Firm Knowledge and International Business Cycles

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DISCUSSION PAPER Nº IDB-DP-584

April 2018

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Abstract*

This paper quantifies the flow of knowledge within U.S. multinational corporations in the United States and European Union. A a general equilibrium model of knowledge flows within multinationals is used to compute the parameter values related to knowledge production such that, in steady state, the model matches the observed factor share differentials between the operations of U.S. multinationals in the United States and European Union. The main assumptions are i) U.S. multinationals produce knowledge in the United States, ii) this knowledge is used by its subsidiaries in the European Union and iii) investment in knowledge is either unobserved or expensed in corporate accounts. The results show that a) the calibrated model matches the observed differentials in the rates of return of U.S. multinational investments in the U.S. and in the European Union, and investment in knowledge is 1.4 times larger than investment in physical capital. Furthermore, it is shown that the model calibrated with these parameter values has quantitative implications for international real business cycles. Accounting for the corporate sector GDP correlation, the model with knowledge flows reduces the distance between the standard international real business cycle model and data by 48 percent.

JEL classifications: F44, F23
Keywords: International real business cycles, Firm knowledge, Foreign direct investment (FDI)

* I am grateful to Manuel Amador, Tim Kehoe, Ellen McGrattan, and Juan Pablo Nicolini for valuable advice and guidance. I also thank Anmol Bhandari, Hernan Boedo, Zhifeng Cai, Daniela Costa, Pablo D'Erasmo, Kyle Herkenhoff, Loukas Karabarbounis, Radek Paluszynski, Joao Pimentel, Pedro Teles, Kei-Mu Yi, participants in the Trade and Development Workshop at the University of Minnesota for helpful comments and discussions. Part of this project was written at the Federal Reserve Bank of Minneapolis and Banco de Portugal, both of which I thank both the hospitality and the financial support. The views expressed herein are mine and do not not necessarily reflect the views of the Inter-American Development Bank. This paper previously circulated under the title "Knowledge Flows and Dark Matter." Author email: joaoay@iadb.org

1 Introduction

The production of knowledge by profit-maximizing firms is at the core of the main theories of economic growth and firm dynamics.[1] Firms invest in knowledge and grow, while spillover effects allow the rest of the economy to benefit from it. Knowledge production also has implications for firms' decisions to access foreign markets through foreign direct investment and trade.[2] Yet, the process through which knowledge is produced is largely unknown. The main reasons are that knowledge is an intangible good, and that part of it is produced and used within firms, so that there are no observable transactions involved in the knowledge production process. In this paper, I abstract from endogenous growth and spillover effects and focus my analysis on knowledge production. I focus on a particular group of firms, multinational corporations, and use data on the operations of U.S. multinationals in the United States and European Union to compute the parameter values related to knowledge production.[3] The results show that the investment in knowledge is 1.4 times larger than the investment in physical capital. Furthermore, I show that the calibrated model with knowledge production has quantitative implications for the cross-country correlation in corporate sector GDP fluctuations. This is another contribution of the paper, since knowledge flows are usually restricted to the analysis of long-run growth patterns.[4]

The production of knowledge and its flow within multinational corporations make up one of the main theories for explaining the existence of multinational corporations in the first place (see, e.g., Markusen (1984) and Helpman (1984)). Knowledge is produced within corporations, and that knowledge can be used in different locations at no additional cost. This creates an incentive for these corporations to expand their operations across different locations, including different countries. This theory is particularly relevant to a study of foreign direct investment between the United States and European Union, which is the focus of this paper. Alternative theories explore the differentials in factor costs across countries, such as labor and tangible capital. While they apply well to the U.S. foreign direct investment in China and Mexico, for example, they do not perform as well in the case of the European Union, since its factor costs are not much different from those in the United States.

The international flow of knowledge through multinational corporations is also one of the leading explanations for the fact that the United States has been a net recipient of investment income despite its increasing external debt position—the United States has

[1] In this paper, knowledge is defined as a nonrival and excludable good. Examples of growth theories with knowledge production are Aghion and Howitt (1992), Grossman and Helpman (1991), and Romer (1986, 1990). See Luttmer (2011) for a theory of firm dynamics with knowledge production.

[2] See Melitz (2003) for an example with trade and Helpman, Melitz and Yeaple (2004) for an example with foreign direct investment.

[3] Multinational corporations are corporations with establishments in multiple countries.

[4] Exceptions are Ramondo and Rappoport (2010) and Menno (2015).

been a net borrower since the early 1980s. Commonly known as dark matter (Hausmann and Sturzenegger (2005)), this literature shows that the positive U.S. net investment income is explained by the high rates of return on U.S. foreign direct investment, and that unobservable flows of intangible goods and capital could account for it (e.g., McGrattan and Prescott (2010)).[5]

In order to be able to compute the parameters related to knowledge production, I assume that multinationals produce knowledge in a single location, the country where its headquarters is located. This assumption is supported by the fact that 86% of the research and development (R&D) expenditures of U.S. multinationals take place in the United States, and that these multinationals export knowledge-intensive services such as royalties and license fees from the United States to their foreign subsidiaries, instead of the other way around.

Under the assumption that U.S. multinationals produce knowledge in the United States and that it is used by its subsidiaries abroad, the theory implies differentials in observed labor shares between the activities of U.S. multinationals in the United States and European Union. In particular, the measured labor share of U.S. multinationals in the United States will be higher than its true counterpart if investment in knowledge is either unobserved or expensed in corporate accounts. In addition, the fact that investment in knowledge is expensed does not affect the observed ratio of capital expenditures to compensation of employees. I use data on the activity of U.S. multinationals in the United States and European Union from the Bureau of Economic Analysis (BEA) and show that this is exactly the pattern observed. The labor share of U.S. multinational operations in the European Union, 46.4%, is significantly lower than the labor share of their operations in the United States, 57.4%, whereas the ratios of capital expenditures to compensation of employees—29.7% in the European Union and 32.8% in the United States—are roughly the same.

In order to estimate the share of knowledge in production and its depreciation rate, I build on the technology capital model of McGrattan and Prescott (2010) and compute the parameter values such that the model matches the observed factor shares of U.S. multinationals in the United States and European Union in steady state. The estimates for the knowledge share and its annual depreciation rate are 30% and 11%, respectively. This share is significantly higher than previous studies suggest. These studies rely on empirical estimates of expenditures in R&D, advertisement, organizational capital, and so on. However, such estimates cannot capture all the investment in knowledge that takes place within a firm. For example, the time that a worker spends in routine tasks versus the time that he spends in creating knowledge cannot be observed. In this sense, I am using a broader definition than technology capital, which I simply call knowledge.

[5] Profit-shifting strategies such as transfer pricing could also generate the same pattern. See Bernard, Jensen and Schott (2006) for a study on transfer pricing.

On the other hand, the estimate for the annual depreciation rate of knowledge, 11%, is close to the value that is usually used in the literature, 15%. While my prior is that a significant fraction of investment in knowledge was not being accounted for by previous studies, there is no reason to think that the depreciation rate of the part of knowledge that is actually observed should be different from the depreciation rate of the part that is not observed.

Next, using the estimated parameter values, I show that the flow of knowledge through multinational corporations have quantitative implications for the international transmission of shocks. More specifically, a negative productivity shock in the headquarters country disrupts its production of knowledge. The lower amount of knowledge causes a reduction in the production of its subsidiaries, and this implies a reduction of economic activity in the host country. At the same time, the negative productivity shock reduces the incentives of multinationals whose subsidiaries are located in that country to produce more knowledge. On top of that, the fact that the knowledge stock of a corporation is present in both regions (its nonrival within the corporation) reduces its incentive to move other factors of productions across countries when the economy is hit by a country-specific shock. I start by documenting the following empirical facts that connect multinational activity to the correlation in GDP fluctuations between the corporate sectors in the United States and European Union: i) the correlation in GDP fluctuations between the United States and major European Union countries during 1995–2007 is explained by the correlation between the corporate sectors of these countries; ii) operations of U.S. multinationals in the United States and European Union are highly correlated during the period 1995–2007; iii) the correlation between corporate sectors increases as a function of the share of U.S. multinationals in the corporate sector GDP. Next, I simulate the model to assess its business cycle properties. The model nests the standard international real business cycle model by Backus, Kehoe, and Kydland (1994) as a particular case. One of the main puzzles in international macroeconomics is the failure of the standard international real business cycle model to account for the observed cross-country correlation in GDP fluctuations. Using the parameter values that I estimate, I show that incorporating knowledge flows within multinational corporations reduces the discrepancy between the correlation implied by the model and the correlation observed in the data by 48%.

1.1 Related Literature

This paper is related to the literature that studies intangible flows within firms and the issues related to its measurement (e.g., Corrado, Hulten, and Sichel (2005)). The closest paper to my analysis is McGrattan and Prescott (2010). In their paper, in order to pin down the values for the parameters related to both knowledge and plant-specific intangible capital, the authors use estimates of expenditures on R&D, advertisement, and so on, and calibrate the remaining parameters using data on the value of corporations. Using these estimates, the authors account for the differentials in tangible capital rates of return between U.S. foreign direct investment and foreign direct investment in the United States. In this paper, I take a different route. I compute the parameter values related to knowledge such that, in steady state, the model matches the moments in the data (labor share differentials and ratio of capital expenditures to compensation of employees), and then I use the results to study international business cycles. Ramondo, Rappoport and Ruhl (2016) use firm-level data on U.S. multinational corporations to document that the median subsidiary ships nothing to the rest of the corporation. Atalay, Hortaçsu and Syverson (2014) find similar evidence for multi-establishment corporations in the United States. These findings can be interpreted as supporting the theory of knowledge flows within corporations.

This paper is also related to the literature on international real business cycles (e.g., Backus, Kehoe and Kydland (1992) and Heathcote and Perri (2002)). In particular, this paper is related to the literature that analyzes the connection between foreign direct investment and international business cycles, which includes the empirical evidence in Kleinert, Martin and Toubal (2015), and Cravino and Levchenko (2016), and the quantitative studies based on general equilibrium models of foreign direct investment, such as Cravino and Levchenko (2016), Ramondo and Rappoport (2010), and Menno (2015). In particular, Menno (2015) was the first to incorporate the technology capital model of McGrattan and Prescott (2010) into an international real business cycle framework. While the author focuses on investment synchronization and uses the parameter values of McGrattan and Prescott (2010), my analysis focuses on corporate sector GDP correlation and uses the parameter values based on the calibration strategy described above.

Finally, this paper is related to the literature on dark matter (e.g., Hausmann and Sturzenegger (2005)), which studies the U.S. external debt position and its net investment income. As mentioned before, leading explanations include the exports of intangibles from the United States to the rest of the world that are not observed in the data, which is a feature of the model that I use in this paper. While there are no global imbalances in my analysis, the empirical facts that I present, together with the large share of knowledge that I estimate, provide further support for this type of explanation.

This chapter is organized as follows: Section 2 presents data on U.S. multinationals regarding intrafirm trade in knowledge intensive services and factor shares in production; Section 3 presents a two-country general equilibrium model of knowledge flows through multinational corporations that replicates the pattern observed in the data; in Section 4, I estimate the parameters related to knowledge production; Section 5 presents data on the correlation between corporate sector GDP fluctuations in the United States and European Union, and between GDP fluctuations of U.S. multinational operations in the

United States and European Union; Section 6 assesses the business cycle properties of the model with knowledge flows; Section 7 concludes.

2 Data on U.S. multinationals

In this section, I document the following facts: i) 86% of R&D expenditures of U.S. multinationals takes place in the U.S.; ii) U.S. multinationals export knowledge-intensive services such as royalties and license fees from the United States to their foreign subsidiaries; iii) the labor share of U.S. multinational operations in the European Union is much lower than the labor share of U.S. multinational operations in the United States, whereas the ratios of physical capital expenditures to compensation of employees are roughly the same. Tables and figures are included at the end of the paper.

Table 1: Data on U.S. multinationals

	Share (%)
Share of U.S. multinationals R&D expenditures that takes place in the United States	85.9
Share of exports of royalties and license fees to the European Union that is intrafirm	66.4
Share of U.S. multinationals in U.S. corporate sector GDP	31.4

Source: BEA.

Note: The table shows that most of R&D expenditures of U.S. multinationals take place in the United States, and that most of exports of royalties and license fees are intrafirm. The share of U.S. multinationals R&D expenditures in the United States is the average share in 2004–2007. The share of intrafirm exports of royalties and license fees to the European Union is the average share in 2006–2007. The share of U.S. multinationals in U.S. corporate sector GDP is the average in share in 1995–2007. The changes in time intervals reflect data availability.

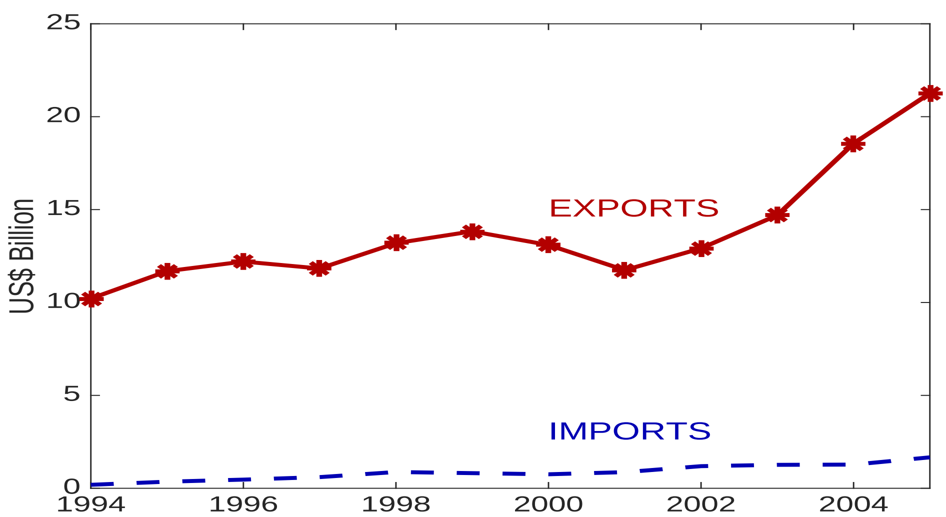
2.1 R&D expenditures and intrafirm trade in royalties and license fees

Table 1 shows statistics on the operations of U.S. multinationals. First, it shows that their R&D expenditures are concentrated in the United States, representing 85.9% of the total. Second, it shows that most of the exports of royalties and license fees from the

United States to the European Union represent exports from U.S. multinationals to their foreign subsidiaries. In this case, intrafirm trade in royalties and license fees accounts for 66.4% of the total exports. Finally, Table 1 shows that U.S. multinationals account for a significant share of the U.S. corporate sector GDP, 31.4%. Figure 1, on the other hand, shows that U.S. multinationals export royalties and license fees from the United States to their affiliates in the European Union, and not the other way around.

Together, these statistics support the model assumptions that I make in Section 3. In particular, I focus on intrafirm flow of knowledge, which is supported by the fact that most of the trade in knowledge intensive services such as royalties and license fees between the United States and European Union represent intrafirm trade. In addition, in the model I assume that the production of knowledge within corporations takes place in a single location, the country where the headquarters is located, which is supported by the fact that U.S. multinationals are net exporters of royalties and license fees (see Figure 1).

Figure 1: U.S. multinationals intrafirm trade of royalties and license fees with the European Union (1994–2005)



Sources: BEA.

Note: The figure shows that U.S. multinationals export royalties and license fees to their subsidiaries in the European Union.

2.2 Labor shares and capital expenditures of U.S. multinationals in the United States and European Union

In this section, I compare the labor shares and the ratio of capital expenditures to compensation of employees between U.S. multinational operations in the United States and European Union. Table 2 summarizes the results. The difference between the shares is significant. While the labor share of U.S. multinationals in the United States is 57.4%, the labor share of their operations in the European Union is much lower, 46.4%.[6] This is surprising, since the labor shares in the United States and European Union corporate sectors are not much different from each other (61.8% and 58.6%, respectively). On the other hand, the ratios between capital expenditures and compensation of employees are similar to each other: 32.8% in the United States versus 29.7% in the European Union.

Figures 2a and 2b show the series. Figure 2a shows that the difference in labor shares is persistent across time, while Figure 2b shows that the ratios of capital expenditures to the sum of capital expenditures and compensation of employees are roughly the same across time.

In the next section, I show that this pattern emerges in a model in which multinationals produce knowledge in their headquarters (home) country and that knowledge is used by their subsidiaries abroad. The investment in knowledge is either unobserved or expensed in corporate accounts, which implies higher (measured) labor shares at home versus abroad.

Table 2: Factor shares: (%) average in 1997–2015

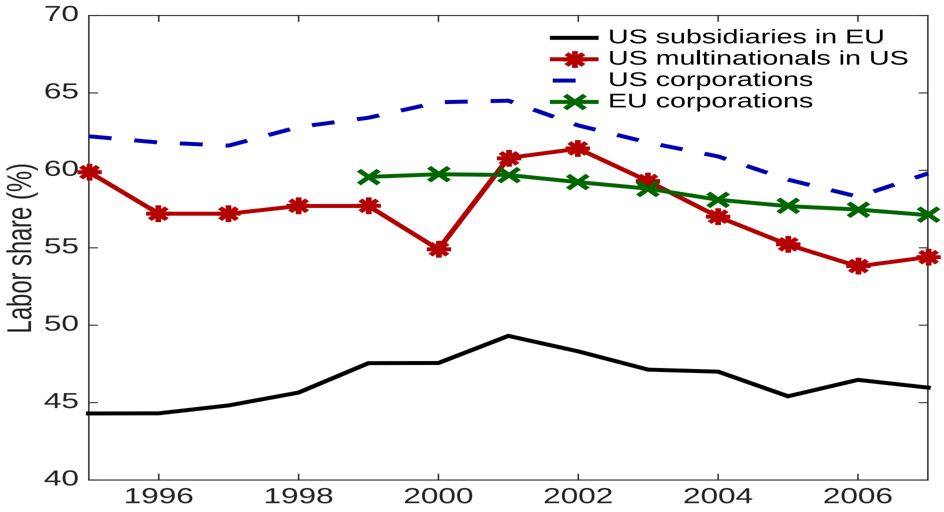
	Labor share (%)	Capital expenditures to compensation of employees ratio (%)
U.S. subsidiaries in EU	**46.4**	29.7
U.S. multinationals in U.S.	**57.4**	32.8
U.S. corporate sector	61.8	-
EU corporate sector (1999:2007)	58.6	-

Source: BEA for the United States, and OECD for the European Union.

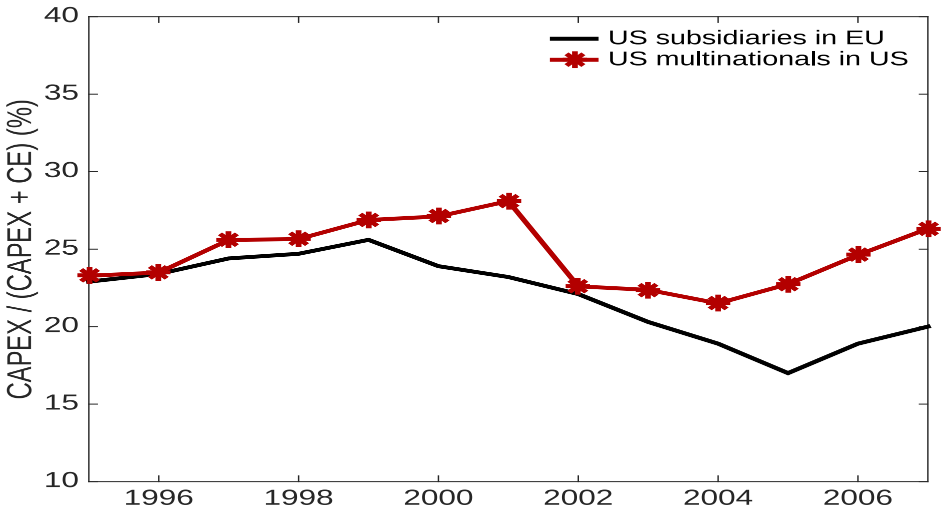
[6] Labor share is defined as the ratio of compensation of employees to gross product (GDP), without adjustments for mixed income. See Mataloni and Goldberg (1994) for a description of the BEA measure of U.S. multinationals value-added.

Figure 2: Factor shares of U.S. multinational operations

(a) Labor share



(b) Capital expenditures



Sources: OECD and BEA.

Note: Figure 4a shows the labor share (compensation of employees over GDP) for four different groups: i) U.S. subsidiaries operating in the European Union; ii) U.S. multinationals operating in the United States; iii) U.S. corporations; iv) EU corporations. Although the labor shares in the U.S. and EU are similar to each other, the labor share of subsidiaries of U.S. multinationals in the European Union is much lower, and this difference is persistent across time. Figure 4b shows the ratio of capital expenditures (CAPEX) to the sum of capital expenditures and compensation of employees (CE), for both U.S. multinationals in the United States and European Union. It shows that they are roughly the same, specially in 1995–2004.

3 Model of international knowledge flows through multinational corporations

In this section, I present a two-country general equilibrium model with knowledge flows within multinational corporations. The model builds on McGrattan and Prescott (2010) by allowing for imperfect substitutability between intermediate goods produced by multinational corporations from different countries. Time is discrete, and the world consists of two symmetric countries ($i = 1, 2$), each characterized by its population size (normalized to one) and a finite measure of locations assumed to be proportional to its population size (the proportionality factor is also normalized to one).[7] To simplify the exposition, I begin by describing the production structure of the economy, and later I describe the households.

Final good production: Final good producers use intermediate goods to produce a nontradable final good that is used for consumption and (tangible) capital investment. I use the final good in country 1 as the numeraire. The nontradable final good in each country is produced according to the following technology:

$$G_1(a_{1,t}, b_{1,t}) = \left(\omega a_{1,t}^{\frac{\sigma-1}{\sigma}} + (1-\omega) b_{1,t}^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}}, \tag{1}$$

$$G_2(a_{2,t}, b_{2,t}) = \left((1-\omega) a_{2,t}^{\frac{\sigma-1}{\sigma}} + \omega b_{2,t}^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}}, \tag{2}$$

where σ, $\sigma \geq 0$, is the elasticity of substitution between the tradable intermediate goods produced by firms from country 1, a, and by firms from country 2, b. The production technology is an Armington aggregator and ω, $0 < \omega < 1$, determines the degree of home bias.[8] In each period t, the final good producers in country 1 solve the maximization problem:

$$\max_{a_{1,t}, b_{1,t} \geq 0} G_1(a_{1,t}, b_{1,t}) - q_{a,t} a_{1,t} - q_{b,t} b_{1,t}. \tag{3}$$

The problem for the final good producer in country 2 is analogous, but note that the price of its final good, $p_{2,t}$, might be different than one.

The structure so far is very close to the standard international real business cycle model (e.g., Backus, Kehoe and Kydland (1994) and Heathcote and Perri (2002)) without

[7] The concept of "location" allows the introduction of a firm-specific nonrival input (knowledge) into a standard model in which agents take prices as given. Corporations are able to use the nonrival input in different locations simultaneously, but the finite measure of locations in each country prevents them from expanding without bound. See McGrattan and Prescott (2009, 2010) for a full description of a model with locations.

[8] In this paper, home bias is not related to the location where the good is produced, but to the nationality of the corporation that produces the good.

aggregate uncertainty.[9] Besides abstracting from aggregate uncertainty, the difference is in the production structure of the intermediate goods a and b, which I describe next.

Instead of restricting intermediate goods a and b to be produced in a single country and then exported, I allow for their production to take place in both countries. The intermediate good producers will then be multinational corporations.

Intermediate good production: Multinational corporations from country 1 produce good a according to the following technology:

$$a_{1,t} + a_{2,t} = y_{11,t} + y_{12,t}, \tag{4}$$

$$y_{11,t} = z_{1,t} F(k^y_{11,t}, m_{1,t}, n^y_{11,t}), \tag{5}$$

$$y_{12,t} = z_{2,t} F(k^y_{12,t}, \theta m_{1,t}, n^y_{12,t}), \tag{6}$$

where $F(k, m, n) = (k^\alpha n^{1-\alpha})^{1-\phi} m^\phi$, $0 < \alpha < 1, 0 < \phi < 1$. Multinational corporations from country 1 produce the intermediate good a using capital k, firm-specific knowledge m, and labor, n. The amount of good a produced in country 1 is y_{11}, and y_{12} is the amount of good a produced by subsidiaries in country 2. Note that m is nonrival, it is used both in (5) and (6).[10] There is a cost of adapting the firm-specific knowledge to a different country, and its denoted by θ, $0 < \theta < 1$. As long as $m > 0$ and $\theta > 0$, the corporation will always choose to operate in both countries. Knowledge is produced according to the following technology:

$$g^m_{1,t} = z_{1,t} F(k^m_{11,t}, m_{1,t}, n^m_{11,t}), \tag{7}$$

$$m_{1,t+1} = (1 - \delta^m) m_{1,t} + g^m_{1,t}, \tag{8}$$

where F is the same production function as in (5) and (6), δ^m is the depreciation rate of knowledge, and g^m is the investment in knowledge.[11] Productivity in countries 1 and 2 are denoted by $z_{1,t}$ and $z_{2,t}$, respectively, and they are assumed to be constant over time and normalized to one, so that $z_{1,t} = z_{2,t} = 1$ for all t.[12] Note that corporations from country 1 produce knowledge only in country 1. The multinational corporation from country 1 then comprises its headquarters in country 1, that produces knowledge, and its subsidiaries in both countries 1 and 2, that produce the intermediate good a.

Given the initial capital and knowledge stocks, multinationals from country 1 choose labor, capital investment, and knowledge investment, in order to maximize the discounted

[9] In Section 6, I introduce aggregate uncertainty and analyze the model's business cycle properties.

[10] It is also used in knowledge production.

[11] Note that I am allowing for the relative price of knowledge with respect to the price of the intermediate good to differ from one.

[12] In Section 6, I allow for productivities to follow an exogenous stochastic process.

value of dividend payments:

$$\max \sum_{t=0}^{\infty} \Lambda_{0,t} D_{1,t}, \tag{9}$$

where $\Lambda_{t_0,t}$ is the pricing kernel.[13] The dividends payment $D_{1,t}$ is given by

$$\begin{aligned} D_{1,t} = \; & q_{a,t}(y_{11,t} + y_{12,t}) + p^m_{1,t} g^{m,s}_{1,t} - (w_{1,t}(n^y_{11,t} + n^m_{11,t}) + w_{2,t} n^y_{12,t}) \\ & -(x^y_{11,t} + p_{2,t} x^y_{12,t} + x^m_{11,t} + p^m_{1,t} g^{m,d}_{1,t}), \end{aligned} \tag{10}$$

where p_2 is the price of the final good in country 2, p^m_1 is the price of the investment in knowledge from multinationals of country 1, and w_1 and w_2 are the wage rates in country 1 and 2, respectively. The capital investment for knowledge production in country 1 is denoted by x^m_{11}, whereas the capital investment for intermediate-good production in countries 1 and 2 are denoted by x^y_{11} and x^y_{12}, respectively. Remember that the capital investment good is nontradable and the corporation must invest in capital in country 2 in order to produce there. I also make a distinction between the amount of knowledge investment supplied by the firm, $p^m_1 g^{m,s}_1$, and the amount of knowledge investment demanded by the firm, $p^m_1 g^{m,d}_1$, so I am assuming there is a competitive market for it and its price will be determined accordingly. In equilibrium, the supply and demand of knowledge investment good will be the same and will cancel each other out.

The production structure and maximization problem for multinationals from country 2 are analogous.

Capital accumulation: Capital evolves according to:

$$k^y_{ij,t+1} = (1-\delta^k) k^y_{ij,t} + x^y_{ij,t}, \tag{11}$$

$$k^m_{ii,t+1} = (1-\delta^k) k^m_{ii,t} + x^m_{ii,t}, \tag{12}$$

for all $i, j = 1, 2$, $i \neq j$, and δ^k, $0 < \delta^k < 1$, is the depreciation rate of capital.

Households: Households from country i value consumption, $c_{i,t}$, and leisure, $1 - n_{i,t}$. The utility function of the household is given by

$$\sum_{t=0}^{\infty} \beta^t U(c_{i,t}, n_{i,t}), \tag{13}$$

where $U(c,n) = \left(c^{\mu}(1-n)^{1-\gamma}\right)^{\gamma}/\gamma$, $0 < \mu < 1$, $\gamma < 1$.

The budget constraint of the household in country 1 is given by

[13] In equilibrium, $\Lambda_{0,t} = \beta^t \frac{U_c(c_{1,t},n_{1,t})}{U_c(c_{1,0},n_{1,0})} = \beta^t \frac{U_c(c_{2,t},n_{2,t})}{U_c(c_{2,0},n_{2,0})}$.

$$c_{1,t} + \sum_{j=1,2} p^v_{j,t}(V^j_{1,t+1} - V^j_{1,t}) = w_{1,t}n_{1,t} + \sum_{j=1,2} V^j_{1,t}D_{j,t}, \tag{14}$$

where w_1 is the wage rate in country 1. Households trade stocks of intermediate-good producing firms (V^j_i denotes the shares of multinational corporations from country j owned by households in country i).[14] The price of the stocks of intermediate-goods producing firms from country j is denoted by p^v_j. The budget constraint of the household in country 2 is analogous. Households maximize their utility subject to the budget constraints, initial stock holdings, and usual non-negativity constraints.

Market-clearing conditions: The market clearing conditions are:

$$\text{(final goods)} \qquad c_{i,t} + x^y_{ii,t} + x^y_{ji,t} + x^m_{ii,t} = G_i(a_{i,t}, b_{i,t}), \tag{15}$$

$$\text{(labor)} \qquad n_{i,t} = n^y_{ii,t} + n^y_{ji,t} + n^m_{ii,t}, \tag{16}$$

$$\text{(intermediate good a)} \qquad a_{1,t} + a_{2,t} = y_{11,t} + y_{12,t}, \tag{17}$$

$$\text{(intermediate good b)} \qquad b_{1,t} + b_{2,t} = y_{21,t} + y_{22,t}, \tag{18}$$

$$\text{(knowledge investment)} \qquad g^{m,s}_{i,t} = g^{m,d}_{i,t} = z_{i,t}F(k^m_{ii,t}, m_{i,t}, n^m_{ii,t}), \tag{19}$$

for all $i, j = 1, 2$, $i \neq j$.

Equilibrium: a competitive equilibrium is a sequence of prices ($p_{2,t}$, $p^m_{1,t}$, $p^m_{2,t}$, $p^v_{1,t}$, $p^v_{2,t}$, $q_{a,t}$, $q_{b,t}$, $w_{1,t}$, $w_{2,t}$), a sequence of labor allocations ($n_{1,t}$, $n_{2,t}$, $n^y_{11,t}$, $n^y_{12,t}$, $n^y_{12,t}$, $n^y_{22,t}$, $n^m_{11,t}$, $n^m_{22,t}$), a sequence of consumption ($c_{1,t}, c_{2,t}$), a sequence of pricing kernel ($\Lambda_{0,t}$), a sequence of intermediate-good allocations ($a_{1,t}$, $a_{2,t}$, $b_{1,t}$, $b_{2,t}$, $y_{11,t}$, $y_{12,t}$, $y_{21,t}$, $y_{22,t}$), a sequence of capital allocations ($k^y_{11,t}$, $k^y_{12,t}$, $k^y_{21,t}$, $k^y_{22,t}$, $k^m_{11,t}$, $k^m_{22,t}$), a sequence of capital investment allocations ($x^y_{11,t}$, $x^y_{12,t}$, $x^y_{21,t}$, $x^y_{22,t}$, $x^m_{11,t}$, $x^m_{22,t}$), a sequence of knowledge allocations ($m_{1,t}$, $m_{2,t}$), a sequence of knowledge investment allocations ($g^{m,d}_{1,t}$, $g^{m,s}_{1,t}$,$g^{m,d}_{2,t}$, $g^{m,s}_{2,t}$), a sequence of dividends ($D_{1,t}$,$D_{2,t}$), and a sequence of stock holdings ($V^1_{1,t}$,$V^2_{1,t}$,$V^1_{2,t}$,$V^2_{2,t}$) such that, given initial capital stocks, knowledge stocks, stock holdings, and given the sequence of prices: i) the allocations solve the households problem; ii) the allocations solve both final-good and intermediate-good firm's problems; iii) market-clearing conditions are satisfied; iv) pricing kernel satisfies $\Lambda_{0,t} = \beta^t \frac{U_c(c_{1,t},n_{1,t})}{U_c(c_{1,0},n_{1,0})}$.

Optimality conditions: final-good producers face a sequence of static problems. The optimality conditions of their profit-maximization problem imply that the following

[14] They also receive profits from the final-good producing firms. It is zero in equilibrium.

relation must hold in equilibrium:

$$\frac{G_{1a}(a_{i,t}, b_{i,t})}{G_{ib}(a_{i,t}, b_{i,t})} = -\frac{q_{a,t}}{q_{b,t}}, \tag{20}$$

for $i = 1, 2$.

Regarding the maximization problem of the intermediate-good producers of country 1 (multinationals from country 1), the following conditions must hold in equilibrium:

$$w_{1,t} = q_{a,t} z_{1,t} F_n(k^y_{11,t}, m_{1,t}, n^y_{11,t}), \tag{21}$$

$$w_{2,t} = q_{a,t} z_{2,t} F_n(k^y_{12,t}, \theta m_{1,t}, n^y_{12,t}), \tag{22}$$

$$w_{1,t} = p^m_{1,t} z_{1,t} F_n(k^m_{11,t}, m_{1,t}, n^m_{11,t}), \tag{23}$$

$$1 = \frac{\Lambda_{0,t+1}}{\Lambda_{0,t}} \left[q_{a,t+1} z_{1,t+1} F_k(k^y_{11,t+1}, m_{1,t+1}, n^y_{11,t+1}) + (1 - \delta^k) \right], \tag{24}$$

$$p_{2,t} = \frac{\Lambda_{0,t+1}}{\Lambda_{0,t}} \left[q_{a,t+1} z_{2,t+1} F_k(k^y_{12,t+1}, \theta m_{1,t+1}, n^y_{12,t+1}) + p_{2,t+1}(1 - \delta^k) \right], \tag{25}$$

$$1 = \frac{\Lambda_{0,t+1}}{\Lambda_{0,t}} \left[p^m_{1,t+1} z_{1,t+1} F_k(k^m_{11,t+1}, m_{1,t+1}, n^m_{11,t+1}) + (1 - \delta^k) \right], \tag{26}$$

$$p^m_{1,t} = \frac{\Lambda_{0,t+1}}{\Lambda_{0,t}} \times \left[\begin{array}{c} q_{a,t+1} z_{1,t+1} F_m(k^y_{11,t+1}, m_{1,t+1}, n^y_{11,t+1}) \\ + \, q_{a,t+1} \theta z_{2,t+1} F_m(k^y_{12,t+1}, \theta m_{1,t+1}, n^y_{12,t+1}) \\ + \, p^m_{1,t+1} \left(z_{1,t+1} F_m(k^m_{11,t+1}, m_{1,t+1}, n^m_{11,t+1}) + (1 - \delta^m) \right) \end{array} \right], \tag{27}$$

together with the transversality conditions.[15] Conditions (21)–(23) and (24)–(26) are the standard optimality conditions for labor and capital investment decisions, respectively.

Equation (27), on the other hand, is not standard. It is the optimality condition with respect to investment in knowledge, and it shows how the price of the knowledge investment good is determined in equilibrium. In particular, the cost of acquiring a unit of knowledge investment good in period t, $p^m_{1,t}$, must be equal to the benefit of acquiring it. The benefit is given by the extra value of intermediate-good production in the following period (produced by subsidiaries in both countries), the extra value of knowledge production in the following period, and the value of the stock of knowledge after depreciation in the following period, all discounted by $\frac{\Lambda_{0,t+1}}{\Lambda_{0,t}}$ to reflect period t values. The optimality conditions for the multinationals from country 2 are analogous.

Finally, the following optimality conditions are derived from the household maximiza-

[15]The transversality conditions are: $\lim_{t\to\infty} \Lambda_{0,t+1} k^y_{11,t+1} = 0$, $\lim_{t\to\infty} \Lambda_{0,t+1} k^y_{12,t+1} = 0$, $\lim_{t\to\infty} \Lambda_{0,t+1} k^m_{11,t+1} = 0$, and $\lim_{t\to\infty} \Lambda_{0,t+1} p^m_{1,t+1} m_{1,t+1} = 0$.

tion problem:

$$\frac{w_{i,t}}{p_{i,t}} = -\frac{U_n(c_{i,t}, n_{i,t})}{U_c(c_{i,t}, n_{i,t})}, \tag{28}$$

$$1 = \beta\frac{U_c(c_{i,t+1}, n_{i,t+1})}{U_c(c_{i,t}, n_{i,t})}(D_{j,t+1} + p^v_{j,t+1}), \tag{29}$$

for $j = 1, 2$, together with the transversality conditions.[16] Equation (28) is the standard intratemporal Euler equation, and (29) is the standard intertemporal Euler equation with respect to stock holdings of corporations from country j, $j = 1, 2$.

Therefore, the equilibrium is characterized by the optimality conditions (20)–(29), together with the budget constraints of the households (14), the constraints of the multinational corporations (4)–(8), the transversality conditions of both the household and multinational corporation problems, the capital accumulation equations (11)–(12), the market clearing conditions (15)–(19), and $\Lambda_{0,t} = \beta^t \frac{U_c(c_{1,t},n_{1,t})}{U_c(c_{1,0},n_{1,0})}$.

In the next section, I analyze the steady state properties of the model outlined above. In this environment, the competitive equilibrium is equivalent to a solution to the planner's problem that maximizes a weighted sum of households utility in country 1 and 2 subject to the resource constraints (equal to the market clearing conditions). In other words, the welfare theorems apply. I analyze the solution to the problem in which the planner gives equal weight to both countries.

4 Steady state, measurement, and knowledge production

In steady state, allocations and prices are constant. I compute the steady state equilibrium of the planner's problem in which the planner gives equal weight to households from both countries. The symmetric steady-state allocation is computed in Appendix A.

As discussed in the introduction, in this section I compute the parameter values for the share of knowledge in production, ϕ, and the depreciation rate of knowledge, δ^m, such that, in steady state, the model matches the factor shares observed in the data. But first, I make a distinction between the true measures in the model and their data-equivalent counterparts.

Let country 1 represent the United States, and country 2 the European Union. An important observation is that, in the model, I define a multinational corporation as a U.S. multinational corporation if it produces knowledge in the United States.[17] This definition

[16] The transversality conditions are: $\lim_{t\to\infty} \Lambda_{0,t+1}p^v_{j,t+1}V^j_{1,t+1} = 0$, $j = 1, 2$.

[17] In the case of imperfect substitutability between intermediate goods produced by multinational corporations from different countries, the nationality of each multinational is also directly related to the good it produces.

has no direct relation to its ownership structure. On the other hand, BEA defines U.S. multinationals according to its ownership structure.[18] Therefore, when using the data on U.S. multinational corporations, I am making the assumption that both definitions coincide.[19]

Another observation is that the measurement issues that I discuss below have no effect on the actual allocation of resources. However, I show that I can use the measures in the data to infer the parameters from the model.

Royalties and license fees: According to the model, the true value-added of U.S. multinational subsidiaries in the European Union is:

$$gdp_{12,t} = \underbrace{q_{a,t}y_{12,t}}_{\text{output approach}} = \underbrace{w_{2,t}n^y_{12,t} + r^k_{12,t}k^y_{12,t} + r^m_{12,t}m_{1,t}}_{\text{income approach}},$$

where $r^k_{12,t} = q_{a,t}z_{2,t}F_k(k^y_{12,t}, \theta m_{1,t}, n^y_{12,t})$ is the return on capital invested in U.S. multinational subsidiaries in the European Union, and $r^m_{12,t} = q_{a,t}\theta z_{2,t}F_m(k^y_{12,t}, \theta m_{1,t}, n^y_{12,t})$ is the return on knowledge investment from operations of U.S. multinational subsidiaries in the European Union. However, in the data, a fraction τ of the return on knowledge, $\tau r^m_{12,t}m_{1,t}$, is treated as exports of royalties and license fees—see Section 2.1. In this case, the measured value-added of U.S. subsidiaries in the European Union is:

$$\widetilde{gdp}_{12,t} = \underbrace{q_{a,t}y_{12,t} + \tau_1 r^m_{12,t}m_{1,t}}_{\text{output approach}} = \underbrace{w_{2,t}n^y_{12,t} + r^k_{12,t}k^y_{12,t} + (1-\tau_1)r^m_{12,t}m_{1,t}}_{\text{income approach}}.$$

Therefore, the first adjustment that I make is to add the value of net exports of royalties and license fees from U.S. multinationals to their foreign affiliates in the European Union to the value-added of these subsidiaries, and to subtract its value from the value-added of U.S. multinational operations in the United States.[20]

Knowledge investment: Investment in knowledge by private corporations is either unobserved or expensed in corporate accounts. Regarding the data on U.S. multinational operations for the period 1995–2007, value-added (GDP) does not include expenditures for R&D. Data on U.S. corporate sector GDP, on the other hand, includes investment in R&D.[21] However, I am assuming that the investment in R&D that is observed in the data on U.S. corporate sector GDP does not necessarily account for all the investment

[18] In particular, BEA uses the 10% ownership threshold to define foreign direct investment.

[19] This will be the case if knowledge is produced by the headquarters of the multinational corporation.

[20] One can also think of $\widetilde{p} = \tau_1 r^m_{12,t}$ as the transfer price of royalties and license fees.

[21] After BEA's 2013 comprehensive revision, expenditures for R&D started to be treated as investment in intellectual property rights and to be included in private fixed investment. See McCulla, Holdren, and Smith (2013) for a description of the revision.

in knowledge that takes place within U.S. corporations. In order to make the data on U.S. corporate sector GDP comparable to the data on value-added (GDP) of U.S. multinationals, I subtract the investment in intellectual property rights by private businesses from corporate sector GDP. Therefore, the true and the measured value-added of operations of U.S. multinational corporations in the United States are (I use tilde for measured values):[22,23]

$$\begin{aligned} gdp_{11,t} &= q_{a,t}y_{11,t} + p^m_{1,t}g^m_{1,t}, \\ \widetilde{gdp}_{11,t} &= q_{a,t}y_{11,t}, \end{aligned}$$

where GDP is computed according to the output approach. So measured GDP does not include investment in knowledge, $p^m_{1,t}g^m_{1,t}$, whereas true GDP does.

Labor share: The fact that measured GDP is different from its true counterpart has implications for the observed labor shares, which is defined as the ratio of compensation of employees to GDP. The payments to employees are observable, so true and measured compensation of employees are the same. The fact that measured GDP does not include investment in knowledge implies that measured GDP is higher than its true counterpart if investment in knowledge is positive, so that the measured labor share is higher than the true labor share.

According to the model presented above, the true labor share of domestic operations, LS_{11}, and foreign operations, LS_{12}, of U.S. multinationals are both equal to $(1-\alpha)(1-\phi)$. This comes from the fact that I am assuming a Cobb-Douglas production function, $F(k,m,n) = (k^\alpha n^{1-\alpha})^{1-\phi}m^\phi$, in which $\alpha(1-\phi)$, $(1-\alpha)(1-\phi)$, and ϕ are the actual factor shares of capital, labor, and knowledge, respectively. So the true labor shares are the same in both countries. The assumption that production functions are the same in both countries is supported by the fact that measured factor shares in the United States and European Union corporate sectors are similar to each other. Their data-equivalent counterparts are:

$$\widetilde{LS}_{11,t} = \frac{w_{1,t}(n^y_{11,t} + n^m_{11,t})}{q_{a,t}y_{11,t}} = (1-\alpha)(1-\phi)\left(1 + \frac{p^m_{1,t}g^m_{1,t}}{q_{a,t}y_{11,t}}\right), \tag{30}$$

$$\widetilde{LS}_{12,t} = \frac{w_{2,t}n^y_{12,t}}{q_{a,t}y_{12,t}} = (1-\alpha)(1-\phi). \tag{31}$$

[22]I am assuming that measured values already include the adjustment for exports of royalties and license fees discussed above.

[23]Note that I am assuming that U.S. corporate sector GDP is divided into two groups: GDP of foreign subsidiaries of EU multinationals, and GDP of U..S. corporations. So I am treating all U.S. corporations that are not EU multinational subsidiaries as U.S. multinationals.

Therefore, the measured labor share of U.S. multinationals in the European Union is not distorted ($\widetilde{LS}_{12,t} = LS_{12,t}$), only their measured labor share in the United States is. The reason why the measured labor share is only distorted in the United States is because I am assuming that investment in knowledge only takes place in the headquarters country, the United States. Since investment in knowledge is positive for $\phi > 0$, it implies that the measured labor share in the U.S. will be higher than in the European Union. Therefore, the model with unobserved knowledge investment replicates the pattern observed in the data, and I will use this information to compute both the share of knowledge in the production function, ϕ, and the depreciation rate of knowledge, δ^m.

4.1 Steady-state and knowledge production

I use the steady-state equilibrium of the model to pin-down the parameter values related to knowledge production. In particular, I use the following relations:[24]

$$\widetilde{LS}_{11} = (1-\alpha)(1-\phi)\left(1+\frac{\delta^m\phi\beta(1+\theta)}{1-\beta+\beta\delta^m(1-\phi)}\right), \tag{32}$$

$$LS_{12} = (1-\alpha)(1-\phi), \tag{33}$$

$$\frac{CAPEX_{11}}{CE_{11}} = \frac{\delta^k\beta}{1-\beta(1-\delta^k)}\frac{\alpha}{1-\alpha}, \tag{34}$$

$$\frac{CE_{12}}{CE_{11}} = \theta\frac{1-\beta(1-\delta^m)-\delta^m\phi\beta}{1-\beta(1-\delta^m)+\delta^m\phi\beta\theta}, \tag{35}$$

where $CAPEX_{11} = x^y_{11} + x^m_{11}$ denotes capital expenditures of U.S. multinationals in the United States, and $CE_{11} = w_1(n^y_{11} + n^m_{11})$ and $CE_{12} = w_2 n^y_{12}$ denote compensation of employees of U.S. multinational operations in the United States and in the European Union, respectively. Note that I have six parameters, $(\alpha, \phi, \delta^k, \delta^m, \beta, \theta)$, and I selected four moments. Parameters β and δ^k are selected according to observed real interest rates and the actual estimates of capital depreciation from BEA. One important thing to note is that the six parameters, $(\alpha, \phi, \delta^k, \delta^m, \beta, \theta)$, will not depend on the remaining parameters, $(\mu, \sigma, \gamma, \omega)$. So I do not need to specify their values in order to compute the share of knowledge, ϕ, and its depreciation rate, δ^m.

Table 3 shows the results. The estimated share of knowledge, $\phi = 0.29$, is much larger than previous studies suggest. For example, McGrattan and Prescott (2010) use 7.0% for the share of technology capital. On the other hand, its depreciation rate, $\delta^m = 0.17$, is close to the value used by the BEA, 15%.

Investment in knowledge: The estimates presented above imply that the invest-

[24]See Appendix A for derivation.

ment in knowledge is actually 1.4 times the investment in capital, whereas the BEA reports that this ratio is only 0.4.[25] Therefore, the data accounts for less than 30% of the total investment in knowledge by U.S. corporations.

Table 3: Matching Moments

Matched moments		Values
Labor share in the U.S. corporate sector		0.58
Labor share of U.S. multinationals in the European Union		0.45
Ratio of capital expenditures to compensation of employees		0.32
Ratio of compensation of employees in subsidiaries over headquarters		0.045
Parameters calibrated to match the moments above		
Share of knowledge	ϕ	**0.30**
Capital/Labor share	α	0.36
Depreciation rate of knowledge	δ^m	**0.11**
Foreign direct investment cost	θ	**0.06**
Other parameters		
Annual depreciation rate of (tangible) capital	δ^k	0.06
Discount factor	β	0.96

International flow of knowledge: Let $r^m_{12,t} m_{1,t}$ denote the flow of knowledge from the United States to the European Union that takes place within U.S. multinationals. The estimates imply that the observed intrafirm net exports of royalties and license fees of U.S. multinationals from the United States to the European Union represent only 13% of the total flow of knowledge. That is, $\tau = 0.13$.

Dark matter (U.S. FDI position): BEA only reports the FDI flows of tangible capital. The value of the stock of knowledge used by U.S. subsidiaries in the European Union can be computed as the present value of the flow of returns on knowledge, $r^m_{12,t} m_{1,t}$, taking into account its depreciation rate. Its values is on average 6.1% of U.S. GDP during the period 1995-2007, which implies that the statistics on U.S. FDI stock are biased

[25]This is the ratio of private domestic investment in intellectual property rights to private domestic nonresidential investment excluding intellectual property rights.

downwards by that amount.

Other implications The high estimates for the share of knowledge have also other important implications. For example, it has implications for international business cycle fluctuations. In Section 6, I simulate the stochastic version of the model outlined above and show that it greatly improves the capacity of the standard international real business cycle model to generate GDP correlations closer to the ones observed in the data. Another example is the welfare implications of the Base Erosion and Profit Shifting (BEPS) action plan by the Organisation for Economic Co-operation and Development (OECD), which can (potentially) have a significant impact on the effective tax rate faced by multinational corporations, therefore affecting the international flow of knowledge. This is part of future work.

5 GDP fluctuations in corporate sectors and U.S. multinationals

Before presenting the business cycle model with international knowledge flows, I present empirical evidence that suggests a central role to U.S. multinationals in accounting for the corporate sector GDP correlation between the United States and European Union. In this section, I document the following facts: i) the correlation in GDP fluctuations between the United States and European Union in 1995–2007 is explained by the correlation between their corporate sectors; ii) fluctuations in the operations of U.S. multinationals in the United States and European Union are highly correlated in 1995–2007; iii) the corporate sector correlation with the United States increases as a function of the share of U.S. multinationals in corporate sector GDP.

5.1 Cross-country correlation in corporate GDP fluctuations

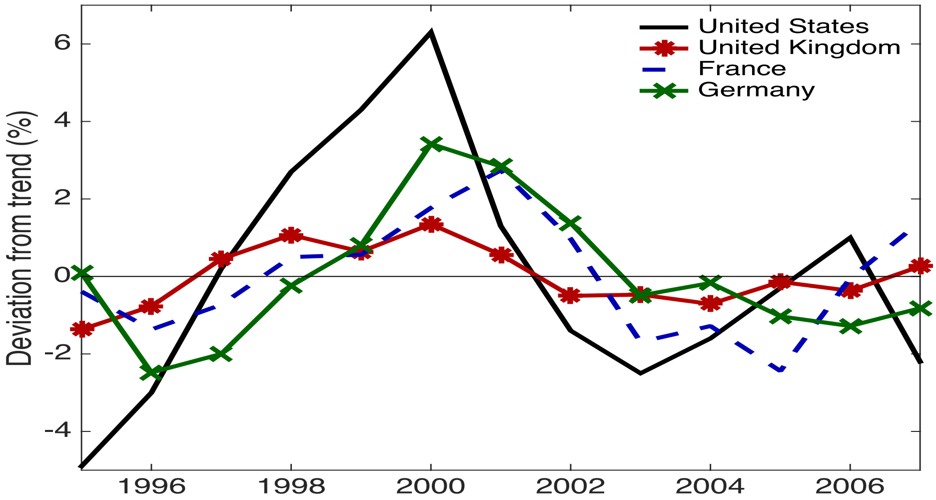
In this section, I document the positive correlation between corporate sector real GDP fluctuations in the United States and major EU countries during the period 1995-2007. Data are from the Organisation for Economic Co-operation and Development (OECD) for EU countries and from the Bureau of Economic Analysis (BEA) for the United States. The corporate sector comprises both financial and nonfinancial corporations.[26] Data for the noncorporate sector are computed by subtracting the corporate sector from the whole economy, and real values are computed using the respective country's GDP deflator. Table 5 reports the cross-country correlations of GDP fluctuations between the United States and major EU countries, together with the share of the corporate sector in each

[26]The noncorporate sector comprises general government, households, and nonprofit institutions serving households. I also separated financial and nonfinancial corporations, and the results did not change.

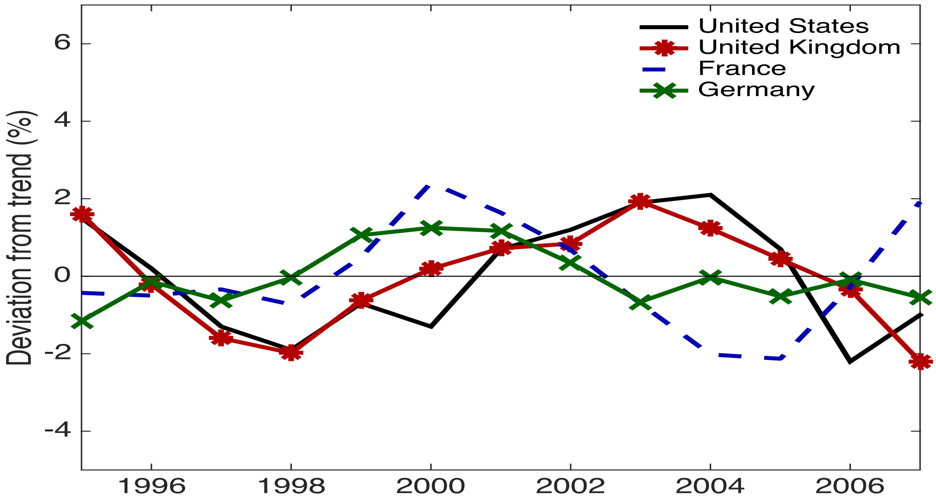
economy. Fluctuations are computed as the difference between log values and a log-linear trend.

Figure 3: Corporate and Noncorporate GDP Fluctuations

(a) Corporate Sector GDP



(b) Noncorporate Sector GDP



Sources: OECD and BEA.

Note: The figure shows the deviation of the log values of real GDP from their log-linear trends. GDP is divided between (a) corporate sectors and (b) noncorporate sectors, and I compute the log-linear trend for each series separately.

The results show a high correlation between corporate sectors' GDP. For example, the corporate sector GDP correlation between Germany and the United States in 1995–2007 is 0.48, whereas the correlation in the noncorporate sector is -0.36. Table 5 also reports the GDP-weighted average of each statistic for a group of 18 EU countries.[27] The average GDP correlation in the corporate and noncorporate sectors are 0.55 and -0.06, respectively.

Although the average share of the corporate sector in the economy is close to one-half (56%), the difference in correlations is striking. While the corporate sectors show a high positive correlation, the noncorporate sectors show a weak negative correlation. However, the negative correlation in the noncorporate sector is not robust to small changes in the sample period. In the case of Germany, for example, the correlation increases from -0.36 to -0.09 once the sample period is expanded to 1991–2007 using data from the German Federal Statistical Office (Destatis).

The high correlation in the corporate sector is also surprising because this period (1995–2007) is part of the period known as the Great Moderation (1985–2007), which is characterized by the low volatility of main macroeconomic variables. Table 4 shows that the standard deviation of the annual U.S. corporate GDP growth rate in that period was 2.2%, which is 35% lower than the standard deviation during the whole sample (1949–2015).[28] These results suggest that the explanation for the comovement based on large correlated shocks is less plausible, since there were no evident large shocks in 1995–2007 compared to the oil price variations in the 1970s or the financial crisis in 2008–2009.

Figure 3 shows the series of fluctuations for the United States, United Kingdom, France, and Germany for both corporate and noncorporate sectors. Figure 3a shows that there was a period of fast growth in corporate GDP between 1995 and 2000 for all countries, followed by a sharp decline in growth afterward. This interpretation is based on the fact that deviations move from negative to positive values between 1995 and 2000, and back to negative values in 2005. On the other hand, in the noncorporate sector, Figure 3b, there is less variation in deviations and no clear pattern such as the one observed in the corporate sector.

[27] Weights are based on the nominal GDP of each country in 2001 in U.S. dollars. I only kept those countries with data for all years in 1995–2007. For example, Spain and Ireland were excluded because their series start in 1999. The countries are Austria, Belgium, Czech Republic, Denmark, Estonia, Finland, France, Germany, Greece, Hungary, Italy, Latvia, Netherlands, Portugal, Slovak Republic, Slovenia, Sweden, and United Kingdom.

[28] Refer to Stock and Watson (2002) for an extensive description of that period.

Table 4: Standard deviation of real GDP growth rates by period (%)

	1949–2015	1962–1984	1985–2007
United States			
Total economy	2.4	2.5	1.2
Corporate sector	3.4	3.1	**2.2**
Noncorporate sector	2.1	2.2	1.6
France			
Total economy	2.2	2.1	1.2
Corporate sector	2.9	2.8	1.8
Noncorporate sector	2.0	2.1	0.9

Source: BEA for the United States, Insee for France.

Therefore, I interpret these results as indicating a close link between the corporate sectors in the United States and in the European Union, a link that I try to account for in this paper by incorporating knowledge flows through multinational corporations. In the next section, I show that the operations of U.S. multinationals in the United States and of their subsidiaries in the European Union are even more correlated, and that the correlation of corporate GDP fluctuations increases as a function of the share of U.S. multinationals in corporate GDP.

5.2 Correlation in GDP fluctuations between U.S. multinational operations in the United States and European Union

Table 6 reports the correlation between fluctuations in economic activity of U.S. multinational operations in the United States and the operations of their subsidiaries in the European Union. Data are from the BEA and cover the period 1983–2013 (except for the GDP series, which starts in 1994). The BEA uses the threshold of 10% of ownership to define foreign direct investment; that is, a foreign enterprise is classified as a subsidiary of a U.S. corporation if more than 10% of its business is owned by a U.S. corporation. However, the BEA reports separate statistics for those subsidiaries that are majority owned (ownership share above 50%), which include a richer set of data. Unless otherwise stated, the statistics in this paper refer to the majority-owned group.[29]

[29]See Mataloni (1995) for a complete description of the data. Note that I am not following the terminology used by the BEA. In particular, foreign direct investment is associated with entities that are not necessarily corporations, although corporations make up the bulk of it.

Table 5: Cross-country correlations in real GDP fluctuations

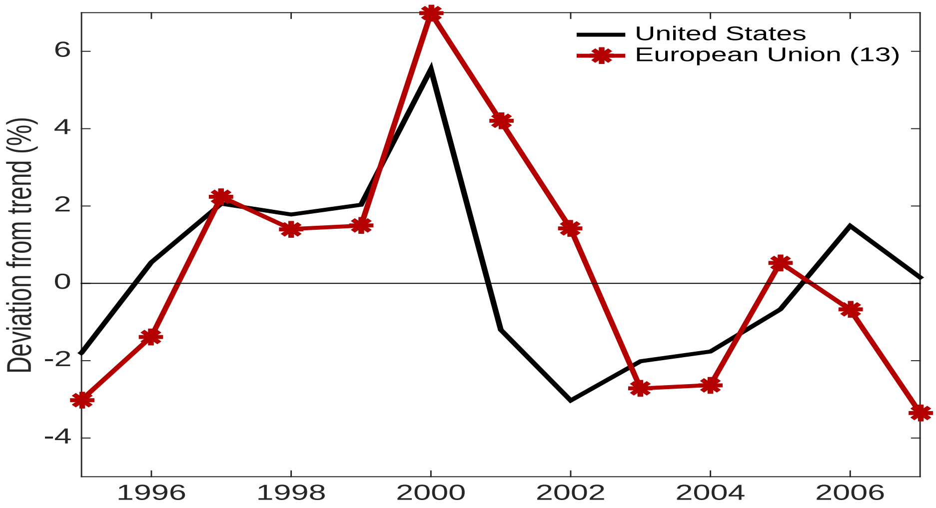
Correlation of GDP fluctuations with the United States				
	Whole economy	Corporate sector	Noncorporate sector	Share of corporate sector in GDP (%)
Period 1995–2007				
GDP-weighted average of 18 EU countries	0.39	**0.55**	-0.06	56
Germany	0.38	0.48	-0.36	59
United Kingdom	0.15	0.88	0.83	58
France	0.76	0.54	-0.21	54
Italy	0.16	0.24	-0.28	50
Netherlands	0.77	0.76	-0.35	63
Sweden	0.15	0.73	0.39	61
Belgium	0.70	0.50	-0.64	59
Austria	0.67	0.65	-0.33	57
Other periods				
Germany (1991–2007)	0.11	0.30	-0.09	64.7
France (1985–2007)	0.48	0.35	-0.04	50.4
France (1949–2007)	0.24	0.22	0.13	54.4

Source: OECD for European Union countries 1995–2007, BEA for the United States 1950–2007. Insee for France 1950–2007, Destatis for Germany 1992–2007.

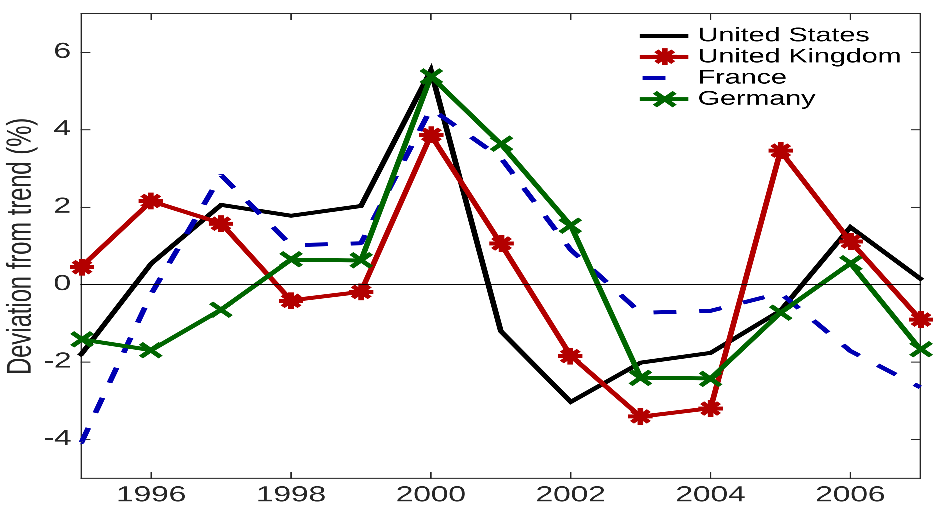
Note: Data frequency is annual. GDP values are in logarithms and the respective GDP deflator is used to compute real values. I assume a log-linear trend, except for Germany (1991–2007) and France (1985–2007 and 1950–2007), where data are HP-filtered with parameter 6.25. The corporate sector comprises both financial and non-inancial corporations (except for Insee data, which contain only nonfinancial corporations). The share of the corporate sector is the average share in the period. For the United States, the average shares are 57.3%, 58.5%, and 58.5% for 1949–2007, 1985–2007, and 1995–2007, respectively. I only included countries with data for all the years in 1995–2007. The 18 EU countries are Austria, Belgium, Czech Republic, Denmark, Estonia, Finland, France, Germany, Greece, Hungary, Italy, Latvia, Netherlands, Portugal, Slovak Republic, Slovenia, Sweden, and United Kingdom. Weights based on U.S. dollar GDP in 2002 from World Bank Development Indicators.

Figure 4: GDP fluctuations of U.S. multinationals in the United States and European Union

(a)



(b)



Sources: BEA.

Note: The figure shows the deviation of the log values of real GDP from their log-linear trends. All series are based on the operations of U.S. multinationals. For example, the series labeled "United States" shows the fluctuations in GDP of their operations in the United States, whereas the series labeled "United Kingdom" shows the fluctuations in GDP of their subsidiaries in the United Kingdom. The European Union (13) includes Austria, Belgium, Denmark, Finland, France, Germany, Greece, Italy, Luxembourg, Netherlands, Portugal, Sweden, and United Kingdom.

Table 6 shows that the activities of U.S. multinationals in the United States and their subsidiaries in the European Union are highly correlated. For example, the correlation in GDP fluctuations between operations of U.S. multinationals in the United States and their subsidiaries in the United Kingdom in 1995–2007 is 0.67. For the aggregate of 13 EU countries, the correlation is 0.66.[30] Table 6 also reports the correlation for other variables related to the economic activity of these multinationals, such as compensation of employees and employment. They follow the same pattern as GDP, that is, the correlation is positive and high.

Table 6: Cross-country correlations of U.S. multinational operations

	Gross domestic product	Compensation of employees	Employment
Period 1995–2007			
Aggregate of 13 EU countries	**0.66**	0.83	0.96
Germany	0.53	0.80	0.50
United Kingdom	0.67	0.76	0.82
France	0.58	0.83	0.87
Period 1983–2007			
Aggregate of 13 EU countries	-	0.59	0.63
Germany	-	0.53	0.22
United Kingdom	-	0.59	0.62
France	-	0.50	0.61

Source: BEA.
Note: Data frequency is annual. The GDP series for multinationals start in 1994. BEA data are in U.S. dollars, and I convert the values to their respective original currency using average market exchange rates. Variables are in logarithms, and the respective GDP deflator is used to compute real values (except for employment). I used the GDP deflator in France in the case of EU-13. I assume a log-linear trend. The 13 EU countries are Austria, Belgium, Denmark, Finland, France, Germany, Greece, Italy, Luxembourg, Netherlands, Portugal, Sweden, and United Kingdom. The years 1999 and 2004 correspond to benchmark revision years in the BEA data, and we can observe large variations in the number of multinationals observed. I opted for using a linear interpolation for these years, where I assumed the average growth rate in the period without including 1994 and 1999. Otherwise, the correlations would be even larger, reflecting large variations in the extensive margin.

Figure 4 shows the series of GDP fluctuations of U.S. multinationals broken down by location (United States, EU-13, United Kingdom, France, and Germany). The series

[30] This group includes Austria, Belgium, Denmark, Finland, France, Germany, Greece, Italy, Luxembourg, Netherlands, Portugal, Sweden, and United Kingdom.

follow a pattern similar to the corporate GDP series; that is, there is a period of fast growth between 1995 and 2000, and a sharp slowdown afterward.

These results are in line with the findings in Cravino and Levchenko (2014), who use the ORBIS database to compute the correlation in sales growth rates between headquarters and subsidiaries of multinational corporations worldwide.[31] They show that sales growth rates of multinational headquarters (operations in the country where the headquarters is located) and their subsidiaries (operations in a foreign country) are positively correlated. However, U.S. corporations do not report unconsolidated financial statements of their operations; that is, their financial statements do not separate U.S. operations from foreign operations. This means that one cannot use the ORBIS database to compare operations of U.S. multinationals in the United States and European Union, so my results complement their analysis.

5.3 Corporate sector GDP correlation and share of U.S. multinationals in corporate GDP

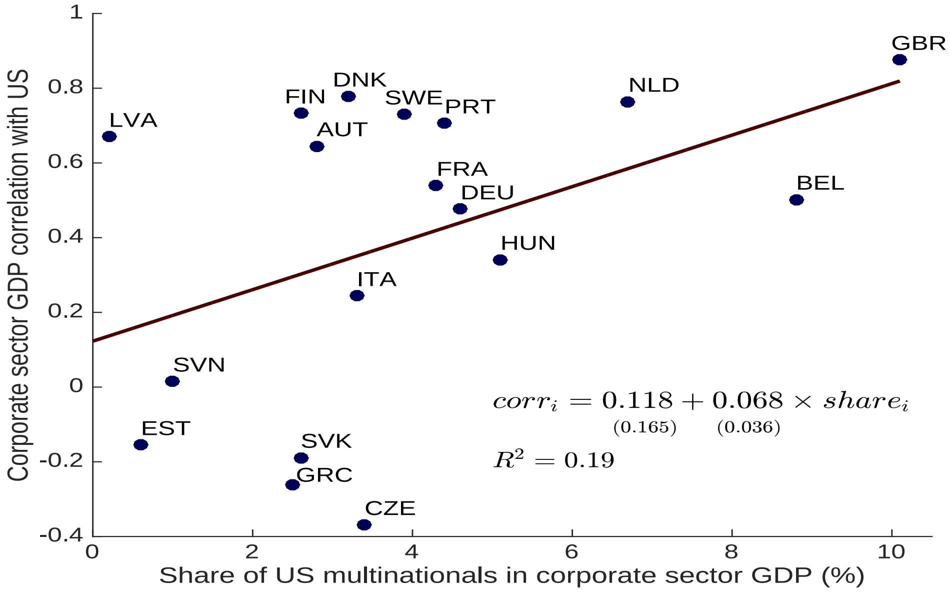
In this section, I document that the corporate sector GDP correlations with the United States described in Section 5.1 are closely related to the share of U.S. multinationals in the corporate sector of the respective country. Figure 5 shows a scatter plot of the bilateral correlations with the United States and the share of U.S. multinationals in each country's corporate sector GDP. They are positively correlated—correlation equal to 0.43.

In the United Kingdom (GBR), for example, the share of U.S. multinationals in corporate sector GDP is 10.1%, the largest share among the countries in my sample. At the same time, its corporate GDP correlation with the United States is 0.88, also the largest among the countries in the sample. Slovenia (SVN) and Estonia (EST), on the other hand, have a low average share of U.S. multinationals in their GDP (less than 1%), and at the same time, the correlation with the United States is also very low (around zero for Slovenia and negative for Estonia).

Latvia (LVA) seems to be an outlier. It has the lowest share of U.S. multinationals in corporate sector GDP, yet its correlation with the United States is relatively high, at 0.67. The results are robust to the inclusion of either the United Kingdom or Latvia. The correlations after excluding either the United Kingdom or Latvia or both are 0.33, 0.53, and 0.46, respectively.

[31] They also analyze value added and employment growth. See Cravino and Levchenko (2016) for a complete description of the ORBIS database.

Figure 5: Correlation with United States versus share of U.S. multinationals in corporate sector GDP



Sources: BEA and OECD.

Note: $corr$ = correlation between corporate sector GDP fluctuations with United States in 1995–2007; $share$ = average share of U.S. multinationals in the corporate sector in the period 1995–2007. The line corresponds to the fitted values of the ordinary least squares regression $corr_i = \alpha + \beta share_i + e_i$. The countries are: Austria (AUT), Belgium (BEL), Czech Republic (CZE), Denmark (DNK), Estonia (EST), Finland (FIN), France (FRA), Germany (DEU), Greece (GRC), Hungary (HUN), Italy (ITA), Latvia (LVA), Netherlands (NLD), Portugal (PRT), Slovak Republic (SVK), Slovenia (SVN), Sweden (SWE), and United Kingdom (GBR).

Again, the empirical findings indicate a close connection between the corporate sectors in these countries and the United States, and that the strength of this connection is associated with the degree of U.S. multinational activities in these countries. Therefore, any theory that attempts to explain the cross-country correlation must be able to account for these patterns, including the activities of multinational corporations.

6 Knowledge Flows and International Real Business Cycles

In this section, I assess the business cycle properties of the model outlined in Section 3 by allowing the productivity parameters to follow a stochastic process. In particular, I am interested in how much GDP correlation the model can generate when compared to the standard international real business cycle (IRBC) model by Backus, Kehoe, and Kydland (1994). As mentioned before, the model of knowledge flows that I present in this paper nests the standard IRBC model. More specifically, the model in this paper is equivalent to the standard IRBC model when I set the knowledge share parameter, ϕ, and the FDI cost, θ, to be equal to zero. Productivity in each country evolves according to:

$$\ln z_{1,t} = \rho \ln z_{1,t-1} + \epsilon_{1,t}, \quad (36)$$

$$\ln z_{2,t} = \rho \ln z_{2,t-1} + \epsilon_{2,t}, \quad (37)$$

where $\epsilon_{1,t}$ and $\epsilon_{1,t}$ follow a multivariate normal distribution, with mean equal to zero and standard deviations such that the standard deviation of GDP in the model matches the standard deviation of U.S. corporate sector GDP fluctuations in the data. The persistence parameter, ρ, is set to be equal to 0.90.

I assume that markets are complete and solve the planner's problem with equal weights. Next, I log-linearize the solution around the steady-state and use the method of undetermined coefficients to compute the dynamics of the model.[32]

Calibration: First, I use the parameter values listed in Table 3. Next, I still have to set values for the remaining parameters that were not used in Section 4. They are: μ, γ, σ, and ω. Table 7 shows their values. The value of μ, $\mu = 0.34$, is chosen such that households allocate 33% of their endowment of time to market activities (work) in steady state. For the home bias parameter, ω, I compute its value in order to match the ratio of the total value of U.S. exports to the European Union net of intrafirm exports plus the GDP of subsidiaries of U.S. multinationals in the European Union to the EU corporate sector GDP. Finally, I set $\gamma = -1$ and $\sigma = 1.5$, equal to the values used by Backus, Kehoe and Kydland (1994).

Results: Table 8 shows the GDP correlation implied by the model, together with the cross-country correlation of other variables such as consumption, labor, and investment. I compare the model with knowledge flows to the model without it (standard IRBC model).

[32]See Uhlig (2002).

Table 7: Calibration

Parameters		
Consumption share	μ	0.34
Home bias	ω	0.78
Share of knowledge	ϕ	**0.30**
Labor/capital share	α	0.35
Depreciation rate of knowledge	δ^m	**0.11**
Foreign direct investment cost	θ	**0.06**
Discount factor	β	0.96
Annual depreciation rate of (tangible) capital	δ^k	0.06
Elasticity of substitution between intermediate goods	σ	1.50
Risk aversion parameter	γ	-1.00
Autocorrelation of exogenous productivity	ρ	0.90
Standard deviation of productivity shocks	σ^ϵ	0.0082
Exogenous correlation of productivity shocks	$corr_\epsilon$	0.30

Table 8 shows that the model with knowledge greatly improves the performance of the model regarding the correlation of GDP fluctuations. As Table 8 shows, the standard IRBC model fails to generate GDP correlations close to the ones observed in the data, even when I allow for positive correlation between the exogenous shocks. On the other hand, the model with knowledge reduces its discrepancy with the data by 50%, i.e., it explains 50% of the distance between the correlation implied by the standard IRBC model and the data.

7 Conclusion

In this paper, I quantified the flow of knowledge within U.S. multinational corporations in the United States and European Union. I built on the general equilibrium model of knowledge flows through multinationals by McGrattan and Prescott (2010) and computed both the share of knowledge in the production function and its depreciation rate such that in steady state the model matches the observed factor share differentials between the operations of U.S. multinationals in the United States and European Union. The main assumptions are: i) U.S. multinationals produce knowledge in the United States;

ii) this knowledge is used by its subsidiaries in the European Union; and iii) investment in knowledge is expensed in corporate accounts. Under these assumptions, the model predicts that the observed labor share of U.S. multinational operations in the United States must be lower than the labor share of their subsidiaries in the European Union, whereas the ratio of capital expenditures to compensation of employees must be the same, both patterns found in the data. The estimated share of knowledge is 30%, and its annual depreciation rate is 11%.

Table 8: Results: cross-country correlations

	Data	with knowledge $\phi = 0.3$	without knowledge $\phi = 0.0$ (BKK-94)
Cross-country correlations			
GDP	0.55	0.13	-0.30
Consumption		0.69	0.86
Labor		-0.25	-0.61
Investment		-0.03	-0.37
Std. relative to output			
Net exports	0.18	0.19	1.15
Investment	1.83	2.64	8.84

The high estimates for the share of knowledge have important implications. For example, I show that the model calibrated with these parameter values has quantitative implications for international real business cycles. I provide empirical evidence that connects the operations of U.S. multinationals in the European Union to the correlation between corporate sector GDP fluctuations in the United State and European Union in 1995–2007. Accounting for the corporate sector GDP correlation, the model with knowledge flows reduces the distance between the standard international real business cycle model and data by 48%.

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A Model of international knowledge flows through multinational corporations

In this section, I solve the planner's problem with respect to the deterministic model presented in Section (3). Let ψ, $0 < \psi < 1$, denote the weight given to households of Country 1. The planner chooses the sequence fo allocations $(c_{i,t}, n_{i,t}, a_{i,t}, b_{i,t}, n^y_{ii,t}, n^y_{ij,t}, n^m_{ii,t}, k^y_{ii,t+1}, k^y_{ij,t+1}, k^m_{ii,t+1}, m_{i,t+1})$ for $i, j = 1, 2$, $j \neq i$, in order to solve the following problem:

$$\max \psi \sum_{t=0}^{\infty} \beta^t U(c_{1,t}, n_{1,t}) + (1-\psi) \sum_{t=0}^{\infty} \beta^t U(c_{2,t}, n_{2,t}) \tag{38}$$

subject to the following constraints:[33]

$$
\begin{aligned}
(\lambda_{1,t}) &: \quad c_{1,t} + k^y_{11,t+1} + k^y_{21,t+1} + k^m_{11,t+1} = G_1(a_{1,t}, b_{1,t}) + (1-\delta^k)(k^y_{11,t} + k^y_{21,t} + k^m_{11,t}), \\
(\lambda_{2,t}) &: \quad c_{2,t} + k^y_{22,t+1} + k^y_{12,t+1} + k^m_{22,t+1} = G_2(a_{2,t}, b_{2,t}) + (1-\delta^k)(k^y_{22,t} + k^y_{12,t} + k^m_{22,t}), \\
(\lambda_{3,t}) &: \quad a_{1,t} + a_{2,t} = z_{1,t} F(k^y_{11,t}, m_{1,t}, n^y_{11,t}) + z_{2,t} F(k^y_{12,t}, \theta m_{1,t}, n^y_{12,t}), \\
(\lambda_{4,t}) &: \quad b_{1,t} + b_{2,t} = z_{1,t} F(k^y_{21,t}, \theta m_{2,t}, n^y_{21,t}) + z_{2,t} F(k^y_{22,t}, m_{2,t}, n^y_{22,t}), \\
(\lambda_{5,t}) &: \quad m_{1,t+1} = z_{1,t} F(k^m_{11,t}, m_{1,t}, n^m_{11,t}) + (1-\delta^m) m_{1,t}, \\
(\lambda_{6,t}) &: \quad m_{2,t+1} = z_{2,t} F(k^m_{22,t}, m_{2,t}, n^m_{22,t}) + (1-\delta^m) m_{2,t}, \\
(\lambda_{7,t}) &: \quad n_{1,t} = n^y_{11,t} + n^y_{21,t} + n^m_{11,t}, \\
(\lambda_{8,t}) &: \quad n_{2,t} = n^y_{22,t} + n^y_{12,t} + n^m_{22,t}.
\end{aligned}
$$

The terms $\lambda_{i,t}$ denote the respective Lagrange multiplier of each constraint. The first-order conditions are:

$$(c_{1,t}) \quad : \quad \beta^t \psi U_c(c_{1,t}, n_{1,t}) = \lambda_{1,t} \tag{39}$$

$$(c_{2,t}) \quad : \quad \beta^t (1-\psi) U_c(c_{1,t}, n_{1,t}) = \lambda_{2,t} \tag{40}$$

$$(n_{1,t}) \quad : \quad -\beta^t \psi U_n(c_{1,t}, n_{1,t}) = \lambda_{7,t} \tag{41}$$

$$(n_{2,t}) \quad : \quad -\beta^t (1-\psi) U_n(c_{1,t}, n_{1,t}) = \lambda_{8,t} \tag{42}$$

[33]And also subject to the non-negativity constraints.

$$(a_{1,t}) \quad : \quad \lambda_{1,t} G_{1a} \left(a_{1,t}, b_{1,t}\right) = \lambda_{3,t} \tag{43}$$

$$(a_{2,t}) \quad : \quad \lambda_{2,t} G_{2a} \left(a_{2,t}, b_{2,t}\right) = \lambda_{3,t} \tag{44}$$

$$(b_{1,t}) \quad : \quad \lambda_{1,t} G_{1b} \left(a_{1,t}, b_{1,t}\right) = \lambda_{4,t} \tag{45}$$

$$(b_{2,t}) \quad : \quad \lambda_{2,t} G_{2b} \left(a_{2,t}, b_{2,t}\right) = \lambda_{4,t} \tag{46}$$

$$(n^y_{11,t}) \quad : \quad \lambda_{3,t} z_{1,t} F_n(k^y_{11,t}, m_{1,t}, n^y_{11,t}) = \lambda_{7,t} \tag{47}$$

$$(n^y_{12,t}) \quad : \quad \lambda_{3,t} z_{2,t} F_n(k^y_{12,t}, \theta m_{1,t}, n^y_{12,t}) = \lambda_{8,t} \tag{48}$$

$$(n^y_{21,t}) \quad : \quad \lambda_{4,t} z_{1,t} F_n(k^y_{21,t}, \theta m_{2,t}, n^y_{21,t}) = \lambda_{7,t} \tag{49}$$

$$(n^y_{22,t}) \quad : \quad \lambda_{4,t} z_{2,t} F_n(k^y_{22,t}, m_{2,t}, n^y_{22,t}) = \lambda_{8,t} \tag{50}$$

$$(n^m_{11,t}) \quad : \quad \lambda_{5,t} z_{1,t} F_n(k^m_{11,t}, m_{1,t}, n^m_{11,t}) = \lambda_{7,t} \tag{51}$$

$$(n^m_{11,t}) \quad : \quad \lambda_{6,t} z_{2,t} F_n(k^m_{22,t}, m_{2,t}, n^m_{22,t}) = \lambda_{8,t} \tag{52}$$

$$(k^y_{11,t+1}) \quad : \quad -\lambda_{1,t} + \lambda_{3,t+1} z_{1,t+1} F_n(k^y_{11,t+1}, m_{1,t+1}, n^y_{11,t+1}) + \lambda_{1,t+1}(1-\delta^k) = 0 \tag{53}$$

$$(k^y_{12,t+1}) \quad : \quad -\lambda_{2,t} + \lambda_{3,t+1} z_{2,t+1} F_n(k^y_{12,t+1}, \theta m_{1,t+1}, n^y_{12,t+1}) + \lambda_{2,t+1}(1-\delta^k) = 0 \tag{54}$$

$$(k^y_{21,t+1}) \quad : \quad -\lambda_{1,t} + \lambda_{4,t+1} z_{1,t+1} F_n(k^y_{21,t+1}, \theta m_{2,t+1}, n^y_{21,t+1}) + \lambda_{1,t+1}(1-\delta^k) = 0 \tag{55}$$

$$(k^y_{22,t+1}) \quad : \quad -\lambda_{2,t} + \lambda_{4,t+1} z_{2,t+1} F_n(k^y_{22,t+1}, m_{2,t+1}, n^y_{22,t+1}) + \lambda_{2,t+1}(1-\delta^k) = 0 \tag{56}$$

$$(k^m_{11,t+1}) \quad : \quad -\lambda_{1,t} + \lambda_{5,t+1} z_{1,t+1} F_n(k^m_{11,t+1}, m_{1,t+1}, n^m_{11,t+1}) + \lambda_{1,t+1}(1-\delta^m) = 0 \tag{57}$$

$$(k^m_{11,t+1}) \quad : \quad -\lambda_{2,t} + \lambda_{6,t+1} z_{2,t+1} F_n(k^m_{22,t+1}, m_{2,t+1}, n^m_{22,t+1}) + \lambda_{2,t+1}(1-\delta^m) = 0 \tag{58}$$

$$(m_{1,t+1}) \quad : \quad \begin{array}{c} -\lambda_{5,t} + \lambda_{3,t+1} z_{1,t+1} F_m(k^y_{11,t+1}, m_{1,t+1}, n^y_{11,t+1}) \\ +\lambda_{3,t+1} \theta z_{2,t+1} F_m(k^y_{12,t+1}, \theta m_{1,t+1}, n^y_{12,t+1}) \\ +\lambda_{5,t+1} z_{1,t+1} F_m(k^m_{11,t+1}, m_{1,t+1}, n^m_{11,t+1}) + \lambda_{5,t+1}(1-\delta^m) \end{array} = 0 \tag{59}$$

$$(m_{2,t+1}) \quad : \quad \begin{array}{c} -\lambda_{6,t} + \lambda_{4,t+1} z_{2,t+1} F_m(k^y_{22,t+1}, m_{2,t+1}, n^y_{22,t+1}) \\ +\lambda_{4,t+1} \theta z_{2,t+1} F_m(k^y_{21,t+1}, \theta m_{2,t+1}, n^y_{21,t+1}) \\ +\lambda_{6,t+1} z_{2,t+1} F_m(k^m_{22,t+1}, m_{2,t+1}, n^m_{22,t+1}) + \lambda_{6,t+1}(1-\delta^m) \end{array} = 0 \tag{60}$$

Together with the transversality conditions.

A.1 Relation to competitive equilibrium

The mapping from competitive prices used in Section (3) to the Lagrange multipliers is the following:[34]

$$w_{1,t} = \frac{\lambda_{7,t}}{\lambda_{1,t}}, \quad q_{a,t} = \frac{\lambda_{3,t}}{\lambda_{1,t}}, \quad p^m_{1,t} = \frac{\lambda_{5,t}}{\lambda_{1,t}}, \quad p_{2,t} = \frac{\lambda_{2,t}}{\lambda_{1,t}},$$

$$w_{2,t} = \frac{\lambda_{8,t}}{\lambda_{1,t}}, \quad q_{b,t} = \frac{\lambda_{4,t}}{\lambda_{1,t}}, \quad p^m_{2,t} = \frac{\lambda_{6,t}}{\lambda_{1,t}}.$$

A.2 Symmetric Steady State

I am solving the planner's problem in which the planner gives equal weight to households of both countries. In this case, the steady state equilibrium is symmetric. The following conditions must hold:[35]

$$\frac{U_n(c_1, n_1)}{U_c(c_1, n_1)} = w, \tag{61}$$

$$G_{1a}\left(a_1, b_1\right) = q, \tag{62}$$

$$G_{1b}\left(a_1, b_1\right) = q, \tag{63}$$

$$qF_n\left(\frac{k^y_{11}}{m_1}, 1, \frac{n^y_{11}}{m_1}\right) = w, \tag{64}$$

$$qF_n\left(\frac{k^y_{12}}{m_1}, \theta, \frac{n^y_{12}}{m_1}\right) = w, \tag{65}$$

$$p^m F_n\left(\frac{k^m_{11}}{m_1}, 1, \frac{n^m_{12}}{m_1}\right) = w, \tag{66}$$

$$qF_k\left(\frac{k^y_{11}}{m_1}, 1, \frac{n^y_{11}}{m_1}\right) = \frac{1-\beta(1-\delta^k)}{\beta}, \tag{67}$$

$$qF_k\left(\frac{k^y_{12}}{m_1}, \theta, \frac{n^y_{12}}{m_1}\right) = \frac{1-\beta(1-\delta^k)}{\beta}, \tag{68}$$

$$p^m F_k\left(\frac{k^m_{11}}{m_1}, 1, \frac{n^m_{12}}{m_1}\right) = \frac{1-\beta(1-\delta^k)}{\beta}, \tag{69}$$

[34] I do not provide a full proof of the equivalence between the competitive equilibrium allocation and the solution to the planner's problem.

[35] I suppress the subscript t for steady-state variables, and use the fact that $z_{1,t} = z_{2,t} = 1$.

$$\frac{q}{p^m}\left[F_m\left(\frac{k^y_{11}}{m_1},1,\frac{n^y_{11}}{m_1}\right)+\theta F_m\left(\frac{k^y_{12}}{m_1},\theta,\frac{n^y_{12}}{m_1}\right)\right]+F_m\left(\frac{k^m_{11}}{m_1},1,\frac{n^m_{12}}{m_1}\right)=\frac{1-\beta(1-\delta^m)}{\beta}, \tag{70}$$

$$c_1+\delta^k\left(k^y_{11}+k^y_{12}+k^m_{11}\right)=G_1\left(a_1,b_1\right), \tag{71}$$

$$n^y_{11}+n^y_{12}+n^m_{11}=n_1, \tag{72}$$

$$a_1+b_1=m_1\left[F\left(\frac{k^y_{11}}{m_1},1,\frac{n^y_{11}}{m_1}\right)+F\left(\frac{k^y_{12}}{m_1},\theta,\frac{n^y_{12}}{m_1}\right)\right], \tag{73}$$

$$\delta^m=F\left(\frac{k^m_{11}}{m_1},1,\frac{n^m_{11}}{m_1}\right), \tag{74}$$

where $q=q_a=q_b$, $w=w_1=w_2$, $p^m=p^m_1=p^m_2$, and I used the fact that $n^y_{12}=n^y_{21}$, $k^y_{12}=k^y_{21}$, and $a_2=b_1$. Above we have a system of 14 equations and 14 variables $(c_1,n_1,a_1,b_1,n^y_{11},n^y_{12},n^m_{11},k^y_{11},k^y_{12},k^m_{11},m_1,w,q,p^m)$.

Next, I use the following functional forms to compute the steady state:

$$\begin{aligned}G_1(a,b) &= \left(\omega a^{\frac{\sigma-1}{\sigma}}+(1-\omega)b^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}},\\ F(k,m,n) &= \left(k^\alpha n^{1-\alpha}\right)^{1-\phi}m^\phi,\\ U(c,n) &= \left(c^\mu(1-n)^{1-\gamma}\right)^\gamma/\gamma.\end{aligned}$$

Under these functional forms, the steady-state can be computed as follows:

Step 1: Prices

$$q = \left(\omega^\sigma+(1-\omega)^\sigma\right)^{\frac{1}{\sigma-1}}, \tag{75}$$

$$p^m = q\frac{\delta^m}{\kappa_1}\left(\kappa_2\kappa_3\right)^{-\alpha(1-\phi)}\left((1-\alpha)(1-\phi)\right)^{\alpha(1-\phi)}, \tag{76}$$

$$w = \left(p^m\right)^{\frac{1}{1-\alpha}}\left(\delta^m\right)^{\frac{-\phi}{(1-\alpha)(1-\phi)}}\left(\kappa_3\right)^{\frac{\alpha}{1-\alpha}}(1-\alpha)(1-\phi), \tag{77}$$

$$\kappa_1 \equiv \frac{1-\beta(1-\delta^m)-\delta^m\phi\beta}{\phi\beta(1+\theta)} \tag{78}$$

$$\kappa_2 \equiv \frac{1-\beta\left(1-\delta^k\right)}{\beta}\frac{1-\alpha}{\alpha} \tag{79}$$

$$\kappa_3 \equiv \frac{\beta\alpha(1-\phi)}{1-\beta\left(1-\delta^k\right)} \tag{80}$$

Step 2: Ratios

$$\frac{n_{11}^m}{m_1} = \frac{p^m}{w}(1-\alpha)(1-\phi)\delta^m, \tag{81}$$

$$\frac{k_{11}^m}{m_1} = p^m\delta^m\kappa_3, \tag{82}$$

$$\frac{k_{11}^y}{m_1} = (w)^{-\frac{(1-\alpha)(1-\phi)}{\phi}}\left(q(1-\alpha)(1-\phi)\right)^{\frac{1}{\phi}}(\kappa_2)^{\frac{(1-\alpha)(1-\phi)-1}{\phi}}, \tag{83}$$

$$\frac{n_{11}^y}{m_1} = \kappa_2\frac{1}{w}\frac{k_{11}^y}{m_1}, \tag{84}$$

$$\frac{k_{12}^y}{m_1} = \theta\frac{k_{11}^y}{m_1}, \tag{85}$$

$$\frac{n_{12}^y}{m_1} = \theta\frac{n_{11}^y}{m_1}, \tag{86}$$

$$\frac{a_1}{m_1} = \left(1+\left(\frac{1-\omega}{\omega}\right)^{\sigma}\right)^{-1}(1+\theta)\left(\frac{k_{11}^y}{m_1}\right)^{\alpha(1-\phi)}\left(\frac{n_{11}^y}{m_1}\right)^{(1-\alpha)(1-\phi)}, \tag{87}$$

where all the ratios are with respect to the stock of knowledge m_1.

Step 3: I compute the stock of knowledge using the following equation

$$m_1 = w\frac{\mu}{1-\mu}\left[\begin{array}{c}\frac{a_1}{m_1}\left(\omega+(1-\omega)\left(\frac{1-\omega}{\omega}\right)^{\sigma-1}\right)^{\frac{\sigma}{\sigma-1}} \\ +\frac{\mu}{1-\mu}w\left((1+\theta)\frac{n_{11}^y}{m_1}+\frac{n_{11}^m}{m_1}\right)-\delta^k\left((1+\theta)\frac{k_{11}^y}{m_1}+\frac{k_{11}^m}{m_1}\right)\end{array}\right]^{-1}.$$

Step 4: Finally, I compute $(a_1, n_{11}^y, n_{12}^y, n_{11}^m, k_{11}^y, k_{12}^y, k_{11}^m)$ using the ratios, and (b_1, c_1, n_1) using the following relations:

$$n_1 = n_{11}^y + n_{12}^y + n_{11}^m, \tag{88}$$

$$c_1 = \frac{\mu}{1-\mu}w(1-n_1), \tag{89}$$

$$b_1 = \left(\frac{1-\omega}{\omega}\right)^{\sigma}a_1. \tag{90}$$

Steady state moments: Given the solution to the steady state equilibrium derived above, the following conditions hold:

$$
\begin{aligned}
\frac{w(n_{11}^y + n_{11}^m)}{qF(k_{11,t}^y, m_{1,t}, n_{11,t}^y)} &= (1-\alpha)(1-\phi)\left(1 + \frac{\delta^m \phi\beta(1+\theta)}{1-\beta+\beta\delta^m(1-\phi)}\right), \\
\frac{wn_{12}^y}{qF(k_{12,t}^y, \theta m_{1,t}, n_{12,t}^y)} &= (1-\alpha)(1-\phi), \\
\frac{\delta^k(n_{11}^y + n_{11}^m)}{w(n_{11}^y + n_{11}^m)} &= \frac{\delta^k\beta}{1-\beta(1-\delta^k)}\frac{\alpha}{1-\alpha}, \\
\frac{wn_{12}^y}{w(n_{11}^y + n_{11}^m)} &= \theta\frac{1-\beta(1-\delta^m)-\delta^m\phi\beta}{1-\beta(1-\delta^m)+\delta^m\phi\beta\theta}.
\end{aligned}
$$