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Abstract

This paper develops an alternative approach for estimating the potential output and the output gap, intending to serve as a good balance between a simple low data requiring method and a powerful but complex structural approach. We rely on the Hamilton's Regression filter properties to generate a statistically robust estimator of the potential Gross Domestic Product level which overcomes the problems associated to the Hodrick-Prescott filter and improves the Production Function Approach (PFA). Furthermore, we use this methodology to update the estimates of the potential output and output gap for the Brazilian economy.

JEL Classification: C53; C45; E17.
Keywords: Macroeconometrics; Output gap; Potential output; GDP; Brazil.

1 Introduction

Potential Output, defined as the output level in which an economy is able to generate sustainable growth without inflationary pressures, is an important component to estimate the output gap: the percent deviation between the observed output and its potential (Okun, 1962; Jahan & Mahmud, 2013). Furthermore, the short-run fluctuations of the observed output around its potential reflects the business cycle of an economy, since an actual output above its potential implies a positive output gap which reflects symptoms of overheating and inflationary pressures while an observed output below its potential represents an economy under idle conditions.

Understanding the dynamics of the output gap is a critical task to assess monetary and fiscal policy (Proietti, Musso & Westermann, 2007; Marifian, 2012). Regarding monetary policy, the output gap provides information to be incorporated in Central Bank's monetary policy decision process, especially if they rely on monetary rules á la Taylor as their main policy instrument (Taylor, 1999)[1]. From the fiscal policy side, the output gap measures the cyclical impact of developments on public finances and is a necessary component for estimating the structural fiscal result, allowing decision makers to visualize the real effort of a government in achieving the sustainability of public finances (CBO, 2001).

Measuring the potential output is not a trivial task since it is an unobservable variable and therefore, there exists some grade of uncertainty around its estimations. The latter is a matter of great relevance that could lead to bad decisions in practice. For example, Orphanides (2003) shows how many estimates of potential output in the United States proved to be more optimistic than they really were during the 1960-1970 period. These measures led the Federal Reserve's authorities to believe that the economy was operating below its potential than it was, contributing to actions that overheated the economy and contributed to a large, sustained increase in inflation (Orphanides, 2002).

The different approaches to obtain precise and reliable estimates for the potential output found in the related literature can be classified between univariate and multivariate models (Proietti, Musso & Westermann, 2007) and could vary depending on if the components used in the estimation process are observable or not; at the same time most of them rely on statistical filtering or the estimation of structural relationships (Cerra & Saxena, 2000). However, since the success of this methodologies are highly dependent to the availability and quality of the data produced by official sources, policymakers face a trade-off between the complexity and the grade of transparency/communication achieved during the estimation process (Orphanides & Van Norden, 2002).

For example, the extraction of the trend component of the GDP through statistical filters (Harvey & Jaeger, 1993; Hodrick & Prescott, 1997) has the advantage of being a simple method

[1]The importance of monitoring the output gap within the monetary policy decision process will depend on the objectives of the monetary authority. For example, the Federal Reserve (FED) possess a dual mandate between maintaining full employment and achieving price stability.

in which large data availability is not required -e.g. for instance, just the real GDP time series in the case of univariate models- at the expense of the absence of economic intuition behind such approaches in addition to the common limitations of the filters used. On the contrary, the use of more structural models such as the Production Function Approach (PFA) or General Equilibrium Models (GEM) could provide a specific framework of economic theory behind at the cost of adding complexity to the procedure, both in terms of data requirements and communication to policymakers. Thus, a midpoint between parsimony and robustness is needed to estimate potential output.

This paper's main contribution to the existing literature is to propose the use of the Hamilton Regression Filter (Hamilton, 2018) to improve the estimation of the potential output and the output gap within the Production Function Approach. In essence, this filter circumvents the statistical problems of the HP Filter as it provides two clear benefits: i) it increases the robustness, stability and accuracy of the univariate estimates of the potential output and its main factors, and ii) it simplifies the estimation of via the PFA, by making the estimation of the trend level of its inputs more direct, while its results are close to the more complex structural estimates existing in the literature.

This methodology could contribute to improve the estimation of the potential output and output gap through statistical filters in countries with low data availability. For countries with a wide availability of information, we believe it could contribute to simplify the estimation of the potential product factors within the PFA while maintaining the accuracy of the measurement. The remaining of this paper is organized as follows: Section 2 briefly presents the most common methodologies used to estimate the potential output. Section 3 provides descriptive statistics for the data used in this paper. Section 4 applies the proposed method and compares its results with other common methods to present an updated estimation for the Brazilian economy. Finally, section 5 concludes.

2 Methods to estimate the potential output

Although a large literature has arisen around the measurement of potential output, there is still no consensus on which methodology gives the most reliable results and therefore, there exists high uncertainty around its estimations (Marcellino & Musso, 2011). For instance, a distinction can be made between univariate and multivariate models and if they rely on the estimation of observed or unobserved components.

The univariate approach simplifies the estimation of the potential output to a trend-decomposition of an aggregate economic activity time series (Proietti, Musso & Westermann, 2007). The most common and perhaps most mechanical methodology is associated with the HP filter and other unobserved component procedures (Watson, 1986; Clark, 1987; Harvey, 1989; Harvey & Jaeger, 1993; Hodrick & Prescott, 1997). However, this approach has received criticism on the reliability of its estimates due to the large revisions and of the end-of-sample bias to which the estimates are exposed (Orphanides & Van Norden, 2002). Guay and St-Amant (1996) also question the extent to which the HP filter can extract business cycle frequencies from macroeconomic time series. Another limitation of this type of approaches is that it could lack of economic theory behind it, reducing the potential output estimation process to a merely statistical procedure.

The use of multivariate models provides a better understanding of potential output, since economic relationships could be defined within the estimation process (e.g., Phillips curve, Okun's law). In this sense, Clark (1989) estimates a bivariate model of U.S real output and unemployment in the spirit of Okun (1962). Laxton and Tetlow (1992) introduces a multivariate-filtering technique that generalizes the HP univariate filter, noting that if movements in potential output have a different effect on inflation than do cyclical movements in output, then information on inflation may be useful in identifying potential output. Kuttner (1994) uses a bivariate unobserved-components model for estimating potential output in which the latent variable is modeled as an unobserved stochastic trend and deviations of GDP from potential affect inflation trough an aggregate supply relationship. Butler (1996) proposes an extended multivariate (EMV) filter to exploit demand-side and supply-side theoretical relationships on Canada's potential output. Gerlach & Smets (1999), Apel & Jansson (1999) and Scott (2000) also rely in the use of multivariate filters to exploit structural macroeconomic relationships to obtain estimates of potential output for U.S, U.K, Canada, and New Zealand. Basistha & Startz (2008) uses a multivariate unobserved-component model that includes inflation to extract an estimated NAIRU[2]. Although these approaches try to circumvent the univariate method's limitations, the usefulness of multivariate filters depend on several factors, including the reliability and information content of the structural relationships and the calibration process (St-Amant & Van Norden, 1997).

Proietti, Musso & Westermann (2007) suggest that another common approach found in the related literature is associated with the use of observed component models, which rely on the Beveridge and Nelson (1981) decomposition and on structural vector autoregressive (VAR) models. For instance, Evans (1989) uses a bivariate VAR model incorporating the changes in real output and the unemployment rate to find an estimate of the potential and cyclical components of the U.S. real GNP. St-Amant & Van Norden (1997) employs a structural VAR approach to estimate the output gap for the Canadian economy while Astley & Yates (1999) use the same approach to estimate the potential output for the U.K. More structural and complex methods such as Dynamic Stochastic General Equilibrium (DSGE) models are also used to incorporate economic relationships within the estimation framework. This is the case of Vetlov et al (2011) who provides historical estimates of potential output and output gaps for the Euro area, Czech Republic, and Hungary.

A more empirical -yet with some degree of economic theory within it- method to estimate potential output is the Production Function Approach (PFA). This procedure tries to balance methodological abundance with requirements for policy advice (Cotis, Elmeskov & Mourougane,

[2]Non-Accelerating Inflation rate of unemployment.

2004) and is used by the Congress of the United States Congressional Budget Office (CBO) to estimate U.S. potential output, output gap and to make 10-year projections (CBO, 2001). Artus (1977), Giorno et al (1995), De Masi (1997) and most government Committees such as the EU economic Policy Committee have also relied on this approach due to its key advantages. In specific, Arnold (2009) mentions how this approach: i) looks explicitly at the supply side of the economy, ii) its allowance for a transparent accounting of the sources of growth, iii) supplies a projection for potential output -e.g., that is consistent with the CBO projection for the federal budget in the case of the U.S.-, iv) by using a disaggregated approach, the method can reveal more insights about the economy than a more-aggregate model would. Notwithstanding, the author also mentions some limitations regarding how the simplicity of the method could be perceived as a drawback since some of the parameters included -e.g., share coefficients of labor and capital within the production function- are imposed rather than estimated, and how the capital stock and the use of deterministic time trends to cyclically adjust most of the variables in the model may introduce measurement errors.

3 Data

This paper relies on official data from Ipeadata and Instituto Brasileiro de Geografía e Estatistica (IBGE, for its acronym in portuguese) to compute estimations in annual basis from 1991 to 2021 and in quarterly basis from 2012Q1 to 2022Q1. Table 1 below specifies the variable's frequency and sources, while table 2 exhibits their main descriptive statistics.

Table 1: Main variables and sources

Variable	Period	Source
Panel A. Annual basis		
Real GDP	1991-2021	Ipeadata
Capital Stock	1991-2021	Ipeadata
Installed Capacity	1991-2021	Ipeadata
Labor Force	1991-2021	IBGE
Unemployment rate	1991-2021	IBGE
Panel B. Quarterly basis		
Real GDP	2012Q1 - 2022Q1	IBGE
Capital Stock	2012Q1 - 2022Q1	Ipeadata
Installed Capacity	2012Q1 - 2022Q1	Ipeadata
Labor Force	2012Q1 - 2022Q1	IBGE
Unemployment rate	2012Q1 - 2022Q1	IBGE

Table 2: Main descriptive statistics

Variable	Units	Obs	Mean	Std. dev.	Min	Max
Panel A. Annual basis						
Real GDP	2010 prices in trillion R$	31	3.3	0.8	2.1	4.3
Capital Stock	2010 prices in trillion R$	31	8.2	1.5	5.8	10.1
Installed Capacity Util.	Percentage, %	31	79.9	4.1	72.0	85.2
Labor Force	Millions of people	31	86.8	14.6	60.2	107
Unemployment	Percentage, %	31	11.2	2.2	7.0	14.7
Panel B. Quarterly basis						
Real GDP	2010 prices in trillion R$	41	954.2	26.7	860.2	998.5
Capital Stock	2010 prices in trillion R$	41	10.0	0.2	9.3	10.1
Installed Capacity Util.	Percentage, %	41	0.8	0.0	0.6	0.9
Labor Force	Millions of people	41	102.1	3.6	95.7	107.8
Unemployment	Percentage, %	41	10.6	2.7	6.3	14.9

The parameters used to calibrate the capital share (α) and the labor share ($1-\alpha$) of the Cobb-Douglas production function, take a value of 0.35 and 0.65, respectively. The latter is consistent with the parameters used by Brazil's Ministry of finance to estimate the fiscal structural result (Ministério da economia, 2022).

4 Applications to the Brazilian Economy

4.1 The Hodrick Prescott filter (HP Filter)

The HP filter intends to separate an original time series into a trend component and a cyclical component, requiring only the data from the series itself. Let be a series y_t, where $t \in [1, ..., T]$, which can be decomposed into:

$$y_t = g_t + c_t + \epsilon_t$$

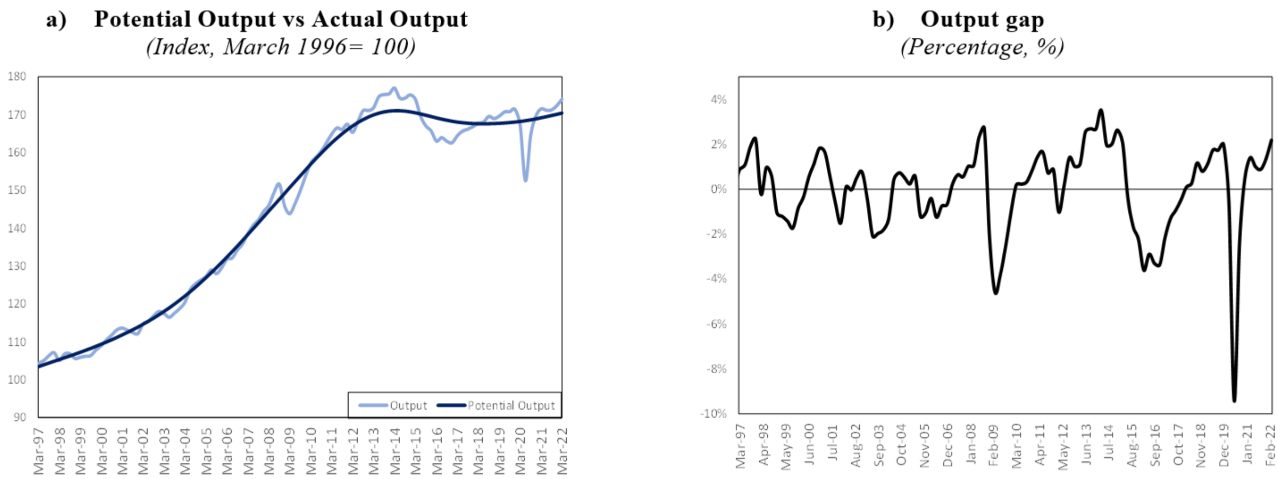
Where g_t represents the trend component, c_t the cyclical component, an ϵ_t the random component. To isolate the trend from a series, the HP filter problem consists in obtaining the series g_t^* that minimizes the following expression:

$$\min_{\{g_t\}_{t=-1}^{T}} \left\{ \sum_{t=1}^{T} (y_t - g_t)^2 + \lambda \sum_{t=1}^{T} [(g_t - g_{t-1}) - (g_{t-1} - g_{t-2})]^2 \right\}$$

In the expression above, the first term represents the problem of minimizing the distance between the trend and the original series, and the second term represents a penalty for trend series that exhibit fluctuations. The *lambda* constant is known as the penalty parameter to fluctuations in the trend term. Hence, when $\lambda = 0$, the trend obtained is exactly equal to the original series and when $\lambda \to \infty$, the solution of the problem is given by a linear trend series. The most common value for the parameter found in the literature is $\lambda = 1600$ for quarterly data and $\lambda = 100$ for yearly data.

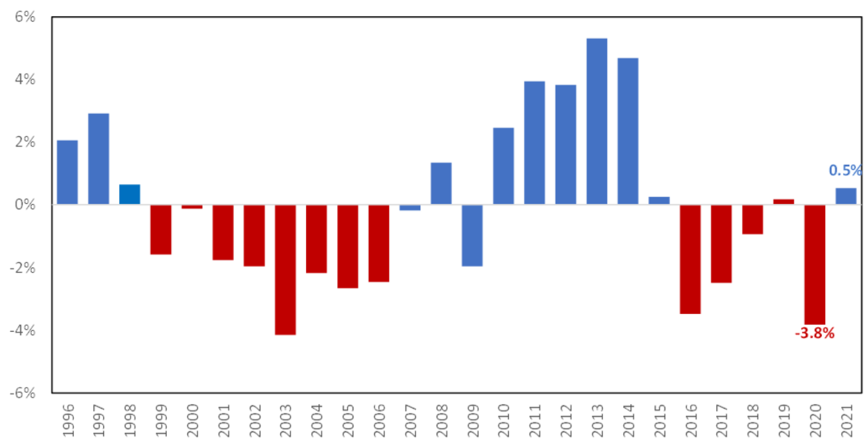
We apply this method for the Brazilian economy using the quarterly GDP series from 1996-Q3 to 2022-Q1. The results suggest a positive output gap averaging 1.2% in 2021 and 2.2% in the first quarter of 2022 (Figure 1). This result is weakly maintained using the procedure on the same time series in a yearly basis, since a positive output gap of 0.5% is obtained for 2021 (Figure 2). The difference in magnitude on the results raise concerns about the robustness of the HP estimates and points out HP filter's main limitation: the end-of-sample bias.

Figure 1: **Quarterly potential output and output gap via HP filter**



Source: Own estimations based on official data.

Figure 2: **Yearly Output gap for Brazil via HP filter**



Source: Own estimations based on official data.

Several explanations have been made towards the HP filter limitations. Hamilton (2018) points out three main statistical disadvantages: i) the HP filter produces series with spurious dynamic relationships that have no basis in the underlying data generation process, ii) the arbitrariness in the choice of the smoothing parameter lambda (λ), since the author highlights there is no way to define a rigorous procedure for selecting the optimal parameter -e.g., when facing a random walk series-., and iii) the end-of-sample bias, which causes variation in the filtered values of the time series at the end of the sample relative from those in the middle besides being characterized by spurious dynamics.

The latter is, perhaps, the most important limitation of the HP filter. It occurs because the filter has the artificial ability to predict the future, since the filter is a function of future realizations. Thus, when applying the HP filter in a full sample, information from all periods -past and future- is used to obtain the trend component at each point over the time horizon. As an example, to estimate the value of the potential GDP for 2010, the procedure uses information from the full sample, including not only the GDP data for years prior to 2010 but also the data afterwards. The bias causes heterogeneity of the trend estimates in each period. In practice, an estimate of potential output made in real time for the year 2022 would be significantly different from a backward-looking estimate for this same period if made a few years later. This limitation is challenging for policymakers and researchers, considering their need to monitor the economy's health to take decisions.

4.2 The Hamilton Filter

Hamilton (2018) proposes an alternative estimator to extract the trend of a time series by suggesting a different concept to define its cyclical component: how different is the value observed at date $t+h$ relative to the value that would have been predicted based on the behavior of the series until date t. Under this idea, this difference is caused by cyclical components in a short time horizon. In this sense, the estimate for the trend component can be obtained from the linear population projection of a y_{t+h} series at a constant a on the four most recent values of the y series available up to date t:

$$y_{t+h} = \beta_0 + \beta_1 y_t + \beta_2 y_{t-1} + \beta_3 y_{t-2} + \beta_4 y_{t-3} + v_{t+h}$$

The latter is a simple approach to extract the trend that achieves the objectives sought by the HP filter while circumventing its problems and limitations. Particularly, these estimates are stable and robust to the end-point-bias, allowing proper real-time monitoring of the economy. For quarterly data, for example, a trend estimation based on pre-views taken at a horizon of two years ahead would be:

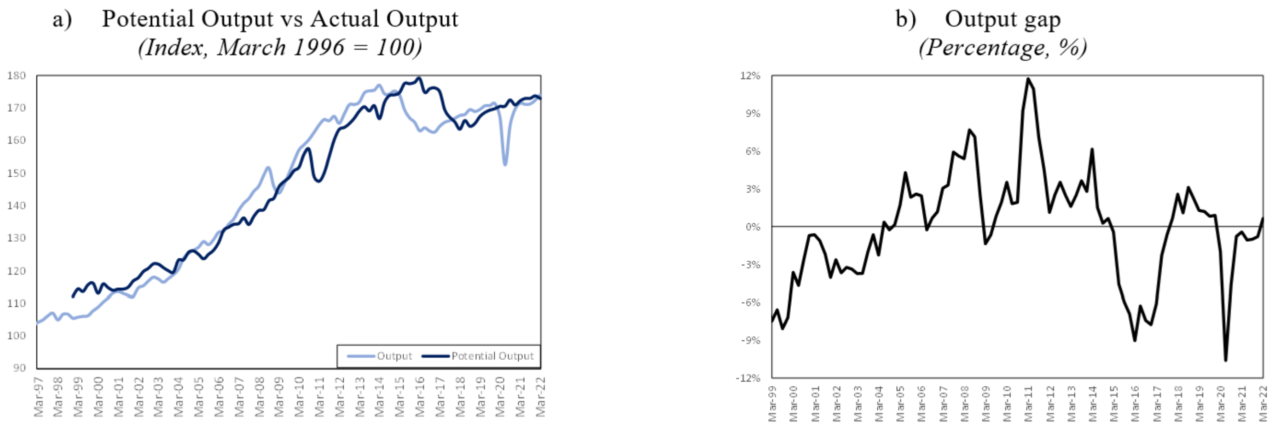
$$y_{t+8} = \beta_0 + \beta_1 y_t + \beta_2 y_{t-1} + \beta_3 y_{t-2} + \beta_4 y_{t-3} + v_{t+8}$$

Which can be rewritten as:

$$y_t = \beta_0 + \beta_1 y_{t-8} + \beta_2 y_{t-9} + \beta_3 y_{t-10} + \beta_4 y_{t-11} + v_t$$

An application of the Hamilton Filter on the Brazilian quarterly GDP series from 1996-Q3 to 2022-Q1 gives as a result that the Brazilian economy operated below its potential during 2021, with an average quarterly output gap of approximately 0.8%. Results for the first quarter of 2022 show a positive output gap around 0.7% (Figure 3).

Figure 3: **Quarterly potential output via Hamilton Filter**

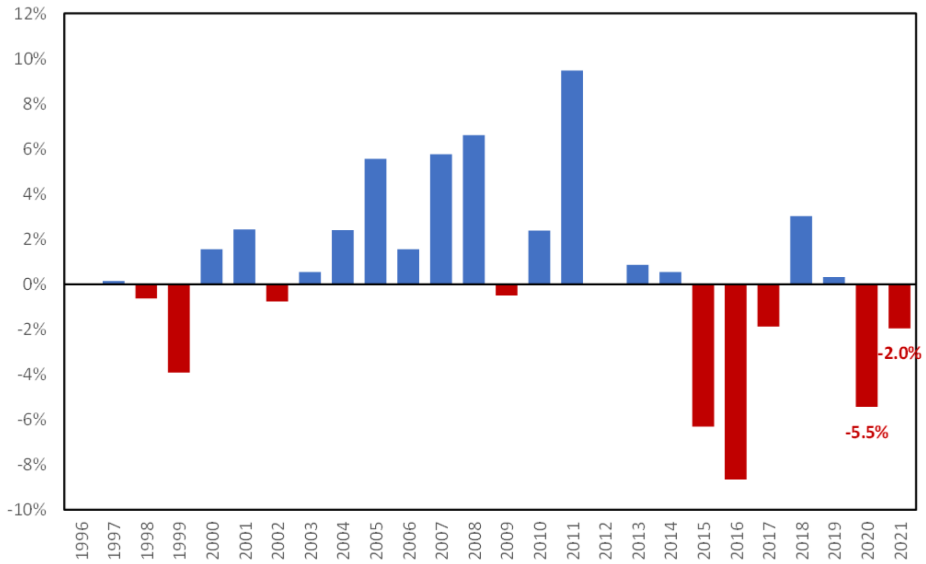


Source: Own estimations based on official data.

These results rise some interesting points. First, the estimate of the potential output via the Hamilton filter is not as smooth as the one obtained using the HP filter. Intuitively, the former method seems to become more adherent to capture changes in the growth path of the potential output, since the Hamilton filter does not use the future information to obtain the trend at each point in time. Second, a difference in the estimates obtained between both filters can be observed at the end of the sample: while the HP filter indicates positive values for the output gap throughout all quarters of 2021, negative values are obtained using the Hamilton filter approach.

Hamilton filter procedure applied to the observed GDP in a yearly basis suggest that the output gap for the Brazilian economy increased 3.5 p.p. from -5.5% in 2020 to -2.0% in 2021 (Figure 4). Unlike what was observed on the HP filter procedure, the results suggest a consistency between the yearly and the quarterly estimates: although the difference in the magnitude of the output gap between both frequencies is not minor, both exercises using the same model exhibit a negative output gap for the Brazilian economy in 2021. These results illustrate the greater consistency of the Hamilton-based estimates relative to the HP filter ones, especially those at the end of the sample.

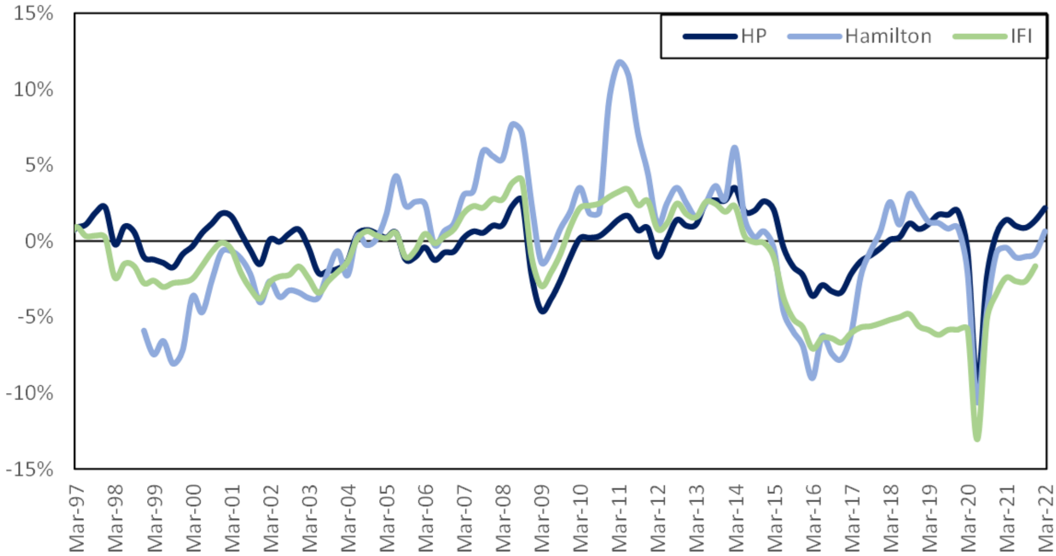
Figure 4: **Yearly Output gap for Brazil via Hamilton filter (Percentage, %)**



Source: Own estimations based on official data.

Figure 5 illustrates a comparison between the output gap estimates using the HP filter, the Hamilton filter, and the model used by the Brazilian Independent Fiscal Institution (IFI) . IFI estimates are used as a benchmark for the Brazilian Economy since they are considered as the most robust and updated methodology following most recent academic developments in the field, despite having a high cost of complexity. In general, throughout most of the analyzed time horizon, the output gap estimate obtained via Hamilton filter is closer to the IFI benchmark relative than the estimates obtained by the HP filter. Furthermore, the Hamilton estimates at the end of the sample are a mid-point between the HP filter and the IFI model. The latter reflects the ability of Hamilton's procedure to approximate the estimates obtained through more complex models, but a lower cost in terms of methodological complexity and data requirements. The Hamilton filter is therefore an ideal approach for countries that do not have a large amount of economic data available but wish to produce an alternative estimate that comes close to the most robust techniques available.

Figure 5: **Output gap estimate comparison (Percentage, %)**



Source: Own estimations based on official data.

4.3 The Production Function Approach (PFA)

Another useful methodology for estimating potential output and output gap is the production function approach (PFA). Although this methodology adds complexity, one of the advantages of this method is related to its structural approach, that is, a method that is based on economic theory; enabling a better understanding of the estimates and allowing researchers and policymakers to decompose the factors that influence the potential output and the output gap.

The standard way of estimating the potential output via the PFA can be made in four steps. The first step specifies a Cobb-Douglas production function assuming constant return to scale on its productive factors: capital (K) and Labor (L).

$$Y_t = A_t K_t^{\alpha} L_t^{1-\alpha}$$

Which can be represented in more detail as:

$$Y_t = A_t \cdot (K_t C_t)^{\alpha} \cdot [L_t (1 - U_t)]^{1-\alpha}$$

Where Y represents the economy's output; A represents total factor productivity; K, the capital stock; C, the level of capacity utilization; L, the labor force; and U denotes the unemployment rate. The constant α represents the share of capital in output.

The second step consists in obtaining an estimate for the Total Factor Productivity (TFP) via the Solow residual since it is an unobservable variable.

$$\ln(A_t) = \ln(Y_t) - \alpha \ln(K_t) - (1-\alpha)\ln(L_t)$$

The following step consists in estimating the trend level of each of the production function's factors. This stage is the most subject to methodological variations, since some researchers rely on the use of simple moving average techniques or the HP filter to detrend the time series, while others add a second layer of theoretical framework adopting structural models to obtain the long-run component of each production factor[3].

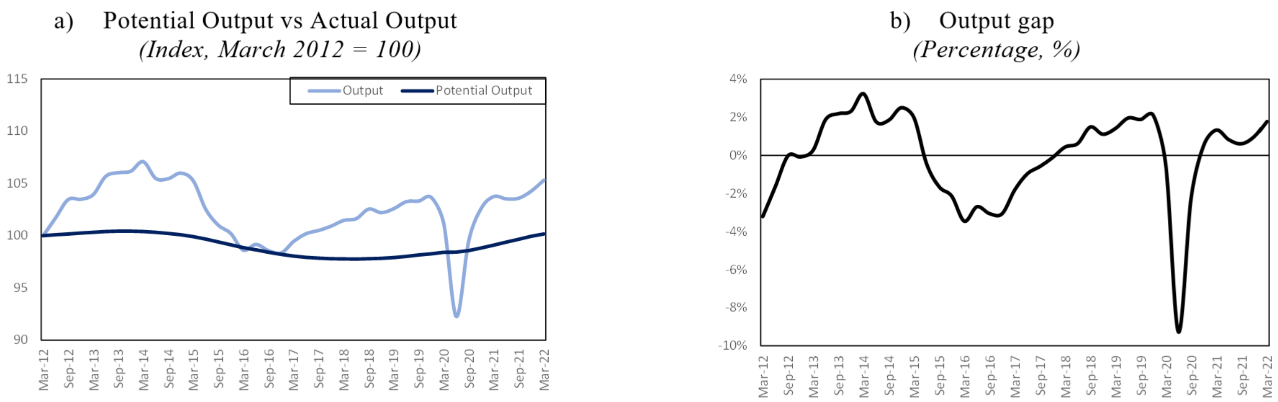
The final step obtains the final estimate of the potential output (Y^*), which is obtained once the long-term components of the production factors are included within the production function.

$$Y_t^* = A_t^* K_t^{*\alpha} L_t^{*1-\alpha}$$

An application of this methodology coupled with the HP filter for the Brazilian economy suggests that, in average, quarterly potential output grew 1.0% and the output gap reached 0.9% in 2021. For the first quarter of 2022, the potential output grew 1.1% and the output gap reached 1.8% (Figure 6). This result is consistent with the HP filter result but not with the results obtained via the Hamilton filter, since the output gap has an opposite sign.

Figure 6: **Potential output and output gap via the PFA-HP filter**

(Precent growth and Percentage, %)



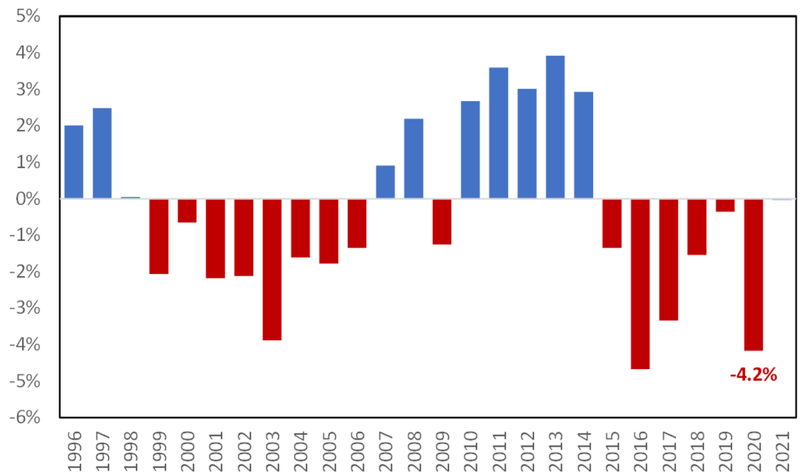
Source: Own estimations based on official data.

Estimates via this methodology using annual data suggest that the potential output grew 0.3% and the output gap reached 0% in 2021 (Figure 7). Furthermore, once again the consistency in the results is not maintained since quarterly estimates exhibit a positive output gap while the annual estimates reflect a gap around zero percent. Part of the literature questions

[3] For instance, the Brazilian Independent Fiscal Institution uses a Phillips curve model to obtain the trend level of the labor force.

whether the problems inherent in the HP filter can somehow "contaminate" the results via the production function. Thus, even with an economic structure behind it, the PFA is high-sensitive to changes in the filtered time series.

Figure 7: **Yearly output gap for Brazil via the PFA-HP (Percentage, %)**



Source: Own estimations based on official data.

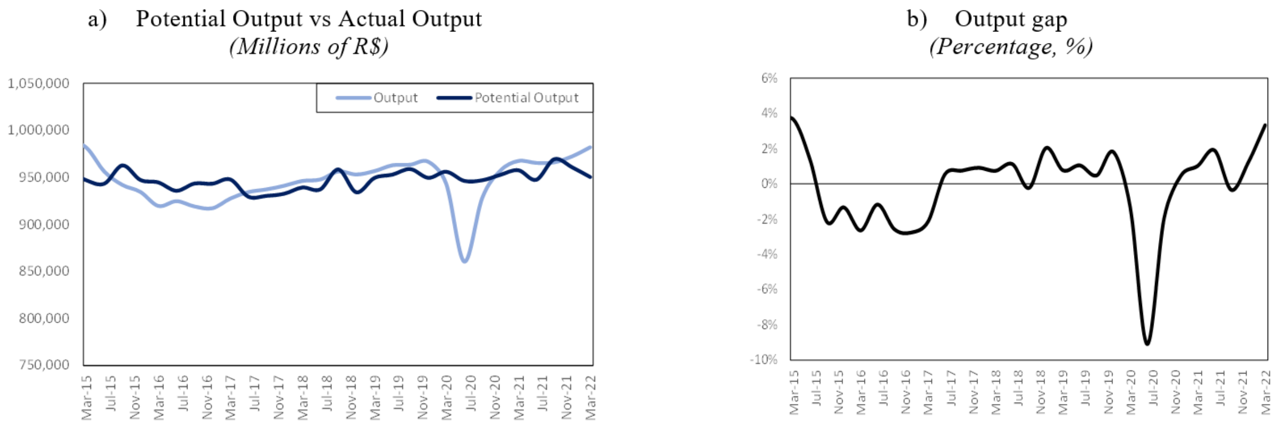
4.4 The PFA with Hamilton Filter

The previous sections argued how Hamilton's regression filter can circumvent the main problems of the HP filter by providing more stable estimates with a lower susceptibility to the end-of-sample bias. Just as using Hamilton's filter can be a good substitute for the HP filter when data is not widely available, it also can be used in cases when information is widely available. In this case, the Hamilton filter is applied within the PFA methodology to obtain the long-run components of the production factors. The latter could make the estimation process more robust and stable, without adding a large level of complexity to the procedure nor requiring the use of additional data sources.

Figure 8 exhibits the results obtained for the variables of interest expressed in a quarterly basis: the output gap averaged 0.9% during 2021 and stood at 3,3% during the first quarter of 2022. In a yearly basis, the PFA coupled with the Hamilton filter suggest that the output gap increased 3.2 p.p. from 2020 (-5.4%) to 2021 (-2.2%) (Figure 9).

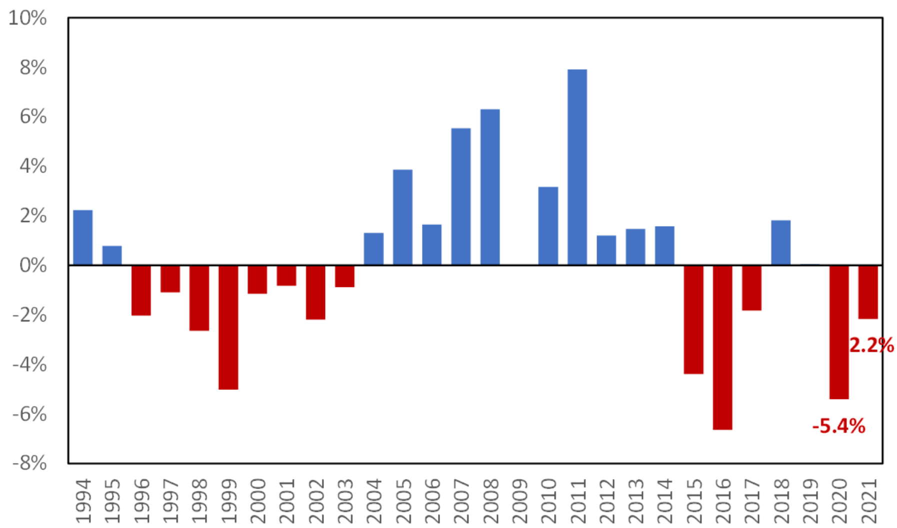
Figure 8: **Potential output and output gap via the PFA-Hamilton filter**

(Perecent growth and Percentage, %)



Source: Own estimations based on official data.

Figure 9: **Yearly output gap for Brazil via the PFA-Hamilton (Percentage, %)**



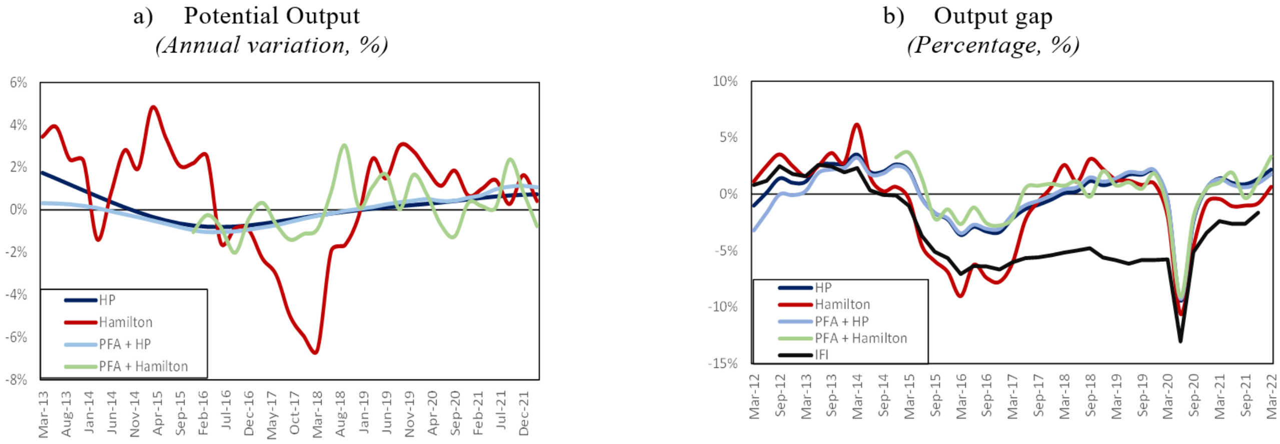
Source: Own estimations based on official data.

4.5 Results Comparison

This paper applied four approaches to update the potential output and output gap for the Brazilian economy. We relied on the Hamilton filter to try to circumvent the end-of-sample bias limitation that the HP filter possess. Regarding the potential output annual growth, both the HP filter and the PFA/HP filter return smooth estimates, while the Hamilton filter and the PFA/Hamilton filter models results seem more volatile. (Figure 10a). Regarding the output gap, both methodologies using HP filter seem to be consistent between each other over the analyzed period. This is not the case where the difference between the single Hamilton filter and the PFA/Hamilton filter seems outstanding, especially in the period 2015-2017 (Figure 10b). Nor of the models seem to approximate IFI estimates over the whole sample.

Figure 10: **Results comparison for quarterly estimates**

(Percent growth and Percentage, %)

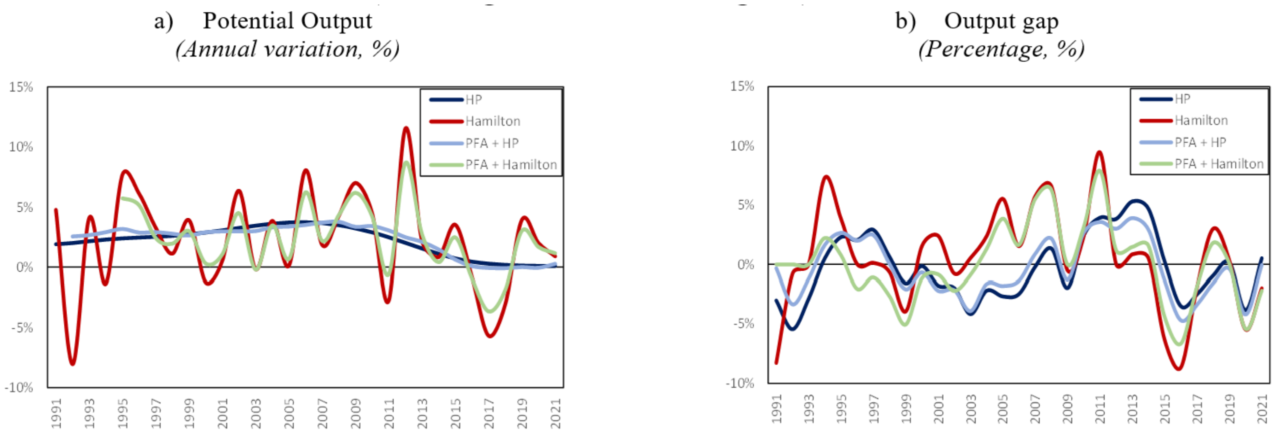


Source: Own estimations based on official data.

In a yearly basis, the models who apply the same filter seem to be more related between them, again sustaining the result that: i) although the Hamilton filter solves the end-of-sample limitation of the HP filter, it brings more volatility in its estimations, and ii) the PFA is highly sensitive to the filtering procedure (Figure 11). The result from the four models suggests that. potential output for Brazil grew around 0.6%-0.9% and the output gap stood between -0.9%-0.6% in 2021, while results for the first quarter of 2022 suggest that potential output grew 0.4% and the output gap stood at 2.0% (Table 1).

Figure 11: **Results comparison for Yearly estimates**

(Percent growth and Percentage, %)



Source: Own estimations based on official data.

Figure 12: Results summary 2021 and 2022-Q1

	Output gap (%)				Potential GDP growth (YoY, %)			
	Quarterly freq.		Annual freq.		Quarterly freq.		Annual freq.	
	2021	2022-Q1	2021	2022-Q1	2021	2022-Q1	2021	2022-Q1
Model 1: HP filter	1.2%	2.2%	0.5%	...	0.7%	0.7%	0.1%	...
Model 2: Hamilton Filter	-0.8%	0.7%	-2.0%	...	1.1%	0.4%	0.9%	...
Model 3: PFA + HP filter	0.9%	1.8%	0.0%	...	1.0%	1.1%	0.3%	...
Model 4: PFA + Hamilton Filter	0.9%	3.3%	-2.2%	....	0.9%	-0.8%	1.2%	....
Average	0.6%	2.0%	-0.9%	...	0.9%	0.4%	0.6%	....

5 Conclusions

This paper presents a methodology that could improve existing methods while finding a good balance between feasibility and complexity. Using a univariate statistical filter could be considered simple and may be subject to limitations of the filter itself, while the PFA could be a more robust alternative with economic theory support behind it, but its use may not be possible in contexts of high requirement for data series. Moreover, even when it is possible to apply the PFA, its level of complexity may vary significantly, since the use of a second layer of structural models to obtain the long-run components of the production factors is a robust and statistically promising approach but it implies a major level of complexity as well as data-requirements for a more precise estimation.

Since the PFA is sensible to the statistical approach used to obtain the long-term components of the production function, the Hamilton filter could be used as an important tool to help obtain more precise and robust estimates for the potential output and the output gap. When a high amount of data is available, the procedure improves the stability of the estimation and can approximate the results obtained by more complex methodologies at a reasonably lower cost. In contexts of sparse data, the Hamilton´s filter could be even more important, due its capability to overcome the main problems of the commonly used HP filter. We find that the potential output for Brazil grew around 0.6%-0.9% and the output gap stood between -0.9%-0.6% in 2021, while results for the first quarter of 2022 suggest that potential output grew 0.4% and the output gap stood at 2.0%.

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