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## Abstract<sup>1</sup>

The effects of capital requirements on risk-taking and welfare are studied in a stochastic overlapping generations model of endogenous growth with banking, limited liability, and government guarantees. Capital producers face a choice between a safe technology and a risky (but socially inefficient) technology, and bank risk-taking is endogenous. Setting the capital adequacy ratio above a structural threshold can eliminate the equilibrium with risky loans (and thus inefficient risk-taking), but numerical simulations show that this may entail a welfare loss. In addition, the optimal ratio may be too high in practice and may concomitantly require a broadening of the perimeter of regulation and a strengthening of financial supervision to prevent disintermediation and distortions in financial markets.

**JEL classifications:** E44, G28, O41

**Keywords:** Capital requirements, Bank risk-taking, Growth and welfare

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# 1 Introduction

The link between financial regulation, risk-taking, and the overall safety of the banking system has been studied in a number of contributions, which include Blum (1999), Diamond and Rajan (2000), Hellmann et al. (2000), Repullo (2004), Kopecky and VanHoose (2006), Gale (2010), Hakenes and Schnabel (2011), De Nicolò and Lucchetta (2012), and more recently Gorton and Winton (2014), Martínez-Miera and Suárez (2014), and Malherbe (2015). Much of this literature has focused on limited liability and explicit or implicit government guarantees, in addition to the degree of market competition, as key factors in creating incentives for banks to engage in excessive (or, more specifically, socially suboptimal) risk-taking.

Another strand of the literature, motivated by the accommodative monetary policy pursued by central banks in advanced economies in the aftermath of the global financial crisis, has focused on the impact of low interest rates on risk taking and the need for coordinating monetary and macroprudential policies to promote macroeconomic and financial stability. These contributions include Agur and Demertzis (2012), Dell’Ariccia et al. (2014), Cociuba et al. (2016), and Collard et al. (2016). Cociuba et al. (2016), for instance, argued that low policy rates have conflicting effects on bank risk-taking: on the one hand, they make riskier assets more attractive than safe bonds; on the other, they reduce the amount of safe bonds available for collateralized borrowing in interbank markets—which facilitates reallocation of resources between financial intermediaries in response to new information about the riskiness of their investments. However, borrowing against safe bonds also allows intermediaries to take advantage of their limited liability and to overinvest in risky projects. Relaxing collateral constraints may thus increase

risk-taking and reduce welfare.

Much of the theoretical literature has focused on capital requirements—in the form of either simple leverage (asset-capital) ratios or risk-based charges—as a way of mitigating the incentives for risk-taking created by limited liability and government guarantees. Some studies, based on both static and dynamic models of banking, have argued that increasing these requirements mitigate moral hazard problems and risk-taking because shareholders have more “skin in the game.” Others, however, have provided more ambiguous support for this proposition. Diamond and Rajan (2000), for instance, argued that capital requirements may have an important social cost because they reduce the ability of banks to create liquidity. It has also been argued that the view that capital requirements create incentives to avoid risks is often based on partial equilibrium analysis and ignores the factors that determine the supply and cost of capital (see Gale, 2010). Indeed, in some of the contributions referred to earlier, capital regulation has no effect on asset returns, while in reality they (especially loan rates) respond to market forces that depend in part on the decisions that banks make when they are faced with changes in capital requirements. In addition, when the cost of capital is high, forcing banks to raise more equity may actually lead to an increase in the probability of default and exacerbate risks to financial stability.

Nevertheless, although some of the early empirical literature on the impact of capital requirements on bank risk-taking is inconclusive (see, for instance, Laeven and Levine, 2009), some recent studies have proved more supportive. In particular, Klomp and de Haan (2014), using data for the period 2002-08, found that stricter capital regulation (as well as tighter supervision) does reduce banking risk.<sup>2</sup>

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<sup>2</sup>They also found that the effect of regulation and supervision on risk-taking depends

Somewhat surprisingly, there have been few contributions aimed at studying the *longer-run* implications (in terms of growth and welfare) of the interactions between financial regulation, risk-taking, and financial stability. This is important because macroprudential financial regulation designed to reduce short-run procyclicality and mitigate the risk of financial crises (the focus of some of the contributions mentioned earlier) could well be detrimental to economic growth in the longer run, as a result of their adverse effect on risk-taking and incentives to borrow and lend. In particular, while the immediate effects of raising capital standards may well be limited (especially if they are implemented gradually), in the longer run it may lead to higher market loan rates, a reduction in lending and investment, and substitution away from risky lending to holding safer assets, as a result of reduced risk incentives and lower resources devoted to monitoring. There may therefore be a potential dynamic trade-off, in terms of both growth and welfare, associated with macroprudential regulation. Understanding the terms of this trade-off is critical to optimally balance benefits and costs when setting the level of macroprudential instruments.

Van den Heuvel (2008) was one of the first to study the welfare effects of macroprudential regulation, in the form of bank capital requirements, in a growth setting. In line with the foregoing discussion, he argues that capital adequacy requirements may have conflicting effects on welfare. On the one hand, by inducing banks to hold less risky portfolios, they mitigate the probability of a financial crisis, which enhances welfare. On the other, by inducing a shift in banks' portfolios away from risky, but more productive, investment projects, toward safer, but less productive, projects, they may hamper economic growth and have an adverse effect on welfare. Capital requirements

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on the level of development, which may be a proxy for administrative capacity.

therefore entail a trade-off between banking efficiency and financial stability. However, a crucial limitation of that paper (and its extension in Van den Heuvel, 2016) is that growth is exogenous; thus, the implications of this trade-off for long-run growth, and the extent to which it can be internalized when setting regulatory policy instruments, cannot be fully explored.<sup>3</sup>

This paper contributes to the ongoing debate on the impact that macroprudential regulatory constraints may have on the risk-taking incentives of financial intermediaries and how, as a result, they may lead to suboptimal levels of lending—with potentially adverse effects on social welfare. Specifically, the paper develops a two-period overlapping generations (OLG) model with competitive banking where growth is endogenized through an Arrow-Romer externality and capital-producing firms can use either a riskless technology or a risky production technology, which depends on an idiosyncratic shock. As in Van den Heuvel (2008) and Collard et al. (2016), banks can make safe or risky loans to entrepreneurs, but the expected return of risky loans is decreasing in the probability of failure. Thus, the model focuses on financial fragility on the asset side of banks’ balance sheets. Due to limited liability and implicit government guarantees, banks have incentives to engage in excessive risk-taking, that is, lending to risky and less productive capital producers. At the same time, banks are subject to capital requirements, which relate equity and loans.

We show that setting the capital adequacy ratio above a structural thresh-

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<sup>3</sup>Agénor (2016), dwelling on the Holmström-Tirole “double moral hazard” setting, uses an endogenous growth model with endogenous bank monitoring to study the growth and welfare effects of reserve requirements. In line with some other contributions in the literature, such as Dell’Ariccia et al. (2014), the intensity of monitoring in Agénor’s model can be viewed as an inverse, and indirect, measure of risk taking by financial intermediaries. By contrast, in this paper banks choose between two types of loans, making the definition of risk-taking more explicit.



old (which depends in particular on marginal monitoring costs and the probability of failure of the risky technology) can eliminate the equilibrium with risky loans, and thus inefficient risk-taking. However, using numerical simulations, we also calculate the optimal capital adequacy ratio that maximizes social welfare (along the balanced growth path) in the equilibrium with no risky loans. If the optimal rate is lower than the threshold value, the economy suffers from a welfare loss. Thus, there may be a trade-off between financial stability and maximizing social welfare. In addition, the optimal capital adequacy ratio may be so high in practice that, due to competitive pressures, they may actually promote the development of shadow banking activities, which may eventually be detrimental to financial stability. To avoid these unintentional consequences, raising capital requirements may necessitate a concomitant strengthening of financial supervision and a broadening of the perimeter of regulation.

The remainder of the paper is organized as follows. Section 2 describes the economic environment and the behavior of agents.<sup>4</sup> The balanced growth path is characterized in Section 3. The welfare-maximizing capital adequacy ratio is established numerically in Section 4. Section 5 discusses the broader policy implications of the analysis. The last section provides some concluding remarks and discusses perspectives for further research.

## **2 Economic Environment**

The economy consists of a continuum of risk-neutral individual agents who live for two periods, adulthood (or young age) and old age, final good pro-

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<sup>4</sup>The OLG structure is employed mainly for tractability, to obtain analytic expressions throughout. It should not be inferred that the intended model period is thus a generation, or 25 to 30 years as is often the case with OLG models. Although we do not match the model directly to the data, the intended model length is of the order of one year.

ducers, and a financial regulator. Population is constant. Individual agents are all endowed with one period of time in adulthood and are of two types: an exogenous fraction  $n \in (0, 1)$  belong to *households*, and the rest are *entrepreneurs*. Without loss of generality,  $n$  is normalized to 0.5 and the measure of each type of agents to one. Households consist of a fixed number of individuals, which is also normalized to unity. When young (period  $t$ ) each household member receives a labor endowment of unity, which is sold in return for wage income  $w_t$  denominated in final goods. At the end of period  $t$ , a fraction  $\varkappa \in (0, 1)$  of household members are randomly selected to become *bankers*, who join together to form a bank, while a fraction  $1 - \varkappa$  becomes *depositors*. Thus, given that the number of households is normalized to unity,  $\varkappa$  is also the share of bankers in the economy. As discussed later,  $\varkappa$  is determined endogenously, through the equilibrium condition of the market for deposits.

Each household divides  $(1 - \varkappa)w_t$  between period- $t$  consumption and saving via deposits, whereas  $e_t = \varkappa w_t$  is used as equity to start the bank. Deposits can be held either at home or abroad; arbitrage implies therefore that both investments yield the same (gross) return,  $R^D > 1$ , which is set on world markets. For simplicity, banking involves no direct time cost, and in each household there is full consumption insurance, that is, depositors and bankers of the same household share consumption equally.

At the end of period  $t$  bankers combine their equity with deposits to lend to entrepreneurs, who invest to produce capital, using either one of two technologies. Capital becomes available at  $t + 1$  and is rented to final good producers, who combine it with the labor endowment of the next generation, to produce a homogeneous final good at  $t + 1$ . In period  $t + 1$  banks receive the return on the loans that they made in period  $t$  and use it to pay back

depositors, returning any profits lump-sum to the now old households, and close their doors. The new generation of young households, having received their wage, then form their own set of banks—which have no direct link to the previous banks—and the process repeats itself.

Entrepreneurs have no resource endowment; to produce capital goods, they must borrow from banks. They have access to two alternative technologies to accumulate capital; one is safe and the other risky. Although (depending on the realization of an idiosyncratic shock) the risky technology may yield more capital than the safe technology when it succeeds, it yields no capital at all when it fails. Nevertheless, limited liability—the ability to default on loans in the event of failure—tempts entrepreneurs to use it. Banks are needed to monitor the entrepreneurs who claim to use the safe technology to ensure that they do so. Banks themselves, however, may have adverse incentives due to limited liability and implicit deposit guarantees, and these adverse incentives create a role for prudential regulation.

At the beginning of period  $t$ , entrepreneurs borrow from banks to finance investment and all agents (households, entrepreneurs, final good producers, and banks) make their optimization decisions. Entrepreneurs using the risky technology are subject to a failure shock that is identically and independently distributed across them. The probability of failure (which is equal to the fraction of risky entrepreneurs who will eventually fail) is known up-front, but the identity of failing entrepreneurs is only discovered after the realization of the shock. That is, in the model, excessive risk-taking arises from limited liability and involves the *type* (not necessarily the *amount*) of credit extended by banks. Risk-taking is thus measured in terms of the *composition* of banks' loan portfolios.

In this setting, the need for capital requirements arises from limited lia-

bility and implicit deposit guarantees. These institutional features truncate the distribution of risky returns facing investors, loans to these investors, and the depositors funding the banks; this is the externality that leads to excessive risk-taking. Excessive risk-taking involves the type of investments that banks may be tempted to finance because limited liability protects them from incurring large losses, and implicit guarantees dissociate their funding costs from their risk-taking. Sufficiently high capital requirements can always force banks to internalize the riskiness of their loans and thus tame risk-taking behavior; the issue, however, is whether doing so entails a cost in terms of growth and welfare.

## 2.1 Households

Households consume both in adulthood and old age. Utility  $U_t$  of a household with all members born in period  $t$  is given by

$$U_t = \ln c_t^t + \Lambda \ln c_t^{t+1}, \quad (1)$$

where  $c_t^{t+j}$  is consumption at  $t+j$ ,  $j = 0, 1$ , and  $\Lambda \leq 1$  is the household's discount factor.

As noted earlier, at the end of period  $t$  a fraction  $\varkappa$  (respectively,  $1 - \varkappa$ ) of household members becomes bankers (respectively, depositors). The representative household's period budget constraints are thus given by

$$c_t^t + d_t = (1 - \varkappa)w_t, \quad (2)$$

$$c_t^{t+1} = R^D d_t + \Pi_{t+1}, \quad (3)$$

where  $\Pi_{t+1}$  denotes expected profits (if any) received from banks.

Solving the household's optimization problem yields the first-order condition

$$\Lambda \frac{R^D}{c_t^{t+1}} = \frac{1}{c_t^t},$$

which, combined with (2) and (3), gives optimal deposits as:

$$d_t = \frac{\Lambda}{1 + \Lambda}(1 - \varkappa)w_t - \frac{1}{1 + \Lambda} \frac{\Pi_{t+1}}{R^D}. \quad (4)$$

## 2.2 Entrepreneurs

Each entrepreneur  $j$ , with  $j \in (0, 1)$ , is also born with one unit of labor time in adulthood, which is used to operate one of two types of technologies, both of which can be used to convert units of the final good into a single capital good: a *safe* technology (identified with the superscript  $S$ ), or a *risky* technology (identified with the superscript  $R$ ), which is subject to an idiosyncratic shock. Because entrepreneurs have limited liability, those using the risky technology will default on their loans in the event of failure.<sup>5</sup> All entrepreneurs produce the same type of capital good and are price takers. For simplicity, there is no aggregate uncertainty, investment entails no costs, and capital goods fully depreciate upon use.

Whatever the technology chosen, operating it generates no income in the first period. Entrepreneurs therefore do not consume in that period and derive utility only from their old-age consumption,  $c_{t+1}^{j,E}$ , which is equal to realized income in old age,  $z_{t+1}^j$ , which is derived later. Thus,

$$U_{t+1}^{j,E} = \ln c_{t+1}^{j,E}. \quad (5)$$

Each entrepreneur invests the amount borrowed from banks,  $l_t^j$ . Thus, capital produced by an entrepreneur  $j$  choosing the safe technology is given by<sup>6</sup>

$$K_{t+1}^j = l_t^j. \quad (6)$$

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<sup>5</sup>An entrepreneur has no incentive to diversify across multiple risky investments (or to combine risky and safe technologies) because in this setting the benefits of limited liability are maximized by undertaking a single risky investment.

<sup>6</sup>Note that there is only one *type* of capital, even though there are two technologies for producing it.

By contrast, entrepreneurs choosing the risky technology are subject to a failure shock,  $\zeta_t^j$ , which is independently and identically distributed across risky producers. Thus, if the investment is successful, capital is given by

$$K_{t+1}^j = \zeta_t^j(1 + \varepsilon)l_t^j, \quad (7)$$

where  $\varepsilon > 0$  is a productivity parameter which ensures that in the absence of failure the risky technology is always more productive than the safe one and,  $\forall j$ ,

$$\zeta_t^j = \begin{cases} 0 & \text{with prob. } p \\ 1 & \text{with prob. } 1 - p \end{cases}, \quad (8)$$

with  $p \in (0, 1)$  denoting the exogenous probability of failure. The failure shock therefore has a discrete distribution with a mean value of  $1 - p$ . Entrepreneur  $j$  chooses whether to use technology  $S$  or technology  $R$  *before* observing the realization of the idiosyncratic shock  $\zeta_t^j$ .

Regardless of the technology used, entrepreneurs rent the capital that they produce to final good producers at the beginning of period  $t + 1$ . The return that they earn from renting is  $R^K > 1$ , the (constant) marginal product of capital in a competitive equilibrium, as defined next.

This setup with two technologies serves to highlight a familiar connection between limited liability and excessive risk taking: if entrepreneurs (borrowers) are not monitored properly, they may take on more risk than a hypothetical social planner would. For simplicity, using the risky technology to *any* degree is assumed to be always inefficient from the perspective of a social planner (the regulator in this setting), as formally stated next. Nevertheless, because of limited liability, entrepreneurs may still have an incentive to use the risky technology. There is consequently a need to monitor those who claim to use the safe technology, and only banks are assumed to have the

skills needed to do so.<sup>3</sup>

To ensure that the risky technology is inefficient, and thus undesirable, from the regulator's perspective, the following condition is imposed:

**Assumption 1.**  $(1 - p)(1 + \varepsilon) < 1$ ,  $\forall \varepsilon > 0$  and  $p \in [0, 1)$ .

The left-hand side of the condition stated in Assumption 1 represents the expected (gross) benefit of allocating one unit of investment to the risky technology, whereas the right-hand side is the (gross) opportunity cost, that is, the output of the safe technology.

Let the (gross) interest rate incurred when choosing technology  $h = S, R$  and borrowing  $l_t^h$  be denoted  $R_{t+1}^h$ .<sup>4</sup> Based on the previous equations, the following proposition can be directly established:

**Proposition 1.** *Entrepreneurs are indifferent between the safe and risky technologies when the lending rate ratio is  $R_{t+1}^R = (1 + \varepsilon)R_{t+1}^S$ . No entrepreneur invests in the risky technology if  $R_{t+1}^R > (1 + \varepsilon)R_{t+1}^S$ .*

Indeed, an entrepreneur choosing the safe technology maximizes expected profits  $R^K K_{t+1}^j - R_{t+1}^S l_t^S$  with respect to  $l_t^S$ , subject to (6); the solution is simply  $R^K = R_{t+1}^S$ . In the same vein, given limited liability, an entrepreneur choosing the risky technology maximizes  $(1 - p)(R^K K_{t+1}^j - R_{t+1}^R l_t^R) + p \cdot 0$  with respect to  $l_t^R$ , subject to (7) and (8). The (interior) solution is now  $R^K(1 + \varepsilon) = R_{t+1}^R$ . Thus, entrepreneurs are indifferent between the two technologies when the first condition stated in Proposition 1 holds; by contrast, when  $R_{t+1}^R > (1 + \varepsilon)R_{t+1}^S$ , no entrepreneur will find it profitable to invest in the risky technology and there will be no demand for risky loans.

Moreover, the model has no equilibrium with  $R_{t+1}^R < R_{t+1}^S$ . Indeed, if that condition were to hold, banks would have no incentive to fund risky

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<sup>3</sup>This explains why there is no direct intermediation from households to entrepreneurs. More generally, as in Gale (2004), households cannot directly invest in risky assets and can only do so through banks.

<sup>4</sup>The loan rate is agreed at time  $t$  but is dated  $t + 1$ , to reflect when loans are repaid.

investments, because safe investments would generate a higher return in every state of nature (that is, whatever the realization of the failure shock  $\zeta_t^j$ ) and there would be no need to monitor them.<sup>5</sup> Thus, given also that (from Proposition 1) there will be no demand for risky loans if the interest-rate ratio  $R_{t+1}^R/R_{t+1}^S$  is strictly higher than the critical value  $1 + \varepsilon$ , there is one and only one possible scenario under which entrepreneurs will use the risky technology—the case where they are indifferent between using either one of them,  $R_{t+1}^R = (1 + \varepsilon)R_{t+1}^S$ .

### 2.3 Final Output

Competitive firms produce the final good (which can be either consumed or used as a production input) by combining labor and capital goods, which become available in each period before production starts. The underlying private technology exhibits constant returns in capital and labor inputs:

$$Y_t = A_t N^{1-\alpha} K_t^\alpha, \quad (9)$$

where  $\alpha \in (0, 1)$ ,  $N$  is the number of workers (or household members),  $K_t = \int_0^1 K_t^j dj$  is the aggregate capital stock, and  $A_t$  a productivity parameter.

There is an Arrow-Romer type externality associated with the capital-labor ratio  $k_t = K_t/N$ , so that

$$A_t = A k_t^{1-\alpha}. \quad (10)$$

Combining (9) and (10) yields, in standard fashion, a linear relationship between (aggregate) production per worker,  $y_t$ , and capital per worker:

$$y_t = A k_t. \quad (11)$$

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<sup>5</sup>This property of the model, which is the same as in Collard et al. (2016), implies that there is no reason for banks to monitor entrepreneurs who claim to use the risky technology. Accordingly, only the cost of monitoring safe technology users is accounted for in studying the behavior of banks later on.



Final goods producers operate in competitive output and input markets so that equilibrium capital rental and wage rates,  $R_t^K$  and  $w_t$ , are determined by their marginal product:

$$R^K = \alpha A, \quad w_t = (1 - \alpha)Ak_t. \quad (12)$$

The following condition is imposed on  $A$ :

**Assumption 2.**  $A > 1/\alpha$ .

This condition ensures that the gross return to capital satisfies  $R^K > 1$ .

## 2.4 Banks and Regulatory Regime

Part of each household's wage income is used to capitalize a bank with net worth  $e_t = \varkappa w_t$ . Using (12), a bank's equity is thus given by

$$e_t = \varkappa(1 - \alpha)Ak_t. \quad (13)$$

However, issuing equity involves a cost, at the rate  $R^E$ . In line with the evidence, this cost rate is taken to exceed the deposit rate, so that  $R^E > R^D$ . For tractability, we assume that this cost is linear (as in Covas and Fujita, 2010, and Nguyen, 2014, for instance) and in what follows set  $R^E = \delta R^D$ , where  $\delta > 1$ .

The bank takes deposits from (other) households and combines them with its own resources to lend to entrepreneurs. Banks are perfectly competitive. They can make safe and risky loans,  $l_t^S$  and  $l_t^R$ , respectively. Each bank extends risky loans to at most one entrepreneur employing the risky technology; this is because the benefits of limited liability are maximized by concentrating the risk in a single loan, given that this maximizes the probability of the worst outcome.<sup>6</sup>

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<sup>6</sup>See Collard et al. (2016) for a further discussion.

Let  $m \in (0, 1)$  denote the exogenous marginal resource cost of monitoring an entrepreneur who claims to use the safe technology.<sup>7</sup> The bank's balance sheet is

$$l_t^S + l_t^R = e_t + d_t - ml_t^S, \quad (14)$$

where  $ml_t^S$  represents the total cost of monitoring safe loans.

Given Assumption 1, risky investments reduce welfare. To rule out the hypothetical case where the regulator directly forbids risk-taking, suppose that, as in Van den Heuvel (2008, 2016) and Collard et al. (2016), banks can hide some risky loans in their portfolio from the regulator. Specifically, suppose that the regulator observes the total amount of loans made by each bank but cannot detect its risky loans up to a given fraction  $\gamma > 0$  of its safe loans.<sup>8</sup> It imposes full capital requirements on risky loans above that fraction,  $l_t^R - \gamma l_t^S$ . The prudential regime is thus characterized by the following formula:

$$e_t \geq \mu(l_t^S + l_t^R) + \max(0, l_t^R - \gamma l_t^S),$$

where  $\mu \in (0, 1)$  is the capital adequacy (or Cooke) ratio. Imposing full capital requirements on risky loans in excess of  $\gamma l_t^S$  are such that they ensure that  $l_t^R \leq \gamma l_t^S$  in equilibrium.<sup>9</sup>

We also assume that banks benefit from an implicit government guarantee on their deposits. Should their gross income from lending be insufficient to fully cover repayment to depositors, they benefit from a cost-free lump-sum

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<sup>7</sup>The cost  $m$  could be endogenous and related in convex fashion to the level of loans. However, this would complicate analytical derivations without adding much insight.

<sup>8</sup>In Van den Heuvel (2008), the threshold  $\gamma$  is a decreasing function of the resources spent on bank supervision. Here it is assumed constant for simplicity.

<sup>9</sup>An additional dimension of risk sensitivity of the regulatory regime could be captured by replacing  $\mu(l_t^S + l_t^R)$  by  $\mu^S l_t^S + \mu^R l_t^R$ , where  $\mu^S, \mu^R \in (0, 1)$  and  $\mu^R > \mu^S$ . However, this would make little substantive difference to the subsequent analysis as long as  $\mu^R$  is specified as a multiple of  $\mu^S$ .

transfer from the regulator, drawn from an initial endowment fund, to make up for the shortfall.<sup>10</sup> The existence of this guarantee, together with the fact that (as noted earlier) equity is more expensive than deposit finance, ensures that banks will hold no more equity than required by regulation—or, equivalently, that they will choose as much leverage as allowed by the financial regulator. The prudential regime can consequently be equally characterized by the binding constraint on equity,

$$e_t = \mu(l_t^S + l_t^R), \quad (15)$$

together with the inequality constraint on risky loans,

$$l_t^R \leq \gamma l_t^S. \quad (16)$$

Given limited liability of risky borrowers and the fact that deposits are fully insured at no cost to the bank, expected bank profits when both types of lending occur can be defined as

$$\Pi_{t+1} = R_{t+1}^S l_t^S + (1-p)R_{t+1}^R \gamma l_t^S - R^D(d_t + \delta e_t). \quad (17)$$

Banks choose either  $l_t^S$  only, or  $l_t^R$  and  $l_t^S$  jointly, to maximize (17) subject to (14), (15), and (16). The solution of the bank's optimization problem is provided in the Appendix. A key implication of the solution of this optimization problem can be summarized in the following proposition:

**Proposition 2.** *Either all banks take no risk ( $l_t^R = 0$ ), or they take the maximum undetectable risk ( $l_t^R = \gamma l_t^S$ ). There are no equilibria with  $0 < l_t^R < \gamma l_t^S$ .*

The intuition, which is fundamentally the same as in Van den Heuvel (2008) and Collard et al. (2016), is as follows. If, given the loan portfolio,

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<sup>10</sup>As in Van den Heuvel (2008) and Gale (2010), for instance, bank contributions to that fund are therefore abstracted from for simplicity.

bank equity is sufficiently small to be wiped out when risky investments fail, then banks do not internalize the cost of additional risk-taking. Additional losses from increasing  $l_t^R$ , if risky investments fail, are truncated by limited liability and the implicit government guarantee on deposits. Consequently, the only equilibrium with the possibility of bank failure involves the solution  $l_t^R = \gamma l_t^S$ . Alternatively, if bank equity is sufficiently large for banks to remain solvent even when risky investments fail, then banks would internalize the cost of additional risk-taking. In that case, given that the risky technology is (relatively) inefficient, banks can increase actual profits by reducing  $l_t^R$ . Accordingly, the only equilibrium without the possibility of bank failure involves the solution  $l_t^R = 0$ .

As shown in the Appendix, in the equilibrium with  $l_t^R = 0$ , the loan spread is given by

$$\left. \frac{R_{t+1}^S}{R^D} \right|_{l_t^R=0} = 1 - \mu + \delta\mu + m, \quad (18)$$

whereas in the equilibrium with  $l_t^R = \gamma l_t^S$ , the spread is

$$\left. \frac{R_{t+1}^S}{R^D} \right|_{l_t^R>0} = \frac{(1 - \mu + \delta\mu)(1 + \gamma) + m}{1 + (1 - p)\gamma(1 + \varepsilon)}. \quad (19)$$

The following proposition can be directly established from (18) and (19):

**Proposition 3.** *Under both equilibria, the spread between the safe interest rate and the deposit rate is increasing in the monitoring cost,  $m$ , and in the capital adequacy ratio,  $\mu$ .*

A higher capital requirement raises the loan rate because, with the level of equity given, the regulatory constraint (15) implies that a higher  $\mu$  raises the funding costs for banks; to keep profits constant, the cost of loans must increase. If, as a result, the demand for loans (both safe and risky, given that  $l_t^R = \gamma l_t^S$ ) falls, higher capital requirements would induce less risk-taking, in line with the common moral hazard argument emphasized in the literature.

In addition, in the risky-loan equilibrium, the spread also depends on the detection threshold,  $\gamma$ , the probability of failure  $p$ , and the productivity of the risky technology,  $\varepsilon$ . In particular, we have the following result:

**Proposition 4.** *In the equilibrium with risky loans, the spread between the safe interest rate and the deposit rate is increasing in the failure probability,  $p$ , and decreasing in the productivity of the risky technology,  $\varepsilon$ .*

Intuitively, as in Collard et al. (2016), the dependence of the spread on the failure probability stems from the fact that making safe loans enables banks to make risky loans—given that hiding the risk associated with these loans is subject to the constraint  $l_t^R \leq \gamma l_t^S$ . The spread is decreasing in the productivity of the risky technology (conditional on it not failing) because a higher value of  $\varepsilon$  raises risk-taking incentives for banks.

## 2.5 Deposit Market Equilibrium

For bankers who provide loans to borrower  $h = S, R$ , a necessary condition for raising deposits to be profitable is  $R_{t+1}^h > R^D$ . Given that there is no equilibrium if  $R_{t+1}^R < R_{t+1}^S$ , and that from (18) and (19)  $R_{t+1}^S > R^D$ , this condition is always satisfied. Thus, the bank will demand deposits up until the point at which the regulatory constraint (15) is indeed binding.

As shown in the Appendix, the *demand* for deposits by banks can be solved residually from the balance sheet and capital requirement constraints (14) and (15) to give

$$d_t = \left( \frac{1+m}{\mu} - 1 \right) e_t, \quad \text{when } l_t^R = 0, \quad (20)$$

$$d_t = \left[ \frac{1+m+\gamma}{\mu(1+\gamma)} - 1 \right] e_t, \quad \text{when } l_t^R = \gamma l_t^S, \quad (21)$$

Because banks make zero profits in equilibrium, from (4) the *supply* of

deposits by households is simply

$$d_t = \frac{\Lambda(1 - \varkappa)}{1 + \Lambda} w_t. \quad (22)$$

In equilibrium, the supply and demand for deposits must be equal. Equations (20) and (22), as well as (21) and (22), can be solved for  $\varkappa$ , the share of income allocated to equity, or equivalently in the present setting, the *size* of the banking system.<sup>11</sup> The Appendix shows that the solutions are

$$\varkappa|_{l_t^R=0} = \frac{1}{\Phi_1} < 1, \quad (23)$$

$$\varkappa|_{l_t^R>0} = \frac{1}{\Phi_2} < 1, \quad (24)$$

where

$$\Phi_1 = 1 + \left(\frac{1 + \Lambda}{\Lambda}\right) \left(\frac{1 + m}{\mu} - 1\right),$$

$$\Phi_2 = 1 + \left(\frac{1 + \Lambda}{\Lambda}\right) \left\{ \frac{(1 - \mu)(1 + \gamma) + m}{\mu(1 + \gamma)} \right\}.$$

From (22), (23) and (24), the following proposition can be established:

**Proposition 5.** *An increase in the capital adequacy ratio,  $\mu$ , lowers the fraction of depositors and increases the share of bankers,  $d\varkappa/d\mu > 0$ .*

Intuitively, a higher capital adequacy ratio raises equity needs for the banks. For a given wage, the equilibrating mechanism operates through a higher share of bankers in each household, who provide the initial net worth that banks use to fund their lending operations. However, the share of depositors falls concomitantly, and this has implications (as discussed later)

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<sup>11</sup>An alternative approach would be to assume, as in Foulis et al. (2015), for instance, that the equilibrium between supply and demand for deposits is achieved through changes in the deposit rate. In the present setting the deposit rate is set exogenously on the world capital market and the equilibrium is obtained through a quantity adjustment. As discussed next, this has important implications for assessing the growth and welfare effects of capital requirements.

for the behavior of household income in response to an increase in the capital adequacy ratio.<sup>12</sup>

### 3 Balanced Growth Path

To establish the balanced growth path, note first that, from (6) and (7), and appealing to the law of large numbers (given that  $\zeta_t$  is independently and identically distributed across investments), capital at  $t + 1$  is given by

$$K_{t+1} = l_t^S, \quad \text{when } l_t^R = 0,$$

$$K_{t+1} = (1 - p)(1 + \varepsilon)(l_t^S + l_t^R), \quad \text{when } l_t^R = \gamma l_t^S.$$

Given that all banks behave in the same fashion, and that their number is normalized to unity, using (13) and (15) these equations yield

$$\frac{k_{t+1}}{k_t} = 1 + g = \frac{\varkappa|_{l_t^R=0}}{\mu}(1 - \alpha)A, \quad (25)$$

$$\frac{k_{t+1}}{k_t} = (1 - p)(1 + \varepsilon)\frac{\varkappa|_{l_t^R>0}}{\mu}(1 - \alpha)A, \quad (26)$$

with  $\varkappa$  defined in (23) and (24).<sup>13</sup> Equations (25) and (26) define the steady-state growth rate of capital and, from (11), final output. In addition, given (13), equity grows at the same rate as well. A comparison of (25) and (26) shows that, given Assumption 1, whether the growth rate is higher when the safe technology is used, compared to the risky technology, depends in part on whether (from (23) and (24))  $\Phi_1$  is higher or lower than  $\Phi_2$ . From the

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<sup>12</sup>A number of other contributions to the literature on capital requirements, such as Gersbach (2013) and Gorton and Winton (2014), also predict a negative impact of higher bank capital requirements on bank deposits.

<sup>13</sup>When  $l_t^R > 0$ , the growth rate  $k_{t+1}/k_t = \zeta_t(1 + \varepsilon)\varkappa\mu^{-1}(1 - \alpha)A$  is stochastic, given that  $\zeta_t$  is a random shock. Because of the linearity of  $\zeta_t$ , moments higher than the mean (such as the variance of  $\zeta_t$ , given by  $p(1 - p)$ ) do not appear in (26).

definitions above, it is easy to establish that  $\Phi_2 < \Phi_1$ , which implies that  $\varkappa|_{l_t^R > 0} > \varkappa|_{l_t^R = 0}$ ; at the same time, however, given Assumption 1,  $(1-p)(1+\varepsilon) < 1$ . Thus, in general the difference in the growth rates in (26) and (25) cannot be signed unambiguously.<sup>14</sup> However, even though the growth rate under the risky technology can be on average higher than under the safe technology, it is socially inefficient for the realized growth rate to fluctuate between zero and positive values. This therefore provides a complementary argument as to why a social planner may want to ensure that entrepreneurs do not use the risky technology.<sup>15</sup>

Equations (25) and (26) also show that the effect of an increase in the capital adequacy ratio on the growth rate depends on the sign of  $d(\varkappa/\mu)/d\mu$ . On the one hand, an increase in  $\mu$  reduces the growth rate directly, because (at the initial level of equity) lending must fall. On the other, as implied by Proposition 5, because banks must now raise more equity it increases the size of the banking system and thus the capacity to lend—thereby promoting growth. As shown in the Appendix, using (23) it can be established that the net effect is positive. Thus, in contrast to other contributions to the literature, in our setting tighter capital requirements promote long-run growth—fundamentally through their effect on the relative size of the banking system,  $\varkappa$ . However, as discussed later, their impact on welfare is nonmonotonic.

The next issue to address is to determine the conditions under which one of the two equilibria (one with safe loans only, the other with both types

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<sup>14</sup>It can be noted that, because  $d(\Phi_2 - \Phi_1)/d\gamma < 0$ , a higher detectability threshold tends to increase the (mean) growth rate under the equilibrium with risky loans, relative to the equilibrium with safe loans only.

<sup>15</sup>Of course, the reason why the realized growth rate can be zero is due to the assumption in (8) that when the risky technology fails (with probability  $p$ )  $\zeta_t^j = 0$ . However, the argument carries through as long the realization of  $\zeta_t^j$  in the good state takes a higher value than in the bad state of nature.



of loans) prevails. As shown in the Appendix, the following proposition can also be established:

**Proposition 6.** *For the equilibrium  $l_t^R = 0$  to hold, a necessary and sufficient condition is  $\mu \geq \max(0, \tilde{\mu} = (\delta - 1)^{-1}[(1 - p)(1 + \varepsilon)(1 + m) - 1]/[1 - (1 - p)(1 + \varepsilon)])$ .*

The first point to know about the expression for  $\tilde{\mu}$  on the right-hand side is that, given Assumption 1, the condition  $(1 - p)(1 + \varepsilon) < 1$  holds; and given that  $\delta > 1$ , the sign of  $\tilde{\mu}$  depends on the sign of  $(1 - p)(1 + \varepsilon)(1 + m) - 1$ . In the trivial case where  $p = \varepsilon = m = 0$ , or if  $(1 - p)(1 + \varepsilon)(1 + m) < 1$ , the restriction  $\mu \geq \tilde{\mu}$  is not binding. For the problem to be interesting, we therefore impose the following condition:

**Assumption 3.**  $(1 - p)(1 + \varepsilon)(1 + m) > 1$ .

Given this condition, the intuition that underlies the result in Proposition 6 is the same as the intuition upon which Proposition 2 is based. Given limited bank liability, for risky lending to occur, banks must make an *expected* profit when risky loans are made; to eliminate the incentives to do so requires setting the capital requirement above  $\tilde{\mu}$ .

It is worth noting that the threshold  $\tilde{\mu}$  is decreasing in the marginal monitoring cost. Note also that, in contrast to Collard et al. (2016), the detectability threshold  $\gamma$  does not affect the threshold value. More importantly, Proposition 6 does not pin down the *socially optimal* value of the capital adequacy ratio; it imposes only a *feasibility constraint* on a policy aimed at eliminating risk-taking. Put differently, if it is legitimate to set  $\mu > \tilde{\mu}$ , how high should  $\mu$  be? Does the regulator face a trade-off between the minimum capital adequacy ratio that eliminates the risky-loan equilibrium and the socially optimal capital adequacy ratio?

## 4 Optimal Capital Requirements

We now derive the welfare-maximizing value of the capital adequacy ratio,  $\mu$ . To do so, suppose that the financial regulator, although concerned primarily with the safety of the financial system, may also seek to maximize social welfare in the absence of conflict between the two objectives. Thus, it may also act as a social planner would, by being far sighted and benevolent—in the sense of taking into account the welfare of all future generations of entrepreneurs and households. To calculate the welfare for each generation, recall that while households consume in both periods, entrepreneurs consume only in adulthood. Thus,  $c_{t+1}^E = z_{t+1}$ , where  $z_{t+1}$  denotes an entrepreneur's realized income in old age. In turn, an entrepreneur's income if the safe technology is chosen (so that  $l_t^R = 0$ ) is  $z_{t+1} = R^K K_{t+1} - R_{t+1}^S l_t^S$ , implying that, using (12), (15), and (18),

$$z_{t+1}|_{l_t^R=0} = [\alpha A - (1 - \mu + \delta\mu + m)R^D]\mu^{-1}e_t.$$

Using (5) and (13), an entrepreneur's indirect utility function is thus

$$V_{t+1}^E|_{l_t^R=0} = V_m^E|_{l_t^R=0} + \ln k_t, \quad (27)$$

where

$$V_m^E|_{l_t^R=0} = \ln\{[\alpha A - (1 - \mu + \delta\mu + m)R^D]\mathcal{Z}(1 - \alpha)A\} - \ln \mu.$$

When  $l_t^R = \gamma l_t^S$ , expected income is  $z_{t+1} = R^K K_{t+1} - (1 - p)R_{t+1}^R l_t^R$ , so that, using Proposition 1,

$$z_{t+1}|_{l_t^R>0} = \frac{\alpha A - (1 - p)(1 + \varepsilon)(1 - \mu + \delta\mu + m)\gamma R^D}{\mu(1 + \gamma)}e_t,$$

or, using again (5) and (13),

$$V_{t+1}^E|_{l_t^R>0} = V_m^E|_{l_t^R>0} + \ln k_t, \quad (28)$$

where now

$$V_m^E|_{l_t^R > 0} = \ln\{\alpha A - (1-p)(1+\varepsilon)(1-\mu + \delta\mu + m)\gamma R^D\}(1-\alpha)A\} \\ + \ln \varkappa - \ln \mu(1+\gamma).$$

For households, given that there are no bequests, the Appendix shows that their indirect utility function takes the form, in both equilibria,

$$V_t^H = V_m^H + (1+\Lambda) \ln k_t, \quad (29)$$

where

$$V_m^H = \ln\left[\frac{(1-\alpha)A}{1+\Lambda}\right] + \Lambda \ln\left[\frac{\Lambda R^D(1-\alpha)A}{1+\Lambda}\right] + (1+\Lambda) \ln(1-\varkappa).$$

Recall that each group represents half of the population. Thus, the welfare criterion is the equally weighted sum within each generation, but discounted sum of utility across an infinite sequence of generations (see De la Croix and Michel, 2002, p. 91):

$$\mathcal{W} = \sum_{h=0}^{\infty} \Omega^h 0.5 (V_{t+h+1}^E + V_{t+h}^H), \quad (30)$$

where  $\Omega \in (0, 1)$  is the regulator's discount factor. From (28) and (29), along the balanced growth path,

$$\mathcal{W} = \sum_{h=0}^{\infty} \Omega^h 0.5 [V_m^E + V_m^H + (2+\Lambda) \ln \tilde{k}_{t+h}]. \quad (31)$$

From (25) or (26),  $k_t$  grows at a constant rate,  $1+g$ , along the balanced growth path. Thus, along the steady-state equilibrium path,  $\tilde{k}_{t+h} = (1+g)^{t+h} k_0$ . Substituting this result in (31) yields

$$\mathcal{W} = \sum_{h=0}^{\infty} \Omega^h 0.5 \{V_m^E + V_m^H + (2+\Lambda)(t+h) \ln(1+g)\}. \quad (32)$$

Given that  $\Omega < 1$ ,  $\mathcal{W}$  is strictly concave and bounded, and the choice set is convex and compact. Thus, the optimization problem  $\max_{\mu} \mathcal{W}$  has a single solution. Solving (32) gives<sup>16</sup>

$$\mathcal{W} \simeq \frac{V_m^E + V_m^H}{1 - \Omega} + \frac{\Omega(2 + \Lambda)}{(\Omega - 1)^2} \ln(1 + g), \quad (33)$$

with  $1 + g$  given in (25) and (26). The optimal value of  $\mu$  is the one for which  $d\mathcal{W}/d\mu = 0$  is obtained. However, the resulting expression is too complex to allow an explicit analytical solution for the optimal value of  $\mu$ .

Before discussing numerical simulations, it is important to note that a potential conflict may emerge between financial stability and welfare maximization. To show this, let  $\mu^*|_{l_t^R > 0}$  and  $\mu^*|_{l_t^R = 0}$  denote the welfare-maximizing solution in the equilibrium with and without risky loans, respectively, and let  $\mathcal{W}^*|_{l_t^R > 0}$  and  $\mathcal{W}^*|_{l_t^R = 0}$  denote the corresponding value of welfare, scaled by the value when  $\mu \simeq 0$ . Let also  $\mathcal{W}(\tilde{\mu})$  denote the (relative) value of welfare when  $\mu = \tilde{\mu}$ . Assuming that the regulator's main goal is to ensure financial stability (by selecting a value of the capital adequacy ratio that is high enough to eliminate incentives to provide risky loans), the solution  $\mu^*|_{l_t^R > 0}$  will *never* be chosen. There are therefore only two relevant outcomes.

1. If  $\mu^*|_{l_t^R = 0}$  is higher than  $\tilde{\mu}$ , the feasibility constraint is not binding; the regulator can select the welfare-maximizing value  $\mu^*|_{l_t^R = 0}$  even if financial stability is the main consideration. There is a *welfare gain* compared to  $\mu = \tilde{\mu}$ , because (by definition of the optimum) welfare is increasing in  $\mu$  up to the point where  $\mu = \mu^*|_{l_t^R = 0}$ . This gain is given by  $\mathcal{W}^*|_{l_t^R = 0} - \mathcal{W}(\tilde{\mu})$ .

2. If  $\mu^*|_{l_t^R = 0}$  is lower than  $\tilde{\mu}$ , the regulator sets  $\mu = \tilde{\mu}$  and welfare is not maximized; ensuring financial stability always entails a *welfare loss*, because

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<sup>16</sup>This derivation uses the standard results  $\sum_{h=0}^{\infty} x^h = 1/(1 - x)$  and  $\sum_{h=0}^{\infty} hx^h = x/(x - 1)^2$  when  $|x| < 1$ .

(again, by definition of the optimum) welfare is decreasing in  $\mu$  beyond the optimal value  $\mu^*|_{l_t^R=0}$ . This loss is given by  $\mathcal{W}(\tilde{\mu}) - \mathcal{W}^*|_{l_t^R=0}$ .

To illustrate these outcomes, we solve for the optimal capital adequacy ratio and welfare levels by performing numerical evaluations based on the following parameter and initial values. Assuming an annual discount factor of 0.04 for both households and the regulator, and interpreting a period as 30 years, yields an intergenerational discount factor  $\Lambda = \Omega = 0.308$ . The elasticity of output to capital is set at  $\alpha = 0.35$ , whereas the gross deposit rate is set at  $R^D = 1.02$ . All these values are fairly standard. Following Collard et al. (2016, Table 1), we set  $p = 0.034$  and  $m = 0.003$ . We also set  $A = 4.5$ ,  $\gamma = 1.0$ ,  $\varepsilon = 0.032$ , and  $\delta = 1.32$ . Thus, the benchmark case considers a situation where the marginal cost of monitoring is fairly low, and the marginal cost rate of issuing equity is 2.6 percent, in line with Gomes and Schmid (2012). For this set of values, assumptions 1, 2 and 3 are all satisfied. The implied value of  $\varkappa$  in the safe-loan equilibrium is 0.0455.

The determination of the optimal capital adequacy ratio in the safe-loan equilibrium is illustrated in Figure 1, which shows that the relationship between social welfare (scaled by its base value when  $\mu$  is close to 0, that is, in the absence of regulation) and the capital adequacy ratio has an inverted-U shape. Intuitively, increases in the value of the capital adequacy ratio improve welfare at first because the net effect on lending is positive. On the one hand, for a given supply of equity, and given the regulatory constraint, a higher ratio tends to reduce the supply of loans, which hampers growth.<sup>17</sup> On the other, the higher demand for equity, for a given capital stock, re-

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<sup>17</sup>As stated in Proposition 3, the higher capital adequacy ratio raises the cost of (both safe and risky) loans. This would normally reduce the demand for loans and mitigate the growth in output, wages, and consumption. In the present setting the amount of lending is determined through the binding regulatory constraint but this adverse effect is the same for a given level of equity.

quires a reduction in the fraction of depositors and an increase in the share of bankers among households (see Proposition 5), which tends to increase lending and promote growth. While in principle the net effect is ambiguous (as noted earlier), in the present case it is positive at low levels of  $\mu$ ; banks lend more, which tends to promote capital accumulation, output growth, and wage income. In turn, this tends to increase consumption and welfare. However, as  $\mu$  increases the reduction in the fraction of depositors tends to lower deposit income and to dampen the initial increase in household consumption and welfare. This effect tends eventually to dominate, even though the growth effect remains positive.<sup>18</sup> Thus, the optimal capital adequacy ratio is pinned down at the point at which its marginal effect on welfare is zero. Importantly, this effect is not due to the fact that capital requirements have an adverse effect on growth due to a negative impact on investment lending—the net effect here is positive—but rather through changes in the composition of households, a proxy in this setting for the *size* of the banking system, and their direct impact on income.<sup>19</sup>

Based on the above parameters, the minimum threshold of the capital adequacy ratio is  $\tilde{\mu} = 0.104$  and the optimal value are  $\mu^*|_{l_t^R > 0} = 0.188$  and  $\mu^*|_{l_t^R = 0} = 0.169$ , which are all significantly higher than the tier-one capital ratios set under the Basel Accord for 2013 (see Basel Committee on Banking Supervision, 2011, Annex 2).<sup>20</sup> As shown in the figure, a lower marginal monitoring cost (from  $m = 0.003$  to  $0.0005$ , for illustrative purposes) would raise both the threshold level of  $\mu$  and its optimal value;  $\mu^*|_{l_t^R = 0}$  increases

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<sup>18</sup>The growth effect actually gets stronger as  $\mu$  increases because, from (23),  $d^2(\varkappa/\mu)/d\mu^2 > 0$ .

<sup>19</sup>Although not shown, a similar inverted U-shaped curve holds for the equilibrium with risky loans.

<sup>20</sup>The tier one capital ratio (which includes common equity) is the relevant concept for comparative purposes, given the simplified bank capital structure of the model.

from 0.169 to 0.183. Intuitively, a lower monitoring cost lowers the cost of borrowing and increases the demand for (both safe and risky) loans; eliminating the risky-loan equilibrium therefore requires a higher capital adequacy ratio.

Moreover, because  $\mu^*|_{l_t^R=0} > \tilde{\mu}$ , the welfare-maximizing rate exceeds the threshold value that eliminates the risky-loan equilibrium; this therefore corresponds to outcome 1, as described above. Calculations also show that  $\mathcal{W}(\tilde{\mu}) = 1.00124$  and  $\mathcal{W}^*|_{l_t^R=0} = 1.0096$ , which implies two results: *a*) compared to the case where capital requirements are (almost) zero, the welfare gain is relatively small; and *b*) setting the capital adequacy ratio to the welfare-maximizing value entails a (relative) welfare gain compared to the threshold value of the order of 0.8 percentage points. Thus, there is no trade-off between financial stability and welfare maximization. Sensitivity analysis suggests that this result is fairly robust with respect to changes in some parameters ( $p$ ,  $\gamma$ , and  $\delta$ , in particular)—as long as Assumptions 1 and 3 are satisfied.

However, outcome 2 (welfare loss) may also occur, depending in particular on the value of  $\varepsilon$ . Indeed, with a higher value of  $\varepsilon$ , the threshold value of the capital adequacy ratio must increase to offset the rise in expected profitability of the risky technology, whereas the growth-maximizing rate in the safe-loan equilibrium—which does not depend on  $\varepsilon$ —remains the same. In particular, with  $\varepsilon = 0.0326$ , we have  $\tilde{\mu} = 0.574 > \mu^*|_{l_t^R=0} = 0.169$ , as well as  $\mathcal{W}(\tilde{\mu}) = 0.977$  and  $\mathcal{W}^*|_{l_t^R=0} = 1.0015$ . The (relative) welfare loss when the regulator chooses  $\tilde{\mu}$  is thus  $-2.4$  percentage points. This loss could be made larger by increasing  $\varepsilon$  further, subject again to Assumptions 1 and 3 being satisfied. The key point, however, is that there is now a trade-off between financial stability and welfare maximization.

In the foregoing discussion, the cost parameter  $\delta$  was kept constant. Suppose now that, in standard fashion, equity becomes more expensive in response to the higher demand for equity induced by increases in  $\mu$ , so that  $\delta = \delta(\mu)$  and  $\delta' > 0$ . Although this assumption would have no effect on the equilibrium growth rate (as can be inferred from (23) and (25)), the adverse effect on welfare alluded to earlier would now be stronger. However, as long as  $\delta'$  is not too large, the inverted U-shape curve relationship between the capital adequacy ratio and welfare would continue to hold.

Conversely, based on the empirical results in Gambacorta and Shin (2016), suppose instead that the cost of equity varies inversely with the ratio of equity to assets, that is,  $e_t/l_t^S$  in the safe equilibrium, due for instance to a signaling effect. From (15), this would imply again that  $\delta = \delta(\mu)$ , but with  $\delta' < 0$ . This would create another channel through which changes in  $\mu$  initially affect welfare positively—and therefore the optimal value of the capital adequacy ratio. The preceding results, however, would again remain qualitatively the same as long as  $\delta'$  is not too large.

In sum, the above experiments suggest that the equilibrium with safe loans provides higher welfare gains, suggesting that there may, or may not, be a conflict between financial stability and welfare. Moreover, even if there is no conflict, the optimal capital adequacy ratio may be fairly high, compared to the 10-12 percent range considered in the Basel Accords. These high values, if implemented, may foster disintermediation and the expansion of the shadow financial system—thereby exacerbating the very financial risks that capital requirements are supposed to address in the first place. We elaborate on these issues in the next section.



## 5 Policy Implications

As noted in the introduction, one of the fundamental roles of capital regulation is to mitigate the moral hazard (or excessive risk-taking) induced by limited liability and government guarantees, which may lead to financial fragility and bank failures. It has been argued, in particular, that capital requirements may be essential to compensate for the possibility of greater risk induced by mispricing of explicit (deposit insurance) or implicit government guarantees. The higher capital requirements are, the more banks internalize the social cost of risk, as they have more “skin in the game.”

However, mitigating risk could unduly affect incentives to lend in the long run. Indeed, some observers have argued that, although there is a risk-reducing effect of capital requirements, it may be achieved at the cost of restricting bank lending, which in turn may hamper growth and reduce welfare. Hence, the regulator is confronted with a difficult trade-off: policies aimed at reducing the likelihood of bank failures and ensuring the safety of the financial system may have persistent, adverse effects on growth and welfare. The foregoing analysis provided a simple analytical characterization of this trade-off in a growing economy, by showing that to mitigate incentives for banks to engage in risky activities, financial regulation may require setting the capital adequacy ratio at a level that is too high compared to its socially optimal value—entailing as a result a welfare loss that may be significant. Thus, even though ensuring bank safety does not necessarily have to come at a high welfare cost—our analysis shows clearly that the outcome depends on the economy’s structural parameters—policies designed to promote financial stability do have the potential to adversely affect the long-run performance of the economy and to reduce social welfare. The potential significance of

this result is hard to overstate, considering that it takes only small changes in the growth rate to produce substantial cumulative gains or losses in output.

At the same time, it is important to keep in mind that in practice, if capital requirements are optimally set at very high levels—as may be the case in the foregoing analysis when the threshold constraint on the capital adequacy ratio is not binding—in an environment where competition forces are strong, they may foster (as indicated earlier) disintermediation and promote the development of the shadow banking sector. In turn, this may distort the functioning of financial markets and ultimately weaken financial stability while hampering growth and reducing welfare.<sup>21</sup> In such conditions, financial supervision may also need to be strengthened, and the perimeter of regulation broadened, to avoid unintended consequences.

## 6 Concluding Remarks

This paper studied, in a growth model where banks serve as financial intermediaries, the welfare effects of banking regulation in the form of capital requirements, which indirectly act as a constraint on banks' portfolios. Because equity is more expensive than deposits, banks choose the minimum amount of capital that is compatible with the capital requirement rule. Entrepreneurs borrow from banks to invest in either a safe or a risky technology, but they are protected by limited liability. The financial regulator's main goal is to promote financial stability by setting the capital adequacy ratio so as to eliminate incentives for banks to provide risky loans.

The main results of the analysis were summarized in the introduction.

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<sup>21</sup>Cizel et al. (2016) provided evidence that macroprudential regulation may lead to substitution effects toward nonbank credit, especially in countries with more developed financial markets. See also Dagher et al. (2016) for a further discussion.

In a nutshell, they show that, depending on the structural features of the economy, there may be a trade-off between financial stability and welfare. In addition, even when there is no trade-off, the level of capital requirements that is necessary to eliminate inefficient risk-taking may be too high in practice and may need to be accompanied by a strengthening of financial supervision to avoid a situation where risks migrate from banks to lightly regulated nonbank financial intermediaries—making the financial system, in the end, more prone to instability and crises.

Our analysis can be extended in several directions. First, in the model, risk-taking is exclusively related to the *type* of credit extended by banks. One extension could be to modify our setup to consider situations in which both the type and the volume of credit matter. To do so the cost of originating and monitoring safe loans could be made an increasing function of the aggregate volume of these loans (as in Christensen et al., 2011, and Collard et al., 2016, for instance). As a result, shocks that affect the volume of safe loans would also affect the cost of these loans and thus banks' risk-taking incentives. Second, the model could be made more realistic by accounting for the possibility of *efficient* risk-taking. To do so would involve adding a third technology that is risky but can be efficiently combined with the safe technology. This would add some desirable risk but would also require now solving a more complex portfolio problem.

A third extension would be to introduce explicitly risk-based deposit insurance premiums and to determine if, as argued by Rochet (1992), for instance, these premiums are a more effective instrument for reducing portfolio risk than capital requirements. Fourth, it may be important to account for adverse selection. Indeed, if higher capital requirements lead to higher lending rates, it may attract lower-quality borrowers who are willing to pay a

high price for their loans, in Stiglitz-Weiss fashion. This may increase bank loan risk and reduce financial stability. As a result, when banks choose to satisfy higher capital requirements by raising equity, lending may decline and riskiness of bank loans may increase, instead of falling. Thus, paradoxically, capital requirements could make banks riskier institutions than they would be in the absence of such requirements. Fifth, our welfare analysis, in standard fashion, used a utility-based metric. However, it would be worth exploring explicitly the case where the social loss function involves not only utility-based welfare but also a measure of financial stability or crisis (such as the leverage ratio or the probability of bank failure), of direct concern to the regulator. This extended welfare criterion may generate an optimal policy that provides a different balance between the costs of capital requirements (lower investment and welfare) and its benefits (reduced risk taking by banks and increased financial stability). However, to the extent that it translates into an optimal capital adequacy ratio that is even higher than what would obtain with a pure utility-based criterion, the risk of disintermediation alluded to earlier may also be magnified.

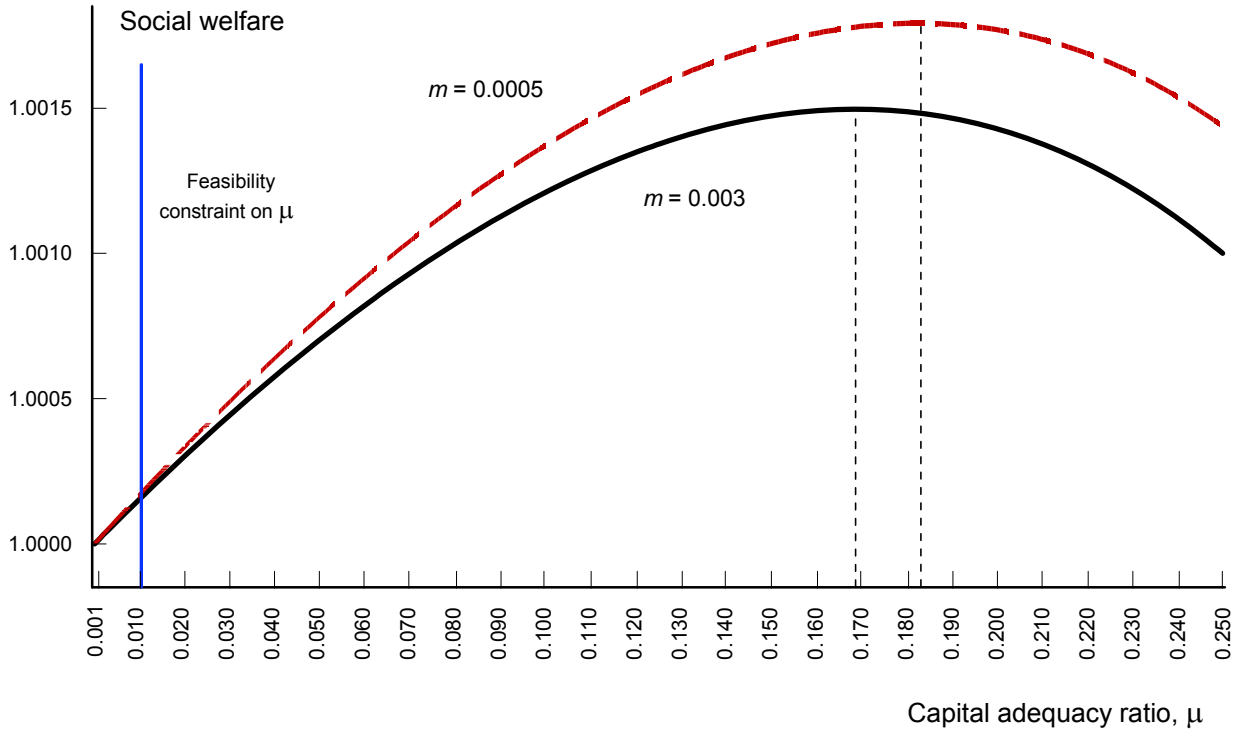
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Figure 1  
Welfare and Optimal Capital Adequacy Ratio  
(Safe-Loan Equilibrium)



Source: Authors' calculations.

Note: Social welfare is normalized by the initial value of welfare for  $\mu = 0.001$ .