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Abstract

This paper presents a comprehensive framework examining fiscal sustainability in developing economies. It integrates public capital, labor informality, and global liquidity shocks in a two-sector DSGE model for a small open economy, revealing their intricate interplay and nonlinear impact on State-Dependent Debt Limits. The framework highlights the significance of initial public capital levels and efficiency in determining the benefits of public investment. High informality rates erode the tax base, compromising the efficiency of public capital for fiscal purposes by weakening revenue generation relative to costs. Adverse global liquidity shocks may significantly contract the fiscal limit distribution only if they are perceived as permanent. Through model calibration and sensitivity exercises on Colombia's fiscal limit distribution, quantitative analyses shed light on underlying mechanisms. Findings challenge the frequent practice of cutting public investment in response to declining revenues, emphasizing it can actually reduce fiscal space. The framework underscores the importance of assessing fiscal pol-icy consolidations aimed at ensuring debt sustainability and responses to global shocks using a structural model, while stressing the fiscal benefits of informality-reducing reforms.

JEL classifications: E32, E62, H20, H30, H50, H60
Keywords: Public debt, Labor informality, Public Investment, Fiscal limit, Fiscal space, Fiscal sustainability, Global liquidity[1]

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1 Introduction

Fiscal stress can come from a variety of local and foreign sources, the latter being particularly influential in Small Open Economies (Schmitt-Grohé (1998); Blankenau et al. (2001); Byrne et al. (2011); Lepetyuk et al. (2020); Levchenko & Pandalai-Nayar (2020)). These different sources bring about nonlinear responses of fiscal variables that are hard to thoroughly examine without a structural model. Therefore, developing a nonlinear framework that appropriately models the channels through which different business-cycle shocks influence fiscal performance is essential to offer a precise quantitative assessment of a country's debt sustainability. In this paper, we develop a two-sector DSGE model for a Small Open Economy (SOE) featuring foreign and local shocks particularly relevant for developing economies' business cycles, such as terms of trade and sector-specific productivity shocks (see Correia et al. (1995); Schmitt-Grohé (1998); Broda (2004); Schmitt-Grohé & Uribe (2018)). Moreover, the model incorporates three paramount features of these economies: public capital, labor informality, and global liquidity shocks.[2] As a case study, the model is calibrated for Colombia and used to compute its fiscal limit distribution. Additional sensitivity exercises shed light on the complex interplay and nonlinear impact that previously highlighted features have on State-Dependent Debt Limits (SDDL). The main insight of these simulations is that cutting public investment in response to recessions, declining fiscal revenues, or a tighter fiscal balance, can actually lead to a less sustainable fiscal equilibrium. Strikingly, the model also uncovers that large labor informality can make public investment expansions analogous to a public consumption upswing that reduces fiscal space. More generally, it brings to light that fiscal multipliers are a negative function of informality and therefore can affect fiscal sustainability beyond the tax base contraction. These interesting interactions and outcomes become clear through the lenses of our framework and can be better understood by examining the underlying mechanisms. First, public investment expansions might differ from those of public consumption and thus yield more sustainable fiscal outcomes for sufficiently high degrees of public capital efficiency. Second, higher formality rates might widen the tax base enough to significantly expand a country's fiscal limit distribution. In turn, more formality also means a larger private capital stock and more productive labor, which together increase the marginal productivity of public capital. As for aggregate output and fiscal revenue, this is equivalent to directly increasing the parameter governing public capital efficiency. On the contrary, adverse global liquidity shocks may drastically contract the Fiscal Limit Distribution. This effect depends more on the permanent nature of the shock to world's

[2] Novelli & Barcia (2021); Ardanaz et al. (2021) argue that public investment is procylical, as it is commonly the main variable of adjustment during recessions and used to comply with fiscal rules, while Ardanaz & Izquierdo (2022) emphasize that this empirical regularity is pervasive in developing countries; papers like Bosch (2006); Restrepo-Echavarria (2014); Granda & Hamann (2020) highlight the relevance of informality in developing economies; and Blankenau et al. (2001); Byrne et al. (2011) find that world interest rates and global liquidity can account for significant fractions of business-cycle dynamics in SOEs.

risk-free interest rate than the magnitude of the shock. The framework proposed in this paper contributes to the fiscal sustainability and sovereign debt literature in a couple of ways. Although this literature has previously used similar structural models and global methods to perform quantitative analyses regarding debt sustainability,[3] our paper enriches this strand of the literature by including labor informality, public capital and global liquidity shocks within the same model. Although each of these features has been addressed before, they have so far only been analyzed in isolation. Moreover, their influence on a country's fiscal limit distributions and its fiscal space has not been evaluated.

First, when studying developing countries this literature often overlooks the cardinal importance of labor informality in the economy (as high as 70% according to Bosch (2006)). Considering the magnitude of informality rates and that informality directly hinders a government's revenue collection, it is imperative to address this omission in the literature. Accordingly, our paper includes exogenous labor informality by modelling two types of households: formal and informal. Formal households offer a labor variety of greater productivity, since formal workers are usually associated with skilled labor as discussed in Amaral & Quintin (2006); Granda & Hamann (2015). Additionally, formal households have full access to financial markets, whereas informal ones are hand-to-mouth consumers (e.g., Granda & Hamann (2015)).[4] Another frequent oversight in the literature is the role of public investment in determining debt sustainability. Provided that public capital is somewhat productive, a higher level of public investment must, as a first-round effect, enhance production. Hence, it is possible in theory that public investment expansions can be self-financed. In fact, Ardanaz et al. (2021) presents some evidence suggesting this possibility. Nevertheless, this possibility critically hinges on the final effect of higher public investment on the tax base. In advanced economies, this effect would largely depend on public capital's initial stock and the efficiency with which it enhances the economy's productive sector. However, in developing economies, informality may play a leading role in how much of each dollar spent in public investment comes back to the government's pocket through greater tax revenue. Our model allows to understand how high informality rates can be equivalent to low public capital efficiency for fiscal sustainability purposes. Thirdly, we consider shocks to the discount factor of foreign investors, which are the only holders of public debt in the model. Shocks to this discount factor enable us to reflect fluctuations in global liquidity. We consider a two-regime switching process for this variable, capturing the low variability in global liquidity during the last decade, as well as its enduring effects on developing economies' access to low financial costs. Finally, an additional contribution of this paper is the estimates for the Colombian economy that stem from our

[3]Some examples can be found in Hamann (2002); Aguiar & Gopinath (2006); Reinhart & Rogoff (2010); Arellano (2008); Cuadra et al. (2010); Bi (2012); Lizarazo (2013); Aguiar et al. (2016); Hürtgen (2020); Mendez-Vizcaino & Moreno-Arias (2021).

[4]Section 1.1 contains a more profound examination of the stylized facts about informal workers that back up our modelling choices.

quantitative analyses. Our calculations add to the existing empirical literature about fiscal sustainability in Latin America, particularly the strand that has focused on offering numerical debt limits and fiscal spaces for countries of the region.[5] Some of these efforts can be found in Lozano-Espitia et al. (2019); Lozano-Espitia & Julio-Román (2020); Lozano-Espitia & Arias-Rodríguez (2020); Mendez-Vizcaino & Moreno-Arias (2021). Because our estimates come from a richer framework, they can deepen future discussions about the relevant quantitative debt limit for Colombia, as well as underlining the relevance of less commonly discussed determinants of this number. What is more, as in Mendez-Vizcaino & Moreno-Arias (2021), a prominent advantage of our method is the resulting distribution of public debt limits, for several default probabilities, rather than the usual point estimate. The remainder of the paper is organized as follows. Section 1.1 discusses the challenges of measuring informality and analyze some empirical facts about labor informality in Latin America and, in greater detail, the available data for Colombia. Section 2 describes the model, offers different empirical and theoretical rationales for some key assumptions, and define the concepts of fiscal limit distribution and fiscal space. Section 3 explains how the model was calibrated and the sources of the data employed. Section 4 presents Colombia's fiscal limit distribution under the baseline calibration and discusses the sensitivity exercises that illustrate the mechanisms through which public capital, labor informality, and global liquidity shocks affect debt sustainability. Finally, Section 5 concludes and suggests some further research opportunities.

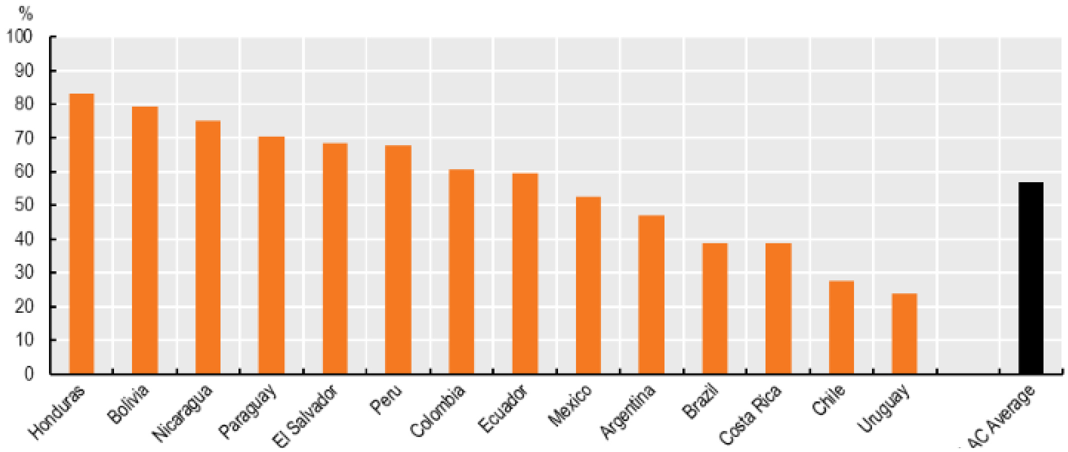
1.1 On the Measure and Importance of Informality

A notable feature of emerging and developing economies is the high degree of labor informality. In Figure 1, we show the percentage of informal employment in Latin America. Although there is high heterogeneity in the share of informal employment in the region, fluctuating between 20% (Uruguay) and 80% (Honduras), these percentages are non-negligible and suggest the presence of weaknesses in country's institutional arrangements. High informality rates are commonly associated with vulnerabilities in the labor markets, limited access to the financial system, and poor tax administration, among other issues.[6] This type of labor usually operates in not legally recognized environments such as the subsistence economy or family-scale businesses, and it is non-compliant with fiscal obligations.

[5] Colombia is one of the five largest economies in Latin America and has never defaulted on its sovereign debt, which makes it a reasonable benchmark for other countries in the region.

[6] For a more detailed analysis onf obstacles to doing business from the perspective of firms, see (Ranasinghe & Restuccia, 2018).

Figure 1: Percentage of Informal Employment in Latin America



Source: OCDE. (2020), COVID-19 in Latin America and the Caribbean: Regional Socio-economic Implications and Policy Priorities

Although this feature is crucial in the economic policymaking decisions in these countries and there have been some efforts to formally introduce the different edges of this phenomenon in economic theory (see for example, (Ulyssea, 2010, 2018; Granda et al., 2019; Granda & Hamann, 2020; Hamann et al., 2021; Restrepo-Echavarria, 2014)), it has not been a broadly studied topic in the context of fiscal sustainability. There are two common definitions of informality based on national surveys on households and firms. The first is based on social security contributions by workers ((Bosch, 2006; Boeri & Garibaldi, 2005; Ulyssea, 2010; Granda & Hamann, 2015)). Under this approach a worker is defined as formal if she contributes to healthcare and/or pension obligations depending on the country's labor regulations. Thus, measures of informality range from the least strict, contributions either to healthcare or pensions, to the most restrictive one, contributions to both healthcare and pensions. Under this approach, National Households surveys play a key role in the identification of agents as formal or informal, as those reports aim to capture the source of the households' income and their characteristics for a representative sample. The second approach takes into account an even more restrictive measure of informality and is related to the existence of a minimum wage. This second approach, explored in Hamann et al. (2021); Arango et al. (2022), is based on workers' minimum wage access. This approach depends on different institutional and regulatory arrangements regarding the compulsory condition of the payment of a minimum wage as part of the obligations a formal firm must fulfill. For instance, an individual who on self-account contributes to pension and healthcare systems but does not earn a minimum wage might be classified as an informal worker, as her earnings do not satisfy part of the regulatory arrangements of formal work within a country.

Using Colombian data of the National Households Survey (GEIH) for 2019, we classify formal and informal workers using both approaches in the literature. We report the estimations in Table 1. We compute the informality rate using four criteria present in Granda & Hamann (2015): i) individuals who contribute to pensions, ii) individuals who contribute to healthcare, iii) individuals who contribute either to pensions or healthcare, and iv) individuals who contribute to both pensions and healthcare systems. Regardless of the criterion, informality in Colombia is high, ranging between 47% and 55%. Under this classification, the least restrictive criterion is the third one, individuals who contribute either to pension or healthcare system (47% of labor informality), while the strictest, as noted earlier, is the fourth, individuals who contribute to both pension and healthcare system (55% of labor informality). Although regulations in the Colombian labor market have improved to enhance formality, high levels of informalityh are a persistent characteristic of this market. When studying the second approach based on minimum wage (MW)[7], the results are strikingly accentuated. When further restricting the fourth criterion to also comply with the minimum wage regulation, informality rises from 55% to 79%.[8] This strict measure points out additional institutional weaknesses in compliance with the regulations of the Colombian labor market.

Table 1: Share of Labor Informality in Colombia with Different criteria

	Criterion	% Informality
1	**Pension**	54
2	**Health**	49
3	**Pension Or Health**	47
4	**Pension And Health**	55
5	**Pension And Health And MW**	79

Source: Authors with information of the GEIH-2019.

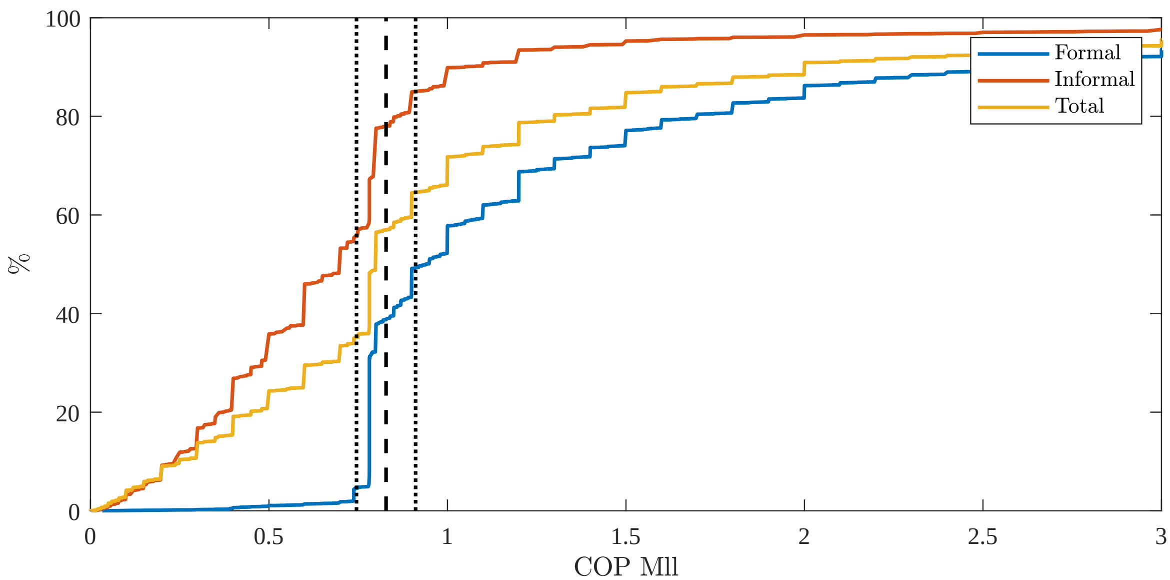
Being informal entails additional features. First, informal income is, on average, lower than the formal income. From the GEIH survey, we find that the average income of the formal workers is roughly 1.76 times greater than the informal workers. In addition, Hamann et al. (2021) report a wage premium of formal agents of 3.71 and the share of labor income of formal agents to total labor income close to 78%. In Figure 2, we show the income distribution for formal, informal and aggregate workers from GEIH 2019 using informality criterion 4. Two features are worth noting. First, monthly aggregate labor income is concentrated in low values of income, around 60% of the workers earn less than the minimum wage. Second, formal labor income distribution is shifted rightwards and has a low mass of individuals (around 5%) below 0.9 times the minimum wage, while the median of the distribution is close to 1.1 times the minimum wage. On the other hand, informal labor income distribution concentrates around 85% of the individuals at this same threshold value of 1.1 times the minimum wage.

[7] Minimum wage in 2019 is around COP$828000, or USD$251.

[8] When we define the criterion with 0.9 of a minimum wage as a commonly used measure in Colombian economy to study the effective minimum wage, informality increases from 55% to 67%.

In addition to the labor income distribution, GEIH survey allows us to disentangle other sources of households' income, such as interest revenues or government transfers. With respect to the former, in GEIH we find that, even though a low share of workers report having interest revenues (around 10%), almost 90% of those workers are formal. Although this measure is only indicative of the depth of the financial market, it does show that informal households might not typically hold capital or financial assets and that this relative lack of access to financial and capital markets might lead agents to smooth consumption through cash (see (Granda et al., 2019)). Additionally, an informal individual does not typically pay direct taxes, as monitoring and enforcement is low in this sector but does benefit from part of government expenditure (social programs, direct transfers, etc.).

Figure 2: Monthly Average Income Distribution from GEIH of Formal and Informal Workers



Source: Authors with information of the GEIH-2019. Using criterion Pension and Health. Vertical black-dashed line depicts the 2019 minimum wage in Colombia, while vertical black-dotted lines represent a threshold of $\pm 0.1MW$,

This introductory characterization of informality allows us to raise questions on the role of informality in fiscal sustainability through i) deterioration of the fiscal balance through reducing the tax base, ii) dampening the multiplier effects of public consumption and investment, and iii) restraining the possible amplifying effects of private investment.

2 Model

This section lays out the agents' optimization problems and explains the market clearing requirements, but all first-order conditions and equations needed to fully characterize the economy's equilibrium can be found in Appendix (A). The model describes a two-sector SOE with labor informality. The economy is populated by three types of agents: firms, households, and government. The two productive sectors are: tradables (T) and nontradables (N). The way this structure is modelled owes most to the framework proposed in Bi et al. (2016b) for the Argentinean economy. Importantly, both types of goods are demanded by private and public agents for investment and consumption. The most notable feature of households is labor informality, which is introduced modelling two types of households: formal and informal. The size of each type of household is assumed to be constant and not an endogenous decision. We favor this modelling approach since we are not interested in studying informality determinants like sector-specific entry costs, social security costs, or formality enforcement (See Ulyssea (2010)), but rather the impact of formality on fiscal sustainability.[9] In this sense, higher formality levels are assumed to affect households on the extensive margin, as they will primarily affect the relative size of formal households and thus increase their weight in aggregate variables. In contrast, firms hire both types of workers, which means they are only able to resort to informality on the intensive margin, as defined in Ulyssea (2018) and following Bosch (2006). Lastly, public consumption, public investment, sector-specific TFP levels, and terms of trade are exogenous variables that follow AR(1) processes.

Households

On the one hand, the two types of households differ in their access to capital markets, productivity, and earned income, beyond their compliance with tax obligations. In particular, only formal households own firms and private capital, and these households have a productivity premium, ϑ, which is consistent with the evidence discussed in Amaral & Quintin (2006); Granda & Hamann (2015). Our paper does not focus on the determinants of labor informality but on the way it affects debt sustainability, thus for simplicity we assume a segmented labor market (See Lewis et al. (1954)) from the perspective of households.[10] All variables directly related to formal households are identified by superscript F, while informal households' variables by superscript I.

[9]Furthermore, as will be later explained, tax rates are assumed to be constant for the computation of fiscal limits, which might be the main driver (although not the only one) of an occupational choice between informality and formality.

[10]For a more detailed discussion on competing views about the frictions that are actually behind the decision of being informal and the observed patterns for developing economies, see the first two sections of Amaral & Quintin (2006).

There is a continuum of formal households of measure Υ and of informal households of measure $1-\Upsilon$. On the other hand, these households are similar in that both maximize the expected present value of their lifetime utility by choosing how much to consume and hours to work. This means their instant utility is:

$$u(c_t^J, l_t^J) = \left[\frac{\left(c_t^J\right)^{1-\sigma^J}}{1-\sigma^J} - \phi^J \frac{\left(l_t^J\right)^{1+\upsilon^J}}{1+\upsilon^J} \right] \quad \forall J \in \{F, I\}$$

with c_t^J being private consumption and l_t^J being the total amount of hours worked. The parameters σ^J, υ^J and ϕ^J are the intertemporal elasticity of substitution, the inverse Frisch elasticity and the scale parameter of labor disutility, respectively.

Formal Households

Formal households have access to capital markets and are able to smooth consumption over time by choosing how much to invest, i_t^F, and capital to hold k_t^F, taking into account its constant depreciation rate, δ, and the capital-adjustment costs governed by parameter κ. The price of private consumption, p_t^C, is the numeraire and normalized to one, while p_t^I represents the relative price of private investment. Formal households earn a wage, w_t^F, for their labor and receive a rental rate of capital, r_t. Additionally, formality implies that households must pay a tax rate, τ_t, for their total income, including their corresponding share of firms' profits. Therefore, the representative formal household's problem is:

$$\max_{c_t^F, l_t^F, i_t^F, k_t^F} \mathrm{E}_t \sum_{t=0}^{\infty} \beta^t \left[\frac{\left(c_t^F\right)^{1-\sigma^F}}{1-\sigma^F} - \phi^F \frac{\left(l_t^F\right)^{1+\upsilon^F}}{1+\upsilon^F} \right]$$

subject to its budget constraint and the law of motion for capital

$$c_t^F + p_t^I i_t^F + \frac{\kappa}{2} \left[\frac{i_t^F}{k_{t-1}^F} - \delta \right]^2 k_{t-1}^F = (1-\tau_t) \left(w_t^F l_t^F + r_t k_{t-1}^F + \Pi_t^{F,N} + \Pi_t^{F,T} \right) \tag{1}$$

$$k_t^F = (1-\delta) k_{t-1}^F + i_t^F. \tag{2}$$

Informal Households

Informal households earn income only from the amount of hours they work, l_t^I, and use it to pay for their consumption, c_t^I. The informal households' problem is:

$$\max_{c_t^I, l_t^I} \mathrm{E}_t \sum_{t=0}^{\infty} \beta^t \left[\frac{\left(c_t^I\right)^{1-\sigma^I}}{1-\sigma^I} - \phi^I \frac{\left(l_t^I\right)^{1+\upsilon^I}}{1+\upsilon^I} \right],$$

subject to the budget constraint,

$$c_t^I = w_t^I l_t^I \tag{3}$$

Notice that these households are hand-to-mouth agents that consume all of their labor income, and that the wage they earn might differ from the wage earned by formal households, since the latter are more productive.

Aggregate Variables

Since there are two types of households and no idiosyncratic risk for either of them, aggregate private quantities like consumption, labor or investment are equal to the continuous sums of the representative individual decisions. Thus, aggregate private consumption is:

$$c_t = \int_{\Upsilon}^{1} c_t^I di + \int_{0}^{\Upsilon} c_t^F di$$

$$c_t = (1-\Upsilon)\, c_t^I + \Upsilon c_t^F$$

Aggregate private capital and investment equal:

$$\begin{aligned} K_t &= \Upsilon k_t^F \\ i_t &= \Upsilon i_t^F \end{aligned}$$

And lastly, aggregate labor supplies per household will be capitalized to distinguish them from individual supplies:

$$\begin{aligned} L_t^F &= \Upsilon l_t^F \\ L_t^I &= (1-\Upsilon) l_t^I \end{aligned}$$

Labor Market

The labor market features two intermediaries, each one on a different side of the market: employment agencies allocate the labor supply of both types of household in the two productive sectors, and labor unions demand formal and informal labor to create a labor bundle per sector that the corresponding firm employs to produce goods. Consequently, there will be two employment agencies (formal and informal) and two labor unions (for tradables and nontradables production). Additionally, either type of household supplies a given amount of hours worked to its corresponding employment agency, but the agency's problem will be to allocate the aggregate labor supply of all its subscribers to each sector. This means that employment agencies will be affected by the extensive and intensive margins of labor supply in the model. For instance, the formal employment agency will not decide using l_t^F but L_t^F, which takes into account the measure of formal households Υ.

Employment Agencies

In this economy, both types of households resort to an employment agency in order to find jobs. The agency's task is to place job seekers in the nontradables and tradables sectors by contacting them with sector-specific labor unions. In this model, there is no labor decision in the extensive margin and each type of household offers a different variety of labor. Therefore, the agency chooses how a household should allocate its working time across productive sectors. This decision is influenced by each household's preferences and the relative wages offered in a perfect competition environment by the labor unions in the nontradables and tradables sectors (indexed by superscripts N and T, respectively). Additionally, agencies use a CES function per type of household to decide sectoral allocations:

$$L_t^J = \left[\gamma^{J\left(-\frac{1}{\xi^J}\right)} \left(L_t^{J,N}\right)^{\frac{1+\xi^J}{\xi^J}} + \left(1-\gamma^J\right)^{\left(-\frac{1}{\xi^J}\right)} \left(L_t^{J,T}\right)^{\frac{1+\xi^J}{\xi^J}} \right]^{\frac{\xi^J}{1+\xi^J}} , \quad \forall \quad J \in \{F, I\}$$

In this function,[11] parameter γ^J reflects the long-run average of the hours worked in the nontradables sector relative to the hours worked by household J, and parameter ξ^J the elasticity of substitution across sectors.[12] The employment agency receives from labor unions the wages $w_t^{J,X} \quad \forall X \in \{N, T\}$, earned by each household in the two sectors and then pays the household a compound wage w_t^J for the total amount of hours worked, L_t^J. Thus, the employment agency solves two problems, one for each type of household, that can be generalized as:

$$\max_{L_t^{J,N}, L_t^{J,T}} w_t^{J,N} L_t^{J,N} + w_t^{J,T} L_t^{J,T} - w_t^J L_t^J, \quad \forall \quad J \in \{F, I\}$$

The relative supplies of labor for each sector can be plugged back into the respective CES function to obtain an expression of the wages earn by formal and informal households:

$$w_t^J = \left[\gamma^J \left(w_t^{J,N}\right)^{1+\xi^J} + \left(1-\gamma^J\right) \left(w_t^{J,T}\right)^{1+\xi^J} \right]^{\frac{1}{1+\xi^J}} , \quad \forall \quad J \in \{F, I\}$$

Labor Unions

The tradables and nontradables sectors do not hire employees directly, but through labor unions. These labor unions receive formal and informal workers and decide how

[11]Notice that because this CES function is homogeneous of degree 1, it is not relevant if the aggregation occurs considering the mass of each kind of worker or not.

[12]To guarantee that each productive sector employs some labor of both types of households, the elasticity of substitution in each CES function is such that hours worked in each sector behave as complements.

to organize them to create a unique labor bundle that firms in each sector employ. This bundle is assembled through a technology described by the following CES function:

$$L_t^X = \left[\left(\varphi^X\right)^{\left(-\frac{1}{\xi^X}\right)} \left(L_t^{I,X}\right)^{\frac{1+\xi^X}{\xi^X}} + \left(1-\varphi^X\right)^{\left(-\frac{1}{\xi^X}\right)} \left(\vartheta L_t^{F,X}\right)^{\frac{1+\xi^X}{\xi^X}} \right]^{\frac{\xi^X}{1+\xi^X}}, \quad \forall X \in \{N,T\}$$

where φ^X is the share of informal labor in aggregate labor of sector "X", ξ^X the elasticity of substitution between formal and informal labor in sector "X". Recall ϑ is the productivity premium of formal workers, as documented in Amaral & Quintin (2006); Granda & Hamann (2015). The labor union in each sector must pay wages to the employment agencies of informal and formal workers, which they finance with the paycheck, $w_t^X L_t^X \quad \forall X \in \{N,T\}$, the representative firm in each sector transfers to its respective union. In that sense, the generalized problem of the labor unions is:

$$\max_{L_t^{I,X}, L_t^{F,X}} w_t^{I,X} L_t^{I,X} + w_t^{F,X} L_t^{F,X} - w_t^X L_t^X, \quad \forall X \in \{N,T\}$$

Replacing the first-order conditions in the CES function that creates each sector's labor bundle, one can derive sectoral wage indexes:

$$w_t^X = \left[\varphi^X \left(w_t^{I,X}\right)^{1+\xi^X} + \left(1-\varphi^X\right) \left(\vartheta^X w_t^{F,X}\right)^{1+\xi^X} \right]^{\frac{1}{1+\xi^X}}$$

Firms

The firms in both sectors, tradables and nontradables, produce and sell their goods in perfectly competitive markets. Additionally, both types of firms benefit from a public capital externality, k_{t-1}^G, supplied by the government with efficiency Φ.

Nontradables Production

The representative firm in this sector combines labor, L_t^N, and capital, K_{t-1}. In production, the firm also faces a variable sector-specific productivity, ζ_t^N, that follows an AR(1) exogenous process. Therefore, the production function is:

$$y_t^N = \zeta_t^N \left(\Phi k_{t-1}^G\right) \left(L_t^N\right)^{\alpha^N} \left(K_{t-1}\right)^{1-\alpha^N}$$

where parameter α^N is the share of labor in nontradables production. The representative firm's optimization problem consists of maximizing its profits:

$$\Pi_t^N = p_t^N y_t^N - w_t^N L_t^N - r_t K_{t-1}$$

Tradables Production

In the tradables sector, the representative firm uses only labor, L_t^T, given its sector-specific productivity, ζ_t^T.[13] The tradables production function is:

$$y_t^T = \zeta_t^T \left(\Phi k_{t-1}^G\right) L_t^T$$

The optimization problem of the representative firm is to maximize its profits:

$$\Pi_t^T = p_t^x y_t^T - w_t^T L_t^T,$$

where p_t^x is the relative price of exports. As in Bi et al. (2016b), all tradables production is exported and local demand for tradables is satisfied through imports bought at the real exchange rate, s_t. This entails that terms-of-trade, ς_t, can be easily modeled like an exogenous AR(1) process and defined as:

$$\varsigma_t \equiv \frac{p_t^x}{s_t}$$

Government

The government obtains revenues by levying taxes on aggregate formal income (labor, capital and firms' profits),[14] and issuing a one-period non-contingent bond, b_t, for risk-neutral foreign investors that demand it at price, q_t.[15] It uses these revenues to pay for the debt service, public consumption, G_t, and public investment, i_t^G. This investment is included in the law of motion for public capital: $k_t^G = \left(1-\delta^G\right) k_{t-1}^G + i_t^G$. As in the private sector, both public consumption and public investment are bundles of tradables and nontradables goods, giving rise to the relative prices p_t^G and p_t^{IG}, respectively. Therefore, the government's budget constraint is:

$$\tau_t \left(w_t^F L_t^F + r_t K_{t-1} + \Pi_t^N + \Pi_t^T\right) + q_t s_t b_t = (1-\Delta_t)\, s_t b_{t-1} + p_t^G G_t + p_t^{IG} i_t^G \qquad (4)$$

Notice that, since the bond is sold in the foreign market, its value in terms of domestic goods must be $q_t s_t b_t$. Moreover, it is key to highlight that at period $t+1$, the government pays as debt service one unit of foreign goods if there is no default on the intensive

[13] For the sake of keeping the nonlinear solution of the model more parsimonious, we assume that the production in tradables does not include capital.

[14] Since we will assume that taxes are constant in the simulations, modelling just one tax rate that is levied on all formal households' income eases the exposition of the structure and facilitates a more general assessment of the tax base responses to various shocks or structural changes. Future work could expand the scope of our analysis to evaluate changes in tax rates and in such a study differentiating tax rates by income source may be valuable.

[15] All debt is assumed to be in foreign currency, which is an extreme version of the stylized fact documented in Hausmann et al. (2001).

margin, Δ_{t+1}. It is worth highlighting that deciding on what fraction of outstanding debt to default is the only endogenous action made by the government in this economy, as the tax rate, public consumption and public investment follow the exogenous AR(1) processes:

$$G_t = \rho^g G_{t-1} + (1-\rho^g)\bar{G} + \varepsilon_t^g$$

$$\tau_t = \rho^\tau \tau_{t-1} + (1-\rho^\tau)\bar{\tau} + \varepsilon_t^\tau$$

$$i_t^G = \rho^{i^G} i_{t-1}^G + \left(1-\rho^{i^G}\right)\bar{i^G} + \varepsilon_t^{i^G}$$

Lastly, following the sovereign default literature, (Arellano, 2008; Bi et al., 2016b),[16] public debt is demanded by risk-neutral foreign investors, which entails that:

$$q_t = \frac{\mathrm{E}_t\left(1-\Delta_{t+1}\right)}{R_t^*},$$

where R_t^* is the risk-free real interest rate. Thus, the equation above implies that the price of government's bonds includes a premium on the risk-free rate depending on the expected government's default. Furthermore, this risk-free rate is modeled through a regime-switching process with two transient states following Bianchi et al. (2016). This enables the model to explore the implications on fiscal sustainability of persistent changes in global liquidity that reflect on this rate. In that sense, the risk-free rate can take two values: one of high liquidity, low interest rate and a high bond price ($R^{*,Low}$); and another of low liquidity, high interest rate and low bond price ($R^{*,High}$):

$$R_t^* = \begin{cases} R^{*,low} & \text{if } S_{R,t} = 1 \\ R^{*,High} & \text{if } S_{R,t} = 2, \end{cases}$$

where the transition between regimes is described by the probability transition matrix:

$$\pi^{S_R}\left(S_R'|S_R\right) = \begin{bmatrix} p^R & (1-p^R) \\ (1-q^R) & q^R \end{bmatrix} \quad \text{for} \quad \{p^R, q^R\} \in [0,1]$$

Goods Bundles

Consumption and investment by both private agents and the government are spent in composite bundles of tradable and nontradable goods and services. This aggregation is performed with a CES technology of the general form:

$$Z_t = \left[\Psi_Z^{\frac{1}{\aleph^Z}}\left(Z_t^N\right)^{\frac{\aleph^Z-1}{\aleph^Z}} + (1-\Psi_Z)^{\frac{1}{\aleph^Z}}\left(Z_t^T\right)^{\frac{\aleph^Z-1}{\aleph^Z}}\right]^{\frac{\aleph^Z}{\aleph^Z-1}}, \quad \forall \quad Z \in \{c, i, G, i^G\}$$

[16] For a detailed discussion on non-risk-neutrality see Lizarazo (2013).

where Z_t^T and Z_t^N are the spending on tradables and nontradables, respectively, of the Z's goods bundle. Also, parameter Ψ_Z is the weight of nontradables on the bundle and parameter $\aleph^Z$ the elasticity of substitution. Using this technology, the generalized intratemporal problem of an agent at the moment of consuming or investing is:

$$\max_{Z_t^N, Z_t^T} P_t^Z Z_t - \left(P_t^N Z_t^N + S_t Z_t^T\right), \quad \forall \quad Z \in \{c, i, G, i^G\}$$

Notice that in this problem the nominal price of nontradables is $\left(P_t^N\right)$ and of tradables is the exchange rate (S_t), irrespective of the type of goods bundle. The solution to this problem yields for each bundle the two optimal relative demands for tradables and nontradables. Replacing each pair on the corresponding CES aggregator, and dividing all nominal prices by the price of the private consumption bundle to obtain relative prices (labeled with lowercase "p"), the generalized price indices equal:

$$p_t^Z = \left[\Psi_Z \left(p_t^N\right)^{1-\aleph^Z} + (1-\Psi_Z)\left(s_t\right)^{1-\aleph^Z}\right]^{\frac{1}{1-\aleph^Z}}, \quad \forall \quad Z \in \{c, i, G, i^G\}$$

Notice that for private consumption, $Z = c$, the relative price, p_t^Z, ends up being equal to 1.

Market Clearing

First, notice that because firms in both sectors operate in perfect competition, in equilibrium their profits must equal zero:

$$\Pi_t^N = \Pi_t^T = 0$$

The economy's total output in local goods units is equal to:

$$y_t = p_t^N y_t^N + \varsigma_t s_t y_t^T,$$

Using this GDP definition, the government's budget constraint and aggregating both types of households' budget constraints, one can obtain a balance-of-payment definition:

$$c_t + p_t^i i_t + \frac{\kappa}{2}\left[\frac{i_t}{K_{t-1}} - \delta\right]^2 K_{t-1} + p_t^{iG} i_t^G + p^G G_t - y_t = q_t s_t b_t - (1-\Delta_t) s_t b_{t-1},$$

From which, one can define for convenience the trade balance deficit as:

$$tb_t = q_t s_t b_t - (1-\Delta_t) s_t b_{t-1}$$

Finally, the clearing condition for nontradable goods must be that:

$$y_t^N = \left(p_t^N\right)^{-\aleph} y_t^{N,D},$$

where the aggregate demand for nontradables, $y_t^{N,D}$ is equal to the sum of the shares of nontradables spending in each expenditure group:

$$y_t^{D,N} = \Psi_c\left[c_t\right] + \Psi_G\left[\left(p_t^G\right)^{\aleph} G_t\right] + \Psi_i\left[\left(p_t^i\right)^{\aleph^i} i_t + \frac{\kappa}{2}\left[\frac{i_t}{K_{t-1}} - \delta\right]^2 K_{t-1}\right] + \Psi_{iG}\left[\left(p_t^{iG}\right)^{\aleph^{iG}} i_t^G\right]$$

Fiscal Limit Distributions and Fiscal Space

The computation of the state-dependent fiscal limit distribution follows the procedure detailed in Bi et al. (2016b). Under that framework, a fiscal limit is defined as the maximum debt level, in units of foreign goods, that a government is able to service entirely (i.e., with zero default). The discounted present value of the stream of maximum fiscal surpluses at each future period, conditional on the initial state of the economy, is that maximum debt level. That approach yields the formula:

$$\mathcal{B}^{max}\left(S_{t+i}\right)=\sum_{i=0}^{\infty}\theta\frac{1}{R_{t+i}^{*}s_{t+i}^{max}\left(S_{t+i}\right)}\left(T_{t+i}^{max}\left(S_{t+i}\right)-p_{t+i}^{G}G_{t+i}-p_{t+i}^{iG}i_{t+i}^{G}\right)$$

where $\mathcal{B}^{max}\left(S_{t+i}\right)$ is the fiscal limit, and $T_{t+i}^{max}=\tau^{max}\left(w_{t+i}^{F}L_{t+i}^{F}+r_{t+i}K_{t+i-1}\right)$ and S_{t+i} are, respectively, the maximum fiscal surplus and the model's state-space at period $t+i$. Also, note that fiscal surpluses are discounted with the exogenous time-varying risk-free rate R_t^* and the endogenous real exchange rate s_{t+i}^{max} when surpluses are maximized. The inverse of these two elements acts as the discount factor because the government only issues foreign debt for risk-neutral investors. Moreover, together they capture the influx of common vulnerabilities of SOEs regarding fiscal limits: real revaluation of external debt due to exchange rate depreciation (Calvo et al. (2003, 2011)), and sudden and persistent increases in financing costs after contractions (or expansions) in global liquidity (Byrne et al. (2011); Blankenau et al. (2001)). Additionally, we include a constant political risk factor, θ, that captures the government willingness to pay rather than its mere ability to do so. Another key point in the formula above, is how maximum fiscal surpluses are computed. Following Bi et al. (2016b), in this model we use as the maximum tax rate, τ^{max}, the highest observed tax revenue as a share of GDP in our sample (2003-2019). Another common approach in the literature is to set, for every period, the tax rate at the peak of a Dynamic Laffer Curve, as done by Bi (2012); Hürtgen (2020); Mendez-Vizcaino & Moreno-Arias (2021). In those papers, this is possible because the model's tractability enables authors to analytically find the revenue-maximizing tax rate. Nonetheless, we depart from this methodology because: i) our model does not offer such tractability and ii) more importantly, the theoretical revenue-maximizing tax rates are usually far above those observed in the data, which is not consistent with the weak revenue collection prevalent in developing economies (Bi et al. (2016b)). Finally, in the literature of fiscal limit distributions it is common to define a country's fiscal space as the difference between the current debt to GDP ratio and the debt limit to GDP associated with a cumulative default probability of 5%. Hence, the fiscal space is a number in percentage points of GDP that serves as a simple metric of the current fiscal sustainability of a government.

3 Calibration

Assigning values to the parameters in a medium-sized DSGE model is a difficult endeavor. Therefore, the strategy to tackling this challenge consists of two steps. First, some parameters that are not easily mapped to available data are given standard values in the literature. Second, the model's deterministic steady state is calibrated to reproduce some features of the Colombian economy at a yearly frequency. Then, the endogenous parameters are used to compute the nonlinear policy function of the model through the procedure specified in Appendix B. The model is solved and simulated at a yearly frequency. Colombia has a high degree of informality regardless of the measure one wants to use (see 1.1). The share of formal workers in the total amount of workers Υ is set to 0.45. For this parameter, we favored the criterion of reported levels of healthcare and pension contributions, which are compulsory for every legal labor contract in Colombia. Nevertheless, using series of wage labor from both Colombia's Central Bank (CCB) and Colombia's National Bureau of Statistics (DANE, by its Spanish initials) one obtains average ratios (2010-2023) of wage labor to total labor of 42% and 48.6%, respectively. The midpoint between these two numbers is 45%, coinciding with our selected criterion. As a matter of fact, including informality brought some additional challenges in the calibration of the model's labor market. First, with regard to labor unions, the share of informal labor in each sector is hard to determine by the very nature of the variables involved. Nevertheless, we were able to cross-reference the self-reported informality of workers in the main longitudinal household survey collected by DANE with the sector classification of reported labor activities performed internally by the CCB.[17] These estimates show that the ratios of informal employment to total employment in tradables and nontradables are 72% and 55%, respectively, hence we set $\varphi^T = 72\%, \varphi^N = 0.55$. As for the elasticity of substitution between informal and formal labor is assumed to be the same in both sectors, and we set them to $\xi^N = \xi^T = 1.50$ following Arango et al. (2022). The parameters that govern the productivity premium of formal labor in both sectors, Ω^N, Ω^T are calibrated in the steady state to match the observed formal-to-informal hourly wage ratios in each sector (1.917 in tradables and 1.104 in nontradables). With reference to employment agencies, the share of nontradable labor to informal and formal labor total supplies of hours worked, $\gamma^I = \gamma^F$, are endogenized in the steady state to enable to model to show the observed ratios of formal wage mass to each sector's total wage mass. These ratios are 0.5819 for nontradables and 0.7065 for tradables. The wage mass and hourly wage ratios come from the same database used to find the sector-specific shares of formal labor. Lastly, the elasticity of substitution between sectors in both employment agencies are the same and equal to -1, so that the CES aggregator collapse to a Cobb-Douglas $\left(\xi^I = \xi^F = -1\right)$. On the firms side, we normalize the share of labor in the tradables sector to 1, since it is the only input we model. This same share in the nontradables sector is calibrated to fix

[17] The further classification into tradables and nontradables follows the same logic of the classification we made for GDP. The databases involved are available upon request.

private investment at a value consistent with a long-run private consumption to GDP of 60% and a trade balance to GDP of 1%. Recall that in our model that the latter is equivalent to the long-run fiscal deficit, because only the government is able to issue foreign debt, and currently the government is committed to keep the deficit at 1% of GDP. The depreciation rate and investment adjustment cost parameters are set to be $\kappa = 1.7$ and $\delta = 0.1$, following Hamann et al. (2021). In our baseline calibration, public capital efficiencies in both sectors are normalized to one, $\Phi^N = \Phi^T = \Phi = 1$.On the households side, their discount factor β is equal to 0.9878, consistent with Hamann et al. (2021) and other structural models for the Colombian economy like Arango et al. (2022). We choose a standard value in the literature for the relative risk aversion coefficient and set it equal for both types of households $\sigma^I = \sigma^F = 2$. The inverse of Frisch elasticities are also taken from Hamann et al. (2021), which are $\upsilon^I = 2.4$ and $\upsilon^F = 3.7$. Considering the aggregation of goods bundles, we equalize all the elasticity of substitution between tradables and nontradables for consumption and investment, both private and public, to take the same value, $\aleph = \aleph^G = \aleph^I = \aleph^{IG} = 0.44$, which Bi et al. (2016b) use for Argentina. To gauge the nontradables share for each of these baskets we use estimations for the Colombian economy based on national accounts. For private consumption $\Psi_c = 0.7$, which is equal to the CCB's own classification for the CPI, and for private investment $\Psi_i = 0.6$, consistent with the estimation of Cárdenas-Hurtado et al. (2018) with data for Colombia during 2001-2017. We take the additional home bias that Bi et al. (2016b) assume for current spending with respect to private consumption (0.13) and add it to our shares for the private counterparts of public consumption and investment, $\Psi_{IG} = 0.73, \Psi_G = 0.83$. As for the exogenous processes, we only set one realization for the tax rate and calibrate the constant value, $\bar{\tau}$, to match the average fiscal observed for the Colombian economy (-2.7%). We also assume only one state for the nontradables productivity shock and its mean values, $\zeta_t^X = \bar{\zeta}, all X \in \{N, T\}$, are calibrated to ensure the model yields in the steady state the average ratios of tradable output and nontradable output to GDP published by DANE's National Accounts. However, the exogenous processes for these shocks are specified, as is common in the literature (e.g., Hernandez & Mendoza (2017)). For the tradables productivity shock, we discretize the cyclical component of the HP-filtered Colombian GDP from 1980-2021, from the WEO data. we obtain a standard deviation of the shock $\sigma^{\zeta^T} = 0.0278$, and the autocorrelation $\rho^{\zeta^T} = 0.56$, and normalize the mean of the shock to 1, $\bar{\zeta^T} = 1$. For the terms of trade shock, we use the HP filtered data of the Terms of Trade Index of the Colombian Central Bank (Banco de la República de Colombia) from the producer-price approach from 1980 to 2022. We obtain a standard deviation of the shock $\sigma^{\varsigma} = 0.0598$, the autocorrelation $\rho^{\varsigma} = 0.85$, and normalize the mean of the shock to 1, $\bar{\varsigma} = 1$. For public consumption and public investment, we use data from the Colombian Ministry of Finance for General Government. For public investment, we use government expenditure in capital. Due to data limitations, we use data from 2007 to 2019, and obtain a standard deviation of the shock $\sigma^{I^G} = 0.163$, the autocorrelation $\rho^{I^G} = 0.375$, and the mean of public investment as share of GDP, $\bar{I^G} = 0.062$. For public consumption, we use government expenditure net of expenditure in capital and debt interests, the process we discretize consists of a standard deviation of the shock $\sigma^G = 0.067$, the autocorrelation coefficient $\rho^G = 0.192$, and the mean of public consumption as share of GDP, $\bar{G} = 0.177$. Note that even if the autocorrelation of the public investment is higher than public consumption, it is considerably more volatile (the coefficient of variation is nearly seven times greater in public investment). All the processes are discretized with a standard Tauchen & Hussey (1991) quadrature algorithm. In relation to the foreign investors' risk-free rate, we use the same calibration as in Bianchi et al. (2016), thus the high-liquidity regime implies a gross real-risk-free rate of $R^{*,Low} = 0.967$ (and a $q^{High} = 1.033$) and the low-liquidity regime a gross real-risk-free rate of $R^{*,High} = 1.014$ (and a $q^{Low} = 0.986$). In addition, during the last 10 years, Colombia's foreign public debt has increased considerably due to terms of trade deterioration, political pressures, recessions like the COVID-19 pandemic, etc. The pandemic was particularly detrimental to Colombia's fiscal sustainability, and in 2021 the country lost its investment-grade rating (note: S&P, Fitch Ratings, and Moody's reduced their ratings on Colombian sovereign debt during the course of 6 months).[18] For the political risk parameter θ, we define a value based on the reported in Bi et al. (2016b) for Argentina (0.665). Due to our data limitations we do not have availability on the International Country Risk Guide's (ICRG) political risk index for Colombia, but we do have information on the Institutional Investor Index (III) for both Colombia and Argentina for the biannual series from 1980 to 2016. This index, although might differ from the ICRG's index captures overall riskiness perceived by investors on the emerging countries. The index rates the countries between 0 and 100, 100 being the best rating a country can obtain, which could be associated with lower risk. The average III for Argentina is 32.9, while that for Colombia is 47.4, meaning that Colombia has approximately a relatively 44% greater rating than Argentina. Applying this value to the one reported in Bi et al. (2016b), would suggest a political-risk parameter of $\theta = 0.958$ for Colombia, which might be consistent with the fact that Argentina has defaulted on its sovereign debt 4 times from 1900 to 2014, while Colombia does not have an external default episode for this period (Reinhart & Rogoff, 2010).

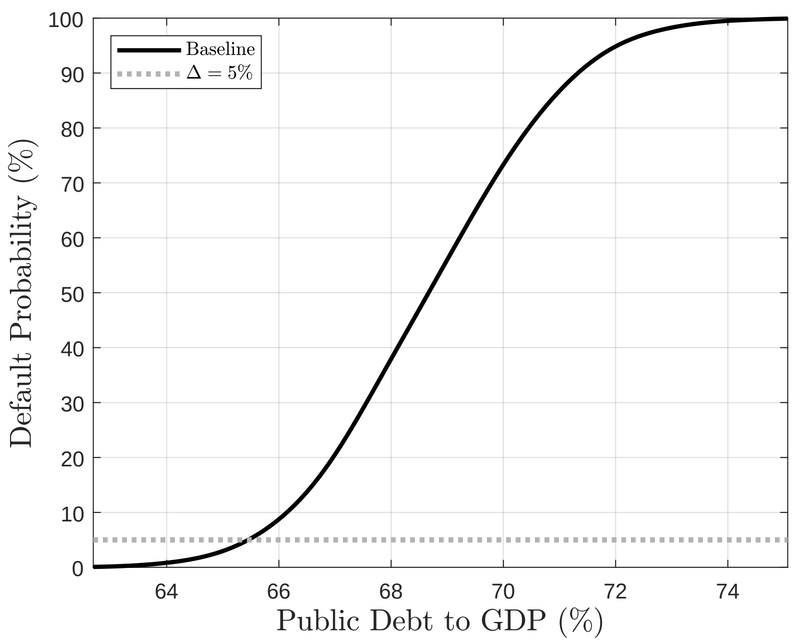
4 Quantitative Analysis

A country's Fiscal Limit Distribution (FLD) provides valuable information about its fiscal sustainability, hence it is the main quantitative tool for supporting the analyses presented in this section. Because this distribution is built using state-dependent debt limits, such sustainability is influenced by the current fiscal stance and state of the economy, as well as by its long-run, structural characteristics. The characteristics that are of main interest in this paper are public investment, labor informality, and global

[18] In Mendez-Vizcaino & Moreno-Arias (2021), the authors present suggestive evidence that this downgrade had to do with the total loss of Colombia's fiscal space during 2020 and 2021.

liquidity shocks. Therefore, the analysis focuses on assessing the sensitivity of the baseline fiscal distribution to these. For each exercise, the distributions are calculated by performing 1,000 simulations of 10,000 periods to obtain well-defined distributions. The first FLD computed and shown in Figure 8 is the baseline distribution for the Colombian economy, using the benchmark calibration explained in the previous section. A convenient way of analyzing the FLD is by plotting the cumulative density function (CDF) of the distribution, as it allows us to easily figure out the default probability for a given debt level. The different public debt-to-GDP ratios in the support of the distribution are along the x-axis, while the accumulated default probabilities are on the y-axis. The non-uniform shape of the CDF responds to the non-linearities of the model, which makes the default rise very quickly, typically after it reaches the debt level associated with a default probability of 10%.

Figure 3: Baseline Fiscal Limit Distribution



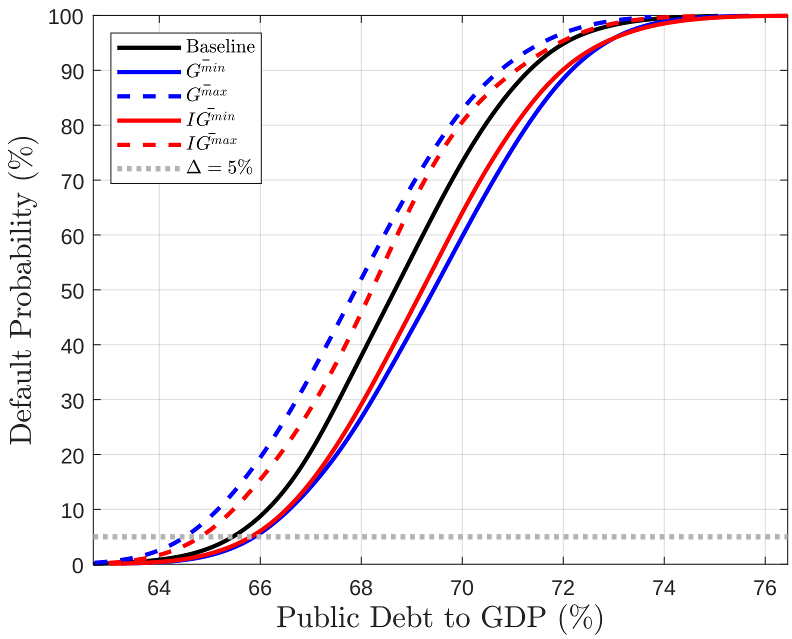
Note: The black solid line describes a scenario in which the variables are on their long-run values for the initial state of the economy. The horizontal gray-dotted line corresponds to a reference line which depicts the 5% probability of default, the usual framework in the literature on fiscal limit distributions.

In particular, for Colombia, debt-to-GDP ratios below 64% yield a default probability close to 0%. Nonetheless, notice how, when the government exceeds this cut-off value, default probability rises rapidly describing a convex path until a little below 70%, from which point it continues drawing a concave path up until the 99% default probability at a 74% debt-to-GDP ratio. As said in subsection , it is common in this literature to summarize the relative position of a country with respect to a risky fiscal scenario by calculating its fiscal space. That metric is defined as the difference between the country's current debt-to-GDP ratio and the ratio with a 5% default probability. The baseline exercise estimates that at a 5% default probability, the sustainable public debt limit is close to 65.5%. As of 2022, Colombia's debt was 62.9% of GDP, which means that its fiscal space is a narrow 2.6% of GDP. Both figures are not far from the values reported recently in Mendez-Vizcaino & Moreno-Arias (2021), which estimated that Colombia's fiscal limit at 5% was of 60.8% in 2020 and its fiscal space of -2.2%. Beyond the different model structures of that paper and this one, some events have transpired that make sense of the differences. On the upside, the post-pandemic recovery of the Colombian economy during 2021 and 2022 helped recover the fiscal revenues lost during 2020 and reduce the debt ratio, which favors the fiscal space. On the downside, in the last two years the Colombian peso has suffered a strong depreciation, causing a negative valuation effect on its foreign debt, and government expenditure, especially due to the downward rigidity of the transfers instated during the pandemic, has remained high. Together these facts help explain why fiscal space, though positive, is still very narrow (it was 21% in 2019, according to Mendez-Vizcaino & Moreno-Arias (2021)).

4.1 Public consumption vs. Public Investment

The first sensitivity exercise explores the effects of assuming different initial state values for public consumption or public investment, as well as the underlying mechanisms that modify the computed FLDs. The blue lines represent simulations in which the initial state of public consumption changes, while the red lines represent a different initial state for public investment. When the initial state of the economy changes with respect to the long-run value (17.7%), there is a direct effect on the fiscal balance that pushes the distribution in the same direction of the change. Consequently, lower (higher) public consumption or investment shifts the FLD rightwards (leftwards) as less (more) pressure is put on the fiscal balance. However, the magnitude of the fiscal balance shift differs between public consumption and public investment. Diminishing public investment to its minimum (3.91%) entails, at a 5% default probability, almost the same expansion of the fiscal limit distribution (50bps) as cutting down public consumption to its minimum (14.6%). Conversely, at the 5% probability increasing public consumption does cause a larger contraction of the FLD than spending more on investment, which confirms the non-linear nature of the model and, particularly, of public capital. Recall that public capital brings about a positive externality on production, which is reinforced as it builds up through higher investment. This creates a multiplier effect when the government allocates more resources on investment, thus mitigating the pressure put on the fiscal balance by the additional spending. This explains that, for a default probability of 5%, the debt limit is 30pbs higher when public investment rises than when public consumption does.

Figure 4: Fiscal Limit: Effects of Variations in Public Consumption and Public Investment



Note: The black solid line describes a scenario in which the variables are on their long-run values for the initial state of the economy. The blue (red) lines depict scenarios in which we only vary the initial state of public investment (consumption) to its minimum (dashed lines) or maximum values (dotted lines), keeping the rest of the remaining states at their long-run values. As before, the horizontal gray-dotted line corresponds to a reference line which depicts the 5% probability of default.

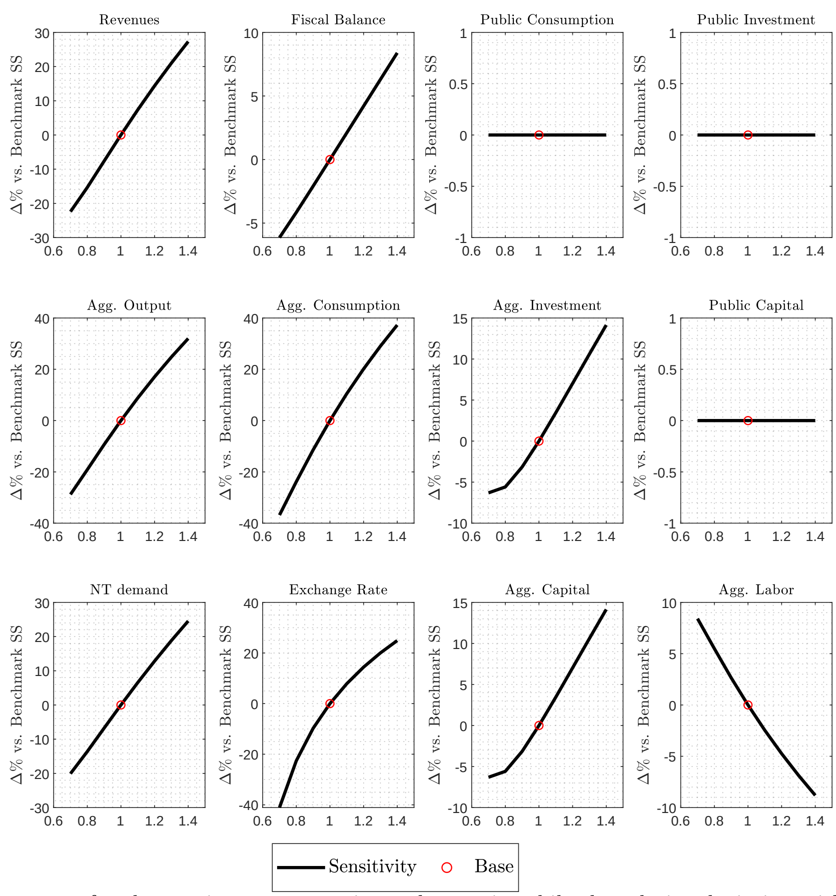
Notice that, irrespective of the type of public expenditure, spending more contracts the FLD and reducing it shifts out the distribution. Although both public investment and public consumption have positive first-round demand effects, neither of the two yields long-run fiscal multipliers greater than one. In the model, public consumption creates greater demand for tradable and nontradable goods, which ultimately stimulates demand for productive factors and improves households' income. Nevertheless, this is true for both types of households, formal and informal, but only the former contribute to the tax system. Speaking just of the first-round channel, this means that for every dollar spent in public consumption, a part of it will not come to enlarge the tax base and increase revenue collection. Moreover, because formal households are Ricardian and feature KPR preferences, they will internalize the adverse wealth-effect by increasing their labor supply, pushing wages down and partially depleting the tax base.

Although these mechanisms apply for both types of spending, the aforementioned positive externality of public capital helps to keep the tax base higher.[19] Particularly, it does so by weakening the effect on wages due to the Ricardian response of formal households. Since public capital enters both production functions, the marginal product of

[19]See Appendix C for the policy functions on variations to public consumption and public investment.

labor is positively affected by it. As a consequence, the extra dollar in public investment shifts out labor demand curves in both sectors beyond the initial shift caused by the first-round increase in demand. The size of this effect depends on the intensity of the externality, which is governed by the so-called public capital efficiency captured by parameter ϕ.

Figure 5: Steady State Sensitivity to Public Capital Efficiency



Note: The support for the varying parameter is on the x-axis, while the relative deviation with respect to the baseline steady state is on the y-axis. The black solid line depicts the functional form obtained when varying the parameter across the support considered, which facilitates the understanding of the way the model responds. The red dot is located at the baseline steady state.

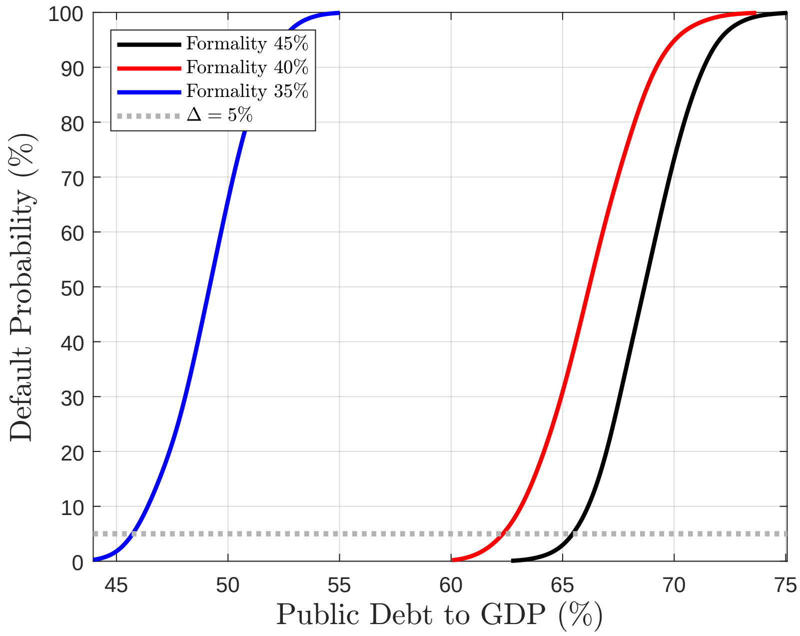
The importance of public capital efficiency for fiscal sustainability can be better illustrated by examining the deterministic steady state's sensitivity to parameter ϕ. Figure 5 depicts how aggregate economic and fiscal variables monotonically increase with efficiency beyond $\phi = 0.6$. Importantly, because public investment is fixed at its baseline steady state and foreign debt is the adjustment variable in long-run fiscal consolidation (note the varying fiscal balance), this exercise abstracts from the first-round demand effect and isolates the supply-side benefit of public investment. Therefore, output, nontradable demand, and fiscal revenues are all increasing because public capital is more efficient in production and not because the government is spending more money. In fact, the fiscal balance monotonically improves with higher ϕ as factor prices increase, generating second-round demand effects, and improving fiscal revenue in spite of the fall in aggregate labor. These results highlight that, if public investment is efficient enough, cutting investment could be counterproductive. The figure points out that for an efficiency level that yields a fiscal multiplier greater than one, reducing investment spending would deteriorate the fiscal balance and, in turn, tighter debt limits. However, the other side of this argument, is that investing in more productive public capital, even without necessarily having to invest more, could enhance fiscal sustainability. The better public investment is allocated, the most likely it is to increase public spending (even on both investment and consumption goods) and at the same time increase debt sustainability.

4.2 Labor Informality

Figure 6 depicts the sensitivity of the FLD to lowering the formality share by 5pp (red line) and 10pp (blue line). As expected, higher informality contracts the fiscal limit distribution. However, the non-linearity of this effect might not be as evident. Notably, at the benchmark 5% probability level, the blue line is about 5 times farther away from the red line than the latter is from the baseline scenario (16.5 vs. 3.5%, respectively). It is straightforward to see that reducing the mass of formal agents directly erodes the tax base and reduces revenue collection. This first-round mechanism puts pressure on the primary fiscal balance and, for the same level of default probability, the fiscal authority will be able to service a lower amount of debt.

The striking magnitude of the nonlinear effect of informality on the FLD is harder to trace. The explanation, though, involves other channels that are activated after formality is lowered, such as a fall in the aggregate productivity of the economy given that the more productive workers (formal) are scarcer. When the mass of formal workers is reduced, their relative aggregate supply falls with respect to that of informal workers, pushing up their wages. Initially, labor unions in the two sectors see the fall in the extensive margin of formal labor and the rise of informal labor and try to stimulate formal labor on the intensive margin by raising their wages relative to informal wages. However, as there is less formal labor, the labor bundle is less productive, which makes firms reduce their demand for labor. The aggregate fall in labor demand forces unions to reduce both wages and demand for hours worked. Additionally, the fall in labor causes nontradables firms to reduce their demand for capital, which translates into a lower rental rate of capital and less private investment. Lower income from capital and wages causes formal consumption to fall as well, which reduces aggregate demand and output.

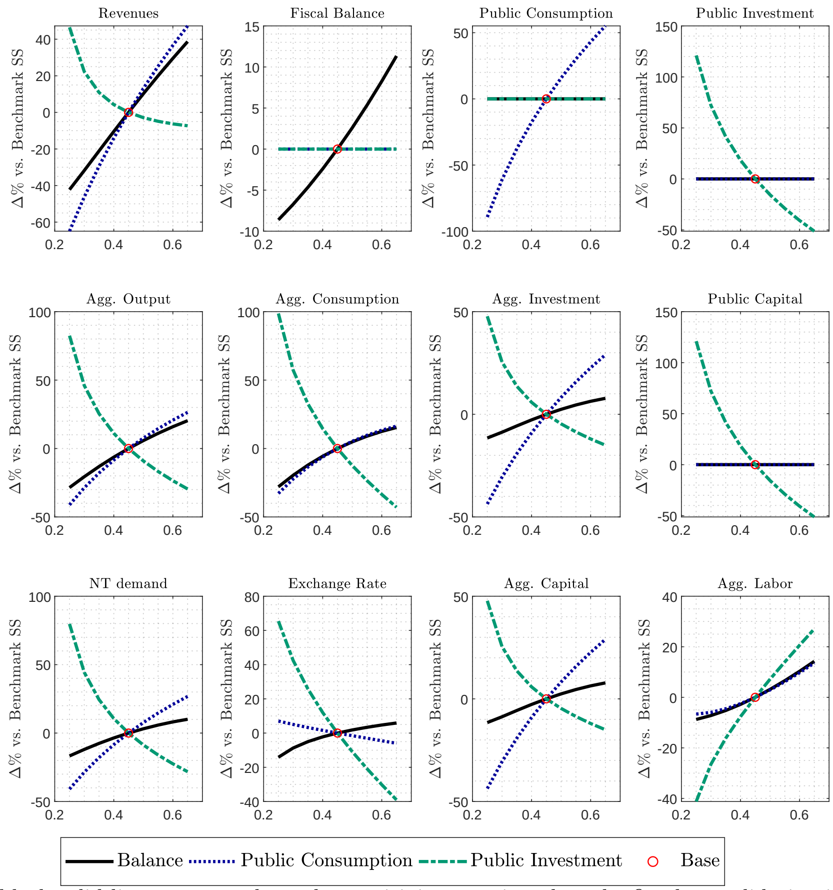
Figure 6: Fiscal Limits: Effects of Varying the Share of Formality



Note: The black solid line describes a scenario in which the variables are on their long-run values for the initial state of the economy. The red (blue) lines refer to scenarios in which we vary the shares of formality to 40% (35%), keeping the rest of the remaining states at their long-run values. The horizontal gray-dotted line corresponds to a reference line which depicts the 5% probability of default.

The key insight to understand this process is that the fall in consumption causes formal households to offer more hours worked, pushing wages further down. Due to the form of the utility function, the wealth effect on labor supply does not exhibit a constant elasticity of substitution with respect to consumption, while featuring a convex response to wages. As a consequence, the fall in formal wages ends up being a convex and decreasing function of the formality rate. Since the government collects revenues according to $\tau_t \left(w_t^F L_t^F + r_t K_{t-1} \right)$, the convex response of wages plus the fall in the rental rate of capital explains the nonlinearity of the deterioration of the fiscal balance as a function of the labor informality rate. Some of the mechanisms explained above can be confirmed looking at Figure 10 in Appendix C. The fiscal deterioration stemming from increasing labor informality comes with a magnifying of "free-riding" problem. At lower formality rates, fiscal multipliers fall as the government is carrying out spending that expands a narrower tax base. As this tax base falls with informality in a convex fashion, so do long-run fiscal multipliers. The effects of labor informality on the tax base have been widely discussed in the literature and policy discussions, but its adverse impact on fiscal multipliers has not been in the spotlight as much, nor have the joint repercussions of these mechanisms for fiscal sustainability. Put together, these results lay out the compelling, less common argument that pursuing informality-reducing policies can significantly improve debt sustainability, especially when small changes in labor informality rate appear to have highly nonlinear effects on FLD.

Figure 7: Steady State Sensitivity to Formality with Different Consolidations



Note: The black solid line corresponds to the sensitivity exercise when the fiscal consolidation is performed through the fiscal balance (equivalently, debt). The dotted blue line corresponds to a fiscal consolidation with public consumption as instrument and the dashed line with public investment. Consolidation through either spending meets the restriction that the fiscal balance should remain constant, thus any variation in revenue due to formality must be compensated by the appropriate response in spending.

The role of informality in fiscal multipliers is of special relevance for public investment. In this case, formality acts as an enhancer of the fiscal returns of public investment. From the government's perspective, increasing formality could bring about the higher tax revenues promised by public capital efficiency. Figure 7 illustrates the different ways in which the economy responds to varying formality among three possible fiscal consolidations. First notice the black line, which is a scenario in which every dollar gained in tax revenue thanks to rising formality goes to repay debt. In that case, the fiscal balance monotonically improves with formality and aggregate variables rise along the way. This is actually what is behind the computation of the FLD, since in that case fiscal instruments beside debt are exogenous, and that is why as formality changes the state-dependent debt limits react. The blue line shows the economy's response when the consolidation is done by adjusting public consumption. As the tax base increases thanks to formality, there is upward pressure on the fiscal balance–as indicated by the black line–that must be compensated by increasing public consumption. This holds along the interval considered for formality shares, even though it does not match the convexity of the fiscal balance in the black line as one could expect. The reason for this discrepancy is the appreciation of the exchange rate, which in turn is explained by the home bias in public consumption (highest of the economy). Additionally, exchange rate appreciation favors investment, as it is the least home-biased of all good bundles and this promotes private capital build-up. The most interesting outcome arises when fiscal consolidation is carried out through public investment. It is notable that most red lines show variables having an inverse response relative to the other two lines, especially because most variables are decreasing, including public investment.[20] Contrary to what could be read at first sight, this occurs because formality is indirectly increasing the efficiency of public capital and making public investment more productive in fiscal terms, but fiscal balance cannot improve. For instance, starting with the baseline level of public investment, when formality increases that same investment level enhances a wider tax base and fiscal balance should improve. To prevent this, public investment should be reduced and offset the initial positive impact on the tax base and other economic variables. By the same token, if informality is higher, then public investment must be increased to mitigate the tax base reduction. Furthermore, the enhancing effects of formality regarding public investment are nonlinear, as can be seen by the convex response of this fiscal instrument to formality. The efforts that government must undertake in terms of investment when informality rises are increasingly higher, whereas it becomes so powerful as formality rises that a relatively smaller reduction keeps fiscal balance constant.

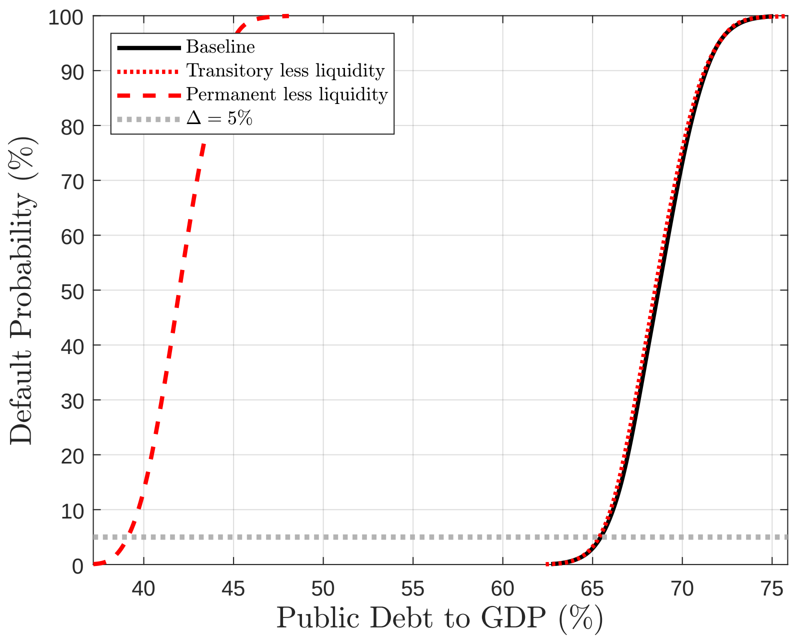
4.3 Global Liquidity

According to Byrne et al. (2011), fiscal sustainability in developing economies like Colombia is highly influenced by global liquidity. Accordingly, this section discusses the impact on the Colombian fiscal limit distribution of fluctuations in world interest rates. Shocks to global liquidity can be either permanent or transitory, and our model elucidates how much this distinct nature can modify the FLD and fiscal space. In particular, the model is shocked with a positive shock to the risk-free interest rate with which foreign investors price Colombian sovereign debt. Figure 8 depicts these results for the baseline FLD and the two alternative natures of the global liquidity shock. First, the transitory contraction of global liquidity is simulated by changing the initial state of the risk-free rate. Under this approach, we define the initial state of the risk-free interest rate at its maximum value, but we let it fluctuate onward following the regime switching dynamics described by the probability transition matrix, $\pi^{S_R}\left(S_R'|S_R\right)$ in Section 2. Second, the permanent change in global liquidity is captured by modifying

[20]The only variable behaving differently is aggregate labor, which once again owes its response to the wealth effect on labor supply for formal households.

the risk-free interest rate and treating it as a parameter onward. In this way, primary surpluses are discounted permanently at a higher interest rate in the computation of the FLD.

Figure 8: Fiscal Limit: Effects of Permanent or Transitory Variations in Global Liquidity



Note: The black solid line describes a scenario in which the variables are on their long-run values for the initial state of the economy. The red lines refer to scenarios in which we only vary the initial state of the risk-free rate to its minimum in a transitory way (dotted lines), or permanently (dashed lines), keeping the rest of the remaining states at their long-run values. The horizontal gray-dotted line corresponds to a reference line which depicts the 5% probability of default.

Remarkably, the leftward shift of the distribution after the transitory shock is negligible (less than 1%), albeit the regime-switching process for the risk-free interest rate is highly absorbent at the low-liquidity regime. A plausible explanation for this result is that transitions between regimes are infrequent but possible, particularly during a very long horizon like the one employed in the computation of the distribution (10,000 periods). If it does fluctuate and if it does so differently across the thousand simulations performed, the initial value of this exogenous state becomes less important in the construction of the fiscal limit as it affects the discounting of a very limited stream of fiscal balances. In contrast, the simulations with the permanent higher interest rate are markedly stronger than the transitory scenario. The fiscal limit distribution contracts by 25% at the 5% probability level, in addition to making the CDF steeper. This sensitivity is strong, but it is consistent with the fact that a permanent shock to global liquidity affects the discounting of the entire set of fiscal balances involved in the computation. In this exercise we focus on the worst-case scenario, a contraction of global liquidity and the implication that when it is permanent debt sustainability suffers a major hit. Nevertheless, it is possible to take a more glass-half-full stance and see that when global liquidity is perceived as permanently higher, foreign investors will allow the country to run very high debt levels. But we focused on the negative shock for a reason. As was mentioned earlier, our estimations suggest that Colombia's fiscal space is a little above 2%, which means that a rapid reversal on low interest rates that have prevailed since the Global Financial Crisis, could mean that the country ends up with a negative fiscal space of 23%. In Mendez-Vizcaino & Moreno-Arias (2021), the authors present suggestive evidence that link the probability of losing the investment grade of sovereign debt with the loss of its fiscal space. What we show with these simulations is that, in the face of this sort of shocks, a permanent contraction can entail that imminent default is just around the corner.

5 Conclusion

This paper presented a theoretical framework with several characteristics typical in developing economies that together influence fiscal sustainability. By integrating public capital, labor informality, and global liquidity shocks, the model unveils their joint impact on fiscal performance and provides a tool to quantify it. A high informality rate, for example, can erode the tax base, reducing the fiscal benefits of expanding public investment and diminishing public capital efficiency in terms of fiscal revenue. However, this outcome crucially hinges on the initial state of the economy, since the productivity of additional public investment depends on its efficiency, the existing stock of public capital, and the initial rate of labor informality. These nonlinear interactions emphasize the importance of employing global methods to precisely compute fiscal limit distributions. The model is calibrated for Colombia to provide some quantitative analyses and shed light on underlying mechanisms through sensitivity exercises, but the framework can be easily used for any economy. Notably, these exercises challenge the conventional approach of cutting back on public investment during fiscal stress, as it can actually be counterproductive for debt sustainability. A decrease in public capital may lead to an output decline that contracts the tax base beyond the initial spending reduction, thereby contracting the fiscal limit distribution. A complementary but broader finding that emerges in the analysis is that fiscal multipliers are a negative function of informality rates and that this is a relevant consideration for fiscal sustainability. In general, the framework facilitates a comprehensive quantitative assessment of fiscal policies, highlighting the importance of structural factors and current economic conditions. Furthermore, we are careful not to assert in the paper that reducing public investment universally contracts fiscal space. Instead, the paper points out that there are reasonable parameter combinations that may yield this outcome. Additionally, it stresses that public policies and institutional reforms targeting enhanced public capital efficiency or reduced labor informality can expand a country's distribution of fiscal limits, strengthening fiscal resilience against adverse external shocks, such as permanent contractions of global liquidity. Future research avenues include estimating the linearized version of the model (like Bi et al. (2016a)) to tackle empirical challenges associated with hard-to-determine or unobservable parameters, such as labor-variety elasticity of substitution and informality rates. Robust estimation would enable a more comprehensive assessment of a country's distribution of fiscal limits and its fiscal space. Another avenue entails expanding the model to incorporate endogenous labor informality decisions, as in Amaral & Quintin (2006), providing insights into labor informality-reducing reforms' impact on fiscal sustainability, an aspect often overlooked in these types of discussions.

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Appendix

A Model's Equilibrium Conditions

Informal Households

$\left[c_t^I\right]$:

$$\lambda_t^{I,c} = \left(c_t^I\right)^{-\sigma^I}$$

$\left[l_t^I\right]$:

$$w_t^{I,}\lambda_t^{I,c} = \phi^I \left(l_t^I\right)^{\upsilon^I}$$

Formal Households

$\left[c_t^F\right]$:

$$\lambda_t^{F,c} = \left(c_t^F\right)^{-\sigma^F}$$

$\left[l_t^F\right]$:

$$\left(1-\tau_t^l\right) w_t^F \lambda_t^{F,c} = \phi^F \left(l_t^F\right)^{\upsilon^F}$$

$[i_t]$:

$$Q_t \equiv \frac{\lambda_t^i}{\lambda_t^c} = p_t^I + \kappa\left(\frac{i_t}{k_{t-1}} - \delta\right)$$

$[K_t]$:

$$Q_t = \beta \mathrm{E}_t \frac{\lambda_{t+1}^c}{\lambda_t^c}\left\{Q_{t+1}\left(1-\delta\right) + \left(1-\tau_{t+1}^k\right) r_{t+1} + \kappa\left(\frac{i_{t+1}}{k_t} - \delta\right)\frac{i_{t+1}}{k_t} - \frac{\kappa}{2}\left(\frac{i_{t+1}}{k_t} - \delta\right)^2\right\}$$

Firms

$\left[L_t^N\right]$:

$$\alpha^N p_t^N y_t^N = w_t^N L_t^N$$

$[K_{t-1}]$:

$$\left(1-\alpha^N\right) p_t^N y_t^N = r_t K_{t-1}$$

$\left[L_t^T\right]$:

$$\alpha \varsigma_t s_t y_t^T = w_t^T L_t^T$$

Employment Agencies

$\left[L^{I,N}\right]$:

$$L_t^{I,N} = \gamma^I \left(\frac{w_t^{I,N}}{w_t^I} \right)^{\xi^I} L_t^I$$

$\left[L^{I,N}\right]$:

$$L_t^{I,N} = \left(1 - \gamma^I\right) \left(\frac{w_t^{I,T}}{w_t^I} \right)^{\xi^I} L_t^I$$

$\left[L^{F,N}\right]$:

$$L_t^{F,N} = \gamma^F \left(\frac{w_t^{F,N}}{w_t^F} \right)^{\xi^F} L_t^F$$

$\left[L^{F,N}\right]$:

$$L_t^{F,N} = \left(1 - \gamma^F\right) \left(\frac{w_t^{F,T}}{w_t^F} \right)^{\xi^F} L_t^F$$

Labor Unions

$\left[L_t^{I,T}\right]$:

$$L_t^{I,T} = \left(\varphi^T\right) \left(\frac{w_t^{I,T}}{w_t^T} \right)^{\xi^T} L_t^T$$

$\left[L_t^{F,T}\right]$:

$$L_t^{F,T} = \left(1 - \varphi^T\right) \left(\frac{w_t^{F,T}}{w_t^T} \right)^{\xi^T} L_t^T \frac{1}{\vartheta}$$

$\left[L_t^{I,N}\right]$:

$$L_t^{I,N} = \left(\varphi^N\right) \left(\frac{w_t^{I,N}}{w_t^N} \right)^{\xi^N} L_t^N$$

$\left[L_t^{F,N}\right]$:

$$L_t^{F,N} = \left(1 - \varphi^N\right) \left(\frac{w_t^{F,N}}{w_t^N} \right)^{\xi^N} L_t^N \frac{1}{\vartheta}$$

B Solution Algorithm

1. Start with a guess of formal capital, $k^F_{t,j}$, relative price of non-tradables, $p^N_{t,j}$, and work in non-tradables in formal and informal, $L^{I,N}_{t,j}$, $L^{F,N}_{t,j}$, and formal work in tradables, $L^{F,T}_{t,j}$ for iteration $j = 1$

2. Find work in non-tradables using the labor union aggregation

$$L^N_{t,j} = \left[\varphi^{N\left(-\frac{1}{\xi^N}\right)} \left(L^{I,N}_{t,j} \right)^{\frac{1+\xi^N}{\xi^N}} + \left(1-\varphi^N\right)^{\left(-\frac{1}{\xi^N}\right)} \left(\vartheta L^{F,N}_{t,j} \right)^{\frac{1+\xi^N}{\xi^N}} \right]^{\frac{\xi^N}{1+\xi^N}}$$

3. Use $p^N_{t,j}$ to find the exchange rate, $s_{t,j}$

$$1 = \left[\Psi_c \left(p^N_t\right)^{1-\aleph} + (1-\Psi_c)\left(s_t\right)^{1-\aleph} \right]^{\frac{1}{1-\aleph}}$$

$$s_{t,j} = \left[\frac{1-\Psi_c \left(p^N_{t,j}\right)^{1-\aleph}}{(1-\Psi_c)} \right]^{\frac{1}{1-\aleph}}$$

4. Use $p^N_{t,j}$ and the exchange rate, $s_{t,j}$to find the price of government consumption, $p^G_{t,j}$, price of government investment, $p^{IG}_{t,j}$, and relative price of investment, $p^I_{t,j}$.

$$p^G_{t,j} = \left[\Psi_G \left(p^N_{t,j}\right)^{1-\aleph} + (1-\Psi_G)\left(s_{t,j}\right)^{1-\aleph} \right]^{\frac{1}{1-\aleph}}$$

$$p^i_{t,j} = \left[\Psi_i \left(p^I_{t,j}\right)^{1-\aleph^i} + (1-\Psi_i)\left(s_{t,j}\right)^{1-\aleph^i} \right]^{\frac{1}{1-\aleph^i}}$$

$$p^{iG}_{t,j} = \left[\Psi_{iG} \left(p^{iG}_{t,j}\right)^{1-\aleph^{iG}} + (1-\Psi_{iG})\left(s_{t,j}\right)^{1-\aleph^{iG}} \right]^{\frac{1}{1-\aleph^{iG}}}$$

5. Also, use the work in tradables, $L^N_{t,j}$, the relative price of non-tradables $p^N_{t,j}$ and the exchange rate, $s_{t,j}$, and the exogenous state variables to find the work in tradables, $L^T_{t,j}$

$$L^T_{t,j} = \left(L^N_{t,j}\right)^{\frac{1+\left(1-\alpha^N\right)\xi}{1+(1-\alpha^T)\xi}} \left[\left(\frac{(1-\varphi)}{\varphi} \right)^{\frac{1}{\xi}} \left(\frac{\alpha^T \varsigma_t s_{t,j}}{\alpha^N p^N_{t,j}} \right) \left(\frac{\zeta^T_t \left(\Phi k^G_{t-1,j}\right)}{\zeta_t \left(\Phi k^G_{t-1,j}\right)\left(L_{t-1,j}\right)^{1-\alpha^N}} \right) \right]^{\frac{\xi}{1+(1-\alpha^T)\xi}}$$

6. With $L^N_{t,j}$, and $L^T_{t,j}$ and the exogenous state variables, find $y^N_{t,j}$, and $y^T_{t,j}$,

$$y^N_{t,j} = \zeta_t \left(\Phi k^G_{t-1}\right) \left(L^N_{t,j}\right)^{\alpha^N} \left(K_{t-1}\right)^{1-\alpha^N}$$

$$y^T_{t,j} = \zeta^T_t \left(\Phi k^G_{t-1,j}\right) \left(L^T_{t,j}\right)^{\alpha^T}$$

7. Now, we can compute the prices of factors $w_{t,j}^N$, $w_{t,j}^T$, $r_{t,j}$

$$w_{t,j}^N = \frac{\alpha^N p_{t,j}^N y_{t,j}^N}{L_{t,j}^N}$$

$$r_{t,j} = \frac{\left(1-\alpha^N\right) p_{t,j}^N y_{t,j}^N}{K_{t-1}}$$

$$w_{t,j}^T = \frac{\alpha \varsigma_t s_{t,j} y_{t,j}^T}{L_{t,j}^T}$$

8. We can now compute the aggregate CES of wages, $w_{t,j}$ and the total output, $y_{t,j}$

$$w_{t,j} = \left[\varphi \left(w_{t,j}^N\right)^{1+\xi} + (1-\varphi)\left(w_{t,j}^T\right)^{1+\xi}\right]^{\frac{1}{1+\xi}}$$

$$y_{t,j} = p_{t,j}^N y_{t,j}^N + \varsigma_t s_{t,j} y_{t,j}^T$$

9. Use the guess of capital, $K_{t,j}$, to find investment, $i_{t,j}$ and the Tobin's Q, $Q_{t,j}$,

$$i_{t,j} = K_{t,j} - (1-\delta) K_{t-1}$$

$$Q_{t,j} = p_{t,j}^I + \kappa \left(\frac{i_{t,j}}{K_{t-1}} - \delta\right)$$

10. Formal and informal households and labor union problems related to job supply

    (a) Given the initial guesses and the non-tradables wage find the informal and formal non-tradable wage, $w_t^{I,N}, w_t^{F,N}$, which needs a non-linear solver solution

$$w_t^{I,N} = \left[\frac{\left(w_t^N\right)^{1+\xi^N} - \varphi^N \left(w_t^{I,N}\right)^{1+\xi^N}}{1+\xi}\right]^{\frac{1}{1+\xi^N}} \left[\frac{\left(1-\varphi^N\right) L_t^{I,N}}{\left(\varphi^N\right) L_t^{F,N}}\right]^{1/\xi^N}$$

$$w_t^{F,N} = \left[\frac{\left(w_t^N\right)^{1+\xi^N} - \varphi^N \left(w_t^{I,N}\right)^{1+\xi^N}}{1+\xi}\right]^{\frac{1}{1+\xi^N}}$$

    (b) With the initial guess of formal work devoted to tradables, $L_t^{F,T}$, find the informal work devoted to tradables $L_t^{I,T}$

$$L_t^{I,T} = \left[\frac{\left(L_t^T\right)^{\frac{1+\xi^T}{\xi^T}} - \left(1-\varphi^T\right)^{\left(-\frac{1}{\xi^T}\right)} \left(\vartheta L_t^{F,T}\right)^{\frac{1+\xi^T}{\xi^T}}}{\left(\varphi^T\right)^{\left(-\frac{1}{\xi^T}\right)}}\right]^{\frac{\xi^T}{1+\xi^T}}$$

(c) With the informal decisions of work tradables and the guess of work in nontradables find $w_t^{I,T}$, the informal wage, w_t^I, informal labor, L_t^I, informal consumption, c_t^{INF}

$$w_t^{I,T} = w_t^{I,N} \left[\frac{\left(1-\gamma^I\right)}{\left(\gamma^I\right)} \frac{L_t^{I,T}}{L_t^{I,N}} \right]^{\frac{1}{\xi^I}}$$

$$w_t^I = \left[\gamma^I \left(w_t^{I,N}\right)^{1+\xi^I} + \left(1-\gamma^I\right) \left(w_t^{I,T}\right)^{1+\xi^I} \right]^{\frac{1}{1+\xi^I}},$$

$$L_t^I = \left[\gamma^{I\left(-\frac{1}{\xi^I}\right)} \left(L_t^{I,N}\right)^{\frac{1+\xi^I}{\xi^I}} + \left(1-\gamma^I\right)^{\left(-\frac{1}{\xi^I}\right)} \left(L_t^{I,T}\right)^{\frac{1+\xi^I}{\xi^I}} \right]^{\frac{\xi^I}{1+\xi^I}},$$

$$c_t^I = w_t^I L_t^I$$

(d) With the formal households decisions with respect to labor in tradables and non-tradables and the initial guesses find formal wages in tradables: $w_t^{F,T}$, aggregate formal wages, w_t^F, aggregate formal labor, L_t^F

$$w_t^{F,T} = w_t^{F,N} \left[\frac{\left(1-\gamma^F\right)}{\left(\gamma^F\right)} \frac{L_t^{F,T}}{L_t^{F,N}} \right]^{\frac{1}{\xi^F}}$$

$$w_t^F = \left[\gamma^F \left(w_t^{F,N}\right)^{1+\xi^F} + \left(1-\gamma^F\right) \left(w_t^{F,T}\right)^{1+\xi^F} \right]^{\frac{1}{1+\xi^F}},$$

$$L_t^F = \left[\gamma^{F\left(-\frac{1}{\xi^F}\right)} \left(L_t^{F,N}\right)^{\frac{1+\xi^F}{\xi^F}} + \left(1-\gamma^F\right)^{\left(-\frac{1}{\xi^F}\right)} \left(L_t^{F,T}\right)^{\frac{1+\xi^F}{\xi^F}} \right]^{\frac{\xi^F}{1+\xi^F}},$$

11. Use the combination of government's BC and BoP definition to find formal consumption, $c_{t,j}^F$ and total consumption

$$k_t = \left(\Upsilon\right) k_t^F$$

$$i_t = \left(\Upsilon\right) i_t^F$$

$$c_t^F = \frac{y_t - p_t^I i_t + \frac{\kappa}{2}\left[\frac{i_t}{K_{t-1}} - \delta\right]^2 K_{t-1} - \tau_t^l w_t^F L_t^F - \tau_t^k r_t K_{t-1} - \left(1-\Upsilon\right) w_t^I L_t^I}{\Upsilon}$$

$$c_t = \left(1-\Upsilon\right) c_t^I + \left(\Upsilon\right) c_t^F$$

12. Compute domestic demand

$$y_{t,j}^{D,N} = \Psi\left[c_{t,j}\right] + \Psi^{G}\left[\left(p_{t,j}^{G}\right)^{\aleph^{G}} g_{t,j}\right] + \Psi^{I}\left[\left(p_{t,j}^{I}\right)^{\aleph^{I}} i_{t,j} + \frac{\kappa}{2}\left[\frac{i_t}{K_{t-1}} - \delta\right]^2 K_{t-1}\right] + \Psi^{IG}\left[\left(p_{t,j}^{IG}\right)^{\aleph^{IG}} i_{t,j}^{G}\right]$$

13. Use formal and informal consumption to find marginal utility of consumption,

$$\lambda_t^{I,c} = \left(c_t^I\right)^{-\sigma^I}$$

$$\lambda_t^{F,c} = \left(c_t^F\right)^{-\sigma^F}$$

14. Interpolate to find variables a $t+1$: Capital, $K_{t+1,j}$, relative price of non-tradables, $p_{t+1,j}^N$, and work in non-tradables, $L_{t+1,j}^N$. formal and informal, and formal work in tradables and then repeat above steps to find the rest of the variables at $t+1$

15. **Update 1:** Use the FOC with respect to capital to find Tobin's Q at time t, for iteration $j+1$, and then the definition of Tobin's Q to update capital at time t, for iteration $j+1$

$$Q_{t,j+1} = \beta \mathrm{E}_t \frac{\lambda_{t+1,j}^{F,c}}{\lambda_{t,j}^{F,c}}\left\{Q_{t+1,j}\left(1-\delta\right) + \left(1-\tau_{t+1}^{k}\right) r_{t+1,j} + \kappa\left(\frac{i_{t+1,j}}{K_{t,j}} - \delta\right)\frac{i_{t+1}}{K_t} - \frac{\kappa}{2}\left(\frac{i_{t+1}}{K_t} - \delta\right)^2\right\}$$

$$K_{t,j+1} = \left[\frac{\left(Q_{t,j+1} - p_{t,j}^{I}\right)}{\kappa} + 1\right] K_{t-1}$$

16. **Update 2:** Use the non-tradable market clearing to update the price of non-tradables for iteration $j+1$

$$p_{t,j+1}^{N} = \left(\frac{y_{t,j}^{N}}{y_{t,j}^{N,D}}\right)^{-\frac{1}{\aleph}}$$

17. **Update 3:** Use the FOC with respect to labor to find formal and informal labor for iteration $j+1$ and then the demand of non-tradable formal and informal labor and demand for formal tradable to update it for iteration $j+1$

$$L_{t,j+1}^{F} = \left[\frac{\left(1-\tau_t^{l}\right)}{\phi^F} w_{t,j}^{F} \lambda_{t,j}^{F,c}\right]^{\frac{1}{v^F}}$$

$$L_{t,j+1}^{F,N} = \gamma^F \left(\frac{w_{t,j}^{F,N}}{w_{t,j}^{F}}\right)^{\xi^F} L_{t,j+1}^{F}$$

$$L^{F,T}_{t,j+1} = \left(1 - \gamma^F\right) \left(\frac{w^{F,T}_{t,j}}{w^F_{t,j}}\right)^{\xi} L^F_{t,j+1}$$

$$L^I_{t,j+1} = \left[\frac{1}{\phi^I} w^I_{t,j} \lambda^{I,c}_{t,j}\right]^{\frac{1}{v^I}}$$

$$L^{I,N}_{t,j+1} = \gamma^I \left(\frac{w^{I,N}_{t,j}}{w^I_{t,j}}\right)^{\xi^I} L^I_{t,j+1}$$

18. Check iteration $j+1$ with respect to iteration j, to check convergence

$$\epsilon^{L^{F,N}}_j = |L^{F,N}_{t,j+1} - L^{F,N}_{t,j}|$$

$$\epsilon^{L^{I,N}}_j = |L^{I,N}_{t,j+1} - L^{I,N}_{t,j}|$$

$$\epsilon^{L^{F,T}}_j = |L^{F,T}_{t,j+1} - L^{F,T}_{t,j}|$$

$$\epsilon^k_j = |k_{t,j+1} - k_{t,j}|$$

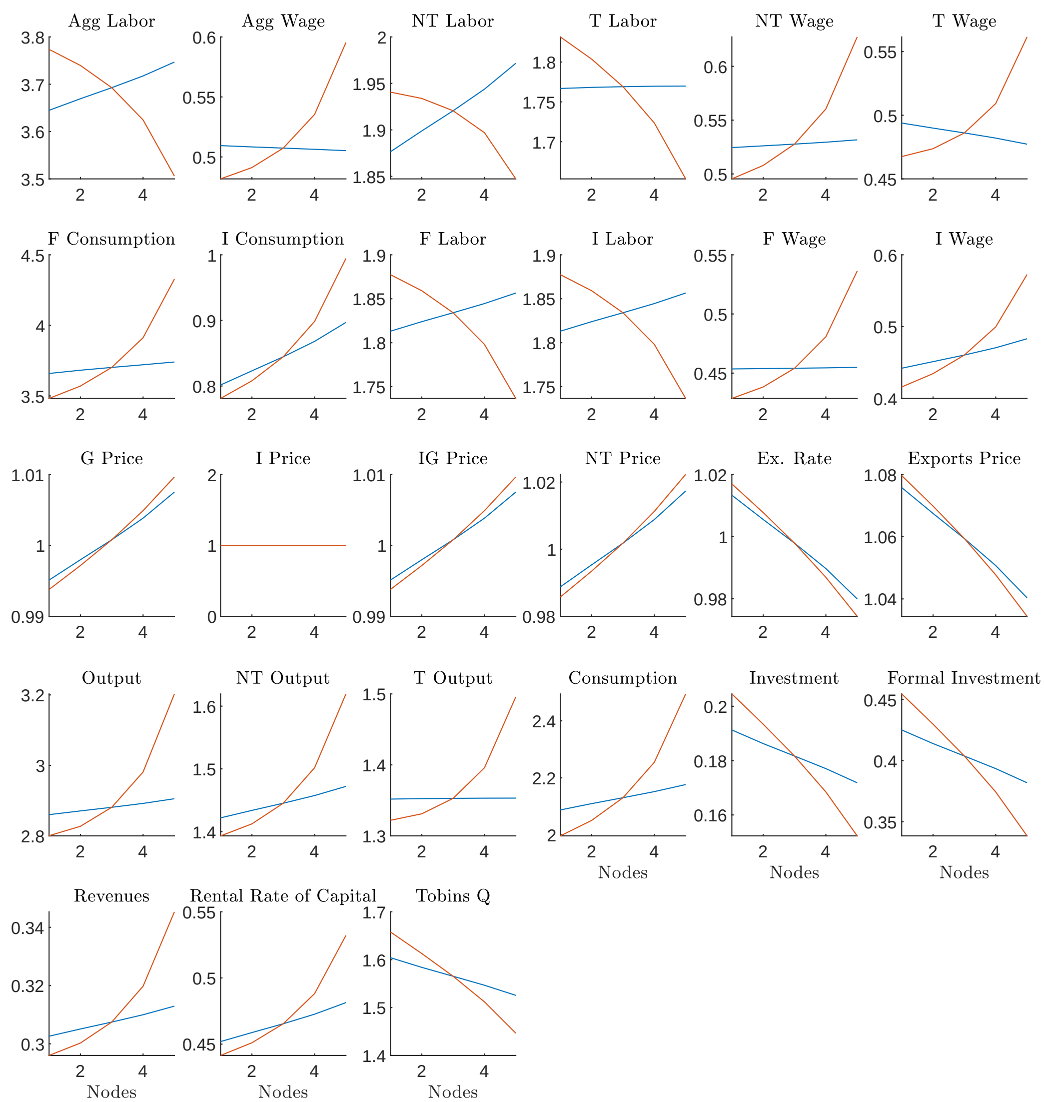
$$\epsilon^{p^N}_j = |p^N_{t,j+1} - p^N_{t,j}|$$

19. Check convergence, if $max\left(\left[\epsilon^{L^{F,N}}_j, \epsilon^{L^{I,N}}_j, \epsilon^{L^{F,T}}_j, \epsilon^k_j, \epsilon^{p^N}_j\right]\right) < tol$, end iterating, otherwise repeat steps 2-16.

C Policy Functions and Complementary Sensitivity Exercises

C.1 Compared Policy Functions for Public Investment and Public Consumption

Figure 9: Policy Functions: Effects of Variations in Public Consumption and Public Investment



Note: The black solid line describes a scenario in which the variables are on their long-run values for the initial state of the economy. The red (blue) lines refer to scenarios in which we vary the shares of formality to 40% (35%), keeping the rest of the remaining states at their long-run values. The horizontal gray-dotted line corresponds to a reference line which depicts the 5% probability of default.

C.2 Sensitivity to Formality

Figure 10: Steady State Sensitivity to Formality Rates