Multiplicity in Sovereign Default Models:

Calvo Meets Cole-Kehoe

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Abstract

This paper proposes a model of sovereign default that features interest rate multiplicity driven by rollover risk. Our core mechanism shows that the possibility of a rollover crisis by itself can lead to high interest rates, which in turn reinforces the rollover risk. By exploiting complementarity between the traditional notions of slow- and fast-moving crises, our model generates a rich simulated dynamics that features frequent defaults and a volatile bond spread even in the absence of shocks to fundamentals. In the presence of risky income, our mechanism amplifies the dynamics of debt and spreads relative to model benchmarks where equilibrium multiplicity relies on the underlying shocks to income.

JEL Classification Numbers: E44, F34

Keywords: Sovereign default, self-fulfilling crises

1 Introduction

Sovereign default crises occur routinely across the emerging and, since recently, also advanced countries, causing significant disruption to economic activity. Economists have long debated whether such events are due to fundamental factors, or can governments find themselves on the brink of bankruptcy merely due to fluctuations in beliefs. The literature on self-fulfilling debt crises has focused on two main sources of multiplicity of equilibria in sovereign debt markets: the interest rate risk as in Calvo (1988), and the rollover risk as in Cole and Kehoe (2000). Existing studies are based on one source of multiplicity only, suggesting that they are seen as substitutes rather than complements. In this paper, we combine both sources of multiplicity and demonstrate how they may complement each other. Specifically, the higher probability of a rollover crisis in the future leads to higher interest rates, which in turn leads to higher probability of a rollover crisis. At the same time, a high interest rate increases the probability of a rollover crisis, which in turn leads to high interest rates. We show that a rich dynamics of the bond spread can be generated in a quantitative model based on this feedback loop alone, without any actual shocks to fundamentals.

This paper is motivated by the fact that the assumptions behind both sources of multiplicity can be adopted jointly. The multiplicity in Calvo (1988) emerges from a restriction in the borrower action space, with the borrower choosing current debt instead of debt at maturity.[1] The amount to be repaid in the future is therefore market-determined and subject to self-fulfilling crises. High interest rates make default more likely, which in turn leads to high interest rates.[2] On the other hand, the multiplicity in Cole and Kehoe (2000) emerges from a timing assumption, in which the borrower issues new debt before making its default decision on maturing debt. In this case, if creditors expect the borrower to default on the previously issued and maturing debt, the price of newly issued debt is zero, indeed pushing the country to default on the maturing debt.[3] In this paper, we explore the consequences of combining the two assumptions. The government chooses current debt and decides whether to default or not on maturing debt after issuing new debt.

We begin our analysis by introducing a simple 3-period model where the borrower issues

[1] Ayres et al. (2018).

[2] This source of multiplicity has been explored in Aguiar and Amador (2020), Lorenzoni and Werning (2019), and Ayres et al. (2018, 2023).

[3] This source of multiplicity has been explored in Conesa and Kehoe (2017), Bocola and Dovis (2019), Aguiar et al. (2022), and Bianchi and Mondragon (2022).

Calvo type debt in the first period and faces a Cole-Kehoe style rollover risk in the second period. Without any income fluctuations and for a risk-neutral borrower, we show analytically that this setup can result in an interest rate multiplicity. If creditors expect a rollover crises in period 2, they will charge a higher interest rate on the debt issued in period 1. The larger debt payments in period 2 in turn will push the borrower to default in case a rollover crisis occurs. We then show numerically that extending this setup to the case of risk-averse borrower, or adding income shocks, results in a wider interval of interest rate multiplicity and more overlapping interest rate schedules.

Guided by the results from our stylized 3-period model, we proceed to test the complementarity between the two notions of multiplicity in a more typical infinite horizon setup with one-period debt. We continue with the assumption of no income shocks; hence the only source of risk comes from the two types of sunspot variables that drive the rollover and interest rate risk sentiments. We find that for any parameterization, there exists a specific probability of a rollover crisis that results in a non-trivial simulated dynamics stemming from the interaction between the two types of multiplicity. Specifically, every simulated path starts with a "slow moving debt crisis", where the borrower's debt accumulation is propelled by a potential bad realization of the Calvo-style sunspot. Then, faced with a high debt and high interest rate burden, the borrower lands in the Cole-Kehoe style crisis zone and remains vulnerable to a rollover crisis which ultimately causes him to default. We show that the model simulations produce non-trivial key statistical moments, with high mean and standard deviation of the bond spread for any targeted level of debt. A comparative statics exercise reveals that by varying the probability of the Calvo-type sunspot, we can obtain different combinations of these moments.

Then, we augment the model with income shocks and show that the feedback loop between both types of multiplicity amplifies the dynamics of debt crises relative to the benchmark models that admit each one of them separately. In particular, the model with both sources of multiplicity attains the high average spread as in the pure Cole-Kehoe setup, and simultaneously generates a high standard deviation of the spread. By contrast, the pure Cole-Kehoe variant of the model features high average spread with zero volatility, while the pure Calvo variant can produce positive combinations of the two moments, but their magnitude is less than half as high as in the baseline. The presence of income shocks also widens the interval of probabilities of a rollover crisis for which the two sunspots interact, relative to model that features non-fundamental shocks only. Income shocks also allow for interesting dynamics of debt and spread that goes in both directions (increases and reductions). As such, the addition of income shock increases the relevance of our core mechanism.

1.1 Literature review

This paper is closely related to the sovereign default literature with self-fulfilling debt crises. The papers related to our work include those that assume equilibrium multiplicity in the tradition of Calvo (1988): Aguiar and Amador (2020), Lorenzoni and Werning (2019), and Ayres et al. (2018, 2023), among others, as well as those that develop models with Cole and Kehoe (2000) type rollover crises: Conesa and Kehoe (2017), Bocola and Dovis (2019), Aguiar et al. (2022), and Bianchi and Mondragon (2022), among others. The paper most closely related to ours is Corsetti and Maeng (2020) which also studies a model with both types of multiplicity to understand the drivers of slow and fast occurring debt crises. By contrast, our paper is interested in the quantitative potential of the complementarity between the two sources of multiplicity. Stangebye (2020) shows that, with long-maturity debt, beliefs about long-term performance of the economy can result in multiplicity of fundamental equilibria.

2 Multiplicity in a three-period model

This section presents a simple three-period environment to illustrate our core mechanism. For simplicity, we present the derivation of our main result for a risk-neutral borrower. In the quantitative section we assume a risk averse borrower.

The borrower receives *deterministic* endowment y in all three periods ($t = 0, 1, 2$). It has zero initial debt and can issue one-period non-contingent bonds to competitive risk-neutral lenders. The borrower is not committed to repay the debt. In the case of default, it is permanently excluded from international financial markets and restricted to consume $y^d < y$. The risk-free gross interest rate is denoted by R^*. To induce borrowing, we assume the borrower has a lower discount factor than the lenders, denoted by β.

As in Cole and Kehoe (2000), we assume the borrower chooses whether to default or not on the previously issued debt *after* the new debt issuance takes place. In this setting, lenders may not roll over the debt if the lack of new borrowing pushes the borrower to default on the old debt, which characterizes the rollover risk. As in Eaton and Gersovitz (1981), we assume that when the bond auction takes place, the borrower moves first by committing to the amount of resources it wishes to raise in the current period, denoted by b. Lenders move next and set the gross interest rate R. These assumptions generate the interest rate multiplicity in Calvo (1988).[4] For a given b, a higher R increases the probability of default because it increases the debt service. In turn, a higher probability of default implies a higher R, as lenders must have expected return equal to R^* in equilibrium.

We present and solve the problem backwards. In period $t = 2$, the only choice for the borrower is whether to pay the debt service on the debt issued in the previous period, $R_2 b_2$, or to default. The borrower defaults if $y^d > y - R_2 b_2$ and repays otherwise.

It follows that in period $t = 1$, if the lenders roll over the debt, the interest rate is uniquely determined. Let's define the threshold $\widetilde{B}_2 \equiv \left(y - y^d\right) / R^*$. For $b_2 \leq \widetilde{B}_2$, there is no default and R_2 must be equal to R^*. For $b_2 > \widetilde{B}_2$, the borrower defaults for sure, so $\widetilde{B}_2$ becomes a borrowing limit.

If lenders do not roll over the debt in $t = 1$, the borrower defaults if

$$v_1^d \equiv (1+\beta) y^d > y - R_1 b_1 + \beta y,$$

where $R_1 b_1$ is the debt service on the debt issued in $t = 0$.[5] A rollover crises may happen only if it pushes the country to default, so the borrower is subject to rollover risk only if

$$R_1 b_1 > (1+\beta)(y - y^d). \tag{1}$$

Note that a rollover crises is equivalent to setting the borrowing limit to zero, a convention we will adopt to simplify the exposition. We can express the problem in period $t = 1$ as:

$$v_1\left(R_1 b_1, s_{ck}\right) = \max\{v_1^{nd}\left(R_1 b_1, s_{ck}\right), v_1^d\},$$

in which

$$v_1^{nd}\left(R_1 b_1, s_{ck}\right) = \max_{b_2 \leq \overline{B}_2(R_1 b_1, s_{ck})} y - R_1 b_1 + b_2 + \beta\left(y - R^* b_2\right).$$

$s_{ck} \in \{0, 1\}$ is a sunspot variable that commands the Cole-Kehoe type of market sentiment. If $s_{ck} = 0$ and condition (1) holds, a rollover crisis happens and the borrowing

[4] See Ayres et al. (2023).

[5] As in Aguiar et al. (2016), we assume the borrower does not keep the proceeds from the new bond auction in case it defaults on the old debt.

limit $\overline{B}_2\left(R_1 b_1, s_{ck}\right)$ equals zero. Otherwise, $\overline{B}_2\left(R_1 b_1, s_{ck}\right) = \widetilde{B}_2$.[6] In addition, note that the condition for the rollover risk in (1) depends on R_1, which gives rise to an interest rate multiplicity. For a given b_1, a higher R_1 makes a rollover crises more likely. In turn, the higher probability of a rollover crisis implies a higher R_1.

We turn to the borrower problem in $t = 0$:

$$v_0(s_c) = \max_{b_1 \leq \bar{b}_1} \; y + b_1 + \beta \sum_{s_{ck} \in \{0,1\}} \pi(s_{ck}) \; v_1\Big(R_1(b_1, s_c) b_1, s_{ck}\Big).$$

$\pi(s_{ck})$ denotes the probability distribution over the values that s_{ck} may take in $t = 1$. We let p denote the probability of the bad sunspot, $\pi(0) = p$. The state variable $s_c \in \{0, 1\}$ denotes the Calvo-type sunspot. In case there are multiple interest rates for a given b_1 such that lenders receive expected return equal to R^*, we use the sunspot variable s_c as a device to select the interest rate. As in Ayres et al. (2023), we will focus on two extreme cases. In the bad sunspot state, $s_c = 0$, R_1 takes the highest possible value. In the good sunspot state, $s_c = 1$, R_1 takes the lowest possible value. Lemma 1 characterizes all the pairs (b_1, R_1) such that lenders receive return R^* in expectation.

Lemma 1 *The pairs (b_1, R_1) in which lenders receive expected return equal to R^* given the borrower's optimal borrowing and default strategies are:*

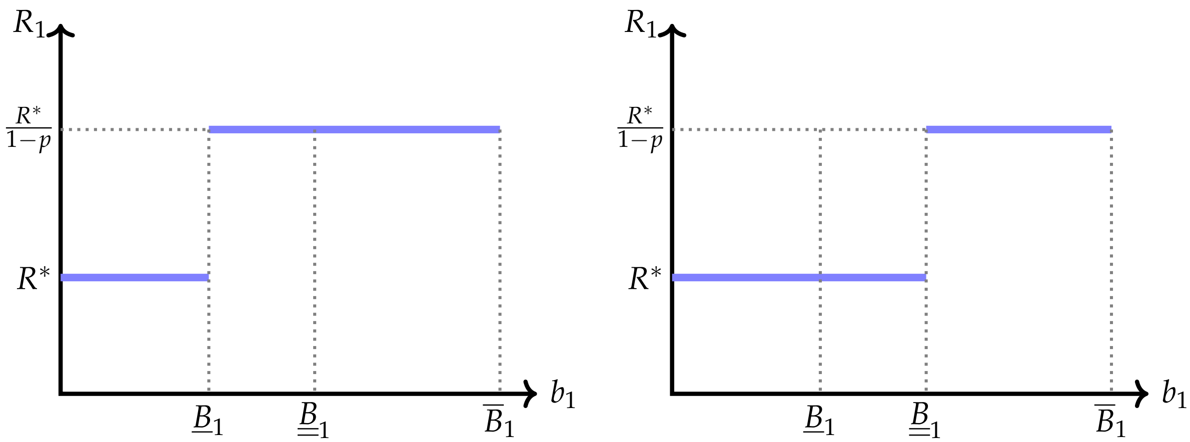
(i) $b_1 \leq \frac{(1+\beta)(y-y^d)}{R^*} \equiv \underline{\underline{B}}_1$ *and* $R_1 = R^*$.

(ii) $\underline{B}_1 \equiv \frac{(1+\beta)(y-y^d)(1-p)}{R^*} \leq b_1 \leq (1-p)(y-y^d)\left(\frac{1}{R^*} + \frac{1}{(R^*)^2}\right) \equiv \overline{B}_1$ *and* $R_1 = \frac{R^*}{1-p}$.

Proof: Appendix A.1

For any debt level smaller than $\underline{B}_1$, the borrower repays even if lenders do not rollover the debt. Hence, the interest rate is unique and equal to the risk-free rate. For a level of borrowing between $\underline{B}_1$ and $\underline{\underline{B}}_1$, however, an interest rate multiplicity arises. It is noteworthy that, in this case, it is the rollover risk in period $t = 1$, rather than an income shock, that generates multiplicity of the interest rate. In other words, for a given $b_1 \in \left[\underline{B}_1, \underline{\underline{B}}_1\right]$, a high or a low interest rate R_1 determines whether the borrower finds itself in a Cole-Kehoe type of crisis zone or not. Finally, if the debt level is sufficiently high, above $\underline{\underline{B}}_1$, then the interest rate is again unique and equal to the high rate $\frac{R^*}{1-p}$. At those debt levels, the borrower defaults even if interest rates were set to R^*.

[6] Note that the optimal strategy for the borrower in this simple case is to set $b_2 = \overline{B}_2\left(R_1 b_1, s_{ck}\right)$.



(a) High interest rate schedule $R_1(b_1, 0)$

(b) Low interest rate schedule $R_1(b_1, 1)$

Figure 1: Stylized illustration of the interest rate schedules

The borrower observes s_c before the bond auction and internalizes how the interest rate will vary with respect to the amount of debt it chooses to issue. Therefore, it considers an interest rate schedule $R_1(b_1, s_c)$ when choosing how much to borrow. That is, a mapping from debt levels into unique interest rate values. Figure 1 presents a stylized illustration of the two interest rate schedules the borrower may face. The one with high rates in Figure 1(a), $R_1(b_1, 0)$, and the other with low rates in Figure 1(b), $R_1(b_1, 1)$. In the following subsection, we provide a numerical example to show that the interest rate multiplicity is numerically significant.

2.1 Numerical example

In this subsection, we provide a simple numerical example to show that the interest rate multiplicity characterized so far is realistic. We also extend the analysis to the case of a risk-averse borrower and show that the result becomes even stronger. The exact derivations for this case are presented in Appendix A.2.

Consider the case of a risk-averse borrower with CRRA utility function of the form $u(c) = \frac{c^{1-\gamma}}{1-\gamma}$. We assume the following, fairly realistic parameterization: $\beta = 0.7$, $\gamma = 3$, $y = 1$, $y^d = 0.95$, $p = 0.8$, $R = 1.03$. Figure 2 presents the interest rate schedules, as well as optimal debt policy for the risk-averse borrower. The solid blue line depicts the lower (risk-free) interest rate, while the red dashed and blue dotted lines represent the upper (risky) interest rate for the case of risk-averse and risk-neutral borrower, respectively. It is immediate to notice that including risk aversion causes the interest rate multiplicity to almost double in size. The presence of this multiplicity also has real consequences for the borrower's actions. When the Calvo sunspot is bad, the government must reduce its debt by around 20%, compared to the case of a good sunspot, to avoid the higher interest rate.

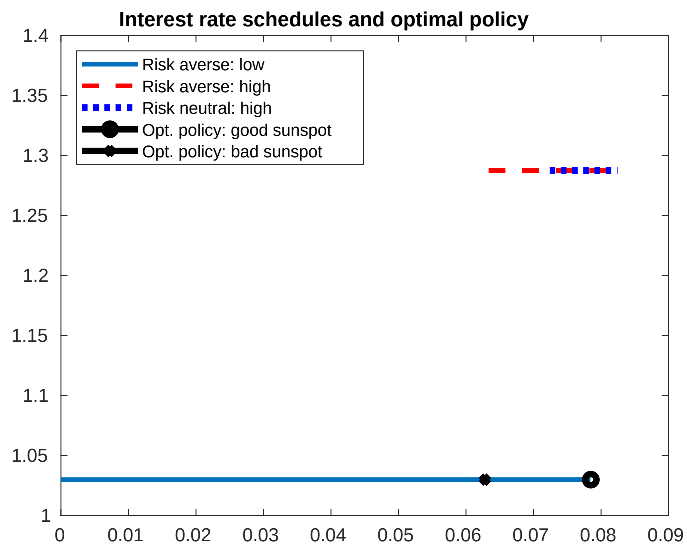


Figure 2: Interest rate schedules and optimal policy

Appendix B extends this model by adding an income shock in period $t = 1$. It shows that in the presence of income shock, in addition to rollover risk, the Calvo and Cole-Kehoe frictions interact and yield a much richer interest rate multiplicity than each of them alone. In the following Section 3 we extend the model to infinite horizon and we show that the rollover multiplicity is quantitatively significant.

3 Infinite horizon model

In this section we develop an infinite-horizon model to study the interaction of interest rate multiplicity with rollover risk.

3.1 Economic environment

Consider a small open economy with a benevolent sovereign that borrows internationally from competitive lenders and receives stochastic endowment. Time is discrete and indexed by $t = 0, 1, 2, ...$ Markets are incomplete and the only asset available for trading is the one-period non-contingent bond. The risk-free gross interest rate is R^*. The representative household has preferences given by the expected utility of the form

$$E_0 \sum_{t=0}^{\infty} \beta^t u(c_t), \tag{2}$$

where we assume the function $u(\cdot)$ is strictly increasing, concave and twice continuously differentiable. The discount factor is given by $\beta \in (0,1)$.

Income process The endowment process consists of both transitory and permanent components. It is given by

$$Y_t = \Gamma_t e^{\sigma \varepsilon_t}, \tag{3}$$

where $\varepsilon_t \sim \mathcal{N}(0,1)$. The permanent component Γ_t evolves according to

$$\Gamma_t = g_t \Gamma_{t-1}, \tag{4}$$

where g_t denotes the growth shock. It can assume two values, g_L and g_H, with $g_H > g_L$. It follows a Markov process with the transition probability matrix given by

$$\Pi = \begin{bmatrix} \pi_L & 1-\pi_L \\ 1-\pi_H & \pi_H \end{bmatrix}, \tag{5}$$

where $Pr(g_{t+1} = g_L | g_t = g_L) = \pi_L$ and $Pr(g_{t+1} = g_H | g_t = g_H) = \pi_H$.

Timing The timing assumptions are the same as in Section 2. The borrower chooses whether to default or not on the debt from previous period *after* the new debt issuance (Cole and Kehoe, 2000). Similar to Calvo (1988), when the bond auction takes place, the borrower moves first by committing to the amount of resources it wishes to raise in the current period, b. Lenders move next and set the gross interest rate R. Shocks are observed in the beginning of the period.

States The set of states is $\{A, Y, \mathbf{s}\}$. $A = RB$ denotes the total debt service to be paid in the current period, Y is the current income, while $\mathbf{s} = \{s_c, s_{ck}\}$ is a vector of sunspot realizations corresponding to the interest rate multiplicity and rollover risk, respectively.

Recursive problem The value function of the government involves a choice of whether to default or not

$$V(A,Y,\mathbf{s}) = \max_{d\in\{0,1\}} \left\{(1-d)V^{nd}(a,Y,\mathbf{s}) + dV^{d}(Y,\mathbf{s})\right\}$$

The value associated with repayment is

$$V^{nd}(A,Y,\mathbf{s}) = \max_{B'\leq\overline{B}(A,Y,\mathbf{s})} \left\{u(C) + \beta\sum_{y'}\sum_{\mathbf{s}'}\Pi(y'|y)p(\mathbf{s}'|\mathbf{s})V\left(B'R(B',Y,\mathbf{s}),y',\mathbf{s}'\right)\right\}$$

subject to

$$C = Y - A + B'.$$

The value associated with default is

$$V^{d}(Y,\mathbf{s}) = u\left(Y(1-\phi)\right) + \beta\sum_{y'}\sum_{\mathbf{s}'}\Pi(y'|y)p(\mathbf{s}'|\mathbf{s})\left\{\theta V\left(0,y',\mathbf{s}'\right) + (1-\theta)V^{d}\left(y',\mathbf{s}'\right)\right\}$$

where ϕ represents the fraction of income lost upon default.

As in Section 2, the borrowing limit $\overline{B}\,(A,Y,\mathbf{s})$ equals zero whenever $s_{ck} = 0$ and the lack of new borrowing pushes the country to default. That happens when the following condition is satisfied:

$$u(Y-A) + \beta\sum_{y'}\sum_{\mathbf{s}'}\Pi(y'|y)p(\mathbf{s}'|\mathbf{s})V\left(0,y',\mathbf{s}'\right) \leq V^{d}(Y,\mathbf{s}).$$

Definition 2 formally introduces an equilibrium in this economy.

Definition 2 *A Markov Perfect Equilibrium for this economy consists of the government value functions $V(A,Y,\mathbf{s})$, $V^{nd}(A,Y,\mathbf{s})$, $V^{d}(Y,\mathbf{s})$; policy functions $B'(A,Y,\mathbf{s})$ and $d(A,Y,\mathbf{s})$; the interest rate schedule $R(B',Y,\mathbf{s})$ and the borrowing limit function $\overline{B}\,(A,Y,\mathbf{s})$ such that:*

1. *Policy function $d(A,Y,\mathbf{s})$ solves the government's default-repayment problem.*

2. *Policy functions $B'(A,Y,\mathbf{s})$ solve the government's consumption-saving problem.*

3. *Interest rate schedules $R(B',Y,\mathbf{s})$ and borrowing limit functions $\overline{B}\,(A,Y,\mathbf{s})$ are such that international lenders receive expected return equal to R^*.*

3.2 Quantification of the model

In this section, we introduce the set of parameters that will be used to evaluate our model quantitatively. Table 1 summarizes the assumed calibration. The structural parameters in the upper panel of the table are the same as in Aguiar et al. (2022) and will be used for all variants of our model presented in the following sections. The lower panel of Table 1 lists the parameters governing the bimodal income process, which are based on estimating a Markov-switching process for growth regimes using Mexico's GDP data in years 1980-2021.[7] The estimation technique is based on Bayesian approach as in Ayres et al. (2023).

Table 1: Parameter values assumed in the quantitative model

Parameter	**Meaning**	**Value**
r	Risk-free rate	0.01
γ	Risk aversion	2.00
β	Discount factor	0.8
ϕ	Income loss in default	0.03
θ	Prob. of reentry	0.125
g_h	High growth	0.96
g_l	Low growth	1.02
π_H	Persistence of high growth	0.8
π_L	Persistence of low growth	0.3

3.3 No growth regimes

As a first step, we evaluate the model with no income shocks (and as a result, no regimes whatsoever). Income is deterministic and equal to 1 in every period. Hence, in this variant of the model, rollover risk is the sole driver of defaults and potential interest rate multiplicity. We use 0.1 as the initial probability of the bad Calvo sunspot realization, and we vary the probability of the Cole-Kehoe sunspot to illustrate how the model works. Table 2 presents the statistics from a simulated ergodic distribution for three different probabilities of a bad Cole-Kehoe sunspot. It is evident that the model admits potentially very different types of behavior for seemingly similar values of this parameter. When the probability is 5.6% or lower, the agent borrows on the higher interest rate schedule and defaults every time the markets are closed. As a result, average spread is roughly equal to the probability of a bad Cole-Kehoe sunspot, while the variance of the spread is zero.

[7] For simplicity, we forgo the transitory shock.

On the other hand, for the probability of 5.8% or higher, the agents borrows on the lower interest rate schedule and reduces debt every time the Calvo sunspots switches to bad in order to avoid the region of multiplicity. As a result, no defaults occur on equilibrium path and the bond spread is always zero. In between the two extremes, there is an interval of Cole-Kehoe sunspot probabilities around 5.7% where interesting action occurs. In this case, the agent initially borrows on the lower interest rate schedule, but then increases the debt and jumps to the higher one when the Calvo sunspot switches to bad (a "slow moving debt crisis"). The borrower remains there until a Cole-Kehoe type rollover crisis occurs and forces him into default. It should be emphasized that while the interval of sunspot probabilities for which interesting behavior occurs is quite narrow, it is so because the model does not feature any other sources of uncertainty. Section 3.4 shows that this interval widens considerably when realistic income shocks are introduced. It is also noteworthy that the average debt-to-income ratio in this case, an untargeted moment, comes out exactly equal to its empirical counterpart of 66% as reported by Aguiar et al. (2022) (for quarterly data).

Table 2: Simulated results with no growth regimes

$\mathbf{P(s_{ck} = 1)}$	0.056	0.057	0.058
E(debt/Y)	18.2	16.5	13.1
E(spread)	6.0	4.4	0.0
σ(spread)	0.0	2.7	0.0
ρ(s,TB)	0.0	-0.59	0.0

Figure 3 plots the interest rate schedule that arises in the intermediate case of $P(s_{ck} = 1) = 0.057$. The clear multiplicity confirms our analytical result from Section 2 which shows that the Calvo action space can combine with rollover crises to generate overlapping interest rate schedules *with no income shocks*. The graph also describes the simple dynamics of borrower's decisions in this model. As the agent accumulates debt starting from zero, he moves along the risk-free interest rate towards the points labeled "A" and "B". The former is chosen if the Calvo sunspot realization is initially bad, while the latter is eventually selected when the realization switches to good. Once the borrower lands at point "B", he will not retreat back to "A" upon another bad Calvo sunspot, but instead will choose to increase the debt all the way to "C" and incur an interest rate spread of 6%. With no additional friction or shocks in the model, the agent stays in point "C" until a Cole-Kehoe rollover crisis occurs, in which case he defaults.[8]

[8] As Section 3.4 shows, the full version of this model with stochastic growth regimes also features en-

An interesting feature of our model is that the interval of Cole-Kehoe sunspot probabilities that generates this dynamics is the same for any Calvo sunspot probability parameter

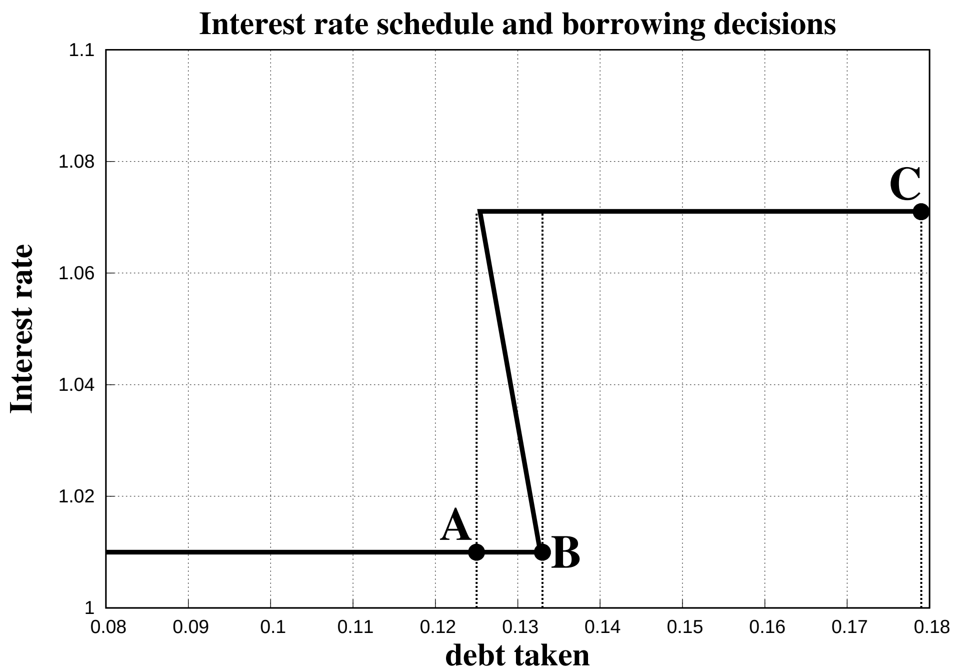
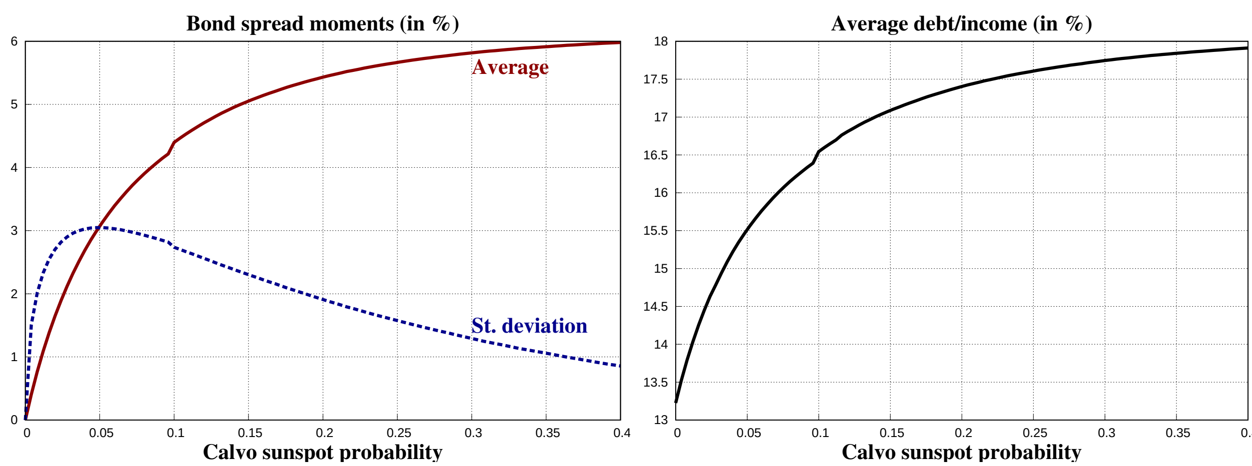


Figure 3: Interest rate schedules and policy functions with no income shocks



(a) Bond spread moments (b) Average debt ratio

Figure 4: Simulated moments as function of the Calvo sunspot probability

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dogenous debt reductions.

that we choose. However, the implications for the resulting simulated moments are quite different as we vary the likelihood of a Calvo-style crisis. Figure 4 explores this comparative statics by plotting the basic moments of the bond spread and debt for a range of values that this parameter can take. Panels 4(a) and 4(b) show that the average spread and average debt ratio are both monotonically increasing in the probability of the bad Calvo sunspot. The intuition is simple: as the switch to the higher interest rate schedule becomes more likely, the agent spends less time at point "B" of Figure 3, characterized by lower debt and the spread of zero, and more time at point "C" with high debt and positive spread. On the other hand, interestingly, the measured volatility of the spread is non-monotonic, initially rising sharply from zero and then falling back gradually. The intuition is straightforward: if a Calvo-style crisis is very unlikely, or if it happens too often, the borrower will end up spending a disproportionate amount of time on the lower or on the upper interest rate schedule, respectively. Hence, there exists an intermediate value of the Calvo sunspot probability that balances the average time spent on the two parts of the schedule, and maximizes the overall bond spread volatility. For the present calibration, we find that standard deviation of the bond spread peaks at the Calvo probability of around 5%.

3.4 Quantitative results with growth regimes

We now evaluate the impact of our mechanism in a model with income shocks. As specified in Table 1, the income shocks are introduced in the form of stochastic growth regimes and the parameters are based on estimating a Markov-switching process for Mexico's economy. Table 3 presents the simulated results across four variants of our model. To offer a meaningful comparison across the different variants of the model, we adjust the sunspot probabilities so that the variants exhibit a similar average debt ratio. For completeness, we also report the results of a model without interest rate multiplicity or rollover risk. In that case, there is no free parameter and debt becomes an untargeted object averaging around twice the level of the baseline, while spreads are essentially zero for reasons similar to what Aguiar and Gopinath (2006) describe.

For our baseline model that combines interest rate multiplicity with rollover risk, we fix the Calvo sunspot probability at 10% and we use a Cole-Kehoe sunspot probability of 4.3% which yields an average debt level of close to 16%. For the pure Calvo and pure Cole-Kehoe variants of the model, we adjust their respective probabilities of a bad sunspot realization upwards so that the average debt ratio in the simulations is similar.[9]

Table 3: Simulated results in the quantitative model

Stat	Both	Only Calvo	Only C-K	None
$P(s_c = 1)$	0.1	0.275	0.0	0.0
$P(s_{ck} = 1)$	0.043	0.0	0.055	0.0
E(debt/Y)	16.1%	16.1%	17.9%	30.5%
E(spread)	4.0%	0.0%	5.9%	0.0%
σ(spread)	1.5%	0.0%	0.0%	0.0%
ρ(TB,y)	−0.22	−0.13	−1.0	−0.68
ρ(s,y)	0.02	0.0	0.0	0.0
ρ(s,TB)	−0.66	0.0	0.0	0.0
% Calvo	10.0	0	0.0	0
% Rollover	100.0	0	100	0
% Low growth	22.4	0	22.0	0

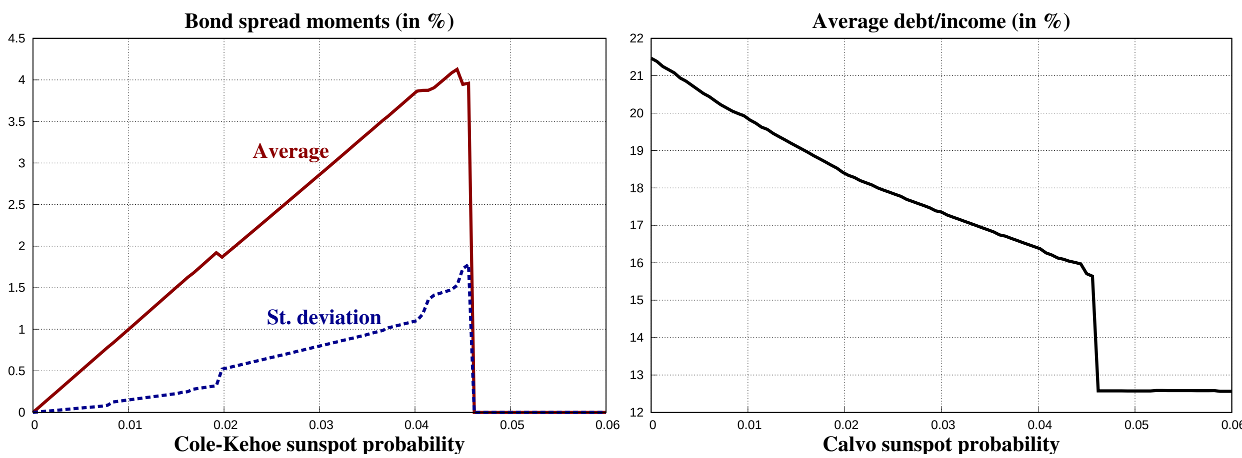
As Table 3 shows, our baseline model with multiplicity and rollover risk generates a simultaneously high average and high volatility of the bond spread. Similar to the case with no fundamental shocks described in Section 3.3, a bad Cole-Kehoe sunspot is needed to trigger a run on the debt and default. However, in this model it is a high growth regime realization that propels the government to accumulate debt and enter the crisis zone under relatively high spreads. By contrast, a bad realization of the Calvo sunspot causes the opposite reaction: debt (and spread) is reduced and the government will repay even if markets do not open the next period. As such, our baseline model with income shocks generates an interesting dynamics of debt and spreads in both directions (accumulation and reduction).

By contrast, in the pure Calvo variant of the model, a high probability of the bad sunspot is required to match the desired level of debt. At that probability, however, the borrower does not default or enter the multiplicity region on the equilibrium path resulting in a zero spread. In the pure Cole and Kehoe variant of the model, a level of debt close to the targeted one is attained for the probability of a bad sunspot of 5.5%, which results in a roughly equal average spread (essentially, the government always defaults when the markets shut down). However, the consequence of this behavior is that the volatility of

[9] Because of a typical knife-edge behavior of such models, it is not necessarily possible to have all three variants deliver exactly the same level of debt. Hence, we seek parameter values that bring debt levels as close to each other as possible.

the spread is zero (Aguiar et al., 2022).

We now turn to the comparative statics analysis for our baseline model with respect to the probability of a bad Cole-Kehoe sunspot. Figure 5 plots the three moments of interest for the sunspot probabilities ranging up to 6%. The main thing to notice is that, in contrast to the variant of our baseline model with no shocks to income (Section 3.3), the range of Cole-Kehoe sunspot probabilities for which we attain interesting debt dynamics is substantially larger. For the interval of such probabilities up to roughly 4.5%, the model generates simultaneously a high mean and high variance of the bond spread (increasing in the probability) with a realistic average debt level. Above that interval the behavior of the borrower is quite standard; the government stays permanently outside the rollover crisis zone and no defaults occur.

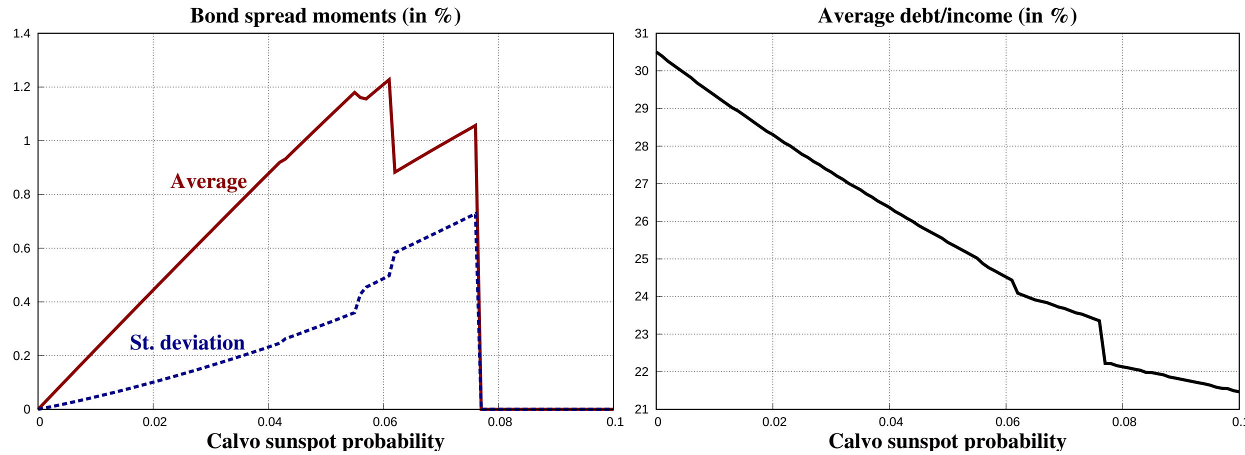


(a) Bond spread moments　　(b) Average debt ratio

Figure 5: Comparative statics in baseline model for Cole-Kehoe sunspot probability

Next, we focus on the pure Calvo variant of the model to compare its ability to generate interesting debt dynamics for a wider range of parameters. Figure 6 presents the comparative statics with respect to the probability of a bad Calvo sunspot, the only non-fundamental variable in that variant. As the figure shows, defaults do occur on equilibrium path and the model can generate non-zero spread for the sunspot probabilities lower than 7.5%, which corresponds to much higher debt-output ratios (23% and above). It is notable, however, that both the average and standard deviation of the bond spread are at least 50% smaller than in our baseline model.

Finally, Figure 7 presents the comparative statics with respect to the sunspot probability

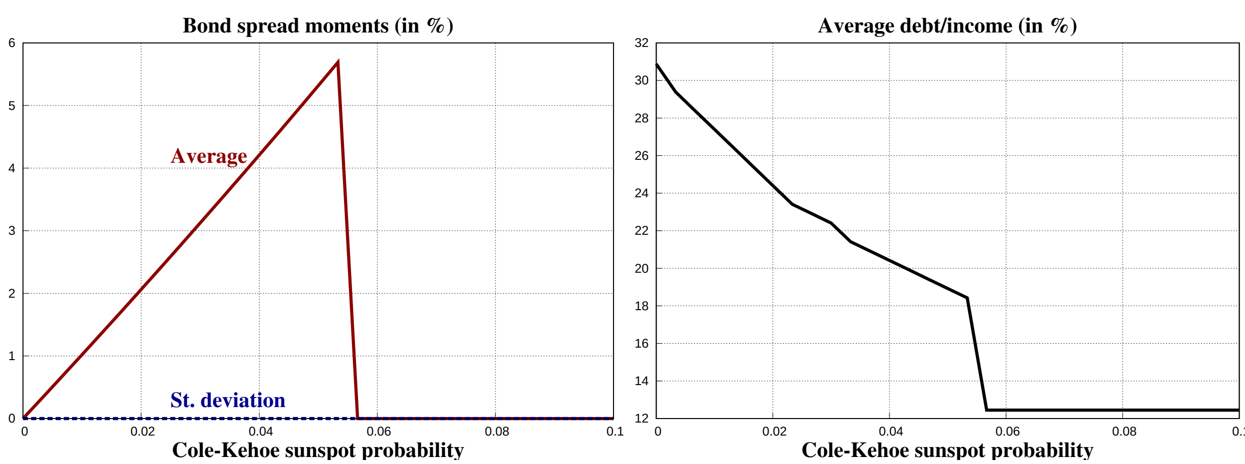


(a) Bond spread moments

(b) Average debt ratio

Figure 6: Comparative statics in the pure Calvo variant of the model

in the pure Cole-Kehoe variant of the model. The government here behaves as expected: for low enough probabilities of a bad sunspot it borrows a lot and always remains in the crisis zone, irrespective of the underlying growth regime. In all these cases, however, the mean spread corresponds directly to the assumed probability of a bad sunspot while the spread volatility is zero (unless a default has already occurred, the equilibrium spread is always a constant). For a sunspot probability greater than roughly 6%, the borrower reduces its debt sharply and stays outside of the crisis zone. As a result, both mean and standard deviation of the spread are zero.



(a) Bond spread moments

(b) Average debt ratio

Figure 7: Comparative statics in the pure Cole-Kehoe variant of the model

4 Conclusion

This paper contributes to the literature of self-fulfilling debt crises by introducing a model with interest rate multiplicity generated by belief-driven runs on government debt. In turn, such runs are justified by a realization of high interest rates that by itself results from pessimistic beliefs. The main achievement of the model is to show that one can generate rich dynamics of sovereign debt and the interest rate spread by combining the notions of slow- and fast moving debt crises *without any underlying shocks to fundamentals*.

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Appendices (for online publication)

A Derivations for the three-period model

In this Appendix, we present the steps to derive the interest rate schedules for our basic three-period model.

A.1 Risk neutrality

Recall that the borrower will default in period $t = 1$ if $R_1 b_1 > (1+\beta)(y - y^d)$. Because there are two possible values of the interest rate, R^* and $R^*/(1-p)$, we have the following two debt thresholds that limit the repayment decision for the case of low and high interest rates:

$$b_1 \leq \frac{(1+\beta)(y-y^d)}{R^*} \equiv \underline{\underline{B}}_1$$
$$b_1 \leq \frac{(1+\beta)(y-y^d)(1-p)}{R^*} \equiv \underline{B}_1$$

Finally, we need to find a debt threshold that makes the borrower indifferent between repaying and defaulting when markets are open in period $t = 1$. The condition is

$$v_1^d = (1+\beta)y^d = y - \frac{R^*}{1-p}b_1 + b_2 + \beta \max\{y_2 - R_2 b_2, y^d\}$$

Under risk neutrality, assuming the borrower is impatient enough, the optimal borrowing in that period is $b_2^* = \frac{y-y^d}{R^*}$. Plugging this value into the indifference condition yields the following upper debt threshold:

$$b_1 = (1-p)(y-y^d)\Big(\frac{1+R^*}{R^{*2}}\Big) \equiv \overline{B}_1$$

A.2 Risk aversion

We now derive the corresponding thresholds for the case of a risk averse borrower. We assume a CRRA utility function of the form $u(c) = \frac{c^{1-\gamma}}{1-\gamma}$, and we analyze the problem backwards. Similar as in the case of risk neutrality, in period $t = 2$ the agent repays if $y - b_2 R^* \geq y^d$. In period $t = 1$, if markets do not roll over the debt, the borrower will default if

$$v_1^d = (1+\beta)u(y^d) > v_1(R_1 b_1, s_1 = 1) = u(y - R_1 b_1) + \beta u(y)$$

If $\gamma > 1$, this condition boils down to

$$R_1 b_1 > y - \left((1+\beta)(y^d)^{1-\gamma}\right)^{\frac{1}{1-\gamma}}$$

which is analogous to the condition we obtained in Section A.1 and shows that the default decision depends on the level of interest rate. Consequently, we have the two debt thresholds that limit the repayment decision for the case of low and high interest rates:

$$b_1 \leq \frac{1}{R^*}\left[y - \left((1+\beta)(y^d)^{1-\gamma} - \beta y^{1-\gamma}\right)^{\frac{1}{1-\gamma}}\right] \equiv \underline{\underline{B}}_1$$
$$b_1 \leq \frac{1-p}{R^*}\left[y - \left((1+\beta)(y^d)^{1-\gamma} - \beta y^{1-\gamma}\right)^{\frac{1}{1-\gamma}}\right] \equiv \underline{B}_1$$

Next, to find the debt threshold that makes the borrower indifferent between repaying and defaulting when markets are open in $t = 1$, we need to find optimal borrowing b_2. Under risk aversion, this entails solving the problem

$$v_1(R_1 b_1, s_1 = 0) = \max_{b_2} \ u(y - R_1 b_1 + b_2) + \beta u(y - R^* b_2)$$

The interior solution to this problem is $b_2^* = \frac{(\beta R^*)^{-1/\gamma} y - (y - R_1 b_1)}{1 + (\beta R^*)^{-1/\gamma} R^*}$, while a corner implies $b_2^* = \frac{y - y^d}{R^*}$. To find threshold $\overline{B}_1$, we need to plug this into the indifference condition in period $t = 1$:

$$v_1^d = (1+\beta)u(y^d) = u(y - \frac{R^*}{1-p} b_1 + b_2^*) + \beta u(y - R^* b_2^*)$$

and solve for b_1. Under risk aversion, this solution cannot be obtained analytically. The results in Section 2.1 present our numerical solution to this problem.

B Three-period model with income shocks

This section illustrates our core mechanism using a simple three-period model with income shocks. Following a general description, we will first introduce two benchmark cases separately—interest rate multiplicity á la Calvo (1988) and rollover risk á la Cole and Kehoe (2000)—and show that they generate equivalent results. Then, we proceed to our main model that combines the two frictions.

A representative agent with CRRA utility function of the form $u(c) = \frac{c^{1-\gamma}}{1-\gamma}$ enters the first period ($t = 0$) with wealth ω and receives endowments y_1 and y_2 in the second ($t = 1$) and third ($t = 2$) periods, respectively. The former is stochastic and can take two values, $y_1 \in \{y_h, y_l\}$, with probabilities $\pi = [p_h, 1 - p_h]$, while the latter is deterministic. The agent can issue one-period non-contingent bonds in $t = 0$ and $t = 1$ to a continuum of foreign risk-neutral lenders, but cannot commit to repay. The risk-free interest rate is denoted as R^*. In the case of default, the agent is permanently excluded from international financial markets and consumes y^d.

B.1 Calvo setup

In the Calvo framework, it is assumed that the lenders move first in offering the borrower an interest rate schedule, who in turn decides whether to default or not and, in case of the latter, chooses the amount of debt revenue b_2 to issue. We present the problem backwards. The borrower arrive in period $t = 2$ with wealth w_2 and chooses whether to repay or default:

$$v_2(w_2) = \max\Big\{u(y_d), u(w_2)\Big\}$$

In period $t = 1$ the borrower with wealth w_1 also chooses between repayment or default by comparing the values associated with both options:

$$v_1(w_1) = \max\Big\{v_1^d, v_1^{nd}(w_1, s_1)\Big\}$$

The value associated with repayment involves the borrower choosing debt revenue b_2, with a corresponding interest rate $R^*(b_2)$:

$$v_1^{nd}(w_1) = \max_{b_2} \; u(w_1 + b_2) + \beta v_2\Big(y_2 - R_2(b_2)b_2\Big)$$

The value associated with default involves permanent autarky:

$$v_1^d = (1 + \beta)u(y_d)$$

Lemma 3 characterizes the optimal borrowing decision in period $t = 1$, along with the threshold level of wealth that makes the borrower indifferent between repaying and defaulting.

Lemma 3 *If the utility function is CRRA, $u(c) = \frac{c^{1-\gamma}}{1-\gamma}$, then the optimal level of debt taken in* *period $t = 1$ is*

$$b_2^*(\omega_1) = \frac{(\beta R^*)^{-\frac{1}{\gamma}} y_2 - \omega_1}{1 + (\beta R^*)^{-\frac{1}{\gamma}} R^*}$$

The threshold level of wealth that makes the government indifferent between repaying and defaulting in period 1 is

$$\overline{\omega}_1 = \left[(1+\beta)(y_d)^{1-\gamma} - \beta \left(y_2 - R^* b_2^*(\overline{\omega}_1) \right)^{1-\gamma} \right]^{\frac{1}{1-\gamma}} - b_2^*(\overline{\omega}_1)$$

Finally, in period $t = 0$ the government chooses an amount to borrow b_1 in the presence of a sunspot variable s_0:

$$v_0(w_0, s_0) = \max_{b_1} \; u(w_0 + b_1) + \beta \sum_{y_1} \pi(y_1)\, v_1\Big(y_1 - R_1(b_1) b_1 \Big)$$

The realization of the sunspot variable s_0 determines which interest rate schedule will apply, in the case of multiplicity. If $s_0 = 0$ then no sunspot occurs and we assume that the lowest possible schedule is in force. If $s_0 = 1$ then the bad sunspot kicks in and we assume that the interest rate switches to the highest-lying schedule. Lemma 4 describes the interest rate schedules that the borrower faces.

Lemma 4 *The interest rate schedules are:*

$$R_1(b_1) = \begin{cases} R^*, & \text{if } b_1 \leq \frac{y_L - \bar{\omega}_1}{R^*} \\ \frac{R^*}{p_H}, & \text{if } \frac{y_L - \overline{\omega}_1}{R^*} p_H \leq b_1 \leq \frac{y_H - \overline{\omega}_1}{R^*} p_H \\ 0, & \text{otherwise} \end{cases}$$

The solution to the borrower's problem in period $t = 0$ is obtained numerically. We present the results in Section B.4.

B.2 Cole-Kehoe setup

In the Cole-Kehoe framework, it is assumed that the borrower moves first and chooses a level of debt obligation, while lenders post a price that reflects the default risk. Self-fulfilling equilibria are generated by a change of timing of the bond auction. It is assumed that the borrower first chooses the new debt issuance, and then decides whether to default or not. The lenders' sentiment about the default decision at the time of pricing the debt in period $t = 1$ is determined by a sunspot variable s_1. In the case of a bad realization, markets shut down in that period and the government is unable to roll over any debt.

The probabilities of the good and bad sunspot realizations are given by $\Gamma = [p_g, 1-p_g]$.

Again, the problem is presented backwards. In period $t = 2$ the government chooses between repayment and default:

$$v_2(b_2) = \max\Big\{u(y_d), u(y_2 - b_2)\Big\}$$

Taking this decision as given, in period $t = 1$ the government solves:

$$v_1(y_1, b_1, s_1) = \max\Big\{v_1^d, v_1^{nd}(y_1, b_1, s_1)\Big\}$$

where

$$v_1^{nd}(y_1, b_1, s_1 = 0) = \max_{b_2}\ u\Big(y_1 - b_1 + b_2\, q_2(b_2)\Big) + \beta v_2(b_2)$$

indicating that the markets are open in that period (i.e. the sunspot realization is good), and

$$v_1^{nd}(y_1, b_1, s_1 = 1) = u(y_1 - b_1) + \beta v_2(0)$$

if the markets do not open in that period. The value of default is

$$v_1^d = (1+\beta)u(y_d)$$

The period $t = 2$ bond price is trivially given by:

$$q_2(b_2) = \begin{cases} \frac{1}{R^*}, & \text{if } b_2 \leq y_L - y_d \\ 0, & \text{otherwise} \end{cases}$$

Hence, the period $t = 1$ bond price is uniquely pinned down in this case by the debt level (no multiplicity) because there is no debt rollover problem in period $t = 2$.

In period $t = 0$ the government chooses:

$$v_0(y_0) = \max_{b_1}\ u\Big(y_0 + b_1\, q_1(b_1)\Big) + \beta \sum_{y_1}\sum_{s_1} \pi(y_1)\Gamma(s_1)\ v_1(y_1, b_1, s_1)$$

Lemma 5 characterizes the bond price q_1 as function of the level of debt chosen by the borrower. The bond price schedule, along with the numerical solution of the dynamic programming problem, are presented in Section B.4.

Lemma 5 *The bond price schedules are:*

$$q_1(b_1) = \begin{cases} \frac{1}{R^*}, & \text{if } v_1^{nd}\Big(y_1, b_1, s\Big) \geq v_1^d \ \forall y_1, s \\ \frac{1-p_B p_L}{1+r^*}, & \text{if } v_1^{nd}\Big(y_L, b_1, 1\Big) < v_1^d, \text{ and } \geq \text{otherwise} \\ \frac{1-p_L}{1+r^*}, & \text{if } v_1^{nd}\Big(y_L, b_1, 0\Big) < v_1^d, \text{ and } \geq \text{otherwise} \\ \frac{1-p_B}{1+r^*}, & \text{if } v_1^{nd}\Big(y_H, b_1, 1\Big) < v_1^d, \text{ and } \geq \text{otherwise} \\ \frac{p_H p_G}{1+r^*}, & \text{if } v_1^{nd}\Big(y_H, b_1, 0\Big) \geq v_1^d, \text{ and } < \text{otherwise} \end{cases}$$

B.3 Calvo and Cole-Kehoe setup

Finally, we combine the two frameworks in the sense that we model the sovereign debt accumulation in the Calvo fashion, but we also allow for the Cole-Kehoe type of rollover crises.

Again, the problem is presented backwards. The choice in period $t = 2$ is the same as in the pure Cole-Kehoe variant. In period $t = 1$ the government chooses whether to repay or default. In the case of a bad sunspot realization, $s_1 = 1$ (that is markets do not open), the government will default if

$$v^{nd}(y_1, b_1, s_1 = 1) = u\Big(y_1 - R_1(b_1)b_1\Big) + \beta u(y_2) \leq (1+\beta)u(y_d)$$

By contrast, if the realized sunspot is good, $s_1 = 0$, then the value of repayment is

$$v^{nd}(y_1, b_1, s_1 = 1) = \max_{b_2} u\Big(y_1 - R_1(b_1)b_1 + b_2\Big) + \beta u(y_2 - R^* b_2)$$

In period $t = 0$ the government chooses:

$$v_0(\omega_0, s_0) = \max_{b_1} \ u\Big(\omega_0 + b_1\Big) + \beta \sum_{y_1}\sum_{s_1} \pi(y_1)\Gamma(s_1) \ v_1(y_1, R_1(b_1, s_0)b_1, s_1)$$

Once again, we assume that the period $t = 0$ sunspot realization determines which interest rate schedule will apply to the borrower. The following Section B.4 presents the schedules and the numerical solution to the borrower's problem.

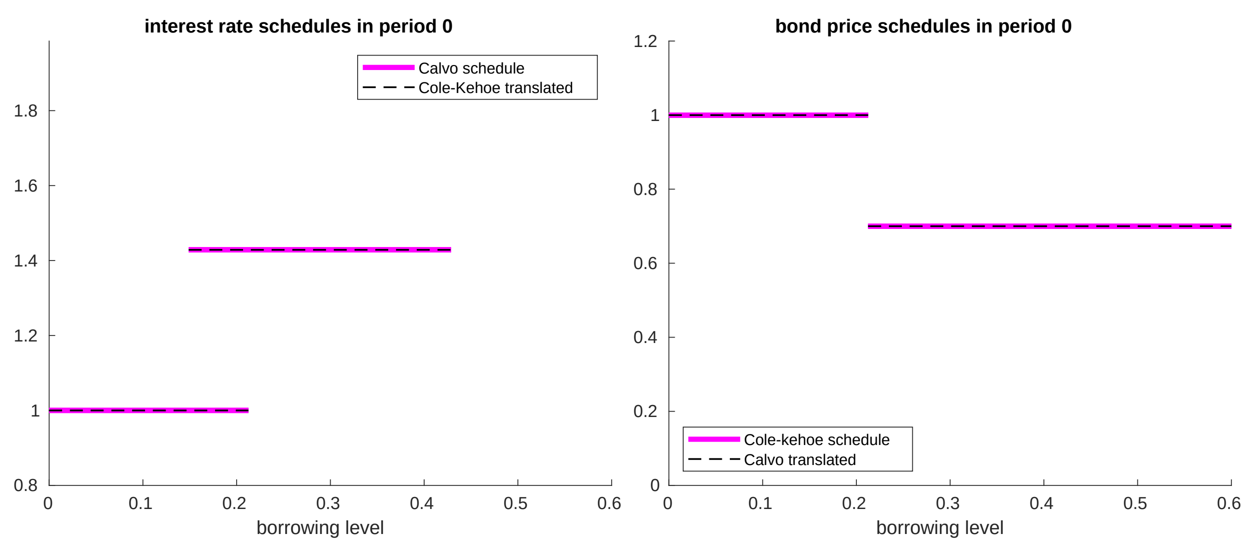
B.4 Numerical example

We will now demonstrate the differences between the models through a simple numerical example. Table 4 contains the parameter values used to solve the model. While these parameters have not been selected in any disciplined way, the following results hold for a wide range of parameterizations.

Table 4: Parameter values assumed in the simple model

Parameter	Meaning	Value
γ	Risk aversion	2.0
β	Discount factor	0.7
y_h	High income	1.2
y_l	Low income	0.8
y_d	Income in default	0.8
y_2	Income in $t = 2$	1.0
p_h	Prob. of low income	0.3
p_g	Prob. of good sunspot	0.8
R^*	Risk-free rate	1.0

Figure 8 shows the interest rate and bond price schedules in the benchmark models of Calvo and Cole-Kehoe, respectively. The first thing to notice are the overlapping interest



(a) Interest rate schedules in Calvo framework (b) Bond price schedules in Cole-Kehoe framework

Figure 8: Interest and bond price schedules in benchmark models

rate schedules in the Calvo model. The overlap determines the region of multiplicity. Second, translating these into the Cole-Kehoe notation yields identical bond price schedules (and vice-versa). In other words, the two models are equivalent. They also yield equivalent policy functions: for the lowest levels of initial wealth, government borrows 0.43 at the higher interest rate of 1.43. As initial wealth rises, the optimal level of debt is either 0.15 or 0.21, depending on the realization of the Calvo sunspot. Equivalently, in the Cole-Kehoe setup, the agent borrows at the price of $q = 0.7$ for the lowest wealth levels, which implies that he defaults in the event of a low income realization whether the market are open or not. As such, the agent avoids the "rollover crisis zone". This means that in both models, as initial wealth rises, the borrower chooses the highest possible level of debt that grants him a risk-free interest rate, and is able to avoid the multiplicity or a self-fulfilling debt crisis altogether.

Now, we present the interest rate schedules in our proposed model that combines the two frameworks. Figure 9 plots the resulting interest rate schedules, along with the ones from the Calvo model without rollover crises depicted in Figure 8(a). It can immediately be noticed that the combined model generates more interest rate schedules which for the most part do not overlap with the original ones. In addition, there are more intervals of multiplicity of equilibrium, which also do not overlap (in terms of the desired borrowing level) with the one predicted by the original model. In fact, in the combined setup multi-

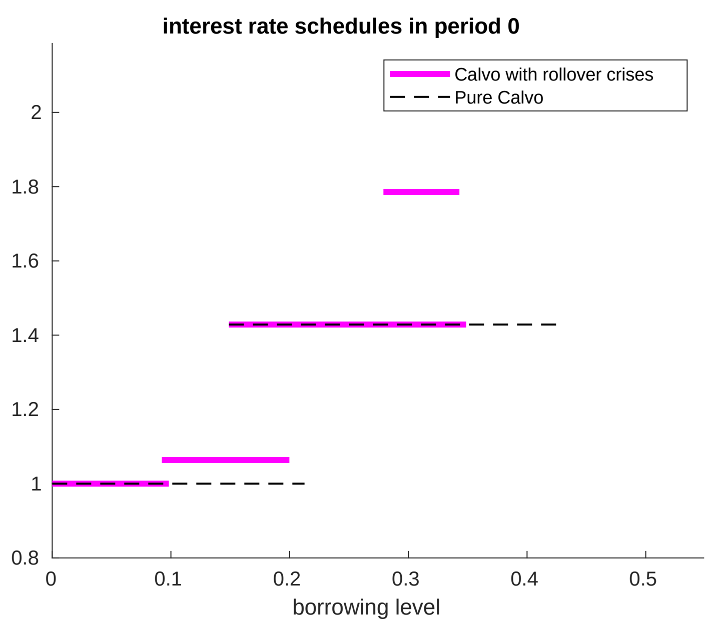
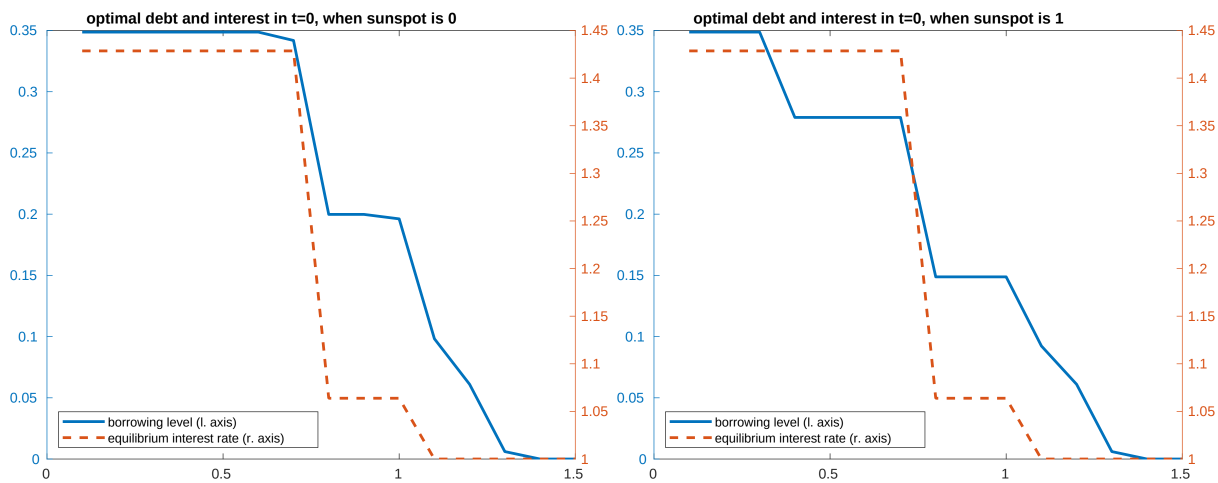


Figure 9: Interest rate schedules in the combined Calvo and Cole-Kehoe model

plicity may occur at much lower levels of borrowing.

Figure 10 plots the optimal borrowing decisions along with equilibrium interest rates for the combined model, as function of the initial wealth level and depending on the initial realization of the Calvo sunspot. It is immediate to notice that the policy functions are more interesting in this setup than in the two benchmarks. In particular, for the case of a good sunspot (panel 10(a)), the agent actively borrows in the region of multiplicity. When the sunspot switches to bad (panel 10(b)), the pattern of borrowing must altered by either borrowing more at a higher interest rate, or borrowing less at a lower one.



(a) Good Calvo sunspot

(b) Bad Calvo sunspot

Figure 10: Optimal borrowing and equilibrium interest rate in the combined model