Inter-American Development Bank

Department of Research and Chief Economist

TECHNICAL NOTE

No. IDB-TN-576

Integrated Template for Debt Sustainability Analysis:

Version 2.0, Instruction Manual, Revised Version

Eduardo Borensztein
Eduardo Cavallo
Paula Cifuentes
Oscar Valencia

September 2013

Integrated Template for Debt Sustainability Analysis:

Version 2.0, Instruction Manual, Revised Version

Eduardo Borensztein
Eduardo Cavallo
Paula Cifuentes
Oscar Valencia

Inter-American Development Bank

2013

Cataloging-in-Publication data provided by the
Inter-American Development Bank
Felipe Herrera Library

Integrated template for debt sustainability analysis: version 2.0, instruction manual, revised version / Eduardo Borensztein, Eduardo Cavallo, Paula Cifuentes, Oscar Valencia.
p. cm. — (IDB Technical Note ; 576)
Includes bibliographic references.
1. Debts, Public—Handbooks, manuals, etc. I. Borensztein, Eduardo. II. Cavallo, Eduardo A. III. Cifuentes, Paula. IV. Valencia, Oscar. V. Inter-American Development Bank. Research Dept. VI. Series.
IDB-TN-576

http://www.iadb.org

Abstract

Instruction Manual for Version 2.0 of the IDB Debt Sustainability Template. The updated Template includes a new interface, more analytical functions, and greater flexibility for the researcher.

JEL classifications: H63, H68
Keywords: Debt sustainability, Fiscal forecasts, Debt management, Public debt, Fan charts

This template has been developed in a joint effort by the Research Department (RES) and the offices of the IDB Regional Economic Advisors (REAs).

The project was coordinated and supervised by Eduardo Cavallo (RES) and Eduardo Borensztein (CSC).

Version 2.0 of the IDB Debt Sustainability Template (updated to June 2013) includes a new interface, more analytical functions, and greater flexibility for the researcher.

The programming was the responsibility of Oscar M. Valencia, Consultant (RES).

This manual was written by Paula Cifuentes, Research Fellow (RES), and Oscar M. Valencia, Consultant (RES).

The following people collaborated on this new stage of the project with a range of comments and suggestions were: Leopoldo Avellan (CAN), Bernardita Piedrabuena (CSC), Leandro Andrián (CAN), and Luis Alejos Marroquín (CID).

Contents

Introduction

Debt Sustainability Analysis (DSA) is a key component of macroeconomic supervision in many countries of Latin America and the Caribbean (LAC). Exposure to risks of debt crisis in the LAC countries is not insignificant, at least in the perception of international financial markets and rating agencies because, when there is a debt crisis, they tend to be costly and their consequences far reaching. In fact, the economic costs of suspension of public debt payments are difficult to estimate accurately, largely because there are many channels and the impact is not always the same (Borensztein and Panizza, 2009). Debt crises are often associated with economic recessions, political crises and institutional degradation, and leave a scar in financial markets at home and the sovereign reputation abroad. Thus, controlling the robustness of LAC economies often requires a careful assessment of the sustainability of the sovereign debt. The IDB is after all a creditor of LAC governments and, despite its special status as a multilateral organization, has to exercise appropriate oversight of the soundness of the finances of its borrowers.

The template with different approaches presented in this paper deals with evaluation of the sustainability of **total government debt,** regardless of whether the creditor is a national or foreign resident, public or private entity or of the jurisdiction where the debt was generated (in the case of debt in bonds). Attention is focused on total government debt, rather than the traditional measure of external debt, in an effort to reflect the evolution of debt management and financial markets over the last two decades. After the revival of the international bond market for LAC with the introduction of "Brady Bonds" in the 1990s, a global market emerged where investors were both national and foreign residents. More recently domestic bond markets have gained importance, and in this case the investor base consists of global institutions and individuals. The prices of instruments reveal a high degree of integration between instruments issued locally and internationally, confirming that aggregate public debt is the relevant measure for focusing the analysis. Moreover, private companies that borrow abroad do so with complete independence from government, unlike the way in which this type of borrowing operated in the 1970s.

The concept of sustainability can be defined in several ways and, in fact, the different approaches to DSA included in the template presented here cover these different aspects. In general terms, debt is considered sustainable when its value as a proportion of GDP follows a

stable path over time. A variant of this concept is evaluation of whether the debt has reached a level beyond the government's capacity to pay, measured as the primary surplus that the government has historically been able to generate. Other approaches emphasize the risks and calculate the probability that specific adverse shocks can raise debt to levels that exceed service capacity or fall into a negative spiral over time.

This template opens access to five different models which evaluate debt sustainability under different approaches. The five DSA models included in the template are: 1) Standard Approach, 2) Endogenous Debt Dynamics, 3) Sudden Stop, 4) Fan Charts, and 5) Natural Debt Limit. The five models are described below in more detail.

The first model, called here **"Standard Approach,"** estimates the level of primary surplus (budget surplus, excluding interest payments on the debt) required to maintain the current debt to GDP ratio constant over the long term. The calculation is based on the expected long-term average levels of the growth rate and the real interest rate, and it is understandably sensitive to those assumptions. The second model - **" Endogenous Debt Dynamics" -** traces the path of the debt/GDP ratio over time in terms of its direct determinants, such as primary surplus, interest rate, exchange rate and growth rate. This model takes a medium-term approach, usually five or ten years, and allows temporary changes in the determinants of the path of the debt. It is also possible to test the sensitivity of the debt path to these assumptions. The third model - **"Fan Chart"** - incorporates the important fact that the future path of the direct determinants of the evolution of the debt - such as interest rates, growth rates, etc. - are uncertain and this uncertainty extends to the path of the relation between debt and GDP. In its most complete version, the model uses a statistical model (autoregressive vectors) to estimate both the variance and correlation between the determining factors of the debt. Instead of projecting a single debt value for each year, the model produces a range of values that the debt/GDP ratio could reach with different probabilities attached to them. This range widens with the projections for future periods because uncertainty is greater, consequently the path of the debt/GDP ratio creates a "fan" on the charts. The fourth model - **"Sudden Stop"** - provides estimates of the impact on debt sustainability of a sudden stop in the capital account, namely, a situation where all external financing freezes. This type of event evokes episodes such as the Tequila crisis and the Russian-Asian crisis, for example, and to much lesser extent the subprime crisis (or mortgage

market crisis). The model considers the different channels through which a sudden stop can affect debt sustainability, including the exchange rate channel, which has a particularly strong effect when a large part of the debt is denominated in foreign currency. The fifth model - **"Natural Debt Limit"** - calculates the maximum debt level that a government would be able to service with absolute confidence. Based on the historical variability of tax revenue and expenditure, the model finds a debt level that the government will always be able to service, even in the worst case scenario of its fiscal situation. This level, where the probability of default is zero, is the debt level that can be compared with current levels to evaluate its sustainability.

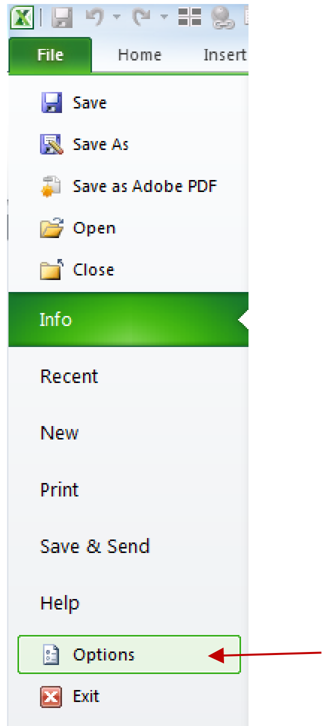
The following sections explain in more detail the five tools for debt sustainability analysis and offer the user a guide on the processes and the technical platform for applying these tools in specific cases in each country. The guide explains the requirements for data entry and presentation of the results for each tool, along with the procedures to modify or enrich the models and make them more consistent with the debt situation of a particular country. In addition, the new variants and improvements to these DSA models suggested by the Bank's economists have been included in this version.

Using the DSA template[1]

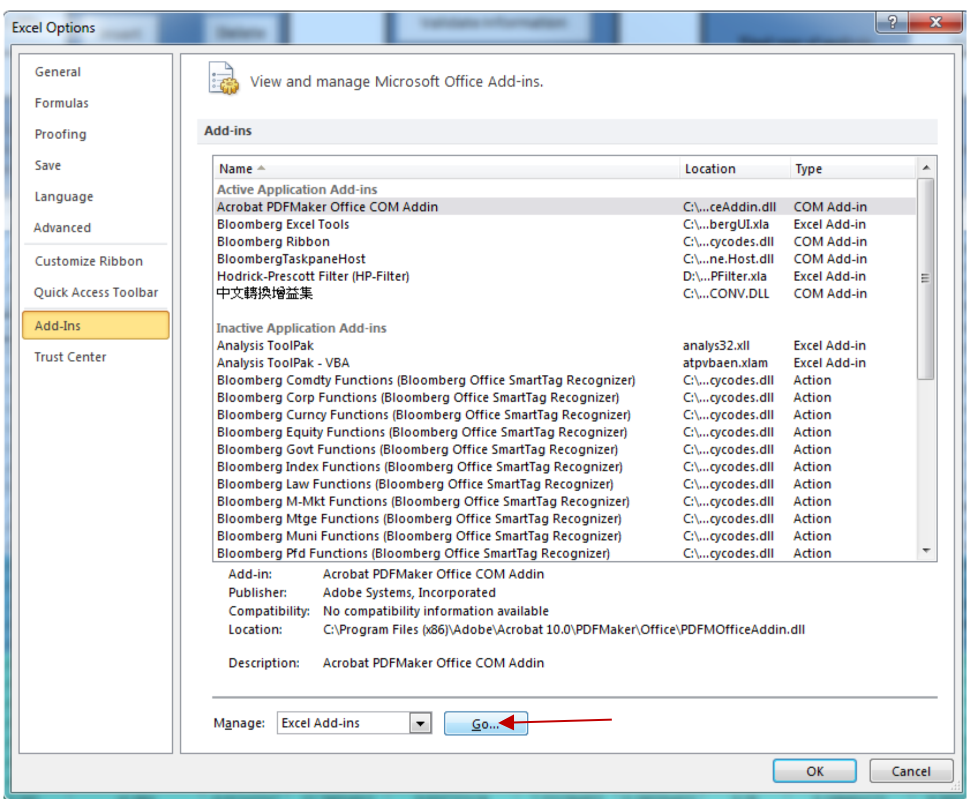
The DSA template was designed to be operated in a version of Excel 2010 VB6, otherwise it could generate errors or malfunction. Additionally, the user must ensure that the add-in of the Hodrick-Prescott filter is installed in Excel. To install the following protocol must be followed:

- Download the **HPFilter.xla** file from the web page where the DSA template was downloaded.
- Open Excel and go to the tools *"File"* tab and select the *"Options"* tab which will display a dialog box.

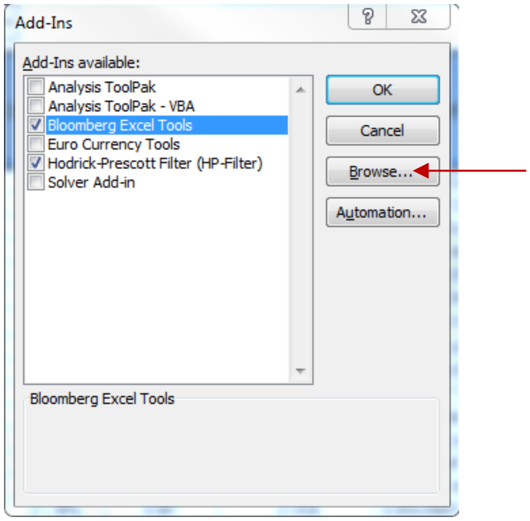
[1] In the case of a technical fault in the template the user must reopen it.



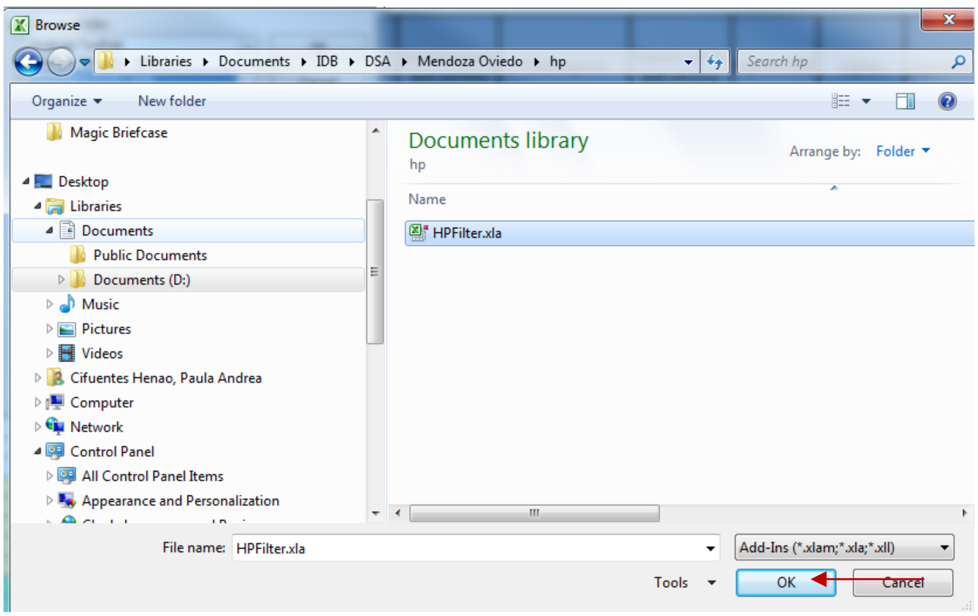
- In this dialog box select the *"Add-Ins"* tab and click on the *"Go"* box at the bottom in *"Manage"*



- A dialog box opens where you click on *"Browse."*



- The user searches for the **HPFilter.xla** file in the folder in which it was previously saved and clicks OK to install it and initiate the use of the DSA template.



- After installation, the HP-Filter is an HP command function (time series such as Range, lambda as Double). The user does not need to execute the command because the template automatically does so when starting the analysis through the *"Star Analysis"* button which will be explained later. However, the user should know that

the "standard" for the Lambda values is 100 for annual data, 1600 for quarterly data and 14,400 for monthly data. Therefore, the filter calculated by the template relates to a Lambda=100 for annual data.

1. Input of general data on the "*Selected Series*" spreadsheet

The *"Selected Series"* spreadsheet is designed with the objective of:

- Allowing the user to enter the data inputs required for the debt sustainability analysis.
- Defining the range of analysis according to the specifics of the case under study and the user's criterion.
- Validating the data entered on the spreadsheet.

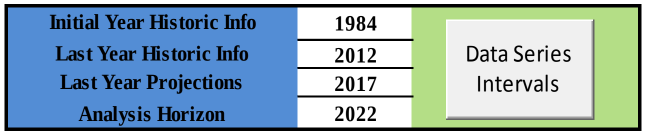
The *"Selected Series"* sheet features 5 major fields of data summarized as:

a. Specifying the interval of the data series
b. Entering the data series
c. Adding data columns
d. Validating the data entered
e. Defining the boundaries for analysis
f. Additional tools
g. Starting the analysis

a. Specification of the data series interval

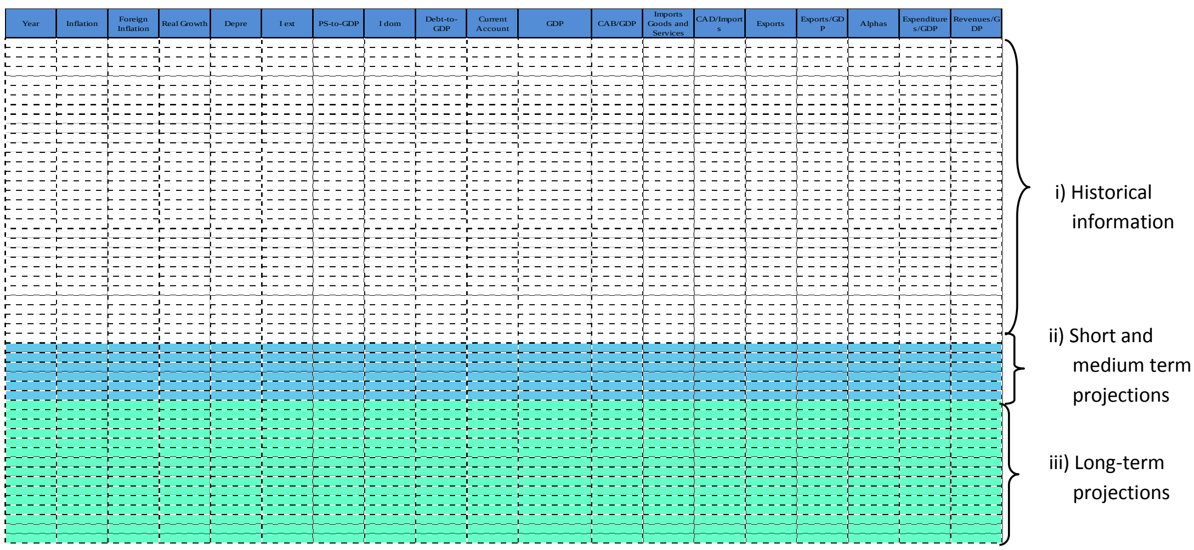
This field relates to the data box located between cells B3 and F6. The following data are entered in the following order, the initial and final years of the historical data provided, last year of projections (at least 5 years of projections), and the analysis horizon that the user considers appropriate. Next, the user clicks on the *"Data Series Intervals"* button to generate the fields in the spreadsheet with their respective formats according to the type of data (historical or projected) (Figure 1).

Figure 1.



The fields where data is entered are automatically differentiated by a color for each group: i) historical data (white); ii) short and medium term projections (CP and MP) (blue); and iii) long term projections (LR) (green) (Figure 2).

Figure 2.



b. Entering the data series

The data series required as input for the 5 different models are added in each column of Figure 2. Next, each of the variables to be included in the *"Select Series"* sheet is listed and explained with their respective location. In addition, some sources are recommended where this data can be obtained. However, the user can use other sources from the country under study, if appropriate.

NOTE: For variables whose source is WEO, click on this link:

http://www.imf.org/external/ns/cs.aspx?id=28

Year (Cell B9)

The years must be entered for use in the sample of historical and projection data.

Inflation (Cell C9)

Annual change in CPI for the economy under study.

Source: WEO

Foreign inflation (Cell E9)

Annual change in CPI of the foreign economy that is taken as reference (usually the United States or a country in Eurozone, although this can be changed by the user).

Source: WEO

Real growth (Cell D9)

Annual change in real GDP in the economy under study.

Source: WEO

Depreciation (Cell F9)

Change in average annual values of the local currency per US dollar. An appreciation is entered with a negative sign (-) and a depreciation with a positive sign (+).

Source:
- LMW

  NXR vis a vis U$S: local currency per US$-annual average[2]
- WEO Confidential

  Exchange rate, national currency per US dollar: Code ENDA
- Bloomberg

Foreign interest (Cell G9)

Nominal interest rate for the total public debt denominated in foreign currency. To calculate the series of foreign interest rates, an option based on WEO data is to take the proportion of interest rates of the external debt payments at time *t (total debt interest paid),* over the total

[2] Typically the bilateral exchange rate versus the US dollar rate if most of the debt is denominated in that currency.

external debt stock over time *t-1* (*total debt outstanding at year-end*). This is known as the implicit interest rate rule.[3]

Source: WEO Confidential.

Total debt interest paid: DSI code[4]

Total debt outstanding at year-end: D code

Primary Fiscal Surplus over GDP (Cell H9)

For most economies this data is obtained from WEO under the name *"General government primary net lending/borrowing - Percent of GDP."* However, depending on the economy analyzed, this data may not be available in WEO and a local source of the country under study may have to be used.

Source: WEO

Domestic interest (Cell I9)

Nominal interest rates for total public debt denominated in local currency. To calculate the domestic interest rate series, an option based on local sources in the country is to take the average annual interest rates applicable to treasury bonds.[5]

Source: Bloomberg

Debt as percentage of GDP (Cell J9)

For most economies this data is obtained from WEO under the heading *"General government gross debt - Percent of GDP."* However, depending on the economy under study, this data may not be available in WEO and a national source in the country under analysis may have to be used.

Source: WEO

Balance on Current Account - CAB (Cell K9)

Value of Current Account Balance in billions of dollars "*Current account balance - Billions of US Dollars*." A surplus must be entered with a positive sign (+) and a deficit with negative sign (-).

Source: WEO

[3] It is assumed that all the external debt is denominated in foreign currency

[4] The codes of the variables in WEO confidential are preceded by the country code

[5] The assumption is that this domestic debt is denominated in local currency.

Gross Domestic Product – GDP (Cell L9)

GDP at current prices in billions of dollars "*Gross domestic product, current prices - Billions of US Dollars.*"

Source: WEO

CAB/GDP (Cell M9)

In this column the values of the Current Account Balance (column K) are divided by the nominal values of GDP (column L) *"Current account balance - Billions of US Dollars/Gross domestic product, current prices - Billions of US Dollars*"

Imports of goods and services - Imports (Cell N9)

Value of imports of goods and services in billions of dollars "*Imports of goods and services - Billions of US Dollars*" (must be entered with a positive sign).

Source: WEO confidential

Imports of goods and services: code BM

CAD/Imports (Cell O9)

In this column the values Current Account Balance (Column K) are taken, preceded by a negative sign, over the value of total imports of goods and services (Column N).

Exports of goods and services – Exports (Cell P9)

Value of exports of goods and services in billions of dollars "*Exports of goods and services - Billions of US Dollars*"

Source: WEO confidential

Exports of goods and services: code BX

Exports/GDP (Cell Q9)

In this column, the values of total exports of goods and services (Column P) are divided by GDP (Column L).

Alphas (Cell R)

This cell contains the ratio of debt denominated in domestic currency over total debt. To calculate this ratio for most countries, the following WEO variables can be used. "*General government gross debt, domestic currency - Billions of US Dollars*" and "*General government gross debt - Billions of US Dollars.*" The alpha of the last year of observed data feeds the "*Currency Composition*" data box located between cells H3:J6, which will be explained in detail later.

Source: WEO confidential

General government gross debt, domestic currency: code GGXWDGCD

General government gross debt: code GGXWDG

Government expenditure as proportion of GDP - Expenditures/GDP (Cell S9)

This data is obtained from the WEO under the name "*General government total expenditure - Percent of GDP*"

Source: WEO

Government revenue as proportion of GDP - Revenue/GDP (Cell T9)

This data is obtained from the WEO under the name "*General government revenue - Percent of GDP*"

Source: WEO

NOTE: applying the Hodrick-Prescott filter to the Expenditures/GDP series (column S) and *Revenue/GDP* (column T) gives the data in columns U and V (tendency), and columns W and X (cyclical component) respectively. This data must not be entered by the user because when *"start analysis"* (Figure 13) is clicked, the template will automatically estimate the data for these columns.

"*Currency composition*" box (Cells H3:K6)

After entering the alpha data in cell J5 of the data box located between H3:K6, the system automatically calculates the proportion of debt denominated in foreign currency, and the proportion of debt denominated in local currency in cell J4. Cells K4 and K5 show the real

domestic and foreign interest rates separately. Finally, in cell J6 the series of domestic and foreign interest rates are converted into real values using the corresponding domestic and foreign inflation rates in order to calculate the long-term (*steady-state*) interest rate as the weighted average of the two values, based on the proportion of debt in local and foreign currencies (Figure 3).

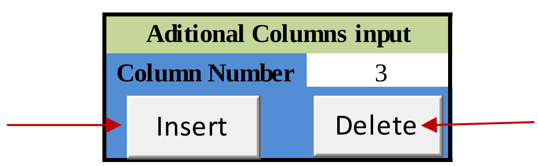
Figure 3.

Currency Composition	%	Interest rate
Domestic Currency	100.0%	3.69%
Foreign Currency	0.0%	-2.80%
Real Interest Rate	**3.69%**	

c. Adding data columns

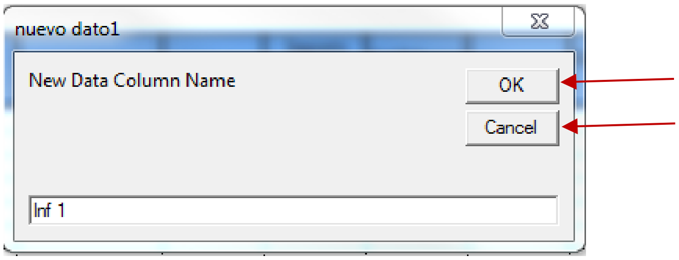
This field was designed to indicate in a methodical and organized way the case where additional variables must be included in the models. This field is designed to allow the estimate of the models to remain consistent after entering additional variables, and not to create problems in the programming code behind the Excel document. In this field, the user has the option of adding columns of data for inclusion in the estimate of any of the models. This relates to the matrix L3:M6 which displays the number of columns the user wishes to add (Figure 4).

Figure 4.



Next clicking on *"Insert"* displays a new dialog box in which the user specifies the name of each variable to be included. Lastly the user must click on the OK link to create the column where data on additional variables can be included, in addition to the variables described previously, in line with the user's needs. However, the option of canceling the entry of new columns is also available (Figure 5).

Figure 5.

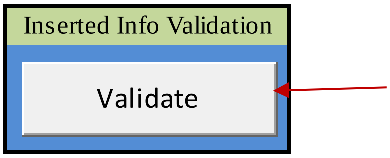


NOTE: The *"Delete"* option in Figure 4 eliminates all the data columns that have been added.

d. Validating entered data

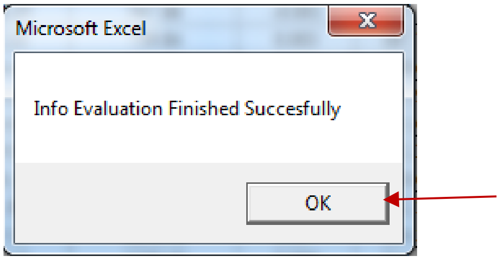
This field is used to validate that the data entered for each variable is correct and is located between cells **O3:P6** (Figure 6).

Figure 6.



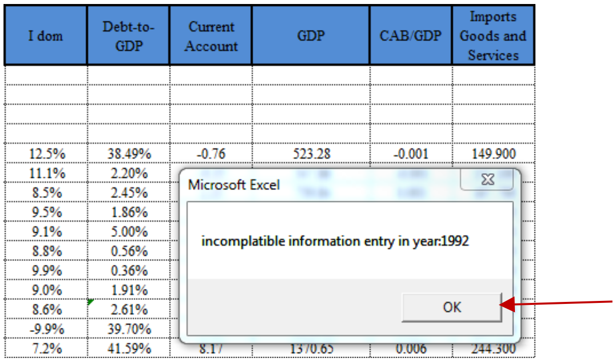
If there is no problem with the data, the user clicks on *"Validate"* to successfully validate the data and a report message is displayed (Figure 7).

Figure 7.



In case of error, the validation displays a dialog box indicating in which year the error in the entering of data was found (Figure 8).

Figure 8.



At this point the user clicks OK, and the template then indicates the location of the spreadsheet containing the error displaying the cell in red (Figure 9).

Figure 9.

I dom	Debt-to-GDP	Current Account	GDP	CAB/GDP	Imports Goods and Services
12.5%	38.49%	-0.76	523.28	-0.001	149.900
11.1%	2.20%	-0.35	547.86	-0.001	157.100
8.5%	2.45%	2.35	759.84	0.003	167.700
9.5%	1.86%	-4.45	918.86	-0.005	180.600
9.1%	5.00%	-4.66	1003.21	-0.005	195.900
8.8%	0.56%	-4.63	1007.94	-0.005	211.600
9.9%	0.36%	-9.87	1247.38	-0.008	222.200
9.0%	1.91%	-6.19	1249.67	-0.005	228.600
8.6%	2.61%	3.84	1375.85	0.003	232.500
-9.9%	39.70%	9.19	1298.36	0.007	224.500
7.2%	41.59%	8.17	1370.65	0.006	244.300

In this example, the format of the entered value is different from the number format. The user must correct the error and perform the validation again, following the steps described above. The user has to wait for the system to display the dialog box indicating successful validation in order to continue the analysis (Figure 7).

The possible errors that the template identifies are: i) incorrect formats, ii) incorrect signs in specific data (e.g. import data must be entered with positive value (+); if for some reason the user enters these values with negative sign, the system reports an error), and iii) data outliers relative to the data that is +/-10 standard deviations from the historical average.

This validation tool ensures that the debt sustainability analysis is compatible with the assumptions of each model and the results are not altered by entering incorrect data (either due to format or sign of the set of variables included).

e. Delimiting the analysis boundaries

In this field, the user can specify the analysis boundaries required. In some cases it is not necessary to use the entire set of data available for a country, or it is necessary to homogenize the dates for which complete data is available. Consequently, in this field the user can specify a sub-sample of the total sample on which to estimate the models (Figure 10).

Figure 10.

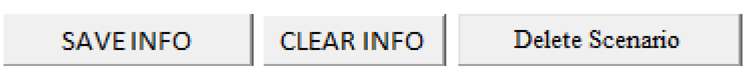
User defined Analysis Boundaries	
Initial year of analysis	**1984**
Final year of analysis	2022

f. Additional tools

The Excel smart document has additional options which enable the user to save, delete and recover the data recorded.

Using the *"SAVE INFO"* box the user can save the data entered on the spreadsheet. Clicking on this box automatically generates a new spreadsheet with the data saved with the name selected by the user. The user can also quickly delete the data entered using the *"CLEAR INFO"* box, but must save it first if required for later use. The *"Delete Scenario"* option eliminates the scenario the user wishes to delete, which had been saved. These options are located under the *"Data Series Intervals"* box (Figure 11).

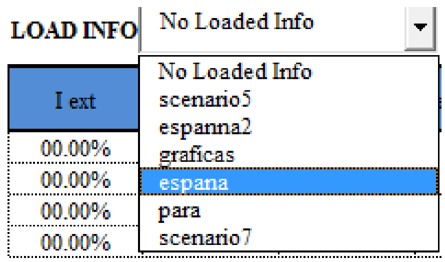
Figure 11.



After clicking on the *"CLEAR INFO"* box the user must delete the data from cell D3 to cell D6.

Lastly, the user who wants to work with data recorded and saved earlier, must go to the *"LOAD INFO"* field and open the menu with the scenarios the user previously saved with all the data. The user then clicks on the one selected for the debt sustainability analysis and the data associated with this scenario is displayed on the intelligent spreadsheet (Figure 12).

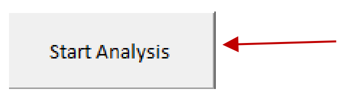
Figure 12.



g. Start of analysis

At this point the user now has the necessary inputs to start the debt sustainability analysis provided by the template by clicking on the *"Start Analysis"* field (Figure 13).

Figure 13.



2. The standard approach

a. Description

The standard approach, sometimes known as the BB approach, following the contributions of Buiter (1985) and Blanchard (1990), is generally regarded as a starting point in fiscal sustainability studies. This approach is based on analysis of the primary surplus required to stabilize the debt/GDP ratio at a certain level (usually current debt levels, or any specific level considered appropriate). As is usual in the literature, the long-term discrete time version of the debt equation is applied (equation 1):

$$f^* = \frac{r-g}{1+g} * d \qquad (\mathbf{1})$$

where d is the debt/GDP ratio, r is the real interest rate, g is growth of real GDP, and f^* is the primary fiscal surplus. This condition has an intuitive interpretation: the sustainable debt/GDP level is where the primary surplus is sufficient to cover the cost of the "effective" interest of the debt service. The effective interest rate is the real interest rate net of the GDP growth rate. Since it is assumed that all the variables are constant over time (or reflect an appropriately calculated average), this equation establishes that the path of the future long-term primary surplus – duly discounted – has to cover total current debt levels (Living with Debt - IDB 2007, chapter 11). In other words, f^* is the primary surplus required to stabilize the debt/GDP ratio for a given interest rate, growth rate of the economy and initial stock of debt (or any given debt level).[6]

A summary of the theoretical model is given in the *"Standard Approach"* spreadsheet located between cells H1:P12 (Figure 14).

Figure 14.

ANALYSIS DESCRIPTION

This approach computes the primary surplus that stabilize the debt to GDP ratio for a given interest rate , growth rate and stock of debt.

$$f^* = \frac{r-g}{1+g}d$$

where:
r is the real interest rate, g is the growth of real GDP, f^* is the primary surplus and d is the debt-to- GDP ratio.

b. Inputs

The required data are historical data on the four variables that enter into the equation **(1).** Of these, the data on real interest rates r have usually been the most difficult to obtain, since they are not generally available "off-the-shelf" and must be calculated from other data. The calculation is further complicated by two factors: first, the currency composition

[6] Note that the required condition for dynamic efficiency is $(r - g) > 0$.

of the public debt is usually a combination of US dollars (or any other foreign currency) and a portion in local currency; second, when the interest rate series are available, they are generally stated in nominal values, which require conversion to real values. The template performs these operations automatically if the user has entered the required data in "selected series."

This minimum input of data automatically feeds the data needed for the analysis in the "Standard Approach" spreadsheet. The template contains data on the steady state debt level between cells E3:E7 of the *"Standard Approach"* spreadsheet (Figure 15) and as mentioned above is fed automatically, with reference to data previously entered on the "*Selected Series*" spreadsheet.*"*

Figure 15.

Standard Approach	
Steady-State Level of Debt	
Debt-to-GDP Ratio	71.63%
Average Real Interest Rate	3.69%
Inflation rate	3.28%
Long-run growth rate	2.60%
Estimated primary surplus for NFPS	-3.52%

c. Outputs

Figure 16 shows the main result of this approach: the required primary surplus as a percentage of GDP (cell E9) and the necessary adjustment of the primary fiscal balance over GDP (E11) which is defined as the difference between the primary surplus estimated in the model (cell E9) and the country's actual primary surplus (cell E7).[7]

[7] Currently the value of the "Actual Primary Surplus" is set as the last value observed. However, this can be changed in order to make a comparison with the external projections for the following year by changing the link of cell "E7" on the "Standard Approach" spreadsheet.

Figure 16.

Required Primary Surplus (% GDP)	0.77%
Required Adjustment (%GDP)	4.29%

Additionally, this approach provides the user with flexibility on two fronts.

i) First, the user can manually override the values of the key variables in equation (1) to check the sensitivity of the results with respect to the baseline hypothesis. Specifically, the user can estimate the required primary surplus as a percentage of GDP and the required adjustment which the country needs to make on its primary fiscal balance as a proportion of GDP, in relation to different values of macroeconomic variables (growth rate of real GDP, real interest rate and debt/GDP ratio).

Similarly, this simulation panel offers the possibility of calculating the primary surplus to achieve a specific debt/GDP target.

This step aims to determine what constant primary surplus is required to achieve a level of debt/GDP target in N periods, calculated as follows:

$$f^{target} = \frac{\mu}{(1+\mu)^N}((1+\mu)^{-N} d_N^* - d_0)$$

Where $= \frac{r-g}{1+g}$, d_N^* is the stock of debt/GDP target in N years and d_0 the initial debt/GDP ratio[8]

All values are entered on the spreadsheet exogenously at the user's discretion (Figure 17).

[8] Escolano, J. 2010. "A Practical Guide to Public Debt Dynamics, Fiscal Sustainability, and Cyclical Adjustment of Budgetary Aggregates." Technical Notes and Manuals International Monetary Fund, 6.

Figure 17.

SIMULATION PANEL	
FIRST SIMULATION	
GDP GROWTH	1.00%
Required Primary Surplues (%GDP)	1.911%
Required Adjustment (%GDP)	5.429%
SECOND SIMULATION	
Interest Rate %	5.00%
Required Primary Surplus (%GDP)	1.68%
Required Adjustment(%GDP)	5.20%
THIRD SIMULATION	
Debt / Y	40.00%
Required Primary Surplus (%GDP)	0.43%
Required Adjustment(%GDP)	3.95%
ALL SIMULATIONS	
Required Primary Surplus (%GDP)	1.58%
Required Adjustment (%GDP)	5.10%
DEBT / Y TARGET	40%
PERIODS TO REACH TARGET	10
Required Primary Surplus (%GDP)	3.78%
Required Adjustment (%GDP)	7.30%

ii) Second, a sensitivity analysis is automatically performed for different assumptions of growth and interest rates. The user has to choose the parameters of the sensitivity analysis. The flexibility provided by the template is very broad at this point, since a sensitivity analysis can be performed in which the interest rate and long-term economic growth rate variables can take values in terms of standard deviations with respect to their average historical value. To specify these criteria, the template offers **four fields** of data where the user must choose between the various options (Figure 18).

The **first field** - ***"Real Interest Rate Deviation Steps"*** - contains options 1, 2 or 3, each of which refers to the data window to be enabled in the matrixes "*Required Primary Surplus (%GDP)*" (B32:B38) and "*Required Adjustment (%GDP)*" (B41:B47) of the "*Real Interest Rate*" variable.

In the **second and third fields - "Standard Deviations"** - the user selects the size of the shocks on the real interest rate and long-term economic growth, respectively, in terms of standard deviations.

The **fourth field -** ***"Long Term GDP Deviation Steps"*** – again contains options 1, 2 or 3, each of which refers to the data window to be enabled in the matrixes "*Required Primary Surplus* (%GDP)" (B32:B38) and "*Required Adjustment (%GDP)*" (B41:B47) of the "*Long Term GDP Growth Rate*" variable.

Figure 18.

Sensitivity Analysis

Long Term GDP Deviation Steps	Standard Deviations	Standard Deviations	Real Interest Deviation Steps
2	1.5	2	3 / 0.0224874

REQUIRED PRIMARY SURPLUS (%GDP)

Real Interest Rate	-4.15%	-0.78%	2.60%	5.97%	11.59%
	Long Term DGP Growth Rate				
16.32%	15.300%	12.345%	9.584%	6.999%	3.038%
12.11%	12.154%	9.306%	6.645%	4.153%	0.336%
7.90%	9.008%	6.267%	3.706%	1.308%	NA
3.69%	5.862%	3.228%	0.767%	NA	NA
-0.52%	2.716%	0.189%	NA	NA	NA
-4.73%	NA	NA	NA	NA	NA
-8.94%	NA	NA	NA	NA	NA

REQUIRED ADJUSTMENT (%GDP)

Real Interest Rate	-4.15%	-0.78%	2.60%	5.97%	11.59%
16.32%	18.818%	15.863%	13.102%	10.517%	6.556%
12.11%	15.672%	12.824%	10.163%	7.672%	3.854%
7.90%	12.526%	9.785%	7.224%	4.826%	#VALUE!
3.69%	9.380%	6.746%	4.285%	#VALUE!	#VALUE!
-0.52%	6.234%	3.707%	#VALUE!	#VALUE!	#VALUE!
-4.73%	#VALUE!	#VALUE!	#VALUE!	#VALUE!	#VALUE!
-8.94%	#VALUE!	#VALUE!	#VALUE!	#VALUE!	#VALUE!

Thus, a sensitivity analysis is automatically performed for different assumptions of growth and real interest rates, producing as output what should be the Primary Fiscal Surplus required for the economy to stabilize its long-term debt/GDP ratio. In addition, the template calculates what adjustment is required on the Primary Fiscal Balance to achieve that target. This data is usually reported as robustness checks in the DSA reports.

NOTE: In cases where r<g violates the condition of invertibility, the template automatically produces "NA" in the respective cells of the *"Required Primary Surplus (%GDP)"* matrix and does not calculate the adjustment required in the *"Required Adjustment (%GDP)"* matrix.

d. Warnings

This is a long term approach since the inputs are usually averages of the historical series. Therefore, any interpretation should assume that the level of inputs will remain approximately constant over a relatively long period of time.

For more data on discussions and examples of this approach see "Living with Debt - IDB (2007) Chapter 11."

3. Endogenous Debt

a. Description

The starting point of this approach is also the discrete time version of equation (1), adjusted to explicitly incorporate the composition of different debt currencies (see equation 2). Therefore:

$$d_t = \left[\alpha \frac{1 + r_t^d}{1 + g_t} + (1 - \alpha) \frac{(1 + r_t^f)(1 + \Delta e)}{(1 + g_t)}\right] d_{t-1} - f_t \, , \qquad (\mathbf{2})$$

where α is the proportion of total public debt denominated in domestic currency and $(1 - \alpha)$ is the proportion of debt denominated in foreign currency. Δe is the annual depreciation (or devaluation) of the reference exchange rate; r^d , r^f and g are, respectively, the interest rate of the domestic currency debt, interest rate on foreign currency debt and the GDP growth rate[9]. Lastly f is the primary fiscal surplus.

A key difference between this approach and the previous one is that this is not a long-term analysis or a steady state approach. Instead, the emphasis here is on the dynamics of the debt over a short-term horizon with a central baseline scenario and discrete sensitivity testing. This analysis is similar to the framework used by the IMF and World Bank, although with some differences explained below.

A summary of the theoretical model is given in the *"Debt Analysis"* spreadsheet located between cells N2:W14 (Figure 19).

[9] Note that when $\alpha = 1$, equation **(2)** is equivalent to equation **(1)**.

Figure 19.

Analysis Description

ANALYSIS DESCRIPTION

This approach follows the debt dynamics equation taking into account the currency composition of the debt as it is presented as follows:

$$d_t = \left[\alpha\left(\frac{1+r_t^d}{1+g_t}\right) + (1-\alpha)\left(\frac{\left(1+r_t^f\right)(1+\Delta e)}{1+g_t}\right)\right] d_{t-1} - f_t$$

Where α is the share of total public debt denominated in domestic currency, $(1-\alpha)$ is the share of debt denominated in foreign currency. Δe is the annual change of the exchange rate . r_t^d,r_t^f, g_t are , respectively, the domestic currency debt, interest rate, foreign currency debt and the growth rate of GDP.

b. Inputs

The *"Debt Analysis"* spreadsheet is automatically fed from the data entered in *"Select Series"* If the user has already entered all the data required for the previous approach (standard approach), only one additional series is required: annual changes in the nominal exchange rate, entered in column F of the *"Selected Series"* spreadsheet[10].

In the **first section - "Simulations Scenarios for shocks analysis"** - the user must select different fields in order to simulate the shocks on some or all of the variables used as inputs in dynamic debt equation **(2),** in different scenarios. Thus, a list of 5 variables[11] on which a shock can be simulated are located between cells B4:B12.

The shock can be enabled by a single click on the box to the left of each variable.

In the example presented in Figure 20, 3 of the 5 options have been selected *("Exchange Rate," "GDP Growth"* and *"Foreign Currency Debt Interest").*

The simulation panel characterizes the shock by its time period. Thus, the user can select the range of years and size in terms of standard deviations with respect to the historical average.

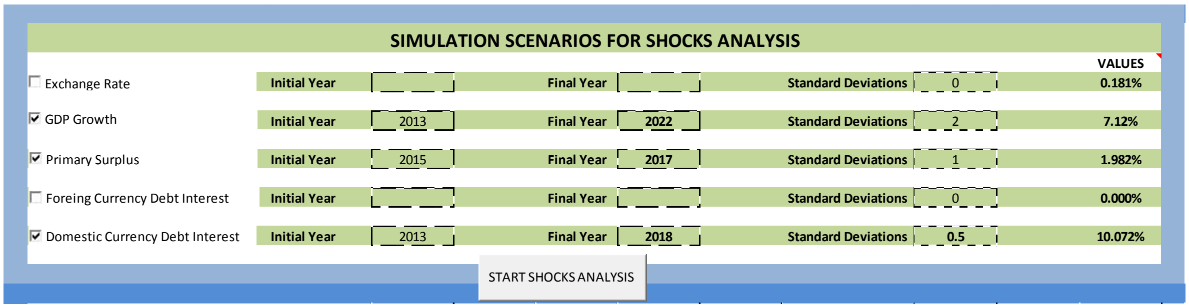
For each selected variable the user must indicate the year when the shock starts (column D), the year when it ends (column G) and its size in terms of standard deviations (column J). At this point, the user clicks on the *"START SHOCKS ANALYSIS"* field, which

[10] Typically the bilateral exchange rate versus the US dollar rate if most of the debt is denominated in that currency.

[11] The variables are exchange rate, GDP growth, primary surplus, foreign current debt interest and domestic currency debt interest.

automatically calculates the average value of the variable plus the size of the shock during the year(s) when it occurred, which is displayed in column L (Figure 20).

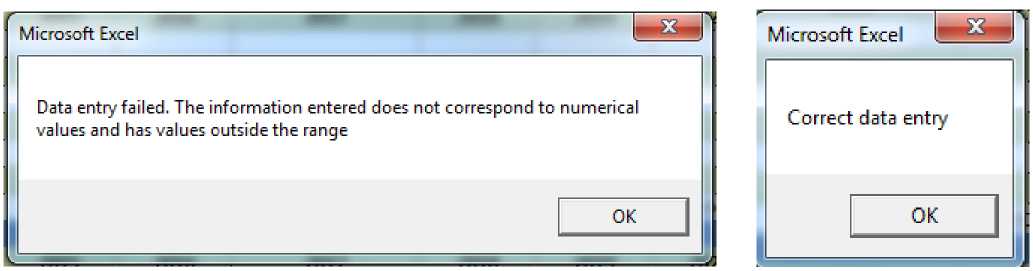
Figure 20.



NOTE: the years when the shocks occur must refer to years that have not yet been observed, otherwise the intelligent spreadsheet will produce an error message (Figure 21a). If the data is entered correctly in the specified fields, the spreadsheet displays a confirmation message (Figure 21b).

Figure 21.

a) **b)**



c. Outputs

The approach produces the following output tables and charts.

A) The central range of the debt in relation to GDP "*BASE LINE (No Shock)*" (Figure 22) relates to the estimated dynamic of the various variables of the model over time under the baseline scenario in the absence of shocks. The data entered for the projection dates (in the example, 2012 onwards) are data forecast in WEO.

Figure 22.

BASELINE (No Shocks)	2011	2012	2013	2014	2015	2016	2017
Real DGP Growth	0.417%	-1.538%	-1.316%	1.004%	1.552%	1.696%	1.734%
Domestic Currency Debt Interest Rate	5.440%	5.000%	5.000%	5.000%	5.000%	5.000%	5.000%
Nominal Exchange Rate Depreciation	-4.637%	9.889%	2.205%	0.313%	0.452%	0.431%	0.409%
Primary Surplus	-5.051%	-3.518%	-1.784%	-0.665%	0.101%	0.863%	1.432%
Inflation	2.356%	3.284%	1.352%	1.446%	1.378%	1.409%	1.424%
Foreign Currency Debt Interest Rate	0.000%	0.000%	0.000%	0.000%	0.000%	0.000%	0.000%
Debt/GDP ratio	49.674%	71.626%	76.978%	79.547%	81.029%	81.636%	81.641%

B) The path of the debt/GDP ratio in the "*SHOCKS ANALYSIS*" sensitivity testing scenarios performed (Figure 23) relates to the path of the debt/GDP ratio over time[12], in relation to a shock in each selected variable in the fields illustrated in Figure 20. The matrix also contains data on the debt/GDP path of the historical scenario, in which the values of the projection dates relate to the historical average of each variable that feeds the debt equation (2). The values recorded in both matrixes were estimated when the user clicked on the "START SHOCK ANALYSIS" field shown in Figure 20.

Figure 23.

SHOCKS ANALYSIS	2011	2012	2013	2014	2015	2016	2017
Real DGP Growth	49.674%	71.626%	71.058%	69.325%	66.930%	63.832%	60.260%
Primary Surplus	49.674%	71.626%	76.978%	79.547%	79.148%	78.602%	78.004%
Domestic Currency Debt Interest Rate	49.674%	71.626%	80.610%	87.258%	93.192%	98.603%	103.753%
Combined Shock	50.010%	52.695%	55.210%	56.588%	55.377%	54.131%	52.861%
Shock 6: Historical Scenario	52.456%	57.679%	62.336%	64.542%	65.726%	66.055%	65.786%
IMF Scenario	49.674%	71.626%	78.243%	80.979%	82.217%	82.800%	82.991%

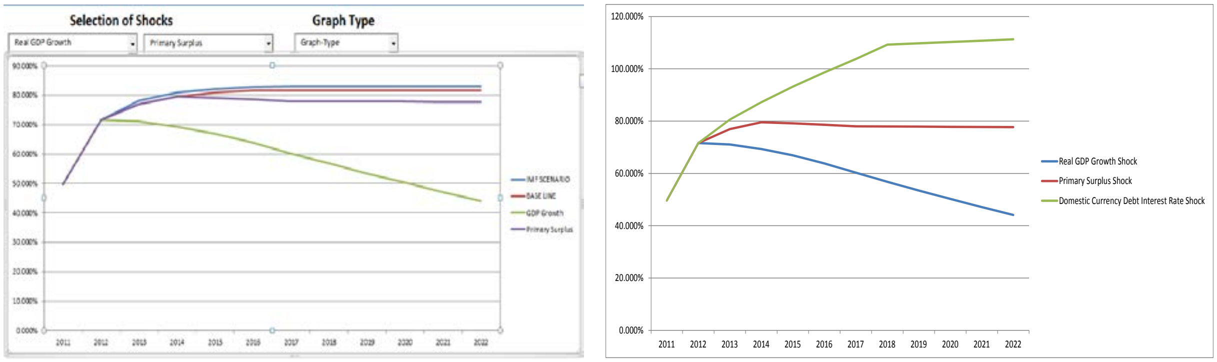
C) The charts of the various paths of the public debt ratio in each scenario (Figure 24a and 24b). The first shows the debt/GDP path in two fixed scenarios ("*Baseline Scenario*" and "*IMF Scenario*"). Additionally, two fields are enabled where the user has the option of choosing two of the simulated shocks, to observe the Debt/GDP path after these shocks and compare each of them with the baseline scenario (Figure 24a).

The second chart shows the debt/GDP path given by each shock that the user previously entered (Figure 20) to obtain a better visualization of the debt/GDP path after each shock (Figure 24b).

[12] The years displayed in the matrix are in line with the information entered in the *"Data Series Intervals"* field of the *"Selected Series"* spreadsheet.

Figure 24.

a) **b)**



D) Path of each variable in the model, incorporating the path of the variables of the debt equation (Figure 25). At this point, the simulated shocks and the path of the debt/GDP ratio over time are considered.[13] In other words, this matrix summarizes the path of the inputs of the debt equation that suffered a shock and the debt/GDP ratio before, during and after the shock.

Figure 25

Comparative Analysis	2011	2012	2013	2014	2015	2016	2017
Nominal Exchange Depreciation Shock	-4.637%	9.889%	2.205%	0.313%	0.452%	0.431%	0.409%
Dept / GDP Ratio							
Real GDP Growth Shock	0.417%	-1.538%	7.117%	7.117%	7.117%	7.117%	7.117%
Dept / GDP Ratio	52.456%	51.230%	71.058%	69.325%	66.930%	63.832%	60.260%
Primary Surplus Shock	-5.051%	-3.518%	-1.784%	-0.665%	1.982%	1.982%	1.982%
Dept / GDP Ratio	52.456%	51.230%	76.978%	79.547%	79.148%	78.602%	78.004%
Foreign Current Debt Interest Rate Shock	0.000%	0.000%	0.000%	0.000%	0.000%	0.000%	0.000%
Dept / GDP Ratio							
Domestic Currency Debt Interest Rate	5.440%	5.000%	0.000%	0.000%	0.000%	0.000%	0.000%
Dept / GDP Ratio	52.456%	51.230%	80.610%	87.258%	93.192%	98.603%	103.753%

[13] The years displayed in the matrix of the fourth section are in line with the information entered in the *"Data Series Intervals"* field of the *"Selected Series"* spreadsheet.

4. Fan Chart in DSA

a. Description

In this model, a probability distribution of the debt-GDP ratio is obtained by combining different methods of multivariate regression analysis and external forecasts. The template permits the use of four methodologies for debt sustainability analysis using fan charts: **1) External projections, 2) VAR Approach, 3) External projections with correlated errors,** and **4) Weighted projections.** These methodologies are explained in more detail in Arizala, Castro, Cavallo and Powell (2010). In addition, a brief description of each methodology is presented in **Appendix A.**

The basic idea in the fan charts is to produce a simulated distribution of debt to GDP based on the dynamic provided by a Vector Autoregressive (VAR)-type econometric model and/or by a series of external forecasts for a group of risk inputs that feed the dynamic debt equation.

The corresponding fan chart is generated for projection of a period of years specified by the user and, finally, it is possible to evaluate the probability that the debt/GDP ratio exceeds various threshold values. Thus, instead of projecting simply a single scenario, the fan chart framework for debt sustainability analysis incorporates the structure of the random shocks that affect the domestic economy in order to obtain a complete distribution of the likely outcomes of the debt to GDP ratio.

This model recognizes that, even if the government is committed to implementing its fiscal policies, the results are subject to significant risks, especially as the planning horizon lengthens.

b. Inputs

The inputs are divided into two categories: deterministic and stochastic

1) **Deterministic:** All components of the law of motion of the debt, as in the case of the endogenous debt approach, for example, GDP, nominal interest rate of foreign currency debt, nominal interest rate of local currency debt, nominal exchange rate, primary surplus, inflation rate, debt/GDP ratio and α (currency composition of public debt portfolio) have all been included in the "Selected Series" spreadsheet.[14]

[14] There is also a column where a series of alphas (α) must be entered if any exogenous norm is found on the

The data in column R of the *"Selected Series"* spreadsheet, series of alphas (α), is also used if there is any exogenous rule on the future path of the currency composition of the debt which must be followed (path subsequent to the last data observed).

The user can also specify the exogenous shocks to the debt stock in some of the projected years. This option can be useful when considering the existence of contingent debt (debt skeletons) and/or the effect of possible future transformation events (for example, a natural disaster, a public infrastructure project, etc.) with direct impact on the accumulation of the public debt.

2) **Stochastic:** Stochastic components are composed of the coefficients and the variance-covariance matrix of a VAR regression. The user must estimate the VAR outside the template, using Eviews or Stata and copy the coefficients resulting from the matrix and the VAR-COV matrix. There are a number of aspects to consider when estimating a VAR (stability, transgression of the classical assumptions on the residuals, etc.), especially bearing in mind that the limitations of the data are significant in the countries of Latin America and Caribbean; therefore the user should consult the relevant literature.

c. Options

The template has the level of flexibility that the user can select from several options listed below:

i. *External projections (with uncorrelated errors):*

•The identity matrix is introduced in place of the VAR-COV matrix.

•The standard deviations column is completed with the historical standard deviations related to each risk input (which can be calculated from the historical series available on the "Selected Series" spreadsheet).

•The Beta Column is completed with 1s.

---

future trend of the currency composition of the debt which must be followed. All this information was previously entered by the user on the “Selected Series” spreadsheet, as explained in the first part of this manual.

ii. VAR Model:

•The coefficients and VAR-COV matrix are entered.

•The standard deviation column must be filled with 1s.

•The Betas column is completed with 0s.

iii. External projections (with correlated errors):

• VAR VOC-matrix is entered.

• The standard deviation column must filled with 1s.

• The Beta column is completed with 1s.

iv. Weighted projections:

• VAR-COV coefficients and matrixes are entered.

• The standard deviation column must be filled with 1s.

• The Beta column must include numbers in the range (0.1).

For currency composition, the user can define different options:

Alpha Option 1: The currency composition is endogenous. The paths of debt in foreign currency and local currency are projected separately.

Monetary composition

The last measure reported in this approach is the maximum, minimum and average composition of local currency during the last year of the projection if alpha is endogenous (alpha option 1). The model calculates the related alphas in accordance with the following equations:

$$\alpha_{min} = \frac{d_{min}^{D}}{d_{min}^{D} + d_{max}^{E}} \quad \textbf{(3)}$$

$$\alpha_{max} = \frac{d_{max}^{D}}{d_{max}^{D} + d_{min}^{E}} \quad \textbf{(4)}$$

$$\alpha_{avg} = \frac{d_{avg}^{D}}{d_{avg}^{D} + d_{avg}^{E}} \quad \textbf{(5)}$$

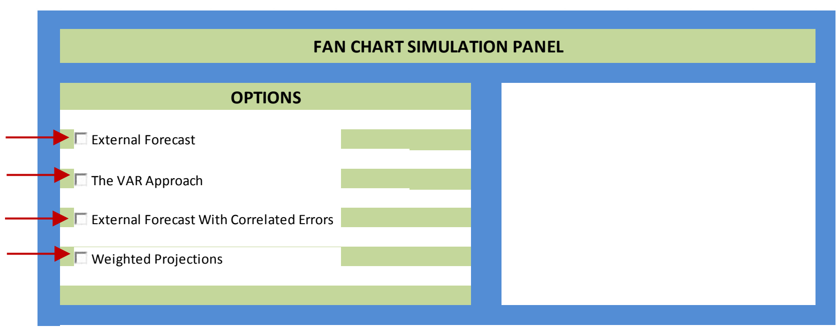
For the minimum percentage of the domestic currency debt, the model takes the 5th percentile for debt in local currency and percentile 95 for the foreign currency debt. In the case of the maximum composition of the public debt, the model takes percentile 95 for the debt in domestic currency and the 5th percentile for the debt in foreign currency.[15]

Alpha Option 2: The currency composition is fixed throughout the projection period and the user defines an exogenous value.

Alpha Option 3: In this case, the user specifies a rule for the currency composition in the "alpha" column of the "Selected Series" spreadsheet[16].

On this basis, the user must first specify which of the four base models for projecting the risk inputs will be selected, to do this the user clicks on each option shown in *"FAN CHART SIMULATION PANEL - OPTIONS"* between cells B6:B12 (Figure 26). Next, the user must follow the instructions given below step by step for each of the four estimation methodologies by clicking on each option.

Figure 26.



[15] In some cases, the highest percentiles of the projections can reach negative values in the debt/GDP ratio; in this case, the corresponding minimum or maximum alpha could report values outside the 0-1 interval.

[16] A specific norm consists of relating the foreign currency-denominated debt $(1-\propto)$ with the behavior of the exchange rate. From $(1-\propto)$ the following is obtained:

$(1-\propto)_t = (1-\propto)_{t-1} + \frac{\partial(1-\propto)_t}{\partial e_t} = (1-\propto)_{t-1} + \frac{\partial\left(\frac{e\times d^f}{d}\right)_t}{\partial e_t}$, where e_t is the nominal bilateral exchange rate with the US dollar, d^f is the debt denominated in foreign currency, d is total public debt, and ∂ is the symbol of partial derivatives. The last equation, using algebraic manipulation, is simplified to: $(1-\propto)_t = (1-\propto)_{t-1} + \%\Delta e_t \times [(1-\propto)_{t-1} - (1-\propto)^2_{t-1}]$, where $\%\Delta$ is the abbreviation for "percentage change."

d. Procedure and outputs

Considering each of the options described above gives:

i. *External projections*

Clicking on *"External Forecast"* displays a dialog box with four data fields (Figure 27).

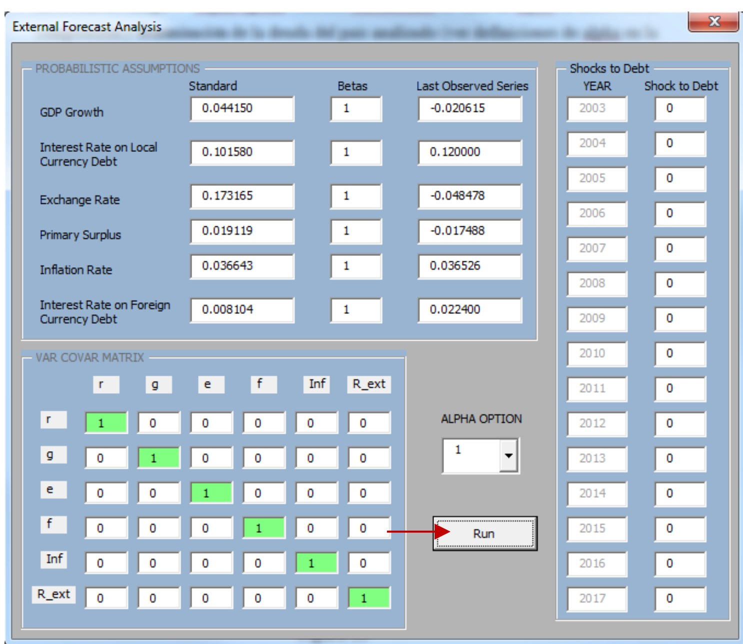
The **first field - *"Probabilistic Assumptions"*** - displays the assumptions of the model for each risk input of the debt equation.

The **second field - *"VAR COVAR Matrix"*** - displays the identity matrix, which is preset and cannot be modified.

The **third field - *"Shocks to Debt"*** – displays some of the observed years (9 years observed and 6 years projected) in a panel where the user can specify the exogenous shocks to the debt stock.

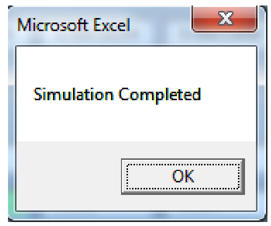
In the **fourth field - "Alpha option" -** the user must select the alpha value in line with the composition and denomination of the debt of the country under review, then click on the box 

Figure 27.



Next, a dialog box appears indicating that the simulation was performed. When OK is clicked (Figure 28), the template finally displays the estimated results of the model graphically (cells B17:I32 - Figure 27a) along with a data box displaying value at risk and sensitivity analysis (cells B33:I44 - Figure 29b).

Figure 28.

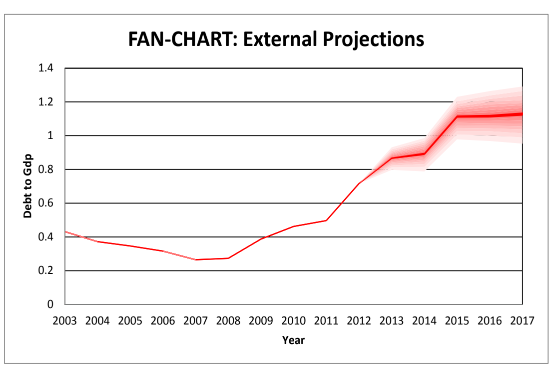


The chart (Figure 29a, 32a, 34a and 36a) produces the observed path of debt to GDP and the simulated distribution for each year of the projected time window, taking into account the uncertainty contained in the set of inputs that feed the debt movement equation.

The box (Figure 29b, 32b, 34b and 36b) calculates the probability that the debt/GDP ratio reaches an exogenous threshold. Specifically, it calculates the frequency with which the debt/GDP ratio exceeds the various threshold values in any of the projected years.

Figure 29.

a)



b)

Value at Risk of the Debt / GDP

Threshold of the debt limit (X)	0.9
Prob(Debt/ GDP) > X	0.7448

Sensitivity Analysis

2012	>0.9	>0.95	>1	>1.05
2013	0.255	0.048	0.006	0.000
2014	0.490	0.242	0.075	0.010
2015	1.000	0.983	0.912	0.786
2016	0.994	0.970	0.901	0.779
2017	0.985	0.951	0.882	0.765
GLOBAL	0.7448	0.6388	0.555200	0.468000

Debt to GDP > X

ii. *VAR Model*

The user must enter the coefficients obtained from modeling VAR (1) in matrix N3:T10 (Figure 30a) and the values of the variance and covariance in matrix V3:AV9 (Figure 30b) of the "*Fan Chart*" spreadsheet. Next, the user selects the *"VAR approach"* box in the *"FAN CHART SIMULATION PANEL - OPTIONS"* (Figure 26), which displays a dialog box with five data fields (Figure 31).

NOTE: If the user wishes to enter additional variables that affect the debt dynamics in the VAR estimate, they must be entered as exogenous variables. Additionally, if there is a variable which, because of the characteristics of the country under study, does not belong to the VAR, the corresponding row/column of the coefficient matrix and VAR COVAR matrix must be completed with zeros.

Figure 30.

a)

Coefficients from VAR (1)

	r	e	g	f	inf	r_ext
r-1	0.446	0.966	-0.252	0.242	0.513	-0.221
e-1	-0.006	0.875	0.092	0.094	-0.000	-0.002
g - 1	0.144	-2.069	0.666	0.606	0.153	0.420
f - 1	-0.285	0.952	-0.133	0.588	-0.001	-0.088
inf-1	1.261	-4.998	-0.726	-0.959	0.145	-0.480
r_ext - 1	0.362	1.504	-0.294	-0.689	-0.177	0.368
c	-0.002	0.149	-0.029	-0.069	-0.013	0.046

b)

Var-Covar Matrix Input

	r	e	g	f	inf	r_ext
r	0.000157	-0.000117	0.000105	3.77E-05	0.00011696	0.000112
e	-0.000117	0.005716	-0.000113	0.000167	-0.00006021	0.000192
g	0.000105	-0.000113	0.000206	0.000215	0.00011458	0.000161
f	3.77E-05	0.000167	0.000215	0.000428	0.00013207	0.00014
inf	0.000117	-6.02E-05	0.000115	0.000132	0.00014249	0.000107
r_ext	0.000112	0.000192	0.000161	0.00014	0.00010691	0.000182

The **first field -** ***"Shocks to Debt"*** - displays some of the observed years (9 years observed and 6 years projected) in a panel where the user can specify the exogenous shocks to the debt stock over GDP.

The **second field -** ***"Probabilistic Assumptions"*** - displays the assumptions of the model for each risk input of the debt equation.

The **third field -** ***"VAR COVAR Matrix"*** - displays the values that were previously entered in the V3:AV9 matrix of the "FanChart" spreadsheet, which is preset in this dialog box and cannot be modified. If any change is required, the user must close the

dialog box, reenter the values in the specified matrix and reselect the *"VAR approach"* option (Figure 26).

The **fourth field** - ***"VAR Analysis"*** - displays the values previously entered in the N3:T10 matrix of the "FanChart" spreadsheet (VAR coefficients), which is preset in the dialog box and cannot be modified. If any change is required, the user must close the dialog box, reenter the values in the specified matrix and reselect the *"VAR approach"* option (Figure 26).

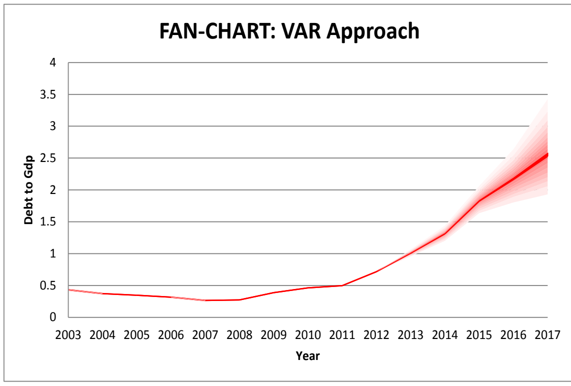
In the **fifth field - "Alpha option" -** the user must select the alpha value in line with the composition and denomination of the debt of the country under review (see definitions of alpha in the description given above in the **Inputs** section), and finally, click on the box 

Figure 31.

Next, a dialog box is displayed indicating that the simulation was performed. When OK is clicked (Figure 26), the template finally shows the estimated results graphically (cells J17:S32 - Figure 32a) along with a data box that summarizes what the chart is displaying (cells K33:R44 - Figure 32b).

Figure 32.

a)



b)

Value at Risk of the Debt / GDP

Threshold of the debt limit (X)	0.9
Prob(Debt/ GDP) > X	1

Sensitivity Analysis

2012	>0.9	>0.95	>1	>1.05
2013	1.000	0.970	0.630	0.125
2014	1.000	1.000	1.000	1.000
2015	1.000	1.000	1.000	1.000
2016	1.000	1.000	1.000	1.000
2017	1.000	1.000	1.000	1.000
GLOBAL	1	0.994	0.926000	0.825000

Debt to GDP > X

iii. *External projections with correlated errors*

The user must enter the values of the variance-covariance matrix in the V3:AV9 matrix (Figure 30b) of the *"FanChart"* spreadsheet. The user must then select the *"External Forecast with Correlated Errors"* box in the *"FAN CHART SIMULATION PANEL - OPTIONS"* (Figure 26), which displays a dialog box with four data fields (Figure 33).

The **first field -** ***"Probabilistic Assumptions"*** - displays the assumptions of the model for each risk input of the debt equation.

The **second field -** ***"VAR COVAR Matrix"*** - displays the values previously entered in the V3:AV9 matrix of the "FanChart" spreadsheet, which is preset in the dialog box and cannot be modified. If a change is required, the user must close the dialog box, reenter the values in the specified matrix and reselect the *"External Forecast with Correlated Errors"* option (Figure 26).

The **third field -** ***"Shocks to Debt"*** - displays some of the observed years (9 years observed and 6 years projected) in a panel where the user can specify the exogenous shocks to the debt stock.

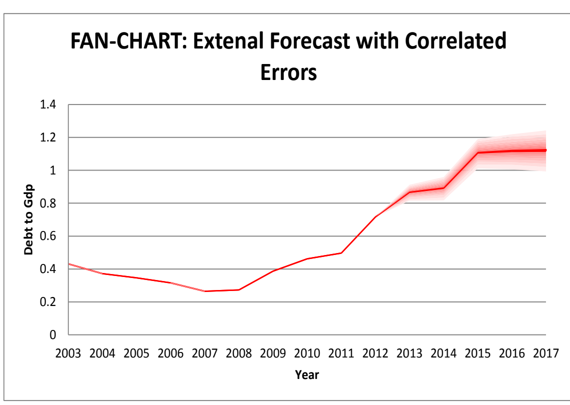
In the **fourth field - "Alpha option"** - the user must select the alpha value in line with the composition and denomination of the debt of the country under review (see

definition of alpha in the theoretical description preceding this section) and then click on the box .

Figure 33.

Next, a dialog box is displayed indicating that the simulation was performed. When OK is clicked (Figure 28), the template finally displays graphically the results estimated by the model (cells T17:AB32 - Figure 34a) along with an data box that summarizes what this chart is displaying (cells U33:AB44 - Figure 34b).

Figure 34.



a)

Value at Risk of the Debt / GDP

Threshold of the debt limit (X)	0.9
Prob(Debt/ GDP) > X	0.7282

Sensitivity Analysis

2012	>0.9	>0.95	>1	>1.05
2013	0.174	0.009	0.000	0.000
2014	0.470	0.146	0.022	0.001
2015	1.000	0.996	0.966	0.860
2016	1.000	0.993	0.963	0.848
2017	0.997	0.986	0.940	0.823
GLOBAL	0.7282	0.626	0.578200	0.506400

Debt to GDP > X

b)

iv. *Weighed projections*

The user must enter the coefficients obtained by modeling the VAR (1) in the N3:T10 matrix (Figure 30a) and the values of the variance-covariance matrix in the V3:AV9 matrix (Figure 30b) of the *"FanChart"* Spreadsheet. Next, the user must select the *"Weighted Projections"* box in the *"FAN CHART SIMULATION PANEL - OPTIONS"* (Figure 26), which displays a dialog box with five data fields (Figure 35).

The **first field - *"Shocks to Debt"*** - displays some of the observed years (9 years observed and 6 years projected) in a panel where the user can specify the exogenous shocks to the debt stock.

The **second field - *"Probabilistic Assumptions"* -** displays the assumptions of the model for each risk input of the debt equation. These assumptions are in three columns, two are preset and cannot be modified by the user *("Standard deviations"* and *"Last observed series")* and the other is a column of betas that can be edited by the user *("Betas")* according to the weighting given by user to the variance-covariance matrix and to the external projections, in an interval defined between 0 and 1.

The **third field - *"VAR COVAR Matrix"*** - displays the values previously entered in the V3:AV9 matrix of the "FanChart" spreadsheet, which is preset in the dialog box and cannot be modified. If a change is required, the user must close the dialog box, re-enter the values in the specified matrix and reselect the *"VAR approach"* option (Figure 26).

The **fourth field - *"VAR Analysis"*** - displays the values previously entered in the N3: T10 matrix of the "FanChart" spreadsheet, which is preset in the dialog box and cannot be modified. If a change is required, the user must close the dialog box, reenter the values in the specified matrix and reselect the *"VAR approach"* option (Figure 26).

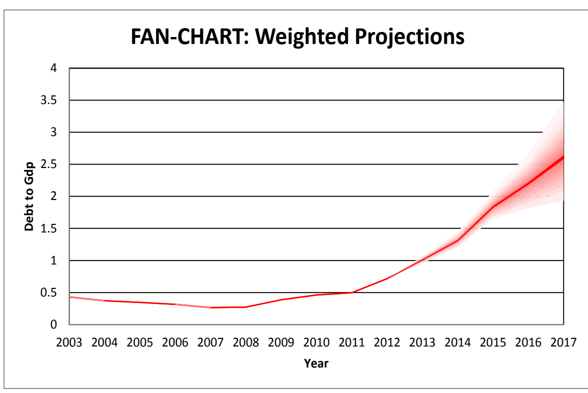
In the **fifth field - "Alpha option,"** the user must select the alpha value in line with the composition and denomination of the debt of the country under review (see definition of alpha in the theoretical description preceding this section) and then click on the box

Figure 35.

Next, a dialog box is displayed indicating that the simulation was performed. When OK (Figure 28) is clicked, the template shows the estimated results of the model graphically (cells B53:I68.- Figure 36a) along with an data box that summarizes what this chart is displaying (cells B70:I81 - Figure 36b).

Figure 36

a)



b)

Value at Risk of the Debt / GDP

Threshold of the debt limit (X)	0.9
Prob(Debt/ GDP) > X	1

Sensitivity Analysis

2012	>0.9	>0.95	>1	>1.05
2013	1.000	0.972	0.665	0.135
2014	1.000	1.000	1.000	1.000
2015	1.000	1.000	1.000	1.000
2016	1.000	1.000	1.000	1.000
2017	1.000	1.000	1.000	1.000
GLOBAL	1.0000	0.9944	0.9330	0.8270

Debt to GDP > X

NOTE: An extract from Arizala, Cavallo, Castro and Powell (2008) presenting the advantages and disadvantages of each of the four base models is reproduced in Table A1 of Appendix A4.

The user will also find 1,000 simulations of the Debt/GDP path for each option or approach of the Fan Chart model in the "Fan Chart Data" spreadsheet.

5. Sudden Stop Approach[17]

a. Description

This approach starts from the observation that persistent sudden stops (SS) of capital flows can cause fundamental changes in the equilibrium real exchange rate (RER)[18]. This in turn can create problems of fiscal sustainability to the extent that the public debt is partially or totally denominated in foreign currencies. A source of vulnerability occurs when the proportion of foreign currency debt becomes significantly higher than the share of tradables production, that is, when a currency mismatch occurs. In these cases, the fluctuation mentioned in the RER effect can generate large debt valuation effects, and thus threaten fiscal sustainability.

When an economy is experiencing a SS, absorption of tradable goods has to fall to restore equilibrium. The size of the adjustment depends on the proportion of absorption of tradable goods which is financed from abroad, or "leveraged." On the assumption of proportionality in consumption between tradables and non-tradables, the fall in demand for tradable goods would have to be accompanied by a fall in demand for non-tradable goods. With a fixed or slow-moving supply, this would lead to changes in relative prices, or the RER. If this were the case, the greater the leverage of tradable goods absorption, the larger is the change in RER that is needed to restore equilibrium after an episode of SS.

In the exercise presented in the *"Sudden Stop Calculations"* spreadsheet, imports (M) are the proxy for absorption of tradable goods; therefore, the current account deficit (CAD) is the leveraged part of this absorption (i.e. not financed by the supply of tradable goods):

$$1 - \omega = \frac{CAD}{M} \qquad (\mathbf{6})$$

In **(6)**, 1- ω represents the fall in the demand for tradable goods following an SS for a given RER. As mentioned, this adjustment should be accompanied by a fall in the demand for

---

[17] See Calvo, Izquierdo and Talvi (2003).

[18] In this case, expectations play a key role in two dimensions. First, the changes to RER (and the product) take place largely because the capital flows drought is unexpectedly sudden. Second, the impact of an SS also depends on the predicted duration, that is, if perceived as temporary or highly permanent.

non-tradable goods. Therefore, if the supply of non-tradable goods is fixed in the short term, the required percentage change in the real exchange rate is given by:

$$-dp = \frac{1-\omega}{\chi}, \quad (\mathbf{7})$$

where p is the relative price of the non-tradable goods, that is, the inverse of the RER, and χ is the price elasticity of the demand for non-tradable goods. This implies that *-dp* is the required depreciation of the RER; therefore this estimate allows to evaluate debt revaluation effects of an SS. The current debt level (b) can be disaggregated as:

$$b = \frac{B + EB^*}{Y + EY^*}, \quad (\mathbf{8})$$

where *E* is the RER, *B, B ** are, respectively, the debt denominated in local and foreign currency (or in terms of tradable goods), *Y, Y** are the production denominated in local and foreign currency. In the current period, the proportion of tradable goods in total production *Y*/(Y + Y *)* is approximated by means of the exports to GDP ratios [19]. Equation (8) shows that the currency mismatch is equivalent to:

$$m = \frac{B/EB^*}{Y/EY^*}, \quad (\mathbf{9})$$

which would determine the effect of debt revaluation that follows an adjustment of RER. When *m =1*, there is no revaluation effect, while if *m* is close to zero (that is, *B /EB** is very small or *Y /EY** is very large) there is "total transmission" of the RER to the debt-GDP ratio. It is important to note that these calculations implicitly assume that the RER shocks are permanent.

In addition to the effects of revaluation of the debt achieved through changes in relative prices, this approach simulates the impact of three shocks usually associated with the occurrence of an SS. These are: i) a rise in nominal interest rates (not offset by inflation), ii) a fall in the growth of the real product and iii) the appearance of contingent liabilities.

[19] Another approximation for tradable inputs is the sum of the value added in the agricultural and manufacturing sectors in GDP.

b. Inputs

The *"Sudden Stop Calculation"* spreadsheet is automatically fed from the following series which have been previously entered on the *"Selected Series"* spreadsheet: current account balance (column K), imports (column N), exports (column P), and the ratio of domestic currency debt to total debt (column R). To calculate the required primary surplus, the exercise requires the inputs used for other approaches, such as the real interest rates for the portion of debt denominated in domestic and foreign currency and the output growth rate. This is located between cells G6:H10 of the *"Sudden Stop Calculation"* spreadsheet (Figure 37).

Figure 37.

DATA INPUT		
B	959.98	Debt denominated in domestic currency (US$ Billions)
*B**	0.00	Debt denominated in foreign currency (US$ Billions)
Y	942.11	Output of non-tradables (domestic-currency output=output - exports)
*Y**	398.16	Output of tradables (foreign-currency output=exports)

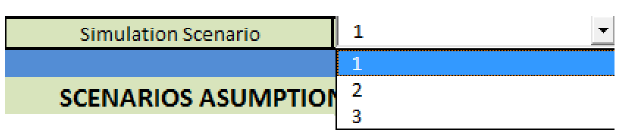
In the field between cells E4 and G4 the spreadsheet is automatically fed with the date of the last year of historical observations entered in *"Selected Series"* (Figure 38).

Figure 38.

Last Observed Year	2012

In cell N4 of the *"Sudden Stop Calculations"* spreadsheet the user can select from three scenarios for these variables, r, r^* and g (Figure 39).

Figure 39.



These scenarios relate to: 1) last values observed, 2) long term averages, and 3) the WEO projections for the following year, scenarios described between cells J6:N10 (Figure 40).

Figure 40.

SCENARIOS ASUMPTIONS				
	Last Observed (1)	**L-R Averages (2)**	**WEO forecats (3)**	**Description**
r	1.7%	3.7%	3.6%	Real Interest rate on domestic currency debt
*r**	1.0%	1.0%	1.0%	Real Interest rate on foreign currency debt
g	-1.5%	2.6%	-1.3%	Real output growth

This selection of assumptions could later be subject to shocks specified by the user as detailed below (Figure 42).

In the parameter section of the "*Sudden Stop Calculation*" spreadsheet between cells B6:B10, the user must enter an estimate of the price elasticity of the demand for non-tradable goods (cell C7); the current reference value for this parameter was taken from Calvo, Izquierdo and Talvi (2003) and is equal to 0.4. The cells with the three remaining parameters $(\alpha, \frac{CAD}{M}, -dp)$ are automatically calculated, so the user does not have to enter additional data in this section (Figure 41).

Figure 41.

PARAMETERS		
χ	0.4	Price Elasticity of demand for nontraded goods
α	100.0%	Share of domestic currency denominated debt in total debt
CAD/M	0.089	Current Account Deficit / Imports (Leveraged tradables absorption)
-dp	22.25%	Required RER change to restore equilibrium after SS

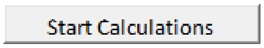
Finally, in the *"Sudden Stop Calculation"* spreadsheet the user should specify the following shocks, other than the RER adjustment, which the economy could undergo in the case of an SS: i) increase in nominal interest rate (cell C13, currently 200bps), ii) decrease in growth (cell C14, currently 1%), and iii) contingent liabilities in national currency (cell C15) (Figure 42).

Figure 42.

SHOCKS INFO		
i shock	200	Shock to the nominal interest rate in bps.
g shock	1.00%	Shock to the real output growth
Conting. Liab Shock	10.00%	Shock to contingent liabilities

Next the user must click on the "Start Calculations" box (Figure 43).

Figure 43.



c. Outputs

The approach presented in the *"Sudden Stop calculations"* spreadsheet has five cumulative exercises and reports the primary surplus required to stabilize the debt after the effects of revaluation described in this section using some of the notions introduced in the approaches presented previously. To perform the analysis the template has a calculator which produces the results of the simulation (Figure 44).

Figure 44.

CALCULATOR				
ITEM	Period Debt to GDP	Required Primary Surplus	Observed Primary Surplus	Primary Surplus Required Adjustment
a) Baseline	71.63%	3.57%	-3.52%	7.34%
b) a + Change RER	67.18%	3.35%	-3.52%	7.11%
c) b + *i* shock	67.18%	4.71%	-3.52%	8.53%
d) c.+ *g* shock	67.18%	5.44%	-3.52%	9.29%
e) d.+ conting. liab	77.18%	6.25%	-3.52%	10.13%

Under scenario No. 1, which uses long-term averages for *r*, *r* * and *g*, the baseline exercise simply reproduces the results of the *Standard Approach*. The exercise of change in relative prices uses the RER adjustment required to reestablish the equilibrium following an SS in order to value the debt and calculate the primary surplus needed to stabilize debt at the new estimated level. The following exercise adds an increase in interest rates to the previous result, and the fourth exercise adds a reduction in the growth of GDP to the previous result. Lastly, the fifth scenario adds a contingent liability for the calculations made in the previous scenario. This approach also reports the currency mismatch stemming from the results of debt revaluation. The outputs of the SS approach are shown in Figure 45.

Figure 45.

RESULTS ANALYSIS

Mismatch Ratio

Results in %GDP

1. Base Line	m	1.42263

Results Sudden Stop Exercise

Results in %GDP

1. Baseline	Debt Level	71.63%
	Primary Surplus	2.33%
2.Changes in Relatives Prices	Primary Surplus	2.18%
3. Scenario 2 + Interest Rate Shock	Primary Surplus	3.55%
4. Scenario 3 + GDP Growth Shock	Primary Surplus	4.27%
5. Scenario 4 + Conting. Liabilities	Primary Surplus	4.91%

6. Mendoza-Oviedo Approach[20]

a. Description

This approach studies the problem of a government which is usually very adverse to the risk of a collapse in its fiscal expenditure. This leads the government to respect a "Natural Debt Limit" (NDL) equal to the annual value of the primary balance in the case of a fiscal crisis. A “fiscal crisis” is defined, in this context, as a long sequence of adverse shocks to fiscal revenue where public expenditure is adapted to a tolerable minimum. That is, the NDL represents a reliable compromise of having the capacity to pay even in a fiscal crisis.[21] In this respect, if the real level of debt continues over the NDL threshold, the government faces a positive probability of default of its sovereign debt.

To generate a fiscal crisis scenario, the probabilistic approach of Mendoza-Oviedo requires data on: (i) volatility of government revenues, (ii) average levels of revenue and expenditure, (iii) the size of the possible adjustments in government spending in case of falling into a crisis state, (iv) the real interest rate on service of the public debt, and (v) the steady state growth rate of the economy. After these assumptions are established, the model is simulated and a set of possible projections of fiscal revenue is generated. Additionally, with these simulations, it is possible to estimate the probability of reaching the debt threshold in the future.

This approach is based on the following assumptions:

1. The path of government revenue is determined exogenously by a Markov process.
2. There is no currency mismatch, which means that the revenue and debt are denominated in the same currency.
3. The variables added, such as the growth rate of the economy and the interest rate, are known with certainty.

The natural debt limit (value of reliable debt repayment) is adjusted to the following conditions:

[20] Mendoza and Oviedo (2003 and 2007)

[21] As the defenders of this approach they suggest, in general, that the NDC concept is not the same as sustainable debt, which is driven by the probabilistic dynamic of the primary balance (Mendoza and Oviedo, 2004).

$$d \leq d^{*} = \frac{\left(t^{min} - e^{min}\right)}{r - g}(1 + g) \quad (\mathbf{10})$$

where d* represents the threshold value for the debt/GDP ratio; t^{min} is the lowest collection of tax revenue with respect to GDP (according to the moments of its distribution); e^{min} represents a minimum level of state spending to GDP after the country enters into a fiscal crisis in which tax revenues peak and stay at t^{min}.

b. Inputs and assumptions

The main data inputs and the main assumptions for this approach are shown in Figure 46.

Inputs: are fed directly from the "*selected series*" spreadsheet or are automatically calculated. They are located between cells D4:D8.

Assumptions: Only three of these assumptions have to be defined by the user: 1) adjustment in government expenditure (cell D14), 2), volatility of fiscal revenue (cell D15), and 3) persistence of tax revenue (cell D16). The user has a graphic tool to help identify which assumptions may be appropriate for the case under study (Figure 47). The other assumptions are fed directly from the "*selected series*" spreadsheet or are calculated automatically.

Figure 46.

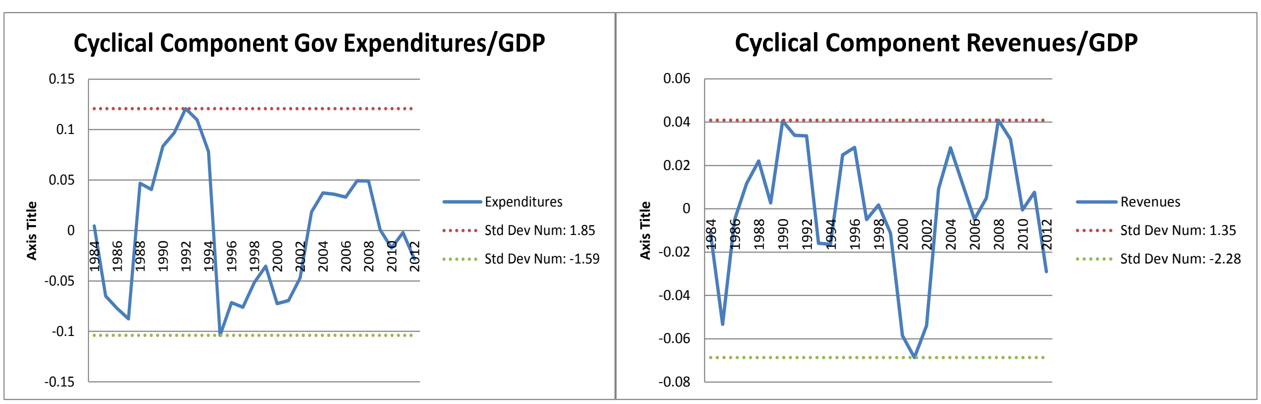
INPUTS	
1+ Real Interest Rate	1.03694
1+ Steady-State GDP Growth	1.02596
Average Levels of non-interest expenditures	24%
Average Levels of Revenues	30%
Initial Level of Debt/GDP	72%

ASSUMPTIONS	
Maximum Expenditures Adjustment	1.59
Adjusted Goverment Expenditure	8.4%
Volatility of goverment revenues	9.0%
Persistence of goverment revenues	1.0%
Std of Minimum levels of Reveneus	2.28
Minimum Levels of Reveneus	10%

OUTCOMES	
Debt Limit	136.7%

SIMULATION PARAMETERS	
Periods	40
Number of States	4
Number of repetitions for the simulations	500

Figure 47.



The main assumptions of the model are the growth rate of the economy in steady state and the long term interest rate. The fiscal variables are also specified in accordance with their levels of steady state and the volatility of revenue. In this respect, expenditure is determined as average levels of expenditure without interest.

The mean levels of revenue and non-interest expenditure as part of GDP are calculated from a data series. The persistence of fiscal revenue in this example is measured as the first order autocorrelation coefficient (adjusting AR (1)) of the cyclical component of revenue/GDP, which in turn is obtained by applying a Hodrick-Prescott filter to the original series.22. The standard deviation of the autoregressive process is taken as the measure of the volatility of public revenue. These two estimates of persistence and volatility constitute the Markov process in the revenue simulations.

With this data, the minimum revenue level can be calculated, taking the mean level and subtracting two standard deviations from it. However, the model also calculates the same value endogenously, using the conditions of the autoregressive process. The NDL is then obtained by means of equation (10).

The maximum adjustment of expenditure in the model is exogenous. The user can calibrate the maximum adjustment of expenditure taking into account the minimum level of collection t^{min} and the highest debt to GDP ratio based on its historical path $\hat{d}$. Therefore, the maximum adjustment of expenditure is calculated as follows:

$$e^{min} = t^{min} - \hat{d}\frac{(r-g)}{(1+g)}$$

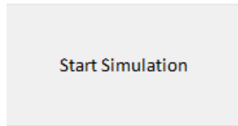
Another way of fixing the maximum adjustment of expenditure is to subtract from the historical value the standard deviations corresponding to the worst result of the expenditure/GDP ratio.

Lastly, the user can specify the number of simulation periods (cell D17), number of states (cell D18) and repetitions for the simulation procedure (cell D19).[23] These values are predetermined in 40, 4 and 500, respectively. The user must then click on the *"Start Simulation"* box (Figure 48).

[22] This particular selection of measures, however, should not be considered as a straight jacket. An alternative is to use the revenue series in levels, at constant prices, and not as portion of GDP, as done by Díaz-Alvarado, Izquierdo and Panizza (2004).

[23] For more detail see the Appendix.

Figure 48



c. Outputs

The model produces the following results:

i. Debt limits with the sensitivity analysis.
ii. Probability of reaching the debt Threshold in *n* periods.

i. Debt limits with the sensitivity analysis.

As mentioned previously, the debt limit is calculated using equation (10). As shown in Figure 49, cells G2:N17 contain a sensitivity analysis with different values for the revenue (in standard deviations) and the expenditure adjustment. The value of the center of the matrix coincides with the value of cell D24 (“*Debt Limit*”) which is the main result of the model.

Figure 49.

DEBT LIMIT THERESHOLDS

	Adjustment of Expenditures						
Std. Dev. Revenues	**7.43%**	**7.76%**	**8.08%**	**8.41%**	**8.73%**	**9.06%**	**9.38%**
10.23%	262%	231%	201%	170%	140%	110%	79%
10.11%	250%	220%	190%	159%	129%	98%	68%
9.99%	239%	209%	178%	148%	118%	87%	57%
9.87%	228%	197%	167%	137%	106%	76%	45%
9.75%	217%	186%	156%	125%	95%	65%	34%
9.63%	205%	175%	145%	114%	84%	53%	23%
9.51%	194%	164%	133%	103%	73%	42%	12%

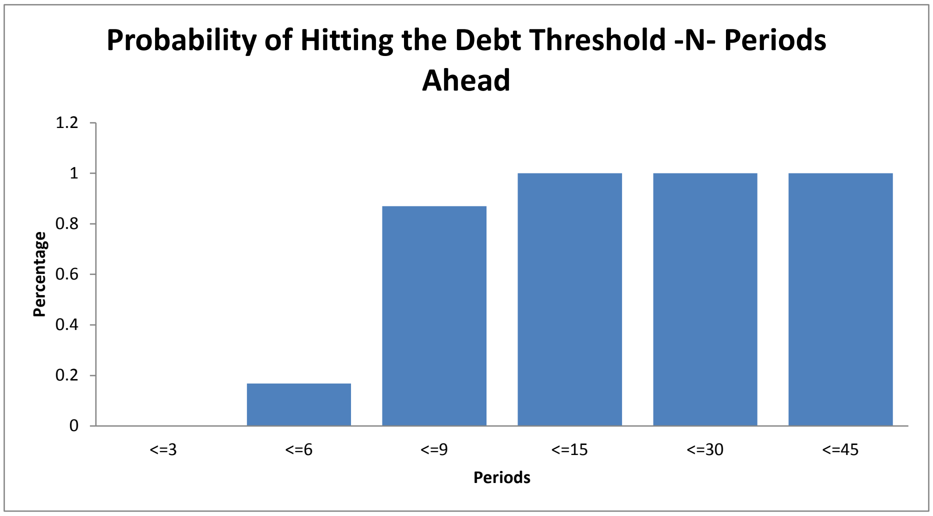
Reveneus		Adjustment of Expenditures	
Standard Deviations	**Revenue Deviation Steps**	**Standard Deviations**	**Expenditures Deviation Steps**
0.04	3	0.05	3

This table provides data on the sensitivity of the debt limit according to the presence of higher (or lower) uncertainty and high (or low) flexibility in the expenditure adjustment after a crash.

ii. Probability of reaching the threshold in "n" future periods.

Figure 50 presents the probabilities that the debt and GDP ratios exceed the NDL threshold in some of the periods projected. In short, the idea is to take the last debt level observed as the initial value and calculate the distribution of the relative frequency of public debt over the GDP ratio for n periods ahead. In this way this frequency distribution can be used to estimate the probability of entering into a fiscal adjustment.

Figure 50.



NOTE: The main source of uncertainty in this approach comes from government revenue. It also omits other sources of uncertainty which can be pertinent for certain countries. The M-O model is very sensitive to the adjustment of public expenditure which is specified by the user, so it is important to "calibrate" this value, based on each country's historical experience.

7. Special Case

BOX 1. DOLLARIZED ECONOMY

In the case of a dollarized economy the following points need to be taken into account:

Selected Series

- The user must make the assumption that $= 1$, therefore in the alpha column (column R) there can only be a 1 in the last cell of historical information.
- As a result, the real interest rate of cell J6 depends only on the interest rate of the debt denominated in local currency.

Debt Analysis

- The equation of the debt dynamic is reduced to:

$$d_t = \left[\alpha\left(\frac{1+r_t^d}{1+g_t^d}\right)\right] d_{t-1} - f_t$$

Fan Chart

- Because debt composition is fixed during the entire time period, the alpha option must consider scenario 2; thus the estimate of the four approximations of the Fan Chart model (external projections, VAR Model, external projections with correlated errors, and weighted projections) must run with Alpha Option =2
- Additionally, given that in a dollarized economy, it is as if the exchange rate were irrevocably fixed, the coefficient matrix and the VAR-COVAR matrix are completed with 0s in the rows and columns that relate to the change in the exchange rate.

Sudden Stop

- Although from the practical point of view it is possible to implement this model, from the conceptual point of view, it is important for the user to bear in mind the assumptions of the model and its limitations. In the template, the (simplified) assumption is made that the foreign currency debt is a proxy for the debt in terms of tradable goods, and that the local currency debt is a proxy for the debt in non-tradables. However, it does not work like this in the case of a dollarized economy.
- Despite this reservation, if the user still wishes to implement the model, the value of α , which is in cell C8, must be equal to 100%, so the user must not enter additional information on the *"Selected Series"* spreadsheet.

8. Guidelines

a. General

1. A problem common to all approaches is the lack of benchmarks to determine which paths are stable or unstable. In some of the cases the problem is not entirely clear. The WB-IMF debt sustainability framework, for example, only establishes thresholds for external debt based on different proportions. A more detailed analysis is recommended for the thresholds for external and domestic debt in Latin America and the Caribbean.
2. Each approach emphasizes a specific aspect of debt sustainability. The more cautious the selection of input data, and their interpretation and the joint analysis of all the approaches, the more reliable the results of the DSA exercise.
3. All econometric procedures (VAR (1) in the fan charts, and AR (1) for revenue in M-O) can easily be run in E-Views or Stata.
4. There is not usually a single bibliographic reference for every model. We have selected a set of five or six documents where each approach, with their methodologies and exercises, are clearly explained. It is important to understand what each approach is actually doing and why, in order to properly interpret the results and consequently adapt and/or adjust the template (including the code) to the specific needs of the analyst
5. All the programming for the fan charts and for the M-O approach (those which use simulations) is in totally open and accessible VBA code, with their respective comments and observations in order to facilitate changes. VBA permits the introduction of "inspections" and various other cleansing tools during execution of the code which are very useful in controlling the values generated by the exercises.

b. Practical:

1. "Suggestions" on how to calculate interest rates in different ways: as interest rate data are often scarce, it is important to clearly establish what is acceptable (for example,

the method of the implicit interest rate) vs. what it is not (for example, simply using a domestic monetary policy rate).

2. There are many definitions of debt which can be confused with each other. These definitions of debt include external vs. domestic debt, debt denominated in foreign currency against debt denominated in local currency, and public versus private debt. Sometimes some of these subsets overlap (for example, domestic debt in foreign currency). For all practical purposes, in this paper we refer only to total public debt (including private debt with public guarantee) and we can break down the data by currency composition. The user must be aware that, to the extent that composition of the debt is broken down into different currencies, each series must have its own interest rate. In some cases, an aggregation rule, although ad hoc, must be defined.

Bibliography

Arizala, F., E. Cavallo, C.E. Castro and A. Powell. 2008. "Debt Sustainability Fan Charts: Combining Multivariate Regression Analysis and External Forecasts." Washington, DC, United States: Inter-American Development Bank, Research Department. www.eduardocavallo.com/FC_05_05_2009_EC_FA.pdf

Blanchard, O.J., 1990. "Suggestions for a New Set of Fiscal Indicators." OECD EconomicDepartment Working Papers 79. Paris, France: OECD Publishing.

Borensztein, E. and U. Panizza, 2009. "The Costs of Sovereign Default." IMF Staff Papers, Palgrave Macmillan Journals, vol. 56(4), pages 683-741.

Buiter, W.H. 1985. "A Guide to Public Sector Debt and Deficits." *Economic Policy* 1(1): 13-79.

Calvo, G., A. Izquierdo and E. Talvi. 2003. "Sudden Stops, the Real Exchange Rate and Fiscal Sustainability: Argentina's Lessons." In: V. Alexander, J. Mélitz and G.M. von Furstenberg, editors. *Monetary Unions and Hard Pegs*. Oxford, United Kingdom: Oxford University Press. Also available as NBER Working Paper 9828.

Celasun, O., X. Debrun and J.D. Ostry. 2006. "Primary Surplus Behavior and Risks to Fiscal Sustainability in Emerging Market Countries: A 'Fan-Chart' Approach." *IMF Staff Papers* 53(3).

Clemen, R.T., and Winkler. 1990. "Unanimity and Compromise among Probability Forecasters." *Management Science* 36: 767–779.

Clemen, R. T., & Winkler, R. L. 1999. "Combining probability distributions from experts in risk analysis." Risk Analysis 19: 187-203.

Díaz-Alvarado, C., A. Izquierdo and U. Panizza. 2004. "Fiscal Sustainability in Emerging MarketCountries with an Application to Ecuador." IDB Research Department Working Paper511. Washington, DC, United States: Inter-American Development Bank.

Escolano, J. 2010. "A Practical Guide to Public Debt Dynamics, Fiscal Sustainability, and Cyclical Adjustment of Budgetary Aggregates." Technical Notes and Manuals International Monetary Fund, 6.

García, M., and R. Rigobón, 2004, "A Risk Management Approach to Emerging Market's Sovereign Debt Sustainability with an Application to Brazilian Data." NBER

Working Paper 10336. Cambridge, United States: National Bureau of Economic Research.

Inter-American Development Bank (IDB). 2007. *Living with Debt: How to Limit the Risks of Sovereign Finance*. Economic and Social Progress in Latin America, 2007 Report. Washington DC, Inter-American Development Bank.

Mendoza, E., and P.M. Oviedo. 2003. “Public Debt Sustainability under Uncertainty.” Washington, DC, United States: Inter-American Development Bank. Mimeographed document.

-----. 2004. “Public Debt, Fiscal Solvency and Macroeconomic Uncertainty in Latin America: The Cases of Brazil, Colombia, Costa Rica, and Mexico.” NBER Working Paper 10637. Cambridge, United States: National Bureau of Economic Research.

Osterholm, P. 2006. “Incorporating Judgement in Fan Charts.” Finance and Economic Discussion Series 2006-39. Washington, DC, United States: Federal Reserve Board. http://www.federalreserve.gov/pubs/feds/2006/200639/index.html

Penalver, A., and G. Thwaites. 2004. “Fiscal Rules for Debt Sustainability in Emerging Markets: The Impact of Volatility and Default Risk.” Working Paper 307. London, United Kingdom: Bank of England.

Sims, C.A. 2002. “The Role of Models and Probabilities in the Monetary Policy Process.”*Brookings Papers on Economic Activity* 2002(2): 1-62.

Tauchen, G. 1986. “Finite State Markov-Chain Approximations to Univariate and Vector Autoregressions.” *Economics Letters* 20(2): 177-181.

APPENDIX

A1. The VAR Approach

Consider the following model:

$$Y_t = \mu_0 + \sum_{k=1}^{p} \mu_k Y_{t-k} + \xi \quad \text{where} \quad Y_t = (r_t, g_t, f_t, \pi_t, e_t);\ \ \xi \propto N(0, \Omega) \qquad (\mathbf{3})$$

Y is a vector that contains the main variables used for forecasting, ξ is a vector error, Ω is the variance-covariance matrix and μ_k is a vector of coefficients. Then 1,000 shocks are simulated for each variable in the VAR, providing a vector of dimension 1,000 x N. Therefore the simulated paths are:

$$x_t^s = x_t^{VAR} + \eta_t$$
$$\eta_{t+1}, \dots \eta_t \; for \; \tau \in [t+1, T] \quad (\mathbf{4})$$
$$\eta_t = W v_t \; where \; v_t \propto N(0,1)$$

The simulated paths come from the estimation of the VAR and the errors generated. *W* is calculated using the Cholesky decomposition of the variance-covariance matrix Ω. The variance-covariance matrix measures the dispersion of the forecast of the Fan Chart. In the following step, these results are used as inputs in the debt equation, whereby we obtain 1,000 paths of the debt-output ratio.

The final step is the construction of the Fan Chart. We first choose the forecast horizon (e.g., T =5), then repeat the procedure with 1,000 new shocks for each period and plot the standard deviation of the forecast.

A2. External Forecast

While the VAR approach requires a substantial amount of data, including periodicity, such data are not always available. In other cases, moreover, it may be desirable to analyze the effect of policies using exogenous variables in the debt equation. Therefore this part of the DSA

includes two components. The first, addressed in this subheading, assumes that all variables are
exogenous and come from the external forecast. In this sense the simulated path is based on:

$$x_\tau^s = x_\tau^{Ext} + e_t, for\ \tau \in [t+1, T]\ where\ e \propto N(0, \overline{\sigma^2}) \quad \textbf{(5)}$$

where x_τ^{Ext} is the external projection using WEO data of WEO or countries' official projections and e is a vector of simulated errors which are not correlated.

A3. External Forecast with Correlated Errors

This is essentially the same approach as described immediately above, the only difference being the inclusion of correlated errors in the law of motion of debt. The structure of the VAR is used to extract the correlation, and the simulated paths are estimated by:

$$x_\tau^s = x_\tau^{Ext} + \eta_t, for\ \tau \in [t+1, T]\ where\ e \propto N(0, \overline{\Omega})$$

where $\overline{\Omega}$ is equal to the matrix variance-covariance of the VAR residual.

A4. Weighted Projections

This case is a combination of the previous cases in which the external forecast is used with zero variances and the VAR forecast:

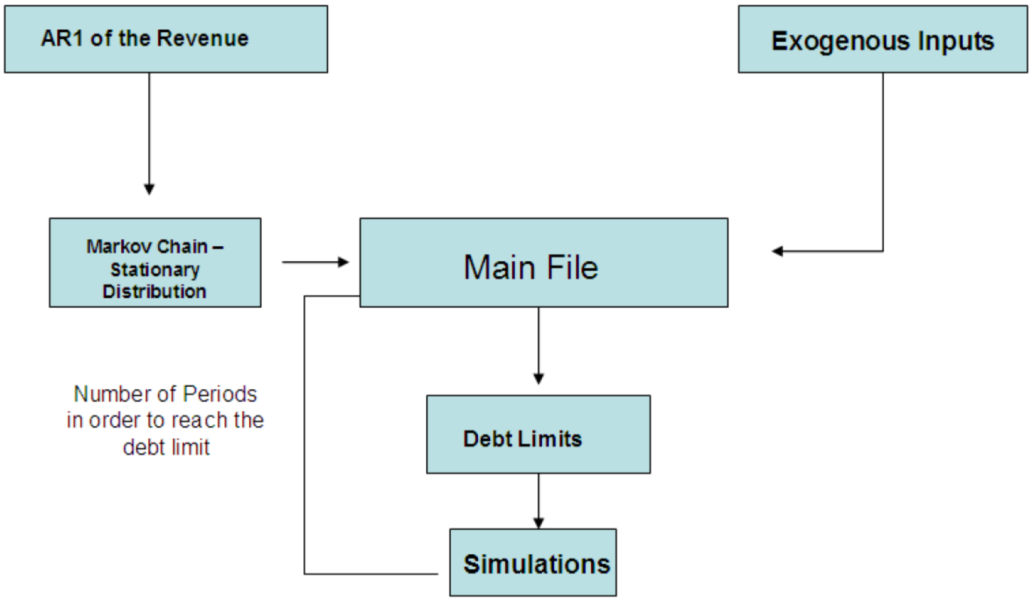
$$x_\tau^s = B^* x_\tau^{Ext} + (1-\beta)^* x_\tau^{VAR} + \eta_t, for\ \tau \in [t+1, T]\ where\ e \propto N(0, \overline{\Omega}) \quad \textbf{(6)}$$

where β is the weighted $\beta \in [0,1]$. When $\beta = 1$, the case collapses to the case with external projection and correlated errors. In the case where $\beta = 0$, the case reproduces the model with VAR projections.

Table A1. Comments on the Fan Chart Approach

Approach	+/-/?	Observation
VAR	+	1. It better captures the structural paths present in the data
		2. It allows for improvements on the baseline econometric estimation technique (for example, with the use of Structural and Bayesian VARs).
		3. There are more and better known measures of performance for the baseline econometric estimation models.
		4. VAR models are flexible and usually have a better fit than alternative approaches.
	-	5. As the VAR model is purely backward-looking, it is not equipped to take into account structural brakes in policies or other structural changes.
		6. Uncertainty and instability in the standard models: the unrestricted VAR approach and the estimated var-cov matrix can be unstable, particularly when applied to emerging economies. Dynamics are different in tranquil times and bad times.
		7. Usually the unrestricted VAR models demand higher-than-annual frequency data (typically quarterly data). Budgetary data of such frequency are often either unavailable or unreliable for the purpose of policy evaluation (Celasun et al., 2006).
	?	8. The central path is completely determined by the econometric model.
External projections	+	1. Forward-looking. Allows the researcher to incorporate possible structural brakes or policy shifts immediately.
		2. Easy implementation. Does not require estimating a VAR.
		3. Flexible introduction of additional shocks (e.g., natural disasters inBorensztein et al. 2009).
	-	4. It is less sensitive to structural paths than econometric models.
		5. Very sensitive to the historical volatility of the series. General problem in developing countries: to include or not to include crisis years?
		6. The uncertainty introduced with the simulation is only attached to empirical facts through the historic volatility of the series. There is no recognition of interactions or correlations among the variables that go into the debt equation (e.g., in many developing countries the lack of automatic stabilizers could be highly relevant).
	?	7. The central tendency is completely driven by external projections. It would be positive or negative depending on the relative importance of structural paths vs. structural brakes.
		8. The whole focus mainly relies on the external projection.
External projections w i	+	1. Has all the positive elements of the external projections approach, plus the additional advantage of the introduction of correlated errors.
	-	2. It is less sensitive to structural paths than econometric models are.
		3. Var-cov matrix could be very unstable during periods of crisis.
Weighted VAR	+	1. In general, this approach combines some of the strengths of econometric-based techniques and of techniques based on external projections. Many of the weak points in the VAR approach are solved by external projections with correlated errors, and vice versa.
	+	2. The importance of a model-consistent consideration of expert subjective beliefs is generally recognized. (See, for example, the discussion of Sims (2002) on the BoE's FC generation process.)
	-	4. Ad hoc selection of the weighted parameter. There are some alternatives in the literature that can be explored; see, for example, Clemen and Winkler (1990 and 1997) and Österholm (2006).
	-	5. It is a less standard approach, so new testing and evaluations tools have to be developed.

A5. Structure of the Mendoza-Oviedo Model



A6. "Discretization" of the State Space: Tauchen (1986) Method

Consider the following stochastic process

$$Z_{t+1} = \rho Z_t + \varepsilon_t \qquad \varepsilon_t \propto N(0, \sigma_\varepsilon^2)$$

The mean is zero and the variance is $\frac{\sigma_\varepsilon^2}{1-\rho^2}$. The idea is to create a grid of equidistant points in which:

$$\Omega = [z_1, z_2, \dots, z_m] \quad where \;\; z_1 < z_2, \dots, < z_m$$
$$such\ that\ z_m = \lambda\sigma_z \qquad z_1 = -z_m$$

where the realization of a given variable is:

$$\text{z} = \rho z_i + \varepsilon_i \;\; \propto N(\rho z_i, \sigma_\varepsilon^2)$$

Let *dz* represent the distance between two points in the grid. The objective is to find that the probability of *z* is in the interval:

This means the probability taking into account the distance between the two points is:

$$prob\left(z_i - dz \le z \le z_j + dz\right) = \pi\left(z_j + dz\right) - \pi(z_j - dz)$$

Therefore the probability between different states is given by the probability distribution in the following support:

$$\left[\frac{z_j - \rho z_i - dz}{\sigma_e}, \frac{z_j - \rho z_i + dz}{\sigma_\varepsilon}\right]$$

Algorithm

In order to "discretize" the state-space of the tax revenues in a Markov chain, the Mendoza-

Oviedo approach uses the algorithm described below.

1) Fix σ, ρ and choose the size of grid λ. The first component of the grid is a multiple of the standard deviation of the process:

$$z_1 = \frac{-\lambda\sigma_\varepsilon}{\sqrt{-1\rho^2}}$$

2) The number of grid points (m) and the different steps between states are chosen

$$step = \frac{-2z_i}{(m-1)} \qquad z_i = z_1 + (i+1)step$$

3) The transition distribution is calculated as follows:

$$P = p_{i,j}$$

$$p_{i1} = \pi\left(\frac{z_1 - pz_i}{\sigma_\varepsilon} + \frac{step}{2\sigma_\varepsilon}\right)$$

$$p_{ij} = \pi\left(\frac{z_j - pz_i}{\sigma_\varepsilon} + \frac{step}{2\sigma_\varepsilon}\right) - \pi\left(\frac{z_j - pz_i}{\sigma_\varepsilon} + \frac{step}{2\sigma_\varepsilon}\right)$$

$$p_{im} = 1 - \sum_{j=1}^{m-1} p_{ij}$$

A7. Index of Matrix Library in VB- Excel

See the webpage http://digilander.libero.it/foxes/Matrix_Functions.htm

MATRIX
Matrix and Linear Algebra Functions for EXCEL

Macros List of MATRIX add-in ver. 2.3 (update of 1-12-2006)

Function/Sub Name	Description
GJstep	Gauss-Jordan algorithm step by step
MAbs	Norm of vector or matrix
MAbsC	Norm of complex vector or matrix
MAdd	Matrix addition
MAddC	Complex matrix addition
MBAB	Similarity transform [B]*[A]*[B]^-1
MCond	Matrix condition number
MpCond	Log10 of matrix condition number
MDet	Determinant
MDetC	Determinant for complex matrix
MDet3	Determinant for tridiagonal matrices
MDetPar	Parametric Determinant
MDiag	Diagonal matrix
MExp	Matrix exponential e^[M]
MExpErr	Error of matrix exponential
MIde	Matrix Identity (I)
MInv	Matrix inverse [A]^-1
MInvC	Complex Matrix inverse [A]^-1
MMultC	Complex matrix multiplication

MMultTpz	Toeplitz matrix and vector multiplication
MMult3	Tridiagonal matrix multiplication
MPow	Matrix power [A]^n
MPowC	Complex matrix power [A]^n
MProd	Matrix product [A]*[B]*[C]*….
MMultS	Matrix scalar multiplication
MMultsC	Complex matrix scalar multiplication
MRank	Rank of matrix
MSub	Matrix subtraction
MSubC	Complex matrix subtraction
MTrace	Matrix Trace
MAdm	Admittance matrix of a linear passive network
MBlock	Block-partitioned matrix
MBlockPerm	Permutation vector of block-partitioned matrix
MCholesky	Cholesky decomposition
MHessemberg	Hessemberg form
MHilbert	Hilbert's matrix
MHilbertinv	Inverse Hilbert's matrix
MHouseholder	Householder matrix
MLeontInv	Leontief inverse matrix for Input Output Analysis
MLU	LU decomposition
MPseudoinv	Moore-Penrose pseudo-inverse
MQH	QH decomposition
MQR	QR decomposition
MQRiter	Diagonalization with the QR iterative method

MTartaglia	Tartaglia’s matrix
MVandermonde	Vandermonde’s matrix
MChar	Characteristic matrix
MCharC	Complex Characteristic matrix
MCharPoly	Characteristic polynomial coefficients
MCharPolyC	Complex characteristic polynomial coefficients
MCmp	Companion matrix
MCorr	Correlation matrix
MCovar	Covariance matrix
MCplx	Converts 2 real matrices into a complex matrix
MDiagExtr	Diagonal extractor
MEigenSortJacobi	Sorts eigenvectors for eigenvalues amplitude
MEigenvalJacobi	Eigenvalues of symmetric matrix with Jacobi algorithm
MEigenvalMax	Dominant eigenvectors with power method
MEigenvalPow	Eigenvectors with power method
MEigenvalQL	Eigenvalues of tridiagonal matrix
MEigenvalQR	Eigenvalues of real matrix with QR algorithm
MEigenvalQRC	Eigenvalues of complex matrix with QR algorithm
MEigenvalTTpz	Eigenvalues of tridiagonal Toeplitz matrix
MEigenvec	Eigenvector
MEigenvecC	Complex eigenvector
MEigenvecJacobi	Eigenvectors of symmetric matrix with Jacobi algorithm
MEigenvecMax	Dominant eigenvalues with power method
MEigenvecPow	Eigenvalues with power method
MEigenvecT	Eigenvectors of tridiagonal matrix

MEigenvecInv	Eigenvector with inverse algorithm
MEigenvecInvC	Complex eigenvector with inverse algorithm
MExtract	Extract sub-matrix
MMopUp	Eliminates round-off errors
MNorm	Vector or Matrix Norm
MNormalize	Vectors Normalization
MNormalizeC	Complex Vectors Normalization
MOrthoGS	Orthogonalization with Gram-Schmidt algorithm
MPerm	Permutation matrix
MRnd	Random matrix
MRndEig	Random matrix with given eigenvalues
MRndEigSym	Random symmetric matrix with given eigenvalues
MRndRank	Random matrix with given rank or determinant
MRndSym	Random symmetric matrix with given rank or det.
MRot	Orthogonal planar rotation matrix
MRotJacobi	Jacobi rotation matrix
MT	Matrix transpose
MTC	Complex Matrix transpose
MTH	Complex Matrix transpose-conjugate
PathFloyd	All-pairs-path matrix with Floyd algorithm
PathMin	Shortest path with Floyd algorithm
PolyRoots	Polynomial rootfinder (Siljak+Ruffini method)
PolyRootsQR	Polynomial rootfinder (QR method)
PolyRootsQRC	Complex polynomial rootfinder (QR method)
ProdScal	Scalar Product (inner)

ProdScalC	Complex scalar product
ProdVect	Vector Product
RegrCir	Circular regression
RegrL	Linear regression with SVD method
RegrP	Polynomial regression with SVD method
Simplex	Linear Optimization with Simplex algorithm
SVDD	SVD: returns D matrix
SVDU	SVD: returns U matrix
SVDV	SVD: returns V matrix
SysLin	Linear System solving
SysLinC	Complex Linear System solving
SysLinIterG	Linear System solving with Gauss-Seidel SOR algorithm
SysLinIterJ	Linear System solving with Jacobi algorithm
SysLinT	Triangular linear system solving
SysLinTpz	Toeplitz Linear System solving
SysLin3	Tridiagonal linear system solving
SysLinSing	Singular Linear System solving
TraLin	Linear Transform
VarimaxIndex	Varimax index of a given Factors matrix
VarimaxRot	Orthogonal rotation with Varimax Kaiser’s
VectAngle	Angle between the two vectors v1, v2
Sub	Matrix Generator
Sub	Macro Toepliz matrix generator
Sub	Macro Shortest-Path
Sub	Macro Draw Graph

Sub	Macro Block reduction
Sub	Macro Matrix Operations
Sub	Macro Complex Matrix Operations
Sub	Macro Random matrix with given eigenvalues
Sub	Matrix Clean-up
Sub	Macro Matrix Round
Sub	Macro Sparse Matrix Operations
Sub	Macro Random sparse matrix