IDB WORKING PAPER SERIES Nº IDB-WP-1514

Fighting for the Best, Losing with the Rest:

The Perils of Competition in Entrepreneurial Finance

Juan Hernández
Daniel Wills

Inter-American Development Bank
Department of Research and Chief Economist

January 2024

Fighting for the Best, Losing with the Rest:

The Perils of Competition in Entrepreneurial Finance

Juan Hernández*
Daniel Wills**

* World Bank
** Universidad de los Andes

January 2024

Cataloging-in-Publication data provided by the
Inter-American Development Bank
Felipe Herrera Library

Hernández, Juan.
Fighting for the best, losing with the rest: the perils of competition in entrepreneurial finance / Juan Hernandez, Daniel Wills.
p. cm. — (IDB Working Paper Series ; 1514)
Includes bibliographical references.
1. Entrepreneurship-Finance-Econometric models. 2. New business enterprises-Finance-Econometric models. 3. Financial risk management-Econometric models. I. Wills, Daniel G. II. Inter-American Development Bank. Department of Research and Chief Economist. III. Title. IV. Series.
IDB-WP-1514

http://www.iadb.org

Abstract

Financiers in early-stage entrepreneurial finance are known for their "spray-and-pray" approach, where they fund multiple start-ups expecting profits on a few to compensate losses on a lot of failed ones. We develop a theoretical framework in which financiers compete to fund entrepreneurs in an environment featuring risk, adverse selection, and limited liability. Financiers use steep payoff schedules to screen entrepreneurs, but limited liability implies they can only do so by giving more to all entrepreneurs. In equilibrium, competition for the best entrepreneurs forces intermediaries to offer better terms to all customers, there is cross-subsidization among entrepreneurs, and intermediation profits are zero. Competition among financial intermediaries always forces them to fund projects with negative expected returns both from a private and from a social perspective. This is an extensive margin inefficiency, as all projects are funded at their efficient scale. The three main features of our framework (competition, adverse selection, and limited liability) are necessary to get the inefficient *laissez-faire* outcome and a role for regulation. The inefficiency shrinks, but some part will always persist, when firms can collateralize some portion of the credit as long as there is still an unsecured fraction. Additional imperfect information, like a credit score, may increase inefficiency. Crucially, a small externality on financiers exacerbates the extensive margin inefficiency, yielding a negative social surplus in the entrepreneurial financing market. [1]

JEL classifications: D82, G14, G28

Keywords: Adverse selection, Entrepreneurial finance, Competition, Extensive margin inefficiency

---

[1] Juan Hernandez: juanhernandez@worldbank.org. Daniel Wills: d-wills@uniandes.edu.co. We thank Hal Cole, Dirk Krueger, Guillermo Ordoñez and Ali Shourideh for their useful comments. We also want to thank participants at the 2021 IDB conference on Open Innovation and Corporate Venturing, the 2021 Latin American Meeting of the Econometric Society, the 2019 Barcelona GSE Summer Forum on Economics, Science and Innovation, the Midwest Macroeconomics Meeting Fall 2017, LACEA-LAMES meeting 2017, and seminars at Universidad de Los Andes and the University of Pennsylvania. Santiago Neira provided exceptional research assistance. The usual waiver of liabilities applies.

1 Introduction

The intricate interplay between risk and adverse selection has historically shaped the contractual approach of start-up financiers, which include Venture Capitalists, Angel Investors, and more recently, private equity firms (Sahlman, 1990; Kaplan and Stromberg, 2001). Beyond screening, the literature has identified the governance aspect of early-stage financing—mentoring, monitoring, and decision-making by financiers—as key determinants of the contractual relation between entrepreneurs and financiers.

However, the past two decades have witnessed a critical transformation in the entrepreneurial finance landscape. This shift is marked by dynamism and intense competition, primarily attributed to financiers' "spray-and-pray" approach. In essence, financiers back a plethora of start-ups, anticipating that profits from a select few successful ventures will offset losses from numerous unsuccessful ones (Ewens et al., 2018). They identify an increase in entry by entrepreneurs who would not have been financed in the past, because their lower expected value. These were deemed "long-shot" start-ups, which although presenting a lower probability of success, promise a high payoff if they do succeed.

This "spray-and-pray" approach has also diluted the governance features of the financing relationship, as investors have less time to engage with each venture. Recent studies, like Khanna and Mathews (2022), have pointed out the welfare costs associated with financiers spreading their skills thin across multiple start-ups.

Our paper focuses on a potentially detrimental effect of this new paradigm: an excess of long-shot start-ups, which could harm welfare. We approach this question from an unrestricted contract space, characterizing the contracts offered in equilibrium by a competitive financial sector to entrepreneurs. In this environment, start-ups are heterogeneous in quality and hence, in their optimal project scale; this information is privately held by the entrepreneur. Financiers compete to attract ventures by posting financing contracts subject to limited liability. We find that the resulting equilibrium sees the enactment of projects with negative expected returns, both from a private and a social perspective. These long-shot ventures are undertaken as entrepreneurs gamble with the financiers' money

Interestingly, if financial intermediaries could collude or if the limited liability clause were removed, the first-best outcome would be achievable. The role of regulation, therefore, becomes pertinent in this scenario, as these market frictions distort the entrepreneurial finance ecosystem. A fixed tax per contract, resembling the red-tape requirements removed by the JOBS Act, could restore efficiency, contradicting the traditional view ofLevine (2005) who argued that such barriers to entry diminish output.

Our model suggests that even when firms can collateralize a portion of the credit, inefficiencies persist if an unsecured fraction remains. Moreover, the presence of imperfect information, such as credit scores, could exacerbate these inefficiencies. Perhaps our most striking finding is how a minor externality on financiers could amplify the extensive margin inefficiency to the point of yielding a negative aggregate social surplus in the entrepreneurial financing market, a departure from the generally accepted view(Samila and Sorenson, 2011).

Next we give a detailed view of the environment and mechanism behind the result. In the basic form of this framework, entrepreneurs are endowed with an idea of quality $\theta \in [0, 1]$ and they lack assets or wealth (to be relaxed later). The value of θ is private information for the entrepreneurs, drawn from a known distribution $G(\theta)$. If entrepreneurs decide to implement their idea, the probability of success depends jointly on its quality and the amount of capital invested, which needs to be provided by a financial intermediary. The entrepreneur can always discard the idea and take its outside option, which provides benefits w with certainty. Financial intermediaries compete for the projects by posting as many contracts as they wish. The contracts stipulate the amount to be invested and repayments in case of success and failure. In the former case, the venture produces profits pi independent of the success probability; in case of failure the venture produces zero output and a limited liability restriction prevents the contracts from demanding any payment from the entrepreneurs to the financiers, but not from the latter to the former. Beyond the information friction and limited liability, the set of contracts to be offered is entirely unrestricted.

Competition among financiers implies all enacted projects are funded at their efficient scale and zero *aggregate* profits for the intermediaries, but not necessarily zero profits *by contract.* Intermediaries will fight for the best entrepreneurs, offering them favorable contract terms. However they also will seek to provide steep incentives, preventing the low (idea) quality entrepreneurs from taking advantage of the "sweet deal"designed for high quality ones. Without limited liability, this would lead to a separating equilibrium with zero profit per contract.

The crucial aspect is that limited liability imposes a lower bound on all contracts' payoffs and this translates into an upper bound on the steepness of the expected utility a set of contracts yields to each entrepreneur. Hence, to be able to offer higher expected utility to high-quality entrepreneurs financiers must also offer better terms to lower-quality ones. The expected utility steepness bound also implies expected surplus per project must grow faster than entrepreneurs' expected utility with respect to the entrepreneur's type θ, whenever the financiers expect to make non-negative profits with that particular type. This means financiers will make profits with all types above that θ, and thus must be losing with the rest (lower types) to obtain aggregate zero profits.

In addition inefficiency arises because some entrepreneurs will enact ides with an expected surplus smaller than the entrepreneur's outside option w. The lowest type that accepts a contract must have an expected utility exactly equal to the outside option w. Since financiers must be losing money with that project, the expected surplus from that project must be lower than w. This inefficiency results from the interaction of several forces: first, there is asymmetric information that introduces a "lemons" problem; second, limited liability puts a bound on the screening that can be done by financial intermediaries;

third, competition among intermediaries introduces profitable deviations from the efficient outcome (a monopolist lender would reach efficiency). Simple entry costs to intermediation or fees per contract could correct the inefficiency.

In the basic model, entrepreneurs cannot collateralize their assets. We extend the model to the case in which entrepreneurs have some collateral. As long as loans cannot be fully collateralized, the inefficiency is reduced but not removed. Additional information that allows to segment markets, like a credit score, generally reduces the inefficiency, although in extreme cases may be detrimental to welfare, reducing aggregate surplus.

We also introduce a small negative externality on financiers not funding a successful startup. This may come come from reputation cost of missing an ex post very publicly successful startup, from lower profits on other firms in its portfolio, or from direct impact on its operations' profits, as is the case for corporate venture capitalists. Crucially, this small negative externality interacts with our entry margin inefficiency and erases all social gains from the entrepreneurial activity. A regulator would rather shut down the market.

Related Literature: This paper relates to the extensive body of literature studying asymmetric information in financial markets, going back to the seminal paper of Stiglitz and Weiss (1981). In their framework, where projects have all the same expected return but differ in their volatility, the market unravels, the interest rate cannot clear the credit market because of a standard "lemons" problem, and there is credit rationing. Subsequent papers allow the financial intermediaries to use other tools to screen borrowers' types.

In contrast, a line of papers started with de Meza and Webb (1987), where heterogeneity among entrepreneurs is due to different expected returns and not only variance. In their model, the market does not collapse and, in the same fashion as we do, they find over-investment in equilibrium. We extend their results by considering the full contracting space and not focusing only in pooling equilibria, in our setup the equilibrium loan size varies and allows us to distinguish among entrepreneurs. Ghatak et al. (2007) extends de Meza and Webb (1987) by incorporating a labor market where demand comes from enacted projects and supply from those potential entrepreneurs choosing to be workers. In their setup there are only two quality types and all types chose the same (labor force) size. But the assumption implies the universally optimal size is dependent on the average quality of active entrepreneurs, which allows them to consider general equilibrium effects of altering the wage (outside option) of potential entrepreneurs. As in our model, they find that there is excess entrance to entrepreneurship, and that a higher wage could correct that inefficiency. Crucially, they show that remaining entrepreneurs also benefit, in a cleansing effect like that of Ates and Saffie (2021). Given that Ghatak et al. (2007) already find equilibrium multiplicity with only two types, we abstained from making the entrepreneurs' outside option w endogenous in our paper to preserve mathematical tractability and equilibrium uniqueness. We believe their results would qualitatively apply to an extended version of our framework with endogenous w if we appropriately refined our equilibrium concept.

Innes (1990) generalizes the environment in de Meza and Webb (1987) by allowing for a distribution of outcomes for each type, ordered by the Monotone Likelihood Ratio Property (MLRP), albeit with a fixed investment level for all projects. The main result is that, despite the continuum of outcomes, the equilibrium contracts are standard debt contracts, and there are still too many entrants in equilibrium. We extend Innes (1990) by allowing for a continuum of both types and investment levels, reaching a *separating* equilibrium. This also allows us to study project size distortions in the presence of externalities.

Another extension of Innes (1990) is the recent paper by Scheuer (2013), who incorporates a second dimension to the private entrepreneur type related to her skill as a worker. In this framework, at any entrepreneurial skill level there are always both potential entrepreneurs who prefer to work, and some who prefer to start a business. That implies an extensive margin inefficiency in equilibrium at all entrepreneurial skill levels, in contrast to our paper where the inefficiency is concentrated on the lower quality entrepreneurs starting a project. As in our setup, taxes can fix the inefficiency. However, but since in equilibrium there is one cutoff per entrepreneurial skill level in Scheuer (2013), the optimal policy is a tax schedule, compared to our framework where there is a single type cutoff that can be corrected with a constant marginal tax rate. Our results are complementary to those in Scheuer (2013), as we both extended Innes (1990) framework but in different dimensions: worker skill and optimal investment/project size.

A strand of subsequent papers allowed intermediaries to use collateral, on top of the interest rate, to screen types. Bester (1985b) turns down the credit rationing result, by enabling intermediaries to offer interest rate/collateral contract pairs. By using collateral in addition to the interest rate, banks can screen borrowers: risky borrowers will accept paying higher interest rates to benefit from a lower collateral requirement. However, in Bester (1985b)'s economy, there is no limit to the amount of collateral that borrowers can post. The question of limits to collateral is studied by Besanko and Thakor (1987). The environment is like Bester (1985b)'s, and safer types will prefer loans with low interest rates and high collateral. Nonetheless, it may be that the borrower does not have enough wealth to provide the required collateral. In that case, the collateral/interest rate pair cannot achieve the sufficient spread in payoffs necessary to separate types. Besanko and Thakor (1987) resolve this issue by allowing the contracts to specify, additionally, the probability of approval. To achieve the required spread of utilities, low interest - high collateral credits will be denied with positive probability. An interesting point in Besanko and Thakor (1987) is that monopoly may lead to higher welfare, depending on parameter values.

Another strand of papers has departed from the Stiglitz and Weiss (1981) result by allowing intermediaries to screen borrowers using the size of the loan. A contract is hence an interest rate - loan size pair. In Milde and Riley (1988), borrowers are entrepreneurs with access to a project with risky returns. As in this paper, the expected return of a project depends on both the borrower's type and the size of the loan. The interaction between type and loan size in the project's payoff allows separating types using interest rate - loan size menus. The outcome, however, depends strongly on the specific function mapping the type and loan size to the return of the project. In general, good types take bigger loans accompanied by higher interest rates. However, Milde and Riley (1988) provide examples of production functions for which the opposite happens: good types take smaller loans and pay lower interest rates. They focus on the intensive margin: in their framework projects will not be funded to their optimal, full-information size because of a signaling distortion a la Riley. We abstract from that distortion to consider an overlooked channel: the extensive margin.

More recently, Martin (2009) uses a similar framework to study the relation between entrepreneurial wealth and aggregate investment. In his model, intermediaries can use both collateral and the size of the loan to screen types. He shows that when entrepreneurial wealth is high, collateral can be used to separate types. When entrepreneurial wealth is low, screening is mainly done by restricting the level of investment and becomes costlier. As a result, in the latter case, a pooling equilibrium is more likely. However, Martin (2009) restricts the interest rate to be un-contingent. We show that, when transfers contingent of the success of the project are allowed (say by a contingent interest rate or an equity-like contract), the intermediaries never distort the level of investment to screen types. Instead, they find it optimal to use the contingent transfer.

Although close in terms of topic, all the papers cited above impose ad hoc restrictions on the space of contracts potentially offered by the financial sector. In contrast, in our model, the set of contracts is only restricted by the features of the environment.

Lester et al. (2019) develop a model where screening contracts are unrestricted. Their environment features adverse selection between informed sellers and uninformed buyers. Beyond asymmetric information, they introduce imperfect information coming from search theoretic frictions. The later feature allows them to do comparative statics on the degree of imperfect competition and how it interacts with the severity of adverse selection. As in our environment, they find that increased competition may reduce welfare when markets are competitive. However, in their framework all trading is welfare-enhancing by assumption, while in our case some socially inefficient projects are available and enacted in equilibrium. In that way, their paper overlooks the extensive margin channel generating our main result.

The rest of the paper is organized as follows: In the next section, we describe the basic model at the core of this paper, and the equilibrium characterization is in Section 3. Section 4 extends the model to allow for imperfect signals about entrepreneurs quality and for entrepreneurs having assets to partially collateralize their loans. Section 5 introduces externalities to non-financing intermediaries from the success of a startup. Concluding remarks are made in Section 7.

2 Basic Model

2.1 Environment

The economy is populated by a continuum of entrepreneurs (or startups) indexed by their heterogeneous ability or quality $\theta \in [0,1]$ and by a plurality of financial intermediaries, two for simplicity. Each entrepreneur can undertake a risky R&D project, whose probability of success depends jointly on her type and the amount of capital invested in the firm. We initially assume the entrepreneur has zero wealth: if she decides to undertake its project, it will have to borrow funds from a financial intermediary. If the entrepreneur does not undertake the project, she gets benefits w with certainty; w is then the opportunity cost of R&D, including forgone revenue from other endeavors or forgone wages the entrepreneur could earn somewhere else, as well as non-monetary costs.

The ability of each entrepreneur, θ, is drawn from a continuous distribution with c.d.f. $G(\theta)$ and p.d.f. $g(\theta)$, known to all agents. This ability will affect the probability that the project succeeds. If the entrepreneur decides to start the project and invests an amount k of capital in the startup, it will be successful with probability $\theta \cdot f(k)$ where $f(\cdot)$ is increasing and strictly concave, and $f(0) = 0$. Hence, the surplus maximizing investment will be increasing in the startup quality. Although we are interpreting θ as entrepreneurial ability, it could include anything that is known by the entrepreneur, but not by the intermediary, and increases the probability that the project succeeds. In line with the endogenous growth literature, we interpret a successful project as the arrival of an innovation, which allows the small firm to create or "steal" some market. We denote π the payoff of a successful project, while in case of failure the payoff is zero.

We are assuming that the probability of success $p(\theta, k)$ is multiplicatively separable: $p(\theta, k) = t(\theta)f(k)$. Without loss of generality, we can call $t(\theta)$ the type instead of θ and adjust the known distribution of types accordingly. The assumption allows us to abstract from Riley-style signaling distortions in investment like those in Milde and Riley (1988) and Martin (2009) and to highlight our main mechanism where competition in the intermediation sector generates over-investment in the *extensive margin*. It also helps to keep the model tractable and allows a simpler characterization of the optimal contract.[2]

[2]Without multiplicative separability the optimal contract is harder to characterize; however, the main message of the paper remains: the optimal contract yields an inefficient outcome. As shown by Bester (1985a), when the cross-partial derivatives of $\ln(p(\theta, k))$ are not zero, entrepreneurs will use k to signal their type as in Riley (1975). As a result, the project size k will be distorted, which will lead to another source of inefficiency. Relative to our results, this will lead to less extensive inefficiency (fewer *ex ante* suboptimal projects are started) but more intensive efficiency (all projects will be run at a suboptimal scale). This is in contrast with the equilibrium of the log-linear environment we present, in which k is always the full-information optimal level.

The financial intermediaries have access to capital at the (gross) risk-free rate R and face a loan demand from entrepreneurs. The financial intermediation market entails two frictions. First, intermediaries cannot observe the entrepreneurs' ability but can see investment in the project, introducing adverse selection. Second, if a project fails, the intermediaries cannot exert any claims on the entrepreneurs because projects in this economy are subject to limited liability.

Intermediaries offer contracts, specifying the size of the loan k, the repayment in case of success x_1, and the repayment if the project fails x_0. Given that there are only two possible outcomes from the project, 0 or π, the contract space is generic. For convenience, we make a linear transformation of the contract that will allow a more straightforward parallel with the mechanism design literature focused on payments to the entrepreneur (agent). Set $x = -x_0$ and $z = \pi - (x_1 - x_0)$, then a contract will be characterized by a triple (k, x, z) prescribes a fixed pay for the agent x, an additional payment contingent on success z and an investment amount k that determines the probability of the contingent payment happening.

Potential entrepreneurs and intermediaries interact in the following way: each intermediary selects a mechanism, and then workers decide which message to send to each intermediary. The mechanism assigns a (k, x, z) contract to each possible message. The agent chooses between the two intermediaries' offers and the outside option of salaried work (which we write as $(0, w, 0)$), accepting one and rejecting the others.

The message space available to the startups could be arbitrarily large, comprising the firm's type, details on the other financing mechanism, cheap talk, or any other unverifiable claim even if it is orthogonal to the project's outcome. Fortunately, Martimort and Stole (2002) developed a version of the revelation and delegation principles for common agency games like this one. They showed that any of those potential interactions could be reduced to games in which each intermediary offers a set of contracts, and the startup chooses a contract inside the available menus, like the one developed here. In that sense, our mechanism and results are generic. We describe these menu games next.

2.2 Menu Games

In this subsection, we specify the contract menus game. For simplicity we assume there are only two identical financial intermediaries, venture capitalists (VC), indexed by $i \in \{1, 2\}$, competing *a la Bertrand* for entrepreneurs.[3] There is a continuum of entrepreneurs indexed by their private type θ drawn from a distribution $G(\theta)$ known to intermediaries. The total mass of entrepreneurs is 1.

Timing: Financial intermediaries move simultaneously, posting unrestricted menus (sets) of contracts. After that, entrepreneurs choose among the available menus and the outside option. Once a

[3]If entrepreneurs observe all offered contracts, the outcome will be the same with more intermediaries, although the optimal strategies of each one may differ. Hence this is just a notational simplification for free entry into the intermediaries sector.

menu is chosen each startup picks a contract inside that set, or the outside option.

Strategies: For intermediaries, the contract menu *space* contains any compact[4] subset of triples (k, x, z). That is, the power set of $\mathbb{R}^+ \times \mathbb{R}^+ \times \mathbb{R}$ denoted $\mathcal{P}(\mathbb{R}^+ \times \mathbb{R}^+ \times \mathbb{R}) = \boldsymbol{\Xi}$. Venture Capitalists post a single contract set $\mathcal{C}_i \in \boldsymbol{\Xi}$.

We call the outside option as intermediary $i = 0$ and define the outside option menu as $\mathcal{C}_0 = \{(0, w, 0)\}$.We define the set of contract menus available to an entrepreneur as $\mathcal{C} = \big(\mathcal{C}_0, \mathcal{C}_1, \mathcal{C}_2\big)$. After observing her private type θ and $\mathcal{C}$, the entrepreneur chooses her preferred contract menu, say $\mathcal{C}_j$, determining her financier, in that case j, and then a contract inside that menu.

Payoffs: All players are risk-neutral and care only about the expected payoff. For a entrepreneur of type θ the expected payoff of signing a contract (k, x, z) is:

$$u\big(\theta, (k, x, z)\big) = \theta f(k) z + x. \tag{1}$$

In particular, if the researcher takes her outside option, her payoff is $u\big(\theta, (0, w, 0)\big) = w$. Conditional on an entrepreneur of type θ signing a contract (k, x, z) with a venture capitalist i, the financier's expected payoff is:

$$v\big(\theta, (k, x, z)\big) := \theta f(k) \cdot (\pi - z) - x - Rk. \tag{2}$$

Adding equations (1) and (2) yields the expected gross surplus of the startup. Among a set of contracts delivering the same expected utility to the entrepreneur, the financier strictly prefers the one maximizing gross surplus. It will be shown that the financier can always break ties while preserving incentives at a negligible cost, which will allow us to collapse the entrepreneur's strategies into choosing exactly one contract per financier. However, entrepreneurs will still randomize among financiers when indifferent.

Strategy Space and Payoffs: As stated before, the strategy space for venture capitalists (VCs) is the power set of $\mathbb{R}^+ \times \mathbb{R}^+ \times \mathbb{R}$ denoted $\boldsymbol{\Xi}$. For the entrepreneur of type θ a strategy is a function s_θ that assigns to each set of available contract menus $\mathcal{C}$ a probability distribution over financiers or the outside option and a probability distribution with support on each contract menu:

$$s_\theta : \boldsymbol{\Xi}^3 \to \Delta\big(\{0, 1, 2\}\big) \times \Delta(\boldsymbol{\Xi})^3, \quad s.t. \quad Supp\{s_\theta(\mathcal{C})_{i+2}\} \subseteq \mathcal{C}_i, \quad \forall i \in \{0, 1, 2\} \tag{3}$$

where $\Delta(\mathbb{X})$ is the set of probability measures over $\mathbb{X}$ and $Supp\{\cdot\}$ denotes the support.

The first component of the entrepreneur's strategy is the distribution over intermediaries: $s_\theta(\mathcal{C})_1 = (q_0, q_1, q_2)$ where q_i is the probability assigned to intermediary i. The expected payoff for an entrepreneur given her type, strategy s_θ and available menus is:

$$U(\theta, s_\theta, \mathcal{C}) = \sum_{i=0}^{2} q_i \int_{(k,x,z)\in\mathcal{C}_i} \big(\theta f(k) z + x\big) ds_\theta(\mathcal{C})_{i+2}[k, x, z], \tag{4}$$

[4]Boundedness is without loss of generality since posting unbounded contract menus implies negative expected profits in equilibrium. Closure is slightly restrictive but necessary for the expected payoffs to both type of players to exist in equilibrium.

where $M[k,x,z]$ denotes the cumulative density at $[k,x,z]$ of the probability distribution M.

The expected payoff to the entrepreneur in equation (4) implies an expected payoff for intermediary i from potentially financing an entrepreneur of type θ of:

$$v_i(\theta, s_\theta, \mathcal{C}) := s_\theta(\mathcal{C})_{(1,i+1)} \int_{(k,x,z)\in\mathcal{C}_i} \big(\theta f(k)(\pi - z) - x - Rk\big) ds_\theta(\mathcal{C})_{i+2}[k,x,z], \tag{5}$$

where $s_\theta(\mathcal{C})_{(1,i+1)} = q_i$ is the prescribed probability of entrepreneur choosing financier i, that is the $(i+1)$-th component of the probability distribution over financiers and the outside option $s_\theta(\mathcal{C})_1$.

Denote the entrepreneurs' strategy profile $\Sigma = \{s_\theta | \theta \in \Theta\}$. The total expected payoff of the intermediary i can be written as:

$$V_i(\Sigma, \mathcal{C}) = \int_{\theta\in\Theta} \big[v_i(\theta, s_\theta, \mathcal{C})\big] dG(\theta), \tag{6}$$

which is just the sum of the expected profits v_i over each type θ.

Equilibrium definition: The equilibrium concept applicable to this framework is the Perfect Bayes Equilibrium.

Definition A strategy profile $\{\mathcal{C}_i^*, s_\theta^*\}$ for all $i \in \{1,2\}$, and $\theta \in \Theta$, is a *Perfect Bayes Equilibrium* if:

1. Entrepreneurs maximize expected utility: For all $\theta \in \Theta$, the strategy for entrepreneur of type θ must have the form defined in equation (3). If it assigns a positive probability to intermediary i, that is $s_\theta^m(\mathcal{C}^m)_{(1,i+1)} = q_i > 0$, then all of i's contracts chosen with positive probability must maximize expected utility among *all* available contracts:

$$Supp\big\{s_\theta^*(\mathcal{C}^*)_{i+1}\big\} \subseteq \underset{(k,x,z)\in\cup_{j=0}^2 \mathcal{C}_j^*}{\arg\max} \; \theta f(k) z + x. \tag{7}$$

2. For each intermediary $i \in \{1,2\}$, given the entrepreneur's strategies $\Sigma^* = \cup_{\theta\in\Theta} s_\theta^*$ and the competitor's contract menus $\mathcal{C}_{-i}^*$, her own contract menu $\mathcal{C}_i^*$ maximizes her expected utility defined in equation (6), that is:

$$\begin{aligned} &\mathcal{C}_i^* \in \underset{\mathcal{C}_i\in\boldsymbol{\Xi}}{\arg\max} V_i(\Sigma^*, \mathcal{C}_i, \mathcal{C}_{-i}^*) \\ &s.t. \forall (k,x,z) \in \mathcal{C}_i : k \geq 0, \quad x \geq 0, \quad x + z \geq 0. \end{aligned} \tag{8}$$

The conditions $x \geq 0$ and $x + z \geq 0$ ensure that limited liability is satisfied: if the project fails, the entrepreneur cannot make any payment to the intermediary, but the intermediary could potentially make a transfer to the entrepreneur. We assume the posted menus are compact sets for the entrepreneur's optimal strategies to exist.[5]

[5] Boundedness is without loss of generality since posting unbounded contract menus implies negative expected profits in equilibrium. Closure is slightly restrictive but necessary to guarantee the least upper bound of expected payoffs is attainable.

3 Equilibrium Contract Characterization

In this section we state a sequence of claims leading to the characterization of the equilibrium contract. As will be shown, intermediaries make zero profits in any equilibrium and the entrepreneur's payoff is linear in their type. Although the equilibrium is by no means unique, all equilibria are payoff equivalent.

Utility Representation

For every nonempty contract menu $\mathcal{C}_i$ offered by intermediary i there exists $U_i(\theta)$ the promised expected utility function for each type is defined as:

$$U_i(\theta; \mathcal{C}_i) = \max_{(k,x,z)\in\mathcal{C}_i} \theta f(k) z + x \tag{9}$$

Without loss of generality the contract menus $\mathcal{C}_i$ can be restricted to include only contracts that belong to the argmax in equation (9) for at least one θ. As $f(0) = 0$ it is also safe to assume whenever a contracts has $k = 0$ it must have $z = 0$.

Let $U(\theta; \mathcal{C}_1, \mathcal{C}_2)$ be the potential payoff that a θ-type agent could get conditional on becoming an entrepreneur (not taking the outside option), when intermediaries post contract sets $\mathcal{C}_1$ and $\mathcal{C}_2$. That is:

$$U(\theta; \mathcal{C}_1, \mathcal{C}_2) = \max_{(k,z,x)\in\mathcal{C}_1\cup\mathcal{C}_2} \theta f(k) z + x \tag{10}$$

In what follows, and to simplify notation, we drop the dependency of U on $\mathcal{C}_1, \mathcal{C}_2$.

For each θ, let $\big(k(\theta), z(\theta), x(\theta)\big)$ be a representative of the equivalence class of maximizers of the problem in equation 10. Under the assumptions for θ and $f(k)$, the Spence-Mirrless conditions (single crossing) hold and local incentive compatibility conditions are equivalent to the global incentive compatibility conditions. Hence, Myerson's envelope condition can be applied:

Claim 1. *Let $\mathcal{C}_1^*$ and $\mathcal{C}_2^*$ be equilibrium contract schedules and, for each θ, let $\big(k^*(\theta), z^*(\theta), x^*(\theta)\big)$ be a representative of the equivalence class of maximizers of the problem in equation 10 with $\mathcal{C}_1^*$ and $\mathcal{C}_2^*$, then :*

$$U(\theta) = U(0) + \int_0^\theta f\big(k^*(s)\big) z^*(s) ds, \tag{11a}$$

$$f\big(k^*(\theta)\big) z^*(\theta) \text{ is non-decreasing.} \tag{11b}$$

$$x^*(\theta) = U(0) + \int_0^\theta f\big(k^*(s)\big) z^*(s) ds - \theta f\big(k^*(\theta)\big) z^*(\theta), \tag{11c}$$

We refer to $\big(k^*(\theta), z^*(\theta), x^*(\theta)\big)$ as an *incentive compatible* contract menu. The equilibrium payoff of a θ-type agent is $\max\{w, U(\theta)\}$.

The Payoff of Intermediaries

Since the market structure resembles Bertrand competition, it is natural to conjecture that if an intermediary were to make profits, his competitor could offer contracts slightly more generous and steal the entire market, the key is to keep incentives aligned and avoid attracting too many low types. This conjecture is correct and stated in the next claim. All proofs are in the Appendix.

Claim 2 (Zero Profit Condition)**.** *In any equilibrium, the profits for intermediaries is zero.*

We now aim to characterize the amount of capital lent to each entrepreneur in equilibrium. First, define $k^F(\theta)$ as the full information optimal investment in a project of type θ.

$$k^F(\theta) = \arg\max_{k\geq 0}\{\theta f(k)\pi - Rk\} \tag{12}$$

Then, let $S(\theta)$ be maximum gross surplus generated by an entrepreneur of type θ,

$$\begin{aligned} S(\theta) &= \max_{k\geq 0}\{\theta f(k)\pi - Rk\} \\ &= \theta f\big(k^F(\theta)\big)\pi - R\cdot k^F(\theta). \end{aligned} \tag{13}$$

$S(\theta)$ is a gross surplus because it does not include the opportunity cost of forgoing the outside option w. Note that under the assumptions for $f(k)$, the optimal project size $k^F(\theta)$ is a continuous and strictly increasing function of θ.

The payoff of the entrepreneur only depends on k through the product $f(k)z$. If a type θ entrepreneur's preferred a contract with a different scale: $k(\theta) \neq k^F(\theta)$, the intermediary could offer a contract with the same x, $k^F(\theta)$ and adjust $\hat{z}(\theta)$ to keep $f\big(k^F(\theta)\hat{z}(\theta) = k(\theta)z(\theta)$. As $f(k)z$ remains constant, the new contract added does not change $U(\theta')$ for any other entrepreneur of type θ' as can be seen in equations (11). After a few details on breaking the tie, it follows that in equilibrium (almost) all projects are enacted at their private optimal size, which is the socially optimal in absence of externalities. Claim 3 formalizes the argument above.

Claim 3. *Contracts signed in equilibrium have $k(\theta) = k^*(\theta)$ for (almost) every θ.*

These results put together imply in equilibrium (almost) all entrepreneurs have a unique maximizer per intermediary and, when multiple, all yield the same payoffs for both entrepreneur and intermediary. Hence, without loss of generality the strategy space for entrepreneur can be simplified to

$$s_\theta : \boldsymbol{\Xi}^3 \to \Delta\big(\{0,1,2\}\big) \times \boldsymbol{\Xi}^3, \quad s.t. \quad s_\theta(\mathcal{C})_{i+2} \in \mathcal{C}_i, \quad \forall i \in \{0,1,2\} \tag{14}$$

3.1 Fighting for the Best, Losing with the Rest

We just showed that entrepreneurs make zero profits. As a result, entrepreneurs have all the bargaining power, and one could suspect that intermediaries will make zero profits type by type and each entrepreneur would receive all the economic surplus she produces, as in Rothschild and Stiglitz (1992). That intuition would be correct in a similar framework without limited liability, or without asymmetric information. However, the interaction between the two frictions does not allow for the former result to hold in our economy. On the contrary, the expected surplus of the project tends to grow much faster than incentives can be provided: whenever expected profits are positive, locally expected revenues increase faster than expected costs. In a nutshell, intermediaries want to invest more in more able types, but cannot increase rewards too fast to keep incentives.

Claim 4. *Suppose $S(\hat{\theta}) \geq U(\hat{\theta})$ for some $\hat{\theta} > 0$. Then, $S(\theta') > U(\theta')$ for almost every $\theta' > \hat{\theta}$.*

Because both intermediaries are making zero profits, and U and S are continuous, there is a θ such that $U(\theta) = S(\theta)$. Heuristically, by the envelope theorem *derivatives* are $U'(\theta) = f\big(k(\theta)\big)z(\theta)$ and $S'(\theta) = f\big(k(\theta)\big)\pi$, and for the financier not to make losses it needs to earn something in case of success $\pi > z(\theta)$. The claim above implies that such type, denoted θ_E is necessarily unique: although intermediaries make zero profits on average, they make strict profits with the best types and strict losses with other types. This cross-subsidization and zero aggregate profit result is akin to those in Rosenthal and Weiss (1984) and Dasgupta and Maskin (1986).

Next, we characterize the set A of types choosing to be entrepreneurs. There will be a cutoff type θ_L such that entrepreneurs with better ideas will enact them and those with weaker prospects will take the outside option.

Claim 5. *In equilibrium, there exists some type θ_L such that (almost) all entrepreneurs with $\theta > \theta_L$ strictly prefer to sign a contract with an intermediary instead of taking the outside option, (almost) all entrepreneurs with $\theta < \theta_L$ take the outside option, and $\theta_L\big[U(\theta_L) - w\big] = 0$.*

Because entrepreneurs are risk neutral, insuring entrepreneurs by giving a fixed pay x is more valuable for low types. On the contrary, intermediaries want the incentives to be as steep as possible to cream-skim the market, making limited liability bind. Heuristically, by limited liability $0 \leq x(\theta) = U(\theta) - \theta f\big(k(\theta)\big)$, using $U'(\theta) = f\big(k(\theta)\big)$ it follows that $U'(\theta) \leq \frac{U(\theta)}{\theta}$. Hence, given a utility level $U(\theta)$ to provide incentives as steep as possible (larger U') limited liability must bind.

Claim 6. *Competition in the intermediation market and the limited liability constraint imply that any contract offered and signed in equilibrium has $x(\theta) = 0$ for all $\theta > \theta_L$.*

It is useful to note that the result above would not hold, absent limited liability. In that case, the optimal mechanism specifies that entrepreneurs get all the profits from the project ($z = \pi$) and pay $x = -R \cdot k^*(\theta)$ regardless of whether the project succeeds or fail. Limited liability puts a bound to the separation of types.

Claim 6 implies that in equilibrium incentives are provided using only k and z. Hence the set of most preferred contracts for an entrepreneur of type θ, defined by:

$$\underset{(k,x,z)}{\arg\max}\, \theta f(k) z \qquad \text{s.t: } (k, 0, z) \in \mathcal{C}_1^* \cup \mathcal{C}_2^*$$

is independent of θ. This means, regardless of their ability, all entrepreneurs will choose one among the contracts offering the highest product $f(k)z$ and in all enacted contracts the product $f\big(k^*(\theta)\big)z(\theta)$ will be the same.

Hence, for a low enough level of skill, agents prefer to be workers rather than entrepreneurs: $U(0) < w$ and then $\theta_L > 0$. Since, $\theta_L f(k(\theta_L))z(\theta_L) = w$, we get from the condition above:

$$f(k(\theta))z(\theta) = \frac{w}{\theta_L}. \tag{15}$$

This together with claims 3 and 6, fully characterize the equilibrium optimal contract.

Proposition 1. *In equilibrium, the contract signed by almost every θ is:*

$$k^*(\theta) = \arg\max_k \{\theta f(k)\pi - Rk\} = k^F(\theta), \qquad z^*(\theta) = \frac{w}{\theta_L f\big(k^F(\theta)\big)}, \qquad x^*(\theta) = 0.$$

where θ_L, the lowest type who accepts the contract, is such that the financial intermediaries make zero profits:

$$\int_{\theta_L}^1 \left\{\theta f\big(k^F(\theta)\big)\pi - Rk^F(\theta)\right\} dG(\theta) - \int_{\theta_L}^1 \theta \frac{w}{\theta_L} dG(\theta) = 0 \tag{16}$$

Proposition 1 implies that the expected payoff for the entrepreneur increases linearly on her type, namely $\frac{\theta}{\theta_L}w$. By contrast, the payoff contingent on success, $\frac{w}{\theta_L f\left(k^F(\theta)\right)}$, is decreasing on her type. In order to induce truth-telling, in case of success, the intermediaries have to reward bad types more highly, as they understand success is harder for them. The expected payoff for the intermediary on a θ-type project is $\theta f\big(k(\theta)\big)\pi - \frac{w\theta}{\theta_L} - Rk(\theta) = S(\theta) - U(\theta)$. It follows from claim 4 that this payoff can be zero only for one particular θ and will be positive for higher types. Moreover $S(\theta) - U(\theta)$ is a convex function of θ.[6] The good types are very valuable for intermediaries: the profits coming from them will compensate for the losses from bad types. The resulting cross-subsidization opens the door for inefficient extensive margin decisions, as will be discussed in the next section.

Interestingly, *firms cannot fight for segments of the market (i.e., try to steal only a subset of $\Theta = [0, 1]$).* If an intermediary were to deviate and offer a more attractive contract for the profitable types, it must be that he posted a contract such that $f\big(k\big)z$ is higher, but then all types get a higher payoff, and hence the contract is better for every θ. In other words, in a world where the limited liability constraint is binding, intermediaries are not able to "cream-skim" up to the point of making zero profits type by type. This is in contrast with Rothschild and Stiglitz (1992).

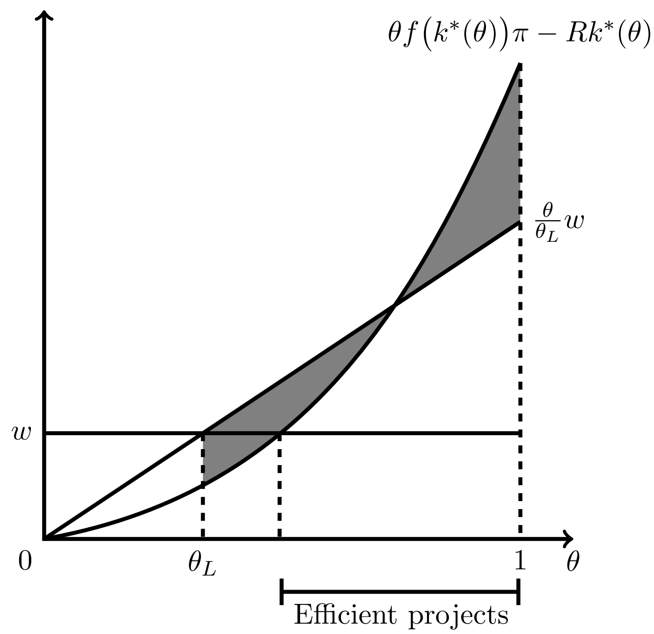
Figure 1 illustrates the equilibrium described by proposition 1. The figure shows several monetary payments as functions of θ. We plot the entrepreneurs' expected payoff $U(\theta) = \frac{\theta}{\theta_L}w$ and their outside

[6] By the envelope theorem, the derivative is $\pi f\big(k^F(\theta)\big) - \frac{w}{\theta_L}$

option w (which is assumed to be independent of θ); the expected surplus of the project, net of financial costs, $S(\theta) = \theta f\big(k^F(\theta)\big)\pi - Rk^F(\theta)$.[7] The area shaded in gray represents the profits of an intermediary. From the figure it is clear that changes in θ_L change the slope of the entrepreneurs expected payoff. These changes in turn modify the intermediaries' payoffs and hence θ_L can be adjusted so that intermediaries make zero profits.

The discussion above illustrates the key force in this economy: to provide the correct incentives, the payoff to entrepreneurs cannot grow as fast as the total surplus does. That means that intermediaries always make higher profits with the highest types. Since high types are so profitable, intermediaries are willing to lose money with low types to offer more attractive contracts to (profitable) high types and maintain incentives.

Figure 1: Equilibrium Contract and Zero Profits



3.2 Welfare and Optimal Policy

The expected social surplus of a project is $\theta f\big(k^*(\theta)\big)\pi - Rk^*(\theta) - w$. Hence an unconstrained social planner (one that could observe agents' abilities) would set $\theta f'\big(k^*(\theta)\big)\pi = R$ as before, but only for those θ such that the expected social benefit is not negative. As the surplus is increasing in θ[8], there is a lower bound θ_P, for those socially valuable projects. Then it is worth for the society to devote resources to all projects with $\theta \geq \theta_P$ where $\theta f\big(k^*(\theta_P)\big)\pi - Rk^*(\theta_P) = w$. By contrast, in the decentralized equilibrium,

[7] Note that the expected surplus is similar to the profit function of a competitive firm facing an output price of θ. It is well known that profit functions are convex in prices and hence the surplus is convex in θ, as displayed.

[8] The surplus is increasing by the envelope theorem

θ_L yields zero profit for intermediaries, but that implies it must be smaller than θ_P.

Claim 7. *In the decentralized solution, socially inefficient projects are always enacted. That is, $\theta_L < \theta_P$. Moreover, efficiency will be restored if any of the following becomes true:*

- *No adverse selection: Types are public information.*
- *No limited liability: Intermediaries are able to recover any contracted amount.*
- *No competition: There is only one intermediary or intermediaries collude.*

The underlying intuition is as follows: if only efficient projects were financed, the intermediaries would make profits in all the projects. But then, positive profits attract new intermediaries who can "steal" the market by offering more generous contracts. More generous contracts involve some cross-subsidization, and as a result socially inefficient projects will be active.

If types are public information but all other conditions remain the same, intermediaries will break even on each type. This implies that all signed contracts are such that $U(\theta) = S(\theta)$, and only types $\theta > \theta_P$ sign contracts. Contracts will not be completely determined, since many combinations of $x(\theta)$ and $z(\theta)$ yield $U(\theta) = S(\theta)$, but $k(\theta) = k^F(\theta)$. The equilibrium payoff of type θ is $U(\theta) = \max\{w, S(\theta)\}$.

If there is no limited liability but all other conditions hold, the only IC contract schedule in equilibrium is $k(\theta) = k^F(\theta)$, $x(\theta) = -R \cdot k^F(\theta)$ and $z(\theta) = \pi$, which implies $U(\theta) = S(\theta)$. This is a risk-free type of contract. As is well understood, when the entrepreneur is risk neutral, transferring her all the risk solves the incentive problem.

If there is only one intermediary, but adverse selection and limited liability still hold, the only equilibrium is as follows: the contract that maximizes profits for the intermediary is $k(\theta) = k^F(\theta)$, $x = w$, $z = 0$ and entrepreneurs with types $\theta \geq \theta_L$ take it, all others reject. The intermediary will take all the surplus and her profits would be $\int_{\theta_P}^{1} \big(S(\theta) - w\big) dG(\theta)$. Interestingly, in the context of our model, the first best outcome is not reached in the competitive equilibrium but is instead attained without competition. The reason is that limited liability constrains the contracts space, but—while this constraint is binding under competition—it is not under collusion.

3.2.1 Optimal Policy

In this section we consider two possibilities for taxation: a fixed sum tax per contract to financial intermediaries ϕ and a tax rate τ on entrepreneurs' income. In both cases the full information optimal allocation can be achieved.

First we consider a *Red Tape Tax*, which is a fixed sum ϕ per contract signed that the intermediary must pay.

Claim 8. *Let ϕ^* be a fixed tax per contract defined by:*

$$\phi^* \equiv \left[\int_{\theta_P}^{1} \left\{ \theta f(k^F(\theta))\pi - R k^F(\theta) \right\} dG(\theta) - \int_{\theta_P}^{1} \theta \frac{w}{\theta_P} dG(\theta) \right] \left[1 - G(\theta_P) \right]^{-1}$$

Then only efficient projects (and all of them) are funded.

The intuition behind the claim above is that a lump-sum tax per contract does not change incentives for entrepreneurs, captured by the expected utility function $U(\theta)$. It will deter entry only by affecting the intermediaries' residual surplus, which is reduced in $\phi\big(1 - G(\theta_L)\big)$. Since there is too much entry into entrepreneurship, a small tax is always desirable, and first best can be attained by setting the right tax.

This tax can also be seen as fixed subsidy on w, or any instrument that increases the outside option of every entrepreneur. However, the revenue implications for the tax authority would be different. Since we do not model the nature of the wage or the outside option, we stick to the contract fee tax interpretation.

Now suppose there is a profit or dividend tax. Intermediaries make zero profit, so they will not be affected by such tax. However, startups do make profits. Hence a tax rate τ on profits would make any contract $\big(k, x, z\big)$ look to the entrepreneurs like $\big(k, x(1-\tau), (1-\tau)z\big)$

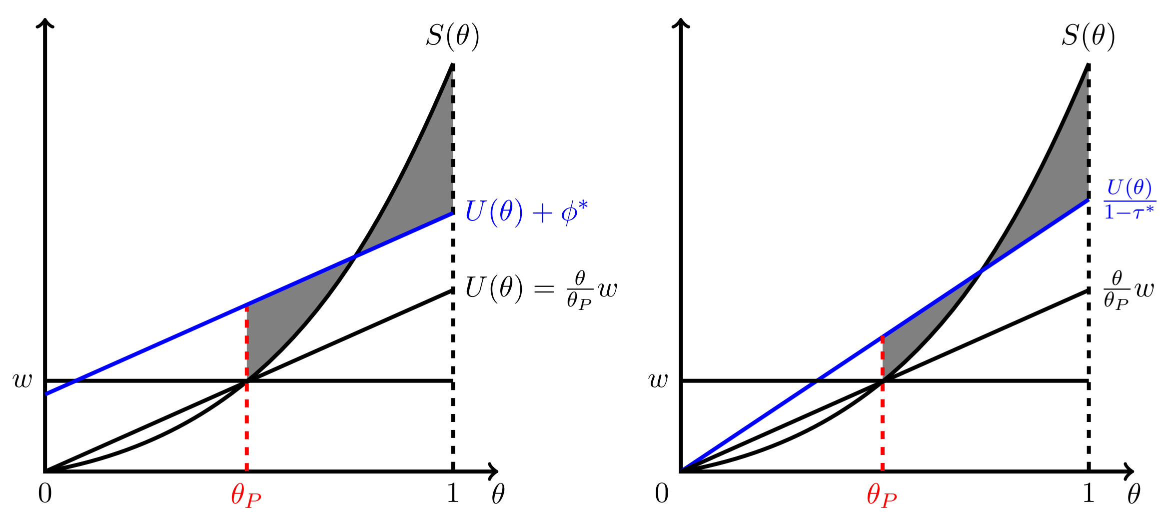
Claim 9. *Let τ^* be the tax rate on profits defined by:*

$$1 - \tau^* \equiv \int_{\theta_P}^{1} \theta \frac{w}{\theta_P} dG(\theta) \cdot \left[\int_{\theta_P}^{1} \left\{\theta f(k^F(\theta))\pi - Rk^F(\theta)\right\} dG(\theta)\right]^{-1}.$$

Then only efficient projects (and all of them) are funded.

A tax on profits homothetically increases the cost for financiers to provide utility to entrepreneurs. To achieve the firs best, the tax rate is the fraction of the efficient gross surplus not captured by the entrepreneurs when an optimal contract menu with $\theta_L = \theta_P$ is enacted. Figure 2 panel B shows both tax distortions. The fact that corporate income tax can be desirable is consistent with the findings of Scheuer (2013) and Ghatak et al. (2007).

Figure 2: Policy Remedies: Lump-sum and Profit Taxes



Taxing R as capital income would be troublesome since it will distort $k^F(\theta, R)$, reducing the total surplus, which is the standard result on capital income taxation. In principle this would be at odds with claim 9, but this is because we assume taxation on capital income is made at the individual level. Hence as long as the intermediaries can aggregate profits on investments before taxes, or claim back taxes on dividends from entrepreneurs' firms in which they have a stake, they will not be affected by the taxation, only the entrepreneurs. So far we have avoided making any interpretation of the legal stance the contracts will have. While we can think of the contracts as debt contracts with limited liability clauses or as equity stakes in a firm, those details are irrelevant for our discussion above but become important once embedded in a complex tax system. The discussion of that topic is far beyond the scope of this paper.

3.3 A Numerical Example

We have established that in the environment described above, competition among financial intermediaries yields to an inefficient outcome. This subsection illustrates the changes in the deadweight loss when various parameters of the model change.

Parameterization

Recall that a project succeeds with probability $\theta f(k)$. We let $f(k) = 1 - \exp(-\beta k^{\alpha})$ for $\beta > 0$ and $\alpha \in (0,1)$. This functional form has several properties. First, it is continuous and strictly increasing and strictly concave on $\mathbb{R}_+$. Second, $f(0) = 0$ and $\lim\limits_{k\to\infty} f(k) = 1$. Third, $\lim\limits_{k\to 0} f'(k) = \infty$. The last condition ensures that for every $\theta > 0$ there is a scale such that the θ-type project is profitable (Inada condition). A way to interpret the above functional form is that the probability of success is the product of θ and the probability that an exponential random variable being smaller than k^{α} [9]. Exponential variables are usually employed for waiting times for a Poisson process; it can be interpreted as the waiting time until the arrival of a new innovation (success). An amount k of capital allows the entrepreneur to run the project for k^{α} periods. Hence the probability of a good idea arriving would be $f(k)$ and the probability of the entrepreneur understanding the idea is θ. The unconditional distribution of θ is $g(\theta) = \frac{1}{\eta}\theta^{\frac{1}{\eta}-1}$. The exponent $\frac{1}{\eta} - 1$ controls the participation of high types on the distribution. The higher is the exponent, the higher will be the density of types higher than a fixed value θ. This distribution has been used in other studies about entry of firms like (Ates and Saffie, 2021). All the parameters used in the numerical example are summarized in Table 1.

We set the entrepreneurs' outside option $w = 15$; the output of the project in case of success, $\pi = 100$;

[9] Equivalently, the probability of success is the product of θ and the probability that a Weibul random variable is lower than k. In that case, the waiting time interpretation would be that the longer it takes for a project to succeed the less likely it will succeed in the future, Jovanovic and Szentes (2013) use a similar approach hence our functional forms may be regarded as a reduced form of their results

Table 1: Benchmark Parameters

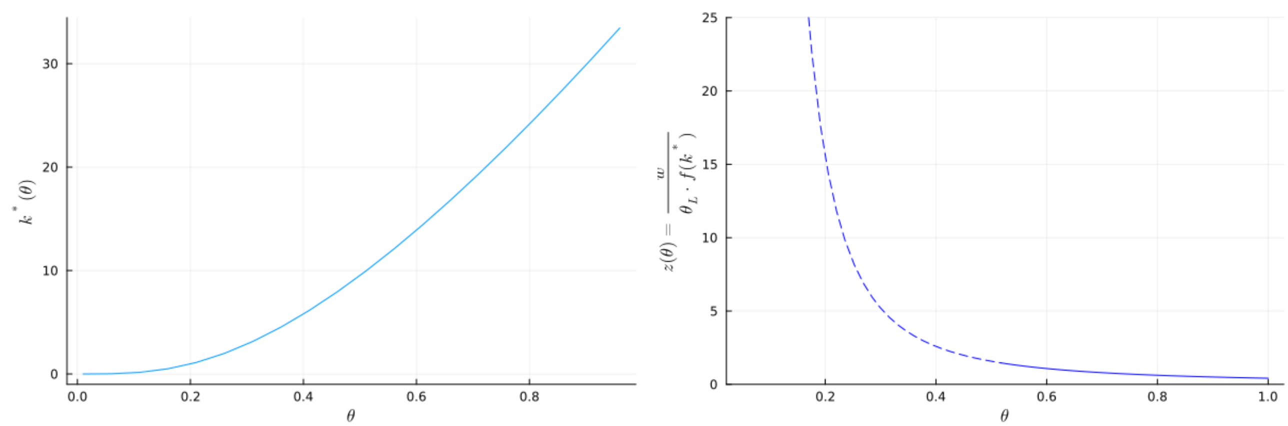
w	π	R	α	β	η
15	100	1.02	0.7	0.1	2

and the gross interest rate is $R = 1.02$. We set $\beta = 0.1$, implying that the latent exponential random variable would have a mean of 10. Under the above interpretation increasing k increases the probability that the random falls below k^{α} at a decreasing rate. We set $\alpha = 0.7$.

Benchmark Results

As defined before, an equilibrium is fully described by the triple $\big(k(\theta), x(\theta), z(\theta)\big)$ defined over all θ greater or equal than the cutoff θ_L. We established that $x(\theta) = 0$, $\forall \theta > \theta_L$. The functions $k(.)$ and $z(.)$ are plotted in Figure 3 below.

Figure 3: Optimal Contract



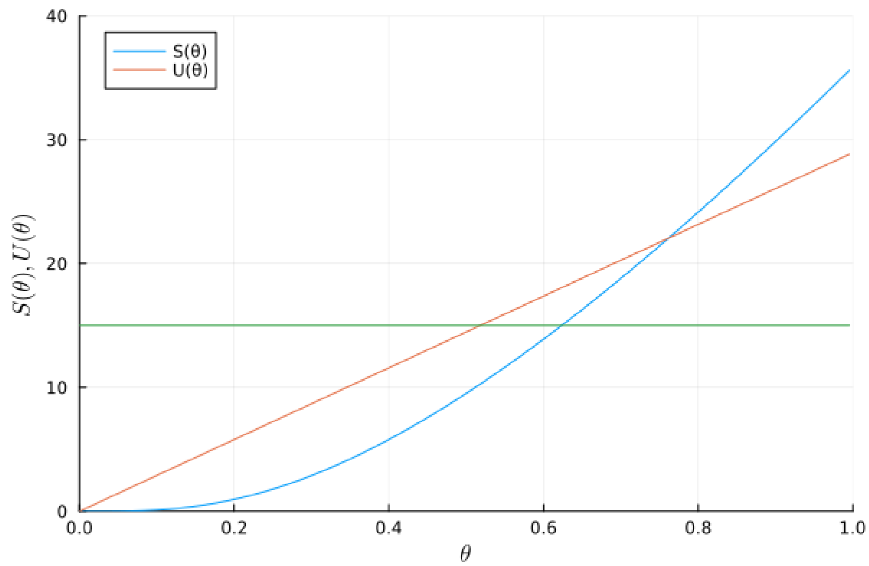
An important equilibrium object is the lowest type taking the contract, θ_L. For this example, the value of θ_L is 0.52, which implies that 28% of the entrepreneurs take the contract. Compare θ_L with the lowest type that would be funded by a social planner $\theta_P = 0.62$.

In Figure 4 we plot $S(\theta) = \pi\theta f\big(k^F(\theta)\big) - Rk^F(\theta)$ as well as the expected payoff of entrepreneurs $U(\theta) = w\theta/\theta_L$. In addition, we plot the the wage (horizontal line) to allow the reader to picture the equilibrium deadweight loss. In what follows we will use the relative inefficiency, defined as the ratio of the deadweight loss to the full information net economic surplus,

$$\mathcal{I} = \frac{\int_{\theta_L}^{\theta_P} \big(w - S(\theta)\big)dG(\theta)}{\int_{\theta_P}^{1} \big(S(\theta) - w\big)dG(\theta)}$$

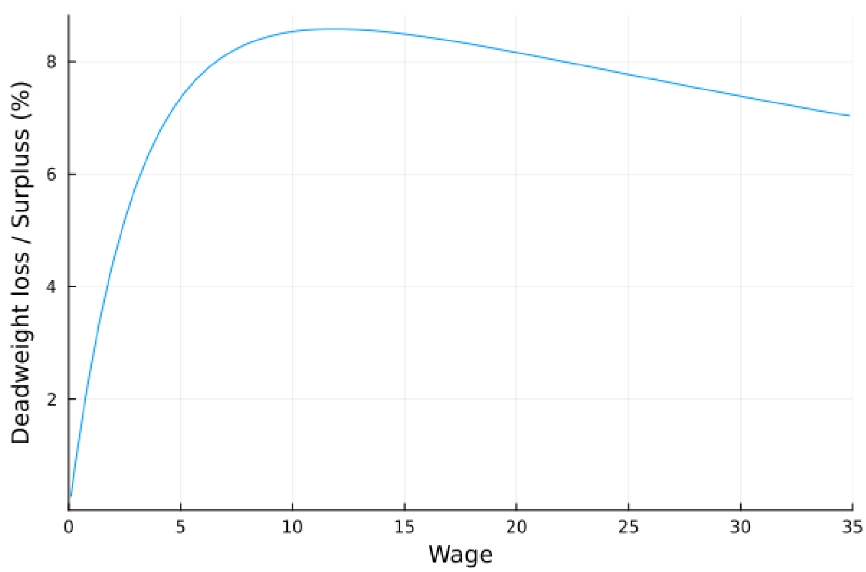
For our benchmark parameterization, the relative inefficiency is $\mathcal{I} = 8.49\%$. Next we change, one by

Figure 4: Surplus, Payoff, Inefficiency



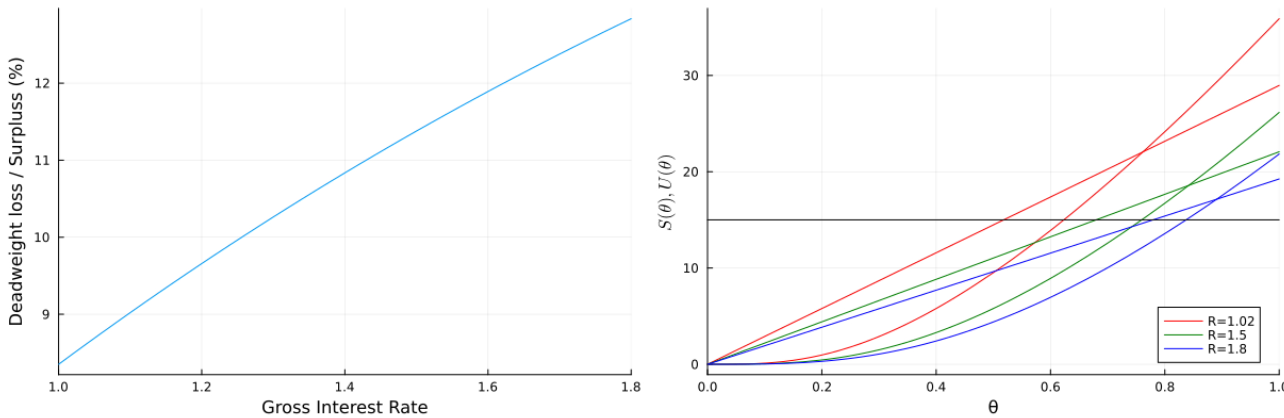
one, all the parameters of the model, holding the other parameters at their respective benchmark values, and study how the inefficiency responds to these changes.

Figure 5: Relative Inefficiency and Wage



The outside option of entrepreneurs, the wage w, is very important because, absent this cost, the economy would not be inefficient. In fact, when $w = 0$, all projects are socially profitable, by assumption, and they are all funded at the optimal scale. At first, an increase in w reduces the net economic surplus, and it also induces increase in θ_L. Coming from low levels, a higher w tends to increase the inefficiency as almost all entrepreneurs prefer to enact a project. However, as the wage keeps growing, starting a project becomes less attractive for most entrepreneurs and intermediaries need to offer more utility to the high types, reducing the profits available for the cross-subsidization at the core of the inefficiency. As illustrated in Figure 5, when the wage is very high, the force is strong enough to actually decrease the inefficiency. Note that if the wage is high enough, all the entrepreneurs take the outside option and the deadweight loss disappears.

Figure 6: Relative Inefficiency and the Interest Rate



The gross interest rate, R, is similar to the wage, in the sense it represents the outside option, or opportunity cost of the capital in hands of financial intermediaries. A higher gross interest rate not only decreases the payoff of intermediaries, but it also decreases the optimal scale of the projects, and hence their expected return. An increase in R can be interpreted graphically as a downward shift of the curve $S(\theta)$, which in turn implies a lower expected utility curve $U(\theta)$ as shown in the right panel of Figure 6. Hence lowering the interest rate decreases both the net surplus and the deadweight loss, everything else equal. However, θ_L will increase to satisfy the zero profit condition. As shown in the left panel of Figure 6, when we let R vary between 1 and 1.8, the relative inefficiency increases up to 13%, at $R = 1.8$.

We move to describing how the inefficiency responds to changes in the shape of function $f(k)$. Figure 4 above suggests that the size of the inefficiency is highly related to the concavity of the function $S(\theta)$. This concavity only depends on the shape of f. In fact,

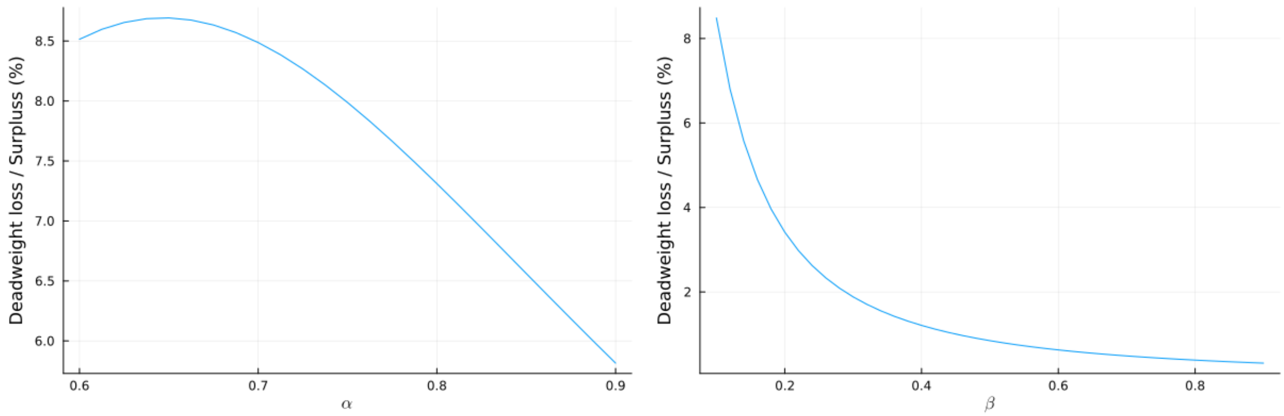
$$S''(\theta) = f'\big(k^F(\theta)\big)\pi \frac{dk^F(\theta)}{d\theta} = -\frac{\big(f'(k^F(\theta))\big)^2}{\theta f''\big(k^F(\theta)\big)}$$

Not surprisingly, the relative inefficiency is quite sensitive to the parameters α and β, which govern the shape of f in the current example. The results are displayed in Figure 7. The relative inefficiency is decreasing in both α and β. It decreases particularly fast as β increases, getting to 0.05% when $\beta = 1$.

Finally, the distribution of types importantly affects the size of the inefficiency. The parameter η governs the shape of the distribution of θ. More precisely, as η increases, the density is shifted toward lower types. Holding the outside option fixed, an increase in η decreases the net surplus, because some density will be shifted from socially profitable projects to unprofitable projects. However, when good types are scarcer, intermediaries will offer less generous contracts, increasing θ_L. The last force tends to

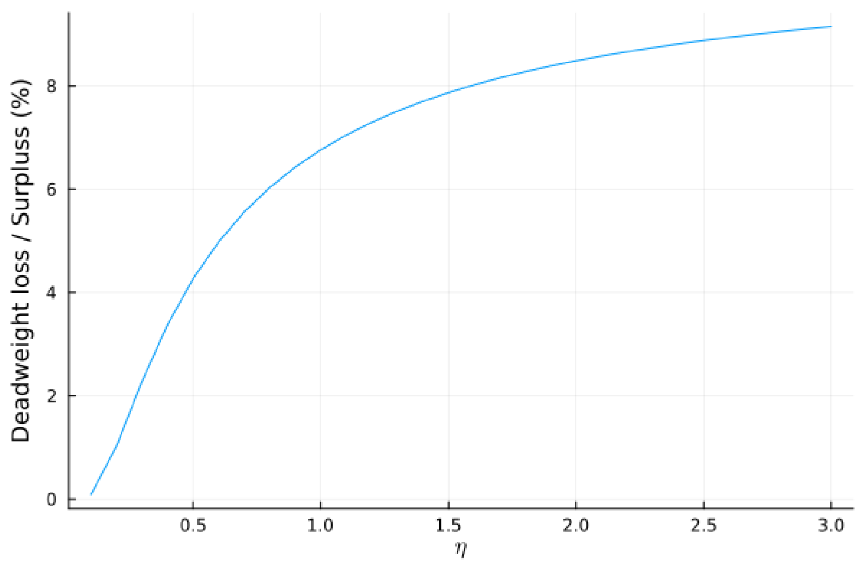
Figure 7: Relative Inefficiency and the Shape of $f(k)$

(a) α-parameter (b) β-parameter



decrease the inefficiency. Figure 8 shows that the relative externality actually increases, reaching 8.5% when $\eta = 3$. On the other hand, for values of η close to zero, the relative inefficiency gets close to zero.

Figure 8: Relative Inefficiency and the Distribution of Types



4 Credit Score

In this section, we will focus on the welfare effects of introducing an imperfect signal about entrepreneurs, akin to a Credit Score. This signal allows intermediaries to split entrepreneurs into groups. It assigns a unique score to each entrepreneur, which is visible to all intermediaries. In our basic example, the possible scores are low or high, $s \in \{l, h\}$, but there could be multiple scores, like all integer numbers between 150 and 850. Once a score is introduced, it segments the markets as intermediaries can now offer contracts contingent on that score. For each signal s they will offer a contract menu and face a conditional distribution $G(\theta|s)$ of entrepreneurs.

A higher score implies a better conditional distribution. However, there are many ways to define an order over distributions. We use a strong definition of distribution quality, the Monotone Likelihood Ratio Property (MLRP), which is the standard in the mechanism design literature. Hence in our basic example with two possible signals, the conditional distributions are such that $G(\theta|h) \succ_{MLRP} G(\theta|l)$. MLRP is a strong assumption but one often used in the information frictions literature because it allows the derivation of neat results. This makes it more challenging to devise a credit score signal that reduces welfare, as will be shown later.

4.1 Equilibrium Characterization with Credit Score

High-Score Equilibrium

As markets split, from Proposition 1, it follows that competition among the two intermediaries for the high signal entrepreneurs will result in an equilibrium contract menu of the following form:

Claim 10. *In equilibrium, startups with high score contract only with intermediaries, and the equilibrium menu $\mathcal{C}_h^*$ is characterized by:*

$$k_h^F(\theta) = k^F(\theta) = \arg\max_k \{\theta f(k)\pi - Rk\}, \qquad z_h^*(\theta) = \frac{w}{\theta_L^h f\big(k^F(\theta)\big)}, \qquad x_h^*(\theta) = 0.$$

where θ_L^h, the lowest type who accepts a contract, is such that the **intermediaries** *make zero profits in the* **high score submarket***, that is:*

$$0 = V_i^h(\Sigma, \mathcal{C}_h^*) = \int_{\theta_L^h}^1 \big[S(\theta) - U_h^*(\theta, \mathcal{C}_h^*)\big] dG(\theta|h) \tag{17}$$

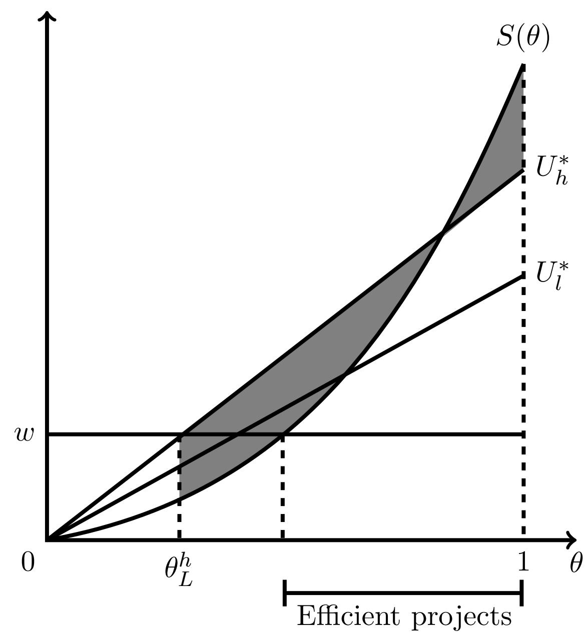
Notice that in equation (17), the integral uses the distribution conditional on a high signal. Again, From proposition 1, competition for the best startups among intermediaries will lead to zero profits for them. Interestingly, if there were another intermediary *without* access to the signal, she would incur losses if she tried to offer similar or better contracts, they would attract not only high but also low-signal types, the unconditional distribution.

Figure 9 shows the equilibrium in the high signal market. The shaded regions represent the difference between the surplus $S(\theta)$ and the expected payment to the startup U_h^*. *Weighted* by the density of types θ conditional on the signal $g(\theta|h) = dG(\theta|h)$, both areas must be equal (zero profit).

Low-Score Equilibrium

In the market with low signal, the equilibrium contract menu offers less utility across the board to all entrepreneurs. The low-signal equilibrium is akin to the original equilibrium without Credit Score,

Figure 9: Equilibrium Contract and Zero Profits with High Score



since all market participants know the distribution of types in this submarket is $G(\theta|l)$. If another intermediary without access to the signal were to enter, she could offer the same menu: high signal types would self-select out. The next claim characterizes the equilibrium:

Claim 11. *In equilibrium, startups with low score face the same contract schedule from intermediaries $\mathcal{C}_l^*$ is characterized by:*

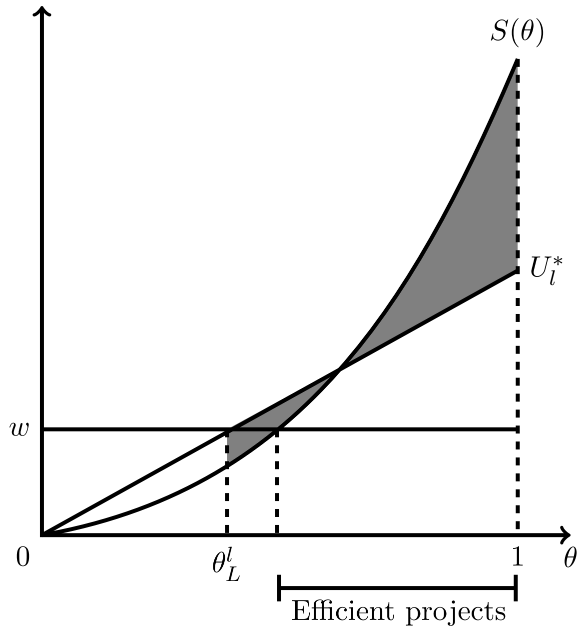
$$k_l^*(\theta) = k^F(\theta) = \arg\max_k \{\theta f(k)\pi - Rk\}, \qquad z_l^*(\theta) = \frac{w}{\theta_L^l f\big(k^F(\theta)\big)}, \qquad x_l^*(\theta) = 0.$$

where θ_L^l, the lowest type who accepts a contract, is such that the **intermediaries** *make zero profits in the* **low score submarket***, that is:*

$$0 = V_i^l(\Sigma, \mathcal{C}_l^*) = \int_{\theta_L^l}^1 \big[S(\theta) - U_l^*(\theta, \mathcal{C}_l^*)\big] dG(\theta|l) \tag{18}$$

This claim follows verbatim from proposition 1. The proof of claim 10 implies that $\theta_L^h < \theta_L^l$ and that $U_h^*(\theta) > U_l^*(\theta)$ for all θ, as seen in Figure 9. Figure 10 illustrates the equilibrium for the entrepreneur with a "low" score. The contract menu offered by intermediaries is $\mathcal{C}_l^*$, generating the curve U_l^*. Therefore, the gray area, weighted by the density $g(\theta|\mathbf{l})$, represents the profits of intermediaries in this sub-market. Although positive expected profit contracts $S(\theta) > U_l^*(\theta)$ seem to be much more than negative expected profit ones, conditional on a low score entrepreneur types are more likely to be in the negative expected profit zone.

Figure 10: Equilibrium Contract and Zero Profits for Low Score



The Credit Score Equilibrium

To sum up, we define the Credit Score Equilibrium, as the whole set of submarkets and contract menus offered.

Proposition 2. *In the Credit Score Equilibrium CSE, there are two contract menus: $\mathcal{C}_h^*$, available only to high-signal entrepreneurs, and $\mathcal{C}_l^*$, posted and available to all entrepreneurs. The contract menus are characterized by:*

$$k^*(\theta) = k^F(\theta) = \arg\max_k \{\theta f(k)\pi - Rk\}, \qquad z_l^*(\theta) = \frac{w}{\theta_L^l f\big(k^F(\theta)\big)},$$

$$x_m^*(\theta) = 0, \forall m \in \{l, h\} \qquad z_h^*(\theta) = \frac{w}{\theta_L^h f\big(k^F(\theta)\big)},$$

where θ_L^m, the lowest type who accepts the contract conditional on obtaining the signal m, is such that $\theta_L^h < \theta_L^l$ and the intermediaries make zero profits in the respective sub-market.

$$0 = V_i^l(\Sigma^l, \mathcal{C}_l^*) = \int_{\theta_L^l}^1 \Big[S(\theta) - \frac{\theta}{\theta_L^l} w\Big] dG(\theta|\mathbf{l}),$$

$$0 = V_i^h(\Sigma^h, \mathcal{C}_h^*) = \int_{\theta_L^h}^1 \Big[S(\theta) - \frac{\theta}{\theta_L^h} w\Big] dG(\theta|\mathbf{h}).$$

4.2 Credit Score and Welfare

This subsection focuses on the welfare effects of having a credit score signaling device. Theoretically, the welfare effect depends on the unconditional distribution and the *quality* of signal. In our numerical examples, it is generally the case that a credit score reduces inefficiency, thus improving welfare. It requires a very steep distribution near the θ_L and a weak signal to yield welfare losses from the introduction of a credit score.

Functional forms and parameters will be the same as in subsection 3.3, except for the type distribution functional form and parameters. To introduce the concept of signal quality, we will use a family of conditional distributions and densities that are ordered by MLRP (Monotone Likelihood Ratio Property) depending on only one parameter $\bar{s}$.

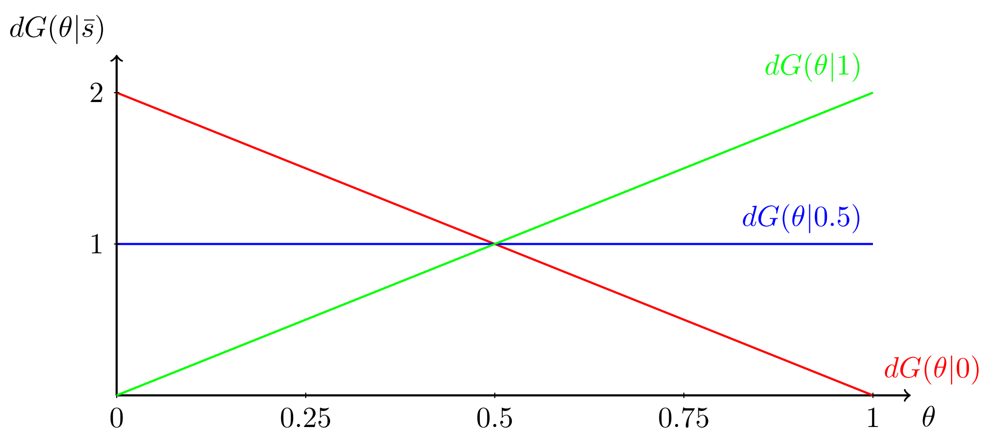
4.2.1 Credit Score Informativeness

To showcase the effect of a credit score informativeness on welfare, we will use a family of conditional distributions as follows:

$$dG(\theta|\bar{s}) = 2[(1-\bar{s}) - (1-2\bar{s})\theta] \quad (\theta, \bar{s}) \in [0,1]^2,$$

where s is the score parameter. When $s = 0.5$, $G(\theta|0,5)$ is an uniform density. These are depicted in figure 11. This family has the property that mixtures among two distributions of the family correspond to the element of the same family, with the parameter being the linear combination of the two parents original parameters, which allows us to easily generate multiple conditional distributions that mix up to the same unconditional one.

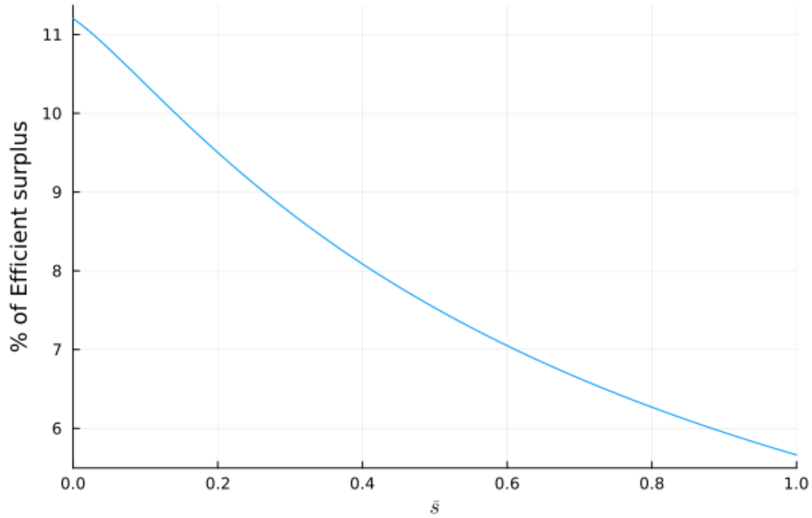
Figure 11: Linear Type Densities



The higher the score s the more right-shifted density function of entrepreneurs' type; that is, higher types become more abundant. This can also be seen through the distribution's skewness. For this family of distributions the skewness is fairly low, $\tilde{\mu}_3(\bar{s}) \in [-0.55, 0.55]$ where $\tilde{\mu}_3(\bar{s})' < 0 \quad \forall \bar{s} \in [0,1]$. If $\bar{s} < 0.5$ the density will be left-shifted ($\mu_3 < 0$), and conversely, when $\bar{s} > 0.5$ the density will be right-shifted ($\mu_3 > 0$). The literature on entry to entrepreneurship, like Ates and Saffie (2021), estimates a scarcity of good entrepreneurs/ideas consistent with very low skewness levels (~ -20).

Figure 12 plots the relative inefficiency $I_{rel}(\bar{s})$ as a function of the parameter $\bar{s} \in [0,1]$. In this case the higher $\bar{s}$ is, the lower the relative inefficiency. With the MLRP, a higher $\bar{s}$ implies low quality

Figure 12: Relative Equilibrium Inefficiency for the Linear Density Family



types are scarcer and high types are more abundant. Competition then drives up the contract's offered utility to bring profits back to zero. But among those types that entail losses to the intermediaries, the MLRP property also implies that socially inefficient types are relatively scarcer than socially efficient but privately costly types. This means more of the cross-subsidization happens among socially efficient types, reducing the inefficiency.

Given a fixed parameter $\bar{s}$ for the unconditional distribution and using the mixture property of the density family, we will consider a variety of mixtures that yield the same unconditional distribution, given some weights for each signal sub-market: ω_h, $\omega_l = 1 - \omega_h$.

$$G(\theta|\bar{s}) = \omega_l G(\theta|l) + (1 - \omega_l)G(\theta|h)$$

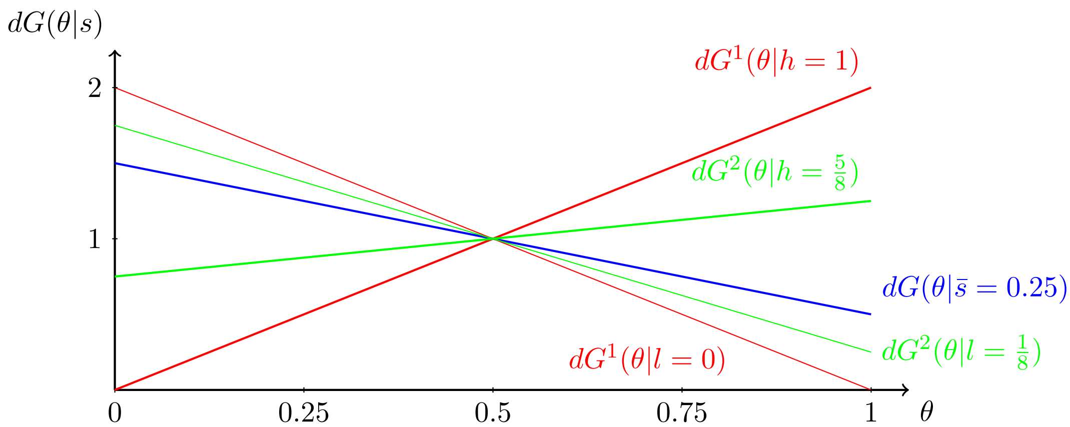
Keeping fixed the unconditional distribution, the bigger the difference among the conditional distributions $G(\theta|l)$ and $G(\theta|h)$ are, the more informative the signal is. In this family, the most informative a signal can be is when the high signal is associated with a parameter of one ($h = 1$) and the low signal with a parameter of zero ($l = 0$). To accommodate that extreme case, weights must be $\omega_h = \bar{s}, \quad \omega_l = 1 - \bar{s}$. However, given these weights there are many pairs of high-low signals that generate the same unconditional distribution. In our case, given $\bar{s}$ and a parameter associated with the high signal $h \in [\bar{s}, 1]$, the corresponding parameter associated with the low signal is $l = \frac{\omega_h}{\omega_l}(1-h)$. Notice that $h = \bar{s}$ implies $l = \bar{s}$ and that the signal is uninformative, so that all distributions are the same. Hence as the parameter for the high signal distribution moves h from $h = \bar{s}$ to $h = 1$, and the corresponding low parameter moves from $l = \bar{s}$ to $l = 0$, the change in aggregate inefficiency will reflect the impact of the signal quality.

For this particular exercise, we set $\bar{s} = 0.25$ to have a relative scarcity of good types. To accommodate the extreme cases this requires $\omega_h = 0.25$ and $\omega_l = 0.75$. Next, for a given high signal h, the low signal that generates the same unconditional distribution is $l = \frac{1}{3}(1-h)$. This implies:

$$G(\theta|0.25) = 0.25 \underbrace{G(\theta|h)}_{high\ signal} + 0.75 \underbrace{G\big(\theta|0.333(1-h)\big)}_{low\ signal}.$$

Figure 13 shows the unconditional density $G(\theta|\bar{s} = 0.25)$ in blue and two pairs of conditional densities, one with $h = 1$ (red) and one with $h = 0.625$ (green), both of which aggregate to the same unconditional one with $\bar{s} = 0.25$ (blue)..

Figure 13: Type Distributions Conditional on Signals



The total Relative Inefficiency for the Credit Score equilibrium is defined as a fraction of the full information surplus.

$$I_{CS}(h, l|\bar{s}) = \frac{\omega_h \int_{\theta_L^h}^{\theta_P} (w - S(\theta))dG(\theta|h) + \omega_l \int_{\theta_L^l}^{\theta_p} (w - S(\theta))dG(\theta|l)}{\int_{\theta_p}^{1} (S(\theta) - w)dG(\theta|\bar{s})}$$

When the unconditional distribution is fixed, the full information efficient surplus (denominator) is the same. Hence the change in the relative inefficiency is only due to the change in the sum of deadweight losses of each sub-market.

Recall $h = \bar{s}$ implies an uninformative signal. Figure 14 plots the relative inefficiency as the parameter for the distribution conditional on the high signal varies from the least informative value $h = 0.25 = bars$ to the most informative $h = 1.0$, and adjusting the low signal conditional distribution parameter as to keep the unconditional distribution unchanged.

In this case there is a clear monotone pattern: the more informative the signal is, the lower the deadweight loss is in the credit score equilibrium. To further look into the equilibrium consequences of the credit score introduction, Table 2 shows the main equilibrium objects for equilibria both with and without Credit Score (CS).

Column θ_L^s in this table shows the lowest type that accepts a contract in each case, corroborating the inequality $\theta_L^h < \theta_L < \theta_L^l < \theta_P$. The aggregate values in the inefficiency correspond to those in Figure 14: 9.1% without credit score vs 7.3% with a credit score. More than half of the inefficiency comes

Figure 14: Relative Inefficiency for Different Credit Score Quality

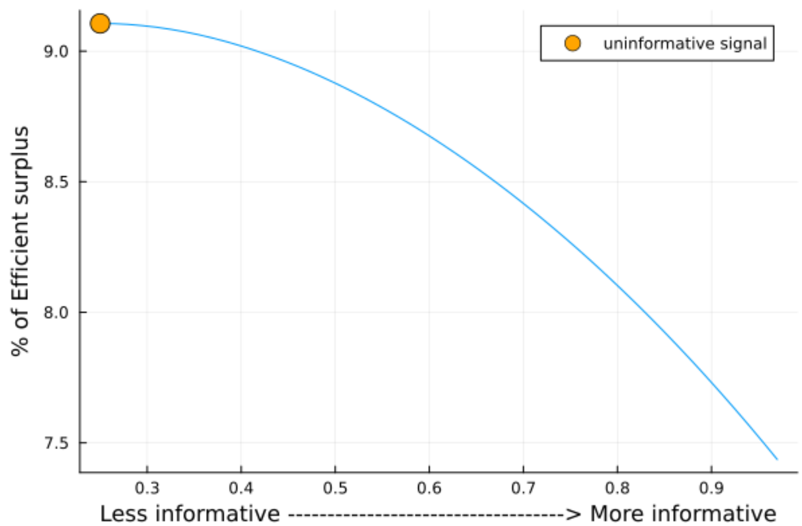


Table 2: Credit Score Equilibrium "Linear Pdf Case"

	Sub-market	θ_L^s	Inefficiency[1]	Surplus[1]
Credit Score	$l = 0$	0.5508	3.3%	26.3%
	$h = 1$	0.50	4.0%	66.4%
	Total	-	7.3%	92.7%
No Credit Score		0.5239	9.1%	90.9%
Efficient Outcome		0.6232	0%	100%

[1] % of aggregate efficient surplus.

from the high score sub-market, but it represents two-thirds of the total surplus. A low score induces the corporation to offer worse terms, increasing the cut-off type accepting a contract. But there are relatively more types around the cutoff, inducing a higher relative inefficiency.

4.2.2 An Extreme Welfare-Reducing Case

While the previous results are encouraging, it is unfortunate that the availability of a strong signal, in the sense of MLRP dominance, cannot guarantee a welfare increase. This is a consequence of the way the type distribution enters the equilibrium objects; that is, only through the zero-profit condition. There is too much leeway for the shape of the distribution of types to generate problems.

While a full numerical results will be presented next, we want to provide a simple intuitive example with a discrete distribution. Assume there are only three types: $\theta_3 = 1.0$ the very good types, some average types θ_2 slightly greater than the lowest socially efficient type θ_P and some bad inefficient types $\theta_1 < \theta_P$. Let the weights on types θ_2 and θ_3 be such that without credit score, the unconditional θ_L is just an $\varepsilon > 0$ bigger than θ_1. This means there is no inefficiency without credit score, just cross-subsidization, as both θ_2 and θ_3 types are socially efficient. Now, introducing some credit score implies $\theta_L^h \theta_L < \theta_L^l$. If it is the case that $\theta_L^h < \theta_1$ then there would be aggregate inefficiency as a few lucky θ_1 types will get the high score and start their projects. In this case, introducing the credit score would be detrimental to welfare.

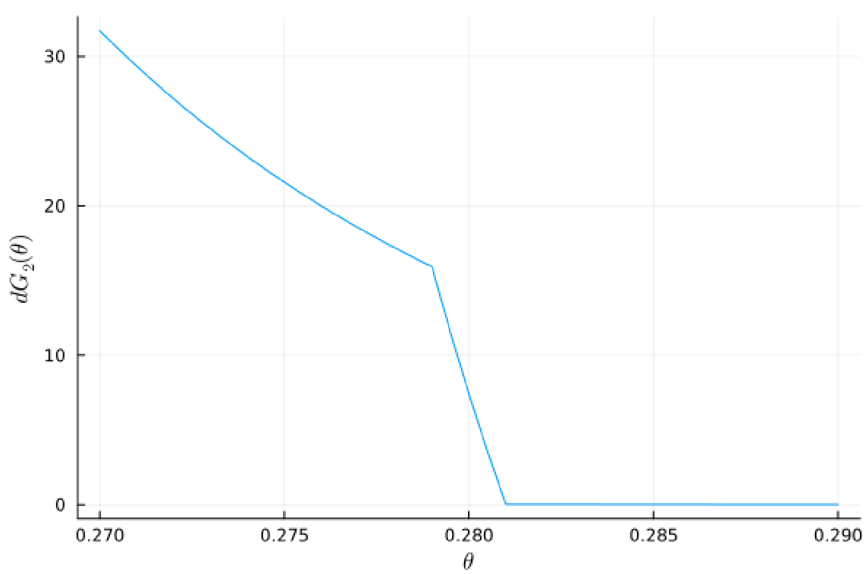
For our numerical example we built a continuous and left-skewed extreme distribution with lots of socially inefficient types. We began with a left-skewed distribution $X(\theta)$. To generate a family, we chose a strictly increasing function $m(\theta)$ with range in $[-1, 1]$ and for each value of the parameter s define a density proportional to a variation of the X distribution as follows:

$$dG_2(\theta|s) \propto \big(1 + (2s - 1) \cdot m(\theta)\big) dX(\theta) \qquad (\theta, s) \in [0, 1] \times [0, 1].$$

This construction generalizes the linear density family used before, which corresponds to $m(\theta) = 2\theta - 1$ and $X(\theta) = \theta$ the uniform distribution. A family built this way has the MLRP property, if $s_a < s_b$ the sign of the likelihood ratio derivative is the same as $(s_a - s_b)m'(\theta)$. Details on the construction of the family, and the linear mixing properties are in Appendix B.1.

We lowered the outside option to make the lowest socially efficient type θ_P to be 0.28. Next we devised the function $m(\theta)$ to be very steep around that same value. This means there are few socially efficient startups while there are many inefficient ones. Figure 15 plots the density around the critical value.

Figure 15: Function $dG_2(\theta|s = 0.0025)$



Under these conditions, the cutoff values are described in the θ_L^s column of table 3. The steep descent in the pdf depicted in Figure 15 begins at $\theta = 0.2790$, which is the cutoff for the high score sub-market θ_L^h. Hence, even though the cutoff for the no credit score equilibrium is $\theta_L = 0.2792$, there is a large concentration of entrepreneurs between these two cuts, relative to those above. Notice that the low score sub-market is way smaller than the high one, as only about 11% of the efficient surplus is generated there.

Table 3: Welfare-Reducing Credit Score Equilibrium

	Sub-market	θ_l^s	Inefficiency[1]	Surplus[1]
Credit Score	$l = 0$	0.2797	2.8%	8.4%
	$h = 1$	0.2790	31.9%	56.9%
	Total	-	34.7%	65.3%
No credit Score		0.2792	33.8%	66.2%
Efficient Outcome		0.28	0%	100%

[1] % of aggregate efficient surplus.

As argued at the start of this sub-section, the *mass* of entrepreneurs just below θ_L causes inefficiency to increase in the high sub-market. The inefficiency was initially small in the low score sub-market due to its size, hence the effect on the high score sub-market dominates, generating the welfare loss.

5 Collateral

As Section 3 shows, for θ high enough, the return of the project for the entrepreneur would be higher if he had access to the risk-free rate. Hence skilled entrepreneurs who own capital are willing to use it on their project. The same will happen if the asset owned is not liquid but is pledgeable: entrepreneurs are willing to pledge their assets if doing so gets them better loan terms. We allow for that possibility in the current section.

Assume entrepreneurs have assets $a \in \mathcal{A}$. Two interpretations are possible, and both will yield the same results : for a project of size k the entrepreneur provides a as capital and the intermediary finances $k - a$ or the intermediary finances k collateralized by Ra from the entrepreneur. In what follows the latter will be used.

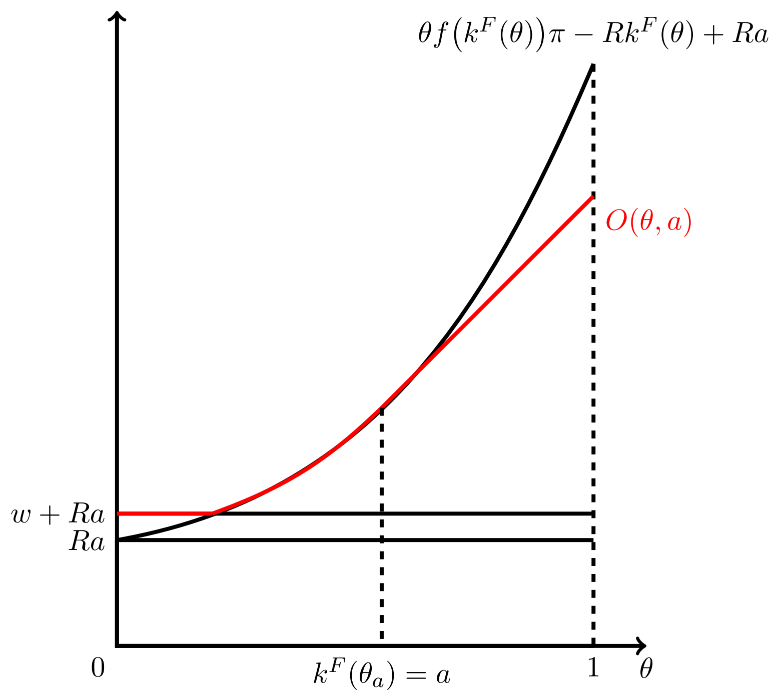
The asset holdings from entrepreneurs serve two purposes: they relax the limited liability constraint for the intermediaries and increase the outside option for the entrepreneurs.

Claim 12. *The outside option $O(\theta, a)$ of an entrepreneur of type θ that holds assets a is characterized* *as follows:*

$$O(\theta, a) = \max\Big\{w + Ra, \theta F\big(\min\{a, k^F(\theta)\}\big)\pi + R\cdot\max\{0, a - k^F(\theta)\}\Big\}$$

The expression for the outside option results from the fact that an entrepreneur of type θ holding a units of capital can work and lend his capital or produce using his skills and own capital investing any excess assets above the optimal capital k^F at the rate R. By the envelope theorem, the line $\theta F(a) - Ra$ is tangent to the surplus curve $S(\theta)$ at the point θ_a uniquely defined by $k^F(\theta_a) = a$. This will be important for the equilibrium contract.

Figure 16: Entrepreneurs' Outside Option with Collateral



As collateral is observable, contract schedules are contingent on a. For each observed collateral level, intermediaries offer sets of contracts. From the intermediaries perspective, the only change in the framework is that now the limited liability condition states $x(\theta) \geq -R \cdot a$.

To facilitate interpretation and map into the intermediaries' valuation, we define the equilibrium payoff of a type θ entrepreneur with assets a be $U(\theta, a) + Ra$, where $U(\theta)$ is the expected utility offered by one of the most preferred contracts. The difference now is that $U(\theta)$ could be negative, as the intermediary can seize the collateral.

Given the notation update, Myerson's lemma and claims A.1 to 2 will still hold for each a, since nothing in their proofs depends on the limited liability condition being exactly zero. With minor changes to the proof we can state:

Proposition 3. *In equilibrium with observable assets, the contract offered by all financial intermediaries* *to almost every* (θ, a) *is:*

$$k^*(\theta,a) = k^F(\theta) = \arg\max_k\{\theta f(k)\pi - Rk\}, \qquad z^*(\theta,a) = \frac{O\big(\theta_L(a),a\big)}{\theta_L(a) f\big(k^F(\theta)\big)}, \qquad x(\theta) = -Ra.$$

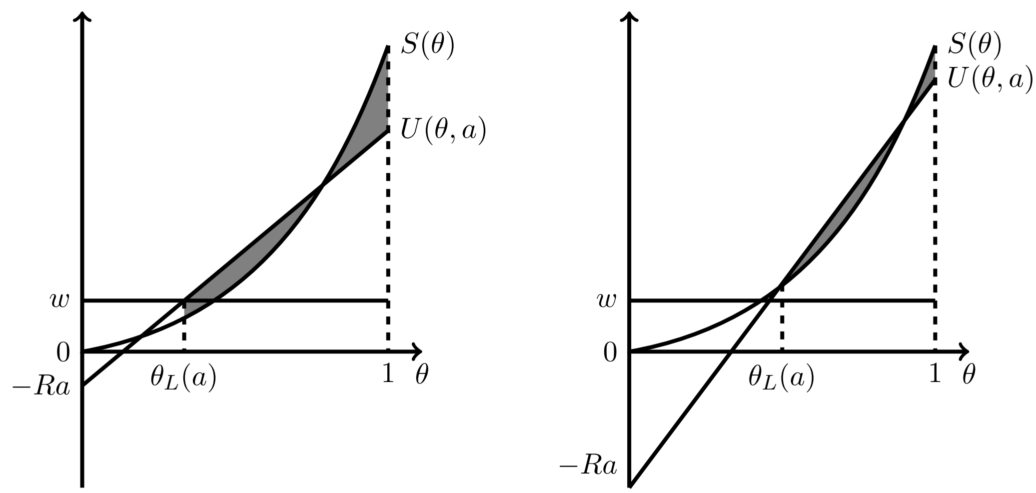
where $\theta_L(a)$*, the lowest* θ *among those entrepreneurs with* a *assets who accepts the contract. For each* a*, it must be the case that:*

$$\int_{\theta_L(a)}^{1} \left\{ \theta f\big(k^F(\theta)\big)\pi - \theta\frac{O(\theta_L(a),a)}{\theta_L} - R\big(k^F(\theta) - a\big) \right\} dG(\theta|a) = 0$$

As before, in all contracts enacted the limited liability constraint binds. Hence the promised expected utility curve $U(\theta, a)$ will be a line with intercept $-Ra$. The following figure illustrates the equilibrium, taking into account that the outside option is not always w.

Figure 17: Equilibrium Contract with Observable Asset Level a

(a) Low a (b) High a



Without assets, we showed the cutoff θ_L was strictly less than the lowest socially efficient type θ_P. With assets, that is not true anymore but a slight variation can be stated:

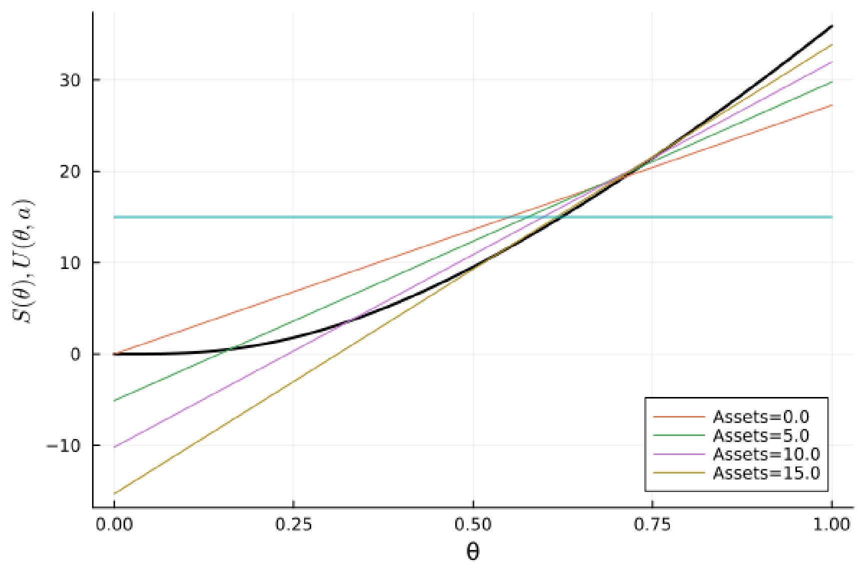
Claim 13. *Let* $\theta_L(a)$ *be the equilibrium cutoff described by proposition 3 when entrepreneurs hold assets* a*. Let* θ_a *be the type whose optimal capital level is* a*, that is* $a = k^F(\theta_a)$*. Then* $\theta_L(a) < \max\{\theta_P, \theta_a\}$*.*

When the asset level is low enough such that $\theta_a < \theta_P$ the equilibrium looks like the Panel A of Figure 17. In the other case, both cases are possible, but for large enough asset holdings the equilibrium must look like Panel B pf Figure 17.

Welfare: Claim 13 implies that as long as $a < k^F(\theta_P)$ there will be socially inefficient projects enacted in equilibrium. If the collateral available is greater than the one required by the lowest socially efficient type, it may be the case that no inefficient project is enacted. However, in that case, without financial intermediaries all efficient projects would be enacted, albeit at an inefficient scale. Hence, if asset holdings are large enough there is not an entry problem to begin with, and introducing financial intermediaries may create one. When the asset level is such that socially efficient holdings cannot be enacted without financing, the introduction of intermediaries does not completely solve the entry problem but only shifts it from under-entry to over-entry.

Next we explore numerically the properties of the equilibrium with assets. Figure 18 shows the equilibrium contract for multiple collateral levels keeping the type distribution constant.

Figure 18: Optimal Contract Under Asset Holdings



The lowest type taking the 0 asset contract is type $\theta = 0.5123$. As collateral becomes available, the corresponding lowest type taking the contract $\theta_L(a)$ also increases. This happens because, as the limited liability always binds, if a sub-market with more collateral had a a lower $theta_L$ the entrepreneurs' surplus would be strictly larger in the high asset sub-market than in the low-asset one. That would then yield negative profits to the financiers, however, since the total surplus depends only on the type distribution.

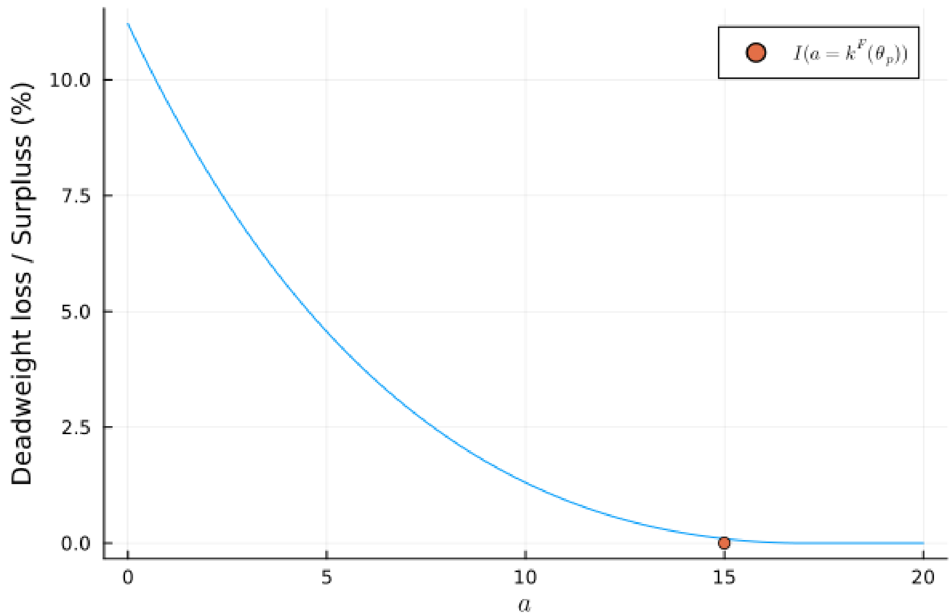
Figure 19 shows that as collateral increases, keeping the distribution constant, the inefficiency is reduced. It also highlights that at the point where $a = k^F(\theta_P)$ the asset holdings inefficiency is low but still positive, becoming zero for assets larger than that.

Finally, asset holdings are not uniformly distributed across entrepreneurs in practice. If entrepreneurs with more collateral were better in the MLRP sense, collateral could be also used as a credit scoring mechanism. That would entail bigger welfare gains, as the credit score part is likely welfare enhancing, and the collateral provision definitely is.

6 Externality to Financiers

The next extension considers the implications of the startup having externalities on intermediaries. For any given financier, whenever there is a contract signed with an entrepreneur, the non-participating

Figure 19: Inefficiency and Asset Distribution



financier(s) will suffer a loss $\pi^e < 0$. In this case, social aggregate surplus from a successful startup Π can be split as $\Pi = \pi + \pi^e$, where π are the profits generated by the startup, and π^e the spillover on other corporations not financing the project.

In the same fashion as in the case without externalities, competition will push intermediaries to offer better contract terms, but now intermediaries may suffer the externality if they lose entrepreneurs, which causes them to offer even better contracts. They will be willing to take some loss as long as it is not worse than taking the negative externality, in equilibrium they will be indifferent between entering the market and losing with failed projects and staying out and losing because of the externality.

Claim 14. *In any equilibrium, financiers will be indifferent between entering the market and staying out and earning the externality.*

In some sense, the externality is changing the financiers' outside option, but their incentives to outbid each other remain as well as their restrictions. In the end the equilibrium will look very similar. In order to characterize it, we define the optimal private investment and surplus:

Definition The private surplus maximizing investment $k^{pr}(\theta)$ is given by:

$$k^{pr}(\theta) = \arg\max_k \{\theta f(k)(\pi) - Rk\}.$$

and π is the profit generated by a successful startup.

Notice that $k^{pr}(\theta)$ is not the socially optimal scale, as it ignores the externality on the other financier.

Claim 15. *Assume in* equilibrium *a type θ entrepreneur obtains investment $k^*(\theta)$ from an intermediary. Then $k^*(\theta) = k^{pr}(\theta)$ for almost all θ*

The proof is exactly the same as in in the case without externalities, Claim 3. Intuitively, the intermediary can always adjust $k^*(\theta)$ and $z^*(\theta)$ proportionally so $U_i(\theta)$ *and incentives* remain constant for *all* types. Then the intermediary would capture the additional surplus. The expected private surplus, not counting opportunity costs or externalities, generated by an entrepreneur of type θ is $S^{pr}(\theta)$, where:

$$S^{pr}(\theta) = \theta f\big(k^{pr}(\theta)\big)(\pi) - Rk^{pr}(\theta).$$

Proposition 4. *In an equilibrium with externalities π^e, there exists a cuttoff θ_L^e such that (almost) every entrepreneur signs a contract $\big(k_e^*(\theta), x_e^*(\theta), z_e^*(\theta)\big)$ if an only if $\theta \geq \theta_L^e$, such that:*

$$k_e^*(\theta) = k^{pr}(\theta) = \arg\max_k\{\theta f(k)(\pi) - Rk\}, \qquad z_e^*(\theta) = \frac{w}{\theta_L^e(\cdot) f\big(k^{pr}(\theta)\big)}, \qquad x_e^*(\theta) = 0.$$

and θ_L^e is such that the corporations are indifferent between entering the market and staying out and earning the externality:

$$\int_{\theta_L^*}^1 \Big\{\theta f\big(k^{pr}(\theta)\big)\pi - Rk^{pr}(\theta)\Big\} dG(\theta) - \int_{\theta_L^e}^1 \theta \frac{w}{\theta_L^e} dG(\theta) = \int_{\theta_L^e}^1 \theta f\big(k^{pr}(\theta)\big)\pi^e dG(\theta). \tag{19}$$

It can be shown that θ_L^e, θ_L when $\pi^e < 0$. This means that if there were other financiers not affected by the externality they would be outbid by those affected. This may explain why many corporations are entering the entrepreneurial finance world, a model known as Corporate Venture Capital.

To show θ_L^e, θ_L when $\pi^e < 0$, just rearrange equation (19) to obtain:

$$\int_{\theta_L^e}^1 \Big\{\theta f\big(k^{pr}(\theta)\big)(\pi - \pi^e) - Rk_c^{pr}(\theta)\Big\} dG(\theta) - \int_{\theta_L^e}^1 \theta \frac{\tilde{\pi}}{\theta_L^e} dG(\theta) = 0$$

and, as $\pi^e < 0$, comparing this with the equation that defines θ_L, (equation 16) the result follows.

6.1 Optimal Social Allocation and Welfare with Externalities

With the equilibrium found in the previous section, we delve into its welfare implications. In order to understand such phenomena, we carefully define the utilitarian planner value for each possible allocation and identify the first best allocation.

For any given type of entrepreneur θ that accepts a contract from a given intermediary, the agents' expected benefits are:

$\theta f(k) z + x$	Entrepreneur
$\theta f(k) \cdot (\pi - z) - x - R \cdot k$	Financing Intermediary
$\theta f(k) \pi^e$	Non-financing intermediary.

As the planner is utilitarian, and z, x are just inner transfers between agents, they are not considered by the planner. This means that the social planner will optimize the set of investments and the allocation sets between types of intermediaries. We first define a generic allocation for this planner.

Definition An allocation for this economy planner is a tuple $\mathcal{L} = \{A, k\}$ where A is the set of entrepreneurs that take investments from any intermediary and $k : \Theta \to \mathbb{R}$ is the function assigning investment for each type $\theta \in A$.

For any given allocation $\mathcal{L}$ the social planner will get net value added surplus, adding the externalities induced by the number of corporations in this market,

$$\mathcal{V}(A, k) = \int_A \Big\{\theta f\big(k(\theta)\big) \cdot \Big[\pi + \pi^e\Big] - R \cdot k(\theta) - w\Big\} dG(\theta)$$

, and the planner will get $\int_0^1 w dG(\theta)$ from all the entrepreneurs.

The Social Planner will optimize:

$$\max_{k, A \subseteq [0,1]} \int_A \Big\{\theta f\big(k(\theta)\big) \cdot \Big[\pi + \pi^e\Big] - R \cdot k(\theta)\Big\} + \int_{[0,1] \setminus A} w dG(\theta) \tag{20}$$

Where $[0, 1] \setminus A$ is the set of entrepreneurs that do not get any investment.

First-Best Allocation

To maximize the planner's objective (20), the planner must maximize pointwise inside the first integral and then choose the set A comparing both integrals point by point. Define the optimal investment $k^s(\theta)$ as:

$$k^s(\theta, \pi, \pi^e) = \arg\max_k \theta f(k) \cdot (\pi + \pi^e) - Rk.$$

The gross social surplus from an entrepreneur of type θ is denoted $\mathcal{S}(\theta, \pi, \pi^e)$, where

$$\mathcal{S}(\theta, \pi, \pi^e) = \theta f(k^s)(\pi + \pi^e) - R \cdot k^s(\theta).$$

The socially optimal set of entrepreneurs enacting a project A^s corresponds to those types whose gross social surplus is equal to or greater than the outside option, that is:

$$A^s = \Big\{\theta \in \Theta : \mathcal{S}(\theta, \pi, \pi^e) \geq w\Big\}.$$

As $\mathcal{S}(0, \pi, \pi^e) = 0$ and $\mathcal{S}(\cdot)$ is strictly increasing in θ, the socially optimal set will be of the form $A^s = [\theta_P(\pi, \pi^e), 1]$, where $\theta_P(\pi, \pi^e)$ is the lowest type entrepreneur whose gross social surplus is equal to the outside option, that is $\mathcal{S}\big(\theta_P(\cdot), \pi, \pi^e\big) = w$.

Welfare

The total net social surplus generated by the socially optimal allocation denoted as $\mathcal{V}_S$ is then:

$$\mathcal{V}_S(\Pi_s) = \int_{\theta_s(\cdot)}^1 \big(\mathcal{S}(\theta, \pi, \pi^e) - w\big) dG(\theta).$$

In the same fashion as in previous sections, the relative inefficiency of any other allocation compares its total net surplus against the optimal net social surplus. For any given allocation $\mathcal{L}' = (A', k')$, the relevant inefficiency is defined as:

$$I(A', k') = 1 - \frac{\mathcal{V}(A', k')}{\mathcal{V}_S}$$

In this framework there are two possible sources of inefficiency:

- **Entry:** The set A may differ from A^s, which implies some socially inefficient projects are enacted.
- **Scale:** The projects enacted may not be run at their efficient scale, which happens when the investment is not $k^s(\theta)$.

Given these two sources, we can split total relative inefficiency as:

$$I(A', k') = \underbrace{\frac{\mathcal{V}_S - \mathcal{V}(A', k^s)}{\mathcal{V}_S}}_{Entry} + \underbrace{\frac{\mathcal{V}(A', k^s) - \mathcal{V}(A', k')}{\mathcal{V}_S}}_{Scale}$$

In the equilibrium without externalities characterized in Section 3, it is shown that all projects are enacted at their privately optimal scale, which is equal to the socially optimal since there is no externality. Hence the inefficiency comes only from the entry term, and there is an over-financing problem, where inefficient entrepreneurs will get financing.

In an equilibrium with externalities an additional source of inefficiency appears. The scale at which projects are enacted is not the socially optimal one as private investment does not take into account the externality, that is $k^{pr}(\theta) \neq k^s(\theta)$.

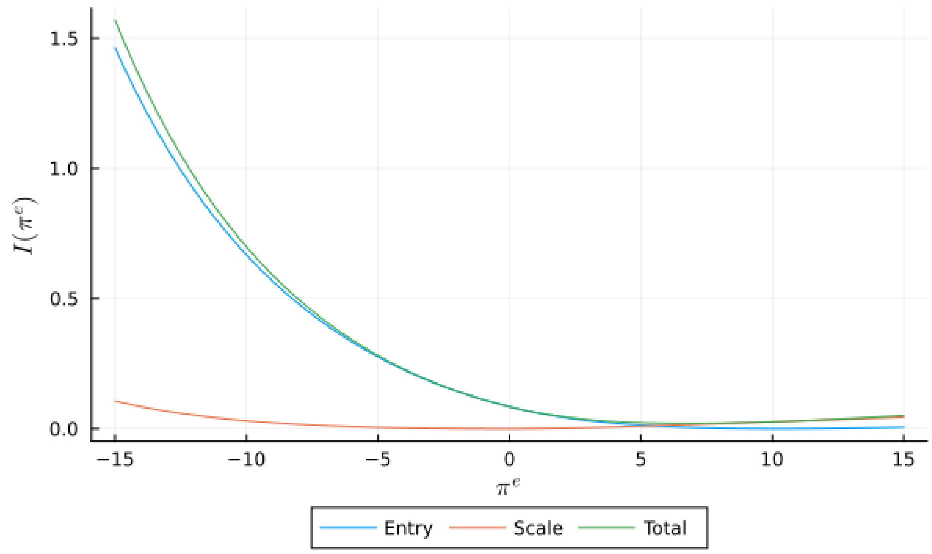
Next we numerically explore the total relative inefficiency and its decomposition for the equilibrium allocation with externalities. We keep the functional forms from Section 3.3 and the parameters set in Table 1. The only new term is the externality, which we vary from -15 to 15. Notice this is small relative to the total profits of 100.

Figure 20 shows a significant loss in total net social surplus when negative externalities are present. Notice the scale is *not* in percentage, but in ratio, hence an inefficiency greater than 1.0 means the total net surplus in equilibrium was negative, i.e., society would have been better without any financing and all entrepreneurs working.

The negative welfare from entrepreneurial finance happens for externalities less than -12, or 12% of the expected startup surplus, which is a small value. If we had used the linear type distribution described in Section 4.2, the negative net social surplus appears for externalities less than -7.5 (7.5% of the possible value of the project).

In the Schumpeterian growth model, new ideas are marginal improvements that steal the whole market, that is $\pi^e = \pi - \varepsilon$. Even in growth models of the Romer style, new ideas generate new intermediate outputs that erode the oligopoly power of incumbents, generating a negative externality on

Figure 20: Inefficiency Decomposition with Externalities ($\pi = 100$)



them. Still, a small externality can cause incumbents to enter the entrepreneurial finance market and significantly reduce if not completely erase the gains from innovation.

7 Conclusion

Over the last two decades, a transformation has unfolded in the entrepreneurial finance landscape, characterized by financiers adopting a "spray-and-pray" strategy, where they support a multitude of start-ups in anticipation that successes will compensate for failures.

In this paper we proposed a model that captures this environment, characterizing financial contracts within a competitive setting, incorporating elements of risk, adverse selection, and limited liability, while acknowledging the inherent heterogeneity of start-ups. Our analysis reveals an intriguing outcome: in equilibrium, inefficient projects (both from private and social standpoints) are enacted. This result is a consequence intricately woven by the interplay of asymmetric information, limited liability constraints, and market competition.

Furthermore, our paper explores various extensions of the basic environment. When firms can partially collateralize credit, inefficiencies persist unless an unsecured fraction remains. The introduction of imperfect information, such as credit scores, has the potential to exacerbate these inefficiencies. Most notably, a minor externality on financiers amplifies extensive margin inefficiencies to the extent of yielding a negative aggregate social surplus in the entrepreneurial financing market.

Overall, our results highlight a potentially detrimental effect of the "spray-and-pray" paradigm in financing ventures: an excess of long-shot start-ups, which could harm welfare. Simultaneously, our results rationalize the necessity of regulatory intervention to alleviate these market frictions. The intricate balance between risk, adverse selection, and limited liability in entrepreneurial finance necessitates a nuanced regulatory approach to foster a more efficient and socially beneficial ecosystem.

References

Ates, S. T. and Saffie, F. E. (2021). Fewer but better: Sudden stops, firm entry, and financial selection. *American Economic Journal: Macroeconomics*, 13(3):304–356.

Besanko, D. and Thakor, A. V. (1987). Collateral and rationing: sorting equilibria in monopolistic and competitive credit markets. *International economic review*, pages 671–689.

Bester, H. (1985a). The level of investment in credit markets with imperfect information. *Zeitschrift für die gesamte Staatswissenschaft/Journal of Institutional and Theoretical Economics*, pages 503–515.

Bester, H. (1985b). Screening vs. rationing in credit markets with imperfect information. *The American Economic Review*, 75(4):850–855.

Dasgupta, P. and Maskin, E. (1986). The existence of equilibrium in discontinuous economic games, i: Theory. *The Review of economic studies*, 53(1):1–26.

de Meza, D. and Webb, D. C. (1987). Too much investment: A problem of asymmetric information. *The Quarterly Journal of Economics*, 102(2):281–292.

Ewens, M., Nanda, R., and Rhodes-Kropf, M. (2018). Cost of experimentation and the evolution of venture capital. *Journal of Financial Economics*, 128(3):422–442.

Ghatak, M., Morelli, M., and Sjostrom, T. (2007). Entrepreneurial talent, occupational choice, and trickle up policies. *Journal of Economic Theory*, 137(1):27–48.

Innes, R. (1990). Limited liability and incentive contracting with ex-ante action choices. *Journal of Economic Theory*, 52(1):45–67.

Jovanovic, B. and Szentes, B. (2013). On the market for venture capital. *Journal of Political Economy*, 121(3):493–527.

Kaplan, S. N. and Stromberg, P. (2001). Venture capitalists as principals: Contracting, screening, and monitoring. *The American Economic Review*, 91(2):426–430.

Khanna, N. and Mathews, R. D. (2022). Skill versus reliability in venture capital. *Journal of Financial Economics*, 145(2, Part A):41–63.

Lester, B., Shourideh, A., Venkateswaran, V., and Zetlin-Jones, A. (2019). Screening and adverse selection in frictional markets. *Journal of Political Economy*, 127(1):338–377.

Levine, R. (2005). Finance and Growth: Theory and Evidence. In Aghion, P. and Durlauf, S., editors, *Handbook of Economic Growth*, volume 1 of *Handbook of Economic Growth*, chapter 12, pages 865–934. Elsevier.

Martimort, D. and Stole, L. (2002). The Revelation and Delegation Principles in Common Agency Games. *Econometrica*, 70(4):1659–1673.

Martin, A. (2009). A model of collateral, investment, and adverse selection. *Journal of Economic Theory*, 144(4):1572–1588.

Milde, H. and Riley, J. G. (1988). Signaling in credit markets. *The Quarterly Journal of Economics*, pages 101–129.

Riley, J. G. (1975). Competitive signalling. *Journal of Economic Theory*, 10(2):174–186.

Rosenthal, R. W. and Weiss, A. (1984). Mixed-strategy equilibrium in a market with asymmetric information. *The Review of Economic Studies*, 51(2):333–342.

Rothschild, M. and Stiglitz, J. (1992). *Equilibrium in competitive insurance markets: An essay on the economics of imperfect information*. Springer.

Sahlman, W. A. (1990). The structure and governance of venture-capital organizations. *Journal of Financial Economics*, 27(2):473–521.

Samila, S. and Sorenson, O. (2011). Venture capital, entrepreneurship, and economic growth. *The Review of Economics and Statistics*, 93(1):338–349.

Scheuer, F. (2013). Adverse selection in credit markets and regressive profit taxation. *Journal of Economic Theory*, 148(4):1333–1360.

Stiglitz, J. E. and Weiss, A. (1981). Credit rationing in markets with imperfect information. *The American economic review*, 71(3):393–410.

A Proofs

A.1 Base Model Proofs

The following is a useful consequence of lemma 1.

Lemma A.1. *In any IC contract schedule, $x(\theta)$ is non-increasing.*

Proof. Let $\theta' > \theta$ from equation (11c) it follows that:

$$x(\theta) = U(0) + \int_0^{\theta} \Big[f\big(k(s)\big)z(s) - f\big(k(\theta)\big)z(\theta)\Big]ds \quad \text{then,}$$

$$x_i(\theta') - x(\theta) = (\theta' - \theta)\Big[f\big(k(\theta)\big)z(\theta) - f\big(k(\theta')\big)z(\theta')\Big] + \int_{\theta}^{\theta'} \Big[f\big(k(s)\big)z(s) - f\big(k(\theta')\big)z(\theta')\Big]ds$$

And both terms in the last equation are negative or zero because of equation (11b) hence the claim holds. □

In any equilibrium, profit maximization by intermediaries implies that, when indifferent among multiple contracts inside a financier's menu, (almost) all entrepreneurs give positive probability only to contracts that maximize joint surplus, which are the contracts most preferred by the financier among those delivering the maximum utility level for the entrepreneur.

Lemma A.2. *Let $\mathcal{C}_i$ be a given set of contracts, U_i be its corresponding entrepreneur utility function from equation* (9) *and for each θ let $\big(k_i(\theta), x_i(\theta), z_i(\theta)\big)$ be a contract of $\mathcal{C}_i$ that attains the maximum in equation* (9) *and $k(\theta) > 0$. For every $\varepsilon > 0$ there exists a contract set $\mathcal{C}_i^{\varepsilon}$ such that for each θ there is a unique maximizer $\big(k_i^{\varepsilon}(\theta), x_i^{\varepsilon}(\theta), z_i^{\varepsilon}(\theta)\big)$ such that for all θ: $k_i^{\varepsilon}(\theta) = k_i(\theta)$, $U(\theta; \mathcal{C}_i^{\varepsilon}) > U(\theta; \mathcal{C}_i)$ and*

$$\int_{\theta\in\Theta} \big[U(\theta; \mathcal{C}_i^{\varepsilon}) - U(\theta; \mathcal{C}_i)\big]dG(\theta) < \varepsilon.$$

Proof. By construction, let $\delta > 0$ be such that $\delta < \varepsilon\big[\int_{\theta\in\Theta}(1+\theta^2)dG(\theta)\big]^{-1}$. The contract set $\mathcal{C}_i^{\varepsilon}$ is the set $\big\{(k_i^{\varepsilon}(\theta), x_i^{\varepsilon}(\theta), z_i^{\varepsilon}(\theta))|\theta \in \Theta\big\}$, where:

$$k_i^{\varepsilon}(\theta) = k_i(\theta) \qquad z_i^{\varepsilon}(\theta) = z_i(\theta) + \frac{\delta\theta}{f\big(k_i(\theta)\big)} \qquad x_i^{\varepsilon}(\theta) = x_i(\theta) + \frac{\delta}{2}(1-\theta^2). \tag{21}$$

If an entrepreneur of type θ takes the contract $\big(k_i^{\varepsilon}(\hat\theta), z_i^{\varepsilon}(\hat\theta), x_i^{\varepsilon}(\hat\theta)\big) \in \mathcal{C}'$, her payoff will be:

$$\theta f(k_i^{\varepsilon}(\hat\theta))z_i^{\varepsilon}(\hat\theta) + x_i^{\varepsilon}(\hat\theta) = \theta f(k_i(\hat\theta))z_i(\hat\theta) + x_i(\hat\theta) + \delta\hat\theta\theta + \frac{\delta}{2} - \frac{\delta\hat\theta^2}{2}.$$

By the assumption on $\big(k_i(\theta), x_i(\theta), z_i(\theta)\big)$ attaining the maximum, The first three terms are *weakly* maximized at $\hat\theta = \theta$. The last three terms $\frac{\delta}{2} + \delta\hat\theta\theta - \delta\hat\theta^2/2$ are uniquely maximized at $\hat\theta = \theta$ which ensures the maximizer is *unique*. The resulting payoff is $U_i^{\varepsilon}(\theta) = U_i(\theta) + \frac{\delta}{2}\big(1+\theta^2\big) > U_i(\theta)$. Thus every entrepreneur strictly prefers the intended contract to any other contract in either $\mathcal{C}_i$ or $\mathcal{C}_i^{\varepsilon}$. Also,

$$\int_{\theta\in\Theta} \big[U(\theta; \mathcal{C}_i^{\varepsilon}) - U(\theta; \mathcal{C}_i)\big]dG(\theta) = \int_{\theta\in\Theta} \big[\frac{\delta}{2}(1+\theta^2)\big]dG(\theta) = \frac{\delta}{2}\int_{\theta\in\Theta}(1+\theta^2)dG(\theta) < \varepsilon,$$

by the assumption on δ. Notice that this lemma can be extended to the case $k(0) = 0$ by defining $z_i^{\varepsilon}(\theta) = z_i(\theta)$. □

While useful for breaking ties inside an intermediary's own contract set, in equilibrium lemma A.2 has some issues. Increasing utility even slightly may strongly entice entrepreneurs to switch from working or from other intermediaries, causing discrete losses for the first intermediary. Since the mass of indifferent entrepreneurs could be very large, the approach will be to take over the whole market, using the fact that other intermediaries do not operate at a loss in equilibrium. This requires matching the other intermediary's contracts and then spending ε to force a favorable tiebreak. If any of the intermediaries were making positive profits, that would yield a profitable deviation, thus in equilibrium intermediaries must make zero profits by entering the market. That is the basis of the zero profit claim (2) proof that follows next:

Proofs of Slaims 2 and 3: Financiers' Zero Profits and Optimal Scale

Proof. By contradiction, assume intermediary 2 is making profits $M > 0$ by entering, we will show intermediary 1 has a profitable deviation. The deviation is basically for intermediary 1 to add all contracts in $\mathcal{C}_2$ to his menu: $\hat{\mathcal{C}}_1 = \mathcal{C}_1 \cup \mathcal{C}_2$. This is not necessarily enough since entrepreneurs may keep choosing the contracts offered by intermediary 2 but, by applying lemma A.2 to $\hat{\mathcal{C}}_1$, intermediary 1 can spend $\varepsilon << M$ of the profits ensuring all entrepreneurs strictly prefer one of his contracts, capturing $M - \varepsilon$ additional profits.

There are two technical details left to address. First, the possibility that for some θ all contracts maximizing equation (10) have $k(\theta) = 0$. Any contract in $\hat{\mathcal{C}}_1$ such that $k(\theta) = 0$ has $z(\theta) = 0$ by assumption, hence the intermediary may include a new contract with the surplus maximizing capital $k(\theta) = k^F(\theta)$ defined in equation (12). That does not change incentives for any entrepreneur since $f(k)z$ still is zero. If anything, it would increase total surplus, of which the intermediary is the residual claimant. In fact the same contract addition can be made for all contracts with $z(\theta) > 0$, in this case new contract has $k^F(\theta)$ and the new $\hat{z}(\theta)$ is such that $F\big(k^F(\theta)\big)\hat{z}(\theta) = F\big(k(\theta)\big)z(\theta)$.

Second, by using lemma A.2 and increasing expected utility for all θ by some $\frac{\delta}{2}(1+\theta^2) \le \delta$ a non-negligible mass of potential entrepreneurs that were not enacting projects may decide to do so, hurting profits. These are those entrepreneurs such that $w - \frac{\delta}{2}(1+\theta^2) \le U(\theta) \le w$, and they can accrue losses of at most $w + \delta$ to the financiers.

If the set $\{\theta : U(\theta) = w\}$ has G-measure zero, it has at most two points since $U(\theta)$ is convex. If it is empty, there is no new entrant problem, as all were entering. If non-empty, by claim 1 for any selection of maximizers $f\big(k(\theta)z(\theta)$ is non-decreasing, its left and right limits exist at every point and these are the left and right derivatives of $U(\theta)$. If any of the side derivatives is zero at $U(\theta) = w$ then U attains a minimum there and almost all entrepreneurs are entering, voiding the problem. Otherwise, $U(\theta)$ is locally invertible at $U(\theta) = w$, and as δ shrinks, the pre-image $U^{-1}([w-\delta, w])$ shrinks as well, bounding the mass of new entrants.

If the set $\{\theta : U(\theta) = w\}$ has positive measure, it is an interval of the form $[\theta_L^l, \theta_L^h] \in [0,1]$. For every $\theta \in (\theta_L^l, \theta_L^h)$, the contracts attaining $U(\theta) = w$ must have $x(\theta) = w$ and $f\big(k(\theta)\big)z(\theta) = U'(\theta) = 0$ which implies $k(\theta) = k^F(\theta)$ and $z(\theta) = 0$. Let θ_P be the lowest type that enacts a project in a full information equilibrium, which means the gross surplus at the full information capital is equal to the outside option $S(\theta_P) = w$. If $\theta_L^l \geq \theta_P$ then $S(\theta_L^l) \geq S(\theta_P) = w$ and there is no problem with potential new entrants. Since they enact profitable projects, the potential additional losses to the financier are still bounded by $\frac{\delta}{2}(1 + \theta^2)$.

If $\theta_L^h < \theta_P$, then all entrepreneurs with $U(\theta) = w$ entail losses to the intermediaries, the profitable deviation should exclude their contracts. Let $U(\theta_P) = u_P > w = U(\theta_L^h)$ and take any contract in $\mathcal{C}_1 \cup \mathcal{C}_2$ attaining $\theta_P f(k)z + x = u_P$, it must have a strictly positive slope $f(k)z = \gamma > 0$ since $\theta_L^h f(k)z + x \leq w$. The deviating contract set contains only those contracts where the slope is at least γ. That is:

$$\hat{\mathcal{C}}_1 = \big\{(k,x,z) \in \mathcal{C}_1 \cup \mathcal{C}_2 : f(k)z \geq \gamma\big\}.$$

By lemma 1 the maximizer contracts attaining the same $U(\theta)$ for all $\theta \geq \theta_P$ are still included. Also, $\theta < \theta_P$ and any maximizer in $\hat{\mathcal{C}}_1$ must have $f(k)z = \gamma$ and $U(\theta; \hat{\mathcal{C}}_1) = u_P - \gamma(\theta_P - \theta)$. In addition, since all contracts were previously available, for all $\theta < \theta_L^h$, $\theta f(k)z + x < \theta_L^h f(k)z + x \leq w$. The only possible issue is that for $\theta \in [\theta_L^h, \theta_P]$ the set $\hat{\mathcal{C}}_1$ excluded their preferred contracts and some of them still enter and have to operate with a different capital. To address that, for each $\theta < \theta_P$ the contract that attains $U(\theta; \hat{\mathcal{C}}_1) = u_P - \gamma(\theta_P - \theta)$ with slope γ but uses the full information capital. That is:

$$\hat{\mathcal{C}}_1 = \big\{(k,x,z) \in \mathcal{C}_1 \cup \mathcal{C}_2 : f(k)z \geq \gamma\big\} \cup \left\{\left(k^F(\theta), u_P - \gamma\theta_P, \frac{\gamma}{f\big(k^F(\theta)\big)}\right) : \theta \in [0, \theta_P]\right\},$$

then this case reduces to the one where the set $\{\theta : U(\theta; \hat{\mathcal{C}}_1) = w\}$ has measure zero.

Finally, if $\theta_P \in (\theta_L^l, \theta_L^h]$ then the deviation contract set must guarantee only those with $\theta < \theta_P$ are excluded and that the others enter for sure. For every small $\gamma_0 > 0$ let $\theta_\gamma = \max\{\theta : \forall x < \theta, U'(x) \leq \gamma_0\}$. Notice that θ_γ could be 1 if the (left and right) derivatives $U'(\theta)$ are less than γ_0 for all θ, also $\theta_\gamma \geq \theta_L^h$ since In this case $U'(\theta) = 0$ for all $\theta \in (\theta_L^l, \theta_L^h)$. Define the deviation contract set as:

$$\hat{\mathcal{C}}_1 = \big\{(k,x,z) \in \mathcal{C}_1 \cup \mathcal{C}_2 : f(k)z \geq \gamma_0\big\} \cup \left\{\left(k^F(\theta), w - \gamma_0\theta_P, \frac{\gamma_0}{f\big(k^F(\theta)\big)}\right) : \theta \in [0, \theta_\gamma]\right\},$$

in the same way as the previous case, the old maximizer contracts attaining $U(\theta)$ for all $\theta > \theta_\gamma$ are included in the first set. The second set of contracts is such that the utility they promise to entrepreneurs is linear in θ, has slope γ_0 and $U(\theta_P = w$, which means no takers for $\theta < \theta_P$. Under the original contract menus, all entrepreneurs with productivity θ_P or above attain utility at least w, under the new contract set $\hat{\mathcal{C}}_1$ they may earn at most an extra $\gamma_0(\theta - \theta_P)$ which is bounded by γ_0. Also as before, since all contracts in the first set have positive slope and were available before, for all $\theta < \theta_L^l$ the utility provided by contracts in that set is less than w, hence they do not take any of the contracts in $\hat{\mathcal{C}}_1$. Hence by applying lemma A.2 on $\hat{\mathcal{C}}_1$ the additional profits are $M - \epsilon - \gamma_0$, constituting a profitable deviation. $\square$

As a corollary from the previous proof, profit maximization implies (almost) all signed contracts with $z(\theta) \geq 0$ must have $k(\theta) = k^F(\theta)$, otherwise there would be a strictly profitable deviation, the profits would come not from taking over the market share of the other intermediary but from having all entrepreneurs produce at their efficient scale. To show all projects are enacted at their optimal scale (claim 3), we are left to prove no contract with negative slope is signed in equilibrium.

Lemma A.3. *In equilibrium, (almost) all signed contracts have $z \geq 0$.*

Proof. By contradiction, assume there is some contracts signed in equilibrium with $z < 0$. By lemma 1 the contract slope $f\big(k(\theta)\big)z(\theta)$ is non-decreasing in θ. Let θ^- be the least upper bound of types signing a contract with negative z. It follows that $U(\theta^-) \geq w$ and $U(\theta)$ is strictly decreasing below θ^- thus if any entrepreneur signs a contract with $z < 0$ all lower types would do as well (the outside option is non-decreasing). If $\theta^- = 0$ then almost all contracts have $z \geq 0$ and w.l.o.g we can assume all do.

If $S(\theta^-) \leq w \leq U(\theta^-)$, (almost) all contracts with negative z that are signed accrue losses to an intermediary. Since no potentially profitable contracts has $z < 0$, dropping all of the negative z contracts strictly increase potential profits for at least one intermediary, by getting rid of the positive mass of entrants accruing losses. To assure a strict increase in profits an intermediary must, as in the proof of the zero profit claim (Claim 2), take over the whole market and use the tie-breaking lemma (Lemma A.2) on a (probably) pruned set of contracts to control for potential new entrants, but this can be done at a negligible cost.

If $S(\theta^-) > U(\theta^-) \geq w$, since $S(\cdot)$ in increasing and $U(\cdot)$ is decreasing there is some θ_E such that $S(\theta_E) = U(\theta_E) = u_E > w$. Notice both the left and right derivatives of U at θ_E are negative. Take any contract attaining u_E for θ_E, the financier not offering that contract has a profitable deviation. She can throw her original contracts and offer $z = 0$, $x = u_E$ and their full information capital to all entrepreneurs. No entrepreneur with $\theta < \theta_E$ would take her new contracts, since the original contract attaining u_E has negative slope it gives strictly higher utility than u_E to all of them. On the other hand, any entrepreneur with $\theta > \theta^-$ would accrue strictly positive profits for the intermediary if he takes the contract. At least all $\theta \in (\theta_E, \theta^-)$ strictly prefer the new contracts since $U(\cdot)$ is strictly decreasing in that positive mass interval. Ties inside the new contracts can be broken using lemma A.2. In this case there is no entry problem. □

Combined, these results imply in equilibrium (almost) all entrepreneurs do not randomize inside a financier's contract set. Given a utility level they must pick among contracts with $k = k_F(\theta)$. If there are multiple, these contracts must differ on x and z but attain the same utility level, which implies there is a minimum and a maximum value of $f(k)z$ among them. Then the left derivative of $U(\cdot)$ is the minimum of these $f(k)z$ and the right derivative is the maximum. Hence the multiplicity happens only at the discontinuity points of the derivative of $U(\cdot)$ (kinks), which are countable.

Proof of Claim 4: Increasing Profits by Type

Proof. Suppose $U(\theta') \geq S(\theta')$ for some $\theta' > \hat{\theta}$ such that $k(\theta') = k^*(\theta')$ are not positive. By the envelope theorem $S'(\theta) = f\big(k^*(\theta)\big)\pi$, which is increasing in θ because f is increasing and concave. Hence $S(\theta)$ is convex. Remembering that $S(0) = 0 \leq x(\theta')$, by incentive compatibility:

$$U(\hat{\theta}) \geq \hat{\theta} f\big(k^*(\theta')\big) z(\theta') + x(\theta') = \frac{\hat{\theta}}{\theta'} U(\theta') + \frac{\theta' - \hat{\theta}}{\theta'} x(\theta') \geq \frac{\hat{\theta}}{\theta'} S(\theta') + \frac{\theta' - \hat{\theta}}{\theta'} S(0) > S(\hat{\theta})$$

contradicting $S(\hat{\theta}) > U(\hat{\theta})$. As $k(\theta') = k^*(\theta')$ for almost all θ', the lemma follows. □

Proof of Claim 5

Proof. As $z \geq 0$ in equilibrium, $U(\theta)$ is non-decreasing. As discussed in the proof of claim 2, the pre-image $U^{-1}(w)$ is either empty, the θ_L or an interval $[0, \theta_L^h]$, as $f(k)z$ is both non-negative and non-decreasing. If empty and $U(0) > w$, then $\theta_L = 0$; if $U(1) > w$ there is a profitable deviation with small slope through θ_P. Next, we will show $U^{-1}(w) = [0, \theta_L^h]$ a contradiction.

If $\theta_P < \theta_L^h$ we will show there is a profitable deviation. The construction is similar to that in the proof of claim 2: a set of contracts with $\varepsilon > 0$ slope through θ_P. Let $0 < \gamma < f\big(k^F(\theta_P)\big)\pi$ and consider the contract set:

$$\hat{\mathcal{C}}_1 = \left\{ \left(k^F(\theta), w - \gamma\theta_P, \frac{\gamma}{f\big(k^F(\theta)\big)} \right) : \theta \in [0,1] \right\},$$

this set offers $U_1(\theta, \hat{\mathcal{C}}_1) = w + \gamma(\theta - \theta_P)$ which means no type lower than θ_P would enter, and any taker will yield profits to the financier. In particular, the positive mass of entrepreneurs in $(\theta_P, \theta_L^h]$ will strictly prefer a contract in $\hat{\mathcal{C}}_1$ above any of the older ones that offered w at best, yielding the strictly positive profit to the intermediary after tie-breaking (lemma A.2).

If $\theta_L^h \leq \theta_P$, the financiers would make losses with any entrepreneur of type $\theta < \theta_L^h$ that takes the contract. All these entrepreneurs are indifferent, but if a positive mass M of them chooses to start their project, there is a profitable deviation removing all contracts with slope $f(k)z$ less than some small $\gamma_0 > 0$. If $U(\theta_P) > w$, γ_0 can be the left derivative $U^{'-}(\theta_P)$, which removes only negative profit contracts to the entrepreneur. When $U(\theta_P) = w$ then $\theta_L^h = \theta_P$ and removing contracts with small but positive slope may exclude some profitable entrepreneurs, but as financiers' net profits are zero for θ_P, γ_0 can be chosen small enough such that the potential losses from excluding these entrepreneurs are less than the losses on the mass M of low types taking the contract. □

Proof of Claim 6: No Fixed Payment in Equilibrium ($x = 0$)

As in lemma A.3, the key is to build a profitable deviation set of contracts with a higher slope. Before proving the claim, a lemma is useful:

Lemma A.4. *Let $U(\theta)$ be the expected utility for entrepreneurs from equation* (10)*. Then $\theta^{-1}U(\theta)$ is non-increasing and strictly decreasing when $x^*(\theta) > 0$.*

Proof. Let two entrepreneurs with types $\theta_1 < \theta_2$ and $x(\theta_2)$ be the highest x among those contracts attaining $U(\theta_2)$ for entrepreneur 2. As entrepreneur 1 weakly prefers his contract to the one intended for entrepreneur 2:

$$U(\theta_1) \geq \frac{\theta_1}{\theta_2}\big(U(\theta_2) - x(\theta_2)\big) + x(\theta_2) = \frac{\theta_1}{\theta_2}U(\theta_2) + x(\theta_2)\Big[1 - \frac{\theta_1}{\theta_2}\Big] \geq \frac{\theta_1}{\theta_2}U(\theta_2),$$

and the last inequality follows from $x(\theta_2) \geq 0$ and is strict when $x(\theta_2) > 0$. $\square$

Now the claim's proof:

Proof. Let $\mathcal{C}_1^*, \mathcal{C}_2^*, s^*$ be an equilibrium with corresponding payoffs $v_1^*,\ v_2^*$, and entrepreneur expected utility function $U^*(\theta)$. Claim 4 implies there is a type θ_E such that $U^*(\theta_E) = S(\theta_E) > w$, otherwise contracts would either all yield profits or all yield losses. Let $\tilde{\theta}$ be the least upper bound of those types such that one of the contracts in $\mathcal{C}^*$ that attains $U^*(\theta)$ has $x > 0$. Let the (almost unique) incentive compatible schedule (maximizer contracts) be $k^*(\theta), x^*(\theta), z^*(\theta)$. By lemma A.1 for all $\theta < \tilde{\theta}$ it must happen that $x^*(\theta) > 0$.

Let $u_E = U^*(\theta_E) = S(\theta_E)$ and consider the deviation:

$$\hat{\mathcal{C}}_2 = \left\{ \left(k^F(\theta), 0, \frac{u_E}{\theta_E f\big(k^F(\theta)\big)}\right) : \theta \in [0,1] \right\},$$

which implies for entrepreneurs the expected utility $U(\theta, \hat{\mathcal{C}}_2) = \frac{\theta}{\theta_E} u_E$.

If $\theta_E < \tilde{\theta}$ then the deviation leads to intermediary 2 cream-skimming the market above θ_E. Without loss of generality, assume intermediary 1 offers a contract schedule such that $x^*(\theta_E) > 0$. Any entrepreneur of type $\theta < \theta_E$ strictly prefers that original contract intended for θ_E than any contract in $\hat{\mathcal{C}}_2$ since

$$U^*(\theta) \geq \frac{\theta}{\theta_E}\big(u_E - x^*(\theta_E)\big) + x^*(\theta_E) > \frac{\theta}{\theta_E} u_E = U(\theta, \hat{\mathcal{C}}_2),$$

and the second term is the expected utility for type θ of taking the contract intended for θ_E, which by assumption is still offered by financier 1. The strict inequality follows from $x^*(\theta_E) > 0$. Also for $\theta > \theta_E$, we have $\theta_E^{-1} u_E > \theta^{-1} U^*(\theta)$ since by lemma A.4 $\theta^{-1}U^*(\theta)$ is strictly decreasing in the interval $[\theta_E, \tilde{\theta})$ and non-increasing afterwards, yielding the cream-skimming $U^*(\theta) < U(\theta, \hat{\mathcal{C}}_2)$. After applying lemma A.2 to break ties, the profitable deviation emerges.

If $\theta_L < \tilde{\theta} \leq \theta_E$ then there is a positive mass of entrepreneurs taking contracts with positive x and all of them entailing losses to the financiers. In this case the deviation $\hat{\mathcal{C}}_2$ gets rid of these non-profitable contracts. Without loss of generality assume intermediary 2 funds a positive fraction of them. Since $x^*(\theta) = 0$ for all $\theta > \tilde{\theta}$, $\theta^{-1}U^*(\theta) = f\big(k^*(\theta)\big)z(\theta)$, and this is both non-increasing and non-decreasing, thus constant. Then it follows that $U^*(\theta) = U(\theta, \hat{\mathcal{C}}_2)$ for all entrepreneurs with $\theta > \tilde{\theta}$, which includes the profitable ones. For those entrepreneurs with $\theta < \tilde{\theta}$, lemma A.4 implies $\theta^{-1}U^*(\theta) > \theta_E^{-1}u_E$ which is equivalent to $U^*(\theta) > U(\theta, \hat{\mathcal{C}}_2)$. Hence with $\hat{\mathcal{C}}_2$ intermediary 2 offers strictly less to a positive mass of non-profitable entrepreneurs, erasing at least part of the losses if they still contract with her or the full losses if they switch to intermediary 1 or the outside option. After applying lemma A.2 to break ties, the profitable deviation emerges.

Finally, if $\theta_L = \tilde{\theta}$ only entrepreneur of type $\theta_L = \tilde{\theta}$ could sign a contract with positive x. In this case, in the same fashion as before $U^*(\theta) = U(\theta, \hat{\mathcal{C}}_2)$ for all entrepreneurs with $\theta > \tilde{\theta} = \theta_L$, but this implies the contracts in the incentive compatible schedule $k^*(\theta), x^*(\theta), z^*(\theta)$ are exactly the same as those in $\hat{\mathcal{C}}_2)$ for these types, since they have the same capital and zero fixed pay and attain the same utility. If the contract for θ_L has $x^*(\theta_L) > 0$ it may as well be replaced by the one in $\hat{\mathcal{C}}_2)$ without altering any agent's payoff. In fact, by the compactness assumption of $\mathcal{C}^*$, it must happen that $\hat{\mathcal{C}}_2) \subseteq \mathcal{C}^*$. □

Proof of Claim 7

Proof. Suppose $\theta_L \geq \theta_P$, then $S(\theta_L) \geq S(\theta_P) = w = U(\theta_L)$ by the definition of θ_L and θ_P. But then the intermediary expects not to lose with the type θ_L and, by claim 4, expects strictly positive profits with all $\theta' \in (\theta_L, 1]$. That implies the intermediary is making profits strictly positive aggregate profits, contradicting the zero profit condition.

If types are public, intermediaries must break even with each type, which implies $U(\theta) = S(\theta)$. For those $\theta < \theta_P$, we have $S(\theta) < w$ hence none of them will take any contract. All the rest will accept the contract offered for their type, hence $\theta_L = \theta_P$ and the inefficiency vanishes.

If limited liability is removed, but all other features remain the same, we will show the equilibrium incentive-compatible contract schedule has to be $k(\theta) = k^F(\theta)$, $x(\theta) = -R \cdot k^F(\theta)$ and $z(\theta) = \pi$ for (almost) all $\theta \geq \theta_P$.

We start by showing that the following strategy profile is indeed an equilibrium:

Each intermediary offers $\mathcal{C}_i = \{(k_i(\theta), x_i(\theta), z_i(\theta)) | \theta \in [0, 1]\}$, where,

$$k_i(\theta) = k^F(\theta) \qquad z_i(\theta) = \pi \qquad x_i(\theta) = -R \cdot k^F(\theta) \tag{22}$$

and each θ-typed entrepreneur flips a coin to choose between the corresponding contracts offered by each intermediary $(k_1(\theta), x_1(\theta), z_1(\theta))$ and $(k_2(\theta), x_2(\theta), z_2(\theta))$, but strictly prefers any of the two compared to any $(k_i(\theta'), x_i(\theta'), z_i(\theta'))$ for $\theta' \neq \theta$. As $z_i(\theta)$ is constant, $f\big(k^F(\theta)\big)z_i(\theta)$ is (strictly) increasing, hence the contract satisfies the conditions of lemma 1 and $(k_i(\theta), x_i(\theta), z_i(\theta))$ maximizes entrepreneur θ's utility among the available options. Also $U_i(\theta) = S(\theta)$, and by definition on θ_P, type θ takes the contract if and only if $\theta \geq \theta_P$. If intermediary i is offering the above contract, intermediary j's best response cannot yield her any profit, since she would get only those types such that $U_j(\theta) \geq U_i(\theta) = S(\theta)$, and hence offering the same contract is a best response.

To see that all equilibriums are payoff equivalent, notice that claims 2 and 3 still hold. An intermediary can always add the other's contracts to her own menu and use the tie-breaking lemma (A.2) to take over the market and extract almost all the profits from the intermediary or from enacting projects at the right scale. Now assume in an unlimited liability equilibrium, intermediary 1 offers a contract schedule such that $U_1(\theta) \neq S(\theta)$ for a positive mass of entrepreneurs with $\theta > \theta_P$, we will show intemrediary 2 has a profitable deviation. Let $\hat{U}_2(\theta) = 0.5U_1(\theta)+0.5S(\theta)$, and define $\hat{k}_2(\theta) = k^F(\theta)$. Using the envelope theorem, $\hat{z}_2(\theta) = \hat{U}_2'(\theta) \cdot f\big(k^F(\theta)\big)^{-1}$ and $\hat{x}_2(\theta) = k^F(\theta) - \theta\hat{U}_2'(\theta)$. By definition, $U_2(\theta) = U_1(\theta)$ if and only if $S(\theta) = U_1(\theta)$, and $\hat{U}_2(\theta) > U_1(\theta)$ if and only if $S(\theta) > U_1(\theta)$. Hence if there is a positive mass such that $S(\theta) > U_1(\theta)$, intermediary 2 can make strictly positive profits with this deviation. If that is not the case, then $S(\theta) \leq U_1(\theta)$ for all $\theta < \theta_P$ and the inequality is strict for a positive mass of types. But then intermediaries must be making losses in aggregate, since all entrepreneurs obtain at least $S(\theta)$ and a positive mass strictly more, which contradicts the equilibrium assumption.

Hence for all $\theta \in [\theta_P, 1]$ it must be the case that $U_1(\theta) = U_2(\theta) = S(\theta)$. The envelope theorem for $S(\theta)$ yields $S'(\theta) = f\big(k^*(\theta)\big)\pi$ which implies $z_1(\theta) = z_1(\theta) = \pi$ for (almost) all those θ. That in turn implies $x_i(\theta) = -R \cdot k^*(\theta)$.

Last, if there is a unique intermediary facing limited liability and adverse selection, the unique equilibrium is $k(\theta) = k^*(\theta)$, $z(\theta) = 0$ and $x(\theta) = w$ and (almost) all entrepreneurs with $\theta \geq \theta_P$ take the contract. In this case an equilibrium is a contract schedule and a decision rule for entrepreneurs such that the schedule maximizes profit for the intermediary and the decision rule maximizes return to the entrepreneur. In the proposed equilibrium the intermediary extracts all the aggregate surplus, hence it is profit maximizing and any other contract that achieves this must enact all projects at their optimal scale and offer no more than w in expected utility to each entrepreneur. All entrepreneurs are indifferent between accepting or rejecting the contract, hence they are also maximizing. However it is key that almost all of them take the prescribed contract if and only if the surplus $S(\theta)$ is higher than w and reject anyone otherwise. As in the proof of claim 2, a contract set with a small but positive slope such that $U(\theta_P) = w$ and the tie-breaking lemma (A.2) implies in equilibrium almost all entrepreneurs must take the prescribed action. □

Proof of Claim 8: Red Tape Tax per Contract

Proof. A fixed cost per contract ϕ does not affect the proof of most of the claims. The tie-breaking lemma does not depend on the intermediary's profit level, hence it holds. Intermediaries will still make zero profit, after accounting for the red tape tax ϕ, and claim 2 also carries. Once the new break-even type is defined such that $S(\theta_E) = U(\theta_E) + \phi$, the proof for $x = 0$ follows as well. The only part that changes is the equation that pins down θ_L. The contract offered in equilibrium is still characterized by a modified version of proposition 1. The zero profit condition for θ_L is:

$$\int_{\theta_L}^{1} \left\{\theta f\big(k^*(\theta)\big)\pi - Rk^*(\theta)\right\} dG(\theta) - \int_{\theta_L}^{1} \left[\theta \frac{w}{\theta_L} + \phi\right] dG(\theta) = 0$$

which has a unique solution θ_L since its derivative with respect to θ_L is strictly positive below θ_E. By definition of ϕ, $\theta_L = \theta_P$ solves the equation and is thus the only solution. □

Proof of Claim 9: Profit Tax to Entrepreneurs

Proof. Suppose intermediaries compete with contracts of the form $\big(k, (1-\tau)x, (1-\tau)z\big)$. All can be renamed such that the contracts are after tax payments. The key difference becomes the residual claim of the financier: $S(\theta) - (1-\tau)^{-1}U(\theta)$, where $U(\theta)$ is the after tax expected utility of entrepreneurs. As before, all proofs carry over and then the contract offered in equilibrium is still characterized by proposition 1 but the θ_L now has to solve:

$$\int_{\theta_L}^{1} \left\{\theta f\big(k^*(\theta)\big)\pi - Rk^*(\theta)\right\} dG(\theta) - \int_{\theta_L}^{1} \theta \frac{w}{(1-\tau)\theta_L} dG(\theta) = 0,$$

because intermediaries have to pay $(1-\tau)^{-1}U(\theta)$ if she is supposed to deliver $U(\theta)$ net of taxes to the entrepreneur. As in proposition 1 there is a unique solution for θ_L. Also, fixing $\theta_L = \theta_P$, the left-hand side is strictly monotone in τ and can be solved for τ^*. □

A.2 Credit Score Proofs

Proof of proposition 2: Equilibrium with Credit Score

Proof. Once markets are split, equilibrium inside market must follow proposition 1. It is left to show that $\theta_L^h < \theta_L^l$ and that there is no pooling equilibrium.

To show $\theta_L^h < \theta_L^l$ it suffices to assert that:

$$\int_{\theta_L^h}^{1} \left[S(\theta) - \frac{\theta}{\theta_L^h} w\right] dG(\theta|\mathbf{l}) < 0.$$

Let θ_E^h be the break-even type such that $S(\theta_E^h) = \frac{\theta_E^h}{\theta_L^h} w = U^h(\theta_E^h)$. Because of the MLRP, there exists a non-increasing function $m(\theta)$ such that $dG(\theta|\mathbf{l}) = m(\theta)dG(\theta|\mathbf{h})$. Because of zero profits in the high signal sub-market it follows that:

$$0 = \int_{\theta_L^h}^{1} \left[S(\theta) - \frac{\theta}{\theta_L^h} w\right] dG(\theta|\mathbf{h}) > \int_{\theta_L^h}^{1} \left[S(\theta) - \frac{\theta}{\theta_L^h} w\right] \frac{m(\theta)}{m(\theta_E^h)} dG(\theta|\mathbf{h}) = \frac{1}{m(\theta_E^h)} \int_{\theta_L^h}^{1} \left[S(\theta) - \frac{\theta}{\theta_L^h} w\right] dG(\theta|\mathbf{l}),$$

the strict inequality follows from the fact that the factor $\frac{m(\theta)}{m(\theta_E^h)}$ is equal to or greater than one when $\left[S(\theta) - \frac{\theta}{\theta_L^h} w\right]$ is negative and equal to or smaller than one, and equal to or smaller than one when θ is greater than the break-even: $\left[S(\theta) - \frac{\theta}{\theta_L^h} w\right] > 0$. By enlarging the negative part and diminishing the positive part, the total integral must then decrease.

Notice that the same proof implies $\theta_L^h < \theta_L < \theta_L^l$ where θ_L is the lowest type taking a contract in a pooling/no-signal equilibrium. This is because, if a distribution G_1 dominates another distribution G_2 in the MLRP, $G_1(\theta) \succ_{MLRP} G_2(\theta)$, then G_1 also dominates any mixture among the two $G_1(\theta) \succ_{MLRP} \lambda G_1(\theta) + (1-\lambda)G_2(\theta) \succ_{MLRP} G_2(\theta)$. In particular, the high signal distribution dominates the unconditional one $G(\theta|\mathbf{h}) \succ_{MLRP} G(\theta)$. If $\theta_L^h < \theta_L$, however, then there cannot be a pooling equilibrium since any intermediary could offer a little bit more to high signal types and make strictly positive profits.

□

A.3 Asset Holdings Proofs

A.3.1 Proof of Claim 12

Proof. If $k^*(\theta) \leq a$, the entrepreneur can self-finance the project up to the optimal scale and save the rest, with expected profit $\theta F\big(k^*(\theta)\big)\pi + R\big(a - k^*(\theta)\big)$, which is the best possible outcome for the entrepreneur outside the credit market. If $k^*(\theta) > a$ the project can still be started but at a scale lower than the optimal, concavity of F implies that the best option, conditional on starting the project, is to invest all the assets in it, which yields $\theta F(a)\pi$. In any case, that has to be compared with the option of not doing the project and getting the return on the assets. □

A.3.2 Proof of Proposition 3

Fixing the asset level a, the IC constraint across types θ is the same, but now the limited liability restriction is $x_i(\theta, a) \geq -Ra$. Proofs of lemmas 1, A.2 stay the same. The proof of claims 2 and 3 do not depend on the limited liability condition being set at zero ($x > 0$), the only thing that matters is that new entrants may have an outside option strictly greater than w, but losses with these are bounded by δ. The proof of A.3 carries over, just bear in mind that as θ^- is the least upper bound of entrepreneurs taking the contract, hence $U(\theta^-) \geq O(\theta^-, a) \geq w$. Define the expected utility of a type θ entrepreneur with assets a under contract i as $U_i(\theta, a) + Ra$. Then $x_i(\theta, a)$ is decreasing in θ and in a competitive equilibrium $k_i(\theta, a) = k^F(\theta)$ for (almost) all θ and $U_i(\theta, a)$ is non-decreasing in θ

It still will be that case that, if financiers expect to make profits with type $\hat{\theta}$, they must expect to make profits with all types $\theta' > \hat{\theta}$, as stated in Claim 4 for the case without assets. That means not only showing that the surplus grows faster than the expected payoff to entrepreneurs, but also that these entrepreneurs choose the contract instead of the outside option. The equivalent claim with assets holdings is:

Claim A.1. *Suppose $S(\hat{\theta}) > U(\hat{\theta}, a) \geq O(\hat{\theta}, a)$ for some $\hat{\theta} > 0$. Then for almost every $\theta' > \hat{\theta}$, it must happen that $S(\theta') > U(\theta', a) \geq O(\theta', a)$ and entrepreneurs of type θ' must take the offered contract.*

Proof. Note that because of the outside option, $\hat{\theta} f(a)\pi < U_i(\hat{\theta}, a) + Ra$ must hold. Also for all θ we have $S(\theta) \geq \theta f(a)\pi - Ra$ with equality only for some θ_a such that $k^*(\theta_a) = a$. Crucially, no profits can be made with types $\theta < \theta_a$, as those can fully self-finance at the optimal scale, hence as the intermediary expects to make profits with type $\hat{\theta}$ it must be the case that $\hat{\theta} > \theta_a$. Suppose the expected profits for some $\theta' > \hat{\theta} > \theta_a$ such that $k_i(\theta') = k^*(\theta')$ are not positive, that is $U_i(\theta') \geq S(\theta')$. $S(\theta)$ is still convex. Remembering that $S(\theta_a) = \theta_a f(a)\pi - Ra$, by incentive compatibility of $\hat{\theta}$ not taking the contract for θ':

$$
\begin{aligned}
U_i(\hat{\theta}, a) &\geq \hat{\theta} f\big(k^*(\theta')\big) z_i(\theta', a) + \underline{x}_i(\theta', a) = \frac{\hat{\theta}}{\theta'} U_i(\theta', a) + \frac{\theta' - \hat{\theta}}{\theta'} \underline{x}_i(\theta', a) \\
U_i(\hat{\theta}, a) &\geq \frac{\hat{\theta} - \theta_a}{\theta' - \theta_a} U_i(\theta', a) + \frac{\theta' - \hat{\theta}}{\theta' - \theta_a} \left[\frac{\theta_a}{\theta'} U_i(\theta', a) + \frac{\theta' - \theta_a}{\theta'} \underline{x}_i(\theta', a) \right] \\
U_i(\hat{\theta}, a) &\geq \frac{\hat{\theta} - \theta_a}{\theta' - \theta_a} U_i(\theta', a) + \frac{\theta' - \hat{\theta}}{\theta' - \theta_a} \left[\frac{\theta_a}{\theta'} (\theta' f(a)\pi - Ra) - \frac{\theta' - \theta_a}{\theta'} Ra \right] \\
U_i(\hat{\theta}, a) &\geq \frac{\hat{\theta} - \theta_a}{\theta' - \theta_a} U_i(\theta', a) + \frac{\theta' - \hat{\theta}}{\theta' - \theta_a} \left[\theta_a f(a)\pi - \frac{\theta_a}{\theta'} Ra - \frac{\theta' - \theta_a}{\theta'} Ra \right] \\
U_i(\hat{\theta}, a) &\geq \frac{\hat{\theta} - \theta_a}{\theta' - \theta_a} S(\theta') + \frac{\theta' - \hat{\theta}}{\theta' - \theta_a} \left[\theta_a f(a)\pi - Ra \right] \\
U_i(\hat{\theta}, a) &\geq \frac{\hat{\theta} - \theta_a}{\theta' - \theta_a} S(\theta') + \frac{\theta' - \hat{\theta}}{\theta' - \theta_a} S(\theta_a) > S(\hat{\theta})
\end{aligned}
$$

contradicting the expected profits for $\hat{\theta}$. As $k_i(\theta') = k^F(\theta')$ for almost all θ' the first inequality in the claim follows. The second follows from the next claim, the asset version of claim 5, which states that if a type takes a contract, all better types take a contract. □

Claim A.2. *In equilibrium, there exists some type θ_L such that (almost) all entrepreneurs with $\theta > \theta_L$ strictly prefer to sign a contract with an intermediary instead of taking the outside option, (almost) all entrepreneurs with $\theta < \theta_L$ take the outside option, and $\theta_L\big[U(\theta_L) - O(\theta_L, a)\big] = 0$.*

Proof. First, we show (almost) all those with $O(\theta, a) = \theta' F(a)\pi$ must enter in equilibrium. Notice that $O(\theta, a) = \theta F(a)\pi$ is equivalent to $\theta > \max\{\theta_a, \theta_P\}$, hence $\theta_L \leq \max\{\theta_a, \theta_P\}$. Assume a positive measure of entrepreneurs with outside option $O(\theta, a) = \theta F(a)\pi$ were not entering the market, then a profitable deviation would be available for intermediaries. The deviation contracts have a slope $\gamma_a = F(a)\pi$, that is:

$$
\hat{\mathcal{C}}_1 = \left\{ \left(k^F(\theta), 0, \frac{\gamma_a}{f\big(k^F(\theta)\big)} \right) : \theta \in [0, 1] \right\},
$$

this set offers $U_1(\theta, \hat{\mathcal{C}}_1) = \gamma_a \theta = \theta F(a)\pi$. Hence it attains the outside option for those $\theta > \theta_a$ and is strictly below for the rest, which means no type lower than θ_a would enter. Any taker will yield profits to the financier. In particular, the positive mass of entrepreneurs that were not entering the market would be indifferent between taking the deviation contracts or staying out. Then, using the tie-braking lemma (A.2) a strictly positive deviation exists.

If $\theta_a < \theta_P$ then the outside option for each θ is either w or $\theta F(a)\pi$. Since those in the latter case always enter, the existence of θ_L reduces to the case with no assets where $U(\theta_L, a) = w$ or $\theta_L = 0$.

If $\theta_a \geq \theta_P$, then (almost) all types $\theta > \theta_a$ must enter and no profits can be made with types $\theta < \theta_a$ as that implies $O(\theta, a) \geq S(\theta)$. Let θ_L be least upper bound of $U(\theta, a) = O(\theta, a)$, in the set $0 \leq \theta \leq \theta_a$. By definition, all $\theta' > \theta_L$ enter since it implies either $\theta' > \theta_a$ or $U(\theta', a) > O(\theta', a)$. Left to show that (almost) no $\theta < \theta_L$ takes the contract in equilibrium. If $\theta_L = 0$ there are no lower types entering. If $U(\theta_L, a) = w$ this reduces to the case with no assets. Otherwise $U(\theta_L, a) = O(\theta_L, a) > w$, and the derivative of $O(\theta_L, a)$ is strictly positive but less or equal to the right derivative of $U(\theta, a)$. Since the outside option derivative is zero between 0 and θ_P and strictly increases between θ_P and θ_a, this deviation keeps every type below θ_L since the (left-)derivative of U is greater than that of the outside option in $[0, \theta_L]$. Then as before, removing all contracts with slope strictly less than $U'^{+}(\theta_L)$ can only dismiss non-profitable entrepreneurs of types $\theta < \theta_L$, which would be a profitable deviation if a positive mass of them were entering. $\square$

Notice that the proof of existence of a θ_L also shows it must be smaller than $\max\{\theta_a, \theta_P\}$. This implies that when $\theta_a \leq \theta_P$ then $\theta_L < \theta_P$; that is when assets are not enough for every socially efficient type to start the project on her own, in equilibrium there are socially inefficient types starting a project.

The proof of the analogous of claim 6 follows, which in this case it that the limited liability condition $x(\theta) = -Ra$ must bind. The key fact is that lemma A.4 has to be augmented to state $\theta^{-1}\big[U(\theta) + Ra\big]$ is non-increasing.

Lemma A.5. *Let $U(\theta)$ be the expected utility for entrepreneurs. Then $\theta^{-1}\big[U(\theta)+Ra\big]$ is non-increasing and strictly decreasing when $x^*(\theta) > -Ra$.*

Proof. Let two entrepreneurs with types $\theta_1 < \theta_2$ and $x(\theta_2)$ be the highest x among those contracts attaining $U(\theta_2)$ for entrepreneur 2. As entrepreneur 1 weakly prefers his contract to the one intended for entrepreneur 2:

$$U(\theta_1)+Ra \geq \frac{\theta_1}{\theta_2}\big(U(\theta_2)-x(\theta_2)\big)+x(\theta_2)+Ra = \frac{\theta_1}{\theta_2}\big[U(\theta_2)+Ra\big]+\big[x(\theta_2)+Ra\big]\left[1-\frac{\theta_1}{\theta_2}\right] \geq \frac{\theta_1}{\theta_2}\big[U(\theta_2)+Ra\big],$$

and the last inequality follows from $x(\theta_2) \geq -Ra$ and is strict when $x(\theta_2) > -Ra$. $\square$

Once the previous lemma is established, the proof for a binding limited liability constraint is analogous to the one without assets: it uses contradiction and builds a deviation to a set of contracts where $\theta^{-1}\big[U(\theta) + Ra\big]$ is constant to cream-skim the market, or drops the contracts with non-binding limited liability clause.

B Details for Numerical Exercises

B.1 Welfare-Reducing Credit Score Example

There is a constant factor omitted in the definition of $dG_2(\theta|s)$, which is $c_s = \big[1+(2s-1)MX\big]^{-1}$ where:

$$MX = \int_0^1 m(\theta)dX(\theta).$$

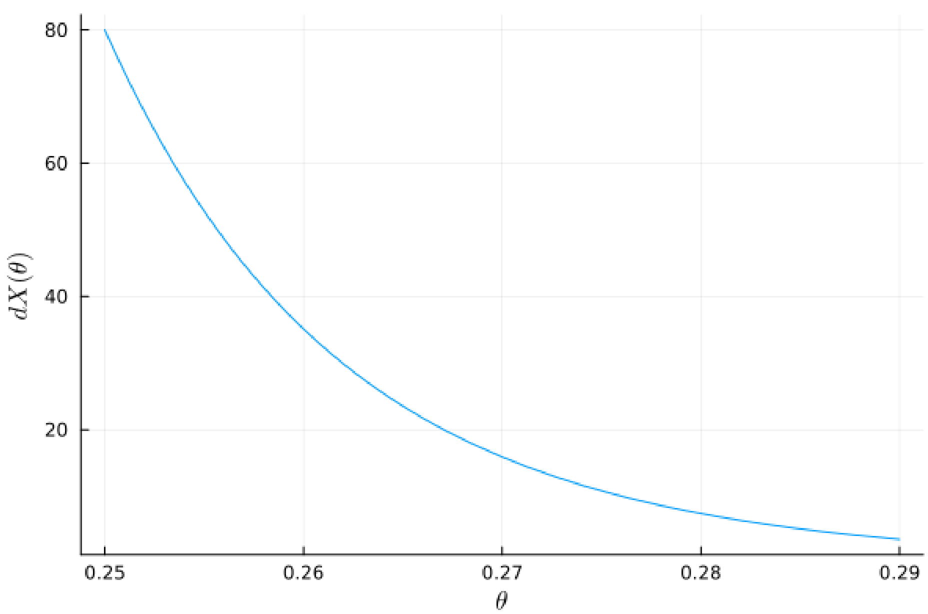
When mixing $dG_2(\theta|s)$ for two parameters s_a, s_b and weights $\omega_a, \omega_b = 1 - \omega_a$ the mixed distribution will correspond to a linear combination of the parameters with weights proportional to:

$$\omega_a\big[1+(2s_a-1)MX\big]^{-1}, \quad \omega_b\big[1+(2s_b-1)MX\big]^{-1}.$$

Hence we set $\omega_a = \frac{1+(2s_a-1)MX}{2+(2s_a+2s_b-2)MX}$, with these weights the mixed distribution is $dG_2\big(\theta|0.5(s_a+s_b)\big)$, that is, the parameter for the mixed distribution corresponds to the simple average of the two conditional distribution parameters $\frac{s_a+s_b}{2}$.

We set X as a Pareto distribution with minimum value $x_m = 0.25$ and curvature parameter $\eta = 20$, that is $X \sim Pareto^T(0.25, 20)$. Next we truncate by removing all mass above $\theta = 1$, keeping the support $[0.25, 1]$. This distribution has considerable mass between 0.25 and 0.3, while having very few high quality types, Figure 21 shows the pdf in that range.

Figure 21: Pdf for $dX(\theta)$ where $X \sim Pareto^T(0.25, 20)$

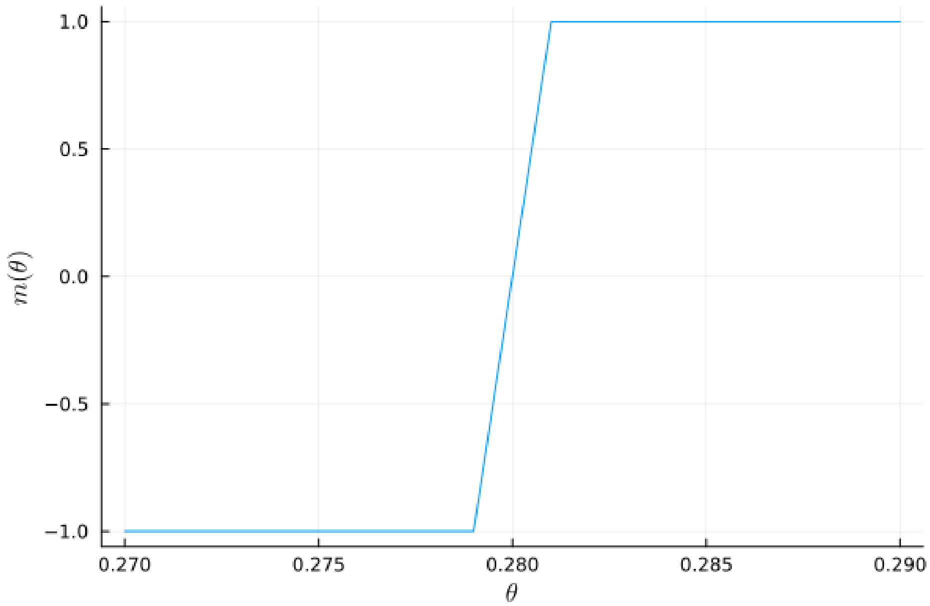


The function $m(\theta)$ is such that it barely increases in the whole domain except for a steep derivative around $\theta = 0.28$. The function is plotted in Figure 22 and its formula is:

$$m(\theta) = \max\Big\{-1+\frac{1}{1000}\theta, \min\{\frac{999}{1000}+\frac{1}{1000}\theta, 1000(\theta-0.28)\}\Big\}.$$

Under this construction, the unconditional distribution will be $dG(\theta|s = 0.0025)$ and it will be such

Figure 22: Function $m(\theta)$



that

$$dG_2(\theta|0.0025) = \omega_h \underbrace{dG_2(\theta|h = 0.005)}_{h=0.005} + \omega_l \underbrace{dG_2(\theta|0)}_{l=0}.$$

For this particular example, we modified the parameters not related to the distribution. We set $\alpha = 1.0$ to have an closed form solution for the optimal capital level and expected surplus. This allows to significantly reduce the numerical error in the calculations, essential for the precision of the results. We set w to 7.5 to make the lowest socially efficient type θ_P to be 0.28; together with the unconditional distribution, this means there are few socially efficient startups while there are many inefficient ones. The parameters are described in Table 4

Table 4: Alternative Parameters

w	π	R	α	β
7.5	100	1.02	1.0	0.1