Optimal Commodity Price Hedging

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Abstract

The dependence of many countries in the region on oil exports makes those countries vulnerable to oil price volatility. In particular, the sharp declines in price observed between 2014 and 2016 resulted in the weakening of public finances in these countries, which saw significant debt increases. A strategy to mitigate the effect of sharp falls in oil prices would enable oil-exporting countries to reduce the impact on their public finances. This paper shows that using put options to insure against oil price hikes lowers public debt and fiscal deficits.

1 Introduction

Commodities have been the cornerstone of the development history of the Latin America and Caribbean (LAC) region and an essential part of its integration into the world economy. Commodity prices are highly interconnected with economic activity in this region and have been a leading indicator of economic performance (Fernández et al., 2018). In the past, favorable terms of trade resulting from higher commodity prices provided tailwinds to propel economic growth: average growth in LAC was on average 4.04 percent during the international commodity price boom of 2003–2011, compared with an average growth rate of 1.29 percent observed between 2012 and 2018.

Despite the importance of commodity exports, LAC countries remain vulnerable to price fluctuations. However, mechanisms such as hedges are rarely used. The purpose of this note is to present a quantitative model that enables the assessment of the impact of hedging on the debt profile of a commodity-exporting country. We build a model and calibrate it to Colombia and find that hedging benefits the government budget by reducing greatly the debt burden and increasing the sales of commodity stock when that commodity's real price is depressed. We show quantitatively that the debt to GDP ratio can be reduced about 10 percentage points, reducing the fiscal deficit about half of a percentage point of GDP and the likelihood of a financial crisis.

Commodity dependence and concentration in LAC are high. Most countries in the region are net commodity exporters, with net commodity exports representing, on average, 10 percent of gross domestic product (GDP). In terms of concentration, in 14 LAC economies more than 50 percent of the gross exports are commodities. The dependence on particular commodity categories varies widely across countries. For instance, the Andean countries have a strong dependence on minerals and metals (10 percent of total exports), while Argentina, Uruguay, and Paraguay are concentrated in agricultural products (8 percent of total exports). The most prominent net exporters of energy, particularly oil, are Venezuela, Colombia, and Ecuador (24 percent of total exports). For some countries, the concentration occurs not only in particular product categories, but also in specific products. The most prominent examples are copper in Chile (45 percent), soybeans in Paraguay (32 percent), gas in Bolivia (32 percent), oil in Ecuador and Colombia (29 percent and 28 percent, respectively) and soybean meal in Argentina (15 percent).

Beyond their impact through international trade, commodities also affect the fiscal accounts of LAC countries, both directly through the impact on fiscal revenues from commodity-related sectors and indirectly through the effect on overall economic activity. Consider first the direct linkages: revenues from commodities are, on average, 3 percent of GDP in LAC, though this figure hides a wide heterogeneity across countries. The revenues originated in the energy sector can be sizeable, as with Ecuador (5.6 percent of GDP), Bolivia (5.2 percent of GDP), and Mexico (4.8 percent of GDP). With regard to other sectors, the direct effects are less prominent, though still significant. Chile, Bolivia, and Peru (0.9 percent, 0.8 percent, and 0.6 percent of GDP, respectively) stand out in the case of mining, while Argentina and Paraguay (0.5 percent and 0.2 percent of GDP, respectively) do so in the case of agricultural products.

Indirect effects resulting from the interlinkages between the commodity sector and the rest of the economy may also be significant, leading to a larger overall impact of commodity prices on fiscal accounts. LAC countries' fiscal positions react strongly to commodity price shocks. Samaké and Spatafora (2012) find that a 10 percent increase in export prices leads to an increase in tax revenue ranging between 0.5 and 0.6 percent of GDP. Medina (2016) also finds that commodity price fluctuations have large effects on government spending, notably the encouragement of procyclical fiscal policy. As a result of budget uncertainty and procyclical spending, volatility in commodity prices may threaten fiscal sustainability.

Excessive commodity price volatility might have severe adverse consequences for the macroeconomic stability and economic development of countries in the region. Governments should carefully assess their exposure to these shocks and evaluate the available risk mitigation mechanisms in this context. Governments in the region have relied mainly on policy buffers and nonfinancial hedging to mitigate the effects of fluctuations in commodity prices. Governments have strived to create fiscal buffers in the form of fiscal space through fiscal consolidations, fiscal rules, and debt repayment. Other buffers have included commodity-based stabilization and saving funds, such as, in the case in Chile, the Economic and Social Stabilization Fund, which aims to shield fiscal revenues from the volatility of copper prices. The effectiveness of these schemes to manage the effects of price volatility is linked to strong governance and transparency.

Why is hedging an attractive solution to address LAC's exposure to risk? Besides the results on the debt profile and the likelihood of crisis highlighted in this paper, the literature offers several reasons that justify the use of hedging. First, reducing income volatility guarantees a stable path of consumption and enhances welfare. Because countries in the region are highly procyclical, hedging reduces the need for social expenditure adjustments. Second, a more stable stream of income reduces the need for stabilization schemes based on savings. If anything, the effectiveness of savings is exacerbated by the ex ante mitigation of risk. For instance, Samaké and Spatafora (2012) find that international reserves are ineffective at dampening the effects of commodity price volatility by themseles. Third, reducing income risks sends a positive signal to the markets, impacting credit conditions and improving countries' ability to borrow abroad (Borensztein et al., 2013).

Despite its potential benefits, the hedging of commodity price risk is not a widespread practice among commodity-dependent governments. One notable exception is Mexico's government, which has implemented the world's top sovereign derivative trade. Mexico hedged its oil exports revenue in 2019 by spending US$1.23 billion on put options to assure an average price of US$55 per barrel. Another is Brazil, which, through its state-controlled company Petrobras, spent US$320 million in 2019 on put options to secure a minimum price of US$60 per barrel. These options were sold in August 2019 amid a sharp fall in crude prices.

The rest of this note is organized as follows: in Section 2, we discuss the main features of our quantitative model. Section 3 describes the calibration and discusses the main findings. Finally, section 4 concludes.

2 Model

In this section, we propose a real model of a small open economy (SOE) facing fluctuations in the real price of oil. The real price of oil is represented by p_t and follows a stochastic process across time. The SOE is populated by a representative household, a representative tradable good firm, a representative nontradable good firm, and a government.

Representative household. The representative household in the SOE has preferences throughout time deriving utility over the stream of private and public consumption goods and labor hours supplied. The preferences across time are represented by

$$\mathbb{E}_0\left[\sum_{t=0}^{\infty}\beta^t U\left(x_t, L_t\right)\right], \tag{1}$$

where $\beta \in (0,1)$ is the discount factor for the utility of future periods, x_t a final composite of consumption goods, and L_t labor hours supplied in the economy. Furthermore, let the utility function be characterized by the following preferences as

$$U\left(x_t, L_t\right) \equiv \frac{x_t^{1-\sigma}}{1-\sigma} - \chi\frac{L_t^{1+\nu}}{1+\nu},$$

where $\frac{1}{\sigma} > 0$ represents the intertemporal elasticity of substitution, $\frac{1}{\nu} > 0$ the Frisch elasticity of substitution, and $\chi > 0$ a parameter capturing the disutility of labor.

The final composite of consumption goods is characterized by an Armington aggregator C_f over private and public consumption goods in every period as

$$x_t = C_x\left(c_t, g_t\right) \equiv \left(\omega_x\left(c_t\right)^{-\eta_x} + \left(1-\omega_x\right)\left(g_t\right)^{-\eta_x}\right)^{-\frac{1}{\eta_x}}, \tag{2}$$

where $\omega_x \in (0,1)$ represents the preferences weight over private consumption goods and $\eta_x > -1$ the elasticity of substitution between private and public consumption goods.

The private consumption goods c_t is also characterized by an Armington aggregator C_p over tradable and nontradable private consumption goods in every period as

$$c_t = C_p\left(c_t^T, c_t^N\right) \equiv \left(\omega_p\left(c_t^T\right)^{-\eta_p} + \left(1-\omega_p\right)\left(c_t^N\right)^{-\eta_p}\right)^{-\frac{1}{\eta_p}}, \tag{3}$$

where $\omega_p \in (0,1)$ represents the preferences weight over private tradable consumption goods and

$\eta_p > -1$ the elasticity of substitution between private tradable and private nontradable consumption goods.

The representative household receives a wage compensation net of taxes $(1-\tau)w_t$ for every labor hour supplied to the economy. All the income is exhausted by purchasing tradable and nontradable goods at the real prices p_t^T and p_t^N, respectively. Summarizing these components, the real budget constraint of the representative household for every period is defined as

$$p_t^T c_t^T + p_t^N c_t^N = (1-\tau) w_t L_t.. \tag{4}$$

We assume that the representative household does not have access to financial markets. This assumption implies that the representative household is hand-to-mouth every period contingent to the government policies. The representative household maximizes (1) by aggregating consumption goods using (2) and (3), while exhausting the budget constraint (4) every period.

The optimality conditions of the representative household problem yield

$$\frac{1-\omega_p}{\omega_p}\left(\frac{c_t^T}{c_t^N}\right)^{1+\eta_p} = \frac{p_t^N}{p_t^T} \qquad \text{and} \qquad \frac{\chi}{\omega_x}\left(\frac{c_t}{x_t}\right)^{1+\eta_x} x_t^{\sigma} L_t^{\nu} = (1-\tau)\frac{w_t}{p_t^C}, \tag{5}$$

where p_t^C represents the consumer price index in the model. This consumer price index in every period can be expressed as a weighted aggregation of tradables and nontradable prices as

$$p_t^C = \mathcal{P}_C\left(p_t^T, p_t^N\right) \equiv \left(\omega_p^{\frac{1}{1+\eta_p}}\left(p_t^T\right)^{\frac{\eta_p}{1+\eta_p}} + (1-\omega_p)^{\frac{1}{1+\eta_p}}\left(p_t^N\right)^{\frac{\eta_p}{1+\eta_p}}\right)^{\frac{1+\eta_p}{\eta_p}}. \tag{6}$$

Tradable firm. The tradable firm in the SOE produces tradable goods using labor and the SOE commodity as inputs. The production technology we assume has constant returns to scale and a productivity parameter z. For every unit of labor hired from representative agents and purchases of the SOE commodity from the government in every period, the tradable firm compensates the representative agents and the government with a real wage w_t and the exogenous commodity real price p_t, respectively. Considering this, the profit maximization problem of the tradable firm can be expressed period-by-period as

$$\begin{aligned} &{}_{Y_t^T, L_t^T, M_t^T}\left\{p_t^T Y_t^T - w_t L_t^T - p_t M_t^T\right\} \\ &\text{subject to } Y_t^T = z\left(\omega_T\left(L_t^T\right)^{-\eta_T} + (1-\omega_T)\left(M_t^T\right)^{-\eta_T}\right)^{-\frac{1}{\eta_T}}, \end{aligned}$$

where $\omega_T \in (0,1)$ represents the weight of labor in tradable production and $\eta_T > -1$ a parameter the captures the elasticity of substitution between labor and the commodity inputs in the tradable sector. The optimality conditions of the tradable goods firm can be expressed as

$$\frac{w_t}{p_t^T} = \omega_T \left(\frac{Y_t^T}{L_t^T}\right)^{1+\eta_T} \qquad \text{and} \qquad \frac{p_t}{p_t^T} = (1-\omega_T)\left(\frac{Y_t^T}{M_t^T}\right)^{1+\eta_T}.. \tag{7}$$

Nontradable firm. The nontradable firm in the SOE produces nontradable goods using labor and the SOE commodity as inputs. The production technology we assume has constant returns to scale and we normalize the productivity parameter. For every unit of labor hired from representative agents and purchases of the SOE commodity from the government in every period, the nontradable firm compensates the representative agents and the government with a real wage w_t and the exogenous commodity real price p_t, respectively. Considering this, the profit maximization problem of the nontradable firm can be expressed period-by-period as

$$\begin{aligned} &{}_{Y_t^N, L_t^N, M_t^N} \left\{p_t^N Y_t^N - w_t L_t^N - p_t M_t^N\right\} \\ &\text{subject to } Y_t^N = \left(\omega_N \left(L_t^N\right)^{-\eta_N} + (1-\omega_N)\left(M_t^N\right)^{-\eta_N}\right)^{-\frac{1}{\eta_N}}, \end{aligned}$$

where $\omega_N \in (0,1)$ represents the weight of labor in nontradable production and $\eta_N > -1$ a parameter the captures the elasticity of substitution between labor and the commodity inputs in the nontradable sector+. The optimality conditions of the nontradable goods firm can be expressed as

$$\frac{w_t}{p_t^N} = \omega_N \left(\frac{Y_t^N}{L_t^N}\right)^{1+\eta_N} \qquad \text{and} \qquad \frac{p_t}{p_t^N} = (1-\omega_N)\left(\frac{Y_t^N}{M_t^N}\right)^{1+\eta_N}.. \tag{8}$$

Government. The government of the SOE collects revenue from the labor of and the lump-sum taxes levied on the representative household and the sale of a fixed endowment of oil production $\overline{Y}$. In addition, the government incurs in public expenditures of tradable and nontradable goods g_t^T and g_N^t, respectively. The public consumption goods purchased by the households g_t is characterized by an Armington aggregator C_g over tradable and nontradable public expenditures in every period as

$$g_t = C_g\left(g_t^T, g_t^N\right) \equiv \left(\omega_g \left(g_t^T\right)^{-\eta_g} + (1-\omega_g)\left(g_t^N\right)^{-\eta_g}\right)^{-\frac{1}{\eta_g}}. \tag{9}$$

Considering the aggregation technology (9), the government minimizes the public expenditure of tradable and nontradable goods, which ensures a level of public expenditure g_t. This minimization problem period by period is characterized by

$$\min \left\{ p_t^T g_t^T + p_t^N g_t^N \right\}$$
$$\text{s.t. } g_t = C_g \left(g_t^T, g_t^N \right).$$

The optimality condition between the public expenditures of tradable and nontradable goods considering the preferences of the household must satisfy

$$\frac{1-\omega_g}{\omega_g} \left(\frac{g_t^T}{g_t^N} \right)^{1+\eta_g} = \frac{p_t^N}{p_t^T}.. \tag{10}$$

The government has the ability to use two different types of financial instruments: public debt and a put option associated with the real price of oil. The public debt is associated with a domestic interest rate r_t, while the cost for every oil unit safeguarded by the put option is q_t. To sum up, the government budget constraint is expressed in every period as

$$\tau w_t L_t + b_{t+1} + \theta_t \lambda \overline{Y} + p_t (1-\lambda) \overline{Y} = p_t^G g_t + (1+r_t) b_t + \frac{\kappa}{2} \left(b_{t+1} - \overline{b} \right)^2 + q_t \lambda \overline{Y},$$

where θ_t represents the strike price of the put option associated with the real price of oil, $\lambda \in \{0, 1\}$ the government's share of oil production safeguarded by the put option, and p_t^G the government expenditures price index in the model. This government expenditures price index in every period can be expressed as a weighted aggregation of tradable and nontradable prices as

$$p_t^G = \mathcal{P}_G \left(p_t^T, p_t^N \right) \equiv \left(\omega_g^{\frac{1}{1+\eta_g}} \left(p_t^T \right)^{\frac{\eta_g}{1+\eta_g}} + (1-\omega_g)^{\frac{1}{1+\eta_g}} \left(p_t^N \right)^{\frac{\eta_g}{1+\eta_g}} \right)^{\frac{1+\eta_g}{\eta_g}}. \tag{11}$$

International financial institutions. International financial institutions provide a put option financial instrument to the SOE government associated with the real price of oil. International financial institutions are perfectly competitive and risk neutral and have deep pockets. Besides selling the put option to the SOE government, international financial institutions have access to a one-period risk-free asset. The international financial institution behavior can be expressed as a period-by-period profit maximization problem. This problem is characterized by

$$
{}_{y_{t+1}} \left\{ \frac{q_t}{1+\gamma} y_{t+1} + \frac{1}{1+r^*} \mathbb{E}_t \left[\{0, \theta_{t+1} - p_{t+1}\} \, y_{t+1} \right] \right\},
$$

where y_{t+1} represents the stock of oil safeguarded by the put option by the SOE government at a strike price θ_{t+1} and $\gamma > 0$ captures the transactional cost of creating such put options.

The pricing of the put option financial instrument satisfies the following no-arbitrage condition:

$$
q_t = \frac{1+\gamma}{1+r^*} \mathbb{E}_t \left[\{0, \theta_{t+1} - p_{t+1}\} \right], \tag{12}
$$

where r^* is the foreign interest rate associated with the one-period risk-free asset. The no-arbitrage condition (12) states that the expected cost of the put option matches the return of the risk-free asset.

Balance of payments. The SOE keeps the current account and the financial accounts balanced in every period. Note that the commodity fixed stock can be indexed under the spot international market price and the put option strike price purchased the previous period. The following condition describes the balance of payments condition:

$$
\begin{aligned}
p_t^T NX_t^T = & \left[(1+r_t) b_t + \frac{\kappa}{2} \left(b_{t+1} - \bar{b} \right)^2 + p_t \left(M_t^T + M_t^N \right) + q_t \lambda \overline{Y} \right] \\
& - \left[b_{t+1} + \{\theta_t, p_t\} \, \lambda \overline{Y} + p_t (1-\lambda) \overline{Y} \right],
\end{aligned} \tag{13}
$$

where NX_t^T represents the net exports of tradable goods to foreign markets.

2.1 Private equilibrium

The private agents of the model respond optimally using their optimality conditions, taking as given the prices of the economy and the government policies. We define the current account of the model determined by the government decisions as

$$
\Delta_t \equiv ((1+r_t) b_t - b_{t+1}) + \frac{\kappa}{2} \left(b_{t+1} - \bar{b} \right)^2 + (q_t - \phi_t) \, \lambda \overline{Y},
$$

where $\phi_t \equiv \{0, \theta_t - p_t\}$ represents the hedging coefficient derived from the put option associated with the real price of the commodity.

We assume government expenditures are chosen optimally, considering the optimal decisions by the representative household. Considering the aggregation technology (2), the government minimizes the private and public consumption expenditures that ensures a level of final consumption x_t. This minimization problem period by period is characterized by

$$\min \left\{p_t^C c_t + p_t^G g_t\right\}$$
$$\text{s.t. } x_t = C_x\left(c_t, g_t\right).$$

The optimality condition between the private and public expenditures considering the preferences by the household must satisfy

$$\frac{1-\omega_x}{\omega_x}\left(\frac{c_t}{g_t}\right)^{1+\eta_x} = \frac{p_t^G}{p_t^C}. \tag{14}$$

The sequential private equilibrium characterizes the optimal sequences that satisfy in every period the optimality conditions of the households, firms, public expenditures, and the external sector, taking as given the real price of the commodity and the current account position of the SOE. Definition 2.1 formally describes the equilibrium sequences.

[Private competitive equilibrium] Taking as given the sequences of current account, real commodity price, and public expenditure $\{\Delta_t, p_t\}_{t=0}^{\infty}$, a private competitive equilibrium is defined as the representative household sequences $\left\{\hat{x}_t, \hat{c}_t, \hat{c}_t^T, \hat{c}_t^N, \hat{L}_t\right\}_{t=0}^{\infty}$, tradable and nontradable firms sequences $\left\{\hat{Y}_t^T, \hat{L}_t^T, \hat{M}_t^T, \hat{Y}_t^N, \hat{L}_t^N, \hat{M}_t^N\right\}_{t=0}^{\infty}$, government expenditures sequences $\left\{\hat{g}_t, \hat{g}_t^T, \hat{g}_t^T\right\}_{t=0}^{\infty}$, tradable net exports sequence $\left\{N\hat{X}_t^T\right\}_{t=0}^{\infty}$, and real prices $\left\{\hat{w}_t, \hat{p}_t^T, \hat{p}_t^N, \hat{p}_t^C, \hat{p}_t^G\right\}_{t=0}^{\infty}$, such that the following conditions hold:

i) Representative households sequences satisfy the private expenditure aggregations (2) and (3), budget constraint (4), and the optimality conditions (5)

ii) Tradable firm sequences satisfy the tradable production function and (7)

iii) Nontradable firm sequences satisfy the nontradable production function and (8)

iv) Government expenditures sequences satisfy the the public expenditure aggregation (9) and the optimality conditions (10) and (14)

v) The public and private price indexes satisfy (6) and (11) *v*) The balance of payments condition satisfies (13)

vii) The markets of tradable and nontradable goods and labor are cleared

$$\hat{c}_t^T + \hat{g}_t^T + N\hat{X}_t^T = \hat{Y}_t^T, \qquad \hat{c}_t^N + \hat{g}_t^N = \hat{Y}_t^N, \quad \text{and} \quad \hat{L}_t = \hat{L}_t^T + \hat{L}_t^N$$

2.2 Government recursive problem

The government is benevolent and maximizes the welfare of the representative household using two different types of financial instruments: foreign debt and a put option associated with the real price of the commodity. The government maximizes the following problem:

$$\begin{aligned} V(\phi, b, p; \lambda) =_{\Delta,\phi',\theta',b'} & \left\{ U\left(\hat{x}(\Delta, p), \hat{L}(\Delta, p)\right) + \beta \mathbb{E}\left[V(\phi', b', p'; \lambda)\right] \right\} \\ \text{s.t. } \Delta &= \left((1 + r(\Delta, p))\, b - b'\right) + \frac{\kappa}{2}\left(b' - \overline{b}\right)^2 + \left(q(\theta', p) - \phi\right) \lambda p \overline{y} \\ \phi' &= \left\{0, \theta' - p'\right\} \\ b' &\leq \zeta \left(\hat{p}^T(\Delta, p)\, \hat{Y}^T(\Delta, p) + \hat{p}^N(\Delta, p)\, \hat{Y}^N(\Delta, p) + p\overline{Y}\right), \end{aligned} \tag{15}$$

where ζ represents the stringency of the collateral constraints between debt and GDP.

The Markov perfect equilibrium of the government recursive problem is defined in Definition 2.2.

[Markov perfect equilibrium] A Markov perfect equilibrium is defined as a set of government value function $V(\phi, b, p)$, government policy functions $\{b(\phi, b, p), \theta(\phi, b, p), \lambda(\phi, b, p)\}$, and a put-on option price schedule $\kappa(\theta', p)$ that satisfies the following:

i) Given the put-on option pricing schedule, the value and government policy functions solve the Problem (15).

ii) Given the government policy functions, the debt pricing schedules satisfy the following no-arbitrage condition:

$$q(\theta', p) = \frac{1+\gamma}{1+r^*} \mathbb{E}\left[\left\{0, \theta' - p'\right\}\right].$$

3 Quantitative analysis

This section presents a quantitative analysis of the model proposed in Section 2. We follow the Colombian experience and set oil stock as the commodity in the economy. We establish as the benchmark version of the model the environment in which put option instruments cannot be purchased. Afterwards, we perform a counterfactual analysis by allowing the government to purchase a put option instrument for the whole stock of oil in the economy.

3.1 Calibration strategy

We use the model to match Colombian data from 2000 to 2020 on a quarterly frequency. First, we characterize the functional forms of the stochastic process the real price of oil follows and the functional form of the real interest rate contingent to the state of the model. Then we describe the key target moments from the data used to calibrate the parameters in the benchmark model. The benchmark model corresponds to an economy without hedging instruments for the commodity real price, $\lambda = 0.00$. Finally, we perform a counterfactual analysis for an economy with hedging instruments that fully safeguard the commodity stock, $\lambda = 1.00$.

Stochastic process. We assume the fluctuations of the real price of oil follows an AR(1) process.[1] This AR(1) process is described as

$$\ln(p_{t+1}) = \rho \ln(p_t) + \varepsilon_{t+1},$$

where $\rho \in (0, 1)$ is the persistence of the stochastic process and $\varepsilon_p \sim N(0, \sigma_p^2)$ represents the shock where $\sigma_p > 0$ captures the volatility of the stochastic process. The values observed in the data are $\rho_p = 0.812$ and $\sigma_p = 0.116$, respectively.

Real interest rate. We construct a functional form of the real interest rate in the model where the realization of the real price of oil affects it. The real interest rate is characterized by the following expression:

$$r_t = e^{\psi_0 + \psi_1 p_t + \psi_2 \Delta_t} - 1,$$

[1] We use quarterly data of the international price of oil in US dollars, the nominal exchange rate between Colombian pesos and US dollars, and the Consumer Price Index for Colombian oil in local currency to construct the real price of oil, $p_t \equiv \frac{e_t P_t^{Oil,*}}{CPI_t^{Col}}$.

where ψ_0, ψ_1, and ψ_2 represent parameters that capture the mean and standard deviation of the interest rates and the correlation between the public deficit and the interest rates.

Model parameters. We divide the complete set of parameters Θ into three sets that follow different calibration strategies $\Theta = \Theta_1 \cup \Theta_2 \cup \Theta_3$. For the first set of parameters, we fix conservative values consistent with the literature, $\Theta_1 \equiv \{\eta_T, \eta_N, \eta_p, \eta_g, \eta_x, \nu, \tau, \sigma, \zeta\}$. The second set of parameters we calibrate by fixing the steady state levels of the real oil price and the public deficit to target moments in Colombian data, $\Theta_2 \equiv \left\{\omega_p, \omega_g, \omega_x, \omega_T, \omega_N, \chi, z, \overline{Y}\right\}$. The third set of parameters we calibrate by simulating the model and targeting the parameters to match key moments in the data, $\Theta_3 \equiv \left\{\beta, \bar{b}, \psi_0, \psi_1, \psi_2, \kappa\right\}$. Table 1 reports the parameter values for the calibration of the model.

Table 1: Calibration parameters to Colombia from 2000 to 2019

Parameter	Value	Target
η_T	0.000	Unitary elasticity of substitution
η_N	0.000	Unitary elasticity of substitution
η_p	0.000	Unitary elasticity of substitution
η_g	0.000	Unitary elasticity of substitution
η_x	1.500	Complementary elasticity of substitution
ν	0.500	Macro estimate Frisch elasticity of substitution
τ	0.330	Average income Colombian tax rate
σ	2.000	Standard intertemporal elasticity of substitution
ζ	0.700	Collateral constraint of debt to GDP
ω_p	0.348	Share of nontradable production in GDP
ω_g	0.348	Share of public consumption in total consumption
ω_x	0.980	Equalizing public and private shares
ω_T	0.938	Share of energy expenditures in tradable output income
ω_N	0.999	Share of energy expenditures in nontradable output income
χ	2.104	Unitary labor supply
z	0.744	Share of nontradable employment in total employment
$\overline{Y}$	0.014	Share of oil rents in GDP
β	0.955	Average public debt to GDP ratio
$\bar{b}$	0.185	Average public deficit to GDP ratio
ψ_0	0.165	Average interest rate
ψ_1	-0.105	Correlation between real oil prices and interest rates
ψ_2	0.555	Correlation between public deficit and interest rates
κ	1.855	Standard deviation of interest rates

Note: The data targets correspond to Colombia from 2000 to 2019.

Table 2: Simulation descriptive statistics

Parameter	Data	Benchmark	Counterfactual
$p_t^N Y_t^N / \text{GDP}_t$	0.667	0.663	0.670
$p_t^G g_t / p_t^X x_t$	0.175	0.181	0.182
$p_t M_t^T / p_t^T Y_t^T$	0.084	0.088	0.086
$p_t M_t^N / p_t^N Y_t^N$	0.001	0.001	0.002
L_t^N / L_t	0.625	0.636	0.637
$p_t \overline{Y} / \text{GDP}_t$	0.043	0.041	0.046
$\mu\left(b_t / \text{GDP}_t\right)$	0.505	0.524	0.415
$\mu\left(\text{PD}_t / \text{GDP}_t\right)$	0.020	0.029	0.024
$\mu\left(r_t\right)$	0.067	0.065	0.067
$\sigma\left(r_t\right)$	0.027	0.029	0.029
$\rho\left(r_t, p_t\right)$	-0.483	-0.943	-0.949
Constraint episodes		0.046	0.001
Welfare gains			0.004

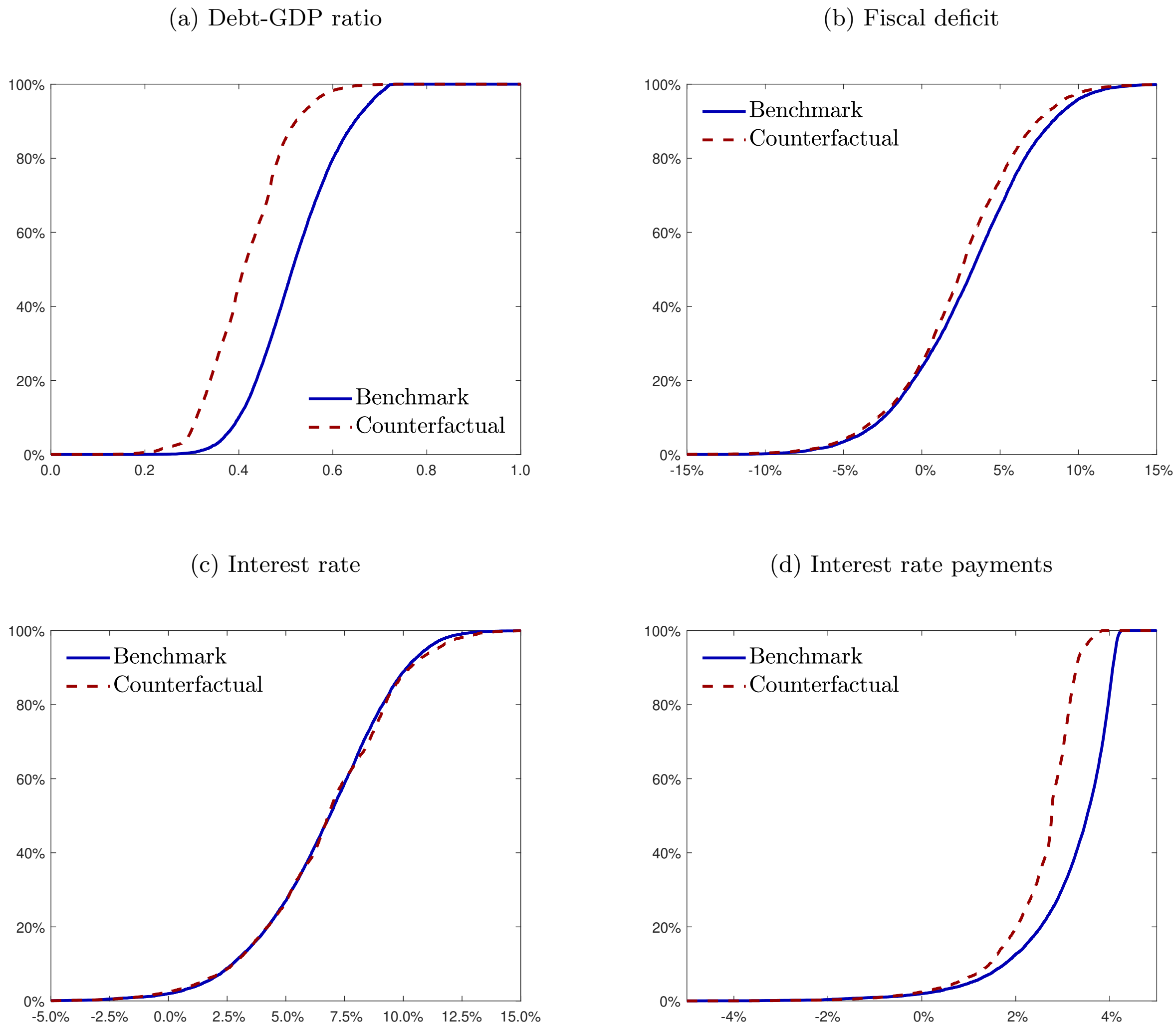
Note: The benchmark corresponds to an economy without hedging instruments for the commodity real price. The counterfactual corresponds to an economy with hedging instruments fully safeguarding the commodity stock ($\lambda = 1.00$). Constraint episodes shows the share of periods in which the economy hits the collateral constraint. Welfare gains correspond to the permanent consumption equivalent of allowing the purchases of the hedging instrument in the benchmark economy for only one period.

We perform a Monte Carlo simulation process to compute the statistics of the benchmark model and counterfactual. We collect 10,000 periods, burning out the first 1,000 episodes to eliminate any initial state bias. We compute a standard average of all the statistics across all the periods and report them in Table 2. The benchmark model performs well in capturing the averages of the share of debt to GDP and spreads. However, the model overestimates the standard deviation of the spreads. The benchmark model only overestimates the negative correlation between the interest rate and the commodity real price. This overestimate is a direct consequence of the functional form imposed for the domestic interest rate.

The counterfactual analysis shows important differences in the moments of the model. The first point we highlight is the drop in the public debt to GDP ratio. The average drops about 10 percentage points compared to the benchmark model. The second point we highlight is the drop in the public deficit to GDP ratio. This average falls about half of a percentage point. The cumulative distributions of these moments can be seen in Panel 1(a) and Panel 1(b). We conclude from these distributions that the ability to purchase hedging instruments for the commodity real price greatly lowers public debt in the economy, improving the government budget and in turn reducing the fiscal deficit. We delve into the fiscal deficit reduction in Panel 1(c) and 1(d). Overall, we do not see a big discrepancy in interest rate performance between the benchmark model and the counterfactual. However, the fall in the public deficit in the counterfactual implies a significant reduction in the interest rate payments experienced by the government.

We continue highlighting the differences between the benchmark model and the counterfactual, specifically, the number of times the collateral constraint binds and the welfare gains allow the benchmark economy to use the hedging instrument for the commodity real price for only one period. In the benchmark model, there are about 50 periods for every 1,000 where the collateral constraint binds. In the counterfactual, there is only 1 period for every 1,000 where the collateral constraint binds. In other words, using hedging instruments for the commodity real price greatly improves financial crises triggered by the high levels of public debt. Lastly, we perform a welfare analysis in which we show how much the benchmark economy gains by allowing the purchases of hedging instruments for only one period. On average, there would be an increase of about half of a percentage point in permanent consumption. In other words, on average households would consume about half of a percentage point more in every subsequent period.

Figure 1: Simulation statistics under different hedging shares

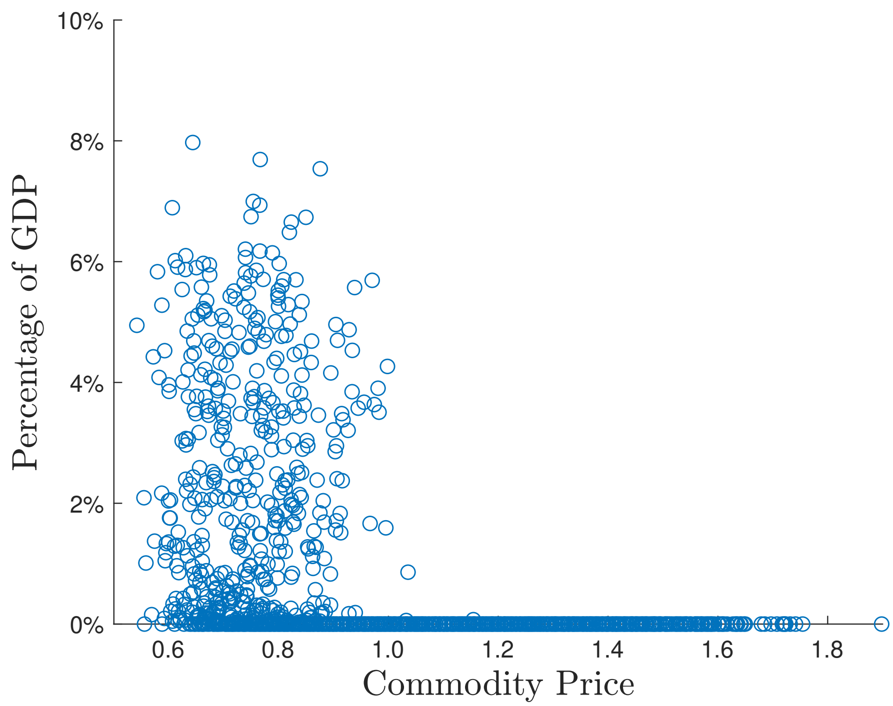


Note: We perform a Monte Carlo simulation and report the cumulative density function. The solid blue line represents the benchmark, an economy without hedging instruments for the commodity real price. The dashed red line represents the counterfactual, an economy with hedging instruments fully safeguarding the commodity stock ($\lambda = 1.00$). with full hedging (counterfactual). Percentage points are plotted on the x-axis in all panels and the moments of the cumulative distribution of the simulation process are plotted on the y-axis .

Lastly, we show how large the benefits of using hedging instruments are. Figure 2 shows the extent of the inflows of the hedging and the episodes in which they are cashed out. Most of the benefit of the hedging instrument appears when the commodity real price falls below the long-run level of the commodity real price, $p = 1.00$. In other words, the hedging instrument benefits the government budget by increasing the income of commodity stock sales by quoting a higher price when the commodity real price is low. These inflows are on average about 4 percentage points of

GDP, but can reach to 8 percentage points.

Figure 2: Hedging inflows



Note: The commodity real price is plotted on the x-axis and the hedging inflows as a percentage of GDP are plotted on the y-axis. By hedging inflows we mean the additional income received by the government from quoting a higher strike price to the spot real commodity price.

4 Conclusion

In this paper we show the consequences of hedging the price of commodities for an SOE and the government fiscal balance. We show that hedging benefits the government budget by greatly reducing the debt burden and increases the sales of commodity stock when its real price is depressed. We show quantitatively that the debt to GDP ratio can be reduced about 10 percentage points, reducing the fiscal deficit by about half of a percentage point of GDP. In addition, we show that using hedging instruments significantly reduces the likelihood of a financial crisis by not hitting collateral constraints in the economy and improves the welfare of the economy as measured by additional permanent consumption for households. Based on our findings, primary commodity exporters, in particular in LAC, should seriously consider the use of hedging instruments.

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