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Carlos da Costa
Lucas Maestri
Cezar Santos

Inter-American Development Bank
Department of Research and Chief Economist

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Carlos da Costa*
Lucas Maestri*
Cezar Santos**

* FGV EPGE
** Inter-American Development Bank and CEPR

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Department of Research and Chief Economist

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Abstract

When searching for employment, workers consider non-wage job characteristics, such as effort requirements or amenities. We study an environment where unemployed workers search for jobs of different quality in a labor market characterized by directed search. In equilibrium, firms are more likely to post vacancies for low-quality jobs, as these are more profitable. Hence, high-quality jobs are hard to come across. The non-observability of these employment contracts influences the optimal unemployment insurance (UI) program, leading to distortionary taxation. Calibrating the model to the U.S. economy, we find that non-observability of employment contracts results in faster-declining UI benefits, steeper taxes upon re-employment, distortionary taxation, and a 10.5% costlier program than an observable contract scenario providing equal welfare.

JEL classifications: H21, J64
Keywords: *Unemployment insurance, Directed search, Intensive margin, Amenities, Hidden savings*

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1 Introduction

The labor market encompasses more than just wage compensation, as jobs vary significantly in terms of amenities, perks, work hours, and effort requirements. When searching for jobs, workers consider these non-pecuniary factors that determine job quality, and firms tailor positions accordingly. Recognizing and understanding these complexities of the labor market is crucial for crafting effective economic policies, particularly in the context of unemployment insurance. Unemployment insurance programs must strike a balance between providing adequate insurance and avoiding disincentives for job search and reemployment. By considering different aspects of job quality, policymakers can develop unemployment insurance policies that better align with the complexities of the modern job market.

In this paper, we study the problem of a government that offers unemployment insurance in a dynamic environment featuring directed search. We innovate by considering non-wage aspects of job quality, which firms can provide at different levels. Firms may expand the supply of vacancies of jobs of lower quality, such as jobs that require more effort and/or provide fewer amenities for the same level of earnings. As a prime example, many people prefer home office jobs due to the flexibility they provide in balancing personal and professional lives, such as eliminating long commutes and accommodating family responsibilities. Furthermore, remote work reduces expenses on commuting, dining out, and professional attire, making it a more economical choice, which motivates individuals to seek out such roles. A growing body of empirical research, discussed in the next section, strongly emphasizes the significance of job quality.

These non-pecuniary dimensions of job quality are important for our analysis. From the workers' perspective, they can reduce the expected unemployment spell if they look for lower-quality jobs. For the design of optimal policies they are important because these adjustments in job quality are typically not controlled by the planner.

We characterize the optimum for general separable preferences when the planner controls the agent's savings. At the optimum, unemployment benefits and net earnings decline with the length of the unemployment spell. The repeated moral hazard nature of the problem implies that, at the optimum, the stochastic process governing consumption satisfies the inverse Euler Equation. In the long run, unemployment benefits converge to zero. The optimal contract also prescribes a positive wedge on the marginal rate of substitution between consumption and job quality, i.e., distortionary taxation. This result materializes even though the planner can use non-distortionary instruments and there is no distributive motive.

The logic is as follows. Consider a firm that increases job quality (lower work requirements or increased amenities) for a fixed level of earnings to attract workers. This increase in the value of the job, which goes under the planner's radar since only earnings are observed, leads to a higher probability of hiring. But, from a worker's perspective, high-quality jobs are harder to find. Because agents searching for a job are entitled to unemployment benefits, high-quality jobs are expensive for the unemployment insurance program. The question is how the planner can discourage firms from offering these high-quality jobs. Now, a worker who manages to get such a job has a higher utility of job quality than those who get lower-quality jobs. In an economy without distortions at the margin, these workers would like the firm to provide less job quality in exchange for a proportional increase in earnings. By taxing earnings at the margin, the planner discourages such a change and makes these high-quality jobs less attractive. This result relies on three realistic assumptions embedded in our framework: directed search, intensive margin adjustments, and unobservability of the details of the employment contract.

We calibrate our model to the U.S. economy. The non-observability of employment contracts has a significant quantitative impact on the optimal unemployment insurance contract designed by the social planner. When contracts are unobservable, the unemployment insurance benefits decline faster, taxes upon re-employment increase more rapidly with unemployment duration, and distortionary tax rates are introduced. These factors lead to an unemployment insurance program that is 10.5% more expensive than one that provides the same level of welfare in a world in which contracts are observable.

To implement the optimal allocation described above, the planner must control the agent's savings, which may not be possible in practice. We take the possibility of hidden savings and borrowing in perfect capital markets into account. For this case, we restrict our analysis to preferences of the Greenwood et al. [1988] type specialized to the case of Constant Absolute Risk Aversion (GHH-CARA preferences). The optimal allocation can be implemented by a simple stationary contract: an upfront unemployment installment, constant gross earnings, and taxes when the agent finds a job. The pattern of declining consumption in both employment and unemployment is achieved by the worker's (dis)savings along the unemployment spell. In this hidden-savings case too, a positive wedge at the margin characterizes the optimum.

Government agencies deploy various strategies to oversee unemployed individuals during their job search to validate eligibility for unemployment benefits. Despite widespread reporting mandates, such as recording job applications and interviews, ensuring job seekers pursue suitable employment proves challenging. Confirming individuals do not solely target highly competitive yet appealing positions that frequently draw numerous applicants presents a significant hurdle. Our research demonstrates that the benefits of establishing an effective unemployment insurance monitoring agency are substantial.

The rest of the paper is organized as follows. After a brief literature review, in Section 2, we motivate empirically the interaction between receipt of unemployment insurance, the likelihood of finding employment, and certain job characteristics. In Section 3, we describe the environment and offer a one-period account of the forces explaining our findings. We derive the properties of an optimal system under the assumption that the planner controls agents' savings in Section 4 and analyze the optimal contract quantitatively. Section 5 describes the optimal contract for the case of hidden savings. Section 6 concludes.

Literature Review

The modern treatment of unemployment insurance program design has its roots in Shavel and Weiss [1979] and found its first canonical treatment in Hopenhayn and Nicolini [1997]. We contribute by focusing on directed search and by introducing the possibility of selecting jobs according to their effort requirements. Acemoglu and Shimer [1999] consider a general equilibrium model of directed search with risk aversion. The static version of our model generalizes theirs by considering the possibility of adjusting the effort requirements of different jobs. Moreover, while their focus is on the general equilibrium aspects of unemployment insurance, we concentrate on the planner's solution to the optimal policy.

Shimer and Werning [2007, 2008] evaluate the consequences of allowing agents to borrow and save in perfect capital markets using McCall's (1970) model of sequential job search. Under CARA preferences, a policy comprised of a constant benefit during unemployment, a constant tax during employment, and free access to a riskless asset is optimal. In our directed search environment with the possibility of intensive margin adjustments once employed, simple stationary policies are also optimal under CARA. We add to the prescription by proving the optimality of introducing distortionary taxation to incentivize search towards easier-to-find jobs.[2]

A strand of the literature investigates redistributive policies in the presence of labor market frictions. Golosov et al. [2013] consider the redistribution of residual income. Under directed search, the optimal redistribution of residual income can be attained with positive unemployment benefits and a positive, increasing, and regressive income tax schedule. They do not consider an intensive margin of non-wage job quality as we do.

[2]We also contribute to the literature that studies the optimal path of UI benefits [e.g., Kolsrud et al., 2018, Lindner and Reizer, 2020]. We add by considering non-observable aspects of job quality.

The presentation of our theory below is centered on variations of effort as the relevant intensive margin adjustment. In practice, workers may adjust their search not only by becoming more selective about wages and how much effort they must exert once employed but also about the quality of their prospective work environment, neither of which is within the reach of policy.[3] We show that the same logic leading to the wedge in effort implies a wedge in the supply of amenities. Recent research shows that job amenities are important for workers [e.g., Sockin, 2022]. For instance, Morchio and Moser [2024] demonstrate that amenities play an important role in explaining the gender pay gap. Bagga et al. [2024] show that increased preferences for telework, a key job amenity, help explain the post-pandemic labor market experience in the United States. We contribute to this literature by showing that these non-wage characteristics of job quality influence the design of the optimal unemployment insurance program.

Kroft et al. [2020] find sufficient statistics for the optimal combination of income taxes and unemployment benefits but do not consider intensive margin adjustments as we do. da Costa et al. [2022] study optimal distributive policies in the presence of labor market frictions. While they emphasize intensive margin choices, their model is static and focused on the interaction between distributive motives and unemployment insurance design. Here, we abstract from redistribution while highlighting the dynamics of insurance when contracts are not observed and there is scope for adjustments in the intensive margin.

2 Empirical Motivation

This section explores data from the United States to check whether there are discernible differences in labor market outcomes between individuals receiving unemployment insurance (UI) and those without such coverage. We rely on data extracted from the March supplement of the Current Population Survey (CPS). This supplement provides data on UI receipts among the unemployed, as well as key characteristics of their job for those currently working. Our analysis encompasses data from 2009 to 2022. We run linear probability regressions to draw comparisons between the labor market trajectories of unemployed individuals benefiting from UI and those without such benefits. Figure 1 provides the main estimates (see Appendix A for the full regressions).

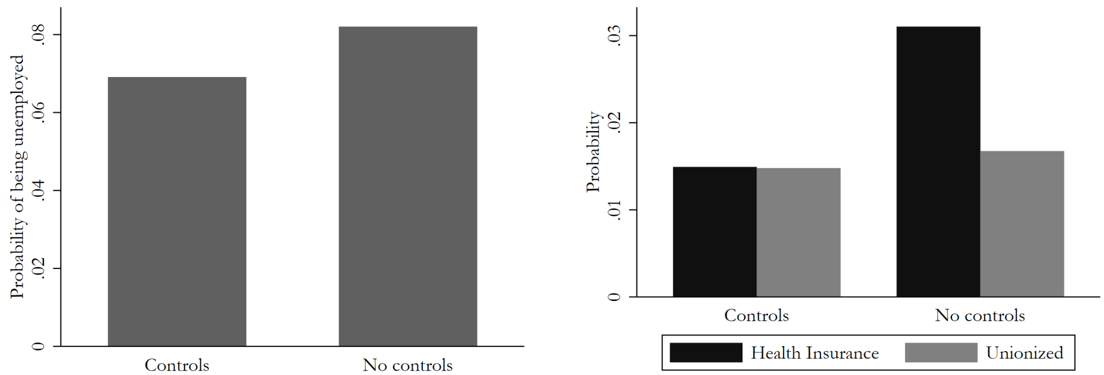
Figure 1(a) plots the difference in the likelihood of being unemployed one year ahead for unemployed workers who receive UI versus those who do not. UI recipients are about

[3] These equalizing differences, surveyed by Rosen [1987], have been shown to be quantitatively important in recent work by Mas and Pallais [2017], Sorkin [2018], Hall and Mueller [2018].

Figure 1: Difference in Outcomes between Unemployed Workers with UI versus those Without

(a) Prob. of unemployment one year later

(b) Prob. of job amenity one year later



Notes: Panel (a): Difference in the probability of being unemployed one year later for the unemployed today that receive UI versus those that do not. Panel (b): Difference in the probability of having a job with certain amenities one year later for the unemployed today who receive UI versus those who do not. Controls: age, gender, and education.

8% more likely to remain unemployed than those without the benefit. Since UI recipients may be different from other unemployed workers, we also control for certain observable characteristics: age, gender, and education. Even controlling for these variables, UI recipients are still around 7% more likely to remain unemployed. Standard moral hazard considerations may explain why UI recipients are less likely to find a job.

Figure 1(b) focuses on the likelihood that unemployed workers who become employed end up working for jobs with certain amenities, i.e., higher-quality jobs. The gray bars show that UI recipients are more likely to find unionized jobs. Presumably, these jobs come with more benefits negotiated by the union. Moreover, the black bars show that UI recipients are more likely to find jobs that provide health insurance, a key amenity in the United States.

Overall, the two graphs in Figure 1 show an interaction between UI, the likelihood of finding employment, and job quality. In the next section, we develop a theory with such interactions and study what the optimal unemployment insurance contract is in such a world.

3 Environment

In what follows, we equate job quality with effort requirements in the job, such that high-quality jobs mean lower-effort jobs. The crucial assumption we make is that the planner either cannot recover the disutility that the agent is incurring from their earnings or cannot condition policy directly on it. While we will mostly use the level of output produced as the driver of disutility, the approach is isomorphic to one in which the level of output associated with each job is fixed but the disutility can be reduced by more amenities.[4]

Time runs for $t = 0, 1, ...,$ and is discounted by $\beta \in (0, 1)$. Preferences are separable across time, states, and between consumption, c, and effort, n. The flow utility generated by (c, n) is given by $U(c, n) = \varphi(c) - \eta(n)$, with $\varphi', -\varphi'', \eta', \eta'' > 0$, satisfying the Inada conditions $\lim_{c \downarrow 0} \varphi'(c) = \infty$ and $\lim_{n \downarrow 0} \eta'(n) = 0$. One unit of effort, n, produces one unit of the consumption good, c, the price of which is normalized to one.

The economy starts with the worker in an unemployment state. A job offer is a contract specifying how much effort, n, the worker must exert if hired and their earnings, y. A labor contract, consisting of the pair (n, y), defines a (sub)market. The probability, p, of receiving a job offer in any market depends on the market tightness, with the implied relationship captured by the function $\varrho : [0, 1] \to [0, \infty)$. This function associates an employment probability, p, with the tightness that generates it. As in most directed search specifications, we assume that ϱ is strictly increasing, twice differentiable, strictly convex, and satisfies $\varrho(0) = 0$, $\lim_{p \uparrow 1} \varrho(p) = \infty$, which implies that $\varrho(p)/p$ is strictly increasing. We also assume that $\phi = \lim_{p \downarrow 0} \varrho(p)/p > 0$.

The following assumption implies that a worker who receives a sufficiently low constant lump-sum payment $\underline{c} > 0$ in every period would look for a job with a positive probability if labor were not taxed:

$$\varphi(\underline{c}) < \max_{y^e} \varphi(\underline{c} + y^e) - \eta(\phi + y^e).$$

This assumption always holds, for instance, when $\lim_{c \downarrow 0} \varphi(c) = -\infty$.

To model a firm's hiring decision, normalize the cost of posting a vacancy to $\kappa/(1-\beta)$. An unemployed worker who applies for a job at time t receives the answer at the beginning of the same period, before collecting unemployment insurance. Firms are risk-neutral, and

[4] Indeed, assume that, if employed, the worker produces a quantity $\bar{z}$ at a disutility cost $\bar{z} - a$, where a denotes the level of amenities, supplied by the firm at a cost $c(a)$, with $c', c'' > 0$, $c(0) = 0$. In Section D of the Appendix, we show that the same is true when *both* the total production and the level of amenities can be adjusted.

a free-entry condition drives expected profits to zero.

We study the cost minimization of a government that must guarantee a lifetime utility, W_0, to the worker. The planner faces the following informational constraints when offering insurance to the agent. First, the planner does not know whether the agent received a job offer and rejected it or whether no offer materialized. Second, conditional on the worker finding a job, the planner does not know the type of contract offered to the agent. More precisely, the planner observes earnings, y, but not effort n. Whereas the first source of informational asymmetry has been extensively studied, the second one is novel. To highlight its role, we first present a one-period version of our economy in which the heuristics for our main findings are simpler to convey.

3.1 A One-period Economy

Consider a simplified version of our model in which an agent lives for a single period split into two sub-periods. In the first sub-period, the agent decides in which sub-market to search, i.e., they choose p. If they find a job, they earn y^e in exchange for producing z^e. If not, they are entitled to an unemployment benefit c^u.

Observable z^e Let us start with the case in which z^e is observed/controlled by the planner. In this case, the optimization problem is to choose (p, c^u, c^e, z^e) to maximize

$$p[z^e - c^e] - (1-p)c^u - \kappa\vartheta(p),$$

subject to a promise keeping restriction,

$$p\left[\varphi(c^e) - \eta(z^e)\right] + (1-p)\varphi(c^u) = V,$$

and an incentive compatibility constraint,

$$\varphi(c^e) - \eta(z^e) \geq \varphi(c^u).$$

The solution of this problem, $(z^e_\circ, c^e_\circ, c^u_\circ, p_\circ)$, is characterized by efficiency at the intensive margin, $\eta'(z^e_\circ) = \varphi'(c^e_\circ)$, a level of search intensity such that $z^e_\circ - (c^e_\circ - c^u_\circ) = \kappa\vartheta'(p_\circ)$ and dispersion in consumption between unemployed and employed: $\varphi(c^e_\circ) - \varphi(c^u_\circ) = \eta(z^e_\circ) > 0$.

Non-observable z^e We now turn to the case in which z^e cannot be observed or controlled by the planner. In this case, the zero profit condition pins down the relationship between output, z^e, earnings, y^e, and the job-finding probability p, $z^e = y^e + \kappa\varrho(p)/p$.

Note two features of the worker's problem. First, contracts are not observable. That is, the planner observes how much an employed worker is paid, y^e, but it cannot monitor how much effort, n^e, a job demands. Second, from the zero profit condition, $n^e - y^e = \kappa\varrho(p)/p$, must hold for any contract, on- and off-the-equilibrium path.

For an employed worker to consume c^e, they must earn $y^e = c^e + T$ and pay taxes, T, to the government. Since the output they produce must also cover the vacancy-related expenditures $n^e > y^e$. So, in what follows, we relegate n^e and T to the background and write the planner's program with the controls c^e, c^u, and y^e.

The planner observes both c^e and y^e. Hence, the only margin in which deviation is possible is the choice of p. Moreover, the effort, $n^e = y^e + \kappa\varrho(p)/p$, is chosen when the worker decides which job to apply to. Because the government observes neither p nor n^e, it can only condition policy on employment status and on earnings, y^e. A worker who chooses a higher matching probability and finds a job must exert more effort for the same level of earnings. The intuition is simple: worse jobs, shorter queues.

Note that (c^e, c^u, y^e) are controlled by the planner. Let, for any $\hat{p}$,

$$U\left(\hat{p}, c^u, c^e, y^e\right) := (1-\hat{p})\varphi(c^u) + \hat{p}\left[\varphi(c^e) - \eta\left(y^e + \kappa\frac{\varrho(\hat{p})}{\hat{p}}\right)\right],$$

and let the agent's optimal choice of $\hat{p}$ be

$$p \in \arg\max_{\hat{p}} U\left(\hat{p}, c^u, c^e, y^e\right). \tag{1}$$

Under the assumption that the solution to the agents' problem is interior, i.e., the worker actively searches for a job, the solution must satisfy the following first-order condition:

$$\varphi(c^e) - \varphi(c^u) - \eta\left(y^e + \kappa\frac{\varrho(p)}{p}\right) - \kappa p\eta'\left(y^e + \kappa\frac{\varrho(p)}{p}\right)\left(\frac{\varrho(p)}{p}\right)' = 0, \tag{2}$$

where the notation

$$\left(\frac{\varrho(p)}{p}\right)' = \frac{d}{dp}\left(\frac{\varrho(p)}{p}\right),$$

is used to simplify the expressions.

It is important to note that at this point that the moral hazard problem is *more severe* due to the intensive margin adjustment. Indeed,

$$\varphi(c_\circ^e) - \varphi(c_\circ^u) = \eta(z_\circ^e) < \eta(z_\circ^e) + \kappa p_\circ \eta'(z_\circ^e)(\vartheta(p_\circ)/p_\circ)',$$

where $(z_\circ^e, c_\circ^e, c_\circ^u, p_\circ)$solves the planner's problem with observable z^e.

To maintain incentive compatibility for the same $p_\circ$ the planner must either further spread consumption or reduce $z_\circ^e$. Second best principles suggest it will do both.[5]

The Pareto frontier can be obtained by maximizing the planner's expected revenue

$$-(1-p)c^u + p\left(y^e - c^e\right), \tag{3}$$

subject to delivering utility U^* to the agent,

$$U\left(p, c^u, c^e, y^e\right) \geq U^*, \tag{4}$$

and to respecting the incentive-compatibility constraint (1).

Due to the concavity of the problem, (4) can be replaced by (2) whenever it is desirable to induce a positive search. If it is not desirable to induce positive search, the solution displays $c^u = c^e = \varphi^{-1}\left(U^*\right)$. Henceforth, we focus on cases where the optimal search is positive.

To incentivize effort, the planner must ensure that $c^e > c^u$, which implies that constraints (2) and (4) bind. This fact, coupled with the concavity of the worker's problem with respect to p, confirms that $c^e - y^e > c^u$. Hence, the planner raises more revenues when the worker finds employment. This is the source of moral hazard in our model. The worker does not internalize the fiscal cost of the insurance provided by the government. Accordingly, the government finds it desirable to induce a higher matching probability.

Turn now to how this impacts the marginal rate of substitution between consumption and leisure. From the first order conditions with respect to c^e and y^e, it is immediate to see that the marginal utility of consumption is greater than the marginal disutility of effort, $\varphi'(c^e) > \eta'\left(y^e + \kappa\varrho(p)/p\right)$; marginal tax rates are positive. Indeed, combining the first order conditions with respect to y^e and with respect to c^e we get

$$1 - \frac{\eta'\left(n^e\right)}{\varphi'(c^e)} = p\eta''(n^e)\kappa\left(\frac{\varrho(p)}{p}\right)'.$$

This implies $\varphi'(c^e) > \eta'\left(n^e\right)$; a positive wedge on the intensive margin.

[5]Of course, p will also be optimally adjusted.

At the optimal allocation, labor effort is distorted downward. To better understand this property, consider an alternative allocation in which this margin is not distorted: $\varphi'(c^e) = \eta'(n^e)$. A small perturbation in which consumption when employed, c^e, and earnings, y^e, are both decreased by some small $\varepsilon > 0$ has no direct fiscal effect and only a second-order effect on the worker's utility. However, it changes the marginal incentive to search for a job. The convexity of the cost of labor and the fact that workers who intend to find a job with higher probability must provide higher effort once employed imply that this perturbation makes it relatively more attractive to search for a job. This relaxes the moral-hazard constraint and allows the planner to save resources.

The planner wants the unemployed worker to search for jobs that are easier to find. As these jobs entail more effort once employed, the planner must provide the worker with incentives to work harder. By imposing an income tax, the planner creates a wedge where the marginal utility of consumption is higher than the marginal disutility of working. Hence, at the margin, exerting more effort is not so costly for the employed worker. Imposing such a wedge is thus optimal even when non-distortionary instruments are available. The next few sections show that this insight carries on to richer environments.

4 Optimal Unemployment Insurance

The one-period version of our model was useful in highlighting the extra margin for deviation when some aspects of jobs cannot be controlled by the planner. Yet, it abstracts from an important aspect of real-world unemployment insurance: the time dimension of policy. This section develops a dynamic version of our model. We start by describing which allocations could be implemented if contract offers were observed. That is, we characterize the benchmark case in which a firm's posted contract is observable, but the planner cannot observe whether or not the worker received an offer. Then, we consider the more realistic case in which not all features of a contract offer are observable by the government. In this section, we also offer the conditions under which the assumptions we made regarding binding constraints hold as a result of the dynamic setting.

4.1 Observable Contracts

For now, assume that the planner observes the details of contracts that are offered. A job contract is a pair, (c^e, n^e), where c^e denotes the consumption for an employed person and n^e the level of effort required from them.

The planner cannot force the agent to find a job if

$$\varphi(c^u) > \varphi(c^e) - \eta\,(n^e)\,.$$

Hence, the program that the planner solves is

$$C(W) = \max_{p,c^e,c^u,n^e,\tilde{W}} \frac{p}{1-\beta}\left[n^e - c^e - \kappa\frac{\varrho(p)}{p}\right] + (1-p)\left[-c^u + \beta C(\tilde{W})\right],$$

subject to the promise-keeping,

$$W = \frac{p}{1-\beta}\left[\varphi(c^e) - \eta\,(n^e)\right] + (1-p)\left[\varphi(c^u) + \beta\tilde{W}\right], \tag{5}$$

and the incentive constraint,

$$\frac{\varphi(c^e) - \eta\,(n^e)}{1-\beta} \geq \varphi(c^u) + \beta\tilde{W}. \tag{6}$$

We can write the program above as the following Kuhn-Tucker problem,[6]

$$\begin{aligned} C(W) = \max_{p,c^e,c^u,n^e,\tilde{W}} & \frac{p}{1-\beta}\left[n^e - c^e - \kappa\frac{\varrho(p)}{p}\right] + (1-p)\left[-c^u + \beta C(\tilde{W})\right] + \\ & \mu\left[\frac{p}{1-\beta}\left[\varphi(c^e) - \eta\,(n^e)\right] + (1-p)\left[\varphi(c^u) + \beta\tilde{W}\right] - W\right] + \\ & \lambda\left[\frac{\varphi(c^e) - \eta\,(n^e)}{1-\beta} - \varphi(c^u) - \beta\tilde{W}\right] \end{aligned}$$

To proceed, we first assess whether the moral hazard and the promise-keeping constraints bind at the optimum. Lemma 4.1 below states that whenever agents search for a job, they are indifferent between doing so and remaining unemployed for another period.

Lemma 4.1 *The promise-keeping constraint (5) binds in every period, and $\mu > 0$. In any period in which there is positive search, the moral-hazard constraint binds, $\varphi(c^e) - \eta\,(n^e) = [1-\beta]\left[\varphi(c^u) + \beta\tilde{W}\right]$, and $\lambda > 0$.*

For every period t in which the moral hazard constraint binds we have

$$\mu_{t+1} = \mu_t - \frac{\lambda_t}{1-p_t},$$

[6]We can rely on Lemma B.3 to write the problem as such. This lemma refers to the case in which contracts are not observed, but the argument is easily adapted to the case with observed contracts.

which implies that unemployment consumption decreases over time,

$$c_{t-1}^{u}=\left(\varphi^{\prime}\right)^{-1}\left(\mu_{t}^{-1}\right)>\left(\varphi^{\prime}\right)^{-1}\left(\mu_{t+1}^{-1}\right)=c_{t}^{u}.$$

Moreover, the consumption process is described by an inverse Euler equation,

$$\frac{1}{\varphi^{\prime}\left(c_{t-1}^{u}\right)}=\mu_{t}=p_{t}\left[\mu_{t}+\lambda_{t}p_{t}^{-1}\right]+\left(1-p_{t}\right)\left[\mu_{t}+\lambda_{t}p_{t}^{-1}\right]=\frac{p_{t}}{\varphi^{\prime}\left(c_{t}^{e}\right)}+\frac{1-p_{t}}{\varphi^{\prime}\left(c_{t}^{u}\right)}.$$

Also, the first-order conditions with respect to c^e and n imply that, in contrast to our one-period model with non-observed contracts, the effort is not distorted at the optimum in the dynamic model with observable contracts. We gather these findings in Proposition 4.1.

Proposition 4.1 *The solution for the planner's problem when contracts are observable has the following properties:*

1. *It entails a zero marginal income tax rate.*

2. *The unemployment insurance is decreasing over time. Moreover, if the agent searches in period t, then the unemployment insurance is strictly lower than the one from the previous period.*

3. *The consumption process is described by an inverse Euler equation.*

To understand 1, note that the incentive-compatibility constraint (6) only depends on the agent's utility when employed, not on how it is generated. Since the government observes contracts, it can choose them to minimize the cost of providing this utility. That is, given any utility level, there is no reason for the government to distort effort, which implies 1. Second, unemployment insurance should decrease over time to make it more costly to turn down employment opportunities, which is the content of 2. Finally, similar to several dynamic moral-hazard models—e.g., Rogerson [1985] —the consumption process is described by an inverse Euler equation.

4.2 Non-observable Contracts

Section 4.1 adopted the strong assumption that the government observes the contracts chosen by workers and hence the disutility of effort from a particular job. We now consider optimal policies under non-observable labor contracts. In this setup, the optimal policy must be based only on whether or not the agent is employed, on their earnings, and the length of the unemployment spell.

If the agent is promised a sufficiently high utility, then there is no search in equilibrium at the solution of the planner's program; it is cheaper to deliver the promised utility if the agent remains unemployed forever; we show this in Lemma B.1 in the Appendix. This is an uninteresting case, and we instead focus on the case in which utility is not too high.

To characterize the optimal unemployment insurance program in this case, we rely on a first-order approach. Lemma B.2 shows that the solution for this relaxed problem is the solution to the original program. Hence, the planner's problem has a recursive structure and can be written as follows,

$$C\left(W\right)=\max_{p,c^e,c^u,y^e,\tilde{W}}\frac{p}{1-\beta}\left(y^e-c^e\right)+(1-p)\left[-c^u+\beta C\left(\tilde{W}\right)\right],$$

subject to a promise-keeping constraint

$$\frac{p}{1-\beta}\left[\varphi(c^e)-\eta\left(y^e+\kappa\frac{\varrho(p)}{p}\right)\right]+(1-p)\left[\varphi(c^u)+\beta\tilde{W}\right]-W\geq 0, \tag{7}$$

and an incentive compatibility constraint

$$\frac{1}{1-\beta}\left[\varphi(c^e)-\eta\left(y^e+\kappa\frac{\varrho(p)}{p}\right)\right]-\varphi(c^u)-\beta\tilde{W} = \frac{p\kappa}{1-\beta}\eta'\left(y^e+\kappa\frac{\varrho(p)}{p}\right)\left(\frac{\varrho(p)}{p}\right)'. \tag{8}$$

Lemma B.3 shows that the planner's problem is differentiable, and hence the optimum must satisfy a constrained optimization in which we write μ and λ for the multipliers relative to the constraints (42) and (43). Both multipliers are strictly positive. If μ were not strictly positive, the planner would be able to save resources by lowering the utility promised to the agent in both states with no consequences for incentives. λ is strictly positive because the worker does not internalize the fiscal externality when unemployed.

Combining the first order conditions with respect to y^e and c^e, one obtains

$$\varphi'(c^e)-\eta'\left(y^e+\kappa\frac{\varrho(p)}{p}\right)=\frac{\lambda p\kappa}{\mu p+\lambda}\eta''\left(y^e+\kappa\frac{\varrho(p)}{p}\right)\left(\frac{\varrho(p)}{p}\right)'>0. \tag{9}$$

The optimal allocation now displays a positive wedge at the intensive margin. The dynamic model inherits the finding from our one-period model. If a firm offers a better job, i.e., one requiring less effort for the same earnings, then it will attract more job candidates. Workers, in turn, will find it harder to land such a job, thus remaining unemployed for a longer horizon. Conditional on getting one of these jobs a worker would have a higher willingness to exert effort compared to someone who got one of the jobs offered by firms along the equilibrium path. To make these deviations less attractive, the planner distorts effort downwards by taxing earnings at the margin.

Since preferences are separable in consumption and effort, it is always feasible to vary the unemployment consumption utility in a period and compensate for it by varying the consumption utility in all states of nature in subsequent periods. Such a strategy changes neither incentives nor expected utility. Thus, these perturbations cannot save resources at the optimum. Because the marginal cost of delivering utility is $1/\varphi'$, the inverse Euler equation ensues.

These findings are summarized in Theorem 4.1, which is proved in the Appendix.

Theorem 4.1 *At the optimum, in every period in which the worker searches,*

1. *the marginal income tax rate is always positive;*

2. *the moral-hazard constraint (43) binds, and the government benefits from strictly increasing p, and;*

3. *conditional on not finding a job at period t, the worker's marginal utility of consumption satisfies the inverse Euler equation,*

$$\frac{1}{\varphi'\left(c_t^u\right)} = \mathbb{E}\left[\frac{1}{\varphi'\left(c_{t+1}\right)}\right].$$

The planner can avoid distorting the effort margin. Taxes may be based on employment, independently of earnings. Moreover, the utility conditional on finding a job depends on $\varphi(c^e) - \eta(n^e)$, regardless of whether c^e and n^e are efficiently chosen. What is then the rationale for distorting the intensive margin prescribed in Theorem 4.1? It is the same as in the static setting. Consider a worker deciding whether to apply for a job in a sub-market that is slightly less tight than what the planner has prescribed $\hat{p} < p$. The planner controls y^e and c^e, but not the amount of effort the agent must make to earn y^e. Upon landing a job in a less tight market, the worker is required to supply effort, $\hat{n} = y^e + \kappa\varrho(\hat{p})/\hat{p} < y^e + \kappa\varrho(p)/p = n$ while receiving the same c^e. This worker, therefore, has a lower marginal disutility of effort than agents who followed the optimal policy. To make this downward deviation less valuable, which is the relevant deviation according to 1, the planner distorts effort downward by introducing a positive wedge. A little less surprising is the fact that, as in Rogerson [1985], Atkeson and Lucas [1995], the Inverse Euler equation characterizes the dynamics of consumption for the unemployed.

When is search optimal? Theorem 4.1 describes the efficient allocation in periods in which there is search. But when is it optimal to search?

Proposition 4.2 *The unemployment benefit is decreasing over time with $c_t^u > c_{t+1}^u$ whenever the worker searches in period $t+1$.*

Moreover, whenever the worker searches in period $t+1$, their consumption from employment is strictly greater than the unemployment benefit from any period $\tau \geq t$.

When the promised utility is very high, the optimal contract provides constant benefits and asks the worker never to search for a job. On the other hand, job search must be incentivized when the government promises a sufficiently low utility to the worker. These two possibilities render the government's cost of providing utility W to the worker not convex in W, in general. As a consequence, we cannot rule out the possibility that the worker does not search for a job in the first period of the optimal contract.

To better understand when it is optimal to search in every period, define $z(W)$ by $z(W) \equiv \text{argmin}_z\, z \quad \text{s.t.} \quad \max_{y^e} [\varphi(y^e + z) - \eta(y^e + \phi)] \geq W$, where, as we recall, $\phi = \lim_{p \downarrow 0} \varrho(p)/p > 0$. Intuitively, $z(W)$ is the minimum amount of resources that would cost the government to motivate the worker to search for employment if their unemployment continuation utility were W, assuming that the labor market was competitive. To see this, we use the fact that $\varrho(p)/p$ is increasing in p. Hence, to find a job with probability p the worker would have to pay $\varrho(p)/p > \phi$ to the firm upon landing a job. Let also $c^u(W)$ by $\varphi(c^u(W)) = W$, the cost of providing utility W for a worker who never searches for a job.

We show in Lemma B.1, in the Appendix, that there is a level of utility, W^*, above which $z(W) > c^u(W)$ and below which $z(W) < c^u(W)$. Lemma 4.2 below shows that, if the initial unemployment insurance provides less utility than W^*, then the worker must search for a job in every period.

Lemma 4.2 *Assume that $\varphi(c_0^u) < W^*$. Then, $p_t > 0$ in every period, t.*

When the initial utility, W_0, is smaller than W^*, the initial contract must induce search in some period. Moreover, whenever the worker searches in some period the unemployment benefits eventually fall so that $\varphi(c_t^u) < W^*$ for some period t. Hence, the worker searches in every period, $\tau > t$, which is the content of Lemma 4.3, below.

Lemma 4.3 *The following conditions hold in any optimal contract:*

a) Assume that $W_0 < W^*$. Then, there is $t > 0$ such that $\varphi\left(c_t^u\right) < W^*$. Hence, the worker who is unemployed in any period $\tau > t$ searches for a job.

b) Assume that the worker searches for a job in some period t. Then there is $T > t$ such that the unemployed worker searches in any period $\tau > T$.

In this case, according to Proposition 4.2, $c_t^u > c_{t+1}^u$ for all t. Therefore, the unemployment benefit converges to a non-negative number. Proposition 4.3 shows that this number is 0.

Proposition 4.3 *Assume that $W_0 < W^*$, then unemployment benefits converge to zero.*

We have focused thus far on the case of separable preferences between consumption and effort. This has been the most frequently studied case in the literature. In Appendix C, we study non-separability for the case of GHH-CARA utility $\mathcal{U}(c, n) = -\exp\left\{-\alpha\big[c - \eta(n)\big]\right\}$.[7] These preferences will be the focus of our analysis when we assume that savings cannot be controlled by the planner. The results of this section carry over to the GHH-CARA case. In particular, the optimal policy for this case also prescribes a positive wedge between effort and consumption.

4.3 Quantitative Analysis

In this subsection, we analyze quantitatively the impact of implementing the optimal unemployment insurance (UI) contract derived above. We use the United States as our benchmark. Our initial step involves the calibration of model parameters based on the prevailing policy framework. To do this, we first write the problem of the agent under such a policy. An unemployed worker is entitled to UI for a fixed duration of T periods, with a constant benefit payout of b. After T, if the individual remains unemployed, they receive a guaranteed minimum consumption floor of f. The worker chooses in which market to search; that is, they choose the job-finding rate p. Plus, they choose their preferred consumption and savings bundle, (c, a').

[7] The constant absolute risk aversion (CARA) case is the only one for which Shimer and Werning [2007] have theoretical results for the non-observable savings scenario. They offer numerical explorations for the constant relative risk aversion (CRRA) case. Because we are also interested in understanding choices at the intensive margin, we suppress income effects at this margin through the assumption of quasi-linearity, as in Greenwood et al. [1988].

Denote by $V_u(t, a)$ the value function for an unemployed worker that still has t periods of UI and owns assets a. If the worker is still eligible for UI (i.e., $t \geq 0$), their value function reads:

$$
\begin{aligned}
V_u(t, a) = &\max_{p,c,a'} pV_e(a, p) + (1 - p)\left[\varphi(c) + \beta V_u(t - 1, a')\right] \\
&\text{s.t. } c + a' = (1 + r)a + b,
\end{aligned}
$$

where $V_e(a, p)$ denotes the value of being employed in a type-p job with asset level a. The continuation value $V_u(t - 1, a')$ reflects the fact that the worker will have one fewer period of UI next period if they do not find a job in the current period.

The value function for an unemployed worker without UI (i.e., after T periods of unemployment) reads:

$$
\begin{aligned}
V_u(0, a) = &\max_{p,c,a'} pV_e(a, p) + (1 - p)\left[\varphi(c) + \beta V_u(0, a)\right] \\
&\text{s.t. } c + a' = (1 + r)a + f.
\end{aligned}
$$

The value function for an employed worker is given by:

$$
\begin{aligned}
V_e(a, p) = &\max_{c,a'} \varphi(c) - \eta\left(y(p) + \kappa\frac{\varrho(p)}{p}\right) + \beta V_e(a', p) \\
&\text{s.t. } c + a' = (1 + r)a + y(p),
\end{aligned}
$$

where the income $y(p)$ is determined by:

$$
\eta'\left(y(p) + \kappa\frac{\varrho(p)}{p}\right) = \varphi'(c)
$$

We must now set functional forms and parameter values to perform counterfactuals. We assume the utility function for consumption is logarithmic: $\varphi(c) = \log c$. The disutility for effort is given by: $\eta(n) = \eta_1 n^{\eta_2}$. Moreover, the labor market tightness is determined by the function: $\varrho(p) = 1/(1/p - 1)$. Each model period corresponds to one week. Accordingly, we set the discount factor $\beta = 0.96^{1/52}$, a standard value. We set the UI in the benchmark, b, to 40% of the average income, the same ratio as in Shimer [2005]. We assume this benefit lasts for $T = 26$ weeks, as it does in the United States. Additionally, the consumption floor is fixed at 10% of the average income.

Three parameters are chosen internally so that the benchmark model matches certain data targets: the parameters that control the disutility of effort, η_1 and η_2, and the vacancy posting cost κ. These parameters are jointly chosen to match three data targets. The first is the mass of unemployed workers who find a job before the UI expires: 86.8% according to Shimer [2008]. The second data target is the wage markdown; that is, how much lower is the wage relative to the worker's productivity. Berger et al. [2022] report an average wage markdown between 11% and 22%. We target the intermediate value of 16.5%. Finally, we match the relative search effort spent by an unemployed worker at week 26 of unemployment (right before losing the UI benefit) versus week 1. We target 50%, the number reported by Marinescu and Skandalis [2020]. Table 1 reports the parameter values and the model fit.

Table 1: Calibrated Parameters and Model Fit

	Parameters		
	κ	η_1	η_2
	0.1352	0.200	5
	Moments		
	Markdown	% Reemployed	Rel. search effort
Model	0.165	0.867	1.549
Data	0.165	0.868	1.500

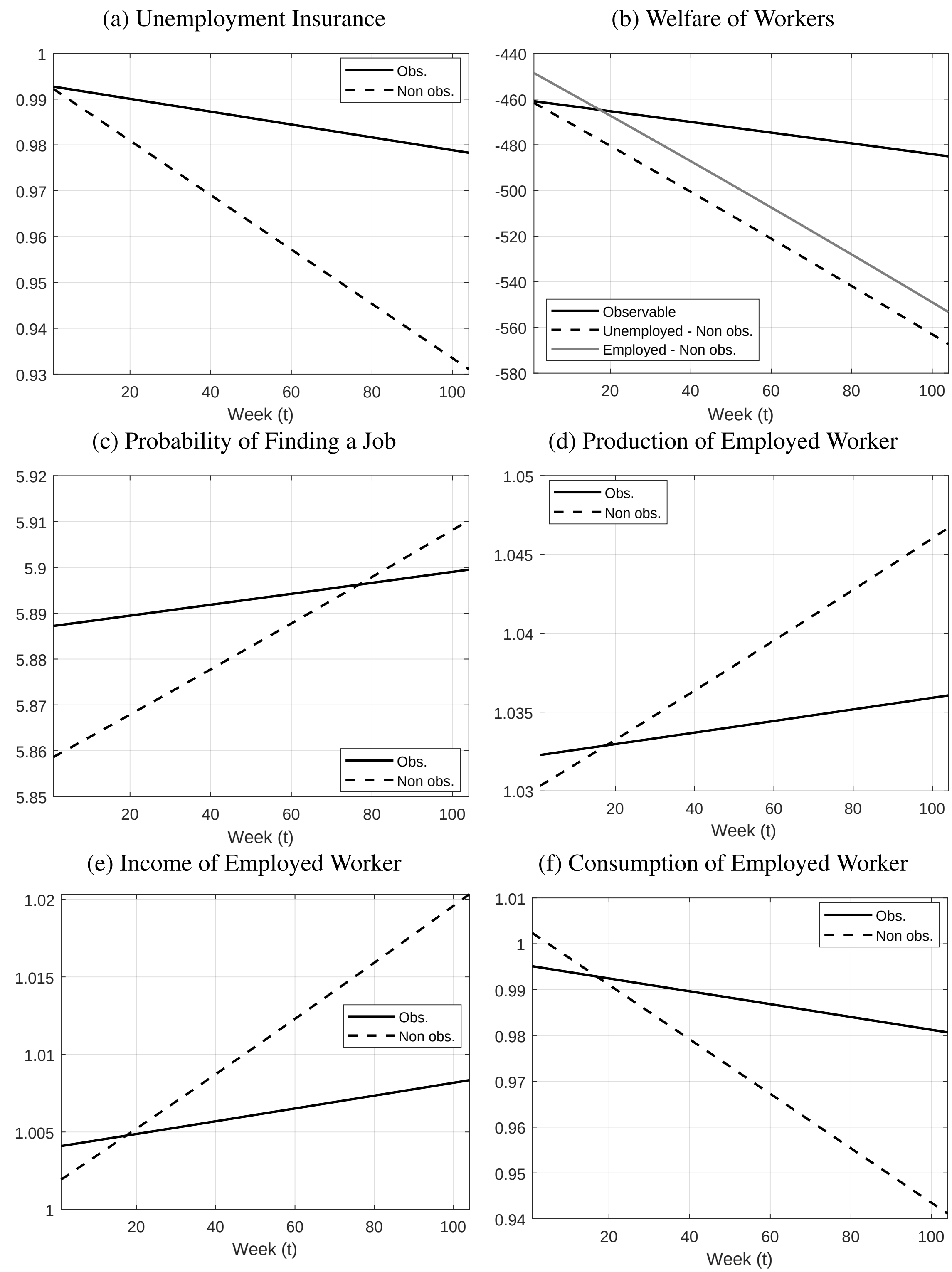
The fit of the model is quite good, as reported in Table 1. From this benchmark, we take the value function of a worker that still has all of their UI payments to receive and owns the average level of assets as in the data: $V_u(26, \bar{a})$.[8] We set this value as the baseline utility that the planner will promise the worker in the optimal unemployment insurance contracts: W_0. We solve for these optimal contracts under two scenarios: one in which the contracts are observable (the contract characterized in Section 4.1) and another in which they are not (Section 4.2).

Under observable contracts, the planner can provide the corresponding contract more cheaply, though the promised utility is the same. Quantitatively, with non-observable contracts, the cost of the program is 10.5% higher, a substantial increase.

Figure 2 reports the comparison of different outcomes under observable versus non-observable contracts. UI declines with the duration of unemployment in both cases. However, the decline is steeper under non-observability (Panel a). This reflects a decreasing promised utility for an unemployed worker throughout the unemployment spell (Panel b).

[8] The level for $\bar{a}$ is calibrated by targeting the level of liquid assets for the median individual in the United States as reported in Kaplan and Violante [2014]. We take their number and divide it by real GDP per worker (USARGDPE from the St. Louis FRED database). The corresponding ratio is 1.59.

Figure 2: Outcomes under the Optimal UI Contract, Observable versus Non-observable



Notes: The outcomes for an unemployed worker are reported for each week t during the unemployment spell. The outcomes for an employed worker are those for an individual who found a job at exactly week t.

Welfare upon employment also goes down with the duration of unemployment, as the planner wants to incentivize the agent to search harder earlier on. The planner achieves this by increasing taxes with the duration of the unemployment spell, as we will see momentarily. In the observable contract case, due to a binding incentive compatibility constraint, the welfare of the unemployed and the employed coincide (so that only one line is displayed in the figure for the observable case).

Lower unemployment insurance and welfare over time incentivize the worker to search harder for a job (Panel c). Therefore, with non-observable contracts, the search effort the worker engages in increases faster over time. This higher search effort (higher probability of finding a job) materializes because the individual is searching for jobs in which they have to work harder and produce more (Panel d). Consequently, the worker is compensated for this higher effort, and their income as an employed worker is higher when they find a job later (Panel e). This happens because search effort increases with time and the worker is compensated for this. However, the consumption of the employed worker is lower for those who find a job later (Panel f), implying the tax increases with the duration of unemployment. As seen in the previous sections, the planner finds it optimal to increase taxes over the duration of unemployment to incentivize search. Again, under non-observable contracts, the variation throughout unemployment is steeper.

With non-observable contracts, the tax collected by the planner increases with the unemployment spell. This transpires by putting together Panels (e) and (f) in Figure 2. In the previous section, we proved that, in this non-observable case, the planner imposes a distortionary tax rate to incentivize the effort supplied by the worker. We can compute such a tax rate in our numerical exercise: 0.03%. So, though the planner does distort the intensive margin of the worker, it does so with a somewhat low tax rate. This low level of the distortionary tax materializes because, in our model, once employed, the worker never loses their job. Hence, the planner must take into account that, by imposing this distortion, the worker will face it forever. Were the worker at risk of losing their job and searching again, this tax rate would have been higher. This distortionary tax rate also increases with the duration of the unemployment spell.

In sum, the non-observability of the employment contract has an important quantitative effect on the optimal UI contract offered by the planner. With non-observable contracts, the planner decreases UI faster and increases taxation upon employment faster with the duration of unemployment (including adding a distortionary component to the tax). These changes all add up to a considerably more expensive UI program.

5 Hidden Savings

In Section 4, we have followed most of the literature [e.g., Hopenhayn and Nicolini, 1997] in assuming that the planner controls the worker's savings. This allowed us to define a one-to-one mapping from unemployment benefits to consumption when unemployed, c^u, and from after-tax earnings, $y^e - T$, to consumption when employed, c^e. What happens if this is not the case? If the government does not control savings, do our results remain valid?[9]

To study the optimal design of an unemployment insurance program for the case where the government does not observe agents' savings, we assume that agents have access to perfect capital markets with an interest rate $r = \beta^{-1} - 1$. Because consumption and earnings need not coincide due to the possibility of borrowing and saving, a crucial distinction between the optimal consumption path and the optimal transfer path arises, as highlighted by Shimer and Werning [2007] in the context of unemployment insurance. We follow their lead in restricting our analysis to the case of preferences that do not exhibit income effects; i.e., we assume that preferences are of the GHH-CARA form.

The planner's program is to minimize the expected cost of delivering utility W_0 for the unemployed agent subject to providing incentives for them to follow the optimal search strategy.

Assume that the worker starts with assets, A_0. In a deterministic mechanism, the government adds liquidity, $a_0 - A_0$, at time $t = 0$ and transfers, b_t, to the unemployed in period $t \geq 0$. If a job is found at period t the government demands the amount of work y_t^e and makes a net transfer T_t^e (which may be negative) in every subsequent period.[10]

Recall that $p \to (\varrho(p)/p)$ is strictly increasing and strictly convex. We make the following additional assumption, which guarantees that it can be optimal for the agent to search for a job.

Assumption H1: *There exists y and $p > 0$ such that*

$$y > \eta \left(y + \kappa \frac{\varrho(p)}{p} \right).$$

Intuitively, if H1 were violated, the benefit of exerting more effort would never com-

[9]As we know from Allen [1985], Cole and Kocherlakota [2001], hidden savings represent an important constraint for designing optimal policies.

[10]Once the worker finds a job, there is no further need to provide incentives and utility provision is optimally accomplished by a time-invariant allocation. Hence, the time index in y_t^e and T_t^e refers to the period t in which the job is found.

pensate for its disutility. Optimal programs would thus entail no vacancy creation.

A ***policy*** is a tuple, $(a_0, \{y^e_t, T^e_t, b_t\}^\infty_{t=0})$, where a_0 is the agent's initial asset holdings, y^e_t is the amount of effort demanded in every future period if a job is found at period t, T^e_t are the taxes to be paid in every future period if a job is found at t, and b_t is the value of unemployment insurance at period t.

The consumption sequence $\{c^u_t, c^e_t\}^\infty_{t=0}$ is feasible under the policy $(a_0, \{y^e_t, T^e_t, b_t\}^\infty_{t=0})$ if $\lim \beta^t c^u_t = \lim \beta^t c^e_t = 0$ and there exists $\{a_t\}^\infty_{t=0}$ with $\lim \beta^t a_t = 0$ such that

$$a_{t+1} = \beta^{-1} \left(a_t - c^u_t + b_t\right),$$

and

$$c^e_t = (1 - \beta)\, a_t + y^e_t - T^e_t.$$

Without loss of generality, we restrict our attention to policies that generate feasible consumption sequences.

Next, we define a ***simple policy***, which will play an important role in all that follows.

Definition 5.1 *A simple policy is a triple (a_0, y^e, T^e) in which the earnings, y_t, of an employed agent, are constant, $y^e_t = y^e$ for all t, and the transfers, T^e_t, that the agent makes to the government once employed are also constant, $T^e_t = T^e$.*

Lemma 5.1 below explains how an agent optimally responds to a simple policy. We then show in Theorem 5.2 that the constrained efficient allocation can be implemented by a simple contract which we fully characterize in Section 5.1.

When facing a simple policy, the worker's problem can be written in a recursive form:

$$\begin{aligned} W_t(a_t) = \max_{a_{t+1} \in R,\ p \in [0,1]} \Big\{ & -\exp\left\{-\alpha\left(a_t - \beta^{-1} a_{t+1}\right)\right\} \\ & + \beta\left\{p W^e_{t+1}\left(a_{t+1}, p\right) + (1-p) W_{t+1}\left(a_{t+1}\right)\right\} \Big\}, \end{aligned} \tag{10}$$

subject to

$$W^e_{t+1}\left(a_{t+1}, p\right) = -\frac{1}{1-\beta} \exp\left\{-\alpha\left[y^e - T^e + \left(\beta^{-1} - 1\right) a_{t+1} - \eta\left(y^e + \kappa \frac{\varrho(p)}{p}\right)\right]\right\}.$$

For short, we write $W_t = W_t(a_t)$, $W^e_t = W^e_{t+1}\left(a_{t+1}, p_t\right)$, and $W^u_t = W_{t+1}\left(a_{t+1}\right)$, at the optimal (a_{t+1}, p_t).

Let $\hat{c}^e_{t,\tau}$ be the consumption at τ for an agent who found a job in period $t < \tau$. For all

$\tau > t$, $\hat{c}^e_{t,\tau} = c^e_t$. Hence, we omit the current period subscript, τ, and write c^e_t to denote the time-invariant consumption of an agent who found a job in period t.

Lemma 5.1 *Assume that the planner offers a simple contract, (a_0, y^e, T^e), to the agent. In this case, the agent chooses,*

1. *a stationary p;*

2. *$c^e_{t+1} - c^e_t = -\Delta_c$ and $c^u_{t+1} - c^u_t = -\Delta_c$, for a constant, $\Delta_c > 0$;*

3. $$\frac{W^e_t}{W_t} = \left\{1 + \alpha p(1-p)\kappa\eta'\left(y^e + \kappa\frac{\varrho(p)}{p}\right)\left(\frac{\varrho(p)}{p}\right)'\right\}^{-1} = k_e < 1,$$

   and

   $$\frac{W^u_t}{W_t} = \left(1 + \frac{\alpha p^2\kappa\eta'\left(y^e + \kappa\varrho(p)/p\right)\left(\varrho(p)/p\right)'}{1 + \alpha^2 p^2(1-p)\kappa\eta'\left(y^e + \kappa\varrho(p)/p\right)\left(\varrho(p)/p\right)'}\right) = k_u > 1;$$

4. $$\frac{p}{(1-\beta)W_t}\exp\left\{-\alpha\left[c_e - \eta\left(y^e + \kappa\frac{\varrho(p)}{p}\right)\right]\right\} = 1 - (1-p)\,k_u.$$

According to Lemma 5.1, when facing a simple contract, the agent chooses a constant p, and, for every additional period in unemployment, they reduce both consumption while unemployed, c^u_t, and planned consumption after finding a job, c^e_t, by the same amount Δ_c. Consumption is kept constant after the agent finds a job. As a result, the ratios W^e_t/W_t and W^u_t/W_t remain constant at k_e and k_u, respectively.[11]

Theorem 5.2 *There exists a simple policy that implements the optimal allocation.*

The optimal unemployment contract implements an allocation characterized by a constant search, p, and a constant work effort, $n^e = y^e + \kappa\varrho(p)/p$ after a job is found. To provide incentives for agents to keep searching, one must guarantee that $c^e_t > c^u_t$ in every period t. However, spreading consumption across the two states, unemployment and employment, within a single period is a costly way of delivering promised utility. To reduce this cost, incentives are back-loaded. The promised utility is reduced every time an agent

[11]Because $W \in (-\infty, 0)$, expected utility decreases with the unemployment spell ($k_u > 1$) and increases once a job is found ($k_e < 1$).

fails to find a job, as stated in 3. For GHH-CARA preferences, a lower utility promise with the same p and y^e can be made incentive-compatible by an equal reduction in c^e and c^u, which we show to be optimal in the proof of Theorem 5.2.

Next, we explain the rationale for how the optimal allocation can be implemented with a simple contract in which the agent is given assets a_0 and is promised a labor contract (y^e, c^e) if they manage to find a job. Agents' (dis)savings choices guarantee that c_t^e and c_t^u will follow the path prescribed in Theorem 5.2.

Under Assumption H1 and with GHH-CARA preferences, changes in asset positions have no impact on agents' search choices. However, they imply an adjustment in consumption during the unemployment spell and after a job is found, which leads to a simple scaling of expected utility. Given this simple response to asset position, a Ricardian-equivalence result materializes. Alternative paths are fully characterized by the time in which the worker finds a job and the worker's decision only depends on the present value of transfers associated with each path. By performing simple changes in the timing of payments, simple insurance schemes are optimal.

5.1 The Optimal Policy

Now that we have established that the optimal contract is stationary and of the form (a_0, c^e, y^e), we rely on this simple structure to provide its complete characterization.

We can restrict the search for the optimal contract to that of finding a triple (a_0, c^e, y^e) that solves the problem

$$\max_{(a_0, c^e, y^e)} \left\{ \frac{p\left(y^e, c^e\right)}{1-\beta\left(1-p\left(y^e, c^e\right)\right)} \frac{y^e-c^e}{1-\beta}-a_0 \right\},$$

subject to

$$U\left(y^e, c^e, a_0\right) \geq U_0,$$

where $U\left(y^e, c^e, a_0\right)$ is the value of the worker's program (10) under the simple policy, (y^e, c^e, a_0). The incentive constraint is summarized by the dependence of p on y^e and c^e.

The promise-keeping constraint can equivalently be written as

$$U\left(y^e, c^e\right) \exp\left\{-\alpha\left(1-\beta\right) a_0\right\} \geq U_0,$$

where we use the simplified form $U\left(y^e, c^e\right)$ for $U\left(y^e, c^e, 0\right)$.

If the constraint is binding, we can eliminate a_0 from the maximization program and write the planner's objective as

$$\max_{(c^e,y^e)} \left\{ \frac{p\left(y^e,c^e\right)}{1-\beta\left(1-p\left(y^e,c^e\right)\right)} \frac{y^e-c^e}{1-\beta} - \frac{1}{\alpha(1-\beta)} \ln\left[\frac{U(y^e,c^e)}{U_0}\right] \right\}$$

The first-order condition with respect to c^e is

$$-\frac{p\left(y^e,c^e\right)}{1-\beta\left(1-p\left(y^e,c^e\right)\right)} + \frac{1}{\alpha(1-\beta)} \frac{\partial U\left(y^e,c^e\right)/\partial c^e}{U\left(y^e,c^e\right)} + \frac{\partial}{\partial p}\left[\frac{p}{1-(1-p)\beta}\right] \frac{y^e-c^e}{1-\beta} \frac{\partial p}{\partial c^e} = 0, \quad (11)$$

and, with respect to y^e is

$$\frac{p\left(y^e,c^e\right)}{1-\beta\left(1-p\left(y^e,c^e\right)\right)} + \frac{1}{\alpha(1-\beta)} \frac{\partial U\left(y^e,c^e\right)/\partial y^e}{U\left(y^e,c^e\right)} + \frac{\partial}{\partial p}\left[\frac{p}{1-(1-p)\beta}\right] \frac{y^e-c^e}{1-\beta} \frac{\partial p}{\partial y^e} = 0. \quad (12)$$

Consider the optimality conditions above. The first term regards the direct fiscal cost of an increase in y^e. The second term is the impact on the worker's utility. Both are purely mechanical impacts. The third term summarizes the indirect, behavioral fiscal effects that are present because p is not observable.

As we show, $\partial p/\partial c^e > 0$ and $\partial p/\partial y^e < 0$. The worker's best response to a higher disposable income is to increase the job-finding probability and decrease it in response to a higher gross income. By comparing the absolute value of these two, we can identify the marginal impact on p of slightly increasing both by the same amount and relate it to the labor wedge:

$$1+\frac{\partial U(y^e,c^e)/\partial y^e}{\partial U(y^e,c^e)/\partial c^e} = \left[y^e-c^e\right] \frac{\alpha U(y^e,c^e)}{\partial U(y^e,c^e)/\partial c^e} \frac{\partial}{\partial p}\left[\frac{p}{1-(1-p)\beta}\right] \left[\frac{\partial p}{\partial y^e}+\frac{\partial p}{\partial c^e}\right].$$

The fiscal effect also depends on the sign of $y^e - c^e$; whether the behavioral response translates into a positive or a negative fiscal effect.

Theorem 5.3 *The efficient allocation is characterized by:*

1. $y^e - c^e$ *is strictly positive;*

2. *The labor wedge,*
$$1 + \frac{\partial U(y^e, c^e)/\partial y^e}{\partial U(y^e, c^e)/\partial c^e},$$
is strictly positive;

3. *As the agent fails to find a job by period t, both W_t^* and W_t^{e*} diverge to minus infinity.*

According to 1, $y^e - c^e > 0$. Hence, the worker pays net taxes after finding a job. An increase in y^e raises the job-finding probability, while an increase in c^e reduces it. That is, a worker who provides lower effort responds better to incentives, being more prone to increasing their job-finding rate due to an increase in consumption when employed. As a result, we obtain 2: as in the model with non-hidden savings, the moral hazard problem implies that effort should be discouraged at the margin.

The moral hazard problem does not arise because of positive taxes, $y^e - c^e > 0$. The planner can make taxes dependent only on whether the agent is employed regardless of how much they earn, thus avoiding the distortions at the work effort margin. The fiscal externality is important because it makes it desirable for the planner to induce agents to search harder. It is optimal for the government to distort the effort margin because a positive effort wedge increases the cost of downward deviation of the search margin.

Finally, turn to the last point of the theorem. The worker always expects to find a job with a constant probability in every period. Because of that, they dis-save. Hence, their unemployment consumption decreases along the duration of the unemployment spell. The longer they stay unemployed, the less resources they will have for consumption. Consequently, the lower their expected utility upon landing a job, W_t^{e*} (and their expected utility, W_t^*, since $W_t^* < W_t^{e*}$ for every t.) The absence of income effects in GHH-CARA preferences implies that their consumption diverges to minus infinity as the unemployment spell becomes arbitrarily long, a phenomenon referred to as "immiseration."

6 Conclusion

This paper has developed a dynamic model with directed search to analyze the optimal design of unemployment insurance when accounting for non-pecuniary characteristics of job quality. By allowing firms to adjust these non-wage job characteristics, our framework captures important dimensions that prior research has overlooked. We characterize the planning optimum and find that, even without distributive motives, a wedge arises at the margin under the non-observability of employment contracts. This distortion occurs because unobserved high-quality jobs are effectively subsidized by the unemployment insurance program. Introducing a marginal tax on earnings discourages such jobs, rendering them less attractive despite providing higher non-wage quality. This distortionary tax emerges whether or not the agent can hide their savings from the planner.

We calibrate our model to the U.S. economy. When employment contracts are unobservable to policymakers, several consequences arise: unemployment benefits decline more rapidly, taxes upon re-employment increase more steeply with unemployment duration, and distortionary tax policies become necessary. Collectively, these adjustments result in an unemployment insurance program that, when compared to a scenario where contracts are fully observable, is 10.5% costlier while providing the same level of welfare.

Our findings highlight the need for effective monitoring of the non-wage dimensions of jobs to validate benefit eligibility and deter job seekers from solely pursuing highly desirable yet improbable positions. Implementing robust oversight policies can help mitigate moral hazard and facilitate a more efficient unemployment insurance system aligned with the realities of today's complex labor market.

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A Data Appendix

This appendix provides the full results for the regressions that yield the coefficients from Figure 1 in Section 2. See Tables 2 and 3. These regressions use U.S. data from the March Supplement of the Current Populations Surveys (CPS) between 2009 and 2022. The controls used in some of the regressions are age, gender and education.

Table 2: Linear Probability Model, Probability of Being Unemployed One Year Later

	(1) Unemployed	(2) Unemployed
insurance	0.0820***	0.0691***
	(0.00995)	(0.0101)
Controls	No	Yes
N	11804	11804

Standard errors in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Table 3: Linear Probability Model, Probability of Having a Job with some Characteristics One Year Later

	(1) Unionized	(2) Unionized	(3) Health	(4) Health
insurance	0.0167***	0.0148***	0.0310	0.0149
	(0.00431)	(0.00439)	(0.0366)	(0.0373)
Controls	No	Yes	No	Yes
N	7422	7422	670	670

Standard errors in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

B Theoretical Appendix

B.1 Proofs of Section 4.1

Proof of Lemma 4.1. First, we show that the constraint (5) binds. The first order condition with respect to c^u reads

$$\varphi'(c^u) = \frac{1-p}{\mu(1-p)-\lambda} > 0.$$

If $\mu \leq 0$ then $\lambda < 0$ and thus $p > 0$ and then using the first order condition w.r.t. c^e we obtain

$$\varphi'(c^e) = \frac{p}{p\mu+\lambda} < 0,$$

a contradiction.

Next, towards a contradiction, assume that, without loss of generality, the constraint (6) does not bind at $t = 0$, $\varphi'(c_0^u) = \varphi'(c_0^e) = \mu_0^{-1} = \eta'(n_0)$. In this case,

$$\varphi(c_0^e) - \eta(n_0) < \varphi(c_0^u). \tag{13}$$

The moral hazard constraint must bind for some $t > 0$, otherwise, $\varphi(c_t^u) = \mu_0^{-1}$, for every t. This means that getting a job in period zero is worse than being unemployed forever.

Assume that the first period in which the constraint binds is $t = 1$ (the other case is analogous). We have $\mu_1 = \mu_0$, $\lambda_1 > 0$ and, hence,

$$\varphi'(c_1^e) = \frac{p_1}{p_1\mu_0+\lambda_1} = \eta'(n_1).$$

Therefore,

$$\varphi(c_0^e) - \eta(n_0) < \varphi(c_1^e) - \eta(n_1) \tag{14}$$

Hence, using (13) and (14) we obtain

$$\frac{\varphi(c_0^e) - \eta(n_0)}{1-\beta} < \varphi(c_0^u) + \beta\frac{\varphi(c_1^e) - \eta(n_1)}{1-\beta},$$

which, using the fact that the moral hazard constraint was binding in the second period, implies that the worker strictly prefers being unemployed to getting a job at zero, a contradiction. ■

B.2 Proofs of Section 4.2

Lemma B.1 *Both mappings, $z(\cdot)$ and $c^u(\cdot)$, are strictly increasing, twice differentiable, and strictly convex. Moreover, there exists W^* such that $z(W^*) = c^u(W^*)$, $z(W) > c^u(W)$, for all $W > W^*$, and $z(W) < c^u(W)$, for all $W < W^*$.*

Proof of Lemma B.1. Let $y^e(W)$ be given by

$$\underset{y^e}{\operatorname{argmax}} \left[\varphi\left(y^e + z(W)\right) - \eta\left(y^e + \phi\right)\right],$$

and note that if $\varphi'(z(W)) - \eta'(\phi) \leq 0$, then $y^e(W) = 0$. Otherwise, $y^e(W)$ is given by $\varphi'(y^e + z(W)) - \eta'(y^e + \phi) = 0$.

Hence, because $z(W) + y^e(W) > c^u(W)$, we have

$$z'(W) = \frac{1}{\varphi'(z(W) + y^e(W))} > \frac{1}{\varphi'(c^u(W))} = c^{u\prime}(W).$$

This implies that if $z(\cdot)$ and $c^u(\cdot)$ cross at most once, and $z(W) > c^u(W)$ (resp. $z(W) < c^u(W)$) for every utility greater (resp. lower) than this utility level.

Since $z(W) \to \infty$ as $W \to \infty$, we have $y^e(W) = 0$ for W large enough, which implies $z(W) > c^u(W)$. The existence of a small W such that $z(W) < c^u(W)$ holds by assumption. Therefore, W^* exists by continuity.

It remains to show that both mappings are strictly convex. Since $c^e(W) := z(W) +$

$y^e(W)$ is strictly increasing with positive derivative, we have

$$z''(W) = \frac{-\varphi''(c^e(W))}{\varphi'(c^e(W))^2} c^{e\prime}(W) > 0,$$

and

$$c^{u\prime\prime}(W) = \frac{-\varphi''(c^u(W))}{\varphi'(c^u(W))^2} c^{u\prime}(W) > 0.$$

■

Lemma B.2 *Suppose that, if a worker gets a job, then they must earn $c^e + T$, paying T to the government, to consume c^e, whereas if the worker fails to get a job then they obtain the continuation utility W. Then this problem admits a unique solution. If the solution is interior, it is given by the associated first-order conditions.*

Proof.

Consider the problem

$$\max p\left[\varphi(c^e) - \eta\left(c^e + T + \kappa\frac{\varrho(p)}{p}\right) - W\right]$$

This problem admits an interior solution if and only if $\varphi(c^e) - \eta(c^e + T) > W$.

Assume that this is the case and consider p that makes its derivative equal to zero:

$$\varphi(c^e) - \eta\left(c^e + T + \kappa\frac{\varrho(p)}{p}\right) - W - p\eta'\left(c^e + T + \kappa\frac{\varrho(p)}{p}\right)\kappa\frac{d}{dp}\left(\frac{\varrho(p)}{p}\right) = 0$$

Differentiate the left-hand side again to obtain

$$\begin{aligned}&-2\eta'\left(c^e + T + \kappa\frac{\varrho(p)}{p}\right)\kappa\frac{d}{dp}\left(\frac{\varrho(p)}{p}\right) - p\eta'\left(c^e + T + \kappa\frac{\varrho(p)}{p}\right)\kappa\frac{d^2}{dp^2}\left(\frac{\varrho(p)}{p}\right)\\ &\qquad - p\eta''\left(c^e + T + \kappa\frac{\varrho(p)}{p}\right)\kappa\left[\frac{d}{dp}\left(\frac{\varrho(p)}{p}\right)\right]^2.\end{aligned}$$

To show that the expression above is negative, it suffices to show that

$$\begin{aligned}-2\frac{d}{dp}\left(\frac{\varrho(p)}{p}\right) - p\frac{d^2}{dp^2}\left(\frac{\varrho(p)}{p}\right) < 0 &\Leftrightarrow 2\left(\frac{\varrho'(p)p - \varrho(p)}{p^2}\right) + p\frac{d}{dp}\left(\frac{\varrho'(p)p - \varrho(p)}{p^2}\right) > 0\\ &\Leftrightarrow p^2\frac{d}{dp}\left[\varrho'(p)p - \varrho(p)\right] > 0 \Leftrightarrow \frac{\varrho''(p)}{p} > 0.\end{aligned}$$

■

Lemma B.3 *For every W, let $C(W)$ be the planner's cost of providing utility W. The mapping $C(\cdot)$ is differentiable at W_t for every $t > 0$.*

Proof. We prove that C is differentiable at W_t. For that, we assume that $p_t > 0$ as the other case is analogous. Consider any small $\epsilon \in \mathbb{R}$ and note that the following perturbation is feasible:

$$\left(\underline{\tilde{u}}_{t-1}, \underline{\tilde{u}}_t, \tilde{c}_t^e\right) = \left(\underline{u}_{t-1} + \epsilon, \underline{u}_t - \epsilon\beta^{-1}, \varphi^{-1}\left(\varphi(c_t^e) + \epsilon\right)\right).$$

One can thus apply the argument in Clausen and Strub to conclude that

$$C'\left(W_t\right) = -c'(\underline{u}_t) = \frac{1}{\varphi'(\underline{u}_t)}.$$

■

Lemma B.4 *The multipliers, μ and λ, are strictly positive if there is a search.*

Proof. First, notice that

$$\left[\mu(1-p) - \lambda\right]\varphi'(c^u) = 1 - p$$

and

$$\frac{p\mu + \lambda}{1-\beta}\varphi'(c^e) = \frac{p}{1-\beta}$$

Hence, $\mu = 0$ implies $\varphi'(c^u)\varphi'(c^e) \leq 0$, which is absurd.

Hence assume towards a contradiction that $\lambda_0 \leq 0$. Clearly, there is a last period at which $\lambda_t \leq 0$ and $\lambda_{t+1} > 0$. Otherwise, as we will verify below, $c_t^u \geq c_t^e$ for every t, and hence there is no search. Assume that $\lambda_1 > 0$ (case in which $\lambda_s \leq 0$ for all $s < t$ and $\lambda_t > 0$ for some $t > 1$ can be handled analogously).

From the first-order condition with respect to p, we get

$$\varphi'(c^u) = \frac{1}{\mu - \lambda(1-p)^{-1}} \leq \frac{1}{\mu + \lambda p^{-1}} = \varphi'(c^e).$$

Hence, $c^u \geq c^e$.

Moreover, notice that from the first order condition we have

$$C'\left(W_1\right) = -\mu_0 + \frac{\lambda_0}{(1-p)} = -\mu_1,$$

which implies

$$\mu_1 = \mu_0 - \frac{\lambda_0}{(1-p)} \geq \mu_0.$$

This and $\lambda_0 \leq 0 < \lambda_1$ imply

$$\varphi'(c_1^e) = \frac{1}{\mu_1 + p_1^{-1}\lambda_1} < \frac{1}{\mu_0 + p_0^{-1}\lambda_0} = \varphi'(c_0^e).$$

Hence,

$$c_1^e > c_0^e. \tag{15}$$

We can rearrange the first order condition with respect to y^e to get

$$\mu\eta'\left(y^e + \kappa\frac{\varrho(p)}{p}\right) = 1 - \lambda\eta''\left(y^e + \kappa\frac{\varrho(p)}{p}\right)\kappa\frac{d}{dp}\left(\frac{\varrho(p)}{p}\right) - \frac{\lambda}{p}\eta'\left(y^e + \kappa\frac{\varrho(p)}{p}\right).$$

Therefore, $\lambda_0 \leq 0 < \lambda_1$ imply

$$\eta'\left(y_1^e + \kappa\frac{\varrho(p_1)}{p_1}\right) < \mu_1^{-1}.$$

Similarly,

$$\eta'\left(y_0^e + \kappa\frac{\varrho(p_0)}{p_0}\right) \geq \mu_0^{-1}.$$

Since $\mu_1 \geq \mu_0$, this implies

$$y_1^e + \kappa\frac{\varrho(p_1)}{p_1} < y_0^e + \kappa\frac{\varrho(p_0)}{p_0},$$

and

$$\eta\left(y_1^e + \kappa\frac{\varrho(p_1)}{p_1}\right) < \eta\left(y_0^e + \kappa\frac{\varrho(p_0)}{p_0}\right),$$

because η is strictly convex.

Since $p_0 > 0$, by the assumption of the lemma, we have

$$
\begin{aligned}
0 < \frac{1}{1-\beta}\left[\varphi(c_0^e) - \eta\left(y_0^e + \kappa\frac{\varrho(p_0)}{p_0}\right)\right] - [\varphi(c_0^u) + \beta W_1] \\
= \frac{1}{1-\beta}\left[\varphi(c_0^e) - \eta\left(y_0^e + \kappa\frac{\varrho(p_0)}{p_0}\right)\right] - \varphi(c_0^u) \\
- \beta\left[p_1\frac{1}{1-\beta}\left[\varphi(c_1^e) - \eta\left(y_1^e + \kappa\frac{\varrho(p_1)}{p_1}\right)\right] + (1-p_1)\left[\varphi(c_1^u) + \beta W_2\right]\right] \\
= \varphi(c_0^e) - \eta\left(y_0^e + \kappa\frac{\varrho(p_0)}{p_0}\right) - \varphi(c_0^u) + \beta\Bigg[\frac{1}{1-\beta}\left[\varphi(c_0^e) - \eta\left(y_0^e + \kappa\frac{\varrho(p_0)}{p_0}\right)\right] \\
- p_1\frac{1}{1-\beta}\left[\varphi(c_1^e) - \eta\left(y_1^e + \kappa\frac{\varrho(p_1)}{p_1}\right)\right] - (1-p_1)\left[\varphi(c_1^u) + \beta W_2\right]\Bigg] \\
= \frac{1}{1-\beta}\left[\varphi(c_0^e) - \eta\left(y_0^e + \kappa\frac{\varrho(p_0)}{p_0}\right)\right] - \varphi(c_0^u) \\
- \beta\left[p_1\frac{1}{1-\beta}\left[\varphi(c_1^e) - \eta\left(y_1^e + \kappa\frac{\varrho(p_1)}{p_1}\right)\right] + (1-p_1)\left[\varphi(c_1^u) + \beta W_2\right]\right] \quad (16)
\end{aligned}
$$

Since $p_1 > 0$, due to $\lambda_1 > 0$, we have

$$
\frac{1}{1-\beta}\left[\varphi(c_1^e) - \eta\left(y_1^e + \kappa\frac{\varrho(p_1)}{p_1}\right)\right] > \varphi(c_1^u) + \beta W_2
$$

Hence,

$$
\begin{aligned}
\varphi(c_0^e) - \eta\left(y_0^e + \kappa\frac{\varrho(p_0)}{p_0}\right) - \varphi(c_0^u) + \beta\Bigg\{\frac{1}{1-\beta}\left[\varphi(c_0^e) - \eta\left(y_0^e + \kappa\frac{\varrho(p_0)}{p_0}\right)\right] \\
- p_1\frac{1}{1-\beta}\left[\varphi(c_1^e) - \eta\left(y_1^e + \kappa\frac{\varrho(p_1)}{p_1}\right)\right] - (1-p_1)\left[\varphi(c_1^u) + \beta W_2\right]\Bigg\} \\
< \varphi(c_0^e) - \eta\left(y_0^e + \kappa\frac{\varrho(p_0)}{p_0}\right) - \varphi(c_0^u) + \beta\left[\frac{1}{1-\beta}\left[\varphi(c_0^e) - \eta\left(y_0^e + \kappa\frac{\varrho(p_0)}{p_0}\right)\right] - \left[\varphi(c_1^u) + \beta W_2\right]\right]
\end{aligned}
$$

Since the first line from the last term is negative, the entire term is less than

$$
\beta\left[\frac{1}{1-\beta}\left[\varphi(c_0^e) - \eta\left(y_0^e + \kappa\frac{\varrho(p_0)}{p_0}\right)\right] - \left[\varphi(c_1^u) + \beta W_2\right]\right],
$$

which is less than

$$\frac{1}{1-\beta}\left[\varphi(c_0^e)-\eta\left(y_0^e+\kappa\frac{\varrho(p_0)}{p_0}\right)\right]-\left[\varphi(c_1^u)+\beta W_2\right],$$

since the term is positive.

Since $\varphi(c_0^e)<\varphi(c_1^e)$, and

$$\eta\left(y_0^e+\kappa\frac{\varrho(p_0)}{p_0}\right)>\eta\left(y_1^e+\kappa\frac{\varrho(p_1)}{p_1}\right),$$

this is less than

$$\frac{1}{1-\beta}\left[\varphi(c_1^e)-\eta\left(y_1^e+\kappa\frac{\varrho(p_1)}{p_1}\right)\right]-\left[\varphi(c_1^u)+\beta W_2\right].$$

Hence, using the first-order conditions with respect to p, the algebra just performed means that

$$\frac{p_1}{1-\beta}\eta'\left(y_1^e+\kappa\frac{\varrho(p_1)}{p_1}\right)\kappa\left(\frac{\varrho(p_1)}{p_1}\right)>\frac{p_0}{1-\beta}\eta'\left(y_0^e+\kappa\frac{\varrho(p_0)}{p_0}\right)\kappa\left(\frac{\varrho(p_0)}{p_0}\right). \tag{17}$$

Since

$$y_1^e+\kappa\frac{\varrho(p_1)}{p_1}<y_0^e+\kappa\frac{\varrho(p_0)}{p_0},$$

If $y_1^e\geq y_0^e$, we will have $p_1<p_0$ which together contradict (17). We conclude that $y_1^e<y_0^e$.

Finally, notice that $\lambda_1>0$ and the first order condition with respect to p and the fact that p is a local maximum imply

$$\frac{y_0^e-c_0^e}{1-\beta}\leq -c_0^u+\beta C\left(W_1\right). \tag{18}$$

Analogously, in period 1, using $\lambda_0\leq 0$, the first order condition with respect to p implies

$$\frac{y_1^e-c_1^e}{1-\beta}\geq -c_1^u+\beta C\left(W_2\right).$$

But notice that

$$C\left(W_1\right)=p_1\frac{y_1^e-c_1^e}{1-\beta}+\left(1-p_1\right)\left[-c^u+\beta C\left(W_2\right)\right]\leq\frac{y_1^e-c_1^e}{1-\beta} \tag{19}$$

Hence, using (18), we have

$$c_0^u \leq \beta C\left(W_1\right) - \frac{y_0^e - c_0^e}{1-\beta} \Leftrightarrow c_0^u + \left(y_0^e - c_0^e\right) \leq \beta C\left(W_1\right) - \frac{\beta\left(y_0^e - c_0^e\right)}{1-\beta}$$

Since $c_0^u \geq c_0^e$ and $y_0^e \geq 0$ we have

$$0 \leq \beta\left[C\left(W_1\right) - \frac{y_0^e - c_0^e}{1-\beta}\right].$$

Using (19), the last term is less than

$$\beta\left[\frac{y_1^e - c_1^e}{1-\beta} - \frac{y_0^e - c_0^e}{1-\beta}\right] = \beta\left[\frac{y_1^e - y_0^e}{1-\beta} + \frac{c_0^e - c_1^e}{1-\beta}\right].$$

Hence, using (17) and $y_1^e < y_0^e$, we see that $(y_1^e - y_0^e) + (c_0^e - c_1^e) < 0$, a contradiction. ■

Proof of Lemma 4.1. Follows from Lemma B.4. ■

Proof of Theorem 4.1. i) The facts that the moral-hazard constraint (43) binds and the government benefits from strictly increasing p follow immediately from Lemma B.4.

ii) We have

$$\begin{aligned}
\left(\varphi'^e\right) &= \frac{p}{\mu p + \lambda} \\
\eta'\left(y^e + \kappa\frac{\varrho(p)}{p}\right) &= \frac{p}{\mu p + \lambda} - \frac{p\lambda}{\mu p + \lambda}\eta''\left(y^e + \kappa\frac{\varrho(p)}{p}\right)\kappa\left(\frac{\varrho(p)}{p}\right)'
\end{aligned}$$

Hence,

$$1 - \frac{\eta'\Big(y^e + \kappa\varrho(p)/p\Big)}{\left(\varphi'^e\right)} = \lambda\eta''\left(y^e + \kappa\frac{\varrho(p)}{p}\right)\kappa\left(\frac{\varrho(p)}{p}\right)' > 0$$

iii) Using the first-order conditions, we have

$$\frac{1}{\varphi'(c_{t-1}^u)} = \mu_{t-1} + p_{t-1}^{-1}\lambda_{t-1} = \mu_{t-1} - \lambda(1-p_t)^{-1} = \mu_t,$$

and

$$\left(\frac{p_t}{\varphi'(c_t^e)}\right) + \left(\frac{1-p_t}{\varphi'(c_t^u)}\right) = \left(p_t\mu_t + \lambda_t\right) + \left(\mu_t(1-p_t) - \lambda_t\right) = \mu_t,$$

Hence,

$$\frac{1}{\varphi'(c_{t-1}^{u})}=\frac{p_t}{\varphi'(c_t^{e})}+\frac{1-p_t}{\varphi'(c_t^{u})}.$$

■

Proof of Proposition 4.2. Notice that

$$\varphi'(c_t^{u})=\left(\mu_t-\frac{\lambda_t}{1-p_t}\right)^{-1}, \quad \text{and} \quad \mu_{t+1}=\mu_t-\frac{\lambda_t}{(1-p_t)}.$$

Hence,

$$\varphi'(c_{t+1}^{u})-\varphi'(c_t^{u})=\frac{1}{\mu_{t+1}-\frac{\lambda_{t+1}}{1-p_{t+1}}}-\frac{1}{\mu_t-\frac{\lambda_t}{1-p_t}}=\frac{1}{\mu_{t+1}-\frac{\lambda_{t+1}}{1-p_{t+1}}}-\frac{1}{\mu_{t+1}}\geq 0,$$

with strict inequality whenever the worker searches in $t+1$ and hence $\frac{\lambda_{t+1}}{1-p_{t+1}}>0$.

Finally, for the last claim assume that the worker actively searches in period $t+1$, use

$$\frac{p_{t+1}}{\varphi'(c_{t+1}^{e})}+\frac{1-p_{t+1}}{\varphi'(c_{t+1}^{u})}=\frac{1}{\varphi'(c_t^{u})}$$

and $c_{t+1}^{u}>c_t^{u}$ to conclude that $c_{t+1}^{e}>c_\tau^{u}$ for every $\tau\geq t$. ■

Proof of Lemma 4.2. First, we claim that unemployment insurance benefits are weakly decreasing over time. Consider two subsequent periods s and $s+1$. First note that, if there is no search at period $s+1$, then the concavity of φ implies that $\varphi\left(c_{s+1}^{u}\right)=\varphi\left(c_s^{u}\right)$. On the other hand, if there is search at period $s+1$ the result follows from Lemma B.4.

Therefore, since unemployment benefits are weakly decreasing, it suffices to show that, if there is no search in period t, then the planner could profitably deviate by offering a contract in which the worker also searches at t. The (normalized) utility $(1-\beta)\,W_t$ can be written as a convex combination of the terms:

1. $$\varphi\left(c_{t+k}^{e}\right)-\eta\left(y_{t+k}^{e}+\frac{\varrho(p_{t+k})}{p_{t+k}}\right),$$

   which are obtained if the worker finds a job at period $t+k$, and;

2. $\varphi\left(c_{t+k}^{u}\right)$, which are obtained if the worker does not get a job by period $t+k$.

Since $c_{t+k}^{u}\leq c_0^{u}$, this implies that $\varphi\left(c_{t+k}^{u}\right)<W^*$. Hence, the cost of delivering $\varphi\left(c_{t+k}^{u}\right)$ is

less than $Z\left(\varphi\left(c^u_{t+k}\right)\right)$ by Lemma B.1. The cost of providing utility,

$$\varphi\left(c^e_{t+k}\right) - \eta\left(y^e_{t+k} + \frac{\varrho(p_{t+k})}{p_{t+k}}\right),$$

is less than

$$Z\left(\varphi\left(c^e_{t+k}\right) - \eta\left(y^e_{t+k} + \frac{\varrho(p_{t+k})}{p_{t+k}}\right)\right).$$

Since the function Z is strictly convex, by Jensen's inequality and a continuity argument, there exists $\varepsilon > 0$ such that, if the planner offers the contract in which payments $y^e\left(W_t + \varepsilon\right)$ are required from and consumption $c^e\left(W_t + \varepsilon\right)$ is provided to the worker, they will search for a job with a positive probability, will obtain a utility $\tilde{W} > W_t$ from this search, and the government will incur a strictly lower cost.

This strategy makes both the worker as well as the planner better off at period t, but may decrease the worker's incentives at period $t-1$. To avoid that, the planner decreases the worker's unemployment consumption at period $t-1$ up to the point at which the worker is indifferent at period $t-1$. This further improves the planner's utility at $t-1$ by showing a strictly more profitable contract. ■

Proof of Lemma 4.3. a) Assume towards a contradiction that $\varphi\left(c^u_t\right) \geq W^*$ for every t. Therefore, the worker can guarantee a utility at least as large as W^* at every period. Then, if we let

$$w^e_t := \varphi\left(c^e_t\right) - \eta\left(y^e_t + \kappa\frac{\varrho(p_t)}{p_t}\right),$$

incentive-compatibility implies that $w^t_e > W^*$ for every t for which there is a positive search, in which case $C^u\left(w^t_e\right) < Z^e\left(w^t_e\right)$. Thus since $W_0(1-\beta)$ is a convex combination of $\{\varphi\left(c^u_0\right), w^e_0, \varphi\left(c^u_1\right), w^e_1, .., \}$ and $C^u(\cdot)$ is strictly convex, by applying Jensen's inequality, we conclude that the government's cost is strictly less than

$$\frac{C^u(W_0(1-\beta))}{1-\beta},$$

which can be achieved by offering constant unemployment insurance equal to $C^u(W_0(1-\beta))$ and never having the worker search for a job, a contradiction.

Next, since c^u_t is decreasing, a) implies that there exists t such that $\varphi\left(c^u_t\right) < W^*$. Then apply Lemma 4.2. ■

Proof of Proposition 4.3. Notice that unemployment insurance is decreasing. Moreover, since $W_0 < W^*$, there is a period T such that it is strictly decreasing from T on. Suppose towards a contradiction that it converges to $c^u_\infty > 0$. If it does not converge to zero, since $\varphi'(c^u_t) = \frac{1}{\mu_{t+1}}$, we conclude that $\mu_t \to (\varphi'(c^u_\infty))^{-1}$. Therefore, $\frac{\lambda_t}{1-p_t} \to 0$. We claim that $p_t \to 0$. Suppose towards a contradiction that there is a subsequence $p_{t_r} \to \hat{p} > 0$ and notice that, since $\varphi'(c^e_t) = (\mu_t + p_t^{-1}\lambda_t)^{-1}$, we have along the subsequence $\varphi'(c^e_{t_r}) \to \varphi'(c^u_\infty)$, implying $c^e_{t_r} \to c^u_\infty$. By incentive compatibility,

$$\eta\left(y^e_{t_r} + \kappa\frac{\varrho(p_{t_r})}{p_{t_r}}\right) \to 0,$$

which is not possible: a contradiction.

But then by a continuity argument, for every $\varepsilon > 0$, there exists a period t^* such that $t \geq t^*$ implies that the government's utility is ε away from $-c^u_\infty/(1-\beta)$, while the worker's utility is ε away from $\varphi(c^u_\infty)/(1-\beta)$.

Since $\varphi(c^u_\infty) < W^*$, there exists $\varepsilon > 0$ such that $Z\left(\varphi(c^u_\infty) + \varepsilon\right) < c^u_\infty$. Hence, if the government deviates toward a stationary employment contract in which it demands $y^e\left(\varphi(c^u_\infty) + \varepsilon\right)$ (see notation of Lemma B.1) and provides consumption $y^e\left(\varphi(c^u_\infty) + \varepsilon\right) + Z\left(\varphi(c^u_\infty) + \varepsilon\right)$, then the worker searches for a job with probability bounded away from zero (for t sufficiently large). Moreover, for t sufficiently large, this deviation makes both the worker and the government better off, a contradiction. ■

B.3 Hidden Savings

Lemma B.5 *Consider any deterministic mechanism. Assume that the agent starts with income a_0. Let (c^u_t, p_t) be the optimal choices of the agent at period t. The agent who starts with income $\tilde{a}_0$ chooses $(c^u_t + (\tilde{a}_0 - a_0)(1-\beta), p_t)$ in every period t and obtains* $\exp\{-\alpha(1-\beta)(\tilde{a}_0 - a_0)\}W_t$, *where W_t is the utility obtained at t by the agent who starts the game with assets a_t.*

Proof. The proof will be based on the principle of optimality. We will guess and verify that if W_t is the agent's continuation utility when period t is started with income a^1_t, then $W_t \exp\left\{-\alpha(1-\beta)(a^2_t - a^1_t)\right\}$ is the continuation utility when the agent starts period t with a^2_t. Take any optimal strategy $\left\{(c^u_\tau(a_t), p_\tau(a_t))\right\}_{\tau\geq t}$ when period t starts with income $a_t \in \{a^1_t, a^2_t\}$ and let W^i_t be its value. Notice that the worker who starts with assets a^2_t can follow strategy $\left\{(c^u_\tau(a^1_t) + (a^1_t - a^2_t)(1-\beta), p_\tau(a^1_t))\right\}_{\tau\geq t}$. Hence, by revealed preference, $W_t^1 \geq \exp\{-\alpha(1-\beta)(a_t^2 - a_t^1)\} W_t^2$. Similarly, $W_t^2 \geq \exp\{-\alpha(1-\beta)(a_t^1 - a_t^2)\} W_t^1$, and thus $W_t^1 = \exp\{-\alpha(1-\beta)(a_t^2 - a_t^1)\} W_t^2$.

Finally, let W_0 be the value from following the optimal strategy when the initial asset is a_0 and observe that strategy $(c_t^u + (\tilde{a}_0 - a_0)(1-\beta), p_t)$ is feasible and it leads to $W_0 \exp\{-\alpha(1-\beta)(\tilde{a}_0 - a_0)\}$. Hence this strategy is optimal. ■

We start with a lemma that establishes a Ricardian equivalence result for this setting. Take a feasible sequence $\{c_t^u, c_t^e, p_t\}_{t=0}^{\infty}$ under the policy $(a_0, \{y_t^e, T_t^e, b_t\}_{t=0}^{\infty})$. Let $\{W_t, W_t^e\}_{t=0}^{\infty}$ the sequence of indirect utilities when employed and unemployed respectively, which is generated by $\{c_t^u, c_t^e, p_t\}_{t=0}^{\infty}$. Moreover, let W_t^u be the utility from failing to find a job at period t.

Lemma B.6 *The sequence $\{c_t^u, c_t^e, p_t\}_{t=0}^{\infty}$ is optimal if and only if:*

i)

$$p_t = \arg\max -p\frac{1}{1-\beta}\exp\left\{-\alpha\left[c_t^e - \eta\left(y_t^e + \kappa\left(\frac{\varrho(p)}{p}\right)\right)\right]\right\} - (1-p)\, W_{t+1} \quad (20)$$

ii)

$$\exp\{-\alpha c_t^u\} = -(1-\beta)\, W_{t+1}. \quad (21)$$

Proof. In light of Lemma B.5, $\{c_t^u, c_t^e, p_t\}_{t\geq T}^{\infty}$ is optimal at period T if and only if

$$\{c_t^u + (1-\beta)(\tilde{a}_T - a_T), c_t^e + (1-\beta)(\tilde{a}_T - a_T), p_t\}_{t\geq T}^{\infty}$$

when period T starts with assets $\tilde{a}_T$. This allows us to apply the one-shot deviation principle. In our context, this asserts that $\{c_t^u, c_t^e, p_t\}_{t=0}^{\infty}$ is optimal if and only if the optimality conditions w.r.t. p_t and c_t^u hold, which are given by (20) and (21). ■

The following Lemma states a Ricardian equivalence result for our environment:

Lemma B.7 *Assume that the sequence $\{c_t^u, c_t^e, p_t\}_{t=0}^{\infty}$ is feasible under the policies*

$$\left(a_0, \{y_t^e, T_t^e, b_t\}_{t=0}^{\infty}\right) \quad \textit{and} \quad \left(\tilde{a}_0, \left\{\tilde{y}_t^e, \tilde{T}_t^e, \tilde{b}_t\right\}_{t=0}^{\infty}\right).$$

The sequence, $\{c_t^u, c_t^e, p_t\}_{t=0}^{\infty}$, is optimal under $\left(a_0, \{y_t^e, T_t^e, b_t\}_{t=0}^{\infty}\right)$ if and only if it is optimal under $\left(\tilde{a}_0, \left\{\tilde{y}_t^e, \tilde{T}_t^e, \tilde{b}_t\right\}_{t=0}^{\infty}\right)$.

Proof. This result follows immediately from Lemma B.6. ■

Lemma B.8 *Let $(c_t^u, c_t^e, y_t^e, p_t)_{t=0}^{\infty}$ solve the government's problem when the agent starts with utility W_0. Then, $(c_t^u + \Delta, c_t^e + \Delta, y_t^e, p_t)_{t=0}^{\infty}$ solves the government's problem when the agent starts with a utility $\tilde{W} = e^{-\alpha\Delta} W_0$.*

Proof. We claim that $(c_t^u + \Delta, c_t^e + \Delta, y_t^e, p_t)_{t=0}^{\infty}$ is at least as good as any allocation $(\tilde{c}_t^u, \tilde{c}_t^e, \tilde{y}_t^e, \tilde{p}_t)_{t=0}^{\infty}$ that yields utility $\tilde{W}$. Indeed, take $(\tilde{c}_t^u, \tilde{c}_t^e, \tilde{y}_t^e, \tilde{p}_t)_{t=0}^{\infty}$ and notice that $(\tilde{c}_t^u - \Delta, \tilde{c}_t^e - \Delta, \tilde{y}_t^e, \tilde{p}_t)_{t=0}^{\infty}$ generates utility W_0. Hence, the optimality of $(c_t^u, c_t^e, y_t^e, p_t)_{t=0}^{\infty}$ implies

$$\sum_{t=0}^{\infty} \beta^t \left(\sum_{\tau \le t} \left(\Pi_{s<\tau} (1-p_s)\right) p_\tau \left[y_\tau^e - c_\tau^e\right] + \left(1 - \sum_{\tau \le t} \left(\Pi_{s<\tau}(1-p_s)\right) p_\tau\right) \left[-c_t^u\right] \right)$$
$$\ge \sum_{t=0}^{\infty} \beta^t \left(\sum_{\tau \le t} \left(\Pi_{s<\tau}(1-\tilde{p}_s)\right) \tilde{p}_\tau \left[\tilde{y}_\tau^e - (\tilde{c}_\tau^e - \Delta)\right] + \left(1 - \sum_{\tau \le t} \left(\Pi_{s<\tau}(1-\tilde{p}_s)\right) \tilde{p}_\tau \right) \left[-\left(\tilde{c}_t^u - \Delta\right)\right] \right),$$

which holds if and only if

$$\sum_{t=0}^{\infty} \beta^t \left(\sum_{\tau \le t} \left(\Pi_{s<\tau}(1-p_s)\right) p_\tau \left[y_\tau^e - (c_\tau^e + \Delta)\right] - \left(1 - \sum_{\tau \le t} \left(\Pi_{s<\tau}(1-p_s)\right) p_\tau\right) \left(c_t^u + \Delta\right) \right)$$
$$\ge \sum_{t=0}^{\infty} \beta^t \left(\sum_{\tau \le t} \left(\Pi_{s<\tau}(1-\tilde{p}_s)\right) \tilde{p}_\tau \left[\tilde{y}_\tau^e - \tilde{c}_\tau^e\right] + \left(1 - \sum_{\tau \le t} \left(\Pi_{s<\tau}(1-\tilde{p}_s)\right) \tilde{p}_\tau\right) \left[-\tilde{c}_t^u\right] \right),$$

proving the optimality of $(c_t^u + \Delta, c_t^e + \Delta, y_t^e, p_t)_{t=0}^{\infty}$ when the promised utility is $\tilde{W}$. ■

Proof of Lemma 5.1. Consider a simple policy (a_0, y^e, T^e). Let (p_0, c_0^u) be the first-period choices and a_1 be the corresponding level of assets if the agent is unemployed in period 1. Lemma B.5 implies that $W_1 = W_0 \exp\{-\alpha(1-\beta)(a_1 - a_0)\}$. Moreover, $p_1 = p_0$, $c_1^e = c_0^e - (1-\beta)(a_1 - a_0)$, and $c_1^u = c_0^u - (1-\beta)(a_1 - a_0)$. Hence, if we let

$$\Delta := \alpha^{-1} \log\left(\frac{W_1}{W_0}\right),$$

and apply Lemma B.5 inductively, we see that $p_t = p_0$ for every t, $c_t^e = c_0^e - \Delta t$ and $c_t^u = c_0^u - \Delta t$.

The last part of the lemma follows immediately from the agent's first-order condition w.r.t. p_t and straightforward algebra. ■

Proof of Theorem 5.2. Let $\{(p_t^*, y_t^{e*}, c_t^{u*}, c_t^{e*})\}_{t=0}^{\infty}$ be the optimal allocation.

Notice that $W_0^* = p_0^* W_0^{e*} + (1 - p_0^*) W_0^{u*}$. If $W_0^{e*} \leq W_0^{u*}$, then $p_0^* = 0$. In this case, the optimal allocation can be implemented by assets

$$a_0 = \frac{-\alpha^{-1} \log\left(-\left(1-\beta\right) W_0\right)}{1-\beta}$$

and a some pair (y^e, T^e) with $y^e = T^e$. The worker best responds by never searching for a job and consuming $-\left(1-\beta\right)\alpha^{-1}\log\left(-\left(1-\beta\right)W_0\right)$ in every period. According to Lemma B.8 this is optimal.

Next, assume that $W_0^{e*} > W_0^{u*}$. Consider the first order condition:

$$-\frac{1}{1-\beta}\exp\left\{-\alpha\left[c_0^{e*} - \eta\left(y_0^{e*} + \kappa\left(\frac{\varrho(p_0^*)}{p_0^*}\right)\right)\right]\right\} - W_0^{u*} -$$
$$\frac{\alpha p_0^*}{1-\beta}\exp\left\{-\alpha\left[c_0^{e*} - \eta\left(y_0^{e*} + \kappa\left(\frac{\varrho(p_0^*)}{p_0^*}\right)\right)\right]\right\}\eta'\left(y_0^{e} + \kappa\left(\frac{\varrho(p_0^*)}{p_0^*}\right)\right)\kappa\left(\frac{\varrho(p_0^*)}{p_0^*}\right)' = 0,$$

and the following promise-keeping condition, $W_0^* = p_0^* W_0^{e*} + (1 - p_0^*) W_0^{u*}$.

By solving these two equations we obtain:

$$\begin{aligned}
\frac{W_0^{e*}}{W_0^*} &= \left[1 + \alpha p_0^*(1-p_0^*)\eta'\left(y_0^{e*} + \kappa\left(\frac{\varrho(p_0^*)}{p_0^*}\right)\right)\kappa\left(\frac{\varrho(p_0^*)}{p_0^*}\right)'\right]^{-1} \\
\frac{W_0^{u*}}{W_0^*} &= 1 + \frac{\alpha p_0^{*2}\eta'\left(n_0^{e*}\right)\kappa\left(\frac{\varrho(p_0^*)}{p_0^*}\right)}{1 + \alpha p_0^*(1-p_0^*)\alpha\eta'\left(n_0^{e*}\right)\kappa\left(\frac{\varrho(p_0^*)}{p_0^*}\right)}.
\end{aligned}$$

Next, notice that W_0^{e*} delivers c_0^{e*} by

$$-\frac{1}{1-\beta}\exp\left\{-\alpha\left[c_0^{e*} - \eta\left(y_0^{e*} + \kappa\left(\frac{\varrho(p_0^*)}{p_0^*}\right)\right)\right]\right\} = W_0^{e*},$$

which implies

$$c_0^{e*} = -\alpha^{-1}\log\left(-\left(1-\beta\right)W_0^{e*}\right) + \eta\left(y_0^{e*} + \kappa\left(\frac{\varrho(p_0^*)}{p_0^*}\right)\right).$$

We claim that there exists (a_0^*, T^{e*}) that solves the system:

$$c_0^{e*} = (1-\beta) a_0 + y_0^{e*} + T^e \tag{22}$$
$$W_0^{u*} = \max_c \left\{-\exp\{-\alpha c\} + \beta U\left(\beta^{-1}(a_0 - c), y_0^{e*}, T^e\right)\right\}, \tag{23}$$

where $U(a, y_0^{e*}, T^e)$ is the utility of an agent who starts a period unemployed and faces a simple policy, (a, y_0^{e*}, T^e).

If $T^e = y^e$ and $a_0 = \frac{c_0^{e*}}{1-\beta}$, then

$$W_1^* < \max_c \left\{-\exp\{-\alpha c\} + \beta U\left(\beta^{-1}(a_0 - c), y_0^{e*}, T^e\right)\right\}, \tag{24}$$

as the agent can keep consumption constant at c^{e*} even without taking a job.

The individual best responds to that contract by choosing $p = 0$ in every period. From this point, if we decrease a_0 by $-\frac{\varepsilon}{1-\beta}$ and decrease T^e by ε, the planner's payoff is increased by

$$\frac{\varepsilon}{1-\beta}\left(1 - \frac{p(a, y_0^{e*}, T^e)}{1-(1-p(a, y_0^{e*}, T^e))\beta}\right) > 0. \tag{25}$$

Next, notice that, by construction,

$$-\frac{1}{1-\beta}\exp\left\{-\alpha\left[(1-\beta)a_0 + y_0^{e*} + T^e - \eta\left(y_0^{e*} + \kappa\left(\frac{\varrho(p_0^*)}{p_0^*}\right)\right)\right]\right\} = W_0^{e*}.$$

Recall that the inequality (24) implies that $p(a, y_0^{e*}, T^e) < p^*$. We claim that, if we keep decreasing a_0 by $-\frac{\varepsilon}{1-\beta}$ and T^e by ε, we can generate (a_0^*, T^{e*}) satisfying (22) and (23). Otherwise, as we take a_0 to $-\infty$, the planner's revenue goes to infinity while the worker's utility at the beginning remains above W_0^*, a contradiction. From the first order condition, we know that p remains bounded below p^* (and by lemma B.5, this holds in every future period) and the principal obtains infinite profits because of (25). At the same time, the worker's utility remains greater than $pW_0^{e*} + (1-p)W_0^{u*}$, a contradiction.

The reasoning above shows that offering (a_0^*, y^{e*}, T^{e*}) in the first period is optimal to generate utility W_0^*. In this case, Lemma B.8 implies that (a_1^*, y^{e*}, T^{e*}) is optimal to generate utility W_1^*, where a_1^* is the asset holdings chosen by the agent. Inductively, we conclude that (a_t^*, y^{e*}, T^{e*}) is optimal to generate utility W_t^* for every t and hence the simple policy (a_0^*, y^{e*}, T^{e*}) is optimal. ■

Lemma B.9 *We have* $\frac{\partial p}{\partial c^e} > 0$ *and* $\frac{\partial p}{\partial y^e} < 0$.

Proof. We must calculate $\frac{\partial p}{\partial c^e}$ and $\frac{\partial p}{\partial y^e}$. Let $c^e := y^e - T^e$, assume without a loss that the agent starts with zero assets (Lemma B.5) and write W_1 for the payoff of an agent who starts a period of unemployment with zero assets. Start with the first order condition with respect to p:

$$-\frac{1}{1-\beta}\exp\left\{-\alpha\left[c^e-\eta\left(n^e\right)\right]\right\}-\max_{a'}\left[-\exp\left\{\alpha a'\beta\right\}+\beta W_1\exp\left\{-\alpha a'\left(1-\beta\right)\right\}\right]$$
$$\frac{-\alpha p}{1-\beta}\exp\left\{-\alpha\left[c^e-\eta\left(n^e\right)\right]\right\}\eta'\left(n^e\right)\kappa\left(\frac{\varrho(p)}{p}\right)'=0. \quad (26)$$

Next, we remark that the problem is strictly concave in p, and hence the derivative of (26) w.r.t. p is strictly negative. Differentiating this condition w.r.t. c^e we obtain

$$\frac{\alpha}{1-\beta}\exp\left\{-\alpha\left[c^e-\eta\left(n^e\right)\right]\right\}-\frac{d}{dc^e}\left[\max_{a'}\left[-\exp\left\{\alpha a'\beta\right\}+\beta W_1\exp\left\{-\alpha a'\left(1-\beta\right)\right\}\right]\right]$$
$$+\frac{\alpha^2 p}{1-\beta}\exp\left\{-\alpha\left[c^e-\eta\left(n^e\right)\right]\right\}\eta'\left(n^e\right)\kappa\left(\frac{\varrho(p)}{p}\right)'.$$

Now, notice that

$$\frac{d}{dc^e}\left[\max_{a'}\left[-\exp\left\{\alpha a'\beta\right\}+\beta W_1\exp\left\{-\alpha a'\left(1-\beta\right)\right\}\right]\right]<$$
$$-\alpha\max_{a'}\left[-\exp\left\{\alpha a'\beta\right\}+\beta W_1\exp\left\{-\alpha a'\left(1-\beta\right)\right\}\right], \quad (27)$$

where the last number is obtained by the derivative of an increase in c in every state of nature.

Therefore, we have

$$\begin{aligned}
&\frac{\alpha}{1-\beta}\exp\left\{-\alpha\left[c^{e}-\eta\left(n^{e}\right)\right]\right\}-\frac{d}{dc^{e}}\left[\max_{a'}\left[-\exp\left\{\alpha a'\beta\right\}+\beta W_{1}\exp\left\{-\alpha a'\left(1-\beta\right)\right\}\right]\right]\\
&\qquad+\frac{\alpha^{2}p}{1-\beta}\exp\left\{-\alpha\left[c^{e}-\eta\left(n^{e}\right)\right]\right\}\eta'\left(n^{e}\right)\kappa\left(\frac{\varrho(p)}{p}\right)'=\\
&\frac{\alpha}{1-\beta}\exp\left\{-\alpha\left[c^{e}-\eta\left(n^{e}\right)\right]\right\}+\alpha\left[\max_{a'}\left[-\exp\left\{\alpha a'\beta\right\}+\beta W_{1}\exp\left\{-\alpha a'\left(1-\beta\right)\right\}\right]\right]\\
&\qquad+\frac{\alpha^{2}p}{1-\beta}\exp\left\{-\alpha\left[c^{e}-\eta\left(n^{e}\right)\right]\right\}\eta'\left(n^{e}\right)\kappa\left(\frac{\varrho(p)}{p}\right)'\\
&\qquad-\alpha\left[\max_{a'}\left[-\exp\left\{\alpha a'\beta\right\}+\beta W_{1}\exp\left\{-\alpha a'\left(1-\beta\right)\right\}\right]\right]-\\
&\qquad\frac{d}{dc^{e}}\left[\max_{a'}\left[-\exp\left\{\alpha a'\beta\right\}+\beta W_{1}\exp\left\{-\alpha a'\left(1-\beta\right)\right\}\right]\right]=\\
&\qquad-\alpha\left[\max_{a'}\left[-\exp\left\{\alpha a'\beta\right\}+\beta W_{1}\exp\left\{-\alpha a'\left(1-\beta\right)\right\}\right]\right]-\\
&\qquad\qquad\frac{d}{dc_{e}}\left[\left[\max_{a'}\left[-\exp\left\{\alpha a'\beta\right\}+\beta W_{1}\exp\left\{-\alpha a'\left(1-\beta\right)\right\}\right]\right]\right]>0,
\end{aligned}$$

where we have used (26) and (27). Therefore, $\partial p/\partial c^{e}>0$.

Next, differentiating the first order condition with respect to y^{e}, we get

$$\begin{aligned}
&-\frac{\alpha\eta'(n^{e})}{1-\beta}\exp\left\{-\alpha\left[c^{e}-\eta\left(n^{e}\right)\right]\right\}-\frac{d}{dy^{e}}\left[\max_{a'}\left[-\exp\left\{\alpha a'\beta\right\}+\beta W_{1}\exp\left\{-\alpha a'\left(1-\beta\right)\right\}\right]\right]\\
&\qquad-\frac{\alpha^{2}p}{1-\beta}\exp\left\{-\alpha\left[c^{e}-\eta\left(n^{e}\right)\right]\right\}\eta'(n^{e})^{2}\kappa\left(\frac{\varrho(p)}{p}\right)'\\
&\qquad\qquad-\frac{\alpha p\eta''(n^{e})}{1-\beta}\exp\left\{-\alpha\left[c^{e}-\eta\left(n^{e}\right)\right]\right\}\eta'\left(n^{e}\right)\kappa\left(\frac{\varrho(p)}{p}\right)'.
\end{aligned}$$

Notice that

$$\begin{aligned}
&\frac{d}{dy_{e}}\left[\max_{a'}\left[-\exp\left\{\alpha a'\beta\right\}+\beta W_{1}\exp\left\{-\alpha a'\left(1-\beta\right)\right\}\right]\right]>\\
&\qquad\alpha\eta'(n^{e})\left[\max_{a'}\left[-\exp\left\{\alpha a'\beta\right\}+\beta W_{1}\exp\left\{-\alpha a'\left(1-\beta\right)\right\}\right]\right].
\end{aligned}\tag{28}$$

Hence,

$$-\frac{\alpha\eta'(n^e)}{1-\beta}\exp\left\{-\alpha\left[c^e-\eta\left(y^e+\kappa\left(\frac{\varrho(p)}{p}\right)\right)\right]\right\}-$$
$$\frac{d}{dy_e}\left[\max_{a'}\left[-\exp\left\{\alpha a'\beta\right\}+\beta W_1\exp\left\{-\alpha a'\left(1-\beta\right)\right\}\right]\right]$$
$$-\frac{\alpha^2p\eta'(n^e)}{1-\beta}\exp\left\{-\alpha\left[c^e-\eta\left(n^e\right)\right]\right\}\eta'\left(n^e\right)\kappa\left(\frac{\varrho(p)}{p}\right)'$$
$$-\frac{\alpha p\eta''(n^e)}{1-\beta}\exp\left\{-\alpha\left[c^e-\eta\left(n^e\right)\right]\right\}\eta'\left(n^e\right)\kappa\left(\frac{\varrho(p)}{p}\right)'=\frac{-\alpha\eta'\left(n^e\right)}{1-\beta}\exp\left\{-\alpha\left[c^e-\eta\left(n^e\right)\right]\right\}-$$
$$\alpha\eta'\left(n^e\right)\left[\max_{a'}\left[-\exp\left\{\alpha a'\beta\right\}+\beta W_1\exp\left\{-\alpha a'\left(1-\beta\right)\right\}\right]\right]$$
$$-\frac{\alpha^2p\eta'\left(n^e\right)}{1-\beta}\exp\left\{-\alpha\left[c^e-\eta\left(n^e\right)\right]\right\}\eta'\left(n^e\right)\kappa\left(\frac{\varrho(p)}{p}\right)'$$
$$\alpha\eta'(n^e)\left[\max_{a'}\left[-\exp\left\{\alpha a'\beta\right\}+\beta W_1\exp\left\{-\alpha a'\left(1-\beta\right)\right\}\right]\right]-$$
$$\frac{d}{dy_e}\left[\max_{a'}\left[-\exp\left\{\alpha a'\beta\right\}+\beta W_1\exp\left\{-\alpha a'\left(1-\beta\right)\right\}\right]\right]$$
$$-\alpha p\eta''\left(n^e\right)\frac{}{1-\beta}\exp\left\{-\alpha\left[c^e-\eta\left(n^e\right)\right]\right\}\eta'\left(n^e\right)\kappa\left(\frac{\varrho(p)}{p}\right)'=$$

$$\alpha\eta'(n^e)\left[\max_{a'}\left[-\exp\left\{\alpha a'\beta\right\}+\beta W_1\exp\left\{-\alpha a'\left(1-\beta\right)\right\}\right]\right]-$$
$$\frac{d}{dy_e}\left[\max_{a'}\left[-\exp\left\{\alpha a'\beta\right\}+\beta W_1\exp\left\{-\alpha a'\left(1-\beta\right)\right\}\right]\right]$$
$$\frac{-\alpha p}{1-\beta}\exp\left\{-\alpha\left[c^e-\eta\left(n^e\right)\right]\right\}\eta''\left(n^e\right)\eta'\left(n^e\right)\kappa\left(\frac{\varrho(p)}{p}\right)'<0,$$

where we have used (26) and (28). ■

Lemma B.10 *We have*

$$\sum_{t=0}^{\infty}p\beta^t\left(1-p\right)^{t-1}\left[1+\frac{\exp\left\{-\alpha\left(1-\beta\right)a_t\right\}}{W_0\left(1-\beta\right)}\exp\left\{-\alpha\left[c^e-\eta\left(n^e\right)\right]\right\}\right]>0.$$

Proof. We have

$$\sum_{t=0}^{\infty} p\beta^t (1-p)^{t-1} \left[\frac{-1}{1-\beta} - \frac{\exp\{-\alpha(1-\beta)a_t\}}{(1-\beta)^2 W_0} \exp\{-\alpha[c^e - \eta(n^e)]\}\right] < 0$$
$$\Leftrightarrow \sum_{t=0}^{\infty} p\beta^t (1-p)^{t-1} \left[-\frac{\exp\{-\alpha(1-\beta)a_t\}}{(1-\beta)\sum_{t=0}^{\infty} p\beta^t (1-p)^{t-1}} \exp\{-\alpha[c^e - \eta(n^e)]\}\right] > W_0,$$

since $z \to -\exp\{-\alpha z\}$ is strictly increasing. Notice that U_0 is the mixture of the distribution F^e over employed payoffs defined above and the distribution over $-\exp\{-\alpha c_t^u\}$, which we call F^u. It follows that if F^e first order stochastic dominates F^u. Hence for any $\lambda \in (0,1)$,

$$\int xd\left[\lambda F^e(x) + (1-\lambda)F^e(x)\right] < \int xdF^e(x).$$

It, therefore, suffices to show that $W_0 < \int xdF^e(x)$.

We have

$$W_0(1-\beta) = p(1-\beta)W_e^0 + (1-p)(1-\beta)\Big[-\exp\{-\alpha c_0^u\}+ \\ \beta\left[pW_1^e + (1-p)\left[-\exp\{-\alpha c_1^u\} + \beta W_2^u\right]\right]\Big].$$

Using $W_e^0 > -\frac{\exp\{-\alpha c_0^u\}}{1-\beta}$ and $-\frac{\exp\{-\alpha c_0^u\}}{1-\beta} = pW_1^e + (1-p)\left[-\exp\{-\alpha c_1^u\} + \beta W_2^u\right]$, we have

$$W_0 < \frac{pW_e^0 + \beta(1-p)\left[pW_1^e + (1-p)\left[\exp\{-\alpha c_1^u\} + \beta W_2^u\right]\right]}{1-(1-p)(1-\beta)}.$$

Proceeding analogously, it follows that the last expression is less than

$$\frac{pW_e^0 + \beta(1-p)\left[pW_1^e + (1-p)\beta W_2^u\right]}{1-(1-p)(1-\beta)-(1-p)^2\beta^2}.$$

Proceeding analogously and taking the limit, we obtain the desired inequality. ■

Proof of Theorem 5.3. Part (i). Recall from (11)

$$\frac{\partial}{\partial p}\left[\frac{p}{1-(1-p)\beta}\right]\left(\frac{y^e-c^e}{1-\beta}\right)\frac{\partial p}{\partial c^e} = \frac{p(y^e,c^e)}{1-(1-p(y^e,c^e))\beta} + \frac{U_{c^e}(y^e,c^e)}{e^{\alpha(1-\beta)a_0}\alpha(1-\beta)W_0}.$$

Since

$$\frac{\partial}{\partial p}\left[\frac{p}{1-(1-p)\beta}\right] > 0 \quad \text{and} \quad \frac{\partial p}{\partial c^e} > 0,$$

$y^e - c^e$ has the same sign as

$$-\sum_{t=0}^{\infty} p\beta^t (1-p)^{t-1} \left[\frac{-1}{1-\beta} - \frac{\exp\left\{-\alpha\left\{c_e - \eta\big(y^e + \kappa\varrho(p)/p\big)\right\}\right\}}{(1-\beta)^2 W_0}\right],$$

by Lemma B.9, which is strictly positive by Lemma B.10.

Part (ii). Consider the problem

$$C(W_0) = \max_{W_1, c_e, y_e} p\left[\frac{y^e - c_e}{1-\beta}\right] + (1-p)\,\beta C(e^{-\alpha a(1-\beta)} W_1),$$

subject to

$$-\frac{p}{1-\beta}\exp\left\{-\alpha\left[c_e - \eta\left(y^e + \kappa\frac{\varrho(p)}{p}\right)\right]\right\} + \\ (1-p)\max_{a'}\left\{-\exp\{\alpha a'\beta\} + \exp\{-\alpha a'(1-\beta)\}\beta W_1\right\} - W_0 = 0$$

and

$$-\frac{1}{1-\beta}\exp\left\{-\alpha\left[c_e - \eta\left(y^e + \kappa\frac{\varrho(p)}{p}\right)\right]\right\} \\ -\max_{a'}\left\{-\exp\{\alpha a'\beta\} + \exp\{-\alpha a'(1-\beta)\}\beta W_1\right\} \\ -\alpha p\frac{1}{1-\beta}\exp\left\{-\alpha\left[c_e - \eta\left(y^e + \kappa\frac{\varrho(p)}{p}\right)\right]\right\}\eta'\left(y^e + \kappa\frac{\varrho(p)}{p}\right)\kappa\left(\frac{\varrho(p)}{p}\right)' = 0.$$

Plugging the last constraint into the problem, one obtains the following Lagrangian

$$C(W_0) = \max_{W_1, c_e, y_e} p\left[\frac{y^e - c_e}{1-\beta}\right] + (1-p)\,\beta C(e^{-\alpha a(1-\beta)} W_1) + \\ \mu\Bigg[-\frac{1}{1-\beta}\exp\left\{-\alpha\left[c_e - \eta\left(y^e + \kappa\frac{\varrho(p)}{p}\right)\right]\right\} - \\ \alpha(1-p)\,p\frac{1}{1-\beta}\exp\left\{-\alpha\left[c_e - \eta\left(y^e + \kappa\frac{\varrho(p)}{p}\right)\right]\right\}\eta'\left(y^e + \kappa\frac{\varrho(p)}{p}\right)\kappa\left(\frac{\varrho(p)}{p}\right)' - W_0\Bigg].$$

Therefore, we have the first order conditions with respect to c^e,

$$p = \mu\alpha \exp\left\{-\alpha\left[c_e - \eta\left(y^e + \kappa\frac{\varrho(p)}{p}\right)\right]\right\}\left[1 + \alpha\left(1-p\right)p\eta'\left(y^e + \kappa\frac{\varrho(p)}{p}\right)\kappa\left(\frac{\varrho(p)}{p}\right)'\right],$$

and with respect to y^e,

$$\begin{aligned} p = \mu\alpha \exp\left\{-\alpha\left[c_e - \eta\left(y^e + \kappa\frac{\varrho(p)}{p}\right)\right]\right\}&\left[\eta'\left(y^e + \kappa\frac{\varrho(p)}{p}\right) + \right. \\ &\left. \alpha\left(1-p\right)p\eta'\left(y^e + \kappa\frac{\varrho(p)}{p}\right)^2\kappa\left(\frac{\varrho(p)}{p}\right)'\right] \\ + \mu\left(1-p\right)p\exp\left\{-\alpha\left[c_e - \eta\left(y^e + \kappa\frac{\varrho(p)}{p}\right)\right]\right\}&\eta''\left(y^e + \kappa\frac{\varrho(p)}{p}\right)\kappa\left(\frac{\varrho(p)}{p}\right)'. \end{aligned}$$

Therefore, we have

$$\eta'\left(y^e + \kappa\frac{\varrho(p)}{p}\right) = 1 - \frac{\left(1-p\right)p\eta''\left(y^e + \kappa\varrho(p)/p\right)\kappa\left(\frac{\varrho(p)}{p}\right)'}{\alpha\left[1 + \alpha\left(1-p\right)p\eta'\left(y^e + \kappa\varrho(p)/p\right)\kappa\left(\frac{\varrho(p)}{p}\right)'\right]}.$$

Part (iii). Notice that $W_t^* < W_t^{e*}$ and hence it suffices to show that $\lim W_t^{e*} = -\infty$. We have

$$\begin{aligned} &\lim(1-\beta)W_t^{e*} = \\ &\quad -\lim\exp\left\{-\alpha\left[c_0^* + \left(1-\beta\right)\bar{a}_0 - \eta\left[y^{e*} + \kappa\left(\frac{\varrho(p^*)}{p^*}\right)\right]\right]\alpha\left(t-1\right)\Delta_c\right\} = -\infty. \end{aligned}$$

■

C Extension: GHH-CARA type and Observable Savings

In this section, we consider the case of observable savings with period utility of the form

$$\mathcal{U}(c,n) = -\exp\left\{-\alpha\left[c-\eta(n)\right]\right\}.$$

We can write the Lagrangean as

$$C\left(W_0\right) = \max \frac{p}{1-\beta}\left(y^e - c^e\right) + (1-p)\left[-c^u + \beta C\left(W_1\right)\right],$$

subject to

$$\frac{p}{1-\beta}\left[-\exp\left\{-\alpha\left[c^e - \eta(n^e)\right]\right\}\right] + (1-p)\left[-\exp\left\{-\alpha\left[c_u\right]\right\} + \beta W_1\right] - W_0 \geq 0,$$

and

$$\begin{aligned}\frac{1}{1-\beta}&\left[-\exp\left\{-\alpha\left[c-\eta\left(y^e+\kappa\frac{\varrho(p)}{p}\right)\right]\right\}\right] + \exp\left\{-\alpha\left[c_u\right]\right\} - \beta W_1 \\ &= \frac{1}{1-\beta}\alpha\eta'\left(y^e+\kappa\frac{\varrho(p)}{p}\right)\left(\frac{\kappa\varrho(p)}{p}\right)'\exp\left\{-\alpha\left[c-\eta\left(y^e+\kappa\frac{\varrho(p)}{p}\right)\right]\right\}.\end{aligned}$$

Let

$$\begin{aligned}U^e &:= \exp\left\{-\alpha\left[c-\eta\left(y^e+\kappa\frac{\varrho(p)}{p}\right)\right]\right\} \\ U^u &:= \exp\left\{-\alpha c^u\right\}.\end{aligned}$$

The first order condition for c_0^e is

$$-p + \mu p \alpha U^e + \lambda \alpha U^e = 0.$$

The first order condition for c_0^u is

$$-\left(1-p\right) + \left(1-p\right)\mu U^u - \lambda \alpha U^u = 0.$$

The first order condition for y^e is

$$p - p\mu\alpha U^e \eta'(n^e) - \lambda\alpha U^e \eta'(n^e)$$
$$-\lambda U^e \left[\alpha\eta''(n^e)\kappa \left(\frac{\varrho(p)}{p}\right)' + \alpha^2 U^e \left[\eta'(n^e)\right]^2 \kappa \left(\frac{\varrho(p)}{p}\right)' \right] = 0$$

From these, we have

$$U^e = \frac{p}{\mu p\alpha + \lambda\alpha}$$
$$U^u = \frac{1-p}{\mu(1-p)\alpha - \lambda\alpha}$$
$$p - \eta'(n^e) U^e \left[p\mu\alpha + \lambda\alpha\right] = \lambda U^e \left[\alpha\eta''(n^e)\kappa \left(\frac{\varrho(p)}{p}\right)' + \alpha^2 U^e \left[\eta'(n^e)\right]^2 \kappa \left(\frac{\varrho(p)}{p}\right)' \right]$$
$$1 - \eta'(n^e) = \frac{\lambda U^e}{p} \left[\alpha\eta''(n^e)\kappa \left(\frac{\varrho(p)}{p}\right)' + \alpha^2 \left[\eta'(n^e)\right]^2 \kappa \left(\frac{\varrho(p)}{p}\right)' \right] \tag{29}$$
$$C'(W_1) = -\mu_0 + \frac{\lambda_0}{(1-p_0)} = -\mu_1,$$

which implies

$$\mu_1 = \mu_0 - \frac{\lambda_0}{(1-p)}.$$

Moreover, the derivative with respect to p implies

$$\frac{p}{1-\beta}(y^e - c^e) = \frac{\lambda\alpha U^e}{1-\beta}\Bigg[\eta'(n^e)\left(\kappa\frac{\varrho(p)}{p}\right)' + \eta''(n^e)\left(\frac{\kappa\varrho(p)}{p}\right)' + \eta'(n^e)\left(\kappa\frac{\varrho(p)}{p}\right)'' + \left[\eta'(n^e)\left(\kappa\frac{\varrho(p)}{p}\right)'\right]^2 \Bigg]. \tag{30}$$

Lemma C.1 *The multipliers μ and λ are strictly positive if there is search.*

Proof. First notice that

$$U_0^u = \frac{1-p_0}{\mu_0(1-p_0)\alpha - \lambda_0\alpha}$$
$$U_0^e = \frac{p_0}{\mu_0 p_0 \alpha + \lambda_0\alpha}$$

hence $\mu_0 = 0$ implies $U_0^u U_0^e \leq 0$, which is an absurd.

Now, assume towards a contradiction that $\lambda_0 \leq 0$. Clearly, there is a last period at which

$\lambda_t \leq 0$ and $\lambda_{t+1} > 0$. Otherwise, as we will verify below, $c_t^u \geq c_t^e$ for every t, and hence there is no search. Assume that $\lambda_1 > 0$ (the case in which $\lambda_s \leq 0$ for all $s < t$ and $\lambda_t > 0$ for some $t > 1$ can be analogously handled).

From the first-order conditions, we have:

$$-U_1^e > -U_0^u \geq -U_0^e. \tag{31}$$

Since the agent searches with positive probability, we have

$$\frac{-U_0^e}{1-\beta} > -U_0^u + \beta \left[p_1 \frac{-U_1^e}{1-\beta} + (1-p_1) \left[-U_1^e + \beta W_2 \right] \right] \tag{32}$$

$$\frac{-U_1^e}{1-\beta} > -U_1^u + \beta W_2. \tag{33}$$

Notice that

$$(U_1^e + \beta W_2) < \frac{U_1^e}{1-\beta}$$

$$(U_1^e + \beta W_2) < \frac{U_0^u}{1-\beta}.$$

Therefore

$$U_1^e + \beta W_2 < U_0^u + \beta \left[p_1 \left(\frac{U_1^e}{1-\beta} \right) + (1-p_1) \left(U_1^e + \beta W_2 \right) \right] \tag{34}$$

$$\frac{y_0^e - c_0^e}{1-\beta} \leq -c_0^u + \beta C\left(W_1\right). \tag{35}$$

$$\frac{y_1^e - c_1^e}{1-\beta} > -c_1^u + \beta C\left(W_2\right). \tag{36}$$

Using (29) and $\lambda_0 \leq 0 < \lambda_1$, we obtain

$$y_1^e + \kappa \frac{\varrho(p_1)}{p_1} < y_0^e + \kappa \frac{\varrho(p_0)}{p_0}. \tag{37}$$

Using (31), (34) and the agent's first order condition w.r.t. p, we obtain

$$\begin{aligned}\frac{1}{1-\beta}\alpha\eta'\left(y_1^e+\kappa\frac{\varrho(p_1)}{p_1}\right)\left(\frac{\kappa\varrho(p_1)}{p_1}\right)'U_1^e &= \frac{-U_1^e}{1-\beta}-\left[-U_1^e+\beta W_2\right]\\ &> \frac{-U_0^e}{1-\beta}-\left[U_0^u+\beta\left[p_1\left(\frac{U_1^e}{1-\beta}\right)+(1-p_1)\left(U_1^e+\beta W_2\right)\right]\right]\\ &= \frac{1}{1-\beta}\alpha\eta'\left(y_0^e+\kappa\frac{\varrho(p_0)}{p_0}\right)\left(\frac{\kappa\varrho(p_0)}{p_0}\right)'U_0^e,\end{aligned}$$

and therefore

$$\eta'\left(y_1^e+\kappa\frac{\varrho(p_1)}{p_1}\right)\left(\frac{\kappa\varrho(p_1)}{p_1}\right)'U_1^e > \eta'\left(y_0^e+\kappa\frac{\varrho(p_0)}{p_0}\right)\left(\frac{\kappa\varrho(p_0)}{p_0}\right)'U_0^e.$$

Since $-U_1^u > -U_0^e$, we have $U_1^u < U_0^e$ and hence

$$\eta'\left(y_1^e+\kappa\frac{\varrho(p_1)}{p_1}\right)\left(\frac{\kappa\varrho(p_1)}{p_1}\right)' > \eta'\left(y_0^e+\kappa\frac{\varrho(p_0)}{p_0}\right)\left(\frac{\kappa\varrho(p_0)}{p_0}\right)'. \tag{38}$$

We claim that (37) and (38) imply $y_1 < y_0$ and $p_1 > p_0$. If $y_1 \geq y_0$, then (37) implies $p_1 > p_0$. These imply

$$\eta'\left(y_1^e+\kappa\frac{\varrho(p_1)}{p_1}\right)\left(\frac{\kappa\varrho(p_1)}{p_1}\right)' < \eta'\left(y_0^e+\kappa\frac{\varrho(p_0)}{p_0}\right)\left(\frac{\kappa\varrho(p_0)}{p_0}\right)',$$

a contradiction. Thus $y_1 < y_0$ and using (37), we also have $p_1 > p_0$. Hence

$$y_1 < y_0 \tag{39}$$

and

$$p_1 > p_0 \tag{40}$$

$$U_0^u \geq U_0^e.$$

Finally, we have

$$\begin{aligned}\frac{y_0^e - c_0^e}{1-\beta} &\leq -c_0^u + \beta C(W_1) \\ &= -c_0^u + \beta \left[p_1 \frac{y_1^e - c_1^e}{1-\beta} + (1-p_1)\left[-c_1^u + \beta C(W_2)\right] \right] \\ &< -c_0^u + \beta \left[p_1 \frac{y_1^e - c_1^e}{1-\beta} + (1-p_1)\left[\frac{y_1^e - c_1^e}{1-\beta}\right] \right] = -c_0^u + \beta \frac{y_1^e - c_1^e}{1-\beta}. \end{aligned} \tag{41}$$

To finish the proof, we consider two cases.

Case 1: $y_0^e - c_0^e \geq y_1^e - c_1^e$ **or** $y_0^e - c_0^e < y_1^e - c_1^e$ **and** $y_1^e - c_1^e < -c_0^u$.
We claim that the planner and the agent are better off if the planner pays constant unemployment insurance equal to c_0^u in each period. In response, the agent never searches. First, (31) and (32) imply that the agent is better off.

To see that the planner is better off, notice that (41) imply that

$$-c_0^u > \max\left\{y_0^e - c_0^e, y_1^e - c_1^e\right\}.$$

This and (36) imply

$$-c_0^u > p_0\left(\frac{y_0^e - c_0^e}{1-\beta}\right) + (1-p_0)\left[-c_0^u + \beta\left[\begin{array}{c} p_1\frac{y_1^e - c_1^e}{1-\beta} \\ +(1-p_1)\left[-c_1^u + \beta C(W_2)\right]\end{array}\right]\right].$$

Case 2: $y_0^e - c_0^e < y_1^e - c_1^e$ **and** $y_1^e - c_1^e \geq -c_0^u$.

Consider a deviation in which the planner gives (y_1^e, c_1^e) to the agent in the first period.

Let $\tilde{p}_0$ be the best response of the agent. The planner's payoff is

$$\begin{aligned}&\tilde{p}_0\left(\frac{y_1^e - c_1^e}{1-\beta}\right) + (1-\tilde{p}_0)\left[-c_0^u + \beta\left[p_1\frac{y_1^e - c_1^e}{1-\beta} + (1-p_1)\left[-c_1^u + \beta C(W_2)\right]\right]\right] \\ &\qquad > -c_0^u + \beta\left[p_1\frac{y_1^e - c_1^e}{1-\beta} + (1-p_1)\left[-c_1^u + \beta C(W_2)\right]\right] \\ &\qquad > p_0\left(\frac{y_0^e - c_0^e}{1-\beta}\right) + (1-p_0)\left[-c_0^u + \beta\left[p_1\frac{y_1^e - c_1^e}{1-\beta} + (1-p_1)\left[-c_1^u + \beta C(W_2)\right]\right]\right],\end{aligned}$$

where the last line uses (35). To show that the agent is better off, notice that p_1 is available and yields

$$
\begin{aligned}
&p_1 \frac{U_1^e}{1-\beta} + (1-p_1)\left(U_0^u + \beta\left[p_1 \frac{U_1^e}{1-\beta} + (1-p_1)\left[U_1^e + \beta W_2\right]\right]\right) \\
&\quad > p_1 \frac{U_0^e}{1-\beta} + (1-p_1)\left(U_0^u + \beta\left[p_1 \frac{U_1^e}{1-\beta} + (1-p_1)\left[U_1^e + \beta W_2\right]\right]\right) \\
&\quad\quad > p_0 \frac{U_0^e}{1-\beta} + (1-p_0)\left(U_0^u + \beta\left[p_1 \frac{U_1^e}{1-\beta} + (1-p_1)\left[U_1^e + \beta W_2\right]\right]\right),
\end{aligned}
$$

which completes the proof. ■

Recall that

$$\phi = \lim_{p \downarrow 0} \varrho(p)/p > 0.$$

We make the following assumption (otherwise working is always inefficient):

Assumption DS (desirable search) Search is desirable,

$$\max y - \eta\,(y + \kappa\phi) > 0.$$

Let

$$y^* = \arg\max y - \eta\,(y + \kappa\phi)$$

or

$$1 = \eta'\,(y^* + \kappa\phi)$$

or

$$y^* = (\eta')^{-1}\,(1) - \kappa\phi.$$

Lemma C.2 *There is positive search in every period.*

Proof. Assume towards a contradiction that there is no search at period t. Let W_t be the agent's utility at t. We have

$$W_t = \mathbb{E}\left[-\sum_{t=0}^{\infty} \beta^t \exp\left(-\alpha c_t + \alpha\eta\,(y_t + \kappa\varrho(p)/p)\right)\right].$$

Let $\chi\,(W)$ be given by

$$-\frac{\exp\left(-\alpha\chi\,(W)\right)}{1-\beta} = W,$$

or

$$\chi\,(W) = -\frac{\log\left(-W\,(1-\beta)\right)}{\alpha}.$$

Let $c(W)$ be given by

$$c(W) = \eta(y^* + \kappa\phi) + \chi(W).$$

Since $x \to -\exp(-\alpha x)$ is strictly concave, it follows that

$$C(W) > \frac{\eta(y^* + \kappa\phi) + \chi(W) - y^*}{1-\beta}.$$

From Assumption DS, there is an $\varepsilon > 0$ such that if the planner demands production y^* in exchange for consumption $\eta(y^* + \kappa\phi) + \chi(W) + \varepsilon$ at period t then the agent searches for a job at t and conditionally on finding a job and both players are better off. If $t > 0$ then the planner can decrease the unemployment insurance at $t-1$ to keep the agent indifferent. This decreases the planner's cost and establishes a contradiction. ■

Theorem C.1 *At the optimum, in every period in which there is a positive search,*

1. *the moral-hazard constraint (43) binds, and the planner benefits from strictly increasing p;*

2. *the marginal income tax rate is always positive, and;*

3. *conditional on not finding a job at period t, the worker's marginal utility of consumption satisfies the inverse Euler equation,*

$$\frac{1}{\varphi'(c_t^u)} = \mathbb{E}\left[\frac{1}{\varphi'(c_{t+1})}\right].$$

Proof. i) The fact that the moral-hazard constraint (43) binds, and the planner benefits from strictly increasing p follows immediately from Lemma C.1.

ii) From (29) and $\lambda > 0$, we have

$$1 - \eta'(n^e) = \frac{\lambda U^e}{p}\left[\alpha\eta''(n^e)\kappa\left(\frac{\varrho(p)}{p}\right)' + \alpha^2\left[\eta'(n^e)\right]^2\kappa\left(\frac{\varrho(p)}{p}\right)'\right] > 0.$$

iii) Using the first-order conditions, we have

$$\frac{1}{\varphi'(c_{t-1}^u)} = \mu_{t-1} - \frac{\lambda_{t-1}}{(1-p_{t-1})} = \mu_t.$$

Hence,

$$\frac{p_t}{\varphi'(c_t^e)} + \frac{1-p_t}{\varphi'(c_t^u)} = (p_t\mu_t + \lambda_t) + (\mu_t(1-p_t) - \lambda_t) = \mu_t = \frac{1}{\varphi'(c_{t-1}^u)}$$

■

Proposition C.1 *The unemployment benefit is decreasing over time with $c_t^u > c_{t+1}^u$.*

Moreover, the worker's consumption from employment at period t is strictly greater than the unemployment benefit from any future period $\tau \geq t$.

Proof.

Notice that $\lambda_{t+1} > 0$ implies

$$\varphi'(c_{t+1}^u) - \varphi'(c_t^u) = \frac{1}{\mu_{t+1} - \frac{\lambda_{t+1}}{1-p_{t+1}}} - \frac{1}{\mu_t - \frac{\lambda_t}{1-p_t}} = \frac{1}{\mu_{t+1} - \frac{\lambda_{t+1}}{1-p_{t+1}}} - \frac{1}{\mu_{t+1}} > 0,$$

and hence $c_{t+1}^u < c_t^u$.

Next, notice that

$$U^u - U^e = \frac{1}{\mu\alpha - \frac{\lambda\alpha}{1-p}} - \frac{1}{\mu\alpha + \lambda\alpha p^{-1}} > 0,$$

which implies $c_t^e > c_t^u + \eta(n_t^e) > c_t^u$. ■

Proposition C.2 *Unemployment benefits converge to zero.*

Proof. Suppose towards a contradiction that $c_t^u \to c_\infty^u > 0$. If it does not converge to zero, since $\varphi'(c_t^u) = \frac{1}{\mu_{t+1}}$, we conclude that $\mu_t \to (\varphi'(c_\infty^u))^{-1}$. Therefore,

$$\frac{\lambda_t}{1-p_t} \to 0.$$

We claim that $p_t \to 0$. Suppose towards a contradiction that there is a subsequence $p_{t_r} \to \hat{p} > 0$ and notice that, since

$$\varphi'(c_t^e) = \frac{1}{\mu_t + p_t^{-1}\lambda_t},$$

we have along the subsequence $\varphi'(c_{t_r}^e) \to \varphi'(c_\infty^u)$, implying $c_{t_r}^e \to c_\infty^u$. By incentive compatibility,

$$\eta\left(y_{t_r}^e + \kappa\frac{\varrho(p_{t_r})}{p_{t_r}}\right) \to 0,$$

which is not possible, a contradiction.

But then, by a continuity argument, for every $\varepsilon > 0$, there exists a period t^* such that $t \geq t^*$ implies that the planner's utility is ε away from $-c^u_\infty/(1-\beta)$, while the worker's utility is ε away from $\varphi(c^u_\infty)/(1-\beta)$.

It follows by Assumption DS that there is $\varepsilon > 0$ such that, if the planner demands production y^* in exchange for consumption $\eta\,(y^* + \kappa\phi) + \chi\,(\varphi(c^u_\infty)/(1-\beta)) + \varepsilon$, then the worker searches with probability bounded away from some $\underline{p} > 0$ for every t large enough. Moreover, this ε can be chosen to make both players better off, a contradiction. ■

D Variable Effort and Amenities

Thus far we have talked about amenities suggesting that their supply plays an analogous role to effort requirements. In reality both dimensions will simultaneously help define what a desirable job is. In this extension, we *add* amenities to the one-period model explicitly connecting it to the effort model we have presented.

Assume that amenities cost a to the firm and lead to a benefit $\phi\,(a)$ by making the working environment more pleasant. We can write the problem as

$$C\,(W_0) = \max \frac{p}{1-\beta}\,(y^e - c^e) - (1-p)c_u$$

subject to[12]

$$p\left[\varphi(c^e) - \eta\left(y^e - \phi\,(a) + (\kappa + a)\,\frac{\varrho(p)}{p}\right)\right] + (1-p)\,c(u) \geq 0, \tag{42}$$

and

$$\left[\varphi(c^e) - \eta\left(y^e - \phi\,(a) + (\kappa + a)\,\frac{\varrho(p)}{p}\right)\right] - c(\underline{u}) = \\ p\,(\kappa + a)\,\eta'\left(y^e - \phi\,(a) + (\kappa + a)\,\frac{\varrho(p)}{p}\right)\left(\frac{\varrho(p)}{p}\right)'. \tag{43}$$

The efficient level of amenities is the solution for

$$a^{fb} := \underset{a}{\operatorname{argmax}}\left\{\phi\,(a) + a\frac{\varrho(p)}{p}\right\}.$$

[12]We are assuming that the cost of amenities must be paid regardless of whether the vacancy is filled.

This implies

$$a^{fb}(p) := (\phi')^{-1}\left(\frac{\varrho(p)}{p}\right),$$

which is decreasing in p.

Suppose the government can choose a. The first order condition implies

$$\begin{aligned}&\left[(\mu+p\lambda)\,\eta'\left(y^e-\phi(a)+(\kappa+a)\,\frac{\varrho(p)}{p}\right)\right.\\&\qquad\left.+\lambda p\,(\kappa+a)\,\eta''\left(y^e-\phi(a)+(\kappa+a)\,\frac{\varrho(p)}{p}\right)\left(\frac{\varrho(p)}{p}\right)'\right]\times\left[\phi'(a)-\frac{\varrho(p)}{p}\right]\\&\qquad\qquad=p\lambda\eta'\left(y^e-\phi(a)+(\kappa+a)\,\frac{\varrho(p)}{p}\right)\left(\frac{\varrho(p)}{p}\right)'\end{aligned}$$

Therefore, the optimal policy implies $\phi'(a) > \varrho(p)/p$, a positive wedge on the optimal level of amenities. The positive wedge on amenities arises whether the effort is another intensive adjustment margin or not.