IDB WORKING PAPER SERIES Nº IDB-WP-782

Instrument Choice and Stranded Assets in the Transition to Clean Capital

Julie Rozenberg
Adrien Vogt-Schilb
Stephane Hallegatte

Inter-American Development Bank
Climate Change and Sustainability

March 2017

Instrument Choice and Stranded Assets in the Transition to Clean Capital

Julie Rozenberg
Adrien Vogt-Schilb
Stephane Hallegatte

March 2017

Cataloging-in-Publication data provided by the
Inter-American Development Bank
Felipe Herrera Library

Rozenberg, Julie.
Instrument choice and stranded assets in the transition to clean capital /
Julie Rozenberg, Adrien Vogt-Schilb, Stephane Hallegatte.
p. cm. — (IDB Working Paper Series ; 782)
Includes bibliographic references.
1. Climatic changes-Government policy. 2. Climatic changes-Risk assessment. 3. Climatic changes-Economic aspects. I. Vogt-Schilb, Adrien. II. Hallegatte, Stéphane. III. Inter-American Development Bank. Climate Change Division. IV. Title. V. Series.
IDB-WP-782

http://www.iadb.org

Copyright © 2017 Inter-American Development Bank. This work is licensed under a Creative Commons IGO 3.0 Attribution-NonCommercial-NoDerivatives (CC-IGO BY-NC-ND 3.0 IGO) license (http://creativecommons.org/licenses/by-nc-nd/3.0/igo/legalcode) and may be reproduced with attribution to the IDB and for any non-commercial purpose, as provided below. No derivative work is allowed.

Any dispute related to the use of the works of the IDB that cannot be settled amicably shall be submitted to arbitration pursuant to the UNCITRAL rules. The use of the IDB's name for any purpose other than for attribution, and the use of IDB's logo shall be subject to a separate written license agreement between the IDB and the user and is not authorized as part of this CC-IGO license.

Following a peer review process, and with previous written consent by the Inter-American Development Bank (IDB), a revised version of this work may also be reproduced in any academic journal, including those indexed by the American Economic Association's EconLit, provided that the IDB is credited and that the author(s) receive no income from the publication. Therefore, the restriction to receive income from such publication shall only extend to the publication's author(s). With regard to such restriction, in case of any inconsistency between the Creative Commons IGO 3.0 Attribution-NonCommercial-NoDerivatives license and these statements, the latter shall prevail.

Note that link provided above includes additional terms and conditions of the license.

The opinions expressed in this publication are those of the authors and do not necessarily reflect the views of the Inter-American Development Bank, its Board of Directors, or the countries they represent.

Instrument Choice and Stranded Assets in the Transition to Clean Capital

By Julie Rozenberg*, Adrien Vogt-Schilb†, and Stephane Hallegatte‡

To mitigate climate change, some governments opt for instruments focused on investment, like performance standards or feebates, instead of carbon prices. We compare these policies in a Ramsey model with clean and polluting capital, irreversible investment and a climate constraint. Alternative instruments imply different transitions to the same balanced growth path. The optimal carbon price minimizes the discounted social cost of the transition to clean capital, but imposes immediate private costs that disproportionately affect the current owners of polluting capital, in particular in the form of stranded assets. A phased-in carbon price can avoid stranded assets but still result in a drop of income for the owners of polluting capital when it is implemented. Second-best standards or feebates on new investment lead to higher total costs but avoid stranded assets, preserve the revenues of vested interests, and smooth abatement costs over individuals and time. These results suggest a trade-off between political feasibility and cost-effectiveness of environmental policies.

JEL: L50, O33, O44, Q52, Q54, Q58
Keywords: public acceptability, policy instrument, climate change, intergenerational equity, energy efficiency standards, mothballing, early-scrapping, premature retirement

This version: March 2017

---

* World Bank, Washington DC, jrozenberg@worldbank.org.
† Corresponding author, Inter-American Development Bank, Washington DC, avogtschilb@iadb.org.
‡ World Bank, Washington DC, shallegatte@worldbank.org.

The authors thank Mook Bangalore, Patrice Dumas, Marianne Fay, Louis-Gaetan Giraudet, Gilles Lafforgue, Ian Parry, Aude Pommeret, Baptiste Perissin-Fabert, Xavier Thimbault and the participants of the RFF seminar, of the 2013 EAERE and AFSE conferences and of the 2014 AEA and SURED conferences for their useful comments on previous versions of this article. All remaining errors are the authors'. The views expressed in this paper are the sole responsibility of the authors. They do not necessarily reflect the views of the Inter-American Bank or the World Bank, their respective boards of directors, or the countries they represent.

For the past centuries, economic growth has involved the accumulation of fossil-fueled capital, such as coal power plants and gasoline-fueled cars, which release greenhouse gases (GHG) to the atmosphere. To stop the resulting climate change and subsequent damages, economies now have to reduce emissions to near-zero levels (IPCC, 2014). Doing so implies a transition from production based on polluting capital to production based on clean, carbon-neutral capital. In principle, the optimal policy to enforce such a transition is to use a carbon price (Pigou, 1932; Nordhaus, 1991; Pearce, 1991), imposed through a carbon market or, perhaps preferably (Goulder and Schein, 2013), a carbon tax. Combined with targeted innovation policies, a carbon price could redirect investment away from polluting and towards clean capital at a relatively low cost (IPCC, 2014).

However, political economy issues may obstruct the implementation of substantial carbon prices (Fay et al., 2015; Jenkins, 2014; Bertram et al., 2015). Indeed, existing capital in the transportation, building, industry, and energy sectors have lifetimes that range from about a decade — in the case of a car — to half a century — for power plants — or even centuries — for city shapes and transportation systems (Davis, Caldeira and Matthews, 2010; Guivarch and Hallegatte, 2011; Sachs et al., 2014). Absent any alternative in the short term, a sudden change in prices induced by environmental taxes may result in the creation of *stranded assets* — polluting capital that has to be discarded prematurely because its continued use is not compatible with climate policies.[1] For instance, Johnson et al. (2015) estimate that a carbon price consistent with the 2°C target will strand at least 165 billion US dollars worth of coal power plants worldwide. In a recent speech, the Governor of the Bank of England has expressed concern that the magnitude of those stranded assets could be a threat for the stability of the financial system (Carney, 2016).

Stranded assets translate into a visible loss of wealth concentrated in a few vested interests, whose owners may oppose the reform — and in some cases may even have the power to veto it (Olson, 1977; Trebilcock, 2014). Furthermore, stranded assets result in a short-term drop of production, and thus income, for the whole economy. In other words, stranded assets set immediate costs on the present generation (and present voters) to the benefits of future generations. In fact, a literature on public attitudes towards environmental taxes suggests that aversion to carbon taxes is partly driven by (i) the perception that they are inefficient, absent clean alternatives to polluting activities; and (ii) the perception that they are unfair, as their cost is perceived to fall disproportionately on a few actors (e.g., Dresner et al., 2006; Kallbekken and Sælen, 2011; Harrison and Peet,

---

[1] The words *stranded assets* are used in the literature on climate change to describe various things: assets that are lost because of the impact of climate change itself (Caldecott et al., 2016), fossil fuel resources that cannot be burnt into the atmosphere if a given climate target is to be reached (Asheim, 2012), also called *unburnable carbon* (Jakob and Hilaire, 2015); and man-made capital that has to be retired early because of climate policies, such as coal power plants that become unprofitable after a carbon price is implemented (Guivarch and Hood, 2010). This paper focuses on stranded man-made capital.

2012). Stranded assets are closely related to both issues, as they are a symptom of limited availability of clean alternatives in the short-term, and they translate into immediate costs concentrated on a few actors.

This paper uses a simple model to investigate how alternative policy instruments may reduce stranded assets—and more generally, concentrated and immediate costs—in the transition to low-carbon capital. We focus on the effect of instruments such as corporate average fuel economy (CAFE) standards in the automobile industry, efficiency standards or mandates for new power plants, buildings and appliances, *feebate* programs that tax energy-inefficient equipment and subsidize energy-efficient equipment, or subsidized loans and tax breaks for energy efficiency investment. All these instruments are similar in that they redirect private investment away from polluting capital and toward clean capital without affecting the existing stock of polluting capital, for instance without providing incentive to drive less or operate existing gas power plants instead of existing coal power plants (that is, without creating stranded assets).

We analyse how using carbon prices, feebates, or performance standards leads to different costs and dynamics of the transition from polluting to clean capital, with a particular focus on stranded assets and the value of existing capital. We compare the first-best carbon price, designed to reduce the discounted cost of the transition, and second-best feebates or mandates designed to minimize costs while avoiding stranded assets. A motivation for comparing these types of instruments is that while some governments have enacted carbon prices (World Bank, 2016), most existing emission-reduction policies regulate only new investment (IEA, 2016; IPCC, 2014, p. 28). In addition, phased-in carbon prices have been proposed as a way to reduce adjustment costs (Williams, 2011); and actual implementation of carbon prices, for instance in British Columbia and France, have involved a phase-in. We thus also look at the second-best phased-in carbon price designed to minimize costs while avoiding stranded assets.

We use a Ramsey model with two types of capital (as in Acemoglu et al., 2012): *polluting* capital, which creates GHG emissions, and *clean* capital, which does not. We disregard knowledge spillovers and we model the climate change constraint as a GHG concentration ceiling. Investment is assumed irreversible (Arrow and Kurz, 1970): existing polluting capital cannot be converted back into consumption or transformed into clean capital. We however allow for under-utilization of existing polluting capital, a feature that is generally omitted in multi-sector growth models. Under-utilization means that emission-reduction effort can be divided between two qualitatively different channels: (i) long-term abatement through accumulation of clean capital instead of polluting capital (e.g. agents buy electric cars instead of gasoline-fueled cars); and (ii) immediate abatement through the underutilization or early decommissioning of polluting capital (e.g. agents drive less or scrap their gasoline cars).

We find that, irrespective of which type of instrument is used, the marginal cost of the climate change policy decomposes as a *technical cost* — the cost of using clean instead of polluting capital — and a temporary *legacy cost* that quantifies society's regret for excessive past investment in polluting capital. The legacy cost comes directly from the irreversibility of past investment in long-lived capital and the fact that polluting capital can become a liability when emission-reduction policies are implemented.

We also find that all policy instruments lead to the same long-term growth path, in which most installed capital is clean and carbon concentration is maintained at its maximum acceptable value. The optimal carbon price and the second-best phased-in carbon price, feebates, and standards however induce different short-term pathways in terms of emissions and costs, and in particular different levels and distribution of legacy costs.

Unsurprisingly, the carbon price minimizes the total discounted cost of the climate change policy. Under a carbon price, investment is redirected towards clean capital until polluting capital has depreciated to a level compatible with the concentration ceiling. In addition, part of the existing polluting capital is stranded if climate policies are stringent — that is, if the carbon price is larger than the marginal productivity of polluting capital over its carbon intensity. Such outcomes are part of the least-cost strategy, because stranding assets reduces legacy costs created by excessive past investment in polluting assets. But this strategy sets a disproportionate cost on the owners of polluting capital (and the workers who depend on it).[2] Even in the absence of stranded assets, the carbon price ties a new cost to the utilization of existing polluting capital and thus decreases its value.

In contrast, standards or feebates on new investment do not prompt producers to underutilize existing polluting capital, and thus do not create stranded assets, and preserve revenues from existing polluting capital. The second-best phased-in carbon price also avoids stranded assets, but interestingly does not preserve the revenues from existing polluting capital: it adjusts at the level where the owners of current capital are indifferent between renting out their assets or scraping them.

However, feebates, standards and phased-in carbon prices are less efficient that the first-best carbon price. They create higher legacy costs: society keeps using obsolete polluting capacities until the end of their lifetime instead of scrapping them — as if refusing to recognize that past accumulation of polluting capital was a mistake. This strategy imposes further abatement efforts on the next generations and increases total inter-temporal abatement costs. With phased-in carbon prices, as with optimal carbon prices, those legacy costs are paid by the owners of polluting capital in the form of reduced rental rates. But with feebates and standards, legacy costs do not reduce the rental rates of existing capacities: they are shared instead by investors building new capacities (that is ultimately on all household's savings). Compared to what happens with the optimal carbon price, the higher cost of the transition is also spread over time. And whatever the instrument chosen, avoiding stranded assets prevents a drop in

---

[2] By disproportionate we mean in relation to their proportion in the population.

national production and thus national income when the policy is implemented.

These results suggest a trade-off between efficiency and political feasibility of climate change mitigation policies. While they are imperfect instruments from the perspective of intertemporal welfare maximization, policy instruments that focus on redirecting investment towards clean capital (such as CAFE standards, energy efficiency requirements for new appliance and buildings, or feebate programs) may reduce opposition to emission-reduction policies, making them easier to implement than a carbon price in the short term. And as they transform progressively the production system, these second-best instruments may prepare the economy and the public to easier implementation of carbon prices in the medium term. Loosely speaking, policy makers face a choice between (1) a higher intertemporal welfare with the optimal carbon price; and (2) less immediate costs, no stranded assets, and lower potential political costs with feebates, standards, and to a lesser extent phased-in prices.

Finally, another important difference between the two types of instruments is their mere *efficacy*. As they do not lead to decommissioning any polluting capital, second-best feebates, standards and phased-in carbon prices reduce emissions slower than the optimal carbon price, and cannot achieve too stringent GHG concentration targets — while the optimal carbon price could reduce emissions arbitrarily quickly. Empirical evidence suggests that it could still be technically possible to reach the 2°C target while avoinding stranded assets. For instance, Davis, Caldeira and Matthews (2010) estimate that emissions embedded in existing long-lived capital and infrastructure in 2010 committed us to a warming of about 1.3°C. However, findings by Rogelj et al. (2013), Johnson et al. (2015), and Iyer et al. (2015) find that the least-cost pathway toward a 2°C-compliant economy does involve stranding assets. Governments willing to limit global warming below 2°C might still have a choice between first-best carbon prices and second-best feebates, standards, or phased-in prices, and this choice implies a trade-off between minimizing discounted costs and avoiding stranded assets.

The remainder of the paper is structured as follows. Section I details our contribution to specific branches of the literature. Section II presents the basic model. Section III presents the decentralized version of the model and solves for the *laissez-faire* equilibrium. In section IV we analyze the least-cost growth path, that can be obtained with a carbon price. In section V, we solve a social planner program where stranded assets are to be avoided, and we look at how second-best standards, feebates, or a phased-in carbon price can decentralize that constrained transition. In section VI, we study the timing issues and risks of lock-in when stranded assets are avoided. Section VII concludes.

I. Contribution to the Literature

This paper relates to several branches of the literature.

First, while the literature on instrument choice for environmental policy has established that the carbon price is the most efficient instrument (Pigou, 1932;

Goulder and Parry, 2008; Fischer and Newell, 2008),[3] less attention has been paid to their different distributional impacts. Most studies focus on the impact of carbon prices, and look at distribution in terms of different income categories (e.g., Rausch et al., 2010; Fullerton, Heutel and Metcalf, 2012; Coady, Flamini and Sears, 2015; Borenstein and Davis, 2015) or different generations (e.g., Karp and Rezai, 2012). Few papers explore how different policies set costs on different sectors of the economy. One is Fullerton and Heutel (2010), who find in a two-sector static model that the additional welfare cost of performance standards, compared to that generated by a carbon price, is not supported by the dirty sector, but spread over the clean one—Giraudet and Quirion (2008) had reached a similar conclusion in the case of policies that promote energy efficiency. We expand this literature by comparing alternative instruments in the dynamic context of the transition to clean capital, showing that the optimal carbon prices and a second-best phased-in carbon prices, feebates, or standards lead not only to different distribution of costs between sectors but also over time.

By modelling investment and production decisions separately, we also show that feebates and standards are not entirely equivalent to a carbon tax plus a production subsidy — as previous research focused on efficiency impacts has found (e.g., Fischer and Newell, 2008; Holland, Hughes and Knittel, 2009; Fullerton and Heutel, 2010). Feebates and standards operate by influencing investment decisions and do not directly reduce income for the owners of existing polluting capital, while a carbon-tax-plus-subsidy scheme operates by influencing production and does reduce the value of the existing stock of polluting capital.

Second, Goulder, Hafstead and Dworsky (2010) have studied how carbon markets can be designed to compensate firms for stranded assets, and find that under a cap-and-trade system, the *owners* of polluting firms may be fully compensated if a fraction of emissions allowances are grandfathered for free—making all permits free is likely to result in substantial windfalls profits for the owners of polluting capital (e.g., Sijm, Neuhoff and Chen, 2006). Goulder and Schein (2013) note that the same result can be obtained with carefully-designed exemptions under a carbon tax.[4] We expand this literature by comparing carbon prices with alternative instruments: instead of offering compensation to the owners, standards and

---

[3] For instance, the extensive literature on CAFE standards stresses that they do not provide incentive to reduce emissions from the existing fleet (Austin and Dinan, 2005), may even create a rebound effect, worsening the effect of unaddressed externalities such as congestion or emission of local pollutants (Anderson et al., 2011) or slow down capital turnover, reducing the speed at which the new, energy-efficient cars enter the fleet (Jacobsen and van Benthem, 2015). All these important considerations are left out of our model.

[4] It is well established that a potential advantage of carbon pricing schemes over regulations — not captured in our model — is that the remaining revenues from carbon pricing can be used to mitigate policy costs by reducing other distortive fiscal policies (Bovenberg and Goulder, 1996; Parry and Bento, 2000; Metcalf, 2014; Rausch and Reilly, 2015). Standards do not have this feature, but feebates may be net revenue raiser (or net spenders, or be revenue-neutral), and phased-in carbon prices do raise revenue. On the other hand, carbon revenues are rents, and they create the potential for rent seeking and lobbying about their distribution among polluters, clean challengers, and any other organized group in the economy (MacKenzie and Ohndorf, 2012), which may or may not be a substantial problem depending on the strength of institutions and the quality of instrument design.

feebates avoid stranded assets and all their social impacts in the first place. The second-best phased-in carbon price also avoids stranded assets, but it adjusts at the maximum level that reduces revenues from polluting capital to zero, which makes little difference from the point of view of their owners. (The government could still use the revenues anticipated from the carbon phase-in, or grandfather the rights in a phased-in market, to compensate the owners of polluting capital.)

Finally, our paper relates to the literature that studies the transition to a clean economy through the lens of the directed technical change theory (e.g. Gerlagh, Kverndokk and Rosendahl, 2009; Kalkuhl, Edenhofer and Lessmann, 2012; André and Smulders, 2014). This literature studies the policy mix to tackle both the climate change externality and sector-specific knowledge accumulation and spillovers. One finding highlighted by Kverndokk and Rosendahl (2007), Grimaud and Lafforgue (2008) and Acemoglu et al. (2012, 2014) is that, in the short term, the least-cost policy relies relatively more on research subsidies in the clean sector than on carbon prices. The reason is that the most powerful lever to reduce GHG emissions is to encourage a structural transformation of the economy over the long term, not to distort production decisions in the short term. Here we disregard knowledge accumulation and path dependence in that process, and focus instead on another feature of Schumpeterian creative destruction: the accumulation of physical capital and the creation of stranded assets. Our findings suggest that feebates and standard help trigger structural change while avoiding immediate disruption of the old sectors.

II. The Social Planner Model

We consider a Ramsey framework with a representative infinitely-lived household or social planner. At time t, consuming c_t provides the household with a utility $u(c_t)$, where the utility function is increasing with consumption, and strictly concave ($u' > 0$ and $u'' < 0$).

The social planner produces one final good y_t, using two types of available capital: polluting capital k_p (e.g., coal power plants, thermal engine vehicles) and clean capital k_c (e.g., renewable power plants, electric vehicles).

Production is used for consumption (c_t) and investment ($i_{p,t}$ and $i_{c,t}$).

$$y_t = c_t + i_{p,t} + i_{c,t} \tag{1}$$

Investment $i_{p,t}$ and $i_{c,t}$ increase the stock of installed capital, which otherwise depreciates exponentially at rate δ (we assume the same depreciation rate for polluting and clean capital to keep notations simple, but this assumption plays no particular role in the analysis):

$$\dot{k}_{p,t} = i_{p,t} - \delta\, k_{p,t} \tag{2}$$

$$\dot{k}_{c,t} = i_{c,t} - \delta\, k_{c,t} \tag{3}$$

The doted variables represent temporal derivatives. Investment is irreversible (Arrow and Kurz, 1970):[5]

$$i_{p,t} \geq 0 \tag{4}$$

$$i_{c,t} \geq 0 \tag{5}$$

This means that for instance, a coal plant cannot be turned into a wind turbine, and only disappears through depreciation. (We leave capital retrofit to further research.) However, the social planner may use only a portion q_t of installed capital k_t to produce the flow of output y_t given by:

$$y_t = F(A_t, q_{p,t}, q_{c,t}) \tag{6}$$

$$q_{p,t} \leq k_{p,t} \tag{7}$$

$$q_{c,t} \leq k_{c,t} \tag{8}$$

where A_t is the exogenous total factor productivity, assumed to increase at an exponential rate over time. F is thus a classical production function, assumed to satisfy the Inada conditions and exhibit decreasing returns to scale, to which we add the explicit assumption that capital can be underutilized.

The substitutability between clean and dirty capital is not important to derive the results in this paper. We assume for short that they are complementary, and that the marginal productivity of both sorts of capital tends to infinity when q tends to 0; doing so allows omitting corner solutions (with $q = 0$ or $k = 0$) in the exposition.

In the remaining of this paper, q_t will be called *utilized capital* and k_t *installed capital*. The underutilization of installed polluting capital can be optimal when facing a constraint on GHG emissions. For instance, all coal plants in the economy can be operated part-time, or some of them can be shut down, if the utilization of the whole capital stock is conflicting with the climate objective. Both cases are captured in aggregate with $q_{c,t} < k_{c,t}$.[6] In this paper, it turns out that underutilization of clean capital is never optimal, so we omit the difference between $q_{c,t}$ and $k_{c,t}$ for short in the remainder of the paper.

Polluting capital used at time t emits greenhouse gases e_t:

$$e_t = G \times q_{p,t}$$

Our modelling of emissions and production captures parsimoniously the functioning of the most energy-intensive sectors of the economy, which are responsible for the bulk of carbon emissions: power generation, transportation, and buildings (light and air conditioning). In these sectors, greenhouse gas emissions depend

---

[5] Following the wording by Arltesou (1999) and Wei (2003), capital is *putty-clay*.

[6] We left decommissioning cost and maintenance costs out of the model. Both could reduce the opportunity of underutilizing dirty capital.

on the technology *embedded* in existing capital (for instance a given type of coal power power plant, light bulb, or car), and how much the capital is used (how many hours per year a light bulb is on, how much electricity is generated from the plant, how many kilometres are travelled by the car). In particular, we omit labour in the production function because substituting labour for capital (e.g. drivers for taxis, operators for coal plants, or domestic workers for light bulbs) is in general not a prominent option to reduce GHG emissions from existing capital.

GHG atmospheric concentration m_t increases with emissions, and decreases with a dissipation rate ε:

$$\dot{m}_t = G \cdot q_{p,t} - \varepsilon m_t \tag{9}$$

The dissipation rate makes it possible to maintain a small stock of polluting capital in the steady state (again simplifying exposition), but the conclusions hold if we assume $\varepsilon = 0$.

III. Decentralized Model and *Laissez-Faire* Equilibrium

A. Decentralized Model

Here we introduce a decentralized version of the previous model, in which we distinguish a representative household and a producer. We use this model to solve for the *laissez-faire* equilibrium, and derive insights that will be useful in the next sections, to analyse how different policy instruments affect the burden sharing across actors in the economy.

Producers. — In the decentralized version, a representative producer produces one final good, from the polluting and clean capital, using the same production function than in the social planner version. They rent the capital $q_{p,t}$ and $q_{c,t}$ (since it is never optimal to underuse clean capial in this paper, we directly substitute $q_{p,t} = k_{c,t}$ for simplicity) from households at the respective rental rates $R_{p,t}$ and $R_{c,t}$, which they take as given. They thus maximize the following profit:

$$\max_{q} \Pi(q_{p,t}, k_{c,t}) = F(A_t, q_{p,t}, k_{c,t}) - R_{c,t} \cdot k_{c,t} - R_{p,t} \cdot q_{p,t} \tag{10}$$

Capitalist households. — Households make money by renting out a portion q of available capacities k to producers to a rental rate they take as given. They use this revenue to purchases goods for consumption c_t or invest in capacities:

$$R_{c,t} \cdot k_{c,t} + R_{p,t} \cdot q_{p,t} = c_t + i_{p,t} + i_{c,t} \tag{11}$$

In the laissez faire equilibrium, the problem of the household is to maximize discounted welfare under budget and capacity constraints:

$$
\begin{aligned}
(12) \qquad & \max_{c,i,q} \int_0^{\infty} e^{-\rho t} \cdot u(c_t)\, dt \\
\text{subject to } & R_{c,t} \cdot k_{c,t} + R_{p,t} \cdot q_{p,t} = c_t + i_{p,t} + i_{c,t} && (\lambda_t) \\
& \dot{k}_{p,t} = i_{p,t} - \delta k_{p,t} && (\nu_t) \\
& \dot{k}_{c,t} = i_{c,t} - \delta k_{c,t} && (\chi_t) \\
& q_{p,t} \leq k_{p,t} && (\beta_t)
\end{aligned}
$$

where ρ is the rate of time preference. We indicated in parentheses the co-state variables and Lagrangian multipliers (chosen such that they are positive): among them, λ_t is the shadow value of income (used as numeraire), ν_t and χ_t are the shadow value of new polluting and clean capital, and β_t is the shadow cost of the polluting capacity constraint.

B. *Laissez-Faire Equilibrium*

Producers. — To maximize profits, producers simply need to observe the rental rates $R_{p,t}, R_{c,t}$ and rent capital up to the point where marginal returns equal the respective rental rates:

$$(13) \qquad \partial_{q_p} \Pi_F = 0 \implies \partial_{q_p} F(q_{p,t}, k_{c,t}) = R_{p,t}$$

$$(14) \qquad \partial_{k_c} \Pi_F = 0 \implies \partial_{k_c} F(q_{p,t}, k_{c,t}) = R_{c,t}$$

Households. — Deriving the FOCs relative to inter-temporal utility maximization by the household leads to (Appendix A):

$$(15) \qquad \lambda_t = \nu_t = \chi_t$$

$$(16) \qquad R_{c,t} = \frac{1}{\lambda_t}[(\delta + \rho)\chi_t - \dot{\chi}_t]$$

$$(17) \qquad R_{p,t} = \frac{1}{\lambda_t}[(\delta + \rho)\nu_t - \dot{\nu}_t] = \frac{\beta_t}{\lambda_t}$$

It is easy to show that since producers are willing to pay a positive rent for polluting capital in the laissez-faire equilibrium, households rent out all of the polluting capacities (Appendix A).

Equation 17 implies that the shadow cost of the capacity constraint equals the revenue from renting out capital $R_{p,t}$ (λ_t is the numeraire): , simply reflecting that the constraint prevents the household from earning more by renting more capital than what is available.

Equation 15 translates that in the laissez faire, for the household, the value of polluting and clean capital are equal, and are both equal to the numeraire. The value of capital influences the trade-off between consumption and investment. It is also linked to the rental rate of capital. As explained by Jorgenson (1967), the relationship between the rental costs ($R_{c,t}$, $R_{p,t}$) and the prices of new capital (χ_t, ν_t) captured by equations (16) and (17) ensures agents would be indifferent between buying and renting capital, given the depreciation rate δ, the pure preference for present ρ, and the future price of capital (implied by $\dot{\chi}_t$ and $\dot{\nu}_t$).

Alternatively, solving these differential equations shows that the shadow values of capacities equal the net present value of future rents received by a deprecating capacity, plus a salvage value:

$$\chi_t = \int_t^\infty e^{-(\rho+\delta)\tau} \lambda_\tau R_{c,\tau} d\tau + \chi_\infty \tag{18}$$

$$\nu_t = \int_t^\infty e^{-(\rho+\delta)\tau} \lambda_\tau R_{p,\tau} d\tau + \nu_\infty \tag{19}$$

Combining all the FOCs 15,16, and 17, one finds that:

LEMMA 1: *In the laissez faire equilibrium, the rental rates of polluting and clean capital are equal, and the marginal productivity of clean and polluting capital are also equal:*

$$R_{c,t} = R_{p,t} \tag{20}$$

$$\partial_{q_p} F(q_{p,t}, k_{c,t}) = \partial_{k_c} F(q_{p,t}, k_{c,t}) \tag{21}$$

This familiar result translates the well known equi-marginal principle.

The laissez-faire equilibrium is simple but provides a useful benchmark. In the following sections, we compare the effect of different social constraints and policy instruments to this benchmark.

IV. Minimizing Inter-temporal Costs in the Transition to Zero Carbon

This section looks at the socially-optimal transition to a clean economy and how it can be decentralized with a carbon tax.

A. *The Centralized First Best Optimum*

Here, we adopt a cost-effectiveness approach (Manne and Richels, 1992; Ambrosi et al., 2003; Weitzman, 2012) and analyze policies that allow maintaining atmospheric concentration m_t below a given ceiling $\bar{m}$:

$$m_t \leq \bar{m} \tag{22}$$

This threshold can be interpreted as a tipping point beyond which the environment and output can be highly damaged, or as an exogenous policy objective such as the Paris 2°C target, or any other temperature target designed to insure society against catastrophic climate change (IPCC, 2014).

The social planner maximizes inter-temporal utility given the constraints set by the economy budget, the capital motion law, investment irreversibility and the GHG ceiling. The social planner program is:

$$\max_{c,i,q} \int_0^\infty e^{-\rho t} \cdot u(c_t)\, dt \tag{23}$$

$$\begin{aligned}
\text{subject to } & F(A_t, q_p, k_c) - c_t - i_{p,t} - i_{c,t} = 0 && (\lambda_t)\\
& \dot{k}_{p,t} = i_{p,t} - \delta k_{p,t} && (\nu_t)\\
& \dot{k}_{c,t} = i_{c,t} - \delta k_{c,t} && (\chi_t)\\
& \dot{m}_t = G\, q_{p,t} - \varepsilon m_t && (\mu_t)\\
& m_t \leq \bar{m} && (\phi_t)\\
& i_{p,t} \geq 0 && (\psi_t)\\
& q_{p,t} \leq k_{p,t} && (\beta_t)
\end{aligned}$$

where μ_t is the shadow price of carbon, expressed in terms of utility at time t.

The Hamiltonian associated to the maximization of social welfare can be found in appendix B.B1. The complementary slackness conditions are:

$$\forall t,\ \psi_t \geq 0 \text{ and } \psi_t \cdot i_{p,t} = 0 \tag{24}$$

$$\forall t,\ \beta_t \geq 0 \text{ and } \beta_t \cdot (k_{p,t} - q_{p,t}) = 0 \tag{25}$$

$$\forall t,\ \phi_t \geq 0 \text{ and } \phi_t \cdot (\bar{m} - m_t) = 0 \tag{26}$$

And the first-order conditions of the social planner's problem boil down to (B.B1):

$$u'(c_t) = \lambda_t = \nu_t + \psi_t = \chi_t \tag{27}$$

$$\partial_{k_c} F = \frac{1}{\lambda_t}\left((\delta + \rho)\chi_t - \dot{\chi}_t\right) \tag{28}$$

$$\beta_t = \left((\delta + \rho)\nu_t - \dot{\nu}_t\right) \tag{29}$$

$$\partial_{q_p} F = \frac{\beta_t}{\lambda_t} + \tau_t \cdot G \tag{30}$$

Where τ_t is the social cost of carbon expressed in dollars per ton:

$$\tau_t := \frac{\mu_t}{\lambda_t} \tag{31}$$

In the right hand side of equations 28 and 29 we recognize the values of the rental rates of clean and polluting capital found in the previous section, that we thus call the implicit rental cost of capital and denote $R^i_{c,t}$ and $R^i_{p,t}$:

$$R^i_{c,t} := \frac{1}{\lambda_t}[(\delta+\rho)\chi_t - \dot{\chi}_t] \tag{16i}$$

$$R^i_{p,t} := \frac{1}{\lambda_t}[(\delta+\rho)\nu_t - \dot{\nu}_t] \tag{17i}$$

The system tends to a final stage which is reached, if ever, at a date that we denote t_{ss}. In the final stage, the carbon budget is binding ($m_t = \bar{m}$), implying that atmospheric emissions are stable ($\dot{m}_t = 0$) and polluting capital is constant at $k_{p,t} = \bar{m}\,\varepsilon/G$. We thus call the final stage *steady state*, even though the rest of the economy ($q_{c,t}$, $k_{c,t}$ and c_t) keeps growing on a balanced growth path, driven by the exogenous productivity growth captured by A_t.

Before the steady state is reached, a classical result (see for instance footnote 11 in Goulder and Mathai, 2000) is that the shadow carbon price grows at the interest rate r_t plus the dissipation rate of GHG (appendix B.B2)

$$\forall t,\ m_t < \bar{m} \implies \dot{\tau}_t = \tau_t\,(r_t + \varepsilon) \tag{32}$$

where the endogenous interest rate r_t is defined as the marginal return from clean investments net from depreciation:

$$r_t := \partial_{k_c} F - \delta \tag{33}$$

These dynamics may be interpreted as a generalized Hotelling rule applied to clean air: along the optimal pathway, and before the ceiling is reached, the discounted abatement costs are constant over time. The appropriate discount rate is $r_t + \varepsilon$, to take into account the natural decay of GHG in the atmosphere. (Rezai and Van der Ploeg (2016) use a more complex climate model and account for fossil reserve depletion, risk aversion, and use a cost-benefit approach, and still find that the optimal carbon price essentially grows exponentially over time before the transition is complete.)

In the *laissez-faire* equilibrium, capital was used up to the point where the marginal productivity of polluting capital was equal to its rental rate. This is no longer the case, since the social planner now accounts for the social cost of carbon when they use polluting capital. They must therefore reduce the amount of polluting capital used for production, to increase its marginal productivity:

LEMMA 2: *Along the socially-optimal path, the marginal productivity of clean capital equals the implicit rental rate of clean capital:*

$$\partial_{k_c} F = R^i_{c,t} \tag{34}$$

The marginal productivity of polluting capital is equal to the rental rate of polluting capital plus the marginal cost of carbon emissions:

$$\partial_{q_p} F = R^i_{p,t} + \tau_t \, G \tag{35}$$

PROOF:

Equation 34 derives from eq. 28 and 16i. Equation 35 is obtained by substituting β_t in eq. 30, using eq. 17i. □

Another difference with the laissez-faire equilibrium is that the implicit rental rate of polluting capital $R^i_{p,t}$ differs from that of clean capital, as it is now affected by a *legacy cost*:

LEMMA 3: *Along the optimal path, the implicit rental rate of polluting capital can be lower than that of clean capital:*

$$R^i_{p,t} = R^i_{c,t} - \ell_t \tag{36}$$

Where we call legacy cost ℓ_t *the monetary impact of the irreversibility constraint on the rental rate of polluting capital:*

$$\ell_t = \frac{1}{\lambda_t} \Big((\rho + \delta)\psi_t - \dot{\psi}_t \Big) \tag{37}$$

where ψ_t *is the Lagrange multiplier associated with the irreversibility constraint* (23)*.*

PROOF:

Equation eq. 36 is obtained by replacing ν_t by $\chi_t - \psi_t$ (eq. 27) in eq. 17. □

Because investment is irreversible, the stock of polluting capital cannot be instantaneously adjusted when the carbon price is implemented. Polluting capital therefore becomes relatively more abundant and a the legacy cost imposes a gap between the implicit rental rates of clean and dirty capacities.

The legacy cost ℓ_t can be seen as the "annualized" version of the shadow cost of the irreversibility constraint ψ_t; similarly to how the implicit rental rate $R^i_{p,t}$ is an "annualized" version of the value of new brown capacities ν_t. The legacy cost quantifies the regret that society has because of excessive irreversible investment in polluting capital (e.g. having built a coal power plant before the climate mitigation policy has been announced).

The next lemma states that the legacy cost is necessarily strictly positive at the beginning of the transition,[7] but then decreases and eventually reaches zero once polluting capital has adjusted through natural depreciation.

---

[7] In the analysis by Arrow and Kurz (1970), the irreversibility constraint can be binding only if the initial capital stock is higher than the steady-state level; here, the irreversibility constraint is binding for any level of initial polluting capital because of the new constraint on emissions.

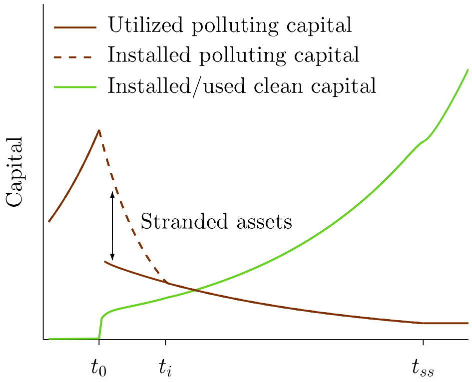

Figure 1. Installed polluting and clean capital, and utilized polluting capital in the least-cost transition to clean capital

Note: Before t_0, the economy is on the *laissez-faire* equilibrium, during which the stock of clean capital is small but not null. At t_0 the carbon price is implemented, investment in polluting capital stops, and polluting capital depreciates until t_i ($\forall t \in (t_0, t_i),\ i_b = 0$). During this period, a portion of polluting capital may be underutilized ($q_{p,t} < k_{p,t}$), becoming stranded assets ($k_{p,t} - q_{p,t}$). Here, the steady state is reached at t_{ss}.

LEMMA 4: *The transition to clean capital goes through a series of phases of two kinds:*

1) *Phases with strictly positive legacy costs, during which the implicit rental price of polluting capital is lower than the rental rate of clean capital and no investment is made in polluting capital:*

$$\begin{aligned} \ell_t &\leq R^i_{c,t} \\ R^i_{p,t} &< R^i_{c,t} \\ i_{p,t} &= 0 \end{aligned}$$

2) *Phases with zero legacy costs, during which the implicit rental rate of polluting capital is equal to the implicit rental rate of clean capital and gross investment in polluting capital is strictly positive:*

$$\begin{aligned} \ell_t &= 0 \\ R^i_{p,t} &= R^i_{c,t} \\ i_{p,t} &> 0 \end{aligned}$$

The transition necessarily starts with a type 1 phase (with positive legacy costs), and ends with a type 2 phase (where rental rates of clean and polluting capital are equal).

PROOF:

Since $R^i_{p,t} = \beta_t \geq 0$ (eq. 29), $\ell_t = R^i_{c,t} - R^i_{p,t} \leq R^i_{c,t}$. Appendix B.B3 proves the rest of the lemma. □

This lemma reflects that the legacy cost are only temporary, because in the long term, excess polluting capital has depreciated to a sustainable level. We show below that this implies that stranded assets are only a temporary phenomena, as all the polluting capital is used during the second type of phase.

Lemma 4 also means that in the social optimum, the maximum possible value for the legacy cost ℓ_t is the marginal productivity of clean capital $\partial_{k_c} F (= R^i_{c,t})$: at worst, the social planner regrets not to have invested in clean instead of polluting capital before t_0. In that case, the rental rate of polluting capital falls down to zero, reflecting that polluting capital is over-abundant and should be underused:

LEMMA 5: *If the carbon price is higher than the marginal productivity of installed polluting capital divided by its carbon intensity, polluting capital is underutilized:*

$$\tau_t\, G > \partial_{k_p} F(k_{p,t}, k_{c,t}) \implies \begin{cases} q_{p,t} < k_{p,t} \\ \ell_t = R^i_{c,t} \\ R^i_{p,t} = 0 \\ \partial_{q_p} F(q_p, k_c) = \tau_t\, G \end{cases} \tag{38}$$

PROOF:

Eq. 35 implies that the implicit rental rate of polluting capital $R^i_{p,t}$ is the difference between the marginal productivity of polluting capital and social cost of carbon. As the implicit rental rate of polluting capital $R^i_{p,t}$ is equal to the positive multiplier associated to the capacity constraint β_t (eq. 29 and 17), when the carbon price is higher than the marginal productivity of installed polluting capital the rental rate of polluting capital is nill and capital is underutilized. □

Lemma 5 means that stopping to use some of the polluting capital that was constructed before the climate policy is enacted can be part of the optimal strategy to reduce the cost of the transition to clean capital. Since in our framework all polluting capital is in aggregate, this lemma can be interpreted as an underutilization of the whole stock of polluting capital, with the rental rate of the whole stock falling to zero. (Another, more realistic interpretation is that the most polluting units of capital, for instance the oldest coal power plants, are decommissioned while the rest of the stock is used at full capacity.)

Underutilization of polluting capital depends on the GHG concentration ceiling $\bar{m}$, on the initial stock of polluting capital k_{b,t_0} and on other parameters of the model such as the functional forms of F and u, on the depreciation rate δ and the preference for the present ρ. As illustrated in figure 2, for a given set of functions and parameters the underutilization of polluting capital happens if initial polluting capital is high (right end of the x-axis) and/or if the ceiling is stringent (lower part of the y-axis).

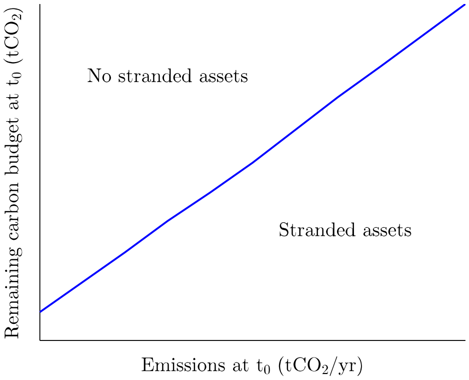

FIGURE 2. UNDER-UTILIZATION OF POLLUTING CAPITAL AS A FUNCTION OF INITIAL EMISSIONS AND THE CEILING.

Note: Depending on initial emissions (i.e. initial brown capital $k_{b,0}$) and on the concentration ceiling ($\bar{m}$), brown capital is underutilized or not in the first-best optimum.

We summarize the findings about the socially-optimal transition to clean capital in the following proposition, illustrated by Figure 1:

PROPOSITION 1: *The optimal transition from polluting to clean capital goes through phases of two kinds.*

1) *In the first type of phase, the irreversibility of investment translates into a positive legacy cost, which itself translates into a gap between the rental rate of polluting and clean capital. As a result, no investment goes to polluting capital during this phase. At worst, the rental rate of polluting capital can drop to zero, and existing polluting capacities can be underused, creating stranded assets.*

2) *In the second type of phase, the rental rates of clean and polluting capital are equal and there are no stranded assets.*

The optimal transition necessarily starts with a phase with positive legacy costs, potentially featuring stranded assets, and ends with a phase with no legacy costs and no stranded assets.

PROOF:

Lemmas 3, 4 and 5. □

The concepts of stranded assets and legacy costs can also be used to decompose the social cost of carbon τ_t (from equations 34, 35 and 36) as a technical marginal abatement cost (e.g. renewable power plants are more expensive than coal power plants) plus the legacy cost:

$$\underbrace{\tau_t}_{\text{Marginal abatement cost}} = \underbrace{\frac{\partial_{q_p} F - \partial_{k_c} F}{G}}_{\text{Technical cost}} + \underbrace{\frac{\ell_t}{G}}_{\text{Legacy cost}} \tag{39}$$

In this section, we thus have found that under irreversible investment, society has to live with past mistakes for a while, once it realizes it has been on a non-optimal growth path. To limit the associated legacy cost, it is efficient to give up part of installed polluting capital in order to reduce emissions faster, that is to create stranded assets. In the next section, we show that a carbon price can decentralize the social optimum, and that the legacy costs are paid in this case by the current owners of polluting capital. Then, we turn to a social planner program where stranded assets have to avoided, and look at policy instruments to decentralize that program.

B. *Decentralization with a Carbon Tax*

Unsurprisingly, the government can trigger the same outcome as in the social optimum in a decentralized economy by imposing a price on carbon emissions. Starting from the decentralized model exposed in Section III, the firm's flow of profit (10) is modified to:

$$\Pi_t = F(A_t, q_{p,t}, k_{c,t}) - R_{c,t} \cdot k_{c,t} - R_{p,t} \cdot k_{p,t} - \tau_t \, G \, q_{p,t} \tag{10tax}$$

Where τ_t is a carbon tax schedule numerically equal to the optimal shadow carbon price in the social planner's program (32).

The FOCs for the producer become:

$$\partial_{q_p} \Pi_F = 0 \implies \partial_{q_p} F(q_{p,t}, q_{c,t}) = R_{p,t} + \tau_t \, G \tag{13tax}$$

$$\partial_{q_c} \Pi_F = 0 \implies \partial_{k_c} F(q_{p,t}, q_{c,t}) = R_{c,t} \tag{14tax}$$

The problem for the household becomes :

$$\max_{c,i,q} \int_0^\infty e^{-\rho t} \cdot u(c_t) \, dt \tag{12tax}$$

$$\begin{aligned}
\text{subject to } & L + R_{c,t} \cdot k_{c,t} + R_{p,t} \cdot q_{p,t} = c_t + i_{p,t} + i_{c,t} && (\lambda_t) \\
& \dot{k}_{p,t} = i_{p,t} - \delta k_{p,t} && (\nu_t) \\
& \dot{k}_{c,t} = i_{c,t} - \delta k_{c,t} && (\chi_t) \\
& i_{p,t} \geq 0 && (\psi_t) \\
& q_{p,t} \leq k_{p,t} && (\beta_t)
\end{aligned}$$

Where L represents lump-sum revenues from the carbon tax. While $L = \tau_t \, G \, q_{p,t}$ in aggregate, atomistic households have a negligible impact on L, which they take as given. Therefore, the representative households also takes L as given.

First order conditions for the investor are the same as in the laissez-faire (15, 16, 17), and combining them with the FOCs for the producer leads to the same set of equations than the FOCs from the planner's program in the previous section — with the implicit rental costs of capital ($R^i_{p,t}$,$R^i_{c,t}$) replaced by the actual rental costs of capital ($R_{p,t}$,$R_{c,t}$), and the social cost of carbon replaced by the actual price of carbon. This means that the carbon price leads to the socially-optimal investment and production decisions.

Applied to the decentralized equilibrium, results from the previous section mean that when the government implements a carbon price, the actual rental rate of polluting capacities is affected by the legacy costs:

$$R_{p,t} = R_{c,t} - \ell_t \tag{36market}$$

At worst, in case of stranded assets, the actual rental rate of polluting capacities can be reduced to zero, jeopardizing the revenues of the owners of polluting capital (lemma 5).

The socially-optimal carbon price may thus turn out to be politically difficult to implement, as it imposes immediate and concentrated costs on a few players, the owners of polluting capital, who can easily organise and oppose the reform (Olson, 1977; Trebilcock, 2014); while its benefits, avoided climate change, are diffuse over all actors and over time, which tends to reduce mobilization to defend the reform.

In the next section, we solve for a constrained equilibrium where stranded assets are to be avoided for political reasons. This increases the total economic cost of the transition, in particular from legacy costs. These legacy costs are paid by different actors, depending on which specific instruments the government uses to enforce the second-best transition.

V. Avoiding Stranded Assets

A. *Social Planner Program*

Here, we solve a new social planner program, identical to the first best optimum, but with the additional political constraint that polluting capital should not be underused:

$$\begin{aligned} (40) \qquad & \max_{c,i,q} \int_0^\infty e^{-\rho t} \cdot u(c_t)\, dt \\ \text{subject to } & F(A_t, q_p, k_c) - c_t - i_{p,t} - i_{c,t} = 0 && (\lambda_t) \\ & \dot{k}_{p,t} = i_{p,t} - \delta k_{p,t} && (\nu_t) \end{aligned}$$

$$\begin{aligned}
&\dot{k}_{c,t} = i_{c,t} - \delta k_{c,t} && (\chi_t)\\
&\dot{m}_t = G\, q_{p,t} - \varepsilon m_t && (\mu_t)\\
&m_t \leq \bar{m} && (\phi_t)\\
&i_{p,t} \geq 0 && (\psi_t)\\
&q_{p,t} \leq k_{p,t} && (\beta_t)\\
&q_{p,t} \geq k_{p,t} && (\alpha_t)
\end{aligned}$$

In this problem, we have left two constraints for analytical purposes: the physical one, that capacity cannot be overused, and the political choice that stranded assets should not occur. The latter is expressed as an inequality which is binding only when there would otherwise be stranded assets, rather than an equality, also for analytical tractability.

First order conditions are available at appendix C. The system tends towards the same steady state as in the first best case. Before the steady state, the carbon price still grows at the interest rate net of carbon dissipation rate.

However, the no-stranded-assets constraint changes the relationship between the shadow cost of the capacity constraints and the value of new capacities (captured by eq.29 in the first best equilibium):

$$\beta_t - \alpha_t = ((\delta + \rho)\nu_t - \dot{\nu}_t) \tag{41}$$

So that defining the implicit rental rates of capital as before (16i, 17i) now yields $\lambda_t R^i_{p,t} = \beta_t - \alpha_t$. The implicit rental cost of polluting capital depends on both the physical constraint that capacities cannot be overused and the political constraint that they shall not be underused. The implicit rental cost of clean capital is as in the first best. Equations 34 and 35 are unchanged:

$$\partial_{k_c} F = R^i_{c,t} \tag{34const}$$

$$\partial_{q_p} F = R^i_{p,t} + \tau_t \cdot G \tag{35const}$$

the marginal productivity of polluting capital is still the sum of the implicit rental price of capital and the social cost of carbon. However, the implicit rental rate of polluting capital can now be negative. Since both β_t and α_t are positive by construction, and $\lambda_t = u'(c_t)$, with $u' > 0$ by assumption (Section II),

$$\alpha = 0 \implies \frac{\beta}{\lambda_t} = \partial_{q_p} F - \tau_t \cdot G > 0 \implies R^i_{p,t} > 0 \tag{42}$$

$$\beta_t = 0 \implies \frac{\alpha}{\lambda_t} = \tau_t \cdot G - \partial_{q_p} F > 0 \implies R^i_{p,t} < 0 \tag{43}$$

If the capacity constraint is not binding, then $\beta_t = 0$. This implies that the social cost of carbon is higher than the marginal productivity of polluting capital, and that the constraint that polluting assets cannot be stranded is binding ($\alpha_t > 0$). In that case, the implicit rental rate of polluting capital is negative. In other words, by refusing to strand polluting assets, society is implicitly subsidizing their utilization. The multiplier associated with the political constraint, α, interprets as that implicit subsidy (λ is the numeraire). The next sections show that depending on what policy instruments the government uses to enforce the constrained transition, this implicit subsidy may or may not translate into an actual subsidy.

On the other hand, when the political no-stranded-assets constraint is not binding, then $\alpha_t = 0$ and, as expected, the implicit rental rate of polluting capital behaves as in the first-best equilibrium ($\beta = R^i_{p,t} > 0$).

Identically to what happens in the first-best pathway, the marginal productivities are differentiated by legacy costs and the social cost of carbon:

$$\partial_{q_p} F = \partial_{k_c} F - \ell_t + \tau_t\, G \tag{44}$$

where legacy costs ℓ_t are defined as previously from the cost of the irreversibility constraint (37), yielding $R^i_{p,t} = R^i_{c,t} - \ell_t$. In the constrained transition, the social cost of carbon τ_t is thus still equal to a technical marginal abatement cost plus a legacy cost, that is equation 39 holds.

But the legacy cost is no longer bounded by $R^i_{c,t}$ as in lemma 4, because $R^i_{p,t}$ can be negative. In particular,

LEMMA 6: *At the beginning of the constrained transition, legacy costs are equal to the carbon price:* $\ell_{t_0} = \tau_{t_0}\, G$.

PROOF:

At t_0, since both clean and polluting capital are fully utilized, $\partial_{q_p} F = \partial_{k_c} F$, and thus $\ell_t = \tau_t\, G$. □

With the first-best carbon price, the maximum regret linked to excess past installation of polluting capital was the opportunity cost of not having invested in clean capital. Here, preventing underutilization is like refusing to recognize that past accumulation of polluting capital was a mistake. When society keeps using obsolete polluting capital instead of early-scrapping it, the legacy cost can be as high as the cost of the carbon emissions generated by the polluting capital, that is much higher than in the first-best transition. Refusing to strand assets thus increases regret from past investment in those assets, as their utilization make the climate target more difficult to achieve.

Positive legacy costs at the beginning of the transition also means that, as in the first best case, the constrained transition starts with a phase with no investment in polluting capacity ($\ell_t > 0 \implies \psi_t > 0 \implies i_{p,t} = 0$). During this phase, since growth happens from clean capital accumulation, a gap between the productivity of clean and polluting capital appears, and grows over time $\partial_{q_p} F > \partial_{k_c} F$, giving space for legacy costs to decrease over time (44).

Before the carbon budget is depleted, there may also be phases where investment in polluting capital is strictly positive (capacity variation net of depreciation can remain negative during those phases). During such phases, legacy cost are necessarily nil ($i_b > 0 \implies \psi_t = 0 \implies \ell_t = 0$). Finally, the system reaches the same steady state as previously, during which investment in polluting capacity maintains polluting capacity at the maximum allowable level, legacy costs are nil, and implicit rental rates are equal for polluting and clean capacities.

Let us compare the constrained transition to the socially-optimal one. Since capacities are not underused, short-term output may be higher in the constrained transition than in the first-best strategy:

LEMMA 7: *In the constrained transition, short-term output is equal or higher than with the first-best carbon price.*

PROOF:

The first-best carbon price may induce underutilization of polluting capital in the short-run ($q_{p,1,t_0} < k_{p,t_0}$). In the second-best solution, capital is not underused ($q_{p,2,t_0} = k_{p,t_0}$). At t_0, production is thus higher in the constrained transition. $F(A_{t_0}, q_{p,2,t_0}, k_{c,t_0}) \geq F(A_{t_0}, q_{p,1,t_0}, k_{c,t_0})$. $\square$

Figure 3 illustrates this result. The figure, derived from numerical simulations, shows not only a higher short-term production but also higher consumption when stranded assets are avoided. But analytically, the effect on consumption is ambiguous because it involves the offsetting impacts from an income effect (short-term output is higher) and two substitution effect (investment in clean capital is cheaper, which tends to decrease short-term investment and thus increase consumption, and investment in polluting capital is more expensive, which tends to increase short-term consumption).

Figure 4 compares the shadow cost of carbon implied by the optimal and constrained transitions. Avoiding stranded assets generate a higher social cost of carbon than the first-best carbon price. However the dynamics of capital accumulation mean that the social cost of carbon at each point in time does not translate into consumption losses at the same point in time (Vogt-Schilb, Meunier and Hallegatte, 2014). In this case, while the constrained transition sets a higher shadow cost of carbon at each time t (figure 4), they lead to higher output, and possibly higher consumption, over the short-run (lemma 7, figure 3).

Results from this section are summarized in the following proposition:

PROPOSITION 2: *The constrained transition leads to the same steady state as the optimal transition, the constrained pathway thus differs only temporarily from the first-best pathway.*

The constrained transition imposes a higher shadow cost of carbon, and an initially higher legacy costs than the optimal transition.

Compared to the optimal transition, the constrained transition smooths social costs: it decreases effort in the short-run (lemma 7), leaves them unchanged in the

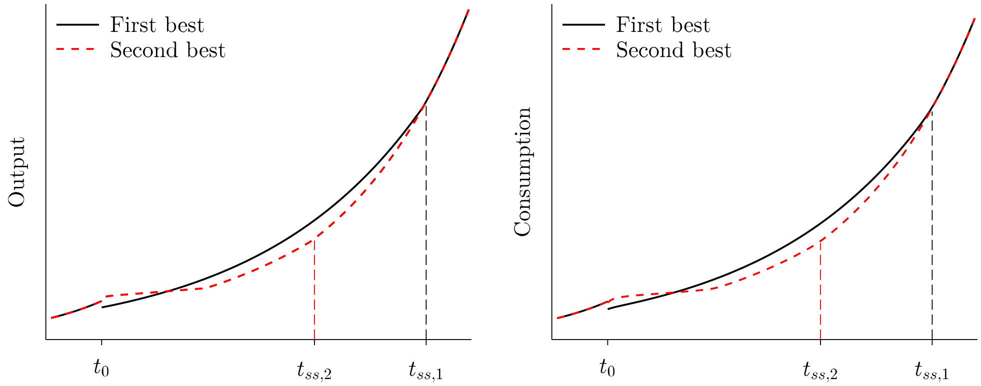

FIGURE 3. OUTPUT AND CONSUMPTION IN THE TWO SIMULATIONS.

Note: The left panel shows output y in the first-best and the constrained transitions. In the short-run output is lower in the first-best case because of stranded assets. On the particular example in the right, short-term consumption c is higher in the second-best case because of a higher output y. t_{ss} is the date at which the steady state is reached, here it is reached sooner in the second-best case ($t_{ss,2} < t_{ss,1}$).

long-run (as the long term steady-state remains unchanged), and thus increases effort in the medium-run.

In the following section, we show that feebates, standards, and phased-in carbon price can all decentralize the constrained transition. But feebates and standards protect revenues for the owners of existing polluting capital, while the second-best phased-in carbon price does not.

B. *Decentralization of the Second-best Equilibrium Combining a Carbon Price and a Subsidy on Polluting Production, or a Phased-in Carbon Price*

One way to decentralize the constrained optimum is to simply transform the social cost of carbon μ and the shadow subsidy for avoiding stranded assets α in an actual carbon tax and an actual subsidy on polluting production.

In that case, the program of the producer becomes:

$$\max_q \pi_t = F(A_t, q_{p,t}, q_{c,t}) - R_{c,t}q_{c,t} - (R_{p,t} + \tau_t G - \alpha_t)q_{p,t} \tag{45}$$

Where R is the market price for renting capacities. This leads to the FOCs for the producer:

$$R_{c,t} = \partial_{k_c} F \tag{46}$$

$$R_{p,t} = \partial_{q_p} F - \tau_t G + \alpha_t \tag{47}$$

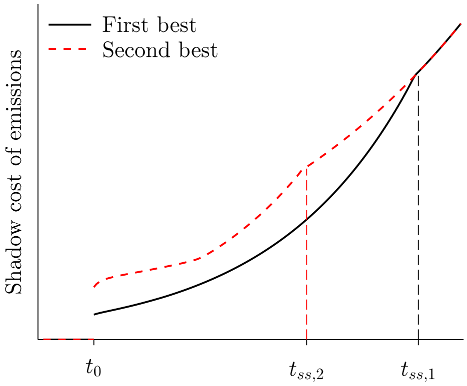

Figure 4. Shadow carbon price in the two simulations.

Note: The shadow price of emissions is higher in the constrained transition. The dates $t_{ss,1}$ and $t_{ss,2}$ denote the moment when the first best transition and the second best transition, retrospectively, reach the steady state.

And the household problem becomes:

$$\max_{c,i,q} \int_0^\infty e^{-\rho t} \cdot u(c_t)\, dt \tag{48}$$

$$\begin{aligned}
\text{subject to } & L - S + R_{c,t} \cdot k_{c,t} + R_{p,t} \cdot q_{p,t} = c_t + i_{p,t} + i_{c,t} && (\lambda_t) \\
& \dot{k}_{p,t} = i_{p,t} - \delta k_{p,t} && (\nu_t) \\
& \dot{k}_{c,t} = i_{c,t} - \delta k_{c,t} && (\chi_t) \\
& i_{p,t} \geq 0 && (\psi_t) \\
& q_{p,t} \leq k_{p,t} && (\beta_t)
\end{aligned}$$

where L represent lump sum revenues from the carbon price, and S is the lump-sum cost of the subsidy, both taken as given by the household. First order conditions for the household are thus unchanged compared to the laissez-faire equilibium or the carbon price.

Thus if the government sets the value of the carbon tax schedule and the subsidy equal to the optimum values of the respective co-state variables from the previous section, the set of FOCs is identical to the one from the constrained transition, where implicit rental rates have been replaced by actual rental rates, the social cost of carbon is replaced by the carbon price, and the implicit subsidy on polluting production is replaced by the actual subsidy, leading to the same transition to a clean economy. (To prove this more formally, Appendix C.C1 shows that the FOCs for this problem and the FOCs for the centralized constrained equilibrium are equivalent). In particular, stranded assets are avoided.

Since these FOCs only depend on $\tau_t G$ and α_t via $(\tau_t G - \alpha_t)$, the government can also decentralize the constrained transition using a single instrument: the phased-in tax scheduled $\tilde{\tau}_t = \tau_t G - \alpha_t$. At the beginning of the transition, $\alpha_t > 0$ reduces the phased-in carbon price below its first-best schedule. Note that the phased-in carbon price increases over time for two reasons: first, efficiency conditions means that the carbon price should increase, basically at the interest rate, as long as the transition to a clean economy is not complete. And second, to avoid stranded assets, the actual carbon price would start at a lower-than-efficient value, and catch-up with the optimal value to give time to players to adjust to the new prices.

Stranded assets can thus be avoided using a phased-in carbon price, or, equivalently, the combination of a carbon price and a subsidy for production from polluting capacity. Both instruments achieve this by imposing a cost to producers, $(\tau_t - \alpha_t/G)$, which is lower than the social cost of carbon τ_t.

It does not follow, however, that these instruments are harmless for the owners of polluting capacities. Indeed, the previous subsection shows that the highest possible value for the second-best subsidy α_t is $\tau_t G - \partial_{q_p} F$. But in that case, the subsidy covers only the gap between marginal productivity of polluting capital and the carbon price, implying that market price for renting polluting capacities is still zero $R_{p,t} = 0$ during this phase (47). Even if they avoid stranded assets, the second best phased-in carbon price does not protect the revenues of the owners of polluting capital. (For the producer, however, the infra-marginal rent is preserved).

This results nuances the claim by Williams (2011) that phasing-in a carbon price is a way of dealing with distributional impacts of climate policies. Our model suggests that while a phased-in carbon tax can avoid an economy-wide drop in production when it is announced, it is set at the level where owners of polluting capacity are just indifferent between renting out their capacities or scrapping them.[8] (As always, the government could still use the revenues from either instrument ex-post, to explicitly offset the losses of the owners of polluting capital.)

In contrast, the next section shows that if government use instruments that regulate new investment decisions instead of production decisions, such as feebate programs or standards on new equipment, then the same transition to a clean economy can be enforced while protecting the owners of polluting capital ex-ante.

We summarize the findings of this section in the following proposition:

PROPOSITION 3: *A phased-in carbon tax is equivalent to a carbon tax complemented with a temporary subsidy on polluting capacity. Both instruments allow decentralization of the second-best transition to clean capital where stranded assets are to be avoided. Second-best phased carbon prices may reduce revenues from existing polluting capital down to zero.*

---

[8] An open question is whether modelling explicitly sunk entry costs in the process of polluting capital accumulation (e.g., Lecuyer and Vogt-Schilb, 2014) would allow the owners of polluting capital to keep some revenue when the second-best phased-in carbon price is implemented.

C. *Decentralization with Feebate or Standards on New Investment*

Current climate mitigation policies are not limited to carbon prices; many governments rely instead on instruments such as energy efficiency standards, direct public investment in "green" sectors such as public transport, and fiscal incentives for green investment such as feebates, which impose additional fees on polluting capital and rebates for clean capital (IEA, 2016). These instruments redirect investment towards clean capital but have no effect on the use of existing capital.

STANDARDS. — One way to regulate investment is with quantity instruments, that is imposing a moratorium on polluting investment and mandating investment in clean capacity. We call these instruments *standards on new investment*.

Standards on new investment may seem extreme in our model, but similar instruments are actually discussed in the field of climate policy. For instance, Bertram et al. (2015) and Pfeiffer et al. (2016) propose to rule out investment in standard coal and gas power plants, and to mandate new power plants to be renewable power, nuclear, or fossil fuel plants equipped with carbon capture and storage.

This example reminds us that while our model only represents "clean" and "dirty" capital, the real-life implementation of standards would not be a pure command and control policy: requiring all new investment to be carbon-free still lets the market chose amongst a range of various complying technologies (Azar and Sandén, 2011). Also, while capital in our model is either perfectly clean or has a carbon intensity of exactly G, in actuality there is a continuum of possible carbon intensities. Standards can thus be set at any values, for instance CAFE standards on new cars or trucks have been progressively tightened over time.

With standards, the household problem becomes:

$$
\begin{aligned}
(49) \qquad & \max_{c,i,q} \int_0^\infty e^{-\rho t} \cdot u(c_t)\, dt \\
\text{subject to } & R_{c,t} \cdot k_{c,t} + R_{p,t} \cdot q_{p,t} = c_t + i_{p,t} + i_{c,t} && (\lambda_t) \\
& \dot{k}_{p,t} = i_{p,t} - \delta k_{p,t} && (\nu_t) \\
& \dot{k}_{c,t} = i_{c,t} - \delta k_{c,t} && (\chi_t) \\
& i_{p,t} \geq 0 && (\psi_t) \\
& q_{p,t} \leq k_{p,t} && (\beta_t) \\
& i_{p,t} \leq s_{p,t} && (\sigma_{p,t}) \\
& i_{c,t} \geq s_{c,t} && (\sigma_{c,t})
\end{aligned}
$$

The standards $s_{p,t}, s_{c,t}$ can be optimally set to equal polluting investments found in section V.A. In this model, $s_{p,t} = 0$ until polluting capacities have depreciated to a level compatible with the carbon ceiling.

First-order conditions for the household can be reduced to the following equations:

$$u'(c_t) = \lambda_t \tag{50}$$

$$\nu_t = \lambda_t - \sigma_{p,t} - \psi_t \tag{51}$$

$$\chi_t = \lambda_t + \sigma_{c,t} \tag{52}$$

$$\lambda_t R_{c,t} = (\delta + \rho)\chi_t - \dot{\chi}_t \tag{53}$$

$$\lambda_t R_{p,t} = \beta_t = (\delta + \rho)\nu_t - \dot{\nu}_t \tag{54}$$

These equations show that the standards $s_{p,t}$ and $s_{c,t}$ impose a shadow cost and a shadow subsidy on investment in new polluting and clean capital respectively. Below, we show that a feebate programs that mimics those shadow values can also decentralize the constrained optimum. Notice that two instruments may be needed here: a moratorium on polluting investment alone imposes a shadow price on investment decision, and can thus result in households consuming too much and saving too little, compared to the second-best constrained transition. The mandate on clean investment compensates that.

With standards, the firms problem is the same as in the laissez-faire:

$$\pi_t = F(A_t, q_{p,t}, q_{c,t}) - R_{c,t}q_{c,t} - R_{p,t}q_{p,t} \tag{55}$$

Now implying that firms are always willing to pay a strictly positive rent for both clean and dirty capacities:

$$\partial_{k_c} F = R_{c,t} \tag{56}$$

$$\partial_{q_p} F = R_{p,t} \tag{57}$$

In particular, the rental rate on polluting capital does not drop when the policy is implemented, and remains strictly positive during the transition. Strictly positive rental rates also imply that with standards on new investment, the household always rents out all the available capital: there are no stranded assets.

Since mandates constrain investment decisions and thus capital stocks, since the household rents out all of those stocks, and since consumption equals total production net of investment, we have shown that:

LEMMA 8: *Well-designed mandates or performance standards can decentralize the constrained transition.*

PROOF:

More details are provided in Appendix C.C2.

FEEBATE. — Unsurprisingly, the same transition can be obtained using price instruments, for instance a so-called feebate program. A feebate is the combination of a subsidy (or rebate) $\theta_{c,t}$ on investment in clean capacity and a tax (or fee) $\theta_{p,t}$ on investment in polluting capacity.[9]

With feebates, the household problem becomes:

$$\max_{c,i,q} \int_0^\infty e^{-\rho t} \cdot u(c_t)\, dt \tag{58}$$

$$\begin{aligned}
\text{subject to } & B + R_{c,t} \cdot k_{c,t} + R_{p,t} \cdot q_{p,t} - c_t - i_{p,t}(1+\theta_{p,t}) - i_{c,t}(1-\theta_{c,t}) = 0 && (\lambda_t)\\
& \dot{k}_{p,t} = i_{p,t} - \delta k_{p,t} && (\nu_t)\\
& \dot{k}_{c,t} = i_{c,t} - \delta k_{c,t} && (\chi_t)\\
& i_{p,t} \geq 0 && (\psi_t)\\
& q_{p,t} \leq k_{p,t} && (\beta_t)
\end{aligned}$$

Where B is the net budgetary impact of the feebate scheme, considered exogenous by the representative household. First order conditions for the household become:

$$u'(c_t) = \lambda_t \tag{59}$$

$$\nu_t = \lambda_t(1+\theta_{p,t}) - \psi_t \tag{60}$$

$$\chi_t = \lambda_t(1-\theta_{c,t}) \tag{61}$$

$$\lambda_t R_{p,t} = \beta_t = (\rho+\delta)\nu_t - \dot{\nu}_t \tag{62}$$

$$\lambda_t R_{c,t} = (\rho+\delta)\chi_t - \dot{\chi}_t \tag{63}$$

To decentralize the constrained social optimum, the government simply needs to set the feebate $(\theta_{p,t}, \theta_{c,t})$ such that the values of clean and polluting investment are the same as in the previous cases; that is choosing $\theta_{p,t}$ such that (60) is equivalent to (51) and choosing $\theta_{c,t}$ such that (61) is equivalent to (52). In that case, the set of equations that describe the response of the household and producer to the feebate program is the same as the set of equation describing their response to the standards or mandates.

Moreover, the firms problem remains unchanged, and the same demonstration than above shows that with a feebate, rents are positive, there are no stranded assets, and the rental rate of polluting capacities does not drop when the policy is implemented:

PROPOSITION 4: *The constrained transition to clean capital where stranded assets are to be avoided can be decentralized with standards on new investment, or with feebates on new investment. Such second-best instruments do not directly affect the revenues of the owners of current polluting capital.*

PROOF:

Appendix C.C2 provides more details.

---

[9] Our model does not capture all important factors in the choice between price and quantity instruments. See Goulder and Schein (2013) for a review of these factors.

Those results extend previous findings, (e.g., Fischer and Newell, 2008; Holland, Hughes and Knittel, 2009; Fullerton and Heutel, 2010), who find, using static models, that performance standards and feebate schemes act as the combination of a carbon tax and a production subsidy. With our dynamic model, we have clarified that while this shadow subsidy protects production and revenues from pre-existing polluting capital, it does not provide incentive to invest in additional polluting capital. The effect of the shadow subsidy is thus only temporary since once the level of polluting capital has decreased to a sustainable path, all instruments are equivalent to a simple carbon tax. And more surprisingly, the incidence of a carbon price plus a temporary subsidy on polluting production differs from the incidence of standards or feebates on investment decisions (Since this result required to model separately investors and producers, it was not found by the above-mentioned papers.)

VI. Committed Emissions, Instrument Choice, and Carbon Lock-in

Since they maintain a full utilization of polluting capital in the short term, feebates, standards, and the second-best phased-in carbon price result in higher short-term emissions than the carbon tax (Lemma 7 and figure 5). These instruments may thus not be sufficient to reach stringent climate objectives if past accumulation of polluting capital is substantial.

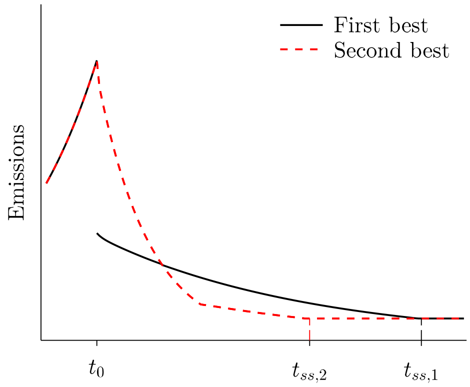

Figure 5. GHG emissions in the two cases.

Note: The first-best carbon prices induces decommission of polluting capital and can thus reduce carbon emissions faster than second-best alternative instruments.

Figure 6 offers a visualization of this issue. At low polluting capital stocks (thus low emissions), a carbon tax does not lead to underutilization of polluting capital. In this case, the first-best carbon price leads to the exact same pathway as second-best feebates or standards (and the phased-in carbon price is simply

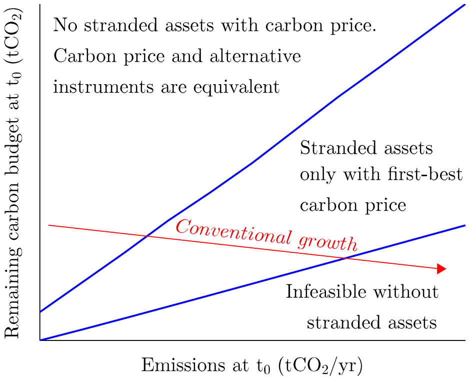

FIGURE 6. UNDER-UTILIZATION OF POLLUTING CAPITAL AND FEASIBILITY OF THE CLIMATE TARGET AVOIDING STRANDED ASSETS AS A FUNCTION OF INITIAL EMISSIONS.

equal to the optimal carbon price). This is a situation of flexibility in which a government can enforce the optimal transition to clean capital using any of the instruments discussed in this paper.

But as long as climate policies are absent or too lax, the economy accumulates polluting capital, making GHG emissions grow and reducing the residual carbon budget for a given climate target (the *conventional growth* arrow).

At one point, the threshold when the marginal productivity of polluting capital is lower than the optimal carbon price is crossed (see eq. 38), meaning that polluting capital should be underutilized and output reduced along the optimal pathway. From there, a carbon price may become even more difficult to implement because of political-economy constraints. But the alternative option of using feebates, standards or phase-in is still available to reach the same carbon budget without immediate drop in income.

There is thus a window of opportunity, during which alternative policy instruments may induce a smooth and maybe politically-easier transition to a low-carbon economy. If this occasion is missed (right hand side, figure 6), it becomes impossible to reach the climate target without underutilization of polluting capital and the alternative instruments are not an option any more (if the climate objective is not revised). In this last area, not only the economic cost of reaching the climate target is higher, but the political economy also creates a carbon lock-in: the only option to reach the climate target involves stranded assets and thus has a significant short-term cost, making it more difficult to implement successfully a climate policy consistent with the target.

The zone in which polluting capital must be underutilized to remain below the ceiling depends on the capital depreciation rate δ, the GHG dissipation rate ε, initial GHG concentration m_0 and initial polluting capital k_0. The lower blue line in figure 6 is expressed analytically in appendix D and can be approximated by:

$$\bar{m} = m_0 + \frac{G\,k_0}{\delta}$$

According to Davis, Caldeira and Matthews (2010), the level of existing polluting infrastructure in 2010 was still low enough to achieve the 2°C target without underutilizing polluting capital. They find that if existing energy infrastructure was used for its normal life span and no new polluting devices were built, future warming would be less than about 1.3°C. While they do not discuss whether the least-cost policy would lead to underutilization — that is, whether we are in the top or the middle triangle in figure 6 — several studies based on integrated assessment models investigate this question. Rogelj et al. (2013) and Johnson et al. (2015) both find that, in most 2°C scenarios, polluting capital (coal power plants in particular) are decommissioned before the end of their lifetime, suggesting that the global economy is in the middle zone in figure 6.

In other words, empirical evidence suggests the optimal pathway to a stabilization of the climate at 2°C involves decommissioning existing capital, but that we can still get there by only reducing the carbon content of new capital — in a recent numerical simulation, Bertram et al. (2015) find that a mix between low carbon prices and technology mandates (in particular a moratorium on coal power plants and a minimum requirement for clean power investment) could indeed deliver the 2°C while substantially limiting stranded assets. For some higher temperature target, feebates or standards and carbon prices are equivalent; while lower temperature targets, such as a 1.5°C target, may now be out of reach if stranded assets are to be avoided — taking into account that since the study by Davis, Caldeira and Matthews (2010), investment in polluting capital has kept growing and adding to committed GHG emissions (Davis and Socolow, 2014).

VII. Conclusion

The present analysis should be interpreted cautiously, as we only explored a few aspects of the transition to clean capital. In particular, our model ignores uncertainty, limited foresight from investors, and limited ability to commit from governments, which can all have important consequences on the comparison between carbon prices, phased-in carbon prices and feebates or standards regulating present-day investment. One possibility for further research is to integrate and quantify the effect of these elements in a single framework.

Despite these limitations, our results highlight that policy makers face a tradeoff between a higher intertemporal efficiency with the optimal carbon price and fewer stranded assets (and perhaps less political costs) with second-best instruments, such as carefully-designed standards in the automobile industry, efficiency standards for new power plants, buildings and appliances, *feebate* programs that tax energy-inefficient equipment and subsidize energy-efficient equipment, subsidized loans and tax breaks for energy efficiency investment, or to a lesser extent a phased-in carbon price. All these instruments are similar in that they redirect private investment away from polluting capital and toward clean capital without providing incentive to drive less or shut down existing coal power plants, that is without creating stranded assets. And as they transform progressively the production system, these instruments may prepare the economy and the public to easier implementation of carbon prices in the medium term.

Finally, the analysis carried here may also be relevant for studying other public economy issues. In essence, we propose a parsimonious model able to analyse structural change triggered by policy changes, its impact on vested interests, and policies to manage the transition. Similar models could be used to study policy reform in other topics, such as deregulation of prices in developing markets or trade liberalization.

REFERENCES

Acemoglu, Daron, Philippe Aghion, Leonardo Bursztyn, and David Hemous. 2012. "The Environment and Directed Technical Change." *American Economic Review*, 102(1): 131–166.

Acemoglu, Daron, Ufuk Akcigit, Douglas Hanley, and William Kerr. 2014. "Transition to Clean Technology." National Bureau of Economic Research Working Paper 20743.

Ambrosi, Philippe, Jean-Charles Hourcade, Stéphane Hallegatte, Franck Lecocq, Patrice Dumas, and Minh Ha Duong. 2003. "Optimal Control Models and Elicitation of Attitudes towards Climate Damages." *Environmental Modeling and Assessment*, 8(3): 133–147.

Anderson, Soren T., Ian W. H. Parry, James M. Sallee, and Carolyn Fischer. 2011. "Automobile Fuel Economy Standards: Impacts, Efficiency, and Alternatives." *Review of Environmental Economics and Policy*, 5(1): 89–108.

André, Francisco J., and Sjak Smulders. 2014. "Fueling growth when oil peaks: Directed technological change and the limits to efficiency." *European Economic Review*, 69: 18–39.

Arltesou, Andrew. 1999. "Models of Energy Use: Putty-Putty Versus Putty-Clay." *The American Economic Review*, 89(4): 1028–1043.

Arrow, Kenneth J., and Mordecai Kurz. 1970. "Optimal Growth with Irreversible Investment in a Ramsey Model." *Econometrica*, 38(2): 331–344.

Asheim, Geir B. 2012. "A Distributional Argument for Supply-Side Climate Policies." *Environmental and Resource Economics*, 56(2): 239–254.

Austin, David, and Terry Dinan. 2005. "Clearing the air: The costs and consequences of higher CAFE standards and increased gasoline taxes." *Journal of Environmental Economics and Management*, 50(3): 562–582.

Azar, Christian, and Björn A. Sandén. 2011. "The elusive quest for technology-neutral policies." *Environmental Innovation and Societal Transitions*, 1: 135–139.

Bertram, Christoph, Gunnar Luderer, Robert C. Pietzcker, Eva Schmid, Elmar Kriegler, and Ottmar Edenhofer. 2015. "Complementing carbon prices with technology policies to keep climate targets within reach." *Nature Climate Change*, 5(3): 235–239.

Borenstein, Severin, and Lucas W. Davis. 2015. "The Distributional Effects of U.S. Clean Energy Tax Credits." National Bureau of Economic Research Working Paper 21437.

Bovenberg, A. Lans, and Lawrence H. Goulder. 1996. "Optimal Environmental Taxation in the Presence of Other Taxes: General- Equilibrium Analyses." *The American Economic Review*, 86(4): 985–1000.

Caldecott, Ben, Elizabeth Harnett, Theodor Cojoianu, Irem Kok, and Alexander Pfeiffer. 2016. *Stranded Assets: A Climate Risk Challenge.* Inter-American Development Bank.

Carney, Mark. 2016. "Resolving the climate paradox." Bank of England Arthur Burns Memorial Lecture, Berlin.

Coady, David P., Valentina Flamini, and Louis Sears. 2015. "The Unequal Benefits of Fuel Subsidies Revisited: Evidence for Developing Countries." *IMF Working Paper No. 15/250.*

Davis, Steven J., and Robert H. Socolow. 2014. "Commitment accounting of CO2 emissions." *Environmental Research Letters*, 9(8): 084018.

Davis, Steven J., Ken Caldeira, and H. Damon Matthews. 2010. "Future CO2 Emissions and Climate Change from Existing Energy Infrastructure." *Science*, 329(5997): 1330 –1333.

Dresner, Simon, Louise Dunne, Peter Clinch, and Christiane Beuermann. 2006. "Social and political responses to ecological tax reform in Europe: an introduction to the special issue." *Energy Policy*, 34(8): 895–904.

Fay, Marianne, Stephane Hallegatte, Adrien Vogt-Schilb, Julie Rozenberg, and Ulf Narloch, ed. 2015. *Decarbonizing development: three steps to a zero-carbon future. Climate change and development series*, Washington, DC:World Bank Group.

Fischer, Carolyn, and Richard G. Newell. 2008. "Environmental and technology policies for climate mitigation." *Journal of Environmental Economics and Management*, 55(2): 142–162.

Fullerton, Don, and Garth Heutel. 2010. "The General Equilibrium Incidence of Environmental Mandates." *American Economic Journal: Economic Policy*, 2(3): 64–89.

Fullerton, Don, Garth Heutel, and Gilbert E. Metcalf. 2012. "Does the Indexing of Government Transfers Make Carbon Pricing Progressive?" *American Journal of Agricultural Economics*, 94(2): 347–353.

Gerlagh, Reyer, Snorre Kverndokk, and Knut Einar Rosendahl. 2009. "Optimal Timing of Climate Change Policy: Interaction Between Carbon Taxes and Innovation Externalities." *Environmental and Resource Economics*, 43(3): 369–390.

Giraudet, Louis-Gaëtan, and Philippe Quirion. 2008. "Efficiency and distributional impacts of tradable white certificates compared to taxes, subsidies and regulations." *Revue d'économie politique*, 118(6): 885–914.

Goulder, Lawrence H., and Andrew R. Schein. 2013. "Carbon taxes versus cap and trade: a critical review." *Climate Change Economics*, 04(03): 1350010.

Goulder, Lawrence H., and Koshy Mathai. 2000. "Optimal CO2 Abatement in the Presence of Induced Technological Change." *Journal of Environmental Economics and Management*, 39(1): 1–38.

Goulder, Lawrence H., Marc A. C. Hafstead, and Michael Dworsky. 2010. "Impacts of alternative emissions allowance allocation methods under a federal cap-and-trade program." *Journal of Environmental Economics and Management*, 60(3): 161–181.

Goulder, L. H., and I. W. H. Parry. 2008. "Instrument Choice in Environmental Policy." *Review of Environmental Economics and Policy*, 2(2): 152–174.

Grimaud, André, and Gilles Lafforgue. 2008. "Climate change mitigation policies : Are R&D subsidies preferable to a carbon tax ?" *Revue d'économie politique*, Vol. 118(6): 915–940.

Guivarch, Céline, and Christina Hood. 2010. "Early retirement of coal-fired generation in the transition to low-carbon electricity systems." *Data and analyses.*

Guivarch, Céline, and Stéphane Hallegatte. 2011. "Existing infrastructure and the 2C target." *Climatic Change*, 109(3-4): 801–805.

Harrison, Kathryn, and Chelsea Peet. 2012. "Historical Legacies and Policy Reform: Diverse Regional Reactions to British Columbia's Carbon Tax." *BC Studies: The British Columbian Quarterly*, 0(173): 97–122.

Holland, Stephen P, Jonathan E Hughes, and Christopher R Knittel. 2009. "Greenhouse Gas Reductions under Low Carbon Fuel Standards?" *American Economic Journal: Economic Policy*, 1(1): 106–146.

IEA. 2016. "Policies and Measures Database." International Energy Agency. http://iea.org/policiesandmeasures.

IPCC. 2014. "Summary for Policymakers." In *Climate Change 2014, Synthesis Report. Contribution of Working Groups I, II and III to the Fifth Assessment Report of the Intergovernmental Panel on Climate Change.* . Cambridge University Press ed. Cambridge, United Kingdom and New York, NY, USA.

Iyer, Gokul C., James A. Edmonds, Allen A. Fawcett, Nathan E. Hultman, Jameel Alsalam, Ghassem R. Asrar, Katherine V. Calvin, Leon E. Clarke, Jared Creason, Minji Jeong, Page Kyle, James McFarland, Anupriya Mundra, Pralit Patel, Wenjing Shi, and Haewon C. McJeon. 2015. "The contribution of Paris to limit global warming to 2°C." *Environmental Research Letters*, 10(12): 125002.

Jacobsen, Mark R., and Arthur A. van Benthem. 2015. "Vehicle Scrappage and Gasoline Policy." *American Economic Review*, 105(3): 1312–38.

Jakob, Michael, and Jérôme Hilaire. 2015. "Climate science: Unburnable fossil-fuel reserves." *Nature*, 517(7533): 150–152.

Jenkins, Jesse D. 2014. "Political economy constraints on carbon pricing policies: What are the implications for economic efficiency, environmental efficacy, and climate policy design?" *Energy Policy*, 69: 467–477.

Johnson, Nils, Volker Krey, David L. McCollum, Shilpa Rao, Keywan Riahi, and Joeri Rogelj. 2015. "Stranded on a low-carbon planet: Implications of climate policy for the phase-out of coal-based power plants." *Technological Forecasting and Social Change*, 90, Part A: 89–102.

Jorgenson, Dale. 1967. "The theory of investment behavior." In *Determinants of investment behavior*. NBER.

Kalkuhl, Matthias, Ottmar Edenhofer, and Kai Lessmann. 2012. "Learning or lock-in: Optimal technology policies to support mitigation." *Resource and Energy Economics*, 34: 1–23.

Kallbekken, Steffen, and Håkon Sælen. 2011. "Public acceptance for environmental taxes: Self-interest, environmental and distributional concerns." *Energy Policy*, 39(5): 2966–2973.

Karp, Larry, and Amon Rezai. 2012. "The Political economy of environmental policy with overlapping generations." *International Economic Review 55 (3), 711-733.*

Kverndokk, Snorre, and Knut Einar Rosendahl. 2007. "Climate policies and learning by doing: Impacts and timing of technology subsidies." *Resource and Energy Economics*, 29(1): 58–82.

Lecuyer, Oskar, and Adrien Vogt-Schilb. 2014. "Optimal transition from coal to gas and renewable power under capacity constraints and adjustment costs." *World Bank Policy Research Working Paper No. 6985.*

MacKenzie, Ian A., and Markus Ohndorf. 2012. "Cap-and-trade, taxes, and distributional conflict." *Journal of Environmental Economics and Management*, 63(1): 51–65.

Manne, Alan S, and Richard G Richels. 1992. *Buying greenhouse insurance: the economic costs of carbon dioxide emission limits.* Cambridge, Mass.:MIT Press.

Metcalf, Gilbert E. 2014. "Using the tax system to address competition issues with a carbon tax." *National Tax Journal*, 67(4): 779.

Nordhaus, William D. 1991. "To Slow or Not to Slow: The Economics of The Greenhouse Effect." *The Economic Journal*, 101(407): 920–937.

Olson, Mancur. 1977. *The logic of collective action: public goods and the theory of groups.* Harvard University Press.

Parry, Ian W. H., and Antonio M. Bento. 2000. "Tax Deductions, Environmental Policy, and the "Double Dividend" Hypothesis." *Journal of Environmental Economics and Management*, 39(1): 67–96.

Pearce, David. 1991. "The Role of Carbon Taxes in Adjusting to Global Warming." *The Economic Journal*, 101(407): 938–948.

Pfeiffer, Alexander, Richard Millar, Cameron Hepburn, and Eric Beinhocker. 2016. "The '2°C capital stock' for electricity generation: Committed cumulative carbon emissions from the electricity generation sector and the transition to a green economy." *Applied Energy*, 179: 1395–1408.

Pigou, A.C. 1932. *The economics of welfare.* London:Macmillan and Co. 4th edition.

Rausch, Sebastian, and John Reilly. 2015. "Carbon Taxes, Deficits, and Energy Policy Interactions." *National Tax Journal*, 68(1): 157–178.

Rausch, Sebastian, Gilbert E. Metcalf, John M. Reilly, and Sergey Paltsev. 2010. "Distributional Implications of Alternative U.S. Greenhouse Gas Control Measures." *The B.E. Journal of Economic Analysis & Policy*, 10(2).

Rezai, Armon, and Frederick Van der Ploeg. 2016. "Intergenerational Inequality Aversion, Growth, and the Role of Damages: Occam's Rule for the Global Carbon Tax." *Journal of the Association of Environmental and Resource Economists*, 3(2): 493–522.

Rezai, Armon, Duncan Foley, and Lance Taylor. 2012. "Global warming and economic externalities." *Economic Theory*, 49(2): 329–351.

Rogelj, Joeri, David L. McCollum, Brian C. O'Neill, and Keywan Riahi. 2013. "2020 emissions levels required to limit warming to below 2C." *Nature Climate Change*, 3(4): 405–412.

Sachs, Jeffrey, Laurence Tubiana, Emmanuel Guerin, Henri Waisman, Carl Mas, Michel Colombier, and Guido Schmidt-Traub. 2014. "Pathways to Deep Decarbonization." Deep Decarbonization Pathways Project 2014 Report.

Sijm, Jos, Karsten Neuhoff, and Yihsu Chen. 2006. "CO2 cost pass-through and windfall profits in the power sector." *Climate Policy*, 6(1): 49–72.

Trebilcock, Michael J. 2014. *Dealing with Losers: The Political Economy of Policy Transitions.* Oxford University Press.

Vogt-Schilb, Adrien, Guy Meunier, and Stéphane Hallegatte. 2014. "Optimal timing, cost and sectoral distribution of emission reductions: abatement cost curves vs. abatement investment." Philadelphia:ASSA.

Wei, Chao. 2003. "Energy, the Stock Market, and the Putty-Clay Investment Model." *The American Economic Review*, 93(1): 311–323.

Weitzman, Martin L. 2012. "GHG Targets as Insurance Against Catastrophic Climate Damages." *Journal of Public Economic Theory*, 14(2): 221–244.

Williams, Roberton C. 2011. "Setting the Initial Time-Profile of Climate Policy: The Economics of Environmental Policy Phase-Ins." *NBER*, 245–254.

World Bank. 2016. *State and Trends of Carbon Pricing 2016.* Washington DC, USA:The World Bank.

APPENDIX

LAISSEZ-FAIRE EQUILIBRIUM (SECTION III.B)

The present value Hamiltonian associated to the maximization of the household's problem (12) is:

$$H_t = e^{-\rho t} \cdot \{u(c_t) + \lambda_t[R_{c,t} \cdot k_{c,t} + R_{p,t} \cdot q_{p,t} - c_t - i_{p,t} - i_{c,t}] + \nu_t[i_{p,t} - \delta k_{p,t}] \\ + \chi_t[i_{c,t} - \delta k_{c,t}] + \beta_t[k_{p,t} - q_{p,t}]\}$$

First order conditions read:

$$\begin{aligned}
&\frac{\partial H_t}{\partial c_t} = 0 \Rightarrow && u'(c_t) = \lambda_t \\
&\frac{\partial H_t}{\partial i_{p,t}} = 0 \Rightarrow && \lambda_t = \nu_t \\
&\frac{\partial H_t}{\partial i_{c,t}} = 0 \Rightarrow && \lambda_t = \chi_t \\
&\frac{\partial H_t}{\partial k_{p,t}} = -\frac{d(e^{-\rho t}\nu_t)}{dt} \Rightarrow && -\nu_t\delta + \beta_t = -\dot{\nu}_t + \rho\nu_t \\
&\frac{\partial H_t}{\partial k_{c,t}} = -\frac{d(e^{-\rho t}\chi_t)}{dt} \Rightarrow && \lambda_t R_{c,t} - \chi_t\delta = -\dot{\chi}_t + \rho\chi_t \\
&\frac{\partial H_t}{\partial q_{p,t}} = 0 \Rightarrow && \lambda_t R_{p,t} = \beta_t
\end{aligned} \tag{A1}$$

And the complementary slackness condition,

$$\beta_t \geq 0 \text{ and } \beta_t \cdot (k_{p,t} - q_{p,t}) = 0 \tag{A2}$$

which, combined with the last equation in A1, shows that as long as the rental rate of polluting capital is positive, the household rents out all the available capital. Since (13) shows that with Inada-compliant production function, the producer is always willing to pay a positive rent on polluting capital, all polluting capacities are always used in the laissez-faire equilibrium.

SOCIAL OPTIMUM (SECTION IV.A)

B1. Efficiency conditions

The present value Hamiltonian associated to the maximization of social welfare (23) is:

$$\begin{aligned} H_t = e^{-\rho t} \cdot \Big\{ & u(c_t) + \lambda_t[F(A_t, q_p, k_c) - c_t - i_{p,t} - i_{c,t}] + \nu_t[i_{p,t} - \delta k_{p,t}] \\ & + \chi_t[i_{c,t} - \delta k_{c,t}] - \mu_t \cdot [G\, q_{p,t} - \varepsilon m_t] + \phi_t \cdot [\bar{m} - m_t] \\ & + \psi_t \cdot i_{p,t} + \beta_t[k_{p,t} - q_{p,t}] \Big\} \end{aligned} \tag{B1}$$

All multipliers are positive.

The complementary slackness conditions are:

$$\forall t,\ \psi_t \geq 0 \text{ and } \psi_t \cdot i_{p,t} = 0 \tag{B2}$$

$$\forall t,\ \beta_t \geq 0 \text{ and } \beta_t \cdot (k_{p,t} - q_{p,t}) = 0 \tag{B3}$$

$$\forall t,\ \phi_t \geq 0 \text{ and } \phi_t \cdot (\bar{m} - m_t) = 0 \tag{B4}$$

First order conditions read:

$$\frac{\partial H_t}{\partial c_t} = 0 \Rightarrow \qquad u'(c_t) = \lambda_t \tag{B5}$$

$$\frac{\partial H_t}{\partial i_{p,t}} = 0 \Rightarrow \qquad \lambda_t = \nu_t + \psi_t$$

$$\frac{\partial H_t}{\partial i_{c,t}} = 0 \Rightarrow \qquad \lambda_t = \chi_t$$

$$\frac{\partial H_t}{\partial k_{p,t}} = -\frac{d(e^{-\rho t}\nu_t)}{dt} \Rightarrow \qquad -\nu_t\delta + \beta_t = -\dot{\nu}_t + \rho\nu_t$$

$$\frac{\partial H_t}{\partial k_{c,t}} = -\frac{d(e^{-\rho t}\chi_t)}{dt} \Rightarrow \quad \lambda_t \partial_{k_c} F(A_t, k_{p,t}, k_{c,t}) - \chi_t\delta = -\dot{\chi}_t + \rho\chi_t$$

$$\frac{\partial H_t}{\partial q_{p,t}} = 0 \Rightarrow \qquad \lambda_t \partial_{q_p} F(A_t, q_{p,t}, k_{c,t}) - \mu_t \cdot G = \beta_t$$

$$\frac{\partial H_t}{\partial m_t} = \frac{d(e^{-\rho t}\mu_t)}{dt} \Rightarrow \qquad -\phi_t + \varepsilon\mu_t = \dot{\mu}_t - \rho\mu_t \tag{B6}$$

If we differentiate eq. B5 with respect to time and substitute λ_t and $\dot{\lambda}_t$, we get the familiar Ramsey formula that links consumption decisions to the interest rate:

$$\frac{c_t \cdot u''(c_t)}{u'(c_t)} \cdot \frac{\dot{c}_t}{c_t} = (\rho + \delta - R_{c,t}) \tag{B7}$$

B2. Social cost of carbon

Eq. B6 gives the evolution of μ_t. Let us define τ_t, the shadow price of carbon expressed in dollars per ton:

$$\tau_t := \frac{\mu_t}{\lambda_t} \tag{31}$$

Using $\dot{\mu_t} = (\dot{\lambda}_t \tau_t + \lambda_t \dot{\tau}_t)$, eq. B5, eq. B7 and eq. 33 yields:

$$\dot{\tau}_t = \tau_t[\varepsilon + r_t] - \frac{\phi_t}{\lambda_t}$$

We assume that GHG concentration reaches the ceiling at a date denoted t_{ss}:

$$\forall t \geq t_{ss},\ m_t = \bar{m}$$

During the steady state, $\dot{m}_t = 0 \implies G\, q_{p,t} = \varepsilon\, \bar{m}$ (eq. 9). In the long run, installed polluting capital has depreciated down to the point where it is not underused, and is thus constant at $k_{p,t} = \bar{m}\,\varepsilon / G$ during the steady state.

Before t_{ss}, $\phi_t = 0$ (B4). The carbon price thus exponentially grows at the endogenous interest rate plus the dissipation rate of GHG until the ceiling is reached:

$$\dot{\tau}_t = \tau_t[\varepsilon + r_t] \tag{B8}$$

Equation B8 gives τ_t off by a multiplicative constant τ_0, which the social planer choses at the lowest value that ensures compliance with the GHG ceiling.

B3. Proof of lemma 4

THE IRREVERSIBILITY CONSTRAINT IS BINDING IN THE SHORT RUN. — A binding GHG ceiling is imposed at t_0. Before that, the economy was in the competitive equilibrium, such that clean and polluting capital have the same marginal productivity and installed capital is fully used (lemma 1):

$$\lim_{t \to t_0^-} q_{p,t} = k_{p,t} \tag{B9}$$

$$\lim_{t \to t_0^-} \partial_{q_p} F(q_{p,t}, q_{c,t}) = \partial_{k_c} F(q_{p,t}, q_{c,t}) \tag{B10}$$

We use a proof by contradiction to show that at t_0^+ (when the constraint is internalized) the irreversibility condition is necessarily binding. Suppose that the transition starts with a phase when the irreversibility constraint is not binding, i.e. $\psi_t = 0$. This would lead to (Propositions 2 and 3):

$$\lim_{t \to t_0^+} \partial_{q_p} F(q_{p,t}, q_{c,t}) = \partial_{k_c} F(q_{p,t}, q_{c,t}) + \tau_{t_0} \cdot G \tag{B11}$$

Besides, investment means that capital is a continuous function of time:

$$\lim_{t \to t_0^+} q_{p,t} = k_{p,t} \tag{B12}$$

If the GHG ceiling is binding then $\tau_{t_0} > 0$ (eq. B8). So from eq. B10 and eq. B11:

$$\lim_{t \to t_0^+} \partial_{q_p} F(q_{p,t}, q_{c,t}) \neq \lim_{t \to t_0^-} \partial_{q_p} F(q_{p,t}, q_{c,t}) \tag{B13}$$

$\partial_{q_p} F$ is a continuous function of $q_{p,t}$ so eq. B13 implies that $\lim_{t \to t_0^+} q_{p,t} \neq \lim_{t \to t_0^+} q_{p,t}$, which is incompatible with eq. B9 and eq. B12. $\square$

The irreversibility constraint is not binding in the long run. — During the steady state, polluting capital is maintained at the maximum level compatible with stabilized GHG concentration, $k_{p,t} = \bar{m}\,\varepsilon/G$, which implies that $i_{p,t} = \delta\bar{m}\,\varepsilon/G > 0$, from which it follows that $\psi_t = 0$ and thus $\ell_t = 0$ by definition (37).

Maximization of social welfare with full utilization constraint

Here, we solve a new social planner program, identical to the first best optimum, but with the additional political constraint that polluting capital should not be underused:

$$\max_{c,i,q} \int_0^\infty e^{-\rho t} \cdot u(c_t)\, dt \tag{C1}$$

$$\begin{aligned}
\text{subject to } & F(A_t, q_p, k_c) - c_t - i_{p,t} - i_{c,t} = 0 && (\lambda_t)\\
& \dot{k}_{p,t} = i_{p,t} - \delta k_{p,t} && (\nu_t)\\
& \dot{k}_{c,t} = i_{c,t} - \delta k_{c,t} && (\chi_t)\\
& \dot{m}_t = G\, q_{p,t} - \varepsilon m_t && (\mu_t)\\
& m_t \leq \bar{m} && (\phi_t)\\
& i_{p,t} \geq 0 && (\psi_t)\\
& q_{p,t} \leq k_{p,t} && (\beta_t)\\
& q_{p,t} \geq k_{p,t} && (\alpha_t)
\end{aligned}$$

The present value Hamiltonian reads:

$$\begin{aligned}H_t = e^{-\rho t} \cdot \{&u(c_t) + \lambda_t[F(A_t, q_p, k_c) - c_t - i_{p,t} - i_{c,t}] + \nu_t[i_{p,t} - \delta k_{p,t}] \\ &+\chi_t[i_{c,t} - \delta k_{c,t}] - \mu_t \cdot [G\, q_{p,t} - \varepsilon m_t] + \phi_t \cdot [\bar{m} - m_t] \\ &+ \psi_t \cdot i_{p,t} + \beta_t[k_{p,t} - q_{p,t}] + \alpha_t[q_{p,t} - k_{p,t}]\}\end{aligned}$$

First order conditions read:

$$\begin{aligned}
&\frac{\partial H_t}{\partial c_t} = 0 \Rightarrow && u'(c_t) = \lambda_t \\
&\frac{\partial H_t}{\partial i_{p,t}} = 0 \Rightarrow && \lambda_t = \nu_t + \psi_t \\
&\frac{\partial H_t}{\partial i_{c,t}} = 0 \Rightarrow && \lambda_t = \chi_t \\
&\frac{\partial H_t}{\partial k_{p,t}} = -\frac{d(e^{-\rho t}\nu_t)}{dt} \Rightarrow && -\nu_t\delta + \beta_t - \alpha_t = -\dot{\nu}_t + \rho\nu_t \\
&\frac{\partial H_t}{\partial k_{c,t}} = -\frac{d(e^{-\rho t}\chi_t)}{dt} \Rightarrow && \lambda_t\partial_{k_c}F(A_t, k_{p,t}, k_{c,t}) - \chi_t\delta = -\dot{\chi}_t + \rho\chi_t \\
&\frac{\partial H_t}{\partial q_{p,t}} = 0 \Rightarrow && \lambda_t\partial_{q_p}F(A_t, q_{p,t}, k_{c,t}) - \mu_t \cdot G = \beta_t - \alpha \\
&\frac{\partial H_t}{\partial m_t} = \frac{d(e^{-\rho t}\mu_t)}{dt} \Rightarrow && -\phi_t + \varepsilon\mu_t = \dot{\mu}_t - \rho\mu_t
\end{aligned} \tag{C2}$$

The complementary slackness conditions are:

$$\forall t,\ \psi_t \geq 0 \text{ and } \psi_t \cdot i_{p,t} = 0 \tag{C3}$$

$$\forall t,\ \beta_t \geq 0 \text{ and } \beta_t \cdot (k_{p,t} - q_{p,t}) = 0 \tag{C4}$$

$$\forall t,\ \alpha_t \geq 0 \text{ and } \alpha_t \cdot (k_{p,t} - q_{p,t}) = 0 \tag{C5}$$

$$\forall t,\ \phi_t \geq 0 \text{ and } \phi_t \cdot (\bar{m} - m_t) = 0 \tag{C6}$$

As before, C6 implies that the carbon price grows at the relevant rate when the carbon budget is not saturated.

C1. Decentralization of the Second Best Equilibrium Combining a Carbon and a Subsidy on Dirty Production, or a Phased-in Carbon Price

On way to decentralize the constrained optimum is to simply transform the social cost of carbon μ and the shadow subsidy for avoiding stranded assets α in an actual carbon tax and subsidy on polluting *production*.

In that case, the household problem becomes:

$$\max_{c,i,q} \int_0^\infty e^{-\rho t} \cdot u(c_t)\, dt \tag{C7}$$

$$\begin{aligned}
\text{subject to } & L - S + R_{c,t} \cdot k_{c,t} + R_{p,t} \cdot q_{p,t} = c_t + i_{p,t} + i_{c,t} && (\lambda_t)\\
& \dot{k}_{p,t} = i_{p,t} - \delta k_{p,t} && (\nu_t)\\
& \dot{k}_{c,t} = i_{c,t} - \delta k_{c,t} && (\chi_t)\\
& i_{p,t} \geq 0 && (\psi_t)\\
& q_{p,t} \leq k_{p,t} && (\beta_t)
\end{aligned}$$

where L represent lump sum revenues from the carbon price, and S is the lump-sum cost of the subsidy, both taken as given by the housold.

First order conditions for the household are thus unchanged compared to the laissez-faire equilibium:

$$u'(c_t) = \lambda_t \tag{C8}$$
$$\nu_t = \lambda_t - \psi_t \tag{C9}$$
$$\chi_t = \lambda_t \tag{C10}$$
$$\lambda_t R_{c,t} = (\delta + \rho)\chi_t - \dot{\chi}_t \tag{C11}$$
$$\lambda_t R_{p,t} = \beta_t = (\delta + \rho)\nu_t - \dot{\nu}_t \tag{C12}$$

The program of the producer however becomes:

$$\max_q \pi_t = F(A_t, q_{p,t}, q_{c,t}) - R_{c,t} q_{c,t} - (R_{p,t} + \tau_t G - \alpha_t) q_{p,t} \tag{C13}$$

Leading to the FOCs for the producer:

$$R_{c,t} = \partial_{k_c} F \tag{C14}$$
$$R_{p,t} = \partial_{q_p} F - \tau_t G + \alpha_t \tag{C15}$$

Thus if the government sets the value of the carbon tax schedule and the subsidy equal to the optimum values of the respective co-state variables from the previous section,it can decentralize the constrained equilibrium. The 7 FOCs C8–C12, C14 and C15 describing the response of the economy to a carbon tax plus subsidy on production are equivalent to the seven FOCs C2 describing the social optimum. Since these differential equations and the boundary conditions entirely describe the system's evolution, both set of equation lead to the same transition to a clean economy.

C2. Decentralization of the Second Best Equilibrium with Investment Standards or Feebates

To decentralize the second best equilibrium, the government needs to make sure that all capacities are used, and that investment mimics investment the second-best investment schedule. One way to achieve this is by regulating new investment only, using either quantity (standards) or price (feebates) based instruments.

With standards, the household problem becomes:

$$\max_{c,i,q} \int_0^{\infty} e^{-\rho t} \cdot u(c_t)\, dt \tag{C16}$$

$$\begin{aligned} \text{subject to } & R_{c,t} \cdot k_{c,t} + R_{p,t} \cdot q_{p,t} = c_t + i_{p,t} + i_{c,t} && (\lambda_t) \\ & \dot{k}_{p,t} = i_{p,t} - \delta k_{p,t} && (\nu_t) \\ & \dot{k}_{c,t} = i_{c,t} - \delta k_{c,t} && (\chi_t) \\ & i_{p,t} \geq 0 && (\psi_t) \\ & q_{p,t} \leq k_{p,t} && (\beta_t) \\ & i_{p,t} \leq s_{p,t} && (\sigma_{p,t}) \\ & i_{c,t} \geq s_{c,t} && (\sigma_{c,t}) \end{aligned}$$

First-order conditions for the household can be reduced to the following equations:

$$u'(c_t) = \lambda_t \tag{C17}$$

$$\nu_t = \lambda_t - \sigma_{p,t} - \psi_t \tag{C18}$$

$$\chi_t = \lambda_t + \sigma_{c,t} \tag{C19}$$

$$\lambda_t R_{c,t} = (\delta + \rho)\chi_t - \dot{\chi}_t \tag{C20}$$

$$\lambda_t R_{p,t} = \beta_t = (\delta + \rho)\nu_t - \dot{\nu}_t \tag{C21}$$

These equations show that the standards $s_{p,t}$ and $s_{c,t}$ impose an shadow cost and shadow subsidy on investment in polluting and clean capital respectively. Below, we show that a feebate programs that mimics those shadow values can also decentralize the constrained optimum.

On the other hand, with standards, the firms problem remains:

$$\pi_t = F(A_t, q_{p,t}, q_{c,t}) - R_{c,t} q_{c,t} - R_{p,t} q_{p,t} \tag{C22}$$

Now implying that firms are always willing to pay a strictly positive rent for renting both clean and dirty capacities:

$$\partial_{k_c} F = R_{c,t} \tag{C23}$$

$$\partial_{q_p} F = R_{p,t} \tag{C24}$$

In particular, the rental rate on polluting capital never drops to zero, and remains strictly positive. Combined with the complementary slackness condition,

$$\beta_t \geq 0 \text{ and } \beta_t \cdot (k_{p,t} - q_{p,t}) = 0$$

and the link between β_t and $R_{c,t}$ (C21), strictly positive rental rates imply that with standards on new investment, the household always rents out all the available capital (there are not stranded assets).

Besides, the standards $s_{p,t}, s_{c,t}$ can be optimally set to equal polluting investments found in the previous section. For instance, $s_{p,t} = 0$ until polluting capacities have depreciated to a level compatible with the carbon ceiling.

Since the investor need to obey the standards, investment in this case is equal to investment in the constrained optimum. (This also leads to the same capital stocks, marginal productivities and rental rates than in the constrained equilibrium.) Since all the production in the economy is used to invest or to consume, the consumption implied by investment standards is also equal to consumption in the constrained equilibrium. To summarize, investment standards can be use to enforce the same consumption, investment, and production decisions than under the constrained transition. Investment standards can thus decentralize the constrained equilibrium.

Feebate. — Unsurprisingly, the same transition can be obtained using a so-called feebate program that subsidizes, that is offers a rebate $\theta_{c,t}$ on investment in clean capacity and taxes, that is imposes a fee $\theta_{p,t}$ on investment in polluting capacity. With feebates, the household problem becomes:

$$\max_{c,i,q} \int_0^\infty e^{-\rho t} \cdot u(c_t)\, dt \tag{C25}$$

$$\begin{aligned}
\text{subject to } & R_{c,t} \cdot k_{c,t} + R_{p,t} \cdot q_{p,t} - c_t - i_{p,t}(1+\theta_{p,t}) - i_{c,t}(1-\theta_{c,t}) = 0 && (\lambda_t)\\
& \dot{k}_{p,t} = i_{p,t} - \delta k_{p,t} && (\nu_t)\\
& \dot{k}_{c,t} = i_{c,t} - \delta k_{c,t} && (\chi_t)\\
& i_{p,t} \geq 0 && (\psi_t)\\
& q_{p,t} \leq k_{p,t} && (\beta_t)
\end{aligned}$$

(C26)

First order conditions for the household become:

$$u'(c_t) = \lambda_t \tag{C27}$$

$$\nu_t = \lambda_t(1+\theta_{p,t}) - \psi_t \tag{C28}$$

$$\chi_t = \lambda_t(1-\theta_{c,t}) \tag{C29}$$

$$\lambda_t R_{p,t} = \beta_t = (\rho+\delta)\nu_t - \dot{\nu}_t \tag{C30}$$

$$\lambda_t R_{c,t} = (\rho+\delta)\chi_t - \dot{\chi}_t \tag{C31}$$

To decentralize the constrained social optimum, the government simply needs to set the feebate $(\theta_{p,t}, \theta_{c,t})$ such that the values of clean and polluting investment are the same as in the previous cases; that is choosing $\theta_{p,t}$ such that (C28) is equivalent to (C18) and choosing $\theta_{c,t}$ such that (C29) is equivalent to (C19). In that case, the set of equations that describe the response of the household and producer to the feebate scheme is the same as the set of equation describing their response to the standards.

Moreover, the firms problem remains unchanged, and the same demonstration than above shows that with a feebate, rents are positive, there are no stranded assets, and the rental rate of polluting capacities stays strictly positive.

SECOND-BEST INFEASIBILITY ZONE

This zone defines the cases when the ceiling is reached before polluting capacities have depreciated to a sustainable level. If no investment is made in polluting capacities, we have:

$$k_{p,t} = k_0\, e^{-\delta t}$$

Therefore, the stock of pollution follows this dynamic:

$$\dot{m} = G\, k_0\, e^{-\delta t} - \varepsilon\, m \tag{D1}$$

The solution to this differential equation is:

$$m_t = \left(m_0 + \frac{G\, k_0}{\delta - \varepsilon}\right) e^{-\varepsilon t} - \frac{G\, k_0}{\delta - \varepsilon}\, e^{-\delta t}$$

This function reaches its maximum m_{max} at the date t_{max} when $\dot{m} = 0$. The maximum date is thus

$$t_{max} = -\frac{1}{\delta}\, \ln(\frac{\varepsilon\, m_{max}}{G\, k_0})$$

The expression of m at the maximum date gives the limit of the infeasibility zone if $m_{max} = \bar{m}$:

$$\bar{m} = \left(m_0 + \frac{G\, k_0}{\delta - \varepsilon}\right) e^{\frac{\varepsilon}{\delta}\ln(\frac{\bar{m}\,\varepsilon}{G\, k_0})} - \frac{G\, k_0}{\delta - \varepsilon}\, e^{\ln(\frac{\bar{m}\,\varepsilon}{G\, k_0})}$$

This can be rewritten:

$$\bar{m} = \left[\left(m_0 + \frac{G\,k_0}{\delta - \varepsilon} \right) \left(\frac{\varepsilon}{G\,k_0} \right)^{\frac{\varepsilon}{\delta}} \left(\frac{\delta - \varepsilon}{\delta} \right) \right]^{\frac{\delta}{\delta - \epsilon}}$$

The “clean incentives infeasibility zone” depends on the capital depreciation rate, the GHG dissipation rate, initial GHG concentration and initial polluting capacities.

Since realistic values for the natural decay of atmospheric GHG ε, less than 0.4% per year (e.g. Rezai, Foley and Taylor, 2012), are negligible with respect to capital depreciation $\varepsilon \ll \delta$, the previous relation can be approximated by $\bar{m} = m_0 + \frac{G\,k_0}{\delta}$, which is also simply the maximum of the solution of (D1) when $\varepsilon = 0$.