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Abstract*

This paper develops a dynamic model of competitive equilibrium in electricity markets with thermal, hydro and intermittent power sources. Thermal generators have positive and increasing costs and use a marketable input. Hydro generators use a free and uncertain input, but one that is storable. Intermittent renewable generators (solar or wind) use a free, uncertain and non-storable input. The competitive equilibrium is characterized, and it is proven to be Pareto optimal for any given technology mix. It is then proved that the optimal capacity matrix exists, is unique and, under reasonable cost assumptions, involves the three technologies. Moreover, the efficient allocation involves using thermal generation in every period. We calibrate our model with data from three countries with very different capacity matrices and diverse natural characteristics, namely, Argentina, Brazil, and Uruguay. For each country we obtain: i) the unrestricted optimal capacity matrix; and ii) the second best matrix, or the optimal investment in capacity given the current matrix. Finally, the numerical results show that when there is an increase in the share of intermittent sources, the profitability of the thermal and hydro increase after some point, following a U-shape relationship, which suggests that the entry of renewable generators does not compromise the system's reliability.

JEL classifications: D24, L94, Q42
Keywords: Electricity markets, Competitive equilibrium, Hydroelectric generation, Intermittent sources

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1. Introduction

The electricity industry has faced important structural changes in the last three decades. There has been a wave of liberalization, privatization and competition in the energy generation sector. New technologies are entering the energy system, and intermittent sources such as wind and solar now account for a significant share of generation in many countries.

The impact of these structural changes is far from clear, mainly because electricity markets function in a very different way than other more commonly studied markets. There are several market failures specific to this industry: for the most part, electricity is non-storable and non-tradable (outside the grid), supply and demand vary substantially across time and the total potential supply is fixed in the short run. Moreover, the different technologies have very distinct features. While thermal sources have a typical neoclassical production function with increasing marginal costs and marketable inputs, renewables rely on non-marketable inputs that are stochastic and non-storable. Moreover, marginal production costs of hydro generators and other renewables are negligible. This poses a challenge to an industry with a diversified portfolio of generators: how much water to store, and how much to produce in any given period? Additionally, the investment decision is also quite complex. A larger share of renewables implies lower expected prices for consumers but also implies less reliability. Thermal sources provide reliability, but they cannot compete with the low prices of renewables. In a country with a diversified portfolio of technologies, how far from efficiency will the competitive equilibrium be? What is the ideal capacity matrix?

We develop a dynamic stochastic model of energy markets to answer such questions. Our model captures the main features of the energy market and provides a tractable framework to analyze i) how the competitive equilibrium works, ii) the optimal capacity matrix, and iii) the impact of a larger share of renewables, in prices and profitability of other sources. Our theoretical model is testable and can provide insights for policy analysis. We are then able to calibrate our model, and we provide numerical exercises to study the optimal matrix in countries with different characteristics. We also provide policy recommendations for investment in capacity in different economies.

Specifically, in our model energy can be produced through hydro, thermal and intermittent power sources. The behavior of the three different types of power generators is quite different. While thermal generators produce energy based on marketable inputs (coal, oil, natural gas), hydro

and intermittent sources produce power based on non-marketable, renewable and free of costs inputs, stochastically delivered by nature: sunlight, wind, and water. However, in contrast to sunlight and wind, water is storable, and that changes the problem completely. Hydro generators face a dynamic problem: using water in the present may prevent them from generating energy in the future. In this economy, the market price correctly signals hydro generators to ration water in the present for future use. We prove a version of the first welfare theorem for this economy: that is, the market equilibrium is Pareto optimal.

Finally, we simulated an economy where hydro capacity is replaced by renewable capacity. The numerical results show that, when we increase the share of intermittent sources, i) the profitability of the thermal and hydro plants follow a U-shape relationship, ii) the mean price drops due to technology diversification and then rises for a high share of intermittent sources, and iii) the profitability of the renewable sources drops steadily. These findings corroborate the idea of a balanced technology mix in long-term equilibrium.

Centralized electricity production has received many criticisms for quite some time. The pioneering work of Joskow and Schmalensee (1998) pointed to efficiency problems in centralized systems and advocated for full liberalization and competition. In fact, perhaps the major motivation for the wave of liberalization that the industry has undergone over the last 25 years was to introduce competition and to move away from centralized decisions.

There is a large literature that analyzes liberalization of power markets in many different contexts and aspects of it. This ranges from Joskow and Schmalensee (1998) to important contributions such as Green and Newbery (1992) and Wilson (2002), among many others. García, Reitzes and Stacchetti (2001) analyzes competition between two hydro generators, and their main concern is market power. Our paper builds on their set-up and extends their analysis to a continuum of generators in a competitive environment.

Crampes and Moreaux (2001) compute the first best, monopoly and duopoly allocation of a market with a thermal and a hydro plant. Ambec and Doucet (2003) analyze decentralization of a hydroelectric industry and show that, while a monopoly brings market power concerns, a decentralized market may have suboptimal use of water resources. Both of these papers have a negatively sloped demand curve, and generators' optimal production tries to equate marginal revenues across periods. Inefficiencies come largely from the fact that generators may not be able to do so.

Our paper has a fundamental distinction to Crampes and Moreaux (2001) and Ambec and Doucet (2003): generators face a discrete choice problem—to produce or not to produce—and not a continuum problem. Production is a binary variable, as well as water storage. Either the generator is full or it is empty. Our model is not aimed at modelling inefficiencies that may arise from limited storage capacity or market power.[1] Instead, it analyzes how a competitive equilibrium works in a continuum of generators with distinct technologies and where most generators have zero marginal cost. We show that, even in a competitive market, prices never go down to zero.

The paper proceeds as follows. Section 2 lays out a benchmark model with hydro and thermal generators only and characterizes the competitive equilibrium. Section 3 derives an important result concerning the effect of a price cap on the competitive equilibrium. Section 4 introduces intermittent sources and analyzes their effects on market equilibrium. Section 5 proves that the first welfare theorem holds in such a market. The last section concludes.

2. Benchmark Model[2]

2.1 Competitive Equilibrium in a Hydro-Thermal Economy

Consider an infinite horizon model in which there is a unitary inelastic demand for electricity, which might be supplied by a thermal production source or by hydroelectric power source. We assume a continuum of identical hydro sources and a single thermal source. Each thermal source can produce an unlimited amount of energy at an increasing and convex marginal cost. The hydro plants are indexed between $[0,\theta]$, where $2 > \theta > 1$. Each hydro plant can generate energy at zero marginal cost, but is capacity constrained and atomistic, that is, if a set $S \subset [0,\theta]$ is producing energy, the energy produced is given by $e(S) = \int_{i\in S} 1di$.

At each period $t = 1,2,...$, a hydro generator might have full capacity or empty capacity, which we will denote by the binary variable $\{0,1\}$. The hydro plant might decide to sell its energy or not. That is, each hydro plant has a discrete choice, and energy is modeled as an indivisible unit. If it does sell, the plant earns the current market price but will finish the period with empty reservoirs. If it does not sell energy, it will enter the following period with full capacity. With these assumptions, we can focus on the extensive margin of energy production.

[1] Garcia, Reitzes and Stacchetti (2001) analyze market power in a setting like the one we use here.
[2] This section was first developed in Moita and Monte (2018).

In order to capture the aggregate uncertainty that is intrinsic to the production of hydroelectric energy, but not to thermal production, we will assume that, at each period t, there are two possible states of the world. Formally, we assume that with probability $\pi \in [0,1]$, which is drawn independently before every period t, the state of nature is $\omega \in \{G,B\}$. Once the state is drawn, it becomes known to everyone. We eliminate any exogenous idiosyncratic uncertainty and assume that if the state of the world is G, every hydro generator has full capacity, while if the state of the world is B, each hydro generator will have full capacity only if it had full capacity in the previous period and decided not to sell its energy in that period. Otherwise, its capacity is empty. The thermal generator has no capacity constraint and faces the same production costs every period.

We also assume a thermal source, whose (static) objective function is: $\max_e \pi^T = pe - c(e)$, where e is the quantity of energy sold, $c(e)$ is the cost function and p is the current market price. For convenience, we think of the thermal source as a price-taker myopic player, capturing the fact that we are actually modeling a representative generator. The hydro's static profit function is given by $\pi^H = pe$. However, recall that i) each hydro is capacity constrained and atomistic and ii) there is a dynamic link, through the hydro's reservoir capacity, between the current period's profit and the expected continuation payoff starting at the subsequent period.

A public history at time t is denoted by h_t and is a sequence of states of the world and energy sold by each player. The set of all public histories at time t is $\mathcal{H}_t$. A behavioral strategy for a hydro generator is defined as:

$$\sigma_H : \cup_{t=0}^{\infty} \mathcal{H}_t \to \{0,1\},$$

and for the representative thermal generator it is:

$$\sigma_T : \cup_{t=0}^{\infty} \mathcal{H}_t \to \mathbb{R}.$$

We consider a Walrasian market in which firms, hydro plants and thermal sources, are price-takers.[3] A competitive equilibrium in our environment is a sequence of prices such that the market clears on a period-by-period basis—that is, in every history demand meets supply. Moreover, the thermal producer maximizes its myopic profit function, and each hydro plant maximizes its dynamic expected continuation payoff.

[3] There are over a thousand hydro plants in Brazil, which is our prototypical example.

Definition 1 (Equilibrium) *An equilibrium is a map* $p : \cup_{t=0}^{\infty} \mathcal{H}_t \rightarrow \mathbb{R}$ *such that the market clears at every period, the thermal generator is maximizing its per-period payoff and each hydro is maximizing its expected present value profit.*

We will denote hydro i^0s expected continuation profit at the start of any period t for which the state of nature is G by V_G. Recall that in state G, every hydro has full capacity, regardless of what happened in the previous period. In contrast, whenever the state of the world is B, each hydro's capacity is dependent on whether it has sold energy in the previous period or not. With slight abuse of notation, we denote by V_B^1 the value function of a given hydro when the state is B, and the hydro has full capacity, while by V_B^0 we denote the value function of the hydro when the state is B and it has empty capacity. The value function at state G can be written as the maximum between selling and not selling the energy:

$$V_G = \max\left\{p_G + \delta\left(\pi V_G + (1-\pi) V_B^0\right), \delta\left(\pi V_G + (1-\pi) V_B^1\right)\right\}. \tag{1}$$

Regardless of the state of the world, whenever a hydro sells its energy at price p, it empties its reservoir, which means that it will only be able to sell energy again whenever the state of the world becomes G again. Thus, we may write the payoff V_B^0 for any period in which the state is B and the hydro has an empty reservoir as:

$$V_B^0 = \delta\left(\pi V_G + (1-\pi) V_B^0\right). \tag{2}$$

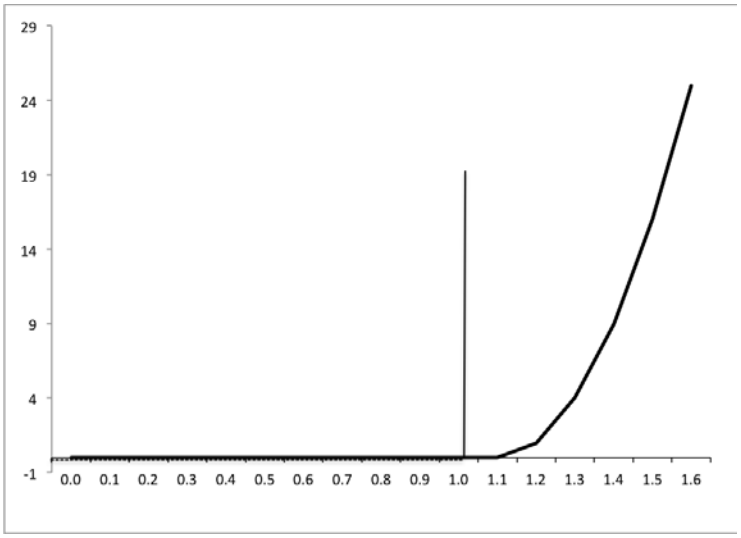
The value function at B when the capacity is full is given by:

$$V_B^1 = \max\left\{p_B + \delta\left(\pi V_G + (1-\pi) V_B^0\right), \delta\left(\pi V_G + (1-\pi) V_B^1\right)\right\}. \tag{3}$$

Before we proceed let us define $p^* = c'(1)$, that is, p^* is the Walrasian price when only the thermal generator supplies the market.

In order to better understand the mechanics of our model, let us first consider the case in which the number of hydro plants selling in a particular given period exceeds the demand. Given that each hydro supplies at a zero marginal cost, the Walrasian price in that period would be zero, as shown in Figure 1.

Figure 1. Only Hydro Production (p=0)



In contrast, if the number of hydro sources selling at a particular period is smaller than the unitary demand, then thermal sources are used to clear the market. Such a scenario is presented in Figures 2 and 3 below.

Figure 2. Equilibrium (t=1): good state

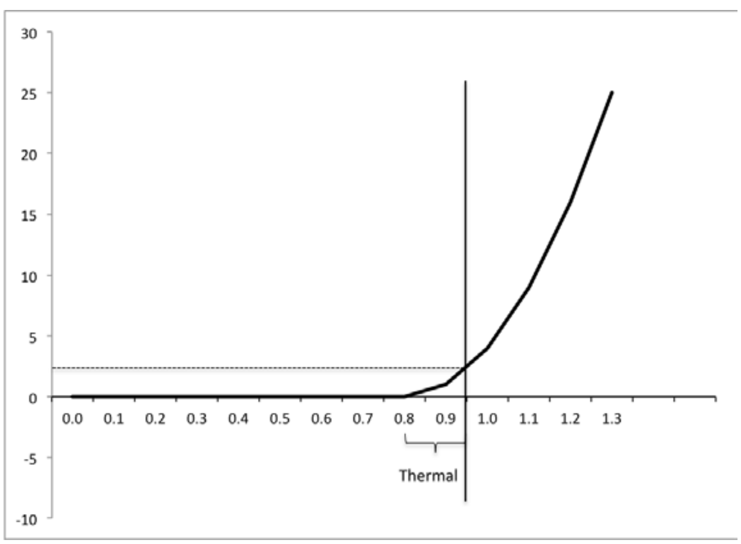
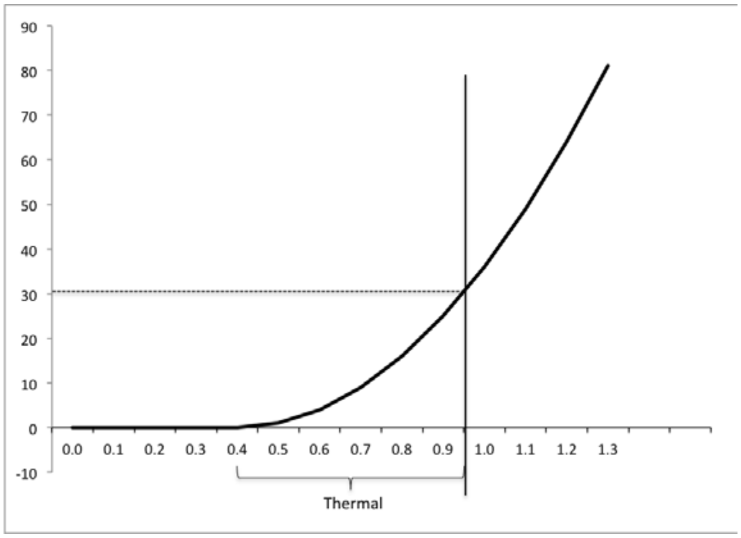


Figure 3. Equilibrium (t=2): Bad State



The market price for such a period in which hydro sources do not supply the entire demand is given by the marginal cost of the thermal production at the market-clearing quantity. In equilibrium, hydro plants must be maximizing their infinite stream of profits discounted by the discount rate $\delta < 1$. For that reason, a price of zero as presented in Figure 1 will not happen in equilibrium, regardless of the history. This is a consequence of the fact that hydro plants are forward-looking and subsequent periods have a positive probability of some water-scarcity. Hydro plants anticipate a positive price in the future and thus prefer to keep water in their reservoir to profit when the price is high. Thus, a first implication of our model is that prices are always positive, despite the fact that at given periods there might be excess supply of zero-marginal-cost suppliers.

Figures 2 and 3 show the prices in a good state and in a bad state, respectively. Note that prices are higher in bad states. That is the intuition of the equilibrium price dynamics in this market.

We are now ready to prove our first two results: first, we prove that an equilibrium always exists in these markets, then we provide a characterization of an equilibrium, that is, a few necessary conditions for equilibrium.

Proposition 1 (Existence) *An equilibrium always exists in this market.*

Proof. The proof is by construction, and we provide it in the Appendix. ■

Lemma 1 (Equilibrium Properties) *An equilibrium must satisfy the following properties:*

i) Prices are weakly lower than the price given by the production of thermal sources exclusively: $\forall h, p_h \leq p^*$.

ii) If the price is the maximal one, $p = p^*$*, then firms have strict incentive to sell energy.*

iii) If at period t firms have strict incentive to sell, then at period $t+1$ *if the state is B* : $p = p^*$.

iv) For any period t, firms weakly prefer to sell energy versus not selling.

v) In equilibrium, at state G, firms are indifferent between selling and not selling energy.

The competitive price is given by the marginal cost of the marginal producer. Recall that in the good state *G*, where all hydro plants have water in their reservoirs, it is not an equilibrium to have only hydro generation. If this were the case, $p = 0$, and the hydro generators would have an incentive to save water for the next period. An important implication of this is that there is always thermal generation, $q_T > 0$, which gives the next lemma.

Lemma 2 (Thermal is the marginal producer) *In equilibrium,* $p_t = C(q_T{}^t)$.

The next lemma characterizes the "off corners" market price dynamics in the competitive equilibrium.[4]

Lemma 3 (Price Dynamics) *Let* p_G *be the price in the good state and* p_B *be the price in a bad state with* $p_B < \delta\,(1-\pi)p^*$. *Then, in equilibrium we have*

$$p_B = \delta\,(1-\pi)p'_{BB} \tag{4}$$

Equation (4) shows the relationship between the price in the good state, p_G, and the price in the bad state, p_B. It shows that the price in the good state is the expected discounted value of the price in the bad state next period. It comes from the fact that hydro generators in the good state are indifferent between selling or not energy. Similarly, equation (4) shows the relationship between

[4] Whenever the water storage is low enough so that the current price using all water left in storage is high enough (namely, when $p > \delta\,(1-\pi)p^*$) the hydro sources with water in storage will strictly prefer to sell their water. For that reason we refer to Lemma 3 as the "off corner" price dynamics.

the price in a bad state following a good state, p_B, and the price in a bad state following a bad state, p_{BB}.

Remark 1 (Price Cap) *To finalize this section, let us consider an important regulatory tool: price caps. Assume that the price of energy is always constrained to be* $p_t \leq \overline{p} < p^*$. *Our next result follows immediately from Lemma 1 (ii).*

Lemma 4 *If* $p_t = \overline{p}$, *then all hydro plants sell their capacity in that period.*

Now, consider a history in which there is a mass μ *of hydro plants with full capacity, and let* $\mu < 1$. *In any such case, if the expected future price is not high enough, the price of energy might be* p^* *(or* $\overline{p}$ *in the case of a binding price cap) in the subsequent period. If, in equilibrium, all hydro plants find it profitable to sell their currently stored reservoirs, that is,* μ *is such that* $e = \mu$, *then we say that we are in an extreme scenario.*

Using Lemma 4 we have that under a binding price cap, hydro plants sell weakly more energy every period than when there is no price cap. This implies that under a price cap, the economy has a higher probability of reaching extreme scenarios. We state this result below.[5]

Proposition 2 Assume that a price cap has been imposed: $\overline{p} < p^*$. *Denote this economy by* E_c *and construct an identical economy, but with no price cap, denoted by* E_f. *Then, whenever the state is G, the energy produced by hydro sources is higher under* E_c *than it is under* E_f, $e_c > e_f$ *moreover,* E_c *has a higher probability of reaching extreme scenarios than f.*

2.2 Intermittent Sources

In our benchmark model we deliberately ignored intermittent power sources. By intermittent sources, we consider all power plants that satisfy two conditions i) a zero marginal cost of producing energy and ii) lack of capacity to store energy. Note that hydro sources satisfy i), but they can store energy in their reservoirs, while thermal sources satisfy ii), but the cost is assumed to be increasing.

Intermittent sources most notably include wind power, solar energy and run-of-river plants (plants that generate hydro energy but have no reservoirs). For environmental reasons, this source

[5] This result is similar to the one obtained by García, Reitzes and Stacchetti (2001), but theirs is a duopoly of hydro generators, while we have a competitive market with a mix of technologies.

of power is generating great interest throughout the world, and it is already quite common in some places. Denmark, for example, generates around 40 percent of its electricity from wind. China, another example, has recently announced ambitious plans to increase wind and solar power capacity. In Brazil, run-of-river plants are common and relatively representative. We will model all of these different sources as a continuum of producers with zero marginal cost and no reservoirs.

In an electricity system with thermal and hydro sources, the entry of intermittent generators has two effects: it decreases the ratio of storage to total capacity, and it diversifies the stochastic processes that generate the input.

In order to have a better understanding of the role of intermittent sources, we will try to disentangle these two effects by analyzing two cases. First, intermittent generators with no storage but the same stochastic process as the hydro sources. We will think of these sources as being run-of-river plants, so that the uncertainty inherent in the production is the same as with the hydro generators. This is useful as a benchmark, but it also provides us with policy recommendations for investment in this type of generator. Second, we study the perhaps more interesting case in which intermittent sources follow a stochastic process that is distinct from that of hydro sources. As an example, wind and rain may have distinct and independent probabilities of occurring. For the first case, we provide an analytical solution, whereas for the second case, the complexity of the problem is substantially increased and, therefore, we solve the model numerically in Section 5.

2.2.1 Run-of-River

Assuming that intermittent sources have mass $\eta < 1$, it is perhaps not surprising that energy should be weakly cheaper in G when all reservoirs are full and the intermittent plants can generate energy.

What is more interesting is that there is a critical level $\bar{\eta}$, such that for any $\eta \leq \bar{\eta}$, the market equilibrium is the same as the one without intermittent sources. Also, this critical capacity level is equal to the amount of energy that is produced by the hydro plants in state G when there is no intermittent capacity installed.

Proposition 3 (Intermittent Power Plants: Threshold capacity for run-of-river plants)
i) There is a critical level of intermittent capacity $\bar{\eta}$*, such that for any* $\eta \leq \bar{\eta}$*, the market equilibrium is unchanged with respect to the benchmark case.*
ii) Let e_h *be the energy generated by the hydro plants in the benchmark case. Then we have,*
$\bar{\eta} = e_h$.

The intuition behind this result is the following. The energy from the intermittent sources will be used in the good state anyway. If their capacity is less than or equal to e_h, the energy produced in state G is still e_h, so that the share of hydro that saves water to the next period is $\theta - e_h$, as in the benchmark case.

For cases where $\bar{\eta} > \eta$, the equilibrium differs. In order to understand the effects of intermittent sources, consider two economies E_i and E_j with a mass of intermittent sources lower in E_i : $\eta_i < \eta_j$. To have a comparable situation, we assume that the total installed capacity of hydro plus intermittent is the same in the two economies, and only the share of each technology differs. That is,

$$\theta_i + \eta_i = \theta_j + \eta_j, \tag{5}$$

and hence, $\theta_i > \theta_j$. Also, for the next proposition assume that at least $\eta_j > e_h$, or both η_i and η_j are greater.

Proposition 4 (Intermittent Power Plants: Higher price volatility) *Consider two economies* E_i *and* E_j *with a mass of intermittent sources lower in* E_i : $\eta_i < \eta_j$, *where* $\eta_j > \eta$. *Also assume that* $\theta_i + \eta_i = \theta_j + \eta_j$. *Then we have the following:*

- *i)* *The equilibrium prices are lower in* E_j *in periods G:* $p_G^j < p_G^i$.
- ii) *The equilibrium prices are higher in* E_j *in periods B:* $p_B^j > p_B^i$.
- iii) *Price volatility is higher in* E_j.

This result has one important policy implication. Competitive energy prices in the good state will be lower with more intermittent sources operating. With no interference, this lower price will signal intermittent sources not to enter the market until perhaps demand increases and it becomes profitable for these sources to enter again.

A subsidy is a common policy for intermittent and environmentally friendly sources. It induces entry of intermittent sources, and it reduces energy prices, as shown. These lower prices require further incentives to induce further entry of clean generators.

Up to this point, the stochastic process governing the delivery of hydro and intermittent energy is identical. It simplifies the problem greatly, but it is not realistic. We will relax this assumption in Section 6, when we will analyze the effect of intermittent sources through a

numerical simulation of the social planner's problem. But to do this, we need to guarantee that the first welfare theorem holds, meaning that the solution to the planner's problem gives rise to the same allocation as that of a market economy. The next section shows that the first welfare theorem holds.

3. Social Planner

3.1 Production Plan

The social planner chooses a production plan to minimize the intertemporal sum of discounted costs.

$$W_t = \sum_{t=0}^{\infty} \delta^t C_t(Q), \tag{6}$$

in which $C_t(Q)$ represents the total cost of producing Q units of energy.

We look at the planner's problem considering a capacity θ of hydro and R of renewable sources. We will treat both θ and R as state variables since this will be essential to study the long-run optimal policy, that is, when the planner is allowed to choose the optimal energy matrix. We will do this in Section 4. In this section we will focus on the case where the matrix is given. This is the short-run problem: for a given matrix, what is the optimal policy of the planner?

We assume that there is a constant inelastic demand D every period, and the cost structure is the following: the hydro plant has zero marginal cost and the thermal plant's cost function is $C(q_T)$, with $C' \geq 0$ and $C'' > 0$. We further assume that $C'(0) = 0$. The thermal plant has enough capacity to supply the entire demand at a cost $C(D)$. We restrict R to the interval $[0,D]$ and θ to the interval $[0,\theta^*]$. Denote by C_G the cost of production when the state is good for renewables and C_B otherwise. The planner's problem in the recursive formulation is:

$$C_i(X,\theta,R) = \min_{q_T,q_H,q_R} \{C(q_T) + \delta\,(\pi\gamma C_G(\theta,\theta,R) + (1-\pi)\gamma C_G(X - q_H,\theta,R) + \pi(1-\gamma)C_B(\theta,\theta,R)) +$$
$$(1-\pi)(1-\gamma)C_B(X - q_H,\theta,R)\}$$

s.t

$$D = q_T + q_H + q_r$$
$$q_H \leq \min\{X,D\}$$
$$q_R \leq \overline{R}$$

=

$$\bar{R} = \begin{cases} u_h R, & \text{if } i = G \\ u_l R, & \text{if } i = B \end{cases}$$

$$X' = \begin{cases} X - q_H, & \text{with probability } 1 - \pi \\ \theta, & \text{with probability } \pi \end{cases}$$

where q_T , q_H, and q_r stand for quantity produced of thermal, hydro and renewable energy, respectively. X and X' denote the total amount of water stored in the reservoirs in the current and subsequent period.

The first restriction says that hydro plus thermal plus intermittent must match total demand, the second states that hydro generation is constrained by the amount of water available and cannot exceed the demand, the third states that the production of intermittent sources cannot exceed the available capacity $\overline{R}$, and the fourth states that the capacity of producing from intermittent sources in a period is given by a factor u_h or u_l (with $u_h > u_l$) multiplied by the total capacity R.[6] Finally, the last equation shows the law of motion of the state variable X.

The total cost function is defined as the sum of thermal plus hydro plus intermittent generation: $C = C_T(q_T)+C_H(q_H)+C_r(q_r)$. Since the costs of the hydro and of intermittent sources are zero, we write the one period cost function as $C = C(q_T)$.

Our next result is straightforward and comes directly from the fact that there is no storage of renewable energy in our model.

Lemma 5 (Full use of renewables) *The planer will always choose* $q_R = \overline{R}$.

The next result states that the planner will always choose to employ some thermal sources in energy production. Before we proceed with the proofs, note that that the cost function C(·) is decreasing in the first argument. The following result shows that it can never be optimal to use no thermal in the current period and some thermal sources on the following period. This result comes from the convexity of the cost function. We prove this result by showing that such a policy has a profitable deviation in which the planner uses some thermal source to produce in the current period and saves water for the future.

[6] For the proofs we use u_h=1 and u_l=0. For the calibration exercise, we use more realistic values.

The next proposition shows that the planner will always use some thermal energy sources. The proof is in the Appendix.

Proposition 5 (Always optimal to use thermal sources) $q_H = D - \overline{R}$ *is never an optimal choice. (As a consequence, thermal is always produced).*

From now on, it will be convenient to consider the equivalent problem of maximizing the function $-C(q_t)$ instead of working with the minimization problem above. Then we have that $-C'(q_t) < 0$ and $-C''(q_t) < 0$, so the objective function becomes a strictly concave function. Next we write the problem in terms of choosing the remaining hydro capacity $X' = X - q_H$. Thus, we can rewrite the above problem as:

$$V^i(X,\theta,R) = \max_{X'}\{-C(D - \overline{R} - X + X^0) + \delta(\pi\gamma V^G(\theta,\theta,R) + (1-\pi)\gamma V^G(X',\theta,R) +$$

$$\pi(1-\gamma)V^B(\theta,\theta,R)) + (1-\pi)(1-\gamma)V^B(X',\theta,R)\} \qquad \text{s.t}$$

$$\max\{0, X + \overline{R} - D\} \leq X' \leq X$$

$$\overline{R} = \begin{cases} u_h R, & \text{if } i = G \\ u_l R, & \text{if } i = B \end{cases}$$

where the maximum is there to ensure that the energy production never exceeds the demand.

We are now able to state our main theoretical result, which combines the results in this section with those of Section 2.1. In the Appendix, the proof provides some properties of the value function, which are necessary for the theorem.

Theorem 1 (First Welfare Theorem) *The competitive equilibrium is Pareto-optimal.*
Proof. See Appendix. ■

In Section 5 we will use the first welfare theorem to implement our numerical analysis. Before we proceed, let us discuss about a very important issue in markets with different technologies.

3.2 Start-up Costs

Start-up costs is a fixed cost paid by a thermal plant that was not producing energy and decides to start producing. That plant needs to be started up, and this fixed cost must be paid before it can feed electricity into the grid.

In our model, we assume that this is an extra cost paid by thermal plants whenever i) their last period's production was zero and ii) their current period's production is positive.

We investigate whether start-up costs would change the planner's solution, or the competitive equilibrium. From a planner's perspective, we know that the minimization of costs leads to thermal sources being used in every period and every state of the world. Thus, it should be clear that an extra cost following an idle period will not alter the planner's optimal allocation.

A similar argument holds for the competitive equilibrium case. Since hydro plants are price-takers in our model, they cannot strategically explore the thermal source's desire to produce even at very low prices. Thus, given that in the competitive equilibrium thermal sources produce every period, the new equilibrium under start-up costs will also have them produce in every period and the allocation is unchanged.

4. Optimal Capacity

We assume that there is an installation cost C_h per unit of hydro capacity and C_R per unit of renewable capacity. Our objective is to show that there is an optimal energy matrix.

We solve this problem from an ex ante perspective. We assume that the planner can choose a capacity in the space E := $[0,\overline{\theta}] \times [0,\overline{R}]$, with $\overline{\theta}$ large enough[7] and $\overline{R} \leq D$, then the planner's problem of choosing the optimal energy matrix will be:

$$\max_{(\theta,R)\in\mathcal{E}} V(X,\theta,R) - C_h\theta - C_R R$$

Theorem 2 (Optimal Capacity Matrix) *There exists a unique optimal capacity matrix* $(\theta^*,R^*) \in$ E.

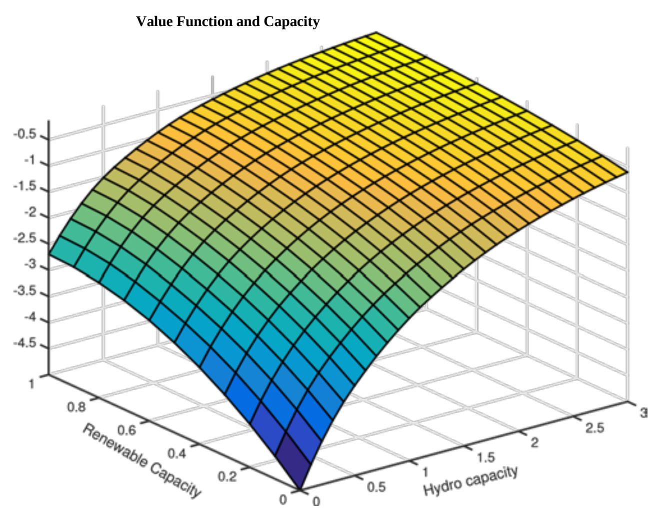
Proof. Note that this problem is well defined since $V^G(\cdot)$ is continuous and bounded and E is compact, so the existence of $(\theta^*,R^*) \in$ E follows directly. Uniqueness comes from the fact that V^g is strictly concave. ■

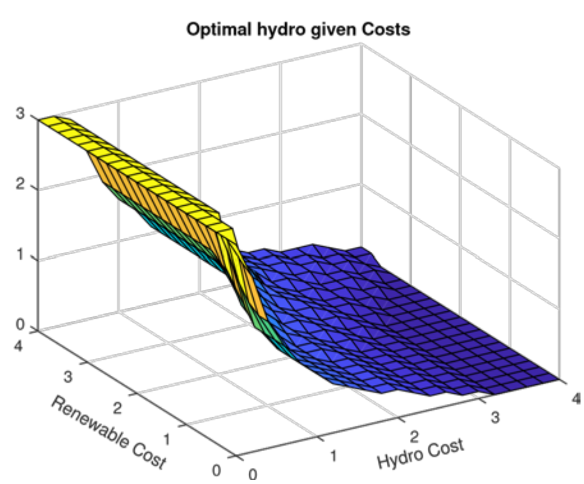
[7] Precisely, we need only that $\overline{\theta} \geq \theta^*$, where θ^* is the optimal capacity in the relaxed problem.

Note that this optimal capacity will depend on the combination of the parameters of the model, which renders an analytic solution infeasible. We can, however, solve this numerically given the parameters.

To illustrate how the optimal capacity relates to the installation costs, we present the following three figures. In the first one, we show the relation of the planner's value function with the installed capacity of hydro and intermittent sources. The main idea to grasp from this figure is that the value function is (obviously) increasing in the installed capacity, but it is also clearly concave. We prove this in Lemma 7.

In the following two figures, we present i) the optimal hydro capacity as a function of the installation costs and ii) the optimal capacity of intermittent sources as a function of these costs.





An important message here is that the optimal matrix will be interior for reasonable cost parameters.

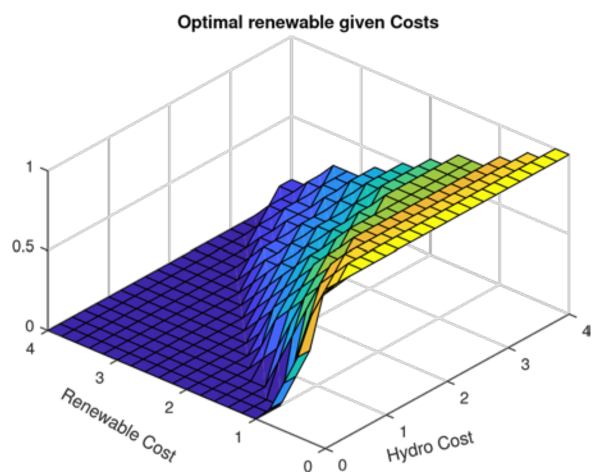
Second best

For a fixed initial matrix (θ,R), with $\theta < \overline{\theta}$ and $R < \overline{R}$. we can ask what is the optimal investment plan. Define now the capacity expansion space as T := [0,$\overline{\theta}-\theta$]×[0,$\overline{R}-R$], the problem is now:

$$\max_{(\Delta\theta,\Delta R)\in\mathcal{T}} V^G(\theta+\Delta\theta,\theta+\Delta\theta,R+\Delta R)-C_h\Delta\theta-C_R\Delta R$$

Again uniqueness comes from the strict concavity of the value function.

Proposition 6 (Second Best Capacity Matrix) *There exists a unique second best capacity* $(\theta_2^*, R_2^*) \in \mathcal{E}$.



5. Quantitative Analysis

We develop a strategy to solve the model numerically. We parameterize the model to data in Brazil, Uruguay and Argentina. These three countries are representative in that they differ significantly in terms of their natural characteristics and in their current energy matrix. We obtain i) the unconstrained optimal capacity matrix for each of these countries and ii) the second best optimal matrix, which is a policy implication: how should these countries invest *given* their current matrix?

5.1 Parameterization

5.1.1 Overall Strategy

Below we describe our calibration strategy. We consider a monthly frequency and use one year of data. We use observed market outcomes from data to calibrate the parameters. Our description is done in the sequence that the parameters are found, so that it also describes our algorithm to retrieve the parameters. In order to obtain the investment cost parameters we assume free entry into the market, so we equate the expected profits with the investment cost. We assume a unitary demand and a quadratic cost function.

- First we choose δ in order to match a target real (gross) interest rate, considering a monthly frequency: $\left(\frac{1}{\delta}\right)^{12} = \bar{R}$
- We set $\gamma = 0.5$. We then sort the monthly observations on the wind capacity factor and choose u_h as the average of the top 6 months and u_l as the average of the bottom 6. This way, the average capacity factor in the model coincides with the one observed in the data.
- R is chosen in order to match the average share of wind energy production in the total energy generation, which is given analytically by $s_r = R(\gamma u_h + (1 - \gamma)u_l)$.
- We set θ to match the observed relation between hydro and wind installed capacities: $\omega = \frac{\theta}{R}$
- We find π in order to match the average share of hydro energy production in the total energy generation, this cannot be done in a closed form, so we do this procedure numerically.[8]
- Given the above calibration we simulate the model 10,000 times to obtain the expected profit for hydro and wind energy sources, P_h and P_r respectively. We then find the cost of each source by equating profits and investment cost:

$$P_h = c_h\theta \quad \text{and} \quad P_r = c_rR$$

[8] We use the Matlab routine fzero to find a zero of a residual function between the average share generated by the model (for a given π) and the target. The routine brackets the value of π in an interval $[a,b]$ and progressively shrinks the bracket until the distance between the extremes is small.

5.1.2 Brazil

Table 1 summarizes the parameters for Brazil used in the quantitative analysis.

Table 1. Parameter Values: Brazil

Brazil		2018
Parameter	Value	Target
Discount factor δ	0.997	Gross interest rate of 1.036 (yearly basis)
Wind scaling factor u_h	52.09%	average wind capacity factor of 41.91%
Wind scaling factor u_l	31.72%	average wind capacity factor of 41.91%
Intermittent capacity R	0.2098	8.79% share of wind in energy production
Hydro Capacity θ	1.4851	theta/R = 7.08
Probability of rain π	73.92%	74.66% share of hydro in energy production
Installation cost of Hydro c_h	17.11	Expected profits equal investment costs
Installation cost of Intermittent c_r	23.2636	Expected profits equal investment costs

5.1.3 Uruguay

In Table 2 below, we summarize the parameters for Uruguay.

Table 2. Parameter Values: Uruguay

Uruguay		2017
Parameter	Value	Target
Discount factor δ	0.997	Gross interest rate of 1.036 (yearly basis)
Wind scaling factor u_h	32.01%	average wind capacity factor of 28.69%
Wind scaling factor u_l	25.38%	average wind capacity factor of 28.69%
Intermittent Capacity R	1.0946	31.41% share of wind in energy production
Hydro Capacity θ	1.1144	theta/R = 1.01
Probability of rain π	81.33%	59.12% share of hydro in energy production
Installation cost of Hydro c_h	11.60	Expected profits equal investment costs
Installation cost of Intermittent c_r	8.86	Expected profits equal investment costs

5.1.4 Argentina

Table 3 summarizes the parameters for Argentina used in the quantitative analysis.

Table 3. Parameter Values: Argentina

Argentina		2017
Parameter	Value	Target
Discount factor δ	0.997	Gross interest rate of 1.036 (yearly basis)
Wind scaling factor u_h	100%	average wind capacity factor of 34.86%
Wind scaling factor u_l	0%	average wind capacity factor of 34.86%
Intermittent Capacity R	0.0129	0.45% share of wind in energy production
Hydro Capacity θ	0.6614	theta/R = 51.24
Probability of rain π	0.4800	28.96% share of hydro in energy production
Installation cost of Hydro c_h	73.07	Expected profits equal investment costs
Installation cost of Intermittent c_r	63.24	Expected profits equal investment costs

5.2 Main Results

In Table 4 we report the optimal capacity matrix and the second best optimal matrix. The former is the unconstrained optimal choice of capacity given the parameters of the economy but disregarding the actual capacity. The latter represents the best capacity matrix given the parameters of the economy but also the current matrix.

Our policy recommendations are based on the second best results, since we do not expect capacity to be removed. The main suggestions are: i) Brazil should invest in renewables, ii) Uruguay is already close to the second best (but has experienced overinvestment in wind capacity and underinvestment in hydro sources) and iii) Argentina needs to massively invest in wind generation. The percentage changes are reported on last column of Table 4.

If we consider the first best, i) Uruguay has overinvested in wind generation, ii) Brazil has overinvested in hydro sources, and iii) Argentina has overinvested in hydro plants.

Table 5 shows the optimal share in terms of quantity generated. Note that the very large percentage increase in Argentina's wind installed capacity represents a 9 percent increase in the total amount generated by this technology.

Table 4. Optimal Capacity Matrix

Source	Actual Capacity	Optimal Capacity	% change	Second Best	% change
Brazil					2018
Hydro	1.4851	1.4293	-3.75%	1.4851	0%
Wind	0.2098	0.3055	+45.63%	0.2658	+26.72%
Uruguay					2017
Hydro	1.1144	1.2012	+7.79%	1.1320	+1.57%
Wind	1.0946	0.9518	-13.05%	1.0946	0%
Argentina					2017
Hydro	0.6614	0.5141	-22.28%	0.6614	0
Wind	0.0129	0.2738	+2020%	0.2147	+1563%

Table 5. Optimal Share of Generation

Source	Actual share	Optimal share	Second Best share
Brazil			2018
Hydro	74.66%	71.43%	73.57%
Wind	8.79%	12.80%	11.14%
Uruguay			2017
Hydro	59.12%	63.46%	60.08%
Wind	31.41%	27.31%	31.41%
Argentina			2017
Hydro	28.96%	23.23%	28.54%
Wind	0.45%	9.54%	7.49%

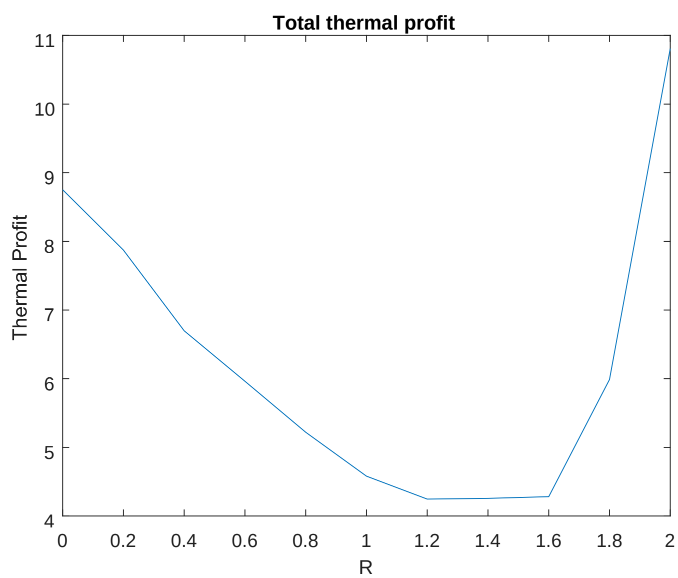
6. Intermittent Power Sources

Our goal in this section is to analyze the impact that the entry of renewable sources has on the reliability of the system. A potential drawback of the entrance of renewables is that by decreasing the average price it might decrease the profitability of other, more reliable sources of energy such as thermal producers. This would, as the argument goes, reduce the reliability of the system due to intermittency. In order to analyze whether this argument is true, we simulate an economy using the data calibrated to Brazil from Table 1 and examine the profitability of thermal planes as a function of the fraction of intermittent sources in the economy. In order to do this, we solve the planner's problem. By the first welfare theorem (Theorem 1), we know that it is also the competitive equilibrium.

As in the previous section, in order to have a reasonable comparison between different industry structures, we assumed that increases in renewable source capacity replaces hydro plants, so that total installed capacity remains constant.

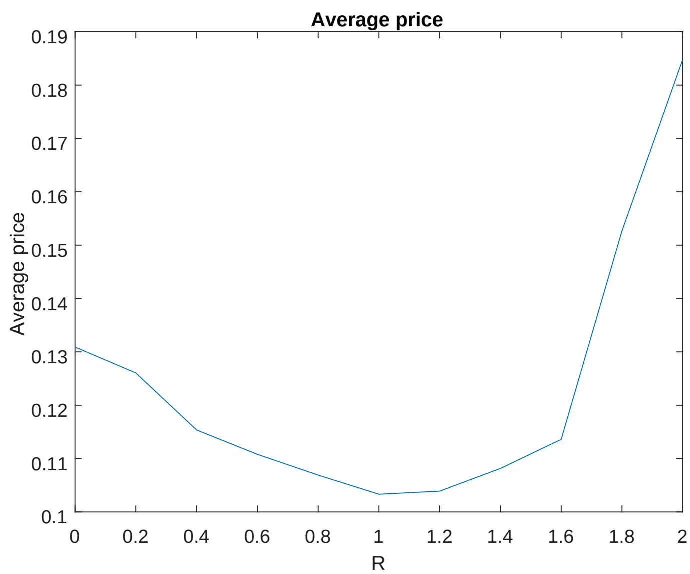
Figure 4 shows the profitability of the thermal plant against the amount of renewable installed. Note that the profitability of the thermal plants follows a U-shape relationship with the amount of intermittent renewable installed.

Figure 4. Thermal Technology Profitability and Share of Renewables



Two forces are at play in this case. First, when wind/solar capacity start replacing hydro capacity, electricity prices drop on average, as shown in Figure 5. It happens because wind and rain follow distinct stochastic processes, and now there is a higher probability that in any period there is either rain or wind or both, lowering electricity cost. This effect dominates at first, and it highlights the gain from a more diverse portfolio of generating technologies.

Figure 5. Average Prices and Share of Renewables



But as we decrease the share of hydro in favor of intermittent renewable capacity, hydro loses the ability to transfer water across time, saving water for scarcity periods. This increases the probability that the thermal plant may find itself in a situation where it has to produce a large quantity at a high price. This second effect dominates the first for higher shares of renewables.

The electricity average price drops steadily until a threshold *R*. For this point on, the loss of storage capacity outweighs the gains from diversification.

Perhaps unexpectedly, price volatility drops until a threshold *R*, then rises sharply. Our first thought was that price volatility would only increase as we decrease the share of hydro plants with storage capacity, since they are the ones which act to smooth the price. Again, the diversification effect is stronger than the smoothing effect for lower levels of renewable capacity.

On the other hand, the profit per unit of installed capacity of wind generators falls continuously as the share of wind increases.

Figure 6. Price Volatility and Share of Renewables

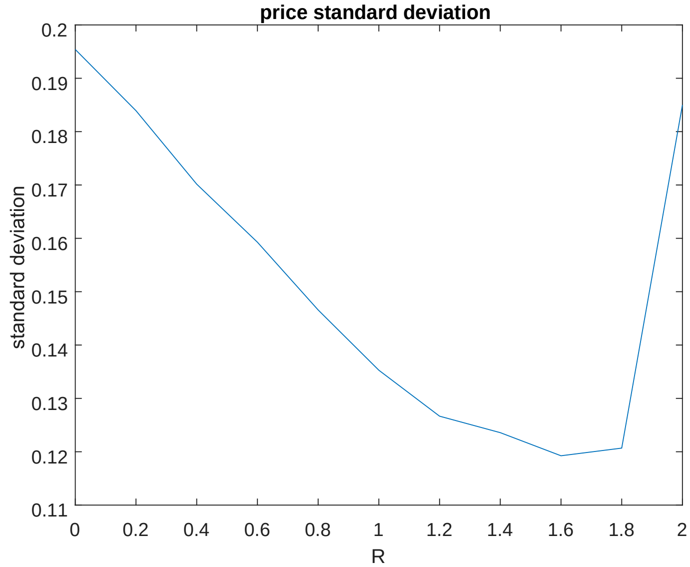


Figure 7. Hydro Technology Profitability and Share of Renewables

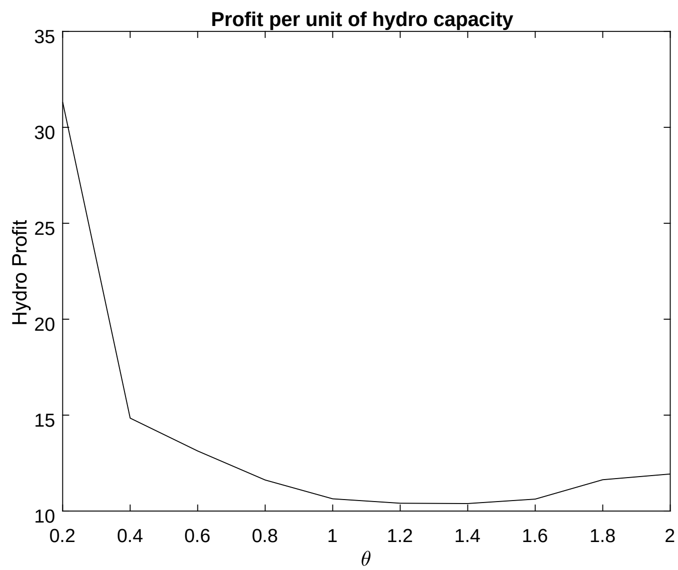
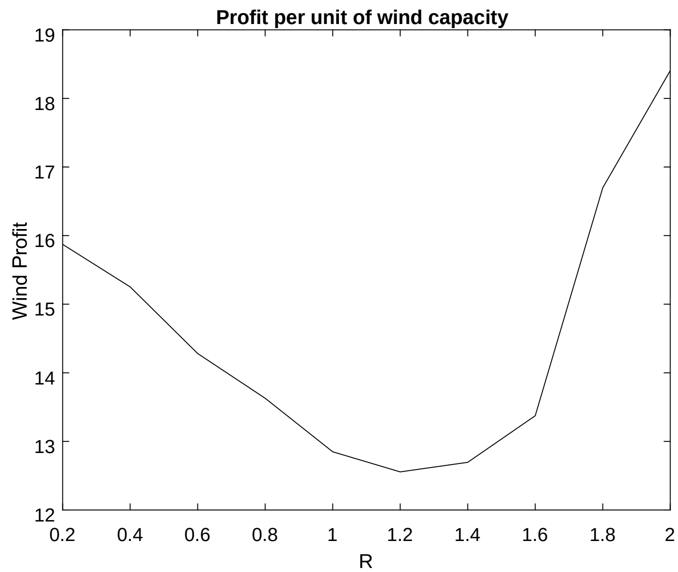


Figure 8. Intermittent Technology Profitability and Share of Renewables



Two long-term conclusions arise from this analysis. First, the generation technology mix tends to be balanced, in the sense that it is unlikely that renewable generation technologies with their zero marginal costs drive other technologies out of the market. Second, the results corroborate the idea of a "clean energy paradox," where the entry of renewable sources becomes progressively harder as their share of the market increases.

7. Conclusion

This paper characterizes the competitive equilibrium of an electricity industry comprised of thermal, hydro and intermittent (solar, wind, etc.) sources and shows that this equilibrium is Pareto optimal.

The price is determined by the marginal generator, usually a thermal plant. The fact that water is storable introduces dynamics into the problem. Even with zero cost, hydro generators do not sell at zero price. An important result shows that the market price signals water scarcity in the future and induces hydro generators to refrain from producing, even if they have lower marginal cost than thermal generators.

A corollary of this result is that a binding price cap may increase water usage today by reducing future peak prices. It reduces the incentive to save water today by decreasing the highest price that may happen in the future.

We use the model to show that there is a unique optimal energy matrix, and we calibrate it with data from Argentina, Brazil and Uruguay. The results for each country show in which technology it is optimal for these countries to invest. We find that Argentina and Brazil should invest in wind generation, while Uruguay should invest in hydro sources.

In another use for the model, we simulated what would happen to the profitability of the more traditional power sources, namely hydro and thermal plants, when we replace the share of hydro with intermittent technology. There is a fear that the expansion of these renewable sources will expel traditional sources from the electricity market, compromising reliability through intermittency. The results show a U-shape relationship between the share of renewables and the profitability of thermal and hydro sources. Profitability first drops due to a decrease in electricity prices, then increases since there are fewer generators able to produce in bad periods, which favors more reliable sources. This positive result suggests that renewables may not be a threat to the reliability of power supply.

Appendix A

Proof of Proposition 1. We provide an algorithm to compute the equilibrium:

The first equation comes from $V_B^0 = \delta\,(\pi V_G + (1-\pi)V_B^0)$:

$$V_B^0 = \frac{\delta\pi}{1-\delta(1-\pi)} V_G \ . \tag{7}$$

The second equation comes from $V_G = p_G + \delta\,(\pi V_G + (1-\pi)V_B^0)$, which leads us to:

$$V_G = \frac{1-\delta\,(1-\pi)}{1-\delta} p_G. \tag{8}$$

Lemma 5 (indifference at G) leads us to:

$$V_G = \delta(\pi V_G + (1-\pi)V_{1,B}^1 \tag{9}$$

where

$$V_{1,B}^1 = p_1 + \delta\left(\pi V_G + (1-\pi)\,V_B^0\right). \tag{10}$$

We have four equations and five unknowns $\left(V_G, V_B^0, V_{1,B}^1, p_G, p_1\right)$. These four equations hold for any equilibrium.

Precisely, we will work with the following steps to find the equilibrium:

Step 1) Assume that $V_{1,B}^1 > \delta\left(\pi V_G + (1-\pi)\,\bar{v}\right)$, where $\bar{v} \equiv p^* + \delta\,(\pi V_G + (1-\pi)V_B^0)$.

Step 2) Compute the share of hydro plants with full capacity in state $(1,B)$.This share is $\theta - 1 + T$, where T is the share of energy produced by a thermal generator. How is T computed? $p_G = c'(T)$. Denote this quantity T by T_{pG}.

Step 3) Compute p_1. This price is given by: $p_1 = c'\,(2-\theta-T)$, where the number in parenthesis is calculated from the supply of thermal that is needed after every hydro with full capacity supplies.

Step 4) We now have p_1, solve the system of four equations and four unknowns above.

Step 5) Check if step 1 is correct, that is, if $V_{1,B}^1 \ > \ \delta\left(\pi V_G + (1-\pi)\,\bar{v}\right)$. If yes, then the equilibrium for the dynamic game has been solved. If no, then from lemma 4 add the following two equations: $V_{1,B}^1 = \delta\left(\pi V_G + (1-\pi)\,V_{2,B}^1\right)$ and $V_{2,B}^1 = p_2 + \delta\left(\pi V_G + (1-\pi)\,V_B^0\right)$.

We now have 6 equations and 7 unknowns. Assume that $V^1_{2,B} > \delta\left(\pi V_G + (1-\pi)\,\bar{v}\right)$ and proceed as before:

Step 2.1) Compute the share of hydro plants with full capacity in state (1,*B*).This share is $\theta - 1 + T_G$, where T_G is the share of energy produced by a thermal generator in state *G*. How is T_G computed? $p_G = c^0\,(T_G)$.

Step 3.1) Compute p_1. This price is given by: $p_1 = c'\,(2 - \theta - T_G)$, where the number in parenthesis is calculated from the supply of thermal that is needed after every hydro with full capacity supplies.

Step 4.1) We now have p_1, solve the system of four equations and four unknowns above.

Step 5.1) Check if step 1 is correct, that is, if $V^1_{1,B} > \delta\left(\pi V_G + (1-\pi)\,\bar{v}\right)$. If yes, then the equilibrium for the dynamic game has been solved. If no, then from lemma 4 add the following two equations: $V^1_{1,B} = \delta\left(\pi V_G + (1-\pi)\,V^1_{2,B}\right)$ and $V^1_{2,B} = p_2 + \delta\left(\pi V_G + (1-\pi)\,V^0_B\right)$. ■

Proof of Lemma 1.

i) $p \le p^*$. Given that we have imposed that the thermal generator is a myopic optimizer, p^* is the maximum possible static price in this dynamic environment.

ii) If $p = p^*$ firms have strict incentive to sell energy. If the firm sells, it cashes in the maximum price possible. Its payoff is: $p^* + \delta\,(\pi V_G + (1-\pi)V_B^0)$. If it does not sell, it has a zero payoff in the current period and in all future periods that it does not sell. If it sells only *t* periods ahead, it will get a payoff $\delta^t p + \delta^{t+1}(\pi V_G + (1-\pi)V_B^0) < p^* + \delta\,(\pi V_G + (1-\pi)V_B^0)$.

iii) At *t* all firms will sell and empty their reservoirs. Thus, all firms will either enter the following period with full reservoirs (*G*) or empty reservoir (*B*). In the latter case, only the thermal generator will supply energy and the price will be p^*.

iv) We need to show that: $p_t + \delta\left(\pi V_G + (1-\pi)\,V^0_B\right) \ge \delta\left(\pi V_G + (1-\pi)\,V^1_{(h,B)}\right)$. Suppose that there exists some period *t* and some history h_t for which selling is strictly worse than not selling energy. This means that nobody sells and $p = p^*$. By lemma 2 we have a contradiction.

v) Suppose that selling is better than not selling at some period *t* and state *G*. Then, all firms will sell and price must be 0. (Recall that the mass of firms exceeds

the demand and they have zero marginal cost). If the next period is *B*, the mass of hydro plants with full capacity is $\theta - 1$, which, by assumption is smaller than 1, that is $\theta - 1 < 1$ and thus, price must be $p > 0$, since thermal production would be needed to clear the spot market. Thus, the payoff from selling would be: $0 + \delta\,(\pi V_G + (1 - \pi)V_B^0)$ which is smaller than waiting a period and with probability π being in the same situation as having sold, but with probability $1 - \pi$ having the possibility of selling energy at a positive price. ■

Proof of Lemma 3. We repeat the Bellman equations of the hydro firm in the three relevant states: good (V_G), bad with water (V_B^1) and bad without water (V_B^0):

$$V_G = \max\left\{p_G + \delta\left(\pi V_G + (1-\pi)\,V_B^0\right), \delta\left(\pi V_G + (1-\pi)\,V_B^1\right)\right\},$$

$$V_B^0 = \delta\left(\pi V_G + (1-\pi)\,V_B^0\right),$$

$$V_B^1 = \max\left\{p_B + \delta\left(\pi V_G + (1-\pi)\,V_B^0\right), \delta\left(\pi V_G + (1-\pi)\,V_B^1\right)\right\}.$$

In equilibrium, on the good state generators need to be indifferent between selling and not selling energy:

$$p_G + \delta\left(\pi V_G + (1-\pi)\,V_B^0\right) = \delta\left(\pi V_G + (1-\pi)\,V_B^1\right),$$

which simplifies to

$$p_G = \delta\,(1-\pi)\left(V_B^1 - V_B^0\right). \tag{11}$$

Applying the same reasoning to generators with water on the bad state we get

$$p_B = \delta\,(1-\pi)\left(V_B^1 - V_B^0\right). \tag{12}$$

Using the definitions of V_B^1 and V_B^0, we have

$$V_B^1 - V_B^0 = p_B + \delta\left(\pi V_G + (1-\pi)\,V_B^0\right) - \delta\left(\pi V_G + (1-\pi)\,V_B^1\right) = p_B. \tag{13}$$

Substituting equation (13) in equations (11) and (12), and letting p'_B be the price on the bad state next period, we have

$$p_G = \delta\,(1-\pi)\,p'_B \tag{14}$$

$$p_B = \delta\,(1-\pi)\,p'_{BB} \tag{15}$$ ■

Proof of Proposition 5.

In order to prove the proposition, we first need to prove the following lemma.

Lemma 6 *We cannot have* $q_H = D - \bar{R}$ *in the current period and* $q_H^0 < D - \bar{R}$ *in the subsequent period if the renewables' state is the same in both periods and there is no rain in the second period.*

Proof. Fix $\theta > 0$ and $R \geq 0$. We will focus on the case of renewable's good state, the proof for the bad state is analogous. Assume by way of contradiction that we have a optimal policy $q_H := q_H(X,\theta,R) = D - u_hR$ and $q'_H := q_H(X - q_H, \theta, R)$, with $q_H > q'_H$. Consider that the planner engages in a deviation of this optimal policy in only two periods, with $q_H^\varepsilon = q_H - \varepsilon$ and $q_H^{\varepsilon}{}' = q'_H + \varepsilon$. If $0 < \varepsilon < D - u_hR$ this policy is feasible. Denote by C^d the associated cost value.

We show that exists ε small enough such that $C^\varepsilon < C$. Note that:

$$C^\varepsilon - C = C(\varepsilon) - C(0) + \delta(1-\pi)((1-\gamma)(C_B(X - q_H + \varepsilon) - C_B(X - q_H))+$$

$$\gamma(C(D - u_hR - q_H^0 - \varepsilon) - C(D - u_hR - q_H^0)))$$

where in the last part of right-hand side we used the fact that in our proposed deviation, nothing changes from the third period onwards (that is, the state variables will be the same in the proposed policy and in the deviation, regardless of the state of the world). Moreover, since C is decreasing in the first argument we have:

$$\mathbb{C}^\varepsilon - \mathbb{C} \leq C(\varepsilon) - C(0) + \delta(1-\pi)\gamma(C(D - u_hR - q'_H - \varepsilon) - C(D - u_hR - q'_H))$$

Since we need to show that $C^\varepsilon - C \leq 0$, it suffices to show that $\exists\ \varepsilon > 0$, such that:

$$\frac{C(\varepsilon) - C(0)}{\varepsilon} < \frac{\delta(1-\pi)\gamma(C(D - u_hR - q'_H) - C(D - u_hR - q'_H - \varepsilon))}{\varepsilon}$$

Both sides are positive since $\forall x > 0\ C^0(x) > 0$. The existence of such an ε comes directly from the fact that $D - u_hR - q'_H > 0$ and , $C^0(D - u_hR - q_H^0) > C^0(0) = 0$. To see why, denote by

$a := \frac{C(\varepsilon) - C(0)}{\varepsilon}$, $b := \frac{\delta(1-\pi)\gamma(C(D - u_hR - q_H) - C(D - u_hR - q_H - \varepsilon))}{\varepsilon}$ and $l = \delta(1-\pi)C^0(D - u_hR - q_H^0)$.

By these previous properties we have that $\forall\gamma > 0$, $\exists\delta > 0$, such that:

$$\varepsilon < \delta \Rightarrow (a < \gamma \quad \wedge \quad l - \gamma < b < l + \gamma) \Rightarrow b - a > l - 2\gamma$$

By picking $\gamma = \frac{l}{4}$ and $0 < \varepsilon < \delta$ we get the desired result. We get then that $C^{\varepsilon} < C$, which is a contradiction with the definition of C. ■

Now assume by way of contradiction that on an arbitrary period t the planner chooses $q_t(X_t,\theta,R) = D - \bar{R}$. Fix $s \in \mathbb{N}$, such that $X_t < s(D - \bar{R})$. Consider a history of s periods of bad states for the hydro power source (and the same states for the renewable source). Then by the previous lemma we have by induction that $q_{t+s} = D - \bar{R}$, which is a contradiction, since it is necessarily true that $q_{t+s} > X_{t+s}$. ■

Proof of Theorem 1.

First we prove that the value function satisfies certain properties.

Lemma 7 *For* $i \in \{G,B\}$, $V^i(X,\theta,R)$ *is bounded, continuous, increasing and concave.*

Proof of Lemma 7. Let D be the space of bounded, continuous, non-decreasing and concave functions defined on $[0,\theta] \times [0,\bar{\theta}] \times [0,D]$, endowed with the sup-norm. D is a complete space, since it is a closed subset of the space of bounded and continuous functions. Since the Cartesian product preserves completeness we have that D ×D is also a complete space. Define the operator T on D × D,

$$T : (f,g) \to (T^G f, T^B g):$$

$$T^G f(X,\theta,R) = \max_{X'}\{-C(D - u_h R - X + X') + \delta(\pi\gamma f(\theta,\theta,R) + (1-\pi)\gamma f(X',\theta,R) +$$

$$\pi(1-\gamma)g(\theta,\theta,R) + (1-\pi)(1-\gamma)g(X',\theta,R)\} \text{ s.t}$$

$$\max\{0, X + u_h R - D\} \leq X' \leq X$$

and

$$T^B g(X,\theta,R) = \max_{X'}\{-C(D - u_l R - X + X') + \delta(\pi\gamma f(\theta,\theta,R) + (1-\pi)\gamma f(X',\theta,R) +$$

$\pi(1-\gamma)g(\theta,\theta,R)) + (1-\pi)(1-\gamma)g(X',\theta,R)\}$ s.t

$\max\{0, X + u_l R - D\} \leq X' \leq X$

Denote by $F^G(X,\theta,R,X')$ and $F^B(X,\theta,R,X')$ the objective function from the problems above, respectively. Also define $\bar{f}(\cdot) := \pi\gamma f(\cdot) + (1-\pi)\gamma f(\cdot) + \pi(1-\gamma)g(\cdot) + (1-\pi)(1-\gamma)g(\cdot)$. Define the following correspondences:

$$\Gamma^G(X,\theta,R) = \{X' \in \mathrm{R}_+ : \max\{0, X + u_h R - D\} \leq X' \leq X\}$$

$$\Gamma^B(X,\theta,R) = \{X' \in \mathrm{R}_+ : \max\{0, X + u_l R - D\} \leq X' \leq X\}$$

Note that both correspondences are non-empty, compact-valued and convex-valued. Since in both cases the functions on each side of inequality are continuous, we conclude that both correspondences are continuous.

(i) $(T^G f, T^B g)$ is bounded and continuous. First note that, since $(f,g) \in \mathrm{D} \times \mathrm{D}$ and by assumption $-C(\cdot)$ is continuous and bounded, then $F^i(X,\theta,R,X')$ is a linear combination of continuous and bounded functions, so it is also continuous and bounded for $i = \{G,B\}$.Then maximizing $F^i(X,\theta,R,X')$ over the compact set $\Gamma^i(X,\theta,R)$ has a solution and by consequence $(T^G f, T^B g)$ is bounded. Moreover since $\Gamma^i(X,\theta,R)$ is compact valued and continuous we can apply Berge's Maximum Theorem to conclude that $(T^G f, T^B g)$ is continuous.

(ii) $T^G f$ is increasing: Fix $(\overline{X},\overline{\theta},\overline{R}) \geq (X,\theta,R)$ with one strict inequality. Pick

$$x' \in \arg\max_{X' \in \Gamma^G(X,\theta,R)} F^G(X,R,\theta,X')$$

Case 1: $x^0 \in \Gamma^G(\overline{X},\overline{\theta},\overline{R})$:

$$T^g f(\overline{X},\overline{\theta},\overline{R}) = \max -C(D - u_h\overline{R} - \overline{X} + X') + \delta\bar{f}(X',\overline{\theta},\overline{R}) \geq -C(D - u_h\overline{R} - \overline{X} + x') + \delta\bar{f}(x',\overline{\theta},\overline{R}) \geq -C(D - u_h R - X + x') + \delta\bar{f}(x',\theta,R) = T^g f(X,\theta,R)$$

where the last inequality comes from the fact that $-C(\cdot)$ is strictly decreasing and $\bar{f}(\cdot)$ is non decreasing. Note that we will have a strictly inequality if

$X > \overline{X}$ or $R > \overline{R}$.

Case 2: $x' \notin \Gamma^G(\bar{X}, \bar{\theta}, \bar{R})$: Then it must be the case that

$\bar{X} + u_h\bar{R} - D > x' \geq 0$:

$T^g f(\bar{X}, \bar{\theta}, \bar{R}) = \max -C(D - u_h\bar{R} - \bar{X} + X') + \delta\bar{f}(X', \theta, \bar{R}) \geq -C(0) + \delta\bar{f}(\bar{X} + \bar{R} - D, \theta, \bar{R}) \geq -C(D - u_h R - X + x^0) + \delta\bar{f}(x^0, \theta, R) = T^g f(X, \theta, R)$

where the last inequality comes from the fact that $-C(0)$ is the maximum of $-C(\cdot)$.

(iii) $T^G f$ is Concave: Fix $(X', \theta', R'), (X'', \theta'', R')$, fix $\alpha \in [0,1]$ and:

$$X' \in \arg \max_{X \in \Gamma^G(X^0, \theta^0, R^0)} F^G(X', \theta', R', X)$$

$$X'' \in \arg \max_{X \in \Gamma G(X'', \theta'', R'')} F^G(X'', \theta'', R'', X)$$

Define $x''' = \alpha x' + (1-\alpha)x''$ and $(X''', \theta''', R''') = \alpha(X', \theta', R') + (1-\alpha)(X'', \theta'', R'')$. Then by the constraints we have:

$$X''' \geq \max\{0, \alpha(X' + u_h R' - D)\} + \max\{0, (1-\alpha)(X'' + u_h R'' - D)\} \geq \max\{0, \alpha(X' + u_h R^0) + (1-\alpha)(X'' + u_h R'') -$$

and we conclude $x''' \in \Gamma^G(X''', \theta''', R''')$. Then

$\max -C(D - u_h R''' - X''' + X) + \delta\bar{f}(X''', \theta''', R''') \geq -C(D - u_h R''' - X''' + x''') + \delta\bar{f}(x''', \theta''', R''') \geq \alpha(-C(D - u_h R' - X' + x') + \delta\bar{f}(x', \theta', R')) + (1-\alpha)(-C(D - u_h R'' - X'' + x'') + \delta\bar{f}(x'', \theta'', R''))$

where the last inequality comes from the fact that $-C(\cdot)$ is concave and $\bar{f}(\cdot)$ is concave. So we conclude that:

$$T^g f(X''', \theta''', R''') \geq \alpha T^g f(X', \theta', R') + (1-\alpha)\, T^g f(X'', \theta'', R'')$$

(iii) $T^B g$ is increasing and Concave: The proof is analogous to the case for $T^G f$.

Using the previous results we can conclude that T:D × D → D × D. Since $\delta < 1$ we can apply Blackwell's Sufficient Conditions to conclude that T is a contraction in a complete space, then we can apply the Banach fixed-point theorem to prove the existence of a unique fixed point of the operator, $(V_G,V_B) \in$ D × D. ■

We can derive further properties if we combine the previous lemma with proposition 5. We know that is always true that $q_H < D - \bar{R}$, then, by definition, it is always true that $X' > X+R-D$.

Proposition 7 *For* $i \in \{G,B\}$, $V^i(X,\theta,R)$ *is bounded, continuous, strictly increasing and strictly concave.*

Proof. By lemma 7 we know that $V^i(X,\theta,R)$ is bounded, continuous, increasing and concave. Then it suffices to show that the function is strictly increasing and strictly concave.

(i) V^g is strictly increasing: The proof that V^g is strictly increasing in X and R is analogous to the proof of lemma 7 but applying Proposition 5 to get the strict inequality. The result that V^g is increasing in θ comes directly from the fact that from a given X, R and $\theta > \bar{\theta}$, $\Gamma^G(X,\theta,R) \subseteq \Gamma^G(X,\theta,\bar{R})$ and the fact that V^g is strictly increasing in X.

(ii) V^g is strictly concave: The procedure is analogous. We first use proposition 5 to show that for a fixed θ,V^g is strictly concave in X and R, and use this fact to show that V^g is strictly concave in (X,θ,R). ■

The following two results will help us describe fully the social planner's optimal policy. The following lemma shows the optimal interior solution to the problem. We already know from Proposition 5 that the planner always uses some thermal source to produce. The next result tells us that off corners, the social planner's optimal policy is to produce with hydro plants such that it equalizes the current thermal plant's marginal cost with next period's expected discounted marginal cost.

Proposition 8 *Fix* $i \in \{G,B\}$, *off corners we have* $\frac{\partial V^i(X,R)}{\partial X} = C'(D - u_h R - X + x')$, *where x' is the optimal choice given the state* (X,R).

Proof. We also have by assumption that $C(\cdot)$ is continuously differentiable on R_+ then it will be continuously differentiable on *int* $\Gamma^G(X,\theta,R)$. With these and the previous results we can apply the Benveniste-Scheinkman Theorem to get the result. ■

Now, consider the situation where there is not much water left on the reservoirs. In this case, when there is no production of intermittent sources, the planner is better off producing using all water left. This is true even though it will imply that in the subsequent period, if there is no rain, all energy production will necessarily come from thermal sources and intermittent sources, at a high cost. The following lemma describes optimal production in this case.

Lemma 8 *There is a storage level* $\tilde{X} > 0$, *such that in a state in which there is no production of intermittent sources, for any amount of storage X in which* $X \leq \tilde{X}$, *all the water is used to produce energy. Moreover, if it is also true that*

$$C'(D - R) \geq \delta\,(1 - \pi)(\gamma C'(D - R) + (1 - \gamma)C'(D)),$$

then there is also a threshold level $\tilde{Y}$ *such that even in a state in which there is full production of intermittent sources, for any amount of storage X in which* $X \leq \tilde{Y}$, *all the water is used to produce energy.*

Proof. Since $C'(D)$ is the highest possible marginal cost, there is a storage level $\tilde{X} > 0$ such that

$$C'(D - \tilde{X}) = \delta\,(1 - \pi)(\gamma C'(D - R) + (1 - \gamma)C'(D))$$

For any level X such that $X < \tilde{X}$, the following holds:

$$C'(D - X) > \delta\,(1 - \pi)(\gamma C'(D - R) + (1 - \gamma)C'(D))$$

Using the convexity of the cost function we get the result. The argument for the case of a good state of intermittent is analogous. ■

We have thus far fully described the social optimum. Lemma 8 shows that there is a critical amount of stored water for each state of intermittent production, such that the planner deploys all

reservoirs in that period. Above these thresholds, we know that production the solution is interior and given by Proposition 8. ■

8. Appendix B: Data Sources

Table 6. Data Sources: Brazil

Brazil 2018

Data	Source	url
Wind Capacity Factor	Operador Nacional do Sistema Elétrico	https://bit.ly/2BVRSu7
Composition of energy production	Operador Nacional do Sistema Elétrico	https://bit.ly/2BVRSu7
Hydro and Wind capacity	Agência Nacional de Energia Elétrica	https://bit.ly/28INSwk

Table 7. Data Sources: Uruguay

Uruguay 2017

Data	Source	url
Wind Capacity Factor	Administración Nacional de Usinas y Trasmisiones Eléctricas	https://bit.ly/2XuLvqU
Composition of energy production	Administracion Nacional de Usinas y Trasmisiones Electricas	https://bit.ly/2EDZoO0
Hydro and Wind capacity	Ministerio de Industria, Energia y Mineria	https://bit.ly/2BZQfvL

Table 8. Data Sources: Argentina

Argentina 2017

Data	Source	url
Wind Capacity Factor	Comisión Nacional de Energía Atómica	https://bit.ly/2Tql7Qu
Composition of energy production	Comisión Nacional de Energía Atómica	https://bit.ly/2Tql7Qu
Hydro and Wind capacity	Comisión Nacional de Energía Atómica	https://bit.ly/2Tql7Qu

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