of inflation risk is preferred, then the cash flows would be expressed in real terms, which simply means that the discount rates $R_{T,h}$ in equation (1) should be real yields instead of nominal bond yields. If a hedge for salary increments in real terms is preferred as well, then real yields could be used in equation (3) instead, and g would be the expected salary growth in real terms. However, having extra hedging for unexpected inflation risk and accounting for real salary growth imply an increment in the costs of financing future consumption, reflected in a higher price B_T. In our empirical analysis, we opt to use nominal yields and set g as the central bank's inflation target. Now, notice that the conversion factor B_T is simply the price of a portfolio of K zero-coupon bonds with maturity dates $T+h$ for h in $\{1, 2, ..., K\}$. Thus, one may calculate the non-arbitrage price of the zero bonds expiring on those same dates but at previous date $t < T$. In that case, the time to maturity for each of those bonds is longer, and the corresponding discount rates are $R_{t,T+h-t}$, which corresponds to the yield of the zero-coupon bonds paying $1, $T+h-t$ periods ahead from current time t. The present value of those cash flows, constitutes the no-arbitrage price of the *retirement bond* at date t:

$$B_t = \sum_{h=1}^{K} (1+g)^{T+h} e^{-(T+h-t)R_{t,T+h-t}} \tag{4}$$

This would be the price of the retirement bond for the cohort of people with retirement date T, and there would be a retirement bond for each cohort (just as there is a target-date fund for each cohort).

The price of these bonds provides individuals with an exact exchange rate between one peso of consumption at any date $t < T$ for a stream of real pesos starting at the individual's retirement date. Hence, for a given lump sum, say a worker's current retirement savings, the individual knows, at any date prior to their retirement date, exactly how much replacement income, in real terms, they can secure until the life expectancy of their cohort, by investing such a lump sum into retirement bonds. This is simply because with the current savings A_t, they could buy $\tilde{N}_t = \frac{A_t}{B_t}$ retirement bonds. Without any further transactions, they will receive $\tilde{N}_t$ indexed pesos per year with certainty from $T+1$ until $T+K$.

Note that this bond only covers retirement consumption from the retirement date until the cohort's life expectancy, leaving the investor exposed to longevity risk. However, if enough people of a given cohort held retirement bonds, then, as a group, they could easily exchange such bonds for (indexed) life annuities with an insurance company, with a rate close to one-to-one in terms of real pesos during retirement. This is because such bonds would represent the perfect hedging asset for an insurance company issuing such annuities for individuals in that cohort because the insurer can use the cash flows of the bonds it obtains from the individ-

uals who die before the cohort's life expectancy to pay the annuity cash flows of the individuals who survive past the life expectancy. Under this one-to-one exchange assumption of retirement bonds per annuity pesos, then, the price of the retirement bond constitutes the cost of financing one unit of retirement consumption throughout retirement.

Notice: In the reminder of the paper, we use the terms retirement bond and retirement unit interchangeably.

DEFINITION 1: Affordable retirement income $\tilde{N}_t$

Let the cost of acquiring one retirement unit at time t be denoted B_t. The value of a portfolio in terms of retirement units (each one paying $1 peso plus COLAs throughout retirement) is denoted:

$$\tilde{N}_t = \frac{A_t}{B_t}$$

The latter also constitutes the number of real pesos of income throughout retirement that the individual could secure at any time at time t during the accumulation phase, by selling all assets and using the proceeds to buy $\tilde{N}_t$ retirement bonds that start paying cash flows at the target retirement date T of the investor.

Although retirement bonds are not currently issued by governments yet, in principle, they are simply a bundle of zero-coupon-indexed bonds, and their payoff can be replicated with some reasonable estimation error, using dynamic interest-rate-hedging strategies. We follow the methodology in Mantilla-Garcia et al. (2019b) for implementing this type of replication/hedging strategies, and in Section 3, we illustrate the replication methodology empirically using standard government bonds as hedging instruments in the case of Colombia.

2.2 Sustainable Pension Liability Definition

The liabilities of DB pension plans usually are defined as the present value of the replacement income cash flows.[14] The benefits are series of periodic (e.g., annual or monthly) pension payments of an amount that is a function of the employee's final salary (or the average of the last few years of salary) and years of contribution. For instance, as Novy-Marx and Rauh (2011) report for the US, in a typical DB plan for state employees, "an active worker accrues the right to a periodic benefit upon retirement that equals a flat percentage of his final (or late-career) salary times his years of service with the employer." For example, if the benefit factor is 2% and an employee has worked for 30 years and had an average wage in the last several years of work equal to $35,000, then the employee will be entitled to a pension of $21,000 (= 2% * 30 * $35,000) per annum when they retire, plus any COLAs their plan offers. Their guaranteed replacement rate then would be 60% = 2% * 30, and the liability of that DB pension plan is the present value of those projected cash flows.

Similarly, in the public nationwide DB pension systems of Brazil, Colombia, and Peru, every employee contributes a fixed percentage of their salary, and after a minimum contribution period to the system (e.g., 25 years), the worker is entitled to a pension that guarantees a given replacement rate (e.g., 64.5%) relative to the average of the last years of salary before retirement (see Altamirano et al., 2018). The root of the financial-sustainability risk of DB plans is the loose relationship between the individual's contributions to the system and the benefits received. As previously noted, the pension benefit depends on the contribution time, instead of the actual funds contributed by the individual (or on behalf of each individual).

In contrast, DC plans do not provide any security such as a defined benefit. The pension amount received is simply equal to the total amount that the worker has accumulated in their retirement account (thus, fund contributions plus their returns) adjusted by the cost of financing an indexed cash-flow stream starting after retirement (i.e., $\tilde{N}_T$). In the absence of retirement bonds (or a replicating investment strategy), this amount is unknown prior to the retirement date T, at which time the assets are converted into pension cash flows. Hence, while there is no financial sustainability risk for the pension plan, the workers bear the risk of ending up with insufficient replacement income. This lack of any "liability" or commitment among DC plans leaves workers without any level of retirement security.

In order to address this central problem of DC plans, Mantilla-Garcia et al. (2019a) propose that an appropriate definition of the "liability" or commitment on the minimum retirement income for pension funds of DC plans is the present value of the cash-flow benefits that can be secured without taking any risk. Indeed, the purpose of investing in risky assets is to seek improvements in terms of retirement income, beyond what is affordable without taking any risk. In that sense, a sensible risk-management objective in this context is to protect at least a percentage of the level of retirement income that is affordable, without taking any risk throughout the accumulation phase.

14. Novy-Marx (2013) established that the correct discount rates to value pension liabilities of DB plans are the yields with lowest risk available, and not the expected return of the pension fund assets.

DEFINITION 2: Affordable retirement income N_t

Let C_t be the value of the contribution to the pension fund at time t in nominal pesos, and $n_t = \frac{C_t}{B_t}$ be the number of retirement bonds that can be afforded with that contribution. If all of the periodic contributions until time t were used to buy retirement bonds, then the number of real pesos of replacement income that the worker could afford without taking risk, at any date during the accumulation phase $t \leq T$ is:

$$N_t = \sum_{i=1}^{t} n_i$$

DEFINITION 3: Liability for funded pension plans in current pesos L_t

Since N_t is the number of real pesos of replacement income affordable for the individual without taking any risk with the contributions that she has done until current time t, hence the liability of the pension fund at time t is defined as the present value of those cash-flows:

$$L_t := N_t B_t \tag{5}$$

which is equal to the current price of N_t retirement bonds.

Notice that L_t also represents the value of a fund in which all contributions have been invested in retirement bonds.

ⓘ **Notice:** The affordable retirement income N_t is the pension liability L_t expressed in number of retirement units, i.e. $N_t = \frac{L_t}{B_t}$

ⓘ **Notice:** If the initial date $t = 1$ at which we make the first estimate of the pension liability is posterior to the creation of the retirement account of the individual, then the assets' value in the account at that initial date (A_1) would be treated as the initial contribution C_1. Hence, at the starting date of a fresh risk management perspective, assets and liabilities would be equal, i.e., $A_1 = C_1 = L_1$.

ⓘ **Notice:** Let us emphasize that this analysis can be performed at the level of each individual account for individual DC plans and also at the aggregate pension-plan level for a collective DC.

2.3 Performance and Risk Metrics for Pension Funds

Funding ratios measure how pensions fund managers are coping with the pension promises (i.e., the defined benefits [DB]). The latter is the target level of replacement income that fund affiliates expect to obtain in retirement by investing in the fund. Hence, funding ratios—which measure the relationship of assets to the present value of the promised level of benefits—have a direct relationship with the expected replacement rate. In that sense, funding ratios are a very sensible metric for evaluating the effectiveness of the investment process, relative to an objective expressed in terms of retirement income. In that sense, we use the previous definition of pension liability for DC plans as follows.

DEFINITION 4: Funding Ratio (FR_t) and Surplus in retirement income (S_t) for funded plans

Once a formal liability process L_t is obtained from the definition in equation (5), the corresponding funding ratio is simply given by:

$$FR_t = \frac{A_t}{L_t} = \frac{\tilde{N}_t}{N_t}$$

and the Surplus/Shortfall in securable retirement income is:

$$S_t = \tilde{N}_t - N_t$$

The former definitions are similar in spirit to the funding ratio and surplus of defined-benefit pension plans. In practice, investment decisions will differ from a portfolio fully invested in risk-free retirement bonds, and the relative performance of a given investment strategy with respect to the risk-free retirement bond portfolio naturally can be summarized in terms of the funding ratio. A ratio greater than 1 means that the securable retirement income $\tilde{N}_t$ has increased since the beginning of the period more than the affordable level of replacement income N_t has, and a ratio lower than 1 indicates that it has decreased. In other words, an investment strategy is successful ex post if it leads to an increase in replacement income relative to what was attainable risk-free (or the lowest possible risk). That occurs if the overall portfolio outperforms the LHP (or the retirement bond) over a given period.

As pointed out by Merton (2014), uncertainty over the securable retirement income is the relevant notion of risk when the final goal is to generate replacement income. To capture such uncertainty, we propose to use variations of the funding ratio, relative to its risk-free level of 1. The following box indicates the proposed metrics.

DEFINITION 5: Variation Metrics of Funding Ratio (FR_t)

For a given funding ratio time series FR_t, relevant risk metrics for a pension fund are:

$$\begin{aligned}\text{MEAD}(FR) &= \; median(|FR_t - 1|)\\ \text{MAAD}(FR) &= \; max(|FR_t - 1|)\end{aligned}$$

where MEAD stands for Median Absolute Deviation, and MAAD for Maximum Absolute Deviation.

Furthermore, we propose using the Maximum Drawdown of the funding ratio series, which we denote $\text{MDD}(FR)$, i.e. the largest cumulative loss in value, expressed as a percentage of the maximum historical funding ratio, and the annualized volatility of the periodic changes in funding ratio.

ⓘ **Notice:** Given that retirement bonds are still not available in the market, a central risk-management task for pension fund managers is to replicate the value of the fund's pension liabilities, as close as possible. Such a task is usually performed in DB plans, in their LHP, using interest-rate-hedging strategies such as duration matching. The latter strategy focuses on hedging parallel shifts in the level of the term structure of interest rates. However, a more complete approach would be to also hedge the impact of the possible changes in the yield curve's shape (as opposed to only parallel levels). The standard model for measuring level and shape variations of the yield curve is Nelson and Siegel (1987). Hence, deviations in exposures to the Nelson–Siegel yield curve factors are risk metrics of the LHP's potential replication error.

DEFINITION 6: Deviations in Yield Curve Factor Exposures

Let the first-order sensitivities of the retirement bond to the level, slope and curvature factors in the Nelson and Siegel (1987) model, be denoted as $D0$, $D1$ and $D2$ respectively. Also, denote as $\hat{D}0(x)$, $\hat{D}1(x)$ and $\hat{D}2(x)$ the first-order sensitivities to the same risk factors, of the liability-hedging portfolio x. Ex ante deviations in these factors exposures measure potential replication error of a given hedging strategy. Hence, to monitor hedging quality, we propose to use:

$$\begin{aligned}\text{MEAD}(Di) &= \; median(|Di_t - \hat{D}i_t(x)|)\\ \text{MAAD}(Di) &= \; max(|Di_t - \hat{D}i_t(x)|)\end{aligned}$$

for $i \in \{1, 2, 3\}$. Furthermore, we propose using the Median Absolute Deviation, and the Maximum Absolute Deviation of the funding ratio series of the LHP, as metrics of the ex-post replication error.

A relevant metric to report to pension fund affiliates is, of course, the value of their retirement account in number of retirement units, $\tilde{N}_t = \frac{A_t}{B_t}$, since this quantity represents the current securable level of retirement income (including the COLAs embedded in the definition of B_t).

Some actuarial methods, such as the projected value of benefits (PVB), estimate DB plans' liability as the present value of beneficiaries' total expected cash flows (see Novy-Marx and Rauh, 2011, for details of the PVB method). In the next box, we define a funding ratio, that is similar in spirit to this type of liability definition, but for funded plans.

DEFINITION 7: Relative Funding Ratio (RFR_t)

The relative funding ratio measures the percentage of the target level of retirement income $\hat{N}_T$, that can be secured given the current level of assets, and interest rates:

$$RFR_t := \frac{A_t}{N_T B_t}$$
$$RFR_t = \frac{\tilde{N}_t}{\hat{N}_T}$$

DEFINITION 8: Affordable Funding Ratio (AFR_t)

The affordable funding ratio measures the percentage of the target level of retirement income $\hat{N}_T$ that could have been afforded until the current date t, if all contributions would have been invested in retirement bonds:

$$AFR_t := \frac{N_t}{\hat{N}_T} \quad (6)$$

Note that $FR_t = \frac{RFR_t}{AFR_t}$. In addition, note that the difference $RFR_t - AFR_t$ is found in the incentives definitions (II.b) and (IV.b) in Section 2.5.

ⓘ **Notice:** Although informative for affiliates, the securable level of retirement income $\tilde{N}_t$, and the relative target ratio RFR_t vary with both, the relative performance of assets with respect to the liabilities, and with the contributions. Hence, these metrics mix the responsibilities of the fund manager (the relative performance) and those of the affiliates (the contributions). In that sense, the funding ratio FR_t and the surplus S_t are better metrics for evaluating pension fund managers' performance because they vary primarily with the portfolio's relative performance and are less dependent on the contributions than $\tilde{N}_t$ and RFR_t[a]. Indeed, the fund management company should be accountable for the performance of the investments it decides to make and not for the lack (or excess) of contributions to the fund.

[a] To see this, note that the contributions cancel out in the dynamics of the surplus (i.e., $\Delta S_t = \Delta N_t - \Delta N_t = \tilde{N}_{t-1} r_t^Z + n_t \text{-} n_t$, where r_t^Z is the relative return of the portfolio's assets with respect to the retirement bond (see Mantilla-Garcia et al., 2019a, for more details).

2.4 Benchmark Portfolios

As discussed in the introduction, portfolio theory ever since Tobin's (1958) two-fund-separation theorem shows that efficient asset-allocation rules split the portfolio into a risky performance-seeking (PSP) block and a safe block with minimum risk. As previously discussed, in the context of pension funds, the safe block is a liability-hedging portfolio (LHP) whose aim is to replicate the retirement bond's payoff as well as possible.

A series of fixed future cash flows, in nominal or real terms, can be replicated with bond-immunization strategies. Such strategies can be used to construct a portfolio of standard bonds as hedging instruments, in a way that matches the sensitivities of the retirement bond's value to the different yield curve factors that drive the changes in the term structure of interest rates. The simplest version of a bond-immunization strategy, known as duration matching,[15] is aimed at immunizing a bond portfolio's value against small parallel shifts in the yield curve. The strategy was extended to account for larger changes in interest rate levels by matching bond-price convexity. However, ample evidence exists that variations in the yield curve's shape, such as slope and curvature movements, are also very significant (see, for instance, Diebold and Li, 2006; Diebold and Rudebusch, 2013), and the latter can have a strong negative influence on the hedging quality of standard-duration and convexity-matching strategies (see Ingersoll et al., 1978; Fong and Vasicek, 1984). We refer the reader to Mantilla-Garcia et al. (2019b) for the implementation details of generalized bond-immunization strategies designed for hedging multiple factors of the yield curve that controls for the potential impact of model-misspecification risk and parameter-estimation errors. In Section 3, we provide an empirical illustration of that replication methodology using standard government bonds as hedging instruments in the case of Colombia.

From the standpoint of the regulator/supervisor of pension funds, perhaps the most natural approach for creating a benchmark index for the LHP is to calculate the no-arbitrage value of the retirement bond corresponding to each cohort, using, for instance, the yield-curve data published by the respective central bank, and let the pension fund managers replicate that theoretical value as best they can with their chosen methodology. An alternative approach is to calculate a benchmark LHP index based on a particular dynamic hedging methodology and use that index's value as the measure of the retirement unit. The advantage of the latter approach over the former one is the higher replicability of the latter's benchmark index.

In this context, pension fund managers have two roles: to construct a LHP that best replicates the value of each cohort's retirement bond and to implement an asset-allocation rule between the LHP and the PSP that helps individuals to aim for a replacement income higher than the affordable retirement income available by investing all their contributions in retirement bonds (i.e., N_T), with a reasonable level of security about the final outcome.

15. Duration-matching was introduced and analyzed in seminal papers by Samuelson (1945), Redington (1952), Fisher and Weil (1971), and Bierwag and Khang (1979). Redington (1952) determined the conditions under which duration matching of multiple cash flows results in local immunization, while Bierwag et al. (1983) established the conditions under which duration matching of multiple liabilities implies global immunization in the presence of level shifts, and Barber (1999) generalized these results to the single-factor affine term-structure model.

ⓘ **Notice:** It is good practice for pension-fund supervisors/regulators to provide at least two types of benchmark portfolios: one for the LHP and one for the overall portfolio strategy.

As shown hereafter, if pension-fund managers construct a LHP to replicate the value of retirement bonds with dynamic hedging, they can help individuals to take risk while providing retirement security by using dynamic-allocation rules that can ensure a minimum level of replacement income.

In what follows, we discuss how to build investment portfolios that allow investors to take risk, but at the same time use retirement bonds (or its replicant LHP) to provide retirement security and control over future retirement income.

Cushion-based portfolio insurance strategies aim to ensure that the portfolio respects a given performance constraint set ex ante, by following an asset-allocation rule that prevents the portfolio's value from falling below a floor value F at all times. To do so, this kind of strategy dynamically allocates the following proportions to the risky PSP and the LHP at time t equal to the following:[16]

$$\omega_t^{PSP} = m_t \times (\tilde{N}_t - F_t)/\tilde{N}_t, m_t > 0, \tag{7}$$

$$\omega_t^{LHP} = 1 - \omega_t^{PSP} \tag{8}$$

For this strategy to insure its risk-management objective at all times, the floor design must satisfy the condition that the LHP process super-replicates the floor value process during time periods when $\tilde{N}_t$ approaches F_t. If that is the case, then continuously reallocating assets to ensure that the exposure to the risky PSP is always equal to (7) maintains a positive cushion (i.e., $C(t) = \tilde{N}_t - F_t > 0$), at all times.

In general, the multiplier parameter that determines the strategy's risk exposure per unit of available cushion may be a non-negative time-varying process (see Martellini and Milhau, 2012; Mantilla-Garcia, 2015, and Mantilla-Garcia et al., 2021b, for methodologies to estimate the optimal multiplier and its upper bound). For $m_t > 1$, the managers can aim to have a higher return by making tactical deviations in the asset allocation or simply having a higher exposure to the PSP's risk premia. However, because assets can only be reallocated in discrete time, this implies a non-zero probability that the portfolio's value will fall below its floor value, before the manager can reallocate assets, which is known as gap risk.

However, gap risk can be eliminated simply by implementing the cushion-based allocation with a multiplier value of $m_t = 1$ at all times. This ensures that the portfolio value always stays above its floor and complies with its minimum performance objective without being subject to any model or estimation risk. To see this,

16. Since the different types of floors used hereafter are defined in numbers of retirement units, the allocation rule (7) was also expressed as a proportion of the number of retirement units that the portfolio is worth. However, note that the proportion of the value (in pesos) allocated to the PSP would be the same. To see this, notice that the number of retirement units allocated to the PSP would be $\omega_t^{PSP} \times \tilde{N}_t$. To convert this quantity to pesos, we multiply by the value of one retirement unit, $\omega_t^{PSP} \times \tilde{N}_t B_t = \omega_t^{PSP} \times A_t$.

note that the number of retirement units invested in the LHP according to the allocation rule (7) implemented with $m_t = 1$ is

$$\omega_t^{LHP} \times \tilde{N}_t = F_t \tag{9}$$

If the PSP's value suddenly drops by 100% at any point in time, all of the retirement units invested in it would be lost. Even in that catastrophic event, the proposed strategies would respect their floor value because the number of retirement units invested in the LHP would be equal to the same floor value, as shown by equation (9).

Hereafter, we present four portfolio-insurance strategies of type (7), introduced in Mantilla-Garcia et al. (2019a) with different floor definitions, designed as benchmark portfolios for pension funds with $m_t = 1$ for all values of t. Note that the floor process is expressed in number of retirement units. We focus on these target income strategies for the reasons explained in the Executive Summary and Section 1.

(I) Strategy securing a minimum funding ratio of κ at all times

- Following the generic portfolio insurance strategy (7) with a floor equal to equation (10) below secures a percentage κ% of the affordable retirement income N_t, i.e., $\tilde{N}_t \geq \kappa N_t$ for all values of t.
- Notice that the insured level of retirement income κN_t never decreases and it increases with every new contribution.
- This strategy also protects the funding ratio from falling below the level κ (i.e., $FR_t \geq \kappa$) for all values of t.
- The floor formula for this strategy is

$$F_t^{(\mathrm{I})} = \kappa N_t \quad \text{where} \quad 0 < \kappa < 1 \tag{10}$$

- Individuals with higher risk aversion should be assigned to a fund with a higher level of protection, e.g. $\kappa = 0.9$, while individuals with lower risk aversion can be put in a fund with a lower level of protection, e.g., $\kappa = 0.7$.
- The benchmark portfolio (I) should hold, at all times, a number of retirement bonds (or shares of the corresponding replicating fund) equal to κN_t.
- While the amounts in pesos and the percentage weights invested in the LHP and PSP constantly vary over time, trading due to asset-allocation changes only happens when the number of retirement bonds (or shares of the corresponding replicating fund) must change according to the strategy's rule. Note that, the number κN_t only varies whenever there are new contributions to the fund.

(II) Strategy securing an increasing funding ratio of κ_t and with decreasing risk-taking

- Following the generic portfolio-insurance strategy (7) with a floor equal to equation (11) below secures an increasing percentage κ_t% of the affordable retirement income N_t , i.e.,$N_t \geq \kappa_t N_t$ for all t.
- The floor formula for this strategy is

$$F_t^{(\mathrm{II})} = \kappa_t N_t \tag{11}$$

- The insured percentage $\kappa_t = 1 - (1-\kappa)\frac{\hat{N}_T}{N_t}$ increases with every new contribution.[a] When the contributions reach the level of planned contributions, i.e. $N_s = \hat{N}_T$, at some date $s \leq T$, then $\kappa_s = \kappa$, and the insurance goal is attained. In other words, at time s, the level of retirement income has been secured to be at least $\tilde{N}_s \geq \kappa \hat{N}_T$ regardless of the returns in the risky assets in the portfolio, and the level of protection can be for instance one of $\kappa \in \{0.9, 0.8, 0.7\}$.
- The target level of affordable retirement income $\hat{N}_T$ (which is also the expected amount of contributions in number of retirement bonds) should be based on each individual's characteristics, particularly on age and expected income.
- The benchmark portfolio (II) should hold, at all times, a number of retirement bonds (or shares of the corresponding replicating fund) equal to $\kappa_t N_t$. Note that both κ_t and N_t only vary whenever there are new contributions to the fund. Hence, trading due to changes in asset allocation only occurs at contribution dates.

[a] If there are frequent contributions, the parameter increases as the retirement date approaches.

(III) Strategy securing a minimum funding ratio of κ at all times plus a ratchet on previous surplus

- Following the generic portfolio-insurance strategy (7) with a floor equal to equation (13) below secures a proportion κ of the maximum level of retirement income ever attained, $\tilde{N}_{t^\star}$ at a previous time $t^\star$, plus the same proportion of all the contributions made to the fund since then, i.e.,

$$\tilde{N}_t \geq \kappa \tilde{N}_{t^\star} + \kappa \sum_{i=t^\star+1}^{t} n_i \tag{12}$$

  The insured level of retirement income (the right hand side in equation 12) never decreases, and increases with every extra contribution and in periods of good relative performance of the risk assets in the fund.

- The floor formula for this strategy is

$$F_t^{(\mathrm{III})} = \kappa_t^\star N_t \tag{13}$$

- The strategy also insures a proportion $\kappa_t^\star = \kappa\left(1+ \frac{S_{t^\star}}{N_t}\right)$ of the affordable retirement income N_t, $\tilde{N}_t \geq \kappa_t^\star N_t \geq \kappa N_t$ for all t.[a] Where $S_{t^\star}$ denotes the maximum level of surplus observed until current time t.

- The surplus in retirement income at time t is $S_t = \tilde{N}_t - N_t$. This quantity does not depend on the contributions but only on the fund's relative performance.

- The protection level can be, for instance, one of $\kappa \in \{0.9, 0.8, 0.7\}$. Individuals with higher risk aversion should be assigned to a fund with a higher level of protection.

- By definition, at the initial date, $\tilde{N}_0 = N_0$, thus $S_{t^\star}$ is always non-negative, and $\kappa_t^\star \geq \kappa$ at all times. Hence, the benchmark portfolio (III) is a more conservative strategy than the benchmark portfolio (I) is.

- The benchmark portfolio (III) should hold, at all times, a number of retirement bonds (or shares of the corresponding replicating fund) equal to $\kappa_t^\star N_t$. This quantity varies whenever there are new contributions to the fund or when a new maximum in the surplus is attained. Hence, trading due to changes in asset allocation only occurs on those dates.

[a] The proportion of $\kappa_t^\star \geq \kappa$ increases each time a new maximum is attained in the level of surplus. If there are extra contributions before a new maximum surplus level is attained, the proportion decreases with each contribution since $\tilde{N}_t$ approaches $\tilde{N}_{t^\star}$.

(IV) Strategy securing an increasing funding ratio of κ_t plus a ratchet on previous surplus

- Following the generic portfolio insurance strategy (7), with the floor formula indicated in equation (15) below, it secures at all times a strictly increasing proportion κ_t plus a proportion κ of the maximum previously attained surplus in retirement income, i.e.,

$$\tilde{N}_t \geq \kappa_t N_t + \kappa S_{t^\star} \tag{14}$$

where $\kappa_t = 1 - (1-\kappa)\frac{\hat{N}_T}{N_t}$ and $S_{t^\star}$ are the maximum surplus attained by time t.

- The floor formula for this strategy is

$$F_t^{(\mathrm{IV})} = \kappa_t^\star N_t \tag{15}$$

where $\kappa_t^\star = \kappa_t + \kappa\left(\frac{S_{t^\star}}{N_t}\right)$

- When the contributions reach the level of planned contributions, i.e. $N_s = \hat{N}_T$, at some date before the target rate $s \leq T$, the insurance goal is attained, i.e.,

$$\tilde{N}_s \geq \kappa \tilde{N}_{s^\star} + \kappa \sum_{i=s^\star+1}^{s} n_i$$

- By definition at the initial date $\tilde{N}_0 = N_0$, thus $S_{t^\star}$ is always non-negative, and $\kappa_t^\star \geq \kappa$ at all times. Hence, the benchmark portfolio (IV) is a more conservative strategy than the benchmark portfolio (II).

- The target level of affordable retirement income $\hat{N}_T$ (which is also the expected number of contributions in the number of retirement bonds) should be based on the characteristics of each individual, particularly on age and expected income.

- The benchmark portfolio (IV) should hold at all times a number of retirement bonds (or shares of the corresponding replicating fund) equal to $\kappa_t^\star N_t$. This quantity varies whenever there are new contributions to the fund or when a new maximum in the surplus is attained. Hence, trading due to changes in asset allocation only happens at those dates.

2.5 Incentives Design

A central question in modern economics is to address agency conflicts through the design of compensation contracts that align the agent's incentives with the interests of the principal. To achieve this alignment objective, contract theory suggests that such incentives should be a function of outcome variables that depend (at least to some extent) on the efforts done by the agent. Furthermore, "incentive contracts in which the agent's payoff is not based on the principal's objective do not in general provide first-best incentives, even when the agent is risk neutral" (Baker, 1992).

Compensation contracts in the hedge fund industry have aimed to achieve incentives alignment using a structure known as 1-20, which combines a fixed annual management fee (of around 1%-2%), with a performance fee, also known as incentive fee, of around 20% of profits over a quarter or year. The payment of the latter is often subject to conditions to ensure that performance fees are only paid on profits exceeding a predetermined high water mark (often being the highest historical portfolio value at each point in time). Such conditional payment is known to induce a "ratchet" effect on wealth. A superficial look at this type of contract might lead to the view that it aligns the incentives of hedge fund managers (the agent) with the objectives of hedge fund investors (the principal). However, recent research has found an important caveat in that type of contract, emerging from the fact that the 20% bit is applied in years when returns are positive but is not applied when returns are negative, and that creates two structural problems.

First, Ben-David et al. (2020) show that such an asymmetric structure in the "incentive" fees creates a large gap between the stated nominal rate of 20% and the effective incentive fee rate. Over a 22-year sample, beginning with 80 funds in 1995 and including nearly 6,000 funds by 2016, they find that incentive fees equaled roughly 50% of hedge fund industry profits in excess of the hurdle rate. Accounting for the annual fixed management fee increases the fraction collected by hedge fund managers over this time period to 64%. That is, for every dollar profit generated by hedge funds, 64 cents of this profit was retained as fees by hedge fund managers and the remaining 36 cents ended up in the pockets of hedge fund investors; a seemingly far cry from the stated promise of 2 and 20."

Second, Dai et al. (2020) point out that the nonlinear asymmetric structure of the performance fee has the same type of payoff of a call option from the standpoint of the manager: the managers get paid a portion of the profits but do not share in the downside. Hence, derivatives pricing theory tells us that "the value of the manager compensation grows almost linearly with the volatility of the underlying strategy, providing the manager with an incentive to increase the volatility of the returns of the portfolio. Similarly, when performance fees are based on relative performance against a benchmark, the value of the performance fees increases with the tracking error against the benchmark, incentivizing more deviation from the benchmark" (Dai et al., 2020).

Indeed, to avoid creating a gap between the stated and effective rates, on the one hand, and to actually align incentives, in what follows we propose compensation contracts with a simpler linear structure. Indeed, such a structure incentivizes the manager to look for excess returns but also to limit losses (instead of simply increasing risk). This recommendation is also motivated by the conclusions of an important strand of the incentives literature, which finds that linear compensation contracts are found to be highly effective (see, for instance, Holmstrom and Milgrom, 1987).

ⓘ **Notice:** Compensation contracts that are a linear function on performance can eventually generate negative "fees" (i.e., penalizations to the asset management company). However, the structures proposed hereafter are designed such that the manager has a clear mechanism to avoid penalization. Such a mechanism consists of using the dynamic asset-allocation strategies presented in Section 2.4, which ensure that cumulative losses in retirement income pesos do not surpass a maximum tolerance level or risk budget. The existence of a risk budget in the compensation function is also crucial to avoid killing the incentives of the fund manager to take on risk. Two of the four proposed fee structures embed the ratchet effect on wealth aforementioned, and as Figure 4 shows, all of them can be written as a linear function of the funding ratio.

ⓘ **Info:** The regulation in four out of nine countries that responded to our survey includes a penalty to pension fund management companies whenever the asset returns (over a rolling one to a few years) are below the peer average (minus a haircut or risk budget). The following incentive structures can also generate penalties, but as indicated below, they also imply a clear path for pension fund managers to avoid penalties at all times. The fee structures proposed induce an alignment of the asset-allocation strategies with the retirement income objective of the fund affiliates, unlike the current asset-only focus implied by the minimum asset return policy mentioned.

ⓘ **Notice:** The management fee functions presented hereafter, denoted as I.a, II.a, III.a, IV.a , are meant to generate periodic payments during the accumulation phase. Given that they are a function of the cumulative assets (on stock), they could be charged on an annual basis. On the other hand, the variations I.b, II.b, III.b, IV.b, are functions of the periodic (e.g., monthly) contributions to the fund (on flows). Of course, the rates to be charged on the flows should be much higher than the rates to be applied on the stock. For a conversion methodology between rates on stock (assets) and flows (contributions), refer to Mantilla-Garcia et al. (2021a).

The next two boxes indicate two possible fee structures that do not generate appropriate incentives, for the reasons mentioned in them, followed by eight boxes, each presenting a fee structure that incentivizes the use of asset-allocation strategies that secure a minimum increasing level of retirement income in pension funds.

Fees proportional to Assets Under Management

- $Q_t^a = b\,\tilde{N}_t$ where $0 < a < 1$
- The fees in current pesos are $Q_t^a \times B_t = bA_t$, hence it induces an asset-only focus, ignoring the variations in the pension liability ✗
- It increases with contributions to the fund, which do not depend on the investment strategy and are mandatory in many cases ✗
- There are no possible penalties to the management company

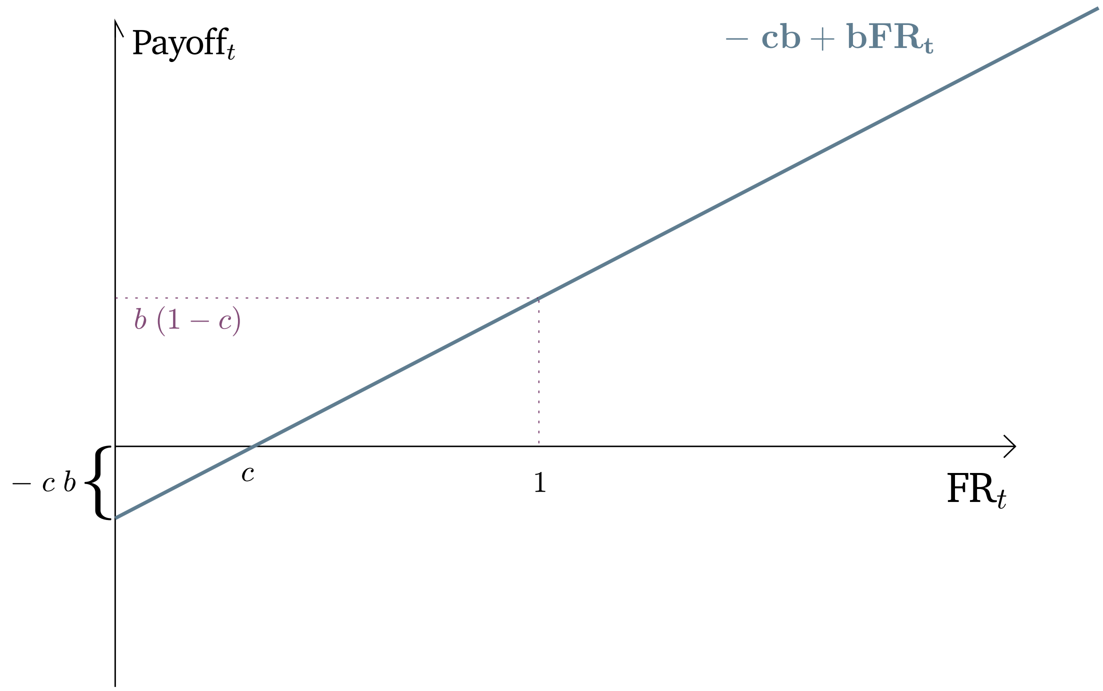
Fees proportional to the surplus in retirement income

- $Q_t^s = b\,(\tilde{N}_t - N_t)$ where $0 < b < 1$
- Performance fees (or penalties) are a constant percentage, b, of the increments (or decreases) in the level of securable retirement income $\tilde{N}_t$
- It only depends on the increments (or decreases) in retirement income generated by the pension fund.
- It does not depend on the mandatory contributions to the fund ✓
- There are possible penalties to the management company. There is no risk budget (penalties can happen even from the first year with a conservative allocation) ✗

(I.a) Performance fees fostering security of retirement income

- $Q_t^{as} = b\,(\tilde{N}_t - \kappa N_t)$ where $0 < \kappa < 1, \;\; 0 < b < 1$
- κ is the risk-aversion parameter, e.g. $\kappa \in \{0.9, 0.8, 0.7\}$, implying a risk budget $x = 1 - \kappa$, e.g. $x \in \{0.1, 0.2, 0.3\}$
- b is a percentage of the adjusted surplus in retirement income.
- There is a penalty to the fund management company, i.e. $Q_t^{as} < 0$ only if the funding ratio falss bellow κ
- The pension fund management company can avoid paying any penalties, i.e. always receiving fees, by using the portfolio insurance strategy (10), which secures an increasing level of minimum retirement income, i.e. $\tilde{N}_t \geq \kappa N_t$
- An equivalent expression is $Q_t^{as} = aN_t + b\,(\tilde{N}_t - N_t)$, with $a = b\,(1 - \kappa)$ hence it is composed by a performance fee proportional to the surplus, and a management fee proportional to cumulative contributions.

FIGURE 4 ▪ **PAYOFF FUNCTION FOR PENSION FUND MANAGEMENT COMPANY**



Payoff for the pension fund management company with management fees equal to Q^i, for $i \in \{a, s, as, as', cs, sr, csr\}$ as a function of the funding ratio $\left(\mathrm{FR}_t = \frac{\tilde{N}_t}{N_t}\right)$. For Q^a, the Payoff$_t$ = Q_t^a/N_t is the linear function shown in the figure with $c = 0$. For Q^s, the Payoff$_t$ = Q_t^s/N_t is the linear function shown in the figure with $c = 1$. For Q^{as}, the Payoff$_t$ = Q_t^{as}/N_t is the linear function shown in the figure with $c = \kappa$. For $Q^{as'}$, the Payoff$_t$ = $Q_t^{as'}/n_t$ is the linear function shown in the figure with $c = \kappa$. For Q^{cs}, the Payoff$_t$ = Q_t^{cs}/N_t is the linear function shown in the figure with $c = \kappa_t$. For Q^{sr}, the Payoff$_t$ = Q_t^{sr}/N_t is the linear function shown in the figure with $c = \kappa\left(1 + \frac{S_{t^\star}}{N_t}\right)$. For Q^{csr}, the Payoff$_t$ = Q_t^{csr}/N_t is the linear function shown in the figure with $c = \kappa_t + \kappa\left(\frac{S_{t^\star}}{N_t}\right)$. In all cases $0 < c$ and $0 < b$.

(I.b) Fees fostering insurance security of retirement income (contribution-based)

- $Q_t^{as'} = b\,\frac{n_t}{N_t}(\tilde{N}_t - \kappa N_t)$ where $0 < \kappa < 1, \quad 0 < b < 1$
- κ is the risk aversion parameter, e.g. $\kappa \in \{0.9, 0.8, 0.7\}$
- $b_t = b\frac{n_t}{N_t}$ is a variable percentage of the adjusted surplus in retirement income. b_t decreases as the retirement date approaches [a]
- There is a penalty to the fund management company, i.e. $Q_t^{as'} < 0$, only if the funding ratio falls below κ.
- The pension fund management company can avoid paying any penalties by using the portfolio insurance strategy (10), which secures an increasing level of minimum retirement income, i.e. $\tilde{N}_t \geq \kappa N_t$
- An equivalent expression is $Q_t^{as'} = an_t + bn_t\,(FR_t - 1)$, with $a = b\,(1-\kappa)$ hence it is composed by a funding ratio based performance fee, and a management fee per contribution

[a] b_t decreases with every new contribution. Assuming there are frequent contributions, the parameter decreases as the retirement date approaches.

(II.a) Fees fostering security of retirement income and decreasing risk-taking

- $Q_t^{cs} = b\,(\tilde{N}_t - \kappa_t N_t)$ where $0 < \kappa_t \leq \kappa, \quad 0 < \kappa < 1 \quad 0 < b < 1$
- κ_t is the risk aversion parameter, which increases as the retirement date approaches [a]
- b is a percentage of the adjusted surplus in retirement income
- There is a penalty to the fund management company, i.e. $Q_t^{cs} < 0$, only if the funding ratio falls below κ_t.
- The pension fund management company can avoid paying any penalties, i.e. always receiving fees, by using the portfolio insurance strategy (11), which secures an increasing level of minimum retirement income, i.e. $\tilde{N}_t \geq \kappa_t N_t$
- An equivalent expression is $Q_t^{cs} = a\hat{N}_T + b\,(\tilde{N}_t - N_t)$, with $a = b\,(1-\kappa)$ hence it is composed by a performance fee proportional to the surplus, and a constant management fee

[a] κ_t increases with every new contribution. Assuming there are frequent contributions, the parameter increases as the retirement date approaches. For full details on κ_t, see section 2.4.

(II.b) Fees fostering security of retirement income and decreasing risk-taking (contribution based)

- $Q_t^{cs'} = b\,\frac{n_t}{\tilde{N}_T}(\tilde{N}_t - \kappa_t N_t)$ where $0 < \kappa_t < \kappa, \quad 0 < \kappa < 1 \quad 0 < b < 1$
- κ_t is the risk aversion parameter, which increases as the retirement date approaches[a]
- b is a percentage of the adjusted surplus
- There is a penalty to the fund management company, i.e. $Q_t^{cs'} < 0$, only if the funding ratio falls below κ_t.
- The pension fund management company can avoid paying any penalties, i.e. always receiving fees, by using the portfolio insurance strategy (11), which secures an increasing level of minimum retirement income, i.e. $\tilde{N}_t \geq \kappa_t N_t$
- An equivalent expression is $Q_t^{cs'} = a n_t + b n_t (RFR_t - AFR_t)$, with $a = b\,(1-\kappa)$ hence it is composed by a performance fee plus a management fee per contribution

[a] κ_t increases with every new contribution. Assuming there are frequent contributions, the parameter increases as the retirement date approaches. For full details on κ_t, see section 2.4.

(III.a) Fees fostering security of retirement income and surplus ratchet

- $Q_t^{sr} = b\,(\tilde{N}_t - \kappa_t^{\star} N_t)$ where $0 < \kappa_t^{\star}, \quad 0 < \kappa < 1 \quad 0 < b < 1$
- $\kappa_t^{\star}$ is the risk aversion parameter that embeds a ratchet effect to preserve $\kappa\%$ of the maximum attained surplus in retirement income[a]
- b is a percentage of the adjusted surplus
- There is a penalty to the fund management company, i.e. $Q_t^{sr} < 0$, only if the funding ratio falls below $\kappa_t^{\star}$
- The pension fund management company can avoid paying any penalties, i.e. always receiving fees, by using the portfolio insurance strategy (13), which secures an increasing level of minimum retirement income, i.e. $\tilde{N}_t \geq \kappa_t^{\star} N_t$
- An equivalent expression is $Q_t^{sr} = aN_t + b\left((\tilde{N}_t - N_t) - \kappa(\tilde{N}_{t^\star} - N_{t^\star})\right)$, with $a = b\,(1-\kappa)$ hence it is composed by a surplus based performance fee plus a management fee proportional to the cumulative contributions
- A performance fee proportional to the excess surplus is paid only if $\kappa\%$ of the maximum historical surplus has been preserved. Losses of more than $(1-\kappa)\%$ of the previously attained surplus actually decrease the total payment

[a] For full details on $\kappa_t^{\star}$, see section 2.4.

(III.b) Fees fostering security of retirement income and surplus ratchet (contribution based)

- $Q_t^{sr'} = b\,\frac{n_t}{N_t}(\tilde{N}_t - \kappa_t^{\star} N_t)$ where $0 < \kappa_t^{\star},\quad 0 < \kappa < 1 \quad 0 < b < 1$
- $\kappa_t^{\star}$ is the risk aversion parameter that embeds a ratchet effect to preserve $\kappa\%$ of the maximum attained surplus in retirement income[a]
- b is a percentage of the adjusted surplus
- There is a penalty to the fund management company, i.e. $Q_t^{sr'} < 0$, only if the funding ratio falls below $\kappa_t^{\star}$
- The pension fund management company can avoid paying any penalties, i.e. always receiving fees, by using the portfolio insurance strategy (13), which secures an increasing level of minimum retirement income, i.e. $\tilde{N}_t \geq \kappa_t^{\star} N_t$
- An equivalent expression is $Q_t^{sr'} = a n_t + b\,\left((FR_t - 1) - \kappa(\tilde{N}_{t^\star} - N_{t^\star})/N_t\right)$ with $a = b\,(1-\kappa)$. Hence it is composed by funding ratio based performance fee plus a management fee per contribution
- A performance fee proportional to the excess surplus is paid only if $\kappa\%$ of the maximum historical surplus has been preserved. Losses of more than $(1-\kappa)\%$ of the previously attained surplus actually decrease the total payment

[a] For full details on $\kappa_t^{\star}$, see section 2.4.

(IV.a) Fees fostering security of retirement income, surplus ratchet and decreasing risk-taking

- $Q_t^{csr} = b\,(\tilde{N}_t - \kappa_t^{\circ} N_t)$ where $0 < \kappa_t^{\circ},\quad 0 < \kappa < 1 \quad 0 < b < 1$
- κ_t° is the risk aversion parameter that embeds a decreasing risk-taking and a ratchet effect to preserve $\kappa\%$ of the maximum attained surplus in retirement income[a]
- b is a percentage of the adjusted surplus
- There is a penalty to the fund management company, i.e. $Q_t^{csr} < 0$, only if the funding ratio falls below κ_t°
- The pension fund management company can avoid paying any penalties, i.e. always receiving fees, by using the portfolio insurance strategy (15), which secures minimum retirement income, i.e. $\tilde{N}_t \geq \kappa_t^{\circ} N_t \geq \kappa_t N_t$
- An equivalent expression is $Q_t^{csr} = a\hat{N}_T + b\,\left((\tilde{N}_t - N_t) - \kappa(\tilde{N}_{t^\star} - N_{t^\star})\right)$ with $a = b\,(1-\kappa)$. Hence it is composed by an excess surplus performance fee, plus a constant management fee
- A performance fee proportional to the excess surplus is paid only if $\kappa\%$ of the maximum historical surplus has been preserved. Losses of more than $(1-\kappa)\%$ of the previously attained surplus actually decrease the total payment

[a] For full details on κ_t°, see section 2.4.

(IV.b) Fees fostering security of retirement income, surplus ratchet and decreasing risk-taking (cb)

- $Q_t^{csr'} = b\,\frac{n_t}{\tilde{N}_T}(\tilde{N}_t - \kappa_t^{\circ} N_t)$ where $\quad 0 < \kappa_t^{\circ}, \quad 0 < \kappa < 1 \quad 0 < b < 1$
- κ_t° is the risk aversion parameter that embeds a decreasing risk-taking and a ratchet effect to preserve $\kappa\%$ of the maximum attained surplus in retirement income[a]
- b is a percentage of the adjusted surplus
- There is a penalty to the fund management company, i.e. $Q_t^{csr'} < 0$, if the funding ratio falls below κ_t°
- The pension fund management company can avoid paying any penalties, i.e. always receiving fees, by using the portfolio insurance strategy (15), which secures a minimum retirement income, i.e. $\tilde{N}_t \geq \kappa_t^{\circ} N_t \geq \kappa_t N_t$
- An equivalent expression is $Q_t^{csr'} = a n_t + b'\; n_t \left((\tilde{N}_t - N_t) - \kappa(\tilde{N}_{t^\star} - N_{t^\star}) \right)$ with constants $a = b\,(1-\kappa)$. Hence it is composed by an excess surplus performance fee, plus a management fee per contribution
- A performance fee proportional to the excess surplus is paid only if $\kappa\%$ of the maximum historical surplus has been preserved. Losses of more than $(1-\kappa)\%$ of the previously attained surplus actually decrease the total payment

[a] For full details on κ_t°, see section 2.4.

3 Empirical Illustration

In this section, we present a historical simulation of the benchmark strategies (I.) to (IV.) described in Section 2.4. These strategies consist of a rule-based dynamic allocation between an LHP and PSP. The PSP can be any diversified portfolio aiming to capture risk premia in a risk-efficient manner. On the other hand, the LHP is a portfolio that aims to replicate the value of a retirement bond, which is by design a bond that matches the cash flows of a generic pension stream. The pension stream starts at a predefined retirement date. Hence, there is a specific LHP per cohort. On the other hand, the PSP can be the same for any cohort.[17] Given that retirement bonds are not available in the market, we use as a proxy the value of a LHP that aims to replicate as close as possible, the value of the retirement bond. The latter can be accomplished using standard duration matching or by minimizing deviations in the sensitivities to the three Nelson-Siegel factors of the LHP, from the respective factor sensitivities of the retirement bond.

In Section 3.1, we present a historical simulation of three possible implementations of the LHP and compare them with the historical returns of the Coltes LP and Coltes UVR indices, published by the Colombian Stock Exchange (Bolsa de Valores de Colombia). The latter have been recently selected by the Colombian pension fund regulation as benchmarks for local fixed income[18]: first, a standard duration–matching strategy[19]; and, second, a strategy hedging the first-order sensitivities to the three Nelson-Siegel factors. The latter follows the methodology in Mantilla-Garcia et al. (2019b), which introduces leverage control through lasso-type regularization constraints. The first version of the strategy is constrained to have a level of leverage below the maximum leverage required by the standard duration–matching strategy, and the second version of the strategy is implemented with a long-only constraint (no leverage). The three strategies are implemented with monthly rebalancing.

Finally, in Section 3.2, we present the result of the historical simulation of the target income benchmark strategies and compare it to the historical performance of the mandatory pension funds in Colombia. As input data, we use the (gross) historical returns of the three types of mandatory pension funds, called "conservador," "moderado," and "mayor riesgo," over the last 10 years (about the time when the "multifondos" started in Colombia).[20] Furthermore, to simulate the value of a retirement account with funds invested in the actual and simulated retirement portfolios, we use estimates of the average income of workers in Colombia, among those who actively contribute to their (mandatory) retirement accounts. The Colombian pension fund supervisor provided both the historical returns of the pension funds and the average income across age groups (*Superin-*

17. In the portfolio theory literature, the PSP is also referred to as the myopic portfolio, as it maximizes short-term expected risk- adjusted return. Hence, its construction is in principle independent of the investment horizon.
18. The source for the historical returns of the Coltes indices is www.bvc.com.co
19. For a textbook presentation of duration matching, see Martellini et al. (2003), Chapter 5.
20. Each of the four pension fund management companies in the country has its own version of each of these three types of funds. We did not observe significant differences in terms of risk and return across the funds in each category, so we used the average return of the four funds in each of the three categories.

tendencia Financiera de Colombia, delegatura para pensiones). The latter varies around two minimum wages (the minimum wage for 2020 was of 877,803 Colombian pesos).

The historical simulation of the retirement accounts is presented in terms of retirement income pesos, $\tilde{N}_t$, and in terms of the funding ratio defined in Section 2. To calculate the funding ratio, we use as the denominator the value of a portfolio that is fully invested in the LHP simulated in 3.1. We find that the ability to control losses of retirement pesos of the target income strategies can provide very substantial increases in retirement income, with a lower level of risk. Furthermore, we find that the three types of multifondos in Colombia have all about the same level of risk, which is relatively high when measured in terms of pesos of retirement income.

3.1 Replicating Retirement Bonds: The Liability-Hedging Portfolio

To estimate the non-arbitrage price of the retirement bond described in Section 2, we use the parameters of the Nelson-Siegel model published daily by the Colombian Central Bank,[21] as well as used an annual COLA factor adjustment of 3% to set the cash flows of the bond, which are distributed over a 20-year payout period.[22] We simulate the retirement bond price for the last 10 years of the accumulation phase of a cohort of individuals retiring on 2020-11-12 (hence, the start of the sample period is 2010-11-12).[23] As hedging instruments, we use a set of 60 Colombian treasury bonds (TES).[24] The number of life bonds over the sample period varies between 22 and 38. The shortest duration of the live bonds in our data set varies from 1 day to 1 year, and the longest duration of live bonds varies between 7.7 and 13.4 years.

We simulate three hedging strategies that aim to replicate the value of the retirement bond, using the available government bonds. All the strategies were simulated daily and implemented them with a monthly rebalancing frequency. At each date, we apply a liquidity filter excluding bonds presenting a return of zero for 5 days in a row on their market price. We first simulated a standard duration–matching strategy, which requires selecting only two bonds among the available set, at each rebalancing date. At the end of each month, we selected the bonds with the longest and shortest available duration.

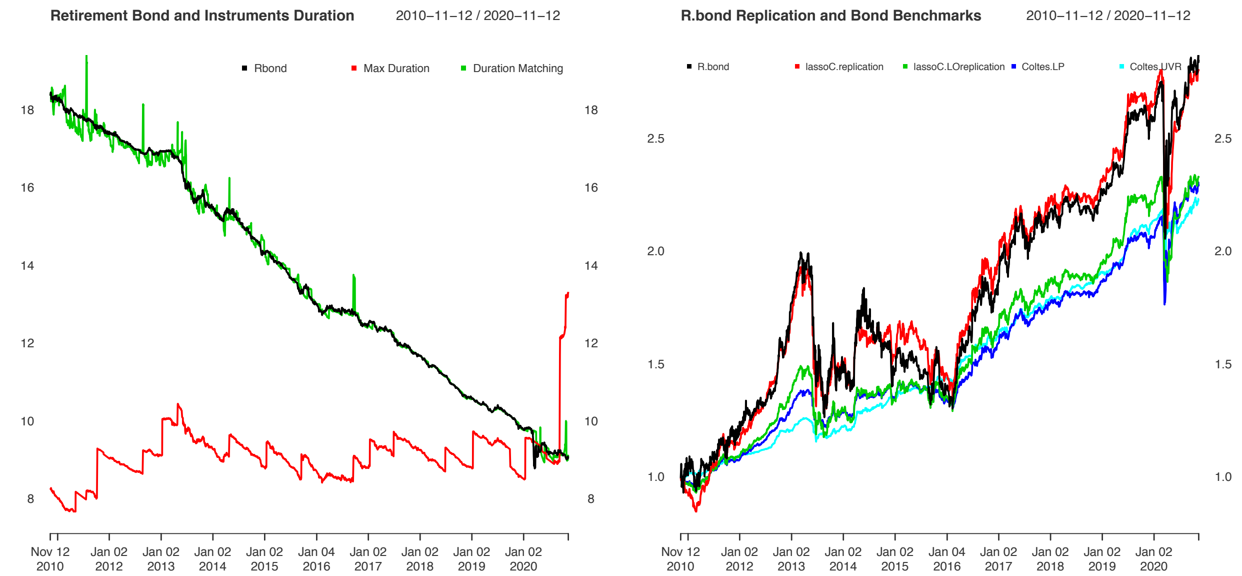
21. The historical time series of the Nelson-Siegel yield curve was downloaded from www.banrep.gov.co/es/estadisticas/tes.

22. We used 20 years of life expectancy after retirement as in Muralidhar (2015); Muralidhar et al. (2016); Merton and Muralidhar (2017); and Kobor and Muralidhar (2018).

23. The latter corresponds to the date when we retrieved the data for this project.

24. We excluded bonds with a floating coupon rate and only considered bonds issued in Colombian pesos. The data set also comprises some strips and short-term TES. We retrieved the data from the data laboratory of Universidad de Los Andes.

FIGURE 5 ▪ **DURATION AND VALUE OF RETIREMENT BOND AND FIXED-INCOME INDICES**



The left panel presents the duration of the retirement bond, the duration-matching strategy, and the maximum duration among the hedging instruments used, at each point in time. The right panel presents the value of the retirement bond (Rbond) and the lasso-type bond replication strategy hedging first-order sensitivities to the three NS factors (introduced in Mantilla-Garcia et al., 2019b). The lassoC refers to an implementation of the latter strategy with a leverage level below the leverage required by the standard duration–matching strategy, and the lassoC.LO is an implementation with a long-only constraint (no leverage). The figure also displays the total return of the Coltes LP and Coltes UVR bond indices, which are currently used as bond benchmarks in the Colombian pension fund regulation.

The left panel of Figure 5 presents the duration of the retirement bond over the sample period, along with the duration of the duration-matching strategy, and the maximum available duration among the set of hedging instruments used. As the figure shows, the initial duration of the retirement bond is 18 years, whereas the longest duration on the first day of the sample, among the set of hedging instruments used, is 8.3 years. That mismatch in duration implies that even a simple duration-matching strategy would require a significant amount of leverage (see, for instance, Martellini et al., 2003, Chapter 5). However, such a mismatch in duration does not exist today, as shown in Table 1, which presents a summary of the durations of all the bonds issued by the Colombian government over the last 11 years. Indeed, the longest duration available in 2020 was 18 years. However, matching the duration of the retirement bonds for younger cohorts, even today, would require leverage (either directly or through a hedging overlay).

To this point, evidence shows that large pension funds around the world that use leverage for interest-rate-hedging purposes have lower risk (Beath, 2019).

TABLE 1 ▪ **DURATION RANGE OF COLOMBIAN GOVERNMENT BONDS OVER TIME**

COLOMBIAN GOVERMENT BONDS DURATION (YEARS)					
Year	Issuances	Minimum	Maximum	Average	Std.Dev.
2009	279	0,21	10,42	2,63	2,92
2010	298	0,22	10,25	3,20	3,28
2011	308	0,24	9,70	3,14	3,07
2012	146	0,25	10,84	5,74	2,76
2013	194	2,51	16,17	7,14	3,43
2014	179	2,15	15,62	7,55	3,54
2015	202	0,76	15,01	7,32	3,55
2016	243	0,76	14,39	6,25	3,82
2017	250	0,77	13,87	6,21	3,88
2018	249	0,77	18,67	6,49	3,89
2019	187	0,73	13,38	5,98	4,07
2020	217	0,77	18,17	8,20	5,13

Source: Colombian Central Bank (Banco de la República)

ⓘ **Notice:** The use of leverage in the LHP aims to reduce risk, while it may increase risk if it is used for performance-seeking purposes in the PSP. It is good practice that the regulation allows for some leverage on the LHP to better control the asset-liability mismatch risk, as some leverage can be necessary to better match the duration of pension liabilities when the longest available duration of regular bonds is shorter than the duration of the pension liabilities. Furthermore, regulation should have a different treatment of the leverage used in the LHP and of the leverage used in the PSP. Nonetheless, too much leverage can hurt replication performance in the LHP. As illustrated in this section, the generalized interest rate hedging strategies introduced in Mantilla-Garcia et al. (2019b) allow to perform a more precise and general hedging of changes of the yield curve, with even slightly lower leverage than the standard duration matching (which hedges only level shifts). In this sense, a simple and sensible regulation rule is to set an upper bound on leverage for liability-hedging purposes equal to the leverage implied by standard duration–matching strategies.

Besides the duration-matching strategy, we simulate the generalized lasso-type hedging strategies introduced in Mantilla-Garcia et al. (2019b). The latter are designed to reduce the model risk and the impact of parameter-estimation errors in the hedging portfolio. Furthermore, these strategies do not require a preselection of hedging instruments (all the bonds available can enter the portfolio), and the maximum leverage of the

portfolio can be set ex ante.[25] Moreover, they can be used to hedge more general movements of the yield curve, unlike duration matching, which only hedges parallel shifts in the curve.[26]

We used the generalized hedging approach of Mantilla-Garcia et al. (2019b) to minimize deviations in the first-order sensitivities to the three Nelson-Siegel factors of the retirement bond. We simulate a version of the lasso strategy with a leverage upper bound equal to the maximum leverage of the standard duration– matching strategy. The average leverage level over the sample period was 118% for the standard duration matching and 110% for the generalized hedging strategy (see last row of Table 2). Given that the longest available duration in government bonds as of 2020 was 18 years, as of today, the strategies would not require any leverage to replicate a retirement bond corresponding to a retirement date 10 years from now. Furthermore, notice that our bond data set does not include all of the bonds that have been issued historically by the Colombian government, and, in fact, the mismatch would have been smaller even historically (to see this, compare the Maximum Duration column in Table 1 with the series of the Maximum Duration series in the left panel of Figure 5).

TABLE 2 ▪ **SUMMARY OF FUNDING RATIO DEVIATIONS OF HEDGING STRATEGIES**

	Duration	lassoC	lassoC.LO	Coltes.LP	Coltes.UVR
MEAD(FR0)	0.09	0.03	0.13	0.16	0.16
MAAD(FR0)	0.17	0.16	0.26	0.31	0.37
MDD(FR0)	0.19	0.20	0.30	0.35	0.42
Mean.Lev	1.18	1.10	0.00	0.00	0.00

Summary statistics of deviations in funding ratio with respect to 100%, for the duration-matching strategy, the lasso-type strategies hedging the first order sensitivities to the three Nelson-Siegel factors, and for the Colombian bond indices Coltes LP and Coltes UVR. Funding ratio calculated from an initial contribution at 2010-11-12, simulated until 2020-11-12. MEAD stands for median absolute deviation and MAAD for maximum absolute deviation (see Definition 5).

TABLE 3 ▪ **DEVIATIONS IN YIELD CURVE FACTOR EXPOSURES OF HEDGING STRATEGIES RELATIVE TO THE RETIREMENT BOND**

	Duration	lassoC	lassoC.LO
D0.MEAD	0.13	0.23	4.24
D0.MAAD	1.44	2.63	10.67
D1.MEAD	0.74	0.63	0.54
D1.MAAD	2.72	2.48	0.98
D2.MEAD	0.43	0.29	0.76
D2.MAAD	1.70	1.21	1.49

Summary of hedging strategies' factor sensitivities deviations relative to the retirement bond sensitivities. MEAD stands for median absolute deviation and MAAD for maximum absolute deviation (see Definition 6).

25. Using more bonds can reduce portfolio turnover relative to duration matching, when applied over relatively long periods. Furthermore, the hedging portfolio is much less concentrated and is more adapted to constitute benchmark bond indices, as it would induce a much lower trading pressure on particular bonds than a standard duration approach.
26. However, during the accumulation phase, the exposure of the retirement bond to changes in slope and curvature is relatively small compared to its sensitivity to changes in the level factor.

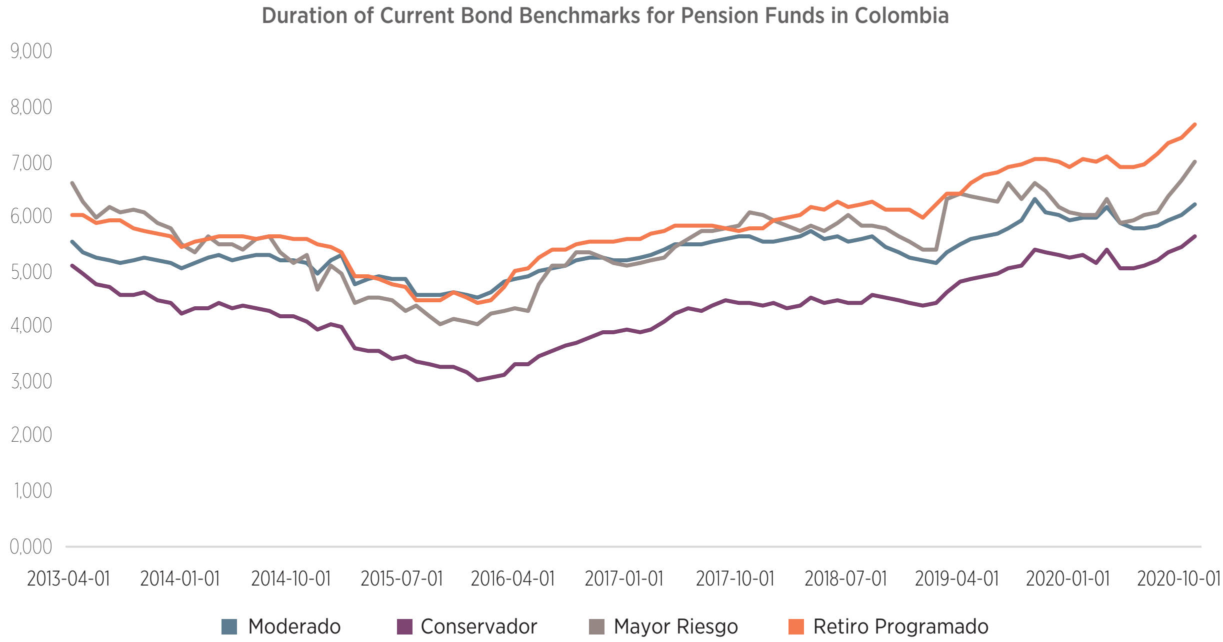
Table 2 also presents the median and maximum absolute deviations in the funding ratio of the aforementioned hedging strategies, as described in Definition 5, relative to the retirement bond. Furthermore, it presents the respective risk metrics for the Coltes LP and Coltes UVR indices over the sample period. Moreover, we simulate another version of the generalized hedging strategy but under the long-only constraint (no leverage), denoted lassoC.LO in Table 2 and Figure 5. The right panel of Figure 5 and Table 2 show that such a strategy presents smaller mismatch to the retirement bond than the Coltes indices. However, the mismatch in ex ante factor exposures (see Table 3), as well as in ex post funding ratio deviations (see Table 2 and the right panel of Figure 5), remains very substantial, especially compared to the levered strategies.[27]

ⓘ **Notice:** The Coltes LP and Coltes UVR are currently defined as benchmarks for the local fixed-income asset class for pension funds in Colombia. It is good practice to have generic asset class benchmarks for the components in the PSP. However, the aforementioned indices are not adequate benchmarks for the liability hedging component of the strategy for two reasons. First, there should be an LHP benchmark per cohort, as the (decreasing) duration of such benchmarks critically depends on the target retirement date of each cohort. Second, the current benchmark indices present a systematic duration mismatch relative to the cash flows that the pension funds owe to finance, as discussed hereafter.

Although the duration of the Coltes indices is not readily available in their provider website, officials from the Superintendencia de Colombia provided the author their estimations for the Duration of the Colombian "Multifondos" Benchmark indices for the period 2013-04-30 to 2020-11-30, presented in Figure 6. The duration of the benchmark indices over that period fluctuated between 3 and 7.7 years. This range is in stark contrast with the duration series of the retirement bond corresponding to the cohort retiring in November 2020 of Figure 5, which starts at 18 years (10 years prior to the retirement date) and ends at 9 years at the retirement date. This gap clearly indicates that there is no duration-matching objective in the construction of those benchmark indices, relative to the duration of the cash flows that pension funds owe to finance. The latter can also be presumed from the fact that such indices are long only (i.e., they do not have any leverage in their position). Another interesting fact observed in Figure 6 is that the duration of the benchmark corresponding to the multifondos conservador is systematically lower than the duration of the other benchmarks. Shorter duration bonds have by construction lower asset-only return volatility than longer duration bonds; however, they present a larger duration mismatch to the cash flows that the pension funds will face. This reveals that, in the current regulatory framework for pension funds, risk is conceived as a variation of asset-only returns instead of the uncertainty in the number of retirement income pesos that they will be able to finance. Moreover, according to the answers from our survey, the absence of regulatory incentives to hedge interest rate risk (such as LHP-type benchmarks), to separate the strategy into LHP and PSP functions, and to report liability-driven risk metrics is very pervasive in the region (see Figures 2 and 7).

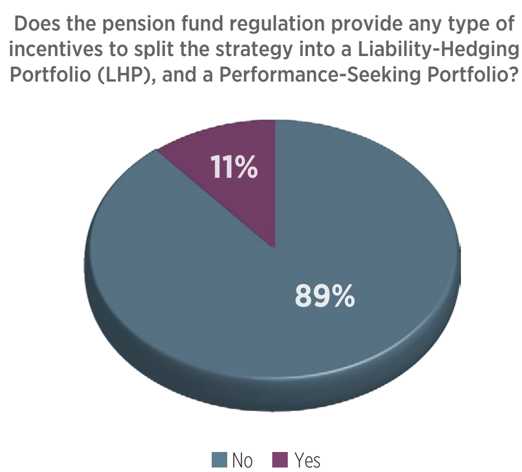
27. Table 3 presents the median and maximum absolute deviations in sensitivities to changes in level, slope, and curvature as captured by the Nelson-Siegel model (see Definition 6 in Section 2.3).

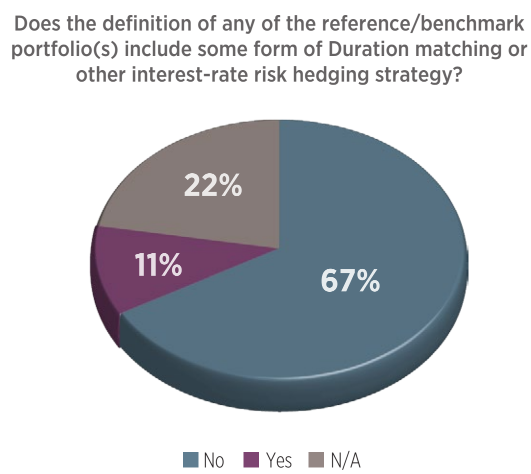
FIGURE 6 ▪ DURATION OF BOND BENCHMARKS FOR "MULTIFONDOS" IN COLOMBIA



Duration of bond benchmarks for multifondos pension funds in Colombia for the period 2013-04-30 to 2020-11-30. Source: Superintendencia Financiera de Colombia, delegatura para pensiones.

FIGURE 7 ▪ SURVEY ANSWERS ON LIABILITY-HEDGING REGULATORY INCENTIVES





3.2 ▸ Target Retirement Income Benchmarks

Here, we present the results of a historical simulation of a retirement account for a typical female worker in Colombia. We assume the worker makes monthly contributions to her retirement account during the 30 years leading to her retirement date on 2020-11-12, when she is 57 years old (the legal retirement age in Colombia for women). To estimate the level of contributions, we use data provided by Superintendencia de Colombia on the average contributions of workers by age group, among workers who presented 100% contribution density in 2020. Information on average contribution levels to retirement accounts is not readily available for previous years; hence, we use historical variations in the minimum wage from 1991 to 2020 to extrapolate the age-contribution structure observed in 2020, adjusted by previous year's income and price levels. This assumes the average worker's contribution level varies in the same percentage as the minimum salary.[28] The average contribution observed in 2020 across age groups between 26 and 56 years implies that the average income among workers with 100% contribution density is between 2.09 and 3.59 times the minimum wage (we deduct this taking into account that net contributions to mandatory pension funds are 11.5% of income). Sources of data containing information about the income distribution across all workers in Colombia show that a large portion of the active population has income levels below the minimum wage due to the high level of labor informality in the country. Hence, the average income in Colombia is significantly lower than the estimated numbers. The reason for the discrepancy in average income is that the population earning below the minimum wage through informal labor arrangements during a significant part of their active lives is severely underrepresented in contribution averages considering full contribution density. We focus on the case of a typical worker with 100% contribution density because the purpose of this exercise is to evaluate the potential value added that pension fund managers can bring using more adapted forms of risk management for a given contribution level. Analyzing the impacts of different contribution patterns and the possibly differentiated parametrization of strategies as a function of the socioeconomic characteristics of individuals with different backgrounds is out of the scope of this study.

We simulate the impact of different investment strategies on the level of retirement income over the last 10 years, which corresponds roughly to the start of the multifondos in Colombia. We assume that all contributions prior to the start of the simulation period (2010-11-10), were invested at the same rate of return for all investment alternatives considered. Assuming an annual return of 8%, we calculated the value of the retirement account 10 years before the retirement date. The resulting value (35,351,406 COP), is the initial value of the fund. In other words, the latter amount is treated as the initial contribution to the fund.

This simulates a situation in which 10 years ago, the assets invested in the same fund, were migrated to different portfolios. We simulate the following different independent alternatives: first, we simulate three different alternatives in which the initial value and later monthly contributions are invested in each of the three aver-

28. The sources used for the historical levels of minimum salary are www.banrep.gov.co/sites/default/files/paginas/ mercado_laboral.xls and www.salariominimocolombia.net/historico.

age multifondos (conservador, moderado, and mayor riesgo).[29] Then, we simulate two different alternatives for each of the four target-income strategies presented in Section 2.4. The first version of the strategy uses the conservador fund as its PSP. As we show in detail, such an alternative does not generate any surplus in retirement income relative to the risk-free alternative of investing 100% in the LHP over the sample period. In unreported results, we found that no simple combination of the current asset class benchmark indices defined by the regulation (i.e., the Colcap index, the MSCI World, the Coltes LP and Coltes UVR, or the Barclays bond global aggregate) consistently outperformed the LHP. However, this is not an indication that in the future the situation will necessarily repeat, particularly taking into account that interest rates are now lower than at the beginning of the sample period. Hence, to better illustrate the behavior of the target income strategies in a scenario in which the PSP outperforms the LHP, we also simulated the four strategies using an ETF that replicates the NASDAQ stock index[30] —which outperformed the LHP over the period of analysis. Again, this does not necessarily indicate that this will occur in the future again, and we are not proposing to use the NASDAQ as a benchmark for an asset class, but these two tests can be interpreted as optimistic and less favorable scenarios. For each configuration, we simulated the target-income strategies for three different levels of risk budget: $\kappa \in \{0.9, 0.8, 0.7\}$. The interpretation of these risk budgets is that the strategy protects at least a proportion κ of the retirement income that would have been generated by taking no risk (i.e., investing 100% of all contributions in the LHP).

The summary statistics of the variations in the number of securable retirement units $\tilde{N}_t$ and the funding ratios of the alternatives we considered are presented in Table 4 for strategies with the conservador fund as PSP, and in Table 5 for strategies with the NASDAQ tracker fund as PSP, for the three levels of risk budget. As presented in Table 4, the monthly retirement income resulting from investing the contributions in the multifondos are 596,104; 594,534; and 583,942 COP for the conservador, moderado and mayor riesgo funds, respectively (see Table 4, first column). Comparing these numbers with the average inflation-adjusted income over the last 10 years of 1,690,479 (implied by the average level of contributions for workers with full contribution density), the three funds generate a replacement rate of about 35%. Furthermore, the three funds present practically the same level of risk, as measured by the volatility of funding ratios (13%, 14% and 15%), the maximum drawdown of funding ratios (45%, 47% and 47%), and the minimum level of funding ratios observed during the sample period (0.64, 0.62, and 0.61), for conservador, moderado, and mayor riesgo, respectively. This illustrates how being unprotected with respect to changes in the discount rates that determine the conversion rate of assets to retirement income almost washes out any differences among the three multifondos.

The resulting replacement rate of 35% for the Multifondos simulation contrasts with the replacement rate of 45% obtained with the risk-free alternative of investing only in the LHP, and with the replacement rates of the target income strategies that range between 42% and 44%, depending on the level of risk budget (see Table 4, third column). Furthermore, the increase in retirement income pesos with respect to the level of income provided by the moderado fund ranges between 18.2% and 25.4% for the target income strategies (see

29. Return series for the multifondos are available from 03-23-2011. Prior to that, there were only moderado funds. In order to complete a period of 10 years of simulation, we use the returns of the average moderado fund for the first 131 days of the sample to complete the return series of the conservador and mayor riesgo funds. This emulates a situation in which the savings of investors who had invested in the moderado fund were migrated after 4 months to the other funds for the rest of the 10-year period. This actually happened for many investors when the multifondos started in Colombia.

30. The ETF used is the Invesco QQQ Trust Series 1.

Table 4, second column). In this less favorable scenario, the increase in retirement income pesos that would have provided the risk-free LHP relative to the moderado fund is 28.1%.

The latter result might be incorrectly seen as an indication that the optimal solution would be to invest all savings in the risk-free LHP alternative. However, as mentioned, prior performance is not a good indicator of future performance, especially considering that the current level of interest rates is lower than the prevailing level at the beginning of the sample period in this simulation. Therefore, to simulate a more optimistic scenario, we present the summary statistics resulting from simulating the target income strategies implemented with an ETF tracking the NASDAQ index as the PSP (see Table 5). By maintaining the same level of protection in terms of minimum retirement pesos, the strategies are able to capture part of the upside performance that the index experienced during the simulation period. This produces an increase in term retirement income pesos relative to the moderado fund of 37.7% to 43.1% for the most conservative strategies ($\kappa = 0.9$), 48.1% to 58.1% for strategies where $\kappa = 0.8$, and an increase of 59.2% to 68.9% for the most aggressive strategies ($\kappa = 0.7$). In terms of replacement rate, the level of 35% obtained using the multifondos contrasts with those obtained by the target income strategies in this more favorable scenario, which range from 48% to 61%.

In terms of risk, the target income strategies present lower annualized volatility of funding ratios than the multifondos. These range from 2% ($\kappa = 0.9$) to 9% ($\kappa = 0.7$) compared to 13% to 15% for the multifondos. Similarly, the maximum drawdown of their funding ratios ranges from 6% ($\kappa = 0.9$) to 21% for the strategies with $\kappa = 0.7$ compared to 45% to 47% for the multifondos. Also note that the minimum funding ratio observed over the sample period for the target income strategies ranges from 0.86 to 0.97, compared to 0.61 to 0.64 for the multifondos.

The funding ratios of the three funds at the end of the sample period were 0.78, 0.78, and 0.77 for the conservador, moderado, and mayor riesgo fund types, respectively. Thus, the level of replacement income produced by the multifondos in this case would have been 22% to 23% less than the retirement income generated by the LHP. This contrasts with the resulting funding ratio of the four target income strategies, which resulted in the less favorable scenario (Table 4) in a funding ratio of 0.92 to 0.94 for the strategies with the largest risk budget of $\kappa = 0.7$, 0.95 to 0.96 for the strategies with $\kappa = 0.8$, and 0.97 to 0.98 for the strategies with $\kappa = 0.9$. In the more favorable scenario (Table 5) the funding ratio of the target income strategies ranges from 1.24 to 1.35 for the strategies with $\kappa = 0.7$, 1.16 to 1.23 for the strategies with $\kappa = 0.8$ and 1.07 to 1.12 for strategies with $\kappa = 0.9$.

ⓘ **Note:** Although of primary importance for the worker, the level of replacement rate, and of retirement income depend on both the level of contributions and the returns on the portfolios managed by the pension fund management company. Hence, the latter metrics mix the manager's responsibility (investment performance), with a factor exogenous to the manager in mandatory funds (the contribution level). In that sense, a more pure metric to evaluate the effectiveness of investment decisions made by a fund manager is the funding ratio, which compares the retirement income generated by the portfolio with the risk-free alternative of investing all contributions in the LHP.

Figures 8 to 10 present the full time series of the funding ratios for all simulated strategies. While the funding ratios of all three funds appear very volatile, the funding ratios of the target income strategies all remain relatively far from the respective floor values that they protect, even in the less favorable ones (right panels). In general, however, it would be possible for these strategies to present lower funding ratios, but never below their protected level $\kappa_t >= \kappa$. The latter characteristic is a clear advantage from a risk management standpoint, as it provides a clear limit to the uncertainty of future retirement income prospects. Furthermore, it facilitates a dialog with fund affiliates, particularly when the floor value is expressed in amount of retirement income (current pesos). As Figures 11 to 13 show, the strategies protect a strictly increasing minimum level of retirement income at all times. On the other hand, the level of securable retirement income from the three multifondos present much more extreme swings over the last 10 years, ending up below the floor value for the strategies with loss aversion parameters of 0.8 and 0.9. They also significantly trespassed the floor values of the strategies with the highest-risk budgets ($\kappa = 0.7$) in early 2013 (see Figures 8 to 13).

ⓘ **Note:** The minimum or Floor level of retirement income secured by the benchmark allocation strategies proposed, when expressed in real pesos, increases exclusively with new contributions from the affiliates for strategies (I.) and (III.), and for strategies (II.) and (IV.), it increases with every contribution as well as during periods of strong relative performance of the PSP, by securing as well a proportion κ of previous cumulative gains. For the communication with investors in real time, the floor value for the retirement income level can be expressed in nominal pesos at each moment in time. In this case, the floor value in nominal pesos increases due to the COLA adjustment even if there are no contributions over a period of time.

Figure 14 presents the time series of the allocation to the PSP of the target income strategies over the less favorable scenario (right panel) and the more favorable scenario (left panel) for the three levels of risk budget. To further characterize the target income strategies, a few remarks are in order. First, note that the strategies with a decreasing risk-taking mechanism (strategies II. and IV.) start with an extra allocation to the PSP relative to the strategies without programmed derisking of 3.2 percentage points for $\kappa = 0.9$, 6.4 percentage points for $\kappa = 0.8$, and 9.6 points for $\kappa = 0.7$. Second, in the less favorable scenario (during which the PSP performed negatively relative to the LHP), all strategies' allocations to the PSP presented downward trends. However, recall that all the proposed benchmark strategies have a multiplier parameter of $m = 1$ (see Equation 7). Hence, the downward trend in PSP allocation we observed is due to the natural drift of weights induced by the variation in the relative value of the PSP versus the LHP, as opposed to the active trend-following behavior that portfolio insurance implemented with $m > 1$. Indeed, when $m = 1$, the proposed strategies do not have the typical trend-following behavior of standard portfolio insurance strategies. To see this, note that for periods when there are no contributions, no trading happens for the strategies that do not embed a ratchet mechanism (strategies I. and II.). In other words, although the relative weights of the LHP and the PSP change over time as their values vary, the number of shares of LHP and PSP held do not change as a function of the relative value movements of the two building blocks of the portfolio. For strategies (III.) and (IV.), the ratchet

effect that they have in fact induces a contrarian trading behavior during periods of strong outperformance of the PSP; this offsets the allocation drift caused by the relative value trend.

Note: Dynamic allocation strategies can be classified into trend-following, contrarian, and buy-and-hold (trend neutral). Trend-following strategies tend to suffer relative to contrarian strategies in the absence of relative trends (see Perold and Sharpe, 1988). The standard constant proportion portfolio insurance strategy (CPPI), which is a particular case of of the allocation class described in Equation (7), are trend-following when implemented with a multiplier parameter $m > 1$. However, for $m = 1$, the CPPI strategy is actually a buy-and-hold strategy (i.e., it is neither trend-following nor contrarian).

4 Conclusion and Policy Recommendations

Our empirical results confirm that the largest investment risk faced by pension funds is their inherent exposure to variations in interest rates. As a matter of fact, interest rates inevitably determine the conversion rate of assets to replacement income that happens at the retirement date,[31] and the variations in that conversion rate are so large (due to the long duration/horizon nature of the retirement investment problem) that they dwarf other investment risks. Indeed, we find that any differences in asset returns across the three types of Colombian multifondos, called conservador, moderado, and mayor riesgo, are almost completely washed out when their value is expressed in their equivalent number of retirement income pesos (instead of current pesos). In other words, when the variations in the 'exchange rate' are taken into account. The reason behind this is that, none of the funds (or their benchmarks) seem to use any form of effective hedging against interest rate movements. Unfortunately, this is a very pervasive problem in DC plans, as pointed out by Merton (2014). Our findings also confirm the results of Mantilla-Garcia et al. (2019a), who show that over the typical length of the accumulation phase, a short-duration bond index is as risky as an equity index in terms of the number of retirement income pesos that an investment in those portfolios can generate. The latter is also in line with Merton (2014), who illustrates that cash is an extremely volatile asset when measured in number of retirement units, and hence a risky investment for pension funds.

In this context, having appropriate metrics to measure the performance and risk of pension funds relative to their objective of financing replacement income, as well as using appropriate risk management tools that allow them to limit the impact of the constant variations in interest rates, are crucial elements for pension fund managers to build efficient investment strategies on behalf of their affiliates. In the historical simulation, we found that the target income strategies proposed would have generated an increase in retirement income pesos of around 20% to 25% in the less favorable scenario relative to the income generated by multifondo portfolios. In the more favorable scenario considered, the increase would have been around 40% to 73%, depending on the strategies' level of risk budget. In all cases, these improvements would have been achieved with a lower level of risk than all the multifondos. In terms of replacement rate, given the level of contributions assumed (which corresponds to 100% contribution density), the level obtained by investing all contributions in the Colombian multifondos is 35% for the three types of funds. In contrast, in the less favorable scenario, the target income strategies would have generated replacement rates between 42% and 45%; in the more favorable scenario, the replacement rates would have been between 50% and 61% depending on the strategies' level of risk budget.

Based on the analysis done in this project, hereafter we provide a series of recommendations for pension fund regulatory and supervisory bodies:

31. With the exception of fund devolution situations that may happen in some countries. In Colombia, the BEPS program provides an alternative form of replacement income that is now used to avoid pension fund devolution for individuals who do not fulfill the legal requirements for entitlement to a regular pension.

- According to our survey, current regulations in Uruguay, the Dominican Republic, Colombia, and Chile define a monetary penalty to pension fund management companies if their managed funds present a return on assets below a peer-based benchmark (minus a haircut). This creates a perverse incentive for pension fund managers, as it forces an asset-only focus in their risk management and asset-allocation strategies. Furthermore, the current structure of management fees is a fixed percentage of the mandatory monthly contributions to the funds. This fee type does not provide any direct incentive for pension fund management companies to improve their investment strategies. Hence, we recommend that regulators remove the minimum asset-only return penalty and replace it with one of the management fee structures proposed in Section 2.5, which foster the risk management strategies illustrated in this document. Furthermore, our proposed compensation rules would create an incentive structure that puts investment strategies at pension fund managers' center of attention, aligning them with the objectives of pension fund affiliates.
- The Liability Hedging Portfolio (LHP), and the Performance Seeking Portfolio (PSP) are two separate elements of the strategy with completely different objectives and characteristics. The LHP seeks to protect investors from the constant variations in interest rates, and the PSP aims to maximize risk-adjusted performance using diversification. It is considered good practice to foster a separate treatment for these two distinct 'defense' and 'attack' functions, within pension fund management companies. Hence, we recommend that supervisors provide separate benchmarks for these two types of portfolios and demand that pension funds report different risk and performance metrics adapted for each block, as well as other metrics for the overall portfolio. In particular, we recommend providing a separate LHP benchmark of the type discussed in Section 3.1 for each retirement date cohort. Furthermore, we recommend using the risk and performance metrics in Sections 2.3 and 3.1 for the LHP and the overall portfolio, as well as providing benchmarks for the overall portfolio similar to the target income strategies presented in Section 2.4.
- We recommend that any regulatory limits on leverage be treated differently if this mechanism is used to reduce interest rate risk exposure within the LHP, or for performance purposes in the PSP. In particular, we recommend allowing for at least some leverage for hedging purposes whenever there is a significant mismatch between the maximum duration among the government bonds available in the market and the duration of the pension stream that each pension fund aims to finance.
- We found no readily available historical data on the duration levels of the current benchmark indices Coltes LP or Coltes UVR on their publisher's website (www.bvc.com.co) or elsewhere, nor could we find clear selection and weighting rules for them. Publishing such information would improve their transparency, which is a crucial value add of benchmark indices in general.

TABLE 4 ▪ RETIREMENT INCOME AND FUNDING RATIO SUMMARY STATISTICS FOR MULTIFONDOS AND TARGET INCOME STRATEGIES WITH CONSERVADOR FUND AS PSP FROM 2010-11-12 UNTIL 2020-11-12

	$\tilde{N}_T$	$\Delta RI\%$	RR	$MDD(\tilde{N}_t)$	FR_T	Vol(FR)	MDD(FR)	Max(FR)	Min(FR)
R.bond	761798.11	28.13	0.45	-0.00	1.00	0.00	-0.00	1.00	1.00
Moderado	594534.52	0.00	0.35	0.40	0.78	0.14	0.47	1.16	0.62
Mriesgo	583942.49	-1.78	0.35	0.40	0.77	0.15	0.47	1.16	0.61
Conservador	596104.49	0.26	0.35	0.38	0.78	0.13	0.45	1.16	0.64
$FR(90)$	745228.74	25.35	0.44	0.01	0.98	0.01	0.05	1.02	0.96
$FR^*(90)$	747863.35	25.79	0.44	0.01	0.98	0.01	0.05	1.02	0.97
$CFR(90)$	742079.21	24.82	0.44	0.01	0.97	0.01	0.07	1.02	0.95
$CFR^*(90)$	745531.40	25.40	0.44	0.01	0.98	0.01	0.06	1.02	0.96
$FR(80)$	728659.38	22.56	0.43	0.03	0.96	0.02	0.10	1.03	0.93
$FR^*(80)$	733371.43	23.35	0.43	0.02	0.96	0.02	0.09	1.03	0.94
$CFR(80)$	722360.31	21.50	0.43	0.04	0.95	0.03	0.13	1.04	0.91
$CFR^*(80)$	728534.78	22.54	0.43	0.04	0.96	0.02	0.11	1.04	0.92
$FR(70)$	712090.02	19.77	0.42	0.06	0.93	0.03	0.15	1.05	0.89
$FR^*(70)$	718311.92	20.82	0.42	0.05	0.94	0.03	0.14	1.05	0.90
$CFR(70)$	702641.41	18.18	0.42	0.08	0.92	0.04	0.19	1.06	0.86
$CFR^*(70)$	710794.54	19.55	0.42	0.07	0.93	0.04	0.17	1.06	0.88

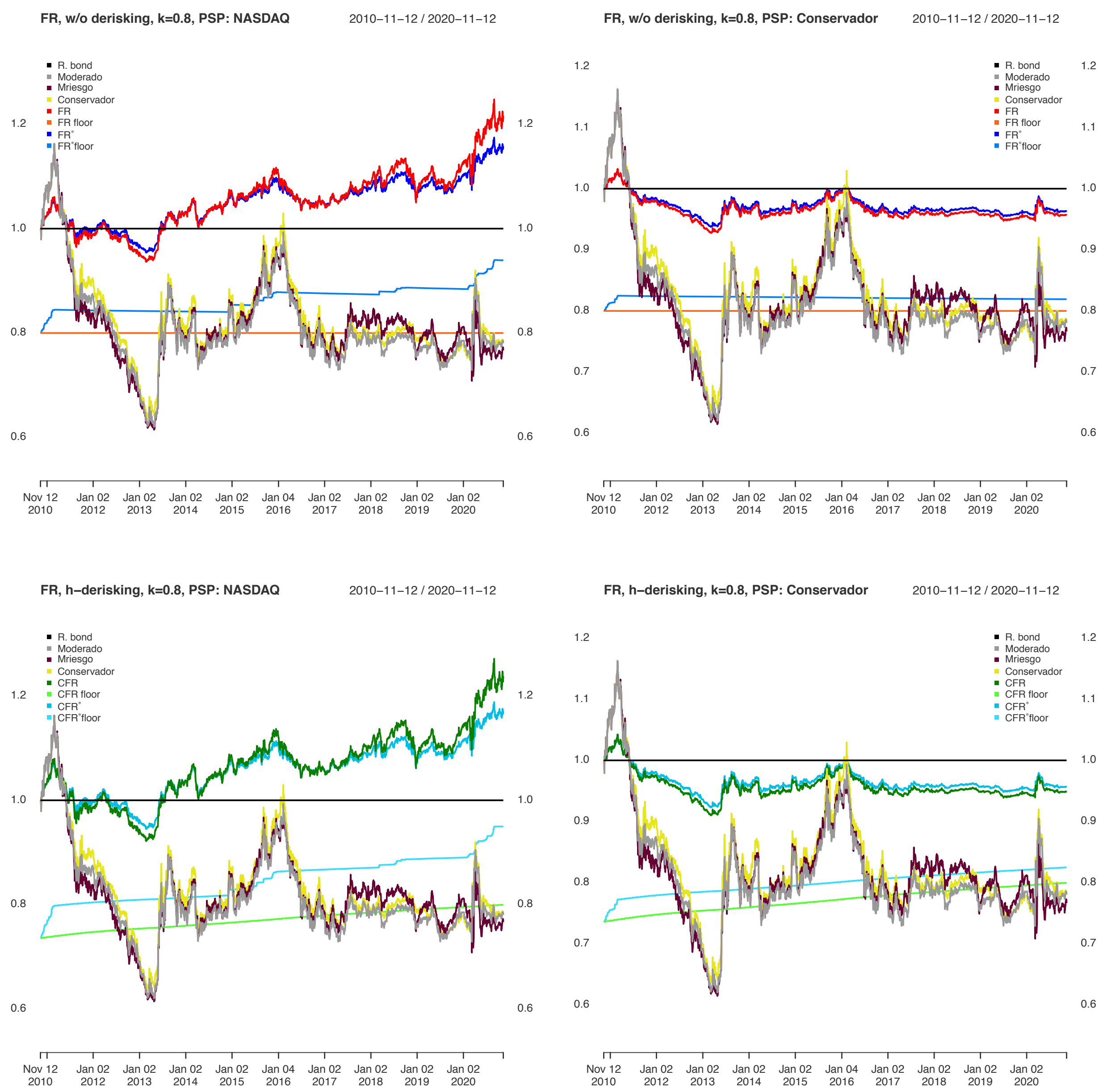
TABLE 5 ▪ RETIREMENT INCOME AND FUNDING RATIO SUMMARY STATISTICS FOR MULTIFONDOS AND TARGET INCOME STRATEGIES WITH NASDAQ FUND AS PSP FROM 2010-11-12 UNTIL 2020-11-12

	$\tilde{N}_T$	$\Delta RI\%$	RR	$MDD(\tilde{N}_t)$	FR_T	Vol(FR)	MDD(FR)	Max(FR)	Min(FR)
R.bond	761798.11	28.13	0.45	-0.00	1.00	0.00	-0.00	1.00	1.00
Moderado	594534.52	0.00	0.35	0.40	0.78	0.14	0.47	1.16	0.62
Mriesgo	583942.49	-1.78	0.35	0.40	0.77	0.15	0.47	1.16	0.61
Conservador	596104.49	0.26	0.35	0.38	0.78	0.13	0.45	1.16	0.64
$FR(90)$	842660.33	41.73	0.50	0.03	1.11	0.03	0.06	1.12	0.97
$FR^*(90)$	818912.05	37.74	0.48	0.02	1.07	0.02	0.05	1.08	0.98
$CFR(90)$	850882.83	43.12	0.50	0.03	1.12	0.03	0.08	1.14	0.96
$CFR^*(90)$	823675.14	38.54	0.49	0.02	1.08	0.02	0.06	1.09	0.97
$FR(80)$	923522.56	55.34	0.55	0.07	1.21	0.06	0.12	1.25	0.94
$FR^*(80)$	880332.56	48.07	0.52	0.05	1.16	0.04	0.10	1.17	0.95
$CFR(80)$	939967.55	58.10	0.56	0.09	1.23	0.06	0.15	1.27	0.92
$CFR^*(80)$	890445.61	49.77	0.53	0.07	1.17	0.05	0.12	1.19	0.94
$FR(70)$	1004384.79	68.94	0.59	0.11	1.32	0.08	0.17	1.37	0.90
$FR^*(70)$	946379.37	59.18	0.56	0.08	1.24	0.06	0.14	1.27	0.93
$CFR(70)$	1029052.27	73.09	0.61	0.14	1.35	0.09	0.21	1.41	0.88
$CFR^*(70)$	962487.35	61.89	0.57	0.11	1.26	0.07	0.18	1.30	0.91

FIGURE 8 ▪ **FUNDING RATIO GENERATED BY STRATEGIES I. TO IV. WITH $\kappa = 0.9$**

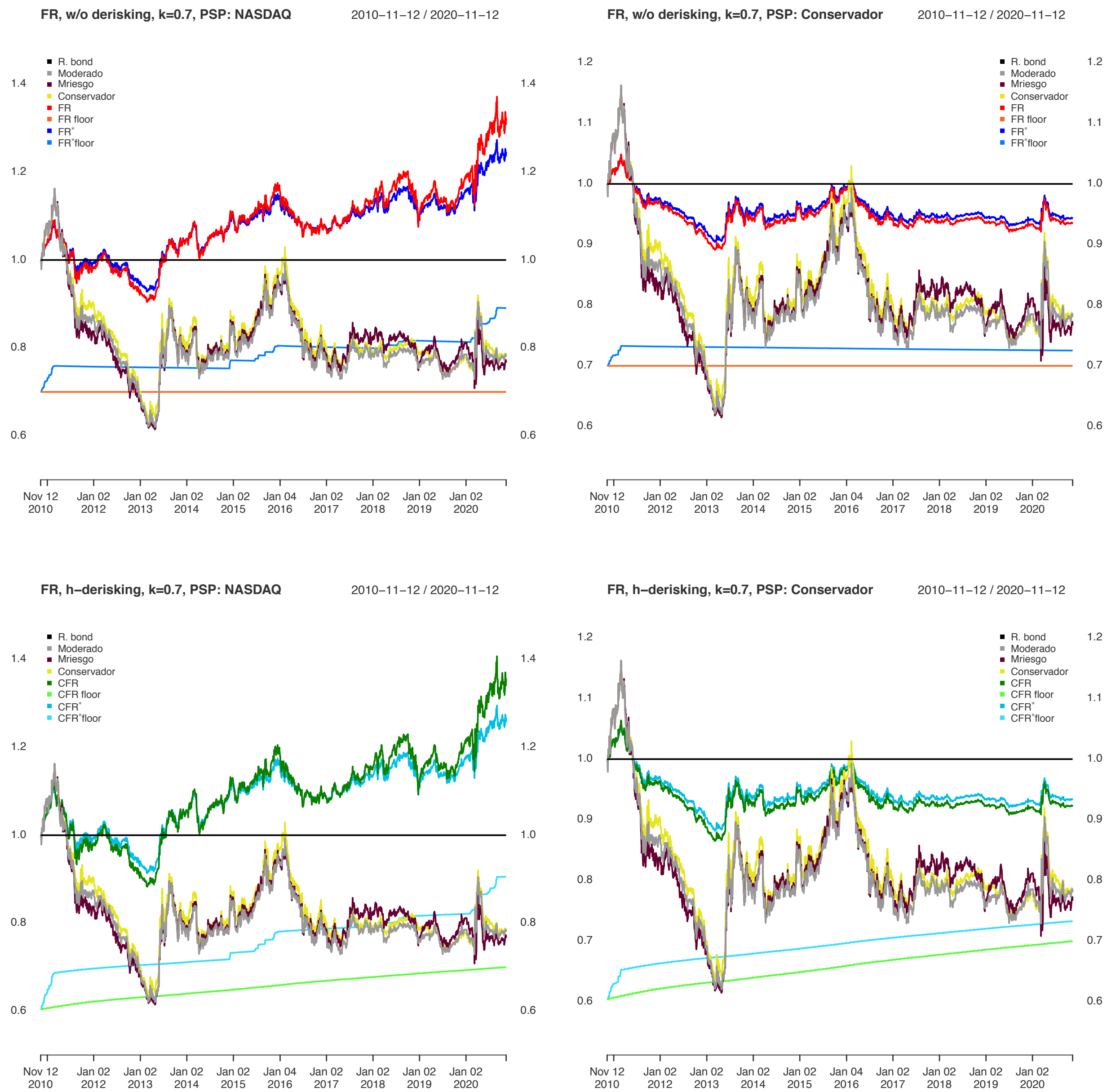
Funding ratio generated by investing every monthly contribution into the Colombian mandatory pension funds 'Conservador', 'Moderado', and 'Mayor Riesgo', and on the target-income strategies I. to IV. with $\kappa = 0.9$. The graph also presents the Floor value of the funding ratio of the latter strategies. The right panels present the historical simulation using as PSP of the target income strategies the 'Conservador' fund. The left panels present the historical simulation using an ETF tracking the NASDAQ index as the PSP.

FIGURE 9 ▪ **FUNDING RATIO GENERATED BY STRATEGIES I. TO IV. WITH** $\kappa = 0.8$



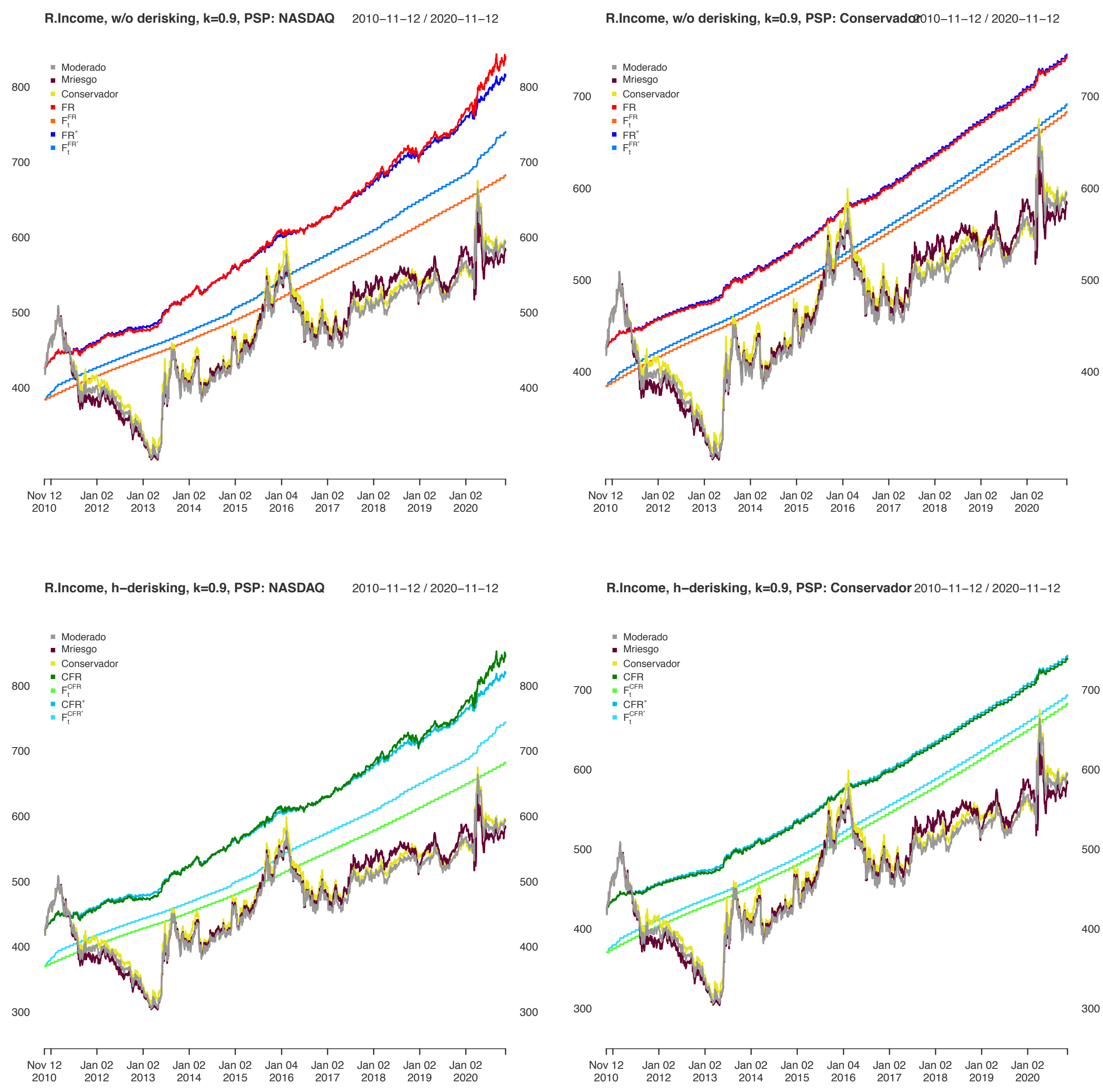
Funding ratio generated by investing every monthly contribution into the Colombian mandatory pension funds 'Conservador', 'Moderado', and 'Mayor Riesgo', and on the target-income strategies I. to IV. with $\kappa = 0.8$. The graph also presents the Floor value of the funding ratio of the latter strategies. The right panels present the historical simulation using as PSP of the target income strategies the 'Conservador' fund. The left panels present the historical simulation using an ETF tracking the NASDAQ index as the PSP.

FIGURE 10 ▪ **FUNDING RATIO GENERATED BY STRATEGIES I. TO IV. WITH $\kappa = 0.7$**



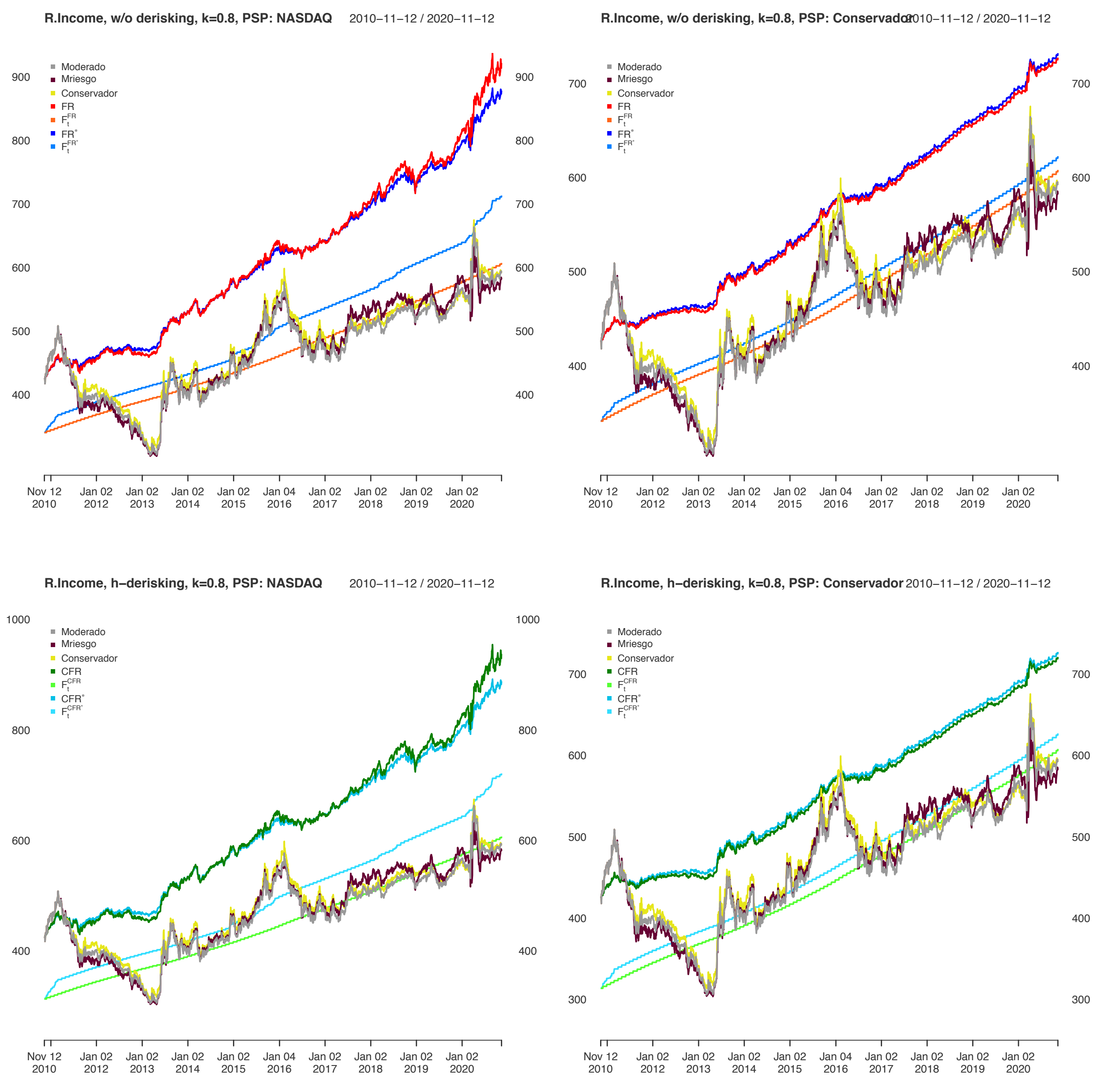
Funding ratio generated by investing every monthly contribution into the Colombian mandatory pension funds 'Conservador', 'Moderado', and 'Mayor Riesgo', and on the target-income strategies I. to IV. with $\kappa = 0.7$. The graph also presents the Floor value of the funding ratio of the latter strategies. The right panels present the historical simulation using as PSP of the target income strategies the 'Conservador' fund. The left panels present the historical simulation using an ETF tracking the NASDAQ index as the PSP.

FIGURE 11 ▪ **NUMBER OF RETIREMENT PESOS GENERATED BY STRATEGIES I. TO IV. WITH $\kappa = 0.9$**



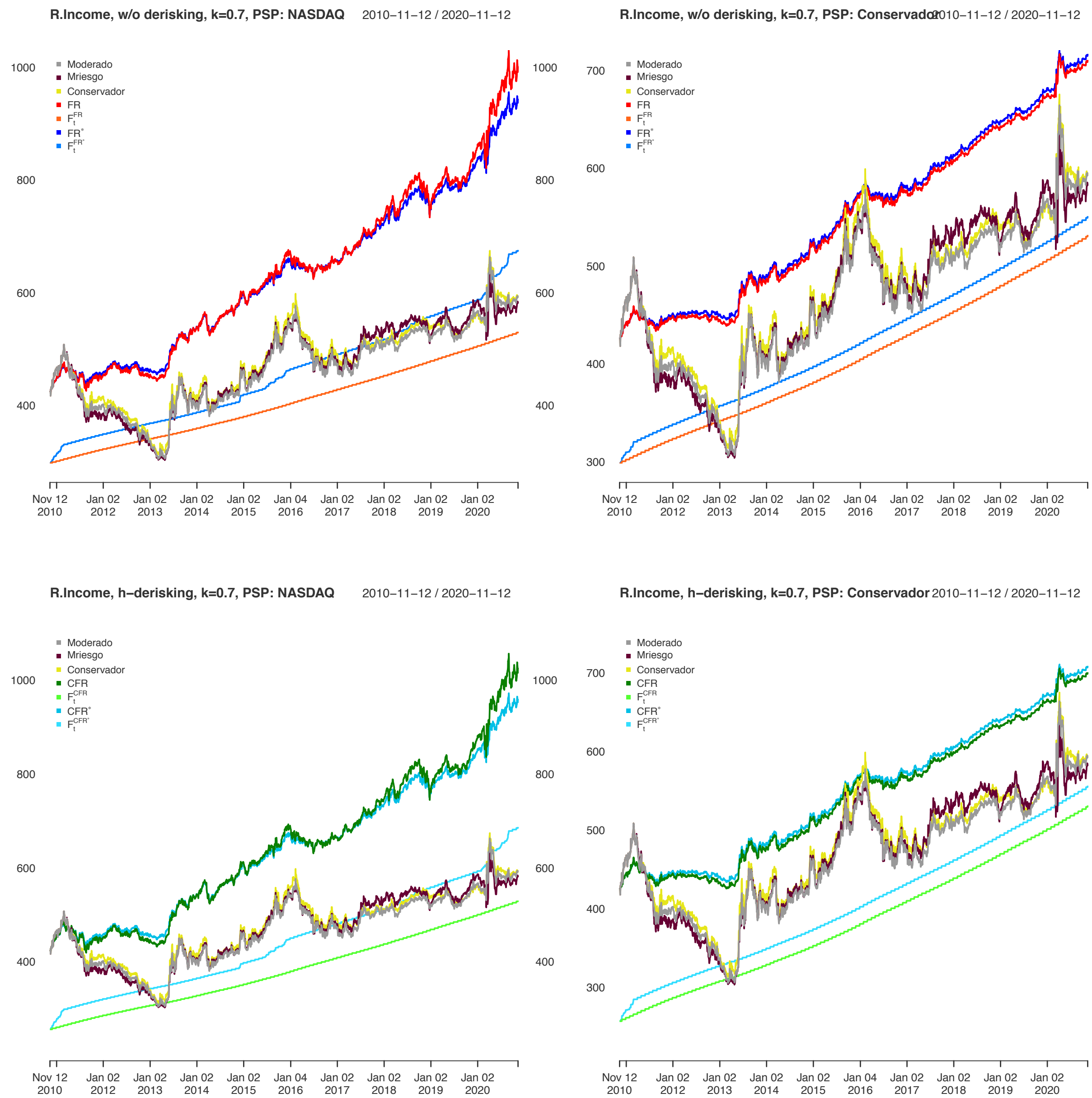
Number of securable retirement income pesos $\tilde{N}_t$ ('000) generated by investing every monthly contribution into the Colombian mandatory pension funds 'Conservador', 'Moderado', and 'Mayor Riesgo', and on target-income strategies I. to IV. with $\kappa = 0.9$. The graphs also presents the Floor value series of the latter strategies. The right panels present the historical simulation using as PSP of the target income strategies the 'Conservador' fund. The left panels present the historical simulation using an ETF tracking the NASDAQ index as the PSP.

FIGURE 12 ▪ **NUMBER OF RETIREMENT PESOS GENERATED BY STRATEGIES I. TO IV. WITH $\kappa = 0.8$**



Amount of securable retirement income pesos $\tilde{N}_t$ ('000) generated by investing every monthly contribution into the Colombian mandatory pension funds conservador, moderado, and mayor riesgo, and on-target-income Strategies I to IV $(\kappa = 0.8)$. Graphs also present the latter strategies' floor value series. Right panels present historical simulation using target income strategies of the conservador fund as PSP. Left panels present historical simulation using an ETF tracking the NASDAQ index as PSP.

FIGURE 13 ▪ **NUMBER OF RETIREMENT PESOS GENERATED BY STRATEGIES I. TO IV. WITH $\kappa = 0.7$**



Amount of securable retirement income pesos $\tilde{N}_t$ ('000) generated by investing every monthly contribution into the Colombian mandatory pension funds conservador, moderado, and mayor riesgo, and on-target-income Strategies I to IV ($\kappa = 0.7$). Graphs also present the latter strategies' floor value series. Right panels present historical simulation using target income strategies of the the conservador fund as PSP. Left panels present historical simulation using an ETF tracking the NASDAQ index as PSP.

FIGURE 14 ▪ **PSP ALLOCATION OF TARGET INCOME STRATEGIES**

PSP allocation of target income strategies. Right panels present historical simulation using conservador fund as PSP (less favorable scenario). Left panels present historical simulation using an ETF tracking the NASDAQ index as PSP (more favorable scenario). Top, middle, and bottom panels correspond to strategies with $\kappa = 0.9$, $\kappa = 0.8$, and $\kappa = 0.7$, respectively.

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