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Minimum Wage, Business Dynamism, and the Life Cycle of Firms*

André Victor D. Luduvice[†] Tomás R. Martinez[‡] Alexandre B. Sollaci[§]

Abstract

This paper studies the effects of the minimum wage on the life cycle of firms. We first build a tractable model where heterogeneous firms have labor market power, invest in innovation, and choose formal or informal sectors. The model predicts that a minimum wage hike not only shrinks young and low-productivity firms but also lowers incentives to innovate, resulting in lower life cycle growth. We then test the predictions of the model using Brazilian administrative and census data leveraging the variation in exposure across establishments and municipalities to the large increase in the minimum wage between 1999 and 2010. At the establishment level, an increase in the minimum wage: i) decreases the growth rates of small and young establishments and ii) increases the growth rates of old and large establishments. When analyzing exposed municipalities, we observe an increase in the earnings of workers in both the formal and informal sectors, as well as informal employment. Our findings suggest that the minimum wage is a possible explanation for the decline in the importance of young establishments and business dynamism in Brazil.

JEL classifications: J38, J42, E24, E26, L25
Keywords: Minimum wage, Business dynamism, Monopsony, Firm heterogeneity, Informality

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1 Introduction

The minimum wage is widely adopted in both advanced and emerging market economies. Its popularity among policymakers stems from the view that it can address income inequality by raising the earnings of low-wage workers while having limited effects on aggregate employment (Cengiz et al., 2019; Engbom and Moser, 2022). However, it can also have effects beyond the earnings distribution, serving as a tool to curtail firms' labor market power and reallocating labor from low-pay to high-pay firms (Dustmann et al., 2021). Nevertheless, it is also the case that, on average, young firms pay lower wages (Babina et al., 2019; Brown and Medoff, 2003; Michelacci and Quadrini, 2009; Dinlersoz et al., 2019) and are therefore more susceptible to increases in the minimum wage policy. Since they also account for the bulk of net job creation and invest relatively more than their older counterparts (Decker et al., 2014), a minimum wage might reallocate jobs away from the most dynamic firms of the economy. This reallocation toward older, less dynamic firms, can overturn the potential positive effects to address inefficiencies generated by the labor market power.

In this paper, we study the impact of the minimum wage on the life cycle of firms. We investigate the case of Brazil, where the real minimum wage almost doubled in the 15 years following the end of the hyperinflation in 1995. Around the same period, we document that Brazil experienced a broad labor reallocation from young to old establishments. This decline in business dynamism can have profound consequences for productivity growth, wage growth, and job creation (Akcigit and Ates, 2021).[1]

Our paper consists of two parts. First, we develop a tractable, two-period model of heterogeneous monopsonistic firms in the spirit of Berger et al. (2022a), augmented to incorporate investment in innovation as well as an endogenous choice into the formal and informal sector. We use the model to build intuition and rationalize how the introduction

[1]In the United States, several studies point out that the entry rate of new businesses, the rate of job reallocation across firms, and the share of the labor force in young firms have decreased over time (Davis and Haltiwanger, 2014; Decker et al., 2016b). This decline in business dynamism has been associated with increases in markups and market concentration, changes in demographics, and changes in technological factors (Karahan et al., 2019; Hopenhayn et al., 2022; De Ridder, 2021; Weiss, 2020).

or an increase in the minimum wage impacts firms' labor allocation, growth, and choice to formalize. The model also allows us to collect empirically testable predictions about the firm's employment growth, which we empirically test in the second part of the paper. The paper contributes to the literature by (a) expanding the contemporary analysis of minimum wage policy in the presence of labor market power in the context of developing economies with large informal sectors, and (b) focusing on the impact of the minimum wage policy on the life cycle of firms.

In our model, firms differ in their productivity level and produce a single homogeneous good that is traded in a competitive market. The labor market, however, is not competitive. Workplaces are heterogeneous, and the imperfect substitutability of jobs results in a positively sloped labor supply curve for each firm. This is a key feature in the model, as the firm's labor market power implies that a minimum wage can play a role in undoing the negative effects of monopsonistic competition and increasing efficiency in the economy.

In the absence of a minimum wage, all firms make use of their market power to keep employment and wages at a low level. After a minimum wage is introduced, the firm's responses will depend on their productivity level (the optimal wage is itself a function of the firm's productivity). Large firms are not directly affected because they already pay wages that are higher than the minimum wage; medium-sized firms grow as the minimum wage erodes their labor market power, leading them to hire more workers; small firms shrink in size as the minimum wage forces them to pay higher wages. At the bottom of the productivity distribution, firms might also decide to completely exit the formal labor market. Because the minimum wage reduces the profits of smaller firms, it increases the productivity cutoff at which informal firms (which are not bound by the minimum wage) decide to formalize.

In addition to the effects described above, which are standard in the literature, we also characterize firms' dynamic choices and how the introduction of a minimum wage affects firms' expected growth rate. In our model, firms make investment decisions that impact the evolution of productivity over time. In the first period, after sectoral choice and production decisions are made, firms can pay a cost to increase their productivity in the second period.

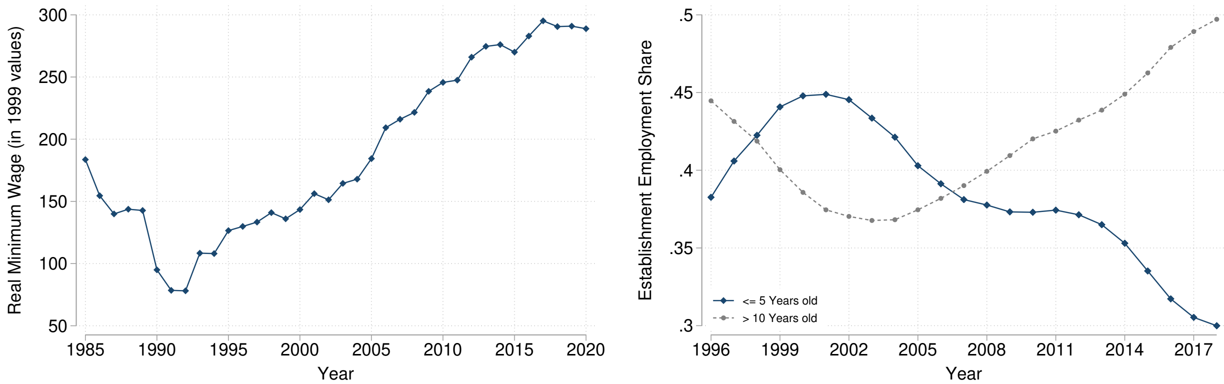
The optimal innovation decision is driven by expected future profits and the employment growth rate can be characterized along the productivity thresholds identified in the static decision.

Large firms that pay higher wages than the minimum wage are once again not directly affected. Medium-sized firms spend more on innovation, but since their optimal employment is already pinned down by the minimum wage, the increase in productivity does not translate to higher employment growth, with a muted effect on the growth rate in the dynamic problem. Small and young firms, on the other hand, decrease their expenditure on innovation and exhibit smaller growth during their life cycle. As a result, the introduction of the minimum wage not only decreases the size of small firms, it also results in them growing at a slower pace. Because young firms also tend to be small, our model predicts that minimum wages not only reallocate labor across the firm size distribution but also across the firm *age* distribution, with potentially long-lasting effects (due to lower growth for young firms).

In the second part of the paper, we empirically test the predictions of our model. We study the impact of a decade-long increase in the federal minimum wage on firm dynamics and employment growth of Brazilian firms, at both the establishment and regional levels, using administrative matched employer-employee and Census data. The Brazilian economy is the ideal laboratory to test our hypothesis: between 1999 and 2010, Brazil's real minimum wage increased by more than 60% through successive and largely unforeseen yearly increases.[2] During the same period, we observe a drop in the entry of new establishments and reallocation of employment from young to old establishments (defined as those that are older than 10 years; see Figure 1).

[2]Recent research has shown that such an increase in the minimum wage accounted for a large share of the reduction of earnings inequality in Brazil, with muted negative effects on employment and output due to the reallocation of workers toward more productive firms (Engbom et al., 2022).

Figure 1: Rise of Federal Minimum Wage (left) and Employment Shares by Establishment's Age (right)



Notes: The left panel shows the real federal minimum wage in Brazil from 1985 to 2020 (values in 1999 BRL). The right panel shows the employment share from 1996 to 2018 for establishments with more than 10 years of age in gray and less than five years in blue. Data are from the RAIS dataset and consider only formal, private, and non-financial establishments.

To isolate the effect of the minimum wage increase and obtain causal estimates, we adopt a two-pronged approach. First, we study the impact of the minimum wage policy in the universe of formal private non-financial establishments founded before 2000, following their employment growth and exit decisions up to 2010. Second, using the Census data, we exploit variation to the minimum wage exposure across Brazilian municipalities to study the impact of the policy on unemployment, labor force participation, and firm entry. Importantly, the Census data allows us to identify workers with informal contracts and self-employed individuals who are not present in the administrative data used in the establishment-level approach.

In the spirit of Card and Krueger (1994), our empirical strategy relies on the fact that establishments and municipalities differ in their pre-policy wage distribution, and therefore in their exposure to the national minimum wage policy. In particular, our treatment is defined as the difference (gap) between the wage of all workers employed in the establishment in 1999 and the minimum wage of 2010. The gap thus captures the proportional increase in an establishment's wage bill required to bring all of its workers employed in 1999 up to the minimum wage in 2010. Importantly, note that this variable captures a decade-long increase in the minimum wage, alleviating concerns that firms may anticipate increases in the wage floor.

We estimate this treatment effect year-by-year from 2000 to 2010 on wage, employment, and firm exit. We find that the establishment employment growth rate between 1999 and 2010 decreased by 0.15 percentage points for every 1 percentage point increase in the minimum wage gap. When focusing only on surviving establishments over the period, we find that the same increase in the minimum wage gap raises the likelihood of exiting by almost 6 percentage points. Furthermore, the policy has markedly different effects on firms depending on their size and age. Large treated establishments experience higher growth than their non-treated counterparts, while the converse effect is observed for small treated establishments. A similar pattern is found when splitting establishments by age, with older treated establishments experiencing higher employment growth relative to the non-treated. Importantly, we also observe that establishments that survive the early years after the treatment show a lower exit probability, indicating employment reallocation from young to older establishments as predicted by the model.

The regional-level approach addresses some of the limitations of the establishment-level analysis. By focusing on the municipality level, we can analyze the effects of the minimum wage increase on informal firms and workers, as well as take into account aggregate and general equilibrium effects at the local level. The treatment is akin to the one used in the establishment-level approach, but accounting for the fact that part of the municipality workers are informally employed. We then follow Dustmann et al. (2021) and use a differences-in-differences framework to estimate the impact of an increase in the minimum wage on firm outcomes at the municipality level. Corroborating the results of the establishment-level approach, we find that firms located in municipalities that are more exposed to the minimum wage rise are older, more likely to exit, and smaller than those in non-exposed municipalities. Concerning employment and occupational choice, we find that a 1 p.p. increase in the gap leads to a 0.16 p.p. rise in the fraction of informal employment and that the minimum wage leads to a rise in the wage of both the formal and informal sectors.

Related Literature. Our paper contributes to several strands of the literature. First, it is closely related to studies on monopsony models of the labor market. Traditionally, most of this literature introduces labor market power through search frictions (Burdett and Mortensen, 1998; Meghir et al., 2015; Jarosch et al., 2019), or heterogeneous preferences for workplaces (Card et al., 2018; Lamadon et al., 2022). In our model, following Berger et al. (2022b) and Hurst et al. (2022), labor market power arises through imperfect substitutability between jobs. Furthermore, most of the literature uses their model to study the redistributive effects of the minimum wage on the wage distribution (Engbom and Moser, 2022; Haanwinckel, 2020), with few papers looking at the allocative effects of this policy. An important exception is Berger et al. (2022a), where misallocation arises because the strategic interactions among firms generate dispersion in markdowns. In our work, the minimum wage distorts investment in innovation and sectoral (formal versus informal) choice.

A recent strand of this literature introduces labor market power in models with informality and self-employment. Informality plays a crucial role in a variety of channels for both firms and workers in developing economies and Brazil (de Paula and Scheinkman, 2011; La Porta and Shleifer, 2014; Meghir et al., 2015; Ulyssea, 2018; Alvarez et al., 2018; Gomes et al., 2020). Notable examples of papers studying labor market power in the presence of informality are Haanwinckel and Soares (2021), Amodio et al. (2023), and Parente (2023). Our analysis differs from these as none of them study the impact of labor market power and the minimum wage on the firm's life cycle. Baumgartner et al. (2020), study employment and firm selection in a similar context, but, differently from our analysis, focus on the effects of a tax reform instead of minimum wage and on a sample period after the one we analyze.

Second, our work is related to the large empirical literature that investigates the effects of a minimum wage increase (Card and Krueger, 1994; Cengiz et al., 2019; Aaronson et al., 2012;

MaCurdy, 2015; Dube et al., 2016).[3] Particularly, this paper is related to a strand of recent studies that investigate the effect of the minimum wage on firm outcomes. Dustmann et al. (2022) document that the introduction of a federal minimum wage in Germany reallocated low-wage workers to more productive, high-paying, establishments. Harasztosi and Lindner (2019) show that Hungarian establishments respond to a minimum wage hike by passing around 75% of their costs to consumers and substituting capital for labor. Draca et al. (2011) find that a minimum wage hike decreases the firm's profit and may increase exit. Similarly, we leverage variation in the firm's pre-policy wage distribution to estimate the effect of the minimum wage on firms' employment and exit decisions in Brazil. This literature focuses on the short-run effects of the minimum wage and abstracts from the effects on the life cycle of firms. We contribute to it by looking at a decade-long increase in the minimum wage and emphasizing the negative effect on young establishments, precisely the most dynamic and high-growth establishments of the economy.

Finally, we contribute to the literature that studies the decline in business dynamism in the US (Decker et al., 2016a; Haltiwanger, 2015) and elsewhere (Calvino et al., 2020; Criscuolo et al., 2014). Among the several possible explanations for this phenomenon, many are connected to an increase in markups and market concentration (Akcigit and Ates, 2021; De Loecker et al., 2021). We contribute to this literature by documenting a fall in the importance of young firms in Brazil and providing a simple model that highlights the role of labor market institutions such as the minimum wage in explaining the decline of business dynamism.

The remainder of the paper is organized as follows. Section 2 describes the model and lays out our theoretical results. Section 3 outlines our empirical strategy and describes the results in our establishment-level and municipal-level approaches. Section 4 concludes the paper.

[3] See Neumark and Shirley (2022) for a recent survey.

2 Model

This section presents a two-period model with heterogeneous monopsonistic firms. The model can be interpreted as a single-market version of Berger et al. (2022a) extended to incorporate informality and investment in innovation. This framework will present intuitively how a minimum wage can affect the firm dynamics. We keep the model simple and purposely focus on the firm's decision in partial equilibrium. In Appendix D, we solve the model in general equilibrium.

The model is cast in discrete time. There is a single labor market with $J < \infty$ firms, indexed by $j = 1, 2, \ldots, J$, with heterogeneous productivity levels $z_j \in \mathbb{R}_0^+$. Firms produce a homogeneous good that can be used to consume or invest and is traded in a competitive market. However, firms have labor market power. Upon entry, they decide to operate in the formal or informal sector, face a static profit maximization, and make a costly investment to improve their productivity. Labor is supplied elastically by a unitary representative household.

2.1 Households

The unitary household has preferences for workplace varieties and chooses labor supply for each firm j to minimize labor disutility and maximize consumption. The elasticity of substitution across firms is given by θ: as $\theta \to \infty$, the labor market becomes competitive. Since the household's problem is static, we omit time subscripts. The problem is given by:

$$\max_{C, \{n_j\}_{j=1}^J} U(C, N) \tag{1}$$

$$\text{s.t.} \qquad N = \left[\sum_{j=1}^{J} n_j^{\frac{1+\theta}{\theta}}\right]^{\frac{\theta}{1+\theta}} \quad \text{and} \quad C = \sum_{j=1}^{J} n_j w_j + \Pi + T,$$

where n_j is the supply of labor to firm j.[4] The aggregate profits, Π, and taxes, T, are rebated to the household. The first-order conditions of the problem imply:

$$U_C(C,N)w_j + U_N(C,N)\left(\frac{n_j}{N}\right)^{\frac{1}{\theta}} = 0. \tag{2}$$

Defining the aggregate wage index as $W \equiv -U_N/U_C$, the solution to this problem is characterized by the inverse labor supply function:

$$w_j = \left(\frac{n_j}{N}\right)^{\frac{1}{\theta}} W \quad \text{and} \quad \sum_{j=1}^{J} w_j n_j = WN. \tag{3}$$

2.2 Firms

Firms live for two periods. Upon entry, they can choose whether to operate formally or informally, but cannot change sectors later. In both sectors, firms face a fixed cost and produce output using only labor according to the decreasing returns to scale technology: $y_j = z_j n_j^{\alpha}$, where $\alpha \in (0,1)$. Formal firms pay taxes on labor and are subject to a minimum wage. Informal firms do not pay taxes, nor are they subject to a minimum wage, but they face a convex and increasing cost of labor as in Ulyssea (2018). We solve firms' problem in two steps. In the first step, we solve the static production plan. In the second step, we solve the investment decision.

2.2.1 Static Profit Maximization

Formal firms. A formal firm j chooses employment n_j to maximize profits taking as given its productivity, z_j, and the household inverse labor supply function. In the absence of the minimum wage, the profit maximization problem reads:

[4] As will become clear later, in the presence of a minimum wage some firms might not want to hire all labor the household is willing to supply and the labor market will fail to clear. Since we will focus on the partial equilibrium effect of the minimum wage on the firm's growth, we abstract from this constraint for simplicity. In Appendix D, following Berger et al. (2022a), we explicitly consider this constraint and solve the model using the shadow wage.

$$
\begin{aligned}
\max_{n_j} \quad & z_j n_j^{\alpha} - (1+\tau) w_j n_j - \kappa \\
\text{s.t.} \quad & w_j = W \left(\frac{n_j}{N}\right)^{\frac{1}{\theta}},
\end{aligned}
\tag{4}
$$

where τ is the payroll tax and κ is the fixed cost in units of the final good. The first-order condition of the problem yields the following labor demand and optimal wage of a formal firm unconstrained by the minimum wage:

$$
n_U(z_j) = \left[\frac{\alpha\theta}{1+\theta}\frac{z_j}{1+\tau}\frac{N^{\frac{1}{\theta}}}{W}\right]^{\frac{1}{1/\theta+1-\alpha}}, \quad w_U(z_j) = \left[\left(\frac{\alpha\theta}{1+\theta}\frac{z_j}{1+\tau}\right)^{\frac{1}{\theta}}\left(\frac{W}{N^{\frac{1}{\theta}}}\right)^{1-\alpha}\right]^{\frac{1}{1/\theta+1-\alpha}} \tag{5}
$$

and the optimal profit function $\pi_U(z_j)$.

Informal firms. Informal firms do not pay taxes but are subject to an increasing convex cost of labor $w_j n_j^{1+\phi}$, where $\phi > 0$.[5] The problem of the informal firm is:

$$
\begin{aligned}
\max_{n_j} \quad & z_j n_j^{\alpha} - w_j n_j^{1+\phi} - \kappa \\
\text{s.t.} \quad & w_j = W \left(\frac{n_j}{N}\right)^{\frac{1}{\theta}}.
\end{aligned}
\tag{6}
$$

The first-order condition characterizes the labor demand for informal firms:

$$
n_j = \left[\frac{\alpha\theta}{1+\theta(1+\phi)} z_j \frac{N^{\frac{1}{\theta}}}{W}\right]^{\frac{1}{1/\theta+\phi+1-\alpha}}, \tag{7}
$$

[5]The convex cost in labor can be interpreted as a reduced form for the probability that informal firms are discovered and fined by labor authorities increases with their size (de Paula and Scheinkman, 2011; Ulyssea, 2018).

and the profits of the informal firm, $\pi_I(z_j) = z_j^{\frac{1/\theta+\phi+1}{1/\theta+\phi+1-\alpha}}\Pi_I(W,N) - \kappa$, where $\Pi_I(W,N)$ is a function of aggregates and parameters.

Introducing a minimum wage. Let z_f be the minimum productivity level such that a firm $z_j \geq z_f$ chooses to formalize (characterized later). Note that the minimum wage, $\overline{w}$, must be at a level equal or higher than the smallest wage paid by the operating formal firms: $\overline{w} > w_U(z_f)$. Otherwise, all formal firms pay salaries higher than $\overline{w}$, and the policy has no effect. Thus, we can define a productivity level $\overline{z}$, given by

$$\overline{w} = w_U(\overline{z}) = \left[\left(\frac{\alpha\theta}{1+\theta}\frac{\overline{z}}{1+\tau}\right)^{\frac{1}{\theta}}\left(\frac{W}{N^{\frac{1}{\theta}}}\right)^{1-\alpha}\right]^{\frac{1}{1/\theta+1-\alpha}}, \tag{8}$$

for which all firms $z_j \geq \overline{z}$ are not directly affected by the minimum wage policy because they pay wages equal to or higher than $\overline{w}$. For formal firms with productivity level $z_j \in (z_f, \overline{z})$, the profit maximization problem becomes

$$\begin{aligned} \max_{n_j} \quad & z_j n_j^{\alpha} - (1+\tau)\overline{w}n_j - \kappa \\ \text{s.t.} \quad & n_j = \min\left\{\left(\frac{\overline{w}}{W}\right)^{\theta} N, \overline{n}_j\right\}, \end{aligned} \tag{9}$$

where $\overline{n}_j = \left(\frac{\alpha}{\overline{w}}\frac{z_j}{1+\tau}\right)^{\frac{1}{1-\alpha}}$ is a rationing constraint on the labor demand. This constraint captures the fact that, for a given minimum wage, there is a level of employment $\overline{n}_j$ where it is not optimal for the firm to raise employment anymore (note that $\overline{n}_j$ is the labor demand of a firm with no market power). It is introduced into the problem because, in equilibrium, households internalize that firms will not hire more employees if it not profitable to do so and thus ration their labor supply to any firm so that it does not exceed $\overline{n}_j$ (even if there is a preference to do so).

As in Berger et al. (2022a), the solution of a firm constrained by the minimum wage is split between two cases.[6] In the first case, even though the firm is constrained to pay a minimum wage, it can still take advantage of its monopsony power to undercut wages below their competitive level. The firm's labor demand is given by

$$n_j = \left(\frac{\overline{w}}{W}\right)^{\theta} N < \overline{n}_j. \tag{10}$$

In contrast, for firms with a sufficiently small productivity z_j, the rationing constraint binds and the firm behaves as if it operated in a competitive labor market, $n_j = \overline{n}_j$, regardless of the labor supply of households. In addition, because $\overline{n}_j$ increases with productivity (while $(\overline{w}/W)^{\theta} N$ does not), there is a unique threshold $\underline{z}$ such that all firms with productivity $z \leq \underline{z}$ operate under the rationed labor supply (assuming $\underline{z} > z_f$). This cutoff is implicitly defined by equalizing the labor supply under the minimum wage with the rationed labor demand:

$$\left(\frac{\overline{w}}{W}\right)^{\theta} N = \left(\frac{\alpha}{\overline{w}}\frac{\underline{z}}{1+\tau}\right)^{\frac{1}{1-\alpha}}. \tag{11}$$

The optimal profit of a formal firm subject to the minimum wage therefore is given by a piece-wise function in three regions: unconstrained, constrained with fixed labor demand, and constrained with competitive labor demand:

$$\pi_F(z) = \begin{cases} \pi_U(z) = z^{\frac{1/\theta+1}{1/\theta+1-\alpha}} \Pi_U(W,N) - \kappa & \text{if } z \geq \overline{z}, \\ \pi_C^f(z) = z\left(\frac{\overline{w}}{W}\right)^{\alpha\theta} N^{\alpha} - (1+\tau)\overline{w}^{1+\theta}\frac{N}{W^{\theta}} - \kappa & \text{if } z \in [\underline{z}, \overline{z}), \\ \pi_C^c(z) = z^{\frac{1}{1-\alpha}} \Pi_C(\overline{w}) - \kappa & \text{if } z \in (z_f, \underline{z}), \end{cases} \tag{12}$$

where $\Pi_U(W,N)$ and $\Pi_C(\overline{w})$ are functions of aggregates and parameters, with $\Pi_C(\overline{w})$ decreasing in the minimum wage.

The results above imply that introducing a minimum wage changes the firm's optimal employment and profits depending on their productivity level, z_j. Keeping aggregate prices

[6] See the Appendix of Berger et al. (2022a) for a complete proof.

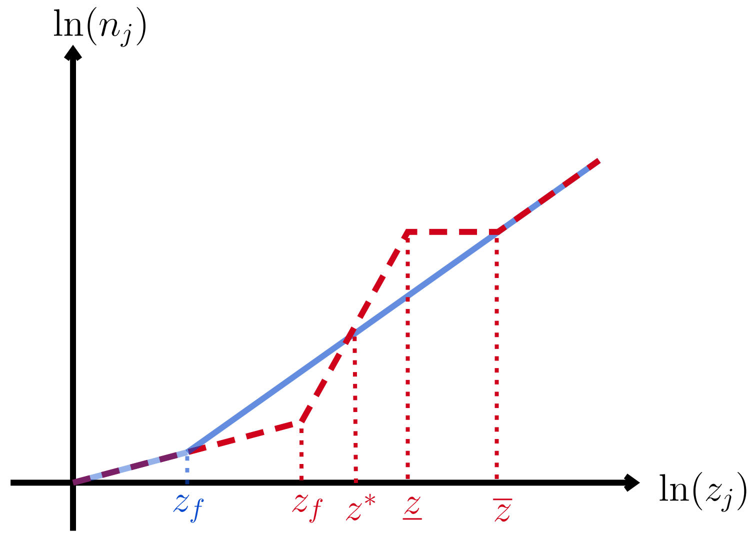
and quantities (i.e., W and N) fixed, (unconstrained) firms above the threshold $\overline{z}$ do not change their behavior, but (constrained) firms $z < \overline{z}$ might increase or decrease in size in the short-run. Specifically, there is a cutoff z^* such that firms $z > z^*$ increase in size and firms $z < z^*$ decrease in size. In the proposition below, we characterize the short-run partial equilibrium effect of the minimum wage on the firm size.

Proposition 1. *Let n_j^{MW} and $n_j^{No\ MW}$ the optimal employment hired by firm j in an economy with and without the minimum wage. Suppose a minimum wage, $\overline{w} > 0$, such that $z^* > z_f$ is introduced. If N and W are fixed, then the optimal firm size changes as follows:*

- $n_j^{MW} = n_j^{No\ MW}$ *if* $z \geq \overline{z}$;
- $n_j^{MW} > n_j^{No\ MW}$ *if* $z \in (z^*, \overline{z})$;
- $n_j^{MW} = n_j^{No\ MW}$ *if* $z = z^*$;
- $n_j^{MW} < n_j^{No\ MW}$ *if* $z \in [z_f, z^*)$.

A formal proof can be found in Appendix A.1; but the intuition behind this result is straightforward. In the absence of the minimum wage, all firms exert their labor market power to reduce employment and keep wages low. When a minimum wage is introduced, large firms ($z \geq \overline{z}$) do not change their labor demand, as they already pay a wage higher than the minimum wage (see equation 5). All other formal firms must increase their wages to comply with the policy; however, firms in this category can no longer affect the wages they pay to workers. A first group of firms, with productivity $z \in [\underline{z}, \overline{z})$, loses part of its monopsony power and thus raises its labor demand (though still lower than the competitive level). The second group of firms, with productivity $z \in (z_f, \underline{z})$ now behaves as if they are in a competitive market. But firms in this group may increase or decrease in size as the minimum wage increases, as the increase in costs might be large enough to offset the effect of the less of market power. In addition, because the elasticity of $\overline{n}_j$ with respect to z is larger than the analogous elasticity in the absence of a minimum wage (equation 5), there is a unique threshold z^* such that firms $z > z^*$ increase and $z < z^*$ decrease in size.

Figure 2: Optimal Firm Size with and without a Minimum Wage



Notes: The figure shows the graph that represents the size determination of the monopsonistic firm in the static problem. The x-axis shows the ln of productivity z_j for a given firm j and the y-axis shows the ln of employment n_j for a given firm j. The solid blue line represents the characterization of the problem for different z_j's without a minimum wage and the dashed red line represents the characterization of the problem after the introduction of a minimum wage. The x-axis exhibits also four different thresholds: z_f, z^*, $\underline{z}$, and $\overline{z}$. The characterization of these thresholds is done in Proposition 1.

2.2.2 Investment Decision

Productivity at the firm level evolves depending on the firm's investment in innovation (Atkeson and Burstein, 2010). In the first period, after the formalization and production decision, a firm with productivity z can pay a cost $z^{\psi}c(p) = z^{\psi}b_1 \exp\{b_2 p\}$ to increase its productivity by a factor of $\lambda > 1$ with probability $p \in [0, 1]$.[7] The cost is specified in units of the final good and scales up as firms become more productive ($\psi > 0$). The parameters $b_1, b_2 > 0$ control the scale and convexity of the cost function. The firm discounts future profits by $\beta \in (0, 1)$.

The value function of a firm operating in the formal sector is therefore

[7]For simplicity, we refrain from adding negative idiosyncratic shocks, as they are inconsequential to our theoretical results.

$$V_F(z) = \max_{p \in [0,1]} \pi_F(z) - z^{\psi} b_1 \exp\{b_2 p\} + \beta \left[p \pi_F(\lambda z) + (1-p)\pi_F(z) \right], \tag{13}$$

where the continuation value uses the fact that the firm only lives for two periods and thus has no incentives to innovate in the second period. The value function of an informal firm, $V_I(z)$, is defined analogously. Assuming an interior solution, the optimal innovation decision is characterized by the following first-order condition:

$$p^*(z) = \frac{1}{b_2} \log \left\{ \frac{\beta}{z^{\psi} b_1 b_2} \left[\pi_F(\lambda z) - \pi_F(z) \right] \right\}. \tag{14}$$

Equation (14) makes clear that the incentives to innovate are related to expected future profits. By changing the returns to productivity, the minimum wage discourages innovation and lowers life cycle productivity growth. As we cannot observe productivity directly, it is useful to characterize the expected growth rate in employment. Define $g_n(z; \overline{w})$ as the expected employment growth of a firm with productivity z. From the results above, we get:

$$g_n(z; \overline{w}) = \mathbb{E}_t \left[\frac{n_{t+1} - n_t}{n_t} \right] = p^*(z; \overline{w}) \frac{n(\lambda z; \overline{w}) - n(z; \overline{w})}{n(z; \overline{w})}. \tag{15}$$

In other words, the minimum wage policy may impact the expected growth rate of firms through both the probability of innovation and the firm's optimal employment size. The following proposition discusses in detail how changes in the minimum wage policy affect the firm's growth rate in each productivity region.

Proposition 2. *Suppose there exists a minimum wage, $\overline{w} > 0$, such that $\underline{z} > z_f$. In addition, assume the productivity growth λ is such that firms do not skip over the cutoffs ($\lambda \underline{z} < \overline{z}$). Let $p^*(z)$ be interior for all $z \geq z_f$ and fix aggregates W and N. Then a marginal increase in $\overline{w}$ has the following effects on the employment growth rate, $g_n(z; \overline{w})$:*

$$\frac{\partial g_n(z;\overline{w})}{\partial \overline{w}} \begin{cases} = 0 & \textit{if } z \geq \overline{z}, \\ = 0 & \textit{if } z, \lambda z \in (\underline{z}, \overline{z}), \\ < 0 & \textit{if } z, \lambda z \in (z_f, \underline{z}). \end{cases} \tag{16}$$

Once again, the proof can be found in Appendix A.2. Proposition 2 states that in our simple two-period model the least productive firms decrease their employment growth rate after an increase in the minimum wage. As neither the profit function nor the optimal employment of large firms ($z \geq \overline{z}$) depends on the minimum wage, a change in their growth rate only happens through general equilibrium effects. Firms with productivity $z \in (\underline{z}, \overline{z})$ increase their innovation expenditure in response to an increase in the minimum wage policy, but since their size does not depend on productivity, the impact on firm growth is irrelevant. Finally, the minimum wage reduces the gains from a productivity increase ($\pi_F(\lambda z) - \pi_F(z)$) for small formal firms ($z \in [z_f, \underline{z})$), leading to lower investment in innovation. Because young firms tend to be small, this leads to slower growth during the firm's life cycle.

2.2.3 Sectoral Choice

Finally, we characterize the sectoral decision of the firm on whether to operate formally or informally. Note that, $\pi_F(z)$ grows faster than $\pi_I(z)$ when productivity z increases. Thus, there is a threshold z^f such that all firms $z_j \geq z^f$ choose to formalize. This threshold is implicitly defined by

$$V_F(z_f) = V_I(z_f). \tag{17}$$

As the optimal profit of a formal firm is a decreasing function of the minimum wage, an increase in the minimum wage (or an introduction) decreases the value of the formal firm V_F, increasing the threshold that a firm operates in the formal sector z_f.

Finally, under positive fixed costs $\kappa > 0$, since the Bellman Equation of the firm, $V(z) = \max\{V_I(z), V_F(z)\}$, is monotonically increasing in the firm's productivity, there will be a threshold, z_e, such that firms $z \geq z_e$ decide to operate. Under free entry, this condition also implicitly characterizes the number J of firms in the economy (note that profits are a function of N and W, which themselves depend on the number of firms that are operating).[8]

2.2.4 Predictions of the Theoretical Model

Even though our theoretical model is simple in many dimensions, it gives rise to testable predictions that will guide our empirical strategy. In particular, the model predicts that the formal firms with the lowest productivity level, in the short run, will decrease in size, while the most productive firms increase in size (or at least do not shrink). Moreover, in the long run, the minimum wage might reduce incentives to innovate, decreasing the life-cycle growth of the least productive firms and increasing incentives to operate in the informal sector. As young firms tend to be small and have low productivity in the data, an increase in the minimum wage may lead to more pronounced effects on young firms.

3 Empirical Evidence

3.1 Institutional Context

The minimum wage policy. The analysis will focus on Brazil, an emerging market country with underdeveloped financial markets and a large informal sector. Brazil's statutory minimum wage is set by the federal government, and it is stated in the monthly wages of a full-time worker (i.e., workers with contracts of 44 hours per week). It applies to all states, industries, and occupations and has historically been re-adjusted year-by-year.[9] As shown in the left panel of Figure 1, after depreciating during the hyperinflation period between

[8]Moreover, with positive fixed costs, it could be the case that $z_e > z_f$ and no informal firm operates.

[9]Since 2000, states are also allowed to set state-specific minimum wages. Five states instituted their own wage floors: Rio de Janeiro and Rio Grande do Sul in 2001, Paraná in 2006, São Paulo in 2007, and Santa Catarina in 2010. In practice, although their wage floors are higher than the federal minimum wage, their growth tracks quite closely to it.

1985 and 1993, the Brazilian real minimum wage experienced a significant increase after the stabilization in 1994. This trend accelerated in the 2000s, at the end of Cardoso's presidency term and at the beginning of Lula's term in 2002, and continued until 2014. Between 1999 and 2010, the real minimum wage increased from 136 BRL to 245 BRL (1999 values), an increase of more than 80% in one decade. Taking together, the minimum wage as a fraction of the median wage increased from around 30% in 1996 to roughly 55% in 2018. This large hike in the real minimum wage is unmatched among developing economies and made Brazil a noteworthy case study in the minimum wage literature (Engbom et al., 2022; Haanwinckel, 2020).

The increase in the minimum wage during 1999-2010 happened during a positive time for the Brazilian economy. The country experienced an average annual GDP per capita growth of 2.8%. Simultaneously, the worker informality rate declined from over 35% to below 25% (see Appendix Figure C.1), accompanied by a reduction in earnings inequality, particularly at the lower end of the distribution. The reduction in earnings inequality is partially attributed to the minimum wage and its spillover effects on both the formal distribution and the informal sector (Engbom et al., 2022; Haanwinckel, 2020).

Until 2008, the determination of the minimum wage in Brazil was an annual process and typically involved a political decision made jointly by Congress and the president during the federal budgeting process. However, starting in 2008, minimum wage growth was governed by a rule considering past inflation and real GDP growth. The substantial increase in the minimum wage during this period resulted from a confluence of political efforts by the ruling Workers' Party,[10] favorable government fiscal outcomes, and economic growth. It is important to note that the Brazilian economic growth in the 2000s is often attributed to external demand shocks, such as increased demand for commodities from China (Benguria et al., 2023), as well as internal supply shocks, such as the rise in educational attainment among the Brazilian labor force. As we will discuss later, this substantial increase in the minimum wage over the next decade was relatively unexpected from the perspective of firms

[10] In Portuguese, *Partido dos Trabalhadores* or PT.

and individuals in 1999.

Business Dynamism. An interesting but lesser-known phenomenon is the decline in business dynamism in Brazil. As illustrated in the right panel of Figure 1, we observe a significant labor reallocation from young to old establishments. In 2000, establishments less than five years old accounted for 45% of total employment in private formal establishments. However, over the course of 18 years, this share declined by 15 percentage points to reach 30%. In contrast, the employment share of older establishments (more than 10 years old) increased from 37% to 50%. This shift toward older establishments can be attributed to a decline in both entry rates and growth rates of young establishments.

In Appendix Figure B.1, we present the decline in the employment share of new establishments (entrants) from nearly 10% to less than 6% in 2015. This decline results from a decrease in the entry of new establishments as well as a reduction in their average size. Moreover, there has been an overall decrease in the growth rates of young establishments. Appendix Figure B.2 compares the growth rates (arc-change) of surviving establishments between 1999-2000 and 2010-2011. Except for one-year-old establishments, the growth rates in 2010-11 were lower for establishments aged 10 years or less. Conversely, the growth rates of mature establishments (more than 10 years old) were higher in 2010-11 compared to 1999-00.

Note that, while the decline in the significance of young firms coincided with the increase in the federal minimum wage, several other aspects of the Brazilian economy changed during the same period. A particularly important factor was the decline in informality rates. In Appendix C.1, we discuss that most of the decline happened because of worker informality. The decline in firm informality was much more subtle. In the following sections, we discuss how the minimum wage and the decline of young firms may be related.

3.2 Data

Our empirical analysis is carried out using two main data sets. The first is the RAIS (*Relação Anual de Informações Sociais*), an administrative matched employer-employee panel data set. It contains all employment spells of the universe of establishments and workers in the Brazilian formal sector, including average gross monthly earnings, selected individual (gender, age, education, hours, occupations, among others), and establishment characteristics (location, industry, and legal status). To construct other establishment-level characteristics such as size and wage bill, we aggregate all workers employed in December of each year.

Based on this information, we build a panel of all non-financial private establishments during the analyzed period. We define an entrant in t if the establishment recorded a positive number of employees for the first time in the sample, and permanent exit if the establishment recorded a positive number of employees in $t-1$ but no employees in t and all years after. The age of the establishment is computed as t minus the entry year.

Crucially, RAIS does not cover employment spells or businesses in the informal sector. Hence, as our second data set, we use the individual 2000 and 2010 Census. Apart from a few questions about self-employed and employers, the census data do not identify businesses. Nevertheless, the data identify workers with formal or informal working contracts, self-employed individuals, and small employers, which tend to be mostly informal businesses. The census data will be particularly useful for constructing municipality-level statistics such as informality shares and unemployment rates. Our census sample is restricted to include only individuals between 15 and 66 years old.

3.3 Establishment-level Approach

In this subsection, we outline the methodology used to estimate the impact of minimum wage increases on individual establishments. Our objective is to identify the effects of these increases on establishment-level growth based on the potential exposure of establishments to the increase in the federal minimum wage policy. Building upon existing literature on the minimum wage (Card and Krueger, 1994; Dustmann et al., 2022; Draca et al., 2011), we construct a measure that captures potential exposure based on the establishment's wage distribution in the pre-policy year. Unlike most of the literature, which focuses on short-run minimum wage hikes, our treatment captures the long-run exposure to a decade of increase in the minimum wage and compares the establishment's wage distribution in December 1999 with the minimum wage in 2010. As the minimum wage policy is readjusted annually to at least account for inflationary losses, firms might anticipate some wage increases from year to year. However, as discussed in the previous section, it was unlikely that establishments anticipated an 80% increase in the real minimum wage in over 10 years, as it was driven by exogenous political and economic factors.[11]

The measure capturing the proportional increase in an establishment's wage bill for all workers to meet the minimum wage in 2010, denoted as GAP_j for establishment j, is defined as follows:

$$\text{GAP}_j = \frac{\sum_{i \in j} \max\{0, \text{MW}_{2010} - w_i\}}{\sum_{i \in j} w_i}, \tag{18}$$

where MW_{2010} is the 2010 national monthly minimum wage (in 1999 values), and w_i is the monthly wage of worker i employed in establishment j in 1999. The establishment wage gap, GAP_j, measures the proportional increase in an establishment's wage bill required to bring all of its workers (in 1999) up to the minimum wage in 2010.

Empirical strategy. We restrict the sample to firms that had at least one employee in 1999 and follow the outcomes in the subsequent years. On average, smaller and younger firms tend to be more exposed and this pattern holds conditional on the average wage (see Appendix Table B.1). Building on Draca et al. (2011) and Harasztosi and Lindner (2019),

[11] The analysis is restricted to 1999-2010 for two reasons: i) in the municipality-level analysis the treatment requires the use of the census, which is available only up to 2010; and ii) in 2008 the government passed a bill adhering the changes to the MW to real GDP growth, increasing the forecasting ability of firms to future changes in the policy.

we estimate a regression of the form:

$$\frac{y_{jt} - y_{j1999}}{y_{j1999}} = \alpha_t + \beta_t \text{GAP}_j + \gamma_t X_{j1999} + \varepsilon_{jt}, \tag{RS1}$$

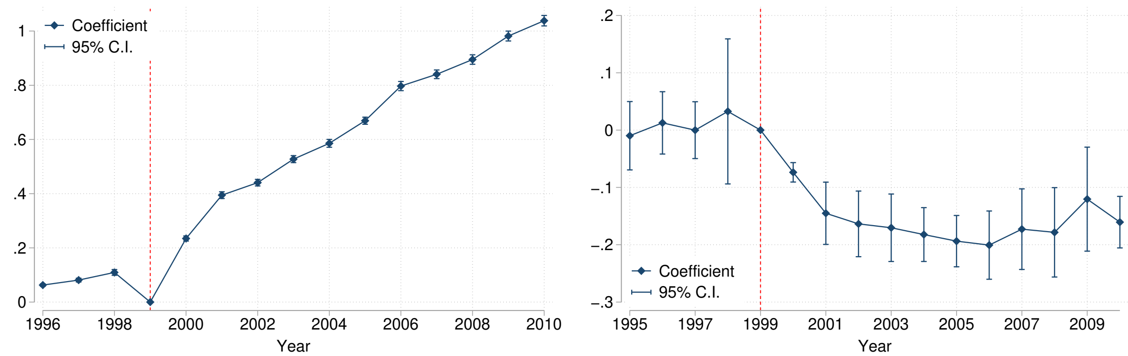
where y_{jt} is the outcome of firm j at time t (either employment or average wage), and X_{j1999} denotes the establishment-level characteristics in 1999. The vector X_{j1999} includes a cubic polynomial in the average wage in 1999, the pre-1999 wage growth, and the interactions of industry-year-size-age-region fixed effects.[12] The parameter β_t identifies the differential employment growth between establishments with high exposure to the minimum wage increase (i.e., high GAP_j) and establishments with low exposure.[13] Crucially, this specification could be estimated with different sample selections to examine the heterogeneous effects on establishment characteristics (e.g., size, age) and different outcomes to uncover adjustment mechanisms.

The empirical strategy relies on the fact that firms differ in their pre-policy wage distribution and, therefore, in their exposure to the national policy. Formally, the underlying identifying assumption is that, in the absence of the minimum wage increase, the outcome in treated firms should evolve in parallel with the outcome in non-treated firms (conditional on the pre-policy characteristics). Given our set of pre-period fixed effects, we are comparing firms within a narrow group of characteristics (age, size, industry, location). As is standard in the literature, identification also requires the absence of spillovers from treated to untreated establishments, as stated by the Stable Unit Treatment Value Assumption (SUTVA).

[12] Industry is defined at 3-digit level, while the region is defined as the micro-region, a set of municipalities akin to the "commuting zone" in the United States. There are seven size categories and six age categories. The size categories are 1-2, 3-5, 6-10, 11-20, 21-50, 51-100, and more than 100 workers, while the age categories are 0 (entrant), 1-2, 3-4, 5-7, 7-9, and 10 or more years old.

[13] This identification strategy can be readily made consistent with our single labor market model—where a firm's size and wage are determined by the same underlying features (and therefore firms of the same size would have the same exposure to the minimum wage)—by including regional labor markets where elasticity of the labor supply faced by firms is different, or several industries where firms' production function varies.

Figure 3: Coefficients β_t: Wage (left) and Employment Growth (right)



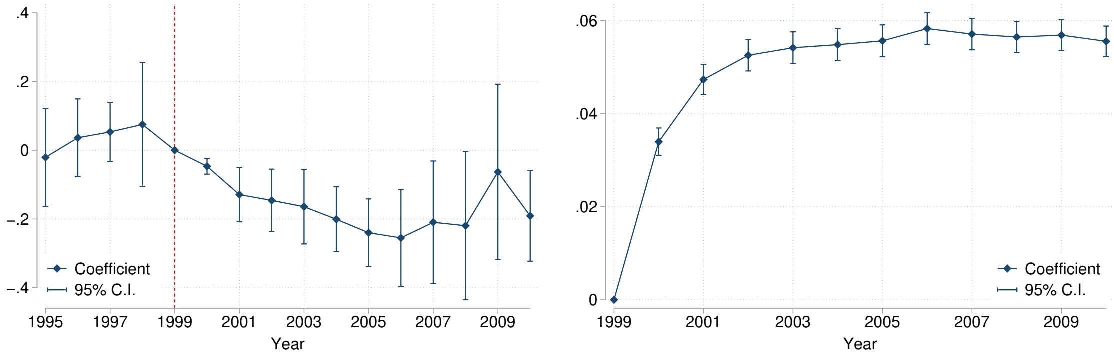
Notes: The figure shows, on the left panel, the estimated coefficient β_t of regression (RS1) with average wage growth as the outcome variable. The right panel shows the estimated coefficient on a regression with establishment employment growth rates as an outcome variable.

Results. We run the regression specification (RS1) year-by-year and present the estimated β_t coefficients along with their confidence intervals in Figure 3.[14] The left panel shows the impact of minimum wage exposure on wage growth, indicating that a 1 percentage point increase in the GAP_j measure leads to a nearly 1 percentage point increase in the average wage growth rate, consistent with full compliance with the minimum wage law. In the right panel, we show the impact of the minimum wage hike on establishment employment growth rates. The findings reveal a negative and statistically significant coefficient, suggesting that treated establishments experienced, on average, lower growth relative to non-treated establishments. Specifically, a 1 percentage point increase in GAP is associated with a 0.15 percentage point decrease in the growth rate between 1999 and 2010.

The analysis above considers both establishments that operated during the entire period and establishments that shut down, reflecting both the extensive margin (exiting) and the intensive margin (hiring and layoff decisions). Figure 4 decomposes the effects in those two margins, showing how employment changes for survivors on the left panel, and the probability of establishment exit on the right panel. The coefficients indicate that exposed establishments had lower employment growth when staying in the market, in a similar magnitude as in the

[14]Standard errors are clustered at the commuting zone level (*regiões geográficas imediatas* as defined by the Brazilian statistical agency).

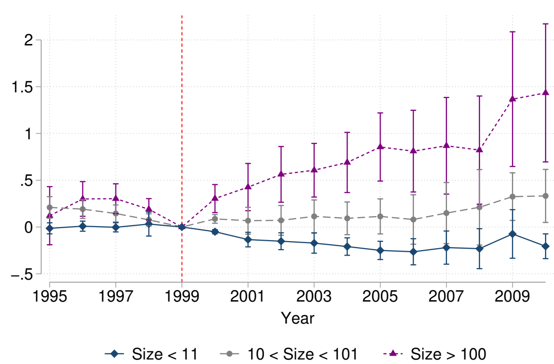
Figure 4: Intensive vs Extensive Margin: Survivors Growth (left) and Exit Probability (right)



Notes: The figure shows, on the left panel, the estimated coefficient β_t of regression (RS1) with employment growth as outcome variable conditional on surviving establishments. The right panel shows the exit probability from a linear probability model adapting the specification in (RS1).

sample with all establishments. To test for the exit probability, we ran a linear probability model using a dummy taking the value one if the establishment is not operating (and zero otherwise) as an outcome. The right panel indicates that a 1 p.p. increase in GAP_j raises the likelihood of exiting the market at almost 6 p.p.

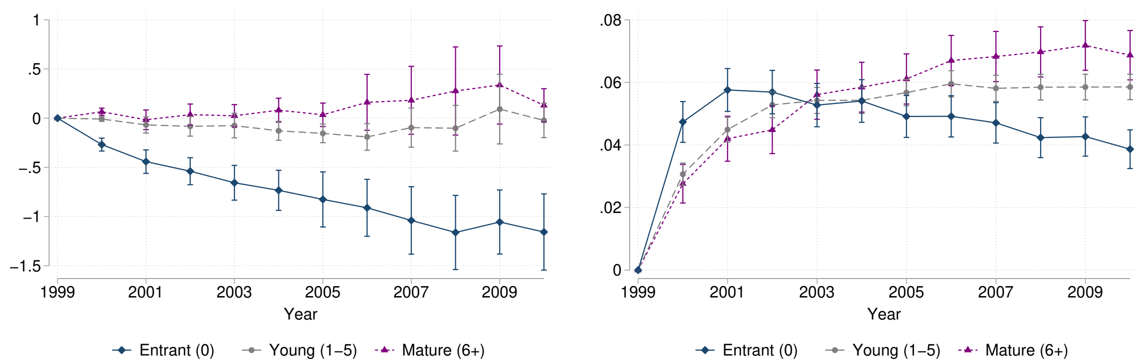
Figure 5: Survivors Employment Growth by Size



Notes: The figure shows the estimated coefficient β_t of regression (RS1) adapted to interact the GAP variable with dummy variables for different firm sizes. The outcome variable is employment growth conditional on surviving establishments.

To explore heterogeneous effects across different establishment types, we extend specification (RS1) by interacting GAP_j with dummy variables representing various establishment groups, such as small and large, or young and old establishments. Our results indicate stark heterogeneity in the effects. Figure 5 presents the coefficients of employment growth for different establishment sizes. Small establishments (with fewer than 11 workers in 1999) exposed to the minimum wage policy experienced lower growth compared to their non-exposed counterparts. In contrast, large exposed establishments (with more than 50 workers) exhibited *higher growth* relative to their non-exposed counterparts. These findings align with the results of Dustmann et al. (2022) when studying the German minimum wage policy. As we show later, the reallocation toward large employers points to a non-linear response of employment to minimum wage changes, consistent with models of monopsonistic labor markets.

Figure 6: Survivors Growth (left) and Exit Probability (right) by Age



Notes: The figure shows results for the regression (RS1) adapted to interact the GAP variable with dummy variables for different firms' age. On the left panel, the figure shows the estimated coefficient for the interacted GAP variable with employment growth as an outcome variable conditional on surviving establishments. The right panel shows the exit probability from a linear probability model.

Finally, Figure 6 highlights heterogeneity in the age of establishments. Treated-young establishments had lower employment growth compared to non-treated-young establishments, while treated-mature establishments experienced *higher* employment growth than their non-treated counterparts (left panel). Furthermore, the right panel shows that treated younger establishments had a higher likelihood of ceasing operations shortly after a minimum wage increase. However, once these establishments survive the early years, their exit probability becomes lower than that of mature establishments. This suggests the presence of selection effects during entry and early years. Overall, these results suggest that the minimum wage policy may contribute to employment reallocation from young to older establishments, as shown in the right panel of Figure 1.

3.4 Regional Approach

Although the establishment-level approach provides valuable evidence of the heterogeneity of minimum wage effects, it has several limitations. First, it focuses exclusively on incumbent establishments, disregarding any post-2000 entrants. Second, it overlooks the informal sector as RAIS data only covers formal firms. Finally, it cannot uncover possible local (i.e., region-level) aggregate and general equilibrium effects, such as unemployment and aggregate wage level effects. To address these limitations, we complement our analysis using national Census data, which allows us to exploit variations in exposure to minimum wage hikes across municipalities and examine evidence along these dimensions.

Treatment and Empirical Strategy. Similar to the firm-level approach, the regional approach constructs a measure of exposure to the minimum wage increase at the municipality level by exploiting its pre-policy wage distribution. In particular, the variable GAP_m captures the proportional increase necessary to bring all workers in the formal sector of municipality m up to the national minimum wage in 2010:

$$\text{GAP}_m = \frac{\sum_{i \in m} \max\{0, \text{MW}_{2010} - w_i^f\}}{\sum_{i \in m} w_i^f + \sum_{i \in m} w_i^i}, \tag{19}$$

where MW_{2010} is the 2010 national monthly minimum wage (in 1999 values), w_i^f is the monthly wage of worker i employed in the formal sector of municipality m, and w_i^i is the monthly wage of worker i employed in the informal sector of municipality m (both in 1999).

To construct the GAP_m variable we make use of both RAIS and the 2000 Census. Using the high-quality administrative data of RAIS, we compute the difference necessary to close the gap between actual wages and the 2010 minimum wage (i.e., the numerator). The denominator requires data on informal wages, which we compute from the 2000 Census using the earnings from informal workers. In particular, we compute:

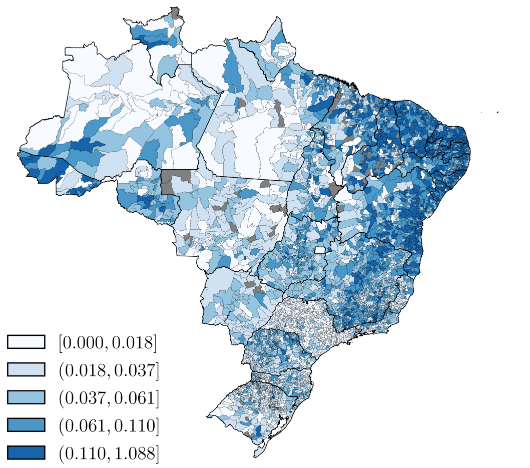
$$\sum_{i \in m} w_i^f + \sum_{i \in m} w_i^i = (1 + \text{RIF}_m) \sum_{i \in m} w_i^f, \tag{20}$$

where $\text{RIF}_m \equiv \sum_{i \in m} w_i^i / \sum_{i \in m} w_i^f$ is the ratio between the sum of informal earnings to the sum of formal earnings in the municipality m. The RIF_m is estimated from the Census, while the sum of formal earnings, $\sum_{i \in m} w_i^f$, is taken from RAIS.

The GAP_m measure is standard and widely used in the literature. One relevant aspect in our setting is the accounting for informal workers. That means that the cross-municipality variation in the treatment stems from two sources: i) the formal wage distribution, which gives how much the minimum wage bites in the formal market; and ii) how large the informal labor market (in earnings) is relative to the formal labor market. Obviously, regions with a large number of workers with low wages in the formal labor market would be largely affected by the minimum wage policy. However, the fact that GAP also depends on the size of the informal labor market induces a non-trivial relationship between the development of the local labor market and the potential exposure to the minimum wage. On one hand, poorer local labor markets tend to have more low-wage workers in the formal sector, increasing their exposure to the national minimum wage policy; on the other hand, their informal sector is large, decreasing their potential exposure.

The measure GAP_m has an (population-weighted) average of 0.072, meaning that if the wages of all workers in the formal labor market were adjusted to comply with national policy, monthly wages would increase, on average, by 7.2%. However, there exists substantial heterogeneity among municipalities, with values ranging from 0 to 1.09. Figure 7 illustrates the spatial distribution of GAP_m across all 5,456 Brazilian municipalities. Darker shades on the map correspond to municipalities with higher exposure to the national minimum wage increase between 1999 and 2010. These highly exposed municipalities are primarily concentrated in the Northeast region, one of the poorest parts of the country.

Figure 7: Exposure to Minimum Wage in Brazilian Municipalities



Notes: The figure plots GAP_m, as defined by equation (19), across Brazilian municipalities.

Following Dustmann et al. (2022), we employ a differences-in-differences framework to estimate the impact of the minimum wage increase during the period 1999-2010 on various outcomes at the municipality level, including establishment entry, informal entrepreneurship, and other relevant variables. We leverage the variation in the bite of the minimum wage across different municipalities captured by GAP_m. The model specification is as follows:

$$y_{jmt} = \alpha_m + \alpha_t + \beta \text{GAP}_m \times \text{Post}_t + \text{controls} + \varepsilon_{jmt} \tag{RS2}$$

where y_{jmt} denotes the outcome of establishment/worker j in municipality m at time t, and α_m and α_t represent municipality and time fixed effects, respectively. The pre-period is defined as 2000 (or December 1999 for RAIS), while the post-period corresponds to 2010. The variable Post_t is an indicator that takes value one if 2010 and zero otherwise. We control for time-varying establishment/individual-level characteristics, as well as a cubic polynomial in municipality income per capita in 2000 interacted with time fixed effects.[15]

[15]The establishment-level controls consist of three-digit industry dummies, whereas individual-level controls include gender, education indicators, age, and age squared.

Akin to the establishment-level approach, the identifying assumption relies on the fact that the outcome of exposed (i.e., high GAP_m) municipalities, conditional on the composition of individuals and establishments, would evolve in parallel with non-exposed municipalities in the absence of the minimum wage. The coefficient, β, measures the effect of increasing 1 percentage point in GAP_m in each outcome after the minimum wage increase of 1999-2010.

Results. Table 1 presents the regression results of the specification (RS2) applied to establishment-level outcomes. The findings align with the previous section. In columns (2), (3), and (4), more exposed municipalities to the minimum wage increase exhibit establishments that are i) more likely to exit, ii) older, and iii) smaller in size compared to non-exposed municipalities. The latter effect of reduction in size can be attributed to the fact that there is a substantially larger number of smaller establishments in our sample. As seen in the previous section in Figure 5, smaller establishments that were treated by the minimum wage gap tend to exhibit lower employment growth when compared to non-exposed establishments. We do not find any significant effects on the share of new establishment entries.

Table 2 presents the coefficient estimates, β, obtained from the individual-level analysis. The results based on the log earnings as the dependent variable revealed that the minimum wage increase had a positive impact on the earnings of both the formal and informal sectors in municipalities with higher exposure. In column (5), a 1 percentage point rise in GAP_m corresponds to approximately a 0.9% increase in average wages of the formal sector, which closely aligns with the expected impact under full compliance with the minimum wage law. Furthermore, as indicated in column (6), the minimum wage also positively influenced the informal sector, albeit to a lesser extent than the formal sector, with a 0.503% increase. This finding supports the notion that the minimum wage might impact wages in the informal sector through general equilibrium effects (e.g., as in Meghir et al., 2015), as well as acts as a reference point even without the need for compliance. As opposed to the effect on workers, the effect of GAP_m on employer earnings is not statistically significant (column 7).

Table 1: Region-level Approach: Firm Results

	(1) Entrant	(2) Exiter	(3) Age > 5 years	(4) log(size)
$GAP_m \times 2010$	-0.0188	0.155***	0.325***	-0.335***
	(0.0290)	(0.0198)	(0.0614)	(0.0918)
Observations	4,707,558	4,707,558	4,707,558	4,707,558
Municipality FE	Yes	Yes	Yes	Yes
Year FE	Yes	Yes	Yes	Yes
Controls	Yes	Yes	Yes	Yes

Notes: The table shows the results for different regressions using specification (RS2) for firms' variables. Column (1) shows the results with the entrant status as dependent variable, column (2) with exiter status as dependent variable, column (3) with age higher than 5 years as dependent variable, and column (4) the log of the size of the firm as dependent variable. Standard errors are clustered at the municipality level. *** p<0.01, ** p<0.05, * p<0.1

Table 2: Region-level Approach: Individual Results

	(1) Informal	(2) Employer	(3) Unemployed	(4) OLF	(5) log($earn$) Formal	(6) log($earn$) Informal	(7) log($earn$) Employer
$GAP_m \times 2010$	0.159***	-0.00884***	0.0481***	0.0268	0.892***	0.503***	0.119
	(0.0236)	(0.00241)	(0.00825)	(0.0174)	(0.0713)	(0.0505)	(0.210)
Observations	7,981,170	7,981,170	13,030,226	13,030,226	3,357,422	2,003,671	175,035
Municipality FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Year FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Controls	Yes	Yes	Yes	Yes	Yes	Yes	Yes

Notes: The table shows the results for different regressions using specification (RS2) for individuals' variables. Column (1) shows the results with the informal worker status as the dependent variable, column (2) with formal worker status as the dependent variable, column (3) with unemployed status as the dependent variable, column (4) with the out of labor force status as dependent variable, column (5) with log of earnings in the formal sector as dependent variable, column (6) with log of earnings in the informal sector as dependent variable, and (7) log of earnings for employers as the dependent variable. Standard errors are clustered at the municipality level. *** p<0.01, ** p<0.05, * p<0.1.

Columns (1) to (4) capture the effects of the minimum wage on occupational choice and employment. In column (1), we consider a binary outcome where a value of 1 represents informal sector employment and 0 otherwise.[16] The coefficient suggests that a 1 percentage point increase in GAP_m leads to a 0.159 percentage point rise in informality. In other words, the minimum wage hike is associated with a 1.5% increase in informality in the 80th percentile municipalities relative to the 20th percentile municipalities based on the GAP_m distribution.[17] Column (2) indicates minimal effects on the probability of being an employer, with less than a -0.1% difference between the 80th and 20th percentiles of GAP_m. Columns (3) and (4) consider all individuals, including the employed, unemployed, and those out of the labor force, estimating the probability of being unemployed and out of the labor force. Consistent with similar studies (Dustmann et al., 2022), both coefficients are economically small.[18]

4 Conclusion

We study the effects of the minimum wage on employment growth and the life cycle of firms. Our focus is the Brazilian economy, where we document a broad labor reallocation from young to old establishments and identify the rise in the minimum wage as one of the main driving mechanisms behind this pattern. We use administrative matched employer-employee and census data to characterize and estimate the impact of this large increase in the federal minimum wage between 1999 and 2010 on firms and workers.

Our analysis is guided by a model of monopsonistic firms that incorporates sectoral choice into formal or informal sectors and investment in innovation. We characterize the firm's decision, finding cutoff strategies on their static and dynamic allocations depending

[16]Informal workers are defined as paid workers without registered employment relationships and self-employed individuals. See Appendix C.1 for more details.

[17]Calculated as $(0.159 \times (0.110 - 0.018))$, where the latter two limits are taken from the percentiles shown in Figure 7.

[18]The employment elasticity with respect to the formal wage is estimated at 0.05 (divide the coefficients of column (3) by column (5)), close to the 0.03 found for Germany by Dustmann et al. (2022).

on their productivity level. The model allows us to make empirically testable predictions and contributes to the contemporary analysis of minimum wage policy in the presence of labor market power by expanding it to the context of developing economies with large informal sectors. Particularly, the model predicts that young and low-productivity firms reduce their incentives to innovate following a minimum wage hike, reducing their size and life cycle growth.

In our empirical analysis, we estimate the impact of the increase in the minimum wage both at the establishment and municipal levels. At the establishment level, we find that an increase in the minimum wage decreases the growth rates of small and young establishments and increases the growth rates of old and large establishments, corroborating the predictions of the model. The establishment-level approach, however, cannot account for effects at entry, informality, or local general equilibrium effects. Using the municipality-level approach, we estimate that an increase in the minimum wage is associated with an increase in the earnings of workers in both the formal and informal sectors as well as the share of informal jobs. We see no evidence that the minimum wage decreases the entry of new establishments.

There are still several avenues for future research. While our theoretical model is qualitatively consistent with our empirical finding, quantitatively matching the extensive changes in the Brazilian economy from 2000 to 2010 remains unexplored. We intend to pursue this line of research in future versions of this work. Finally, we stress that the results presented here are not meant to fully assess the merits of a minimum wage policy, which would take several other factors into account. However, our findings suggest that the minimum wage can produce unintended effects on business dynamism and the life cycle of firms, which should be understood to better conduct policy.

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Online Appendix

A Proof of Propositions

A.1 Proof of Proposition 1

Proposition 1. *Let n_j^{MW} and $n_j^{No\ MW}$ the optimal employment hired by firm j in an economy with and without the minimum wage. Suppose a minimum wage, $\overline{w} > 0$, such that $z^* > z_f$ is introduced. If N and W are fixed, then the optimal firm size changes as follows:*

- $n_j^{MW} = n_j^{\text{No MW}}$ if $z \geq \overline{z}$;
- $n_j^{MW} > n_j^{\text{No MW}}$ if $z \in (z^*, \overline{z})$;
- $n_j^{MW} = n_j^{\text{No MW}}$ if $z = z^*$;
- $n_j^{MW} < n_j^{\text{No MW}}$ if $z \in [z_f, z^*)$.

Proof. First, notice that in the absence of a minimum wage, the optimal hiring decision and wage of all firms in the formal sector is given by equations (5) (i.e., $n_j^{\text{No MW}} = n_U(z_j)$). We can now analyze case-by-case.

If $z \geq \overline{z}$, the firm's optimal wage is higher than the minimum wage, $w_U(z) \geq \overline{w}$, and the minimum wage does not change the firm's labor demand. Therefore, if N and W are fixed, we have that $n_j^{MW} = n_j^{\text{No MW}}$ for all $z \geq \overline{z}$.

If $z \in [\underline{z}, \overline{z})$ the minimum wage binds, and the firm respects the labor supply of the households. Since optimal employment with and without the minimum wage is given by the household's labor supply, $w_j < \overline{w}$, and N and W are fixed, we have that

$$n_j^{MW} = \left(\frac{\overline{w}}{W}\right)^{\theta} N > \left(\frac{w_j}{W}\right)^{\theta} N = n_j^{\text{no MW}},$$

and the firms with $z \in [\underline{z}, \overline{z})$ increase employment after the introduction of the minimum wage.

If $z \in [z_f, \underline{z})$, firms might increase or decrease in size after the introduction of the minimum wage. In this region, all firms are constrained by their labor demand, $n_j = \left(\frac{\alpha}{\overline{w}} \frac{z_j}{1+\tau}\right)^{\frac{1}{1-\alpha}}$. Note that $\overline{n}_j$ increases with z_j at higher rate than the "unconstrained" demand:

$$\frac{\partial \log(\overline{n}_j)}{\partial \log(z_j)} = \frac{1}{1-\alpha} > \frac{1}{1/\theta + 1 - \alpha} = \frac{\partial \log(n_j)}{\partial \log(z_j)}. \tag{A.1}$$

Thus, if $\overline{n}_j$ and n_j (the "unconstrained" labor demand from equation (5)) "cross" for some value $z^* \in (\underline{z}, \overline{z})$, the labor demand for firms $z_j < z^*$ decreases when the minimum wage is implemented, but the labor demand for firms $z_j > z^*$ *increases*. The threshold z^* can be defined by equating n_j and $\overline{n}_j$

$$\left(\frac{\alpha}{\overline{w}}\frac{z^*}{1+\tau}\right)^{\frac{1}{1-\alpha}} = \left[\frac{\alpha\theta}{1+\theta}\frac{z^*}{1+\tau}\frac{N^{\frac{1}{\theta}}}{W}\right]^{\frac{1}{1/\theta+1-\alpha}} \tag{A.2}$$

which gives

$$z^* = \frac{1+\tau}{\alpha}\left[\overline{w}^{1/\theta+1-\alpha}\left(\frac{\theta}{1+\theta}\frac{N^{\frac{1}{\theta}}}{W}\right)^{1-\alpha}\right]^{\theta}. \tag{A.3}$$

Since we assumed that $z^* > z_f$, there are some firms $z \in (z_f, z^*)$ that decreases in size: $n_j^{MW} < n_j^{\text{No MW}}$. Finally, we only need to verify that this threshold, z^*, is "low enough" so that there are some firms that are above it, but still constrained. This requires that $z^* < \underline{z}$, or

$$\overline{w}^{1/\theta+1-\alpha}\left(\frac{1+\tau}{\alpha}\right)^{\frac{1}{\theta}}\left(\frac{\theta}{1+\theta}\frac{N^{\frac{1}{\theta}}}{W}\right)^{1-\alpha} < \overline{w}^{1/\theta+1-\alpha}\left(\frac{N^{\frac{1}{\theta}}}{W}\right)^{1-\alpha}\left(\frac{1+\theta}{\theta}\frac{1+\tau}{\alpha}\right)^{\frac{1}{\theta}}. \tag{A.4}$$

This reduces to $(\frac{\theta}{1+\theta})^{1/\theta+1-\alpha} < 1$, which is always satisfied. Therefore, firms with $z \in (z^*, \overline{z})$ increase in size $n_j^{MW} > n_j^{\text{No MW}}$, and firms with $z = z^*$ stays at the same size.

□

A.2 Proof of Proposition 2

Proposition 2. *Suppose there exist a minimum wage, $\overline{w} > 0$, such that $\underline{z} > z_f$. Let $p^*(z)$ be interior for all $z \geq z_f$ and fix aggregates W and N. Then a marginal increase in $\overline{w}$ has the following effects on the employment growth rate, $g_n(z; \overline{w})$:*

$$\frac{\partial g_n(z;\overline{w})}{\partial \overline{w}} \begin{cases} = 0 & \text{if } z \geq \overline{z}, \\ = 0 & \text{if } z, \lambda z \in (\underline{z}, \overline{z}), \\ < 0 & \text{if } z, \lambda z \in (z_f, \underline{z}). \end{cases}$$

Proof. Let $\mathcal{N}(z;\overline{w}) \equiv \dfrac{n(\lambda z;\overline{w}) - n(z;\overline{w})}{n(z;\overline{w})}$. If $p^*(z;\overline{w})$ is interior, then it only depends on the profit function and parameters:

$$p^*(z;\overline{w}) = \frac{1}{b_2} \log \left\{ \frac{\beta}{z^{\psi} b_1 b_2} \left[\pi_F(\lambda z;\overline{w}) - \pi_F(z;\overline{w}) \right] \right\}. \tag{A.5}$$

Furthermore, we can write the partial derivative as:

$$\frac{\partial g_n(z;\overline{w})}{\partial \overline{w}} = \frac{\partial p^*(z;\overline{w})}{\partial \overline{w}} \mathcal{N}(z;\overline{w}) + p^*(z;\overline{w}) \frac{\partial \mathcal{N}(z;\overline{w})}{\partial \overline{w}}. \tag{A.6}$$

We can now proceed to analyze case-by-case. First, from equations (5) and (12), note that neither $\pi(z)$, nor $p^*(z)$ and $n(z)$ depend on the minimum wage for all $z \geq \overline{z}$. Hence, for a fixed level of W and N:

$$\frac{\partial g_n(z;\overline{w})}{\partial \overline{w}} = 0 \quad \text{if } z \geq \overline{z}.$$

Second, let $z, \lambda z \in (\underline{z}, \overline{z})$. Using equation (12), we have that

$$\pi_F(\lambda z;\overline{w}) - \pi_F(z;\overline{w}) = z(\lambda - 1) \left(\frac{\overline{w}}{W} \right)^{\theta\alpha} N^{\alpha} \tag{A.7}$$

is a positive function of $\overline{w}$ since $\lambda > 1$. Thus, $\partial p^*(z;\overline{w})/\partial \overline{w} > 0$. Moreover, as $z, \lambda z \in (\underline{z}, \overline{z})$, optimal firm size does not depend on z:

$$n = \left(\frac{\overline{w}}{W} \right)^{\theta} N,$$

implying that, for a given minimum wage, $\mathcal{N}(z;\overline{w}) = 0$ and $\partial \mathcal{N}(z;\overline{w})/\partial \overline{w} = 0$. Hence,

$$\frac{\partial g_n(z;\overline{w})}{\partial \overline{w}} = 0 \quad \text{if } z, \lambda z \in (\underline{z}, \overline{z}).$$

Finally, if $z, \lambda z \in (z_f, \underline{z})$, from equation (12):

$$\pi_F(\lambda z; \overline{w}) - \pi_F(z; \overline{w}) = z^{\frac{1}{1-\alpha}} \left(\frac{\alpha}{\overline{w}(1+\tau)} \right)^{\frac{\alpha}{1-\alpha}} (\lambda - 1),$$

which is a decreasing function of $\overline{w}$. Thus, $\partial p^*(z; \overline{w})/\partial \overline{w} < 0$. Since all firms in $z, \lambda z \in (z_f, \underline{z})$ are in their constrained labor demand: $n(z) = \left(\frac{\alpha}{\overline{w}} \frac{z}{1+\tau} \right)^{\frac{1}{1-\alpha}}$, we have that:

$$\mathcal{N}(z, \overline{w}) = \frac{n(\lambda z; \overline{w}) - n(z; \overline{w})}{n(z; \overline{w})} = (\lambda - 1).$$

Since $\partial \mathcal{N}(z; \overline{w})/\partial \overline{w} = 0$ and $\lambda > 1$, we have that the first term of equation (A.6) is negative and the second term zero, implying:

$$\frac{\partial g_n(z; \overline{w})}{\partial \overline{w}} < 0 \quad \text{if } z, \lambda z \in (z_f, \underline{z}).$$

□

B Additional Figures and Tables

Figure B.1: Entry Rate (left) and Entrants Employment Share (right)

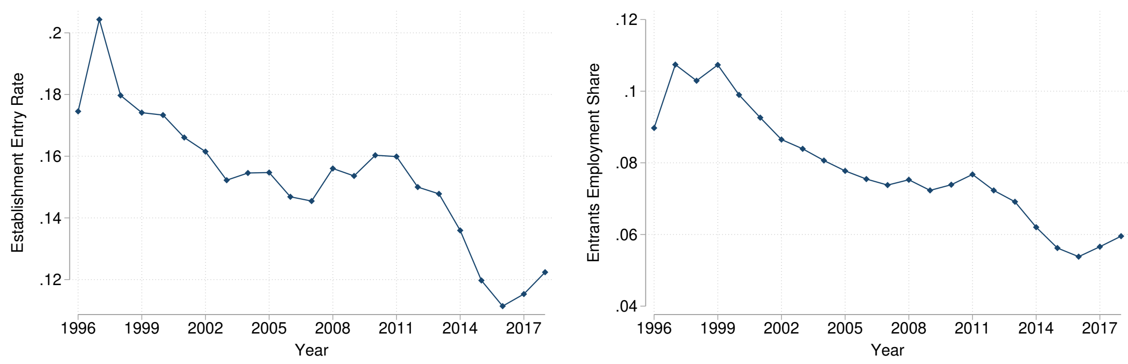


Figure B.2: Establishment Growth Rate by Age: 1999-00 and 2010-11

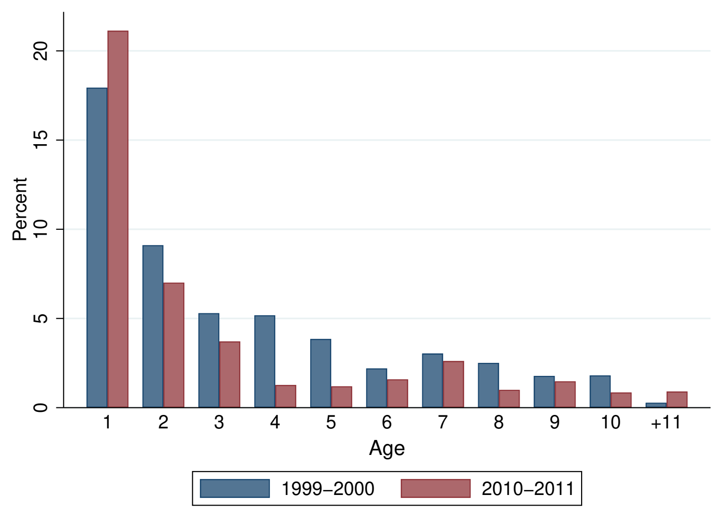


Table B.1: Average GAP_j by selected Characteristics

	Avg. GAP	Observations
All Establishments	0.21	1,809,026
Age ≤ 5	0.24	1,070,155
Age > 5	0.16	738,871
Size ≤ 20	0.22	1,681,936
Size > 20	0.08	127,090

C Data Appendix

C.1 Informality Trends

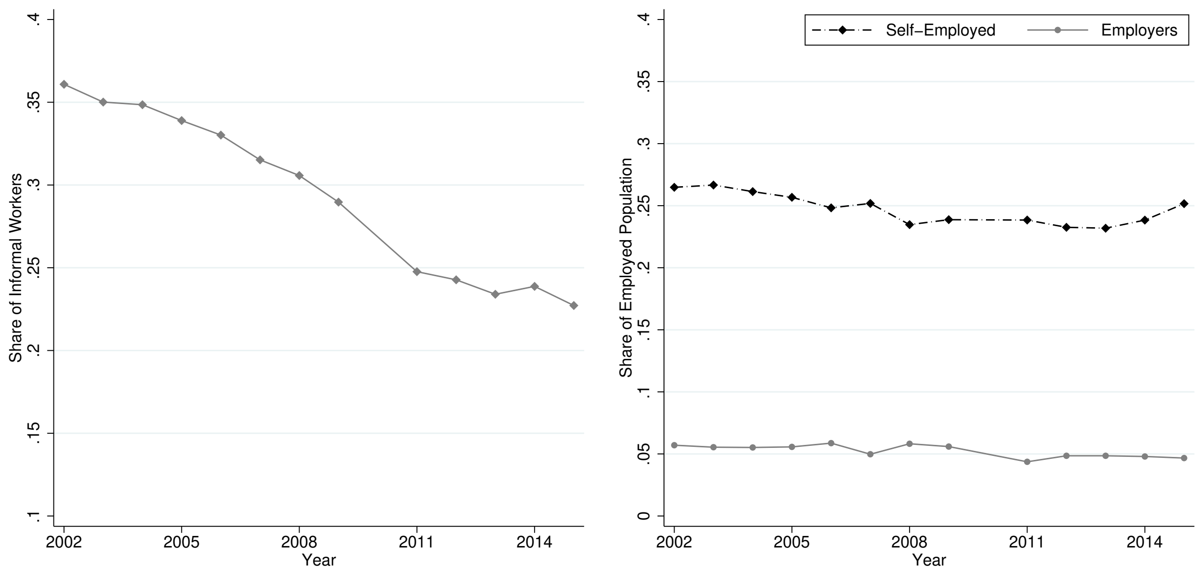
In this subsection, we present an analysis of informality trends in Brazil from 2002 to 2015. To measure the overall trends in informality, we use PNAD (*Pesquisa Nacional por Amostra de Domicílios*), a nationally representative yearly household survey. This survey has traditionally been used to examine long-term shifts in Brazilian demographics, such as age, gender, and education attainment, as well as labor market outcomes including employment, wages, earnings, industry, and occupations. While PNAD does not allow us to track individual workers or identify specific firms like RAIS, it does enable us to identify informal workers and entrepreneurs, including self-employed individuals and employers.[1] Additionally, to gather supplementary information on the formal status of entrepreneurs, we augment PNAD with ECINF (*Pesquisa de Economia Informal Urbana*). ECINF is a cross-sectional matched-employer survey conducted by IBGE, focusing on small urban enterprises. It provides nationally representative data on businesses with up to 5 employees, and for our analysis, we use the latest available year, 2003.

In line with the approach taken by Erosa et al. (2023), we distinguish between workers and entrepreneurs. A worker is considered formal if their labor contract is registered with the Brazilian social security system, which is indicated by the signing in the worker's booklet known as *Carteira de Trabalho*. Meanwhile, a firm is classified as formal if it possesses a tax identification number referred to as CNPJ (*Cadastro Nacional de Pessoa Jurídica*). It is worth mentioning that information regarding the formal status of entrepreneurs is only available in the ECINF dataset, specifically for the year 2003. Unfortunately, PNAD does not provide us with information on the formal status of entrepreneurs.

Figure C.1 illustrates a declining trend in the share of informal workers in Brazil. Over the course of a decade, the informality rate among paid workers decreased from over 35% to approximately 23%. This decline can be attributed to several factors. One significant factor is the demographic change, characterized by an increase in the proportion of younger workers entering the labor market with at least a high school education. Another possible factor is the rise in employment in larger firms, as they tend to have a higher fraction of

[1] Note that in 2012, the Brazilian Bureau of Statistics (IBGE) introduced an updated version of PNAD known as *PNAD-Contínua*. This new version, designed as a nationally representative household rotating panel (five interviews at three-month intervals), replaced the original PNAD. The old PNAD survey was discontinued in 2015.

Figure C.1: Informality in Brazil: 2002-2015



Notes: Workers are defined as paid urban employees in private firms. The share of the employed population is over urban in private firms. PNAD is not available in 2010 since it is a census year. Source: PNAD 2002-2015.

formal workers. Note that during this period, apart from changes in the real minimum wage (which, if anything, would increase informality), there was little change in other labor regulations (Haanwinckel and Soares, 2021).

Measuring the evolution of business informality is challenging due to data limitations. Figure C.1 reveals a slight decline in the share of entrepreneurs in Brazil, from 32% to approximately 30%. This decline primarily stems from a small decline in the number of self-employed individuals, ranging between 26% and 24%. It is noteworthy that the majority of self-employed individuals operate in the informal sector, with around 90% being unregistered, decreasing to 83% in 2012 (Erosa et al., 2023). By comparing entrepreneurs in the ECINF dataset from 2003 with those in the PNAD-*Contínua* dataset, Erosa et al. (2023) argues that the informality rate for firms experienced a slight decline from 30% in 2003 to 28%. Therefore, the decrease in informality is primarily driven by improvements from the worker side, with a minor contribution from the firm side.

D General Equilibrium Model

D.1 The Household's Problem

The household problem is static and can be solved period by period:

$$\max_{C,n_j} \quad \left\{\frac{C^{1-\sigma}}{1-\sigma} - \overline{\nu}\frac{N^{1+\nu}}{1+\nu}\right\}$$
$$\text{s.t.} \qquad C = \int w_j n_j dj + T + \Pi, \qquad N = \left(\int n_j^{\frac{\theta+1}{\theta}} dj\right)^{\frac{\theta}{\theta+1}}, \qquad \text{and} \qquad n_j \leq \overline{n}_j.$$

It is useful to solve the household's problem in two steps. First, we solve an income maximization problem given an aggregate employment index $\overline{N}$. Then, in the second step, we solve for the optimal aggregate labor supply. As in Berger, Herkenhoff, and Mongey (2022a) (BHM), we characterize the first order conditions in terms of shadow wage, $\tilde{w}_j$.

The first step is to solve the following income maximization problem:

$$\max_{n_j} \quad \left\{\int w_j n_j dj\right\} \quad \text{s.t.} \quad \overline{N} = \left(\int n_j^{\frac{\theta+1}{\theta}} dj\right)^{\frac{\theta}{\theta+1}}, \quad \text{and} \quad n_j \leq \overline{n}_j.$$

The first order condition for all j is given by:

$$w_j = \lambda_h \left(\frac{n_j}{\overline{N}}\right)^{\frac{1}{\theta}} + \xi_j, \tag{D.1}$$

where λ_h is the multiplier on the employment index, and ξ_j is the multiplier on the rationing constraint $\overline{n}_j$. Following BHM, we normalize the multiplier on the rationing constraint as $\xi_j = w_j(1-p_j)$. The optimality condition is now written as:

$$\tilde{w}_j \equiv w_j p_j = \lambda_h \left(\frac{n_j}{\overline{N}}\right)^{\frac{1}{\theta}}, \tag{D.2}$$

where $\tilde{w}_j$ is the shadow wage and captures the full extent of the slackness of the rationing constraint. If the constraint is slack, $n_j < \overline{n}_j$, we have $\xi_j = 0$, $p_j = 1$, and the shadow wage is equal to the actual wage: $\tilde{w}_j = w_j$. If the constraint is binding, then $\xi_j \geq 0$, $p_j \leq 1$, and

$\tilde{w}_j \leq w_j$. Manipulating the previous equation and integrating the previous equation:

$$n_j^{\frac{1}{\theta}} = \left(\frac{\tilde{w}_j}{\lambda_h}\right) \overline{N}^{\frac{1}{\theta}}$$
$$\overline{N} = \left(\int \left(\frac{\tilde{w}_j}{\lambda_h}\right)^{\theta+1} \overline{N}^{\frac{\theta+1}{\theta}}\right)^{\frac{\theta}{\theta+1}}$$
$$\lambda_h = \left(\int \tilde{w}_j^{\theta+1} d_j\right)^{\frac{1}{\theta+1}} \equiv \tilde{W},$$

where $\tilde{W}$ is the shadow wage index. Substituting in (D.2), the labor supply function in terms of the shadow wages is given by:

$$n_j = \left(\frac{\tilde{w}_j}{\tilde{W}}\right)^{\theta} \overline{N}. \tag{D.3}$$

The shadow wage aggregates, and it is easy to see that $\int \tilde{w}_j n_j dj = \tilde{W} N$. As explained by BHM, the shadow wage index is allocative, and the household optimizes aggregate labor according to its value. Therefore, we summarize the second step of the household problem as follows:

$$\max_{C,N} \quad \left\{\frac{C^{1-\sigma}}{1-\sigma} - -\overline{\nu}\frac{N^{1+\nu}}{1+\nu}\right\} \quad \text{s.t.} \quad C = \tilde{W}N + T + \Pi.$$

The first-order condition of this problem implies the labor supply condition:

$$\frac{\tilde{W}}{C^{\sigma}} = \overline{\nu} N^{\nu}. \tag{D.4}$$

D.2 The Firm's Problem

Firms are heterogeneous in their productivity. Upon entry, they can choose whether to operate formally or informally, but cannot change sectors later. In both sectors, firms face a fixed cost (possibly different across sectors) and produce output using only labor according to the decreasing returns to scale technology: $y_j = z_j n_j^{\alpha}$, where $\alpha \in (0, 1)$. Formal firms pay taxes on labor and are subject to a minimum wage. Informal firms do not pay taxes nor are subject to a minimum wage, but they face a convex and increasing cost of labor as in Ulyssea (2018). When they operate, firms must pay the fixed cost of production and make the production plan. After, they make an investment decision to upgrade their productivity and decide whether to stay operating or exit the market.

Static problem. The firm's production problem is the same as the one presented in the main text. The only difference is that to make consistent with the shadow wage characterization outlined before, the firm maximizes profits taking as given the labor supply function written in terms of shadow wages (D.3). It is useful to write the problem as a function of the aggregates $X_{agg} \equiv \tilde{W}^{-\theta} N$:

$$n_j = \left(\frac{\tilde{w}_j}{\tilde{W}}\right)^{\theta} N = \tilde{w}_j^{\theta} X_{agg}. \tag{D.5}$$

Note that the profits of the firms depend on the aggregates, X_{agg}. The profits of an unconstrained formal firm:

$$\pi_U(z) = z^{\frac{1/\theta+1}{1/\theta+1-\alpha}} \underbrace{\left(\frac{\alpha\theta}{1+\theta}\frac{1}{1+\tau}X_{agg}^{\frac{1}{\theta}}\right)^{\frac{\alpha}{1/\theta+1-\alpha}} \frac{1+\theta(1-\alpha)}{1+\theta}}_{\Pi_U(\tilde{W},N)} - \kappa_F. \tag{D.6}$$

The profit of a constrained firm on the household labor supply (i.e., the labor demand of the firm is not binding) is

$$\pi_{C,LS}(z) = z\overline{w}^{\alpha\theta} X_{agg}^{\alpha} - (1+\tau)\overline{w}^{1+\theta} X_{agg} - \kappa_F \tag{D.7}$$

Finally, profits of the firms constrained by their labor demand are given by:

$$\pi_{C,LD}(z_j) = z_j^{\frac{1}{1-\alpha}} \left(\frac{\alpha}{\overline{w}(1+\tau)}\right)^{\frac{\alpha}{1-\alpha}} (1-\alpha) - \kappa_F. \tag{D.8}$$

Thus:

$$\pi_F(z) = \begin{cases} \pi_U(z) & \text{if } z \geq \overline{z}, \\ \pi_{C,LS}(z) & \text{if } z \in [\underline{z}, \overline{z}), \\ \pi_{C,LD}(z) & \text{if } z \in (z_f, \underline{z}), \end{cases} \tag{D.9}$$

The profits of an informal firm:

$$\pi_I(z_j) = z_j^{\frac{1/\theta+\phi+1}{1/\theta+\phi+1-\alpha}} \left(\frac{\alpha\theta}{1+\theta(1+\phi)} X_{agg}^{\frac{1}{\theta}} \right)^{\frac{\alpha}{1/\theta+\phi+1-\alpha}} \frac{1+\theta(1+\phi-\alpha)}{1+\theta(1+\phi)} - \kappa. \tag{D.10}$$

Dynamic problem. Productivity at the firm level evolves depending on the firm's investment in innovation (Atkeson and Burstein, 2010). In the first period, after the formalization and production decision, a firm with state z can pay a cost $z^\psi c(p) = z^\psi b_1(\exp\{b_2 p\}$ - 1) to increase their productivity by a factor of e^Δ with probability $p \in [0,1]$, otherwise their productivity decrease by a factor of $e^{-\Delta}$, where $\Delta > 0$. The cost is specified in units of the final good and scales up as firms become more productive (i.e., $\psi > 0$). The parameters $b_1, b_2 > 0$ control the scale and convexity of the cost function. The firm discounts future profits by $\beta \in (0,1)$ and with some probability (δ_F or δ_I, for formal and informal firms) it exits the market with certain. Finally, before production starts, the firm might decide to exit the market.

$$V_F(z) = \max_{p\in[0,1]} \pi_F(z) - z^\psi b_1(\exp\{b_2 p\} - 1) + \beta(1-\delta_F)\left[p\tilde{V}_F(ze^\Delta) + (1-p)\tilde{V}_F(ze^{-\Delta})\right], \tag{D.11}$$

where $\tilde{V}_F(z) = \max\{V_F(z), 0\}$. The value function of an informal firm, $V_I(z)$, is defined analogously. Let $\chi_F(z)$ be the indicator function indicating the exit decision, such that $\chi_F(z) = 1$ represents exit.

Assuming an interior solution, the optimal innovation decision is characterized by the following first-order condition:

$$p_F^*(z) = \frac{1}{b_2} \log\left\{ \frac{\beta(1-\delta_F)}{z^\psi b_1 b_2} \left[\tilde{V}_F(ze^\Delta) - \tilde{V}_F(ze^{-\Delta})\right] \right\}. \tag{D.12}$$

Entry. There is an infinite mass of potential entrants that, upon paying an entry cost c_e (in units of the final good), draw initial productivity from the distribution $G(z)$. After they observe their initial draw, they decide whether to operate in the formal or informal sector. The free entry condition is written as:

$$c_e = \int \max\{V_F(z), V_I(z)\} G(z) dz. \tag{D.13}$$

Denote χ_e as the function indicating the decision to operate in the formal sector, such that $\chi_e(z) = 1$ if the firm is formal, and $\chi_e(z) = 0$ if it is informal.

D.3 Aggregation and Stationary Equilibrium

We will focus on the stationary equilibrium. Let $\Gamma_F(z)$ and $\Gamma_I(z)$ be the stationary distribution of operating firms in the formal and informal firms across the productivity space, and let M_e be the mass of entrants. The stationary distributions must satisfy the following law of motion:

$$\begin{aligned}\Gamma_F(z') =& M^e G(z')\chi_e(z') + (1-\delta_F)(1-\chi_F(z'))p_F(ze^{-\Delta})\Gamma_F(ze^{-\Delta}) \\ &+ (1-\delta_F)(1-\chi_F(z'))(1-p_F(ze^{\Delta}))\Gamma_F(ze^{\Delta}).\end{aligned} \tag{D.14}$$

Similarly, the informal distribution follows:

$$\begin{aligned}\Gamma_I(z') =& M^e G(z')(1-\chi_e(z')) + (1-\delta_I)(1-\chi_I(z'))p_I(ze^{-\Delta})\Gamma_I(ze^{-\Delta}) \\ &+ (1-\delta_I)(1-\chi_I(z'))(1-p_I(ze^{\Delta}))\Gamma_I(ze^{\Delta}).\end{aligned} \tag{D.15}$$

The first term represents the inflow of entrants which is given by the mass of entrants and the distribution of firms entering at productivity level z', accounting for whether the firms are formal or informal. The next two terms represent the inflows of firms shock that i) were at productivity $ze^{-\Delta}$ and drew a positive productivity shock, and ii) were at ze^{Δ} and drew a negative shock. Both terms consider only which survived the exogenous exit shock and endogenously decided to keep operating in the market.

Once we have the distributions $\Gamma_F(z')$ and $\Gamma_I(z')$, together with the free entry condition, we can solve for equilibrium aggregate wage $\tilde{W}$ (as well as all firm's wage w_j), and the mass of entrants M_e.[1] The system is given by:

$$\frac{\tilde{W}}{C^\sigma} = \overline{\nu} N^\nu, \tag{D.16}$$

$$Y = Y_F + Y_I, \tag{D.17}$$

$$Y_F = \int (y(z) - \kappa_F - z^\psi c(p_F^*(z)))\Gamma_F(z)dz, \tag{D.18}$$

$$Y_I = \int (y(z) - \kappa_I - z^\psi c(p_I^*(z)) - w_I(z)n_I(z)(n_I(z)^\phi - 1))\Gamma_I(z)dz, \tag{D.19}$$

$$Y = C + c_e M_e, \tag{D.20}$$

$$\tilde{W} = \left(\int \tilde{w}_j^{\theta+1} d_j\right)^{\frac{1}{\theta+1}}, \tag{D.21}$$

$$c_e = \int \max\{V_F(z), V_I(z)\} G(z)dz. \tag{D.22}$$

[1]Depending on the parameters, there could be an equilibrium where the free entry condition does not hold with equality. We focus only on the equilibrium with positive entry.