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# Can Financial Hedging Serve Macroprudential Objectives?[*]


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**Abstract**

We examine hedging as a macroprudential tool in a Sudden Stops model of an economy exposed to commodity price fluctuations. We find that hedging commodity revenues yields significant welfare gains by stabilizing public expenditure, which heavily depends on these revenues. However, this added stability weakens precautionary motives and exacerbates the pecuniary externality that drives overborrowing in such models. As a result, hedging and traditional macroprudential policy act as *complements* rather than *substitutes*, with more aggressive hedging inducing a stronger macroprudential response. Our findings suggest that while hedging enhances stability and improves welfare, it does not eliminate the need for macroprudential regulation.



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# 1 Introduction

Macroprudential regulation plays a central role in policy frameworks worldwide, aiming to manage credit cycles by curbing excessive borrowing and mitigating financial vulnerabilities leading to Sudden Stops. While research in this area has expanded, much of it has focused on broad, unspecified shocks as triggers for financial instability. A key insight from this literature is the mismatch between private and social borrowing costs, which often justifies the use of Pigouvian taxes as corrective tools. However, when the initial shock is both identifiable and insurable, alternative instruments may also arise as potential macroprudential tools to stabilize credit cycles.

This paper examines financial hedging against commodity price fluctuations as a macroprudential tool for exporting economies. Our focus on these shocks stems from their economic significance and well-documented link to financial instability. Calvo et al. (2004) show that deteriorating terms of trade significantly increase the likelihood of a Sudden Stop, while Caballero and Panageas (2003, 2008) argue that in commodity-dependent economies, falling commodity prices can trigger financial distress and amplify the impact of the fall. More recently, Gazzani et al. (2024) highlight the asymmetric impact of commodity terms-of-trade shocks, finding that negative shocks are associated with trade balance improvements, rising credit spreads, and ultimately, Sudden Stops. Given the potential of such price declines to destabilize credit cycles, managing their effects is crucial for financial stability.[1]

Why can hedging become a macroprudential policy? The link lies in its interaction with fiscal procyclicality. According to Marioli and Vegh (2023), commodity exporters are twice as procyclical as non-commodity exporters, regardless of the specific commodity they export. During commodity booms, rising revenues often lead to increased public spending, which in turn fuels private consumption and heightens financial vulnerabilities (Pieschacón, 2012; Pace et al., 2025). By reallocating windfall revenues to hedge against future price declines, hedging helps mitigate

[1] According to International Monetary Fund (2015), in countries heavily reliant on oil exports, an upswing in oil prices leads to higher revenues, improved fiscal and external positions, and increased government spending. This boosts corporate profitability, raises equity prices, and strengthens bank balance sheets, but can also contribute to the buildup of systemic vulnerabilities in the financial sector.

the expansionary effects of booms. In this way, it serves as a tool for "managing abundance," limiting excessive government spending, private consumption, and credit expansion, while also reducing exposure when the boom ends.

This dynamic is particularly relevant for many Latin American countries, where public finances and external accounts are heavily tied to commodity production and exports. For instance, Bolivia, Colombia, Ecuador, Mexico, Trinidad and Tobago, and Venezuela rely on oil and gas, while Chile and Peru depend on mining. The volatility of commodity prices has profound implications for economic performance, affecting the business cycle, the sustainability of public accounts, and the risk of sudden stops (Leon-Diaz et al., 2022 and Saboin et al., 2024). Given these challenges, hedging can play a crucial role in stabilizing fiscal outcomes and mitigating financial vulnerabilities in commodity-exporting economies.

However, by reducing downside risks, hedging may also lower precautionary saving motives, potentially offsetting its stabilizing effects. This trade-off underscores hedging's dual role: it smooths economic fluctuations but also influences private-sector saving behavior, shaping its overall effectiveness as a macroprudential tool. To analyze the net effect of these two offsetting forces, this paper builds a small open-economy (SOE) model with occasionally binding constraints that includes tradable, non-tradable, and public goods. While the model follows a standard framework for small open economies, it incorporates a distinctive feature: the government provides public goods, such as merit goods like education and health, through a commodity endowment owned by the government. These goods are financed through a government-owned commodity endowment, highlighting the significance of commodity revenues in export-oriented economies.

We solve the model for three scenarios: a decentralized economy with and without hedging and a social planner's solution. We compare them in terms of stability and welfare. While hedging does not significantly reduce the likelihood of Sudden Stops, it improves welfare by stabilizing public expenditure. Government spending is higher and less volatile than in the other two economies, leading to substantial welfare gains that increase with greater hedging coverage. Despite similar average debt-to-GDP levels across models, debt dynamics differ. The social planner limits extreme debt levels and prevents sharp contractions, while the hedge economy stabilizes debt, reducing its dispersion compared to the decentralized case. Welfare gains from hedging are always positive, increasing as a larger share of commodity revenues is insured. Higher hedging levels enhance consumption smoothing and public goods provision, strengthening the benefits of risk management.

When a social planner who internalizes the pecuniary externality is also required to hedge a fraction of its commodity income, the response to the overborrowing externality becomes more aggressive, leading to a higher optimal tax. The optimal tax is non-monotonic in the share of hedged commodity revenues, decreasing when hedging exceeds 60% but remaining above the no-hedge scenario. Welfare gains from the optimal tax consistently increase with the hedged share, reaching about 25% higher under full hedging compared to no hedging. This suggests that stronger macroprudential intervention is desirable as hedging expands. Overall, the results indicate a strong complementarity between hedging and macroprudential policy, with the largest welfare gains occurring under full hedging and a high debt tax (or capital control).

Since the optimal tax is highly nonlinear, state-dependent, and challenging to implement, a constant macroprudential tax provides a more tractable alternative. Our results show that the range of tax rates generating positive welfare gains expands with the share of hedged commodity revenues. This implies that miscalibrating the tax is less likely to result in welfare losses when hedging is higher, highlighting an additional benefit of the complementarity between hedging and macroprudential policy.

In summary, our model suggests that hedging is not a direct macroprudential policy tool, but it complements these policies effectively. While hedging allows governments to increase welfare by buffering against commodity price shocks, it reduces the incentive for precautionary savings, potentially making crises slightly more probable. However, when combined with macroprudential policies, hedging boosts the small gains in welfare associated with prudential measures. This synergy results in lower crisis risks and enhanced welfare, creating a more resilient economic environment with reduced vulnerability to adverse shocks. Moreover, hedging mitigates the risks associated with miscalibrated macroprudential policies, as a larger hedged share expands the range of tax rates that generate positive welfare gains.

**Literature.** This paper also contributes to the literature on the welfare effects of financial hedging. This body of work has focused on how various insurance mechanisms affect macroeconomic conditions and their correlation with commodity prices. Lopez-Martin et al. (2019) present a sovereign default model incorporating financial hedging (along with other instruments), demonstrating that hedging can reduce macroeconomic volatility and enhance welfare. More recently, Ma and Valencia (2024) introduce financial hedging in a sovereign default model, emphasizing its role in improving welfare, primarily by reducing borrowing costs and, secondarily, through a consumption smoothing mechanism. Unlike these studies, our focus is on financial stability and the potential for a financial hedge to reduce the incidence of sudden stops in the presence of fiscal procyclicality, while simultaneously enhancing a country's ability to respond during periods of low commodity prices. This paper is also related to Borensztein et al. (2013) who find that financial derivatives reduce the need for precautionary savings, which impact debt holdings in equilibrium.

Our work is also related to the extensive literature on macroprudential policies. Our paper contributes to this literature by studying the interaction of macroprudential policy and hedging. Our analysis builds on Bianchi (2011), who shows that a decentralized equilibrium can lead to excessive borrowing due to pecuniary externalities. In this context, individual borrowers fail to account for how their actions affect overall credit conditions. Ex-ante macroprudential policies, such as taxes on borrowing, reduce the probability and severity of Sudden Stops. This foundational work has been further refined in the context of flow collateral constraints. For instance, the literature characterizes optimal macroprudential policy in environments with news shocks (Bianchi et al., 2016), liability dollarization (Mendoza and Rojas, 2019), endogenous growth (Ma, 2020), and non-homotheticities with production (Rojas and Saffie, 2022), among others.

A parallel strand of this literature focuses on the optimal design of time-consistent macroprudential policies with a stock collateral constraint. Bianchi and Mendoza (2018) analyze state-contingent debt taxes that balance crisis prevention with long-run economic performance. They conclude that while such policies are effective, they are inherently time-inconsistent. Benigno et al. (2013) adopt a different approach, emphasizing the importance of not only ex-ante but also ex post macroprudential policy. They show that the combination of the two tools can lead to potentially larger welfare gains relative to using only macroprudential policy. In a similar line Jeanne and Korinek (2019) explore optimal capital flow taxation in small open economies. They find that countercyclical capital controls could limit financial instability without significantly disrupting long-term investment. Empirical evidence has supported these theoretical findings, showing that well-designed macroprudential policies can stabilize credit cycles and reduce financial vulnerabilities (Arakelyan et al., 2023). Our work complements this literature by showing that hedging is complementary to ex-ante time-consistent macroprudential policy.[2]

## 2 Model

We consider a small open economy (SOE) model with tradable, non-tradable, and public goods. While it follows a standard framework, a key new feature is that the government provides public goods such as education and health, financed through a government-owned commodity endowment, which highlights the importance of commodity revenues in export-oriented economies. The economy has a fixed commodity supply each period, with its price fluctuating exogenously in international markets. A continuum of identical households inhabits the economy, borrowing externally while facing an occasionally binding borrowing constraint.

We start this section by describing in detail this setup.Then, we analyze an economy where the government can hedge a fraction of its commodity revenues at a financial cost. Finally, we characterize the optimal policy as the solution to a benevolent social planner's problem and describe how its allocations can be decentralized through a macroprudential tax.

[2]Other articles, such as Arce et al. (2019), examine the macroprudential role of alternative instruments like reserves. They show that reserves can be used to replicate the constrained efficient equilibrium of Bianchi (2011).

## 2.1 Baseline Model without Hedging

**Representative household.** The representative household has preferences throughout time deriving utility over the stream of private and public consumption goods. The preferences across time are represented by

$$\mathbb{E}_0 \left[ \sum_{t=0}^{\infty} \beta^t U(x_t) \right], \tag{1}$$

where $\beta \in (0,1)$ is the discount factor for the utility of future periods and $x_t$ represents a final composite of consumption goods. Furthermore, let the utility function be characterized by the following preferences as

$$U(x_t) \equiv \frac{x_t^{1-\sigma}}{1-\sigma},$$

where $\frac{1}{\sigma} > 0$ represents the intertemporal elasticity of substitution.

The final composite of consumption goods is characterized by an Armington aggregator over private $c_t$ and public consumption goods $g_t$ in every period as

$$x_t = \left[\omega_x (c_t)^{-\eta_x} + (1-\omega_x)(g_t)^{-\eta_x}\right]^{-\frac{1}{\eta_x}}, \tag{2}$$

where $\omega_x \in (0,1)$ denotes the weight placed on private consumption goods $c_t$ in preferences, $g_t$ represents the public consumption goods provided by the government, and $\eta_x > -1$ governs the elasticity of substitution $(1/(1+\eta_x))$ between private and public consumption goods. Specifically, we consider $g_t$ as a basket of merit goods (e.g., education and health), which we consider to be complements to private consumption (Fiorito and Kollintzas, 2004).

The private consumption good $c_t$ is also characterized by an Armington aggregator over tradable $c_t^T$ and nontradable private consumption $c_t^N$ goods in every period as

$$c_t = \left[\omega_p \left(c_t^T\right)^{-\eta_p} + (1-\omega_p)\left(c_t^N\right)^{-\eta_p}\right]^{-\frac{1}{\eta_p}}, \tag{3}$$

where $\omega_p \in (0,1)$ represents the preferences weight over private tradable consumption goods and $\eta_p > -1$ rules the elasticity of substitution $(1/(1+\eta_p))$ between (private) tradable and nontradable consumption goods.

The representative household inelastically supplies $\overline{L}$ of labor, for which it receives a total wage compensation $w_t\overline{L}$. Also, every period it receives a stochastic endowment of tradable goods $y_t^T$, earns profits, $\pi_t^N$, from ownership of firms in the nontradable sector, and obtains lump-sum transfers, $T_t$, from the government.

The representative household has access to international financial markets, and can trade one-period non-state contingent bonds $b_t$ with financial intermediaries, which are issued at a gross world interest rate $R^*$. The representative household purchases tradable and nontradable goods taking as given the relative price of nontradable goods $p_t^N$ in terms of tradables. Summarizing these components, the budget constraint of the representative household for every period is defined as

$$c_t^T + p_t^N c_t^N + \frac{b_{t+1}}{R^*} = y_t^T + w_t\overline{L} + \pi_t^N + b_t + T_t. \tag{4}$$

The left hand side of Equation (4) denotes the uses of sources of income: purchases of tradable and nontradables goods, and sales (purchases) of bonds that generate (require) resources $\frac{b_{t+1}}{R^*}$ when $b_{t+1} < 0$ $(b_{t+1} > 0)$.

The households's access to financial markets is limited by a collateral constraint. The sales of bonds in every period are restricted by the gross income available in the period: Households can borrow a fraction $\zeta$ of it. The collateral constraint the representative household faces is represented by

$$\frac{b_{t+1}}{R^*} \geq -\zeta \left( y_t^T + w_t \overline{L} + \pi_t^N + T_t \right), \tag{5}$$

where $\zeta \geq 0$ represents the fraction of the households gross income that can be pledged as collateral, and reflects the tightness in the collateral constraint.

**Nontradable Production Sector** Each period the economy is endowed with $y_t^T$ units of tradable goods. There is a production sector of nontradable goods. This production sector in the SOE is composed by a continuum of identical firms that produce an identical nontradable good using solely labor as input. The production technology exhibits decreasing returns to scale and is described by:

$$F(L_t) = L_t^{\alpha}, \tag{6}$$

where $\alpha \in (0, 1)$ represents the labor curvature in nontradable production. Every unit of labor hired the nontradable firm compensates with real wages $w_t$. Considering this, the profit maximization problem of the nontradable firm is characterized by

$$\max_{y_t^N, L_t} \left\{ p_t^N y_t^N - w_t L_t \right\}$$
$$\text{subject to } y_t^N = F(L_t)$$

**Government.** The government of the SOE collects revenues from selling a fixed endowment of commodity exports $\overline{y}$ at the international commodity price $p_t$, which is in units of the tradable good. Then, the government exhausts the revenues of commodity exports in public expenditures of tradable goods $g_t^T$ and direct transfers to households $T_t$. The period $t$ government budget constraint is:

$$g_t^T + T_t = p_t \overline{y}. \tag{7}$$

We assume that the government allocates a share $\varphi$ of commodity revenues to public expenditures of tradable goods $g_t^T$. Thus, $T_t = (1-\varphi)p_t\overline{y}$.

**Balance of payments.** The SOE keeps the the current account and the financial account balance in every period. The following condition describes the balance of payments

$$NX_t^T = b_t - \frac{b_{t+1}}{R^*} - p_t\overline{y}, \tag{8}$$

where $NX_t^T$ represents the net exports of the tradable endowment that ensures the current account balances the financial account.

**Stochastic processes.** It assumed that both the tradable income and commodity price follow log AR(1) processes given by:[3]

$$\log y_t^T = \rho_{y^T} \log y_{t-1}^T + \varepsilon_t, \quad \varepsilon_t \sim N(0, \sigma_{y^T}^2) \tag{9}$$

$$\log p_t = \rho_p \log p_{t-1} + \epsilon_t, \quad \epsilon_t \sim N(0, \sigma_p^2). \tag{10}$$

#### 2.1.1 Competitive Equilibrium

The representative household maximizes (1) by aggregating public and private consumption goods using (2), private consumption aggregates tradable and nontradable private consumption goods using (3), while exhausting the budget constraint (4), and satisfying the collateral constraint (5) every period. The intratemporal optimality condition of the representative household

[3]The grid of both processes is normalized so that their corresponding long-run mean is equal to 1.

problem is represented by

$$p_t^N = \frac{1-\omega_p}{\omega_p}\left(\frac{c_t^T}{c_t^N}\right)^{1+\eta_p}. \tag{11}$$

In addition, the Euler equations of the representative household's problem is:

$$\frac{\mu_t}{u_T(t)} = 1 - \beta R_t \mathbb{E}_t\left[\left(\frac{x_{t+1}}{x_t}\right)^{1+\eta_x-\sigma}\left(\frac{c_{t+1}}{c_t}\right)^{\eta_p-\eta_x}\left(\frac{c_t^T}{c_{t+1}^T}\right)^{1+\eta_p}\right], \tag{12}$$

where $\mu_t \geq 0$ represents the Lagrange multiplier associated to the collateral constraint, and $u_T(t) = \frac{\partial U(x_t)}{\partial x_t} \cdot \frac{\partial x_t}{\partial c_t} \cdot \frac{\partial c_t}{\partial c_t^T}$ denotes the marginal utility of tradable consumption in period $t$. The slackness condition satisfies

$$\mu_t\left(\zeta\left(w_t\overline{L} + y_t^T + \pi_t^N + T_t\right) + \frac{b_{t+1}}{R^*}\right) = 0. \tag{13}$$

The optimality conditions of the representative firm in the nontradable production sector can be expressed as

$$w_t L_t = \alpha p_t^N y_t^N \qquad \text{and} \qquad \pi_t^N = (1-\alpha) p_t^N y_t^N. \tag{14}$$

Finally, the resource constraint of tradable and nontradable goods as well as government purchases and transfers imply:

$$c_t^T + \frac{b_{t+1}}{R^*} = y_t^T + b_t + T_t \tag{15}$$

$$c_t^N = y_t^N \tag{16}$$

$$T_t = (1-\varphi) p_t \bar{y} \tag{17}$$

## 2.2 Hedge Economy

We consider an alternative environment where the government commits to purchasing hedge contracts against fluctuations in commodity prices. In particular, we assumed that the government can purchase *put* options to cover a fraction $\theta$ of the value of the commodity endowment $p_t \bar{y}$.

Most of the model remains identical to the decentralized economy described in Section 2.1. The key differences are detailed below. The budget constraint of the representative household is

$$c_t^T + p_t^N c_t^N + \frac{b_{t+1}}{R^*} = y_t^T + w_t \overline{L} + \pi_t^N + b_t + T_t, \tag{18}$$

where $T_t$ corresponds to transfers from the government. These transfers are consistent with the budget constraint of the government, which is

$$g_t + q_t \phi \bar{y} + T_t = \phi \max\{\theta, p_t\} \bar{y} + (1 - \phi) p_t \bar{y}, \tag{19}$$

where $\phi$ corresponds to the (time-invariant) fraction of the commodity endowment that is hedged and $q_t$ is the price of the contract. Note that if the government exhausts commodity revenues ($g_t = \varphi(\phi \max\{\theta, p_t\} \bar{y} + (1 - \phi) p_t \bar{y})$), then transfers are

$$T_t = -q_t \phi \bar{y} + (1 - \varphi)(\phi \max\{\theta, p_t\} \bar{y} + (1 - \phi) p_t \bar{y}). \tag{20}$$

Deep-pocket, risk-neutral financial intermediaries set the price of the hedge contract as a function of the solution to the following problem:

$$\max_{l_{t+1}} q_t l_{t+1} - \frac{1}{R^*} \mathbb{E}_t \left[\max\{\theta - p_{t+1}, 0\}\right] l_{t+1}, \tag{21}$$

where $l_{t+1}$ denotes the amount to be hedged and it is assumed that the financial intermediary discounts the future at the world interest rate $R^*$. Note that this problem assumes that the representative financial intermediary lives one period, and then is replaced for an identical individual next period. This can be generalized to an infinite horizon setting, but the outcomes would not change since the representative intermediary is deep-pocketed.

The equilibrium price for the hedge contract is then

$$q_t = \frac{1}{R^*}\mathbb{E}_t\left[\max\{\theta - p_{t+1}, 0\}\right]. \tag{22}$$

The competitive equilibrium characterization is very similar to the one described in the decentralized economy. The key difference is that now the resource constraint of (private) tradable consumption is given by

$$c_t^T + \frac{b_{t+1}}{R_t} = y_t^T + b_t - q_t\phi\bar{y} + (1-\varphi)(\phi\max\{\theta, p_t\}\bar{y} + (1-\phi)p_t\bar{y}), \tag{23}$$

which reflects the spending of tradable resources in purchasing the hedge contract.

## 2.3 Optimal Policy

In this section we characterize the solution to the problem that a benevolent social planner faces. We study the case of the economy where hedging is possible, since this case encompasses the no hedge case when $\phi = 0$. We assume that the social planner sets public expenditures of tradable goods to be equal to the value of commodity exports. Thus, $g_t = \varphi(\phi\max\{\theta, p_t\} + (1-\phi)p_t\bar{y})$.

The recursive formulation of the social planner's problem is then

$$V(y^T, p, b) = \max_{\{x,c,c^T,b'\}} \left\{ u(x) + \beta \mathbb{E}_{y^{T\prime},p'|y^T,p} V(y^{T\prime}, p', b') \right\} \tag{24}$$

s.t.

$$x = \left[\omega_x (c)^{-\eta_x} + (1-\omega_x)\left(\varphi(\phi \max\{\theta, p\} + (1-\phi)p\bar{y})\right)^{-\eta_x}\right]^{-\frac{1}{\eta_x}}$$

$$c = \left[\omega_p \left(c^T\right)^{-\eta_p} + (1-\omega_p)\left(\bar{L}^\alpha\right)^{-\eta_p}\right]^{-\frac{1}{\eta_p}}$$

$$c^T + \frac{b'}{R} = y^T + b + (1-\varphi)(\phi \max\{\theta, p\} + (1-\phi)p\bar{y}) - q\phi\bar{y} \tag{25}$$

$$\frac{b'}{R} \geq -\zeta \left( y^T + \frac{(1-\omega_p)}{\omega_p} \left(\frac{c^T}{\bar{L}^\alpha}\right)^{1+\eta_p} \bar{L}^\alpha + (1-\varphi)(\phi \max\{\theta, p\} + (1-\phi)p\bar{y}) - q\phi\bar{y} \right) \tag{26}$$

where $g = \varphi(\phi \max\{\theta, p\} + (1-\phi)p\bar{y})$, $T = (1-\varphi)(\phi \max\{\theta, p\} + (1-\phi)p\bar{y}) - q\phi\bar{y}$ and $y^N = \bar{L}^\alpha$. The first order conditions (in sequential form) of the social planner's problem are the following:

$$u_T(t) + \mu_t \zeta \left(\frac{1-\omega_p}{\omega_p}\right)(1+\eta_p)\left(\frac{c_t^T}{\bar{L}^\alpha}\right)^{\eta_p} = \lambda_t \tag{27}$$

$$\lambda_t = \mu_t + \beta R_t \mathbb{E}_t\left[\lambda_{t+1}\right] \tag{28}$$

$$\mu_t \left( \zeta \left( y_t^T + \frac{(1-\omega_p)}{\omega_p} \left(\frac{c_t^T}{\bar{L}^\alpha}\right)^{1+\eta_p} \bar{L}^\alpha + (1-\varphi)(\phi \max\{\theta, p\} + (1-\phi)p\bar{y}) - q\phi\bar{y} \right) + \frac{b_{t+1}}{R_t} \right) = 0 \tag{29}$$

where $\lambda_t$ denotes the Lagrange multiplier of the tradable consumption resource constraint and $\mu_t$ the multiplier of the social planner's borrowing constraint. Combining terms, we have that the planner's bond Euler equation can be expressed as

$$u_T(t) + \mu_t \zeta \left(\frac{1-\omega_p}{\omega_p}\right)(1+\eta_p)\left(\frac{c_t^T}{\bar{L}^\alpha}\right)^{\eta_p} = \mu_t + \beta R_t \mathbb{E}_t\left[u_T(t+1) + \mu_{t+1}\zeta\left(\frac{1-\omega_p}{\omega_p}\right)(1+\eta_p)\left(\frac{c_{t+1}^T}{\bar{L}^\alpha}\right)^{\eta_p}\right]. \tag{30}$$

### 2.3.1 Optimal Policy Decentralization

The optimal allocations of the social planner's problem can be decentralized via a macroprudential debt tax $\tau_t$. To see this, consider the budget constraint of the representative household in the decentralized economy that now faces a debt tax:[4]

$$c_t^T + p_t^N c_t^N \frac{b_{t+1}}{(1+\tau_t)R_t} = y_t^T + w_t \bar{L} + \pi_t^N + b_t + T_t. \tag{31}$$

This is, the representative household faces a tax on its new bond issuance. The bonds Euler equation in this economy is now

$$u_T(t) = \mu_t + \beta R_t (1+\tau_t) \mathbb{E}_t \left[ u_T(t+1) \right] \tag{32}$$

Note that when the constraint binds we have that the value of that tax $\tau_t$ is irrelevant. Hence, we can set it to 0 and focus on the *macroprudential* nature of it. Whenever $\mu_t = 0$ we have that the social planner's bond Euler equation is

$$u_T(t) = \beta R_t \mathbb{E}_t \left[ u_T(t+1) + \mu_{t+1} \zeta \left( \frac{1-\omega_p}{\omega_p} \right) (1+\eta_p) \left( \frac{c_{t+1}^T}{\bar{L}^\alpha} \right)^{\eta_p} \right]. \tag{33}$$

The macroprudential tax that implements the social planner's allocation is given by

$$\tau_t = \frac{\mathbb{E}_t \left[ u_T(t+1) + \mu_{t+1} \zeta \left( \frac{1-\omega_p}{\omega_p} \right) (1+\eta_p) \left( \frac{c_{t+1}^T}{\bar{L}^\alpha} \right)^{\eta_p} \right]}{\mathbb{E}_t \left[ u_T(t+1) \right]} - 1 \tag{34}$$

# 3 Quantitative Analysis

This section describes the calibration of the model and the main quantitative results from the three economies described above.

[4] All debt tax revenues are rebated back to households.

## 3.1 Calibration

The model is calibrated at a yearly frequency to match long-run features of the Colombian economy. Table 1 presents a summary of the calibrated parameters of the model. The set of parameters is separated in two: the first one (upper rows) describes parameters that externally calibrated, while the second (lower rows) presents parameters that were either normalized or set to match properties of the Colombian economy.

Table 1: Calibration for the Colombian Economy

| Parameter | Value | Reference |
|---|---|---|
| $\sigma$ | 2.00 | Literature |
| $\eta_x$ | 1.56 | Pieschacon (2012) |
| $\omega_x$ | 0.987 | Pieschacon (2012) |
| $\eta_p$ | 0.205 | Bianchi (2011) |
| $\alpha$ | 0.200 | Durdu et al. (2009) |
| $R_t$ | 0.040 | Bianchi et al. (2016) |
| $\beta$ | 0.930 | Debt-to-GDP ratio |
| $\omega_p$ | 0.300 | Tradable-to-total consumption ratio |
| $\bar{L}$ | 1 | Normalization |
| $\zeta$ | 0.325 | Sudden Stop probability |
| $\rho_{y^T}$ | 0.589 | Colombian tradable GDP |
| $\sigma_{y^T}$ | 0.026 | Colombian tradable GDP |
| $\rho_p$ | 0.664 | Crude Oil WTI |
| $\sigma_p$ | 0.310 | Crude Oil WTI |
| $\bar{y}$ | 0.140 | Colombian share of oil exports/GDP |
| $\theta$ | 1.000 | Average commodity price |
| $\varphi$ | 0.70 | Colombian data |

We start by describing externally calibrated parameters. We set the inverse of the elasticity of substitution $\sigma$ to be 2, which is a standard value in macro literature. We closely follow Pieschacón (2012) and we set $\eta_x = 1.56$ so that the elasticity of substitution between private and public consumption $1/(1+\eta_x)$ is 0.39. We also set $\omega_x = 0.987$ as in Pieschacón (2012). The elasticity of substitution between tradable and nontradable (private) consumption is reported to be 0.83 in the literature (Bianchi, 2011), which implies a value of $\eta_p$ equal to 0.205. Lastly, we set the curvature of the nontradable production function to be $\alpha = 0.2$, in line with Durdu et al. (2009), and a world interest rate of 4% per year, as reported in Bianchi et al. (2016).

The rest of the parameters are estimated directly from the data, normalized, or internally calibrated. We assume that the tradable endowment and commodity price follow a log normal AR(1) process with mean 0. The total labor supply and commodity endowment are set to be 1.[5] The strike price is normalized to be equal to the average commodity price $p$, which is 1. The parameters that govern the tradable endowment stochastic process are set by estimating the first-order autocorrelation and volatility of the cyclical component of Colombian tradable GDP.[6] We find that $\rho_{y^T} = 0.589$ and $\sigma_{y^T} = 0.026$. Regarding the stochastic process for the commodity price, we set $\rho_p = 0.664$ and $\sigma_p = 0.31$. These values are obtained from from calculating the autocorrelation and standard deviation of the cyclical component of log WTI crude oil price (we use the HP filter). The discount factor is set to be $\beta = 0.93$ in order to match an average net foreign asset-to-GDP ratio close to -30%, which is the long-run average reported in Lane and Milesi-Ferretti (2017). $\omega_p$, linked to the long-run tradable-to-total (private) consumption ratio $c^T/(c^T + p^N c^N)$ is set to 0.3 in order to match a target ratio of 30% (Hernandez and Mendoza, 2017). We set $\zeta = 0.325$ so that the Colombian economy experiences a Sudden Stop probability close to 3% in the long-run.[7] Lastly, we set $\bar{y} = 0.14$ so that commodity exports represent 3% of GDP, and we set $\varphi = 0.7$ to reflect a 70% of total government expenditure being transfers to households.

The solution method to solve the model is based on the time-iteration method described in Bianchi et al. (2016). This is a global method, which is suitable for solving quantitative models that feature nonlinear properties and occasionally binding constraints like the one described in this document. A detailed description of the solution algorithm is presented in the Appendix.

[5]We discretize both processes following Tauchen and Hussey (1991).

[6]We follow Bianchi and Sosa-Padilla (2023) in terms of distinguishing between tradable and nontradable sectors. We then proceed to obtain the cyclical component of tradable GDP and estimate the first-order autocorrelation and unconditional standard deviation.

[7]A Sudden Stop is defined as an event where the current account-to-GDP ratio is two standard deviations above its long-run average and the borrowing constraint binds.

## 3.2 Long-Run Moments & Welfare Analysis

After the three versions of the model (decentralized, social planner and hedge economy) are solved, we use the corresponding policy functions to simulate three economies for 100,000 periods.[8] Each economy considers the same underlying sequence of shocks in order to reduce noise at the moment of comparing outcomes. For the social planner's economy we assume that the share of the commodity endowment insured is equal to 1, so $\phi = 0$, while for the hedge economy we assume $\phi = 1$. In Section 4 we explore the case where $\phi = 1$ for the social planner's economy.

Table 2 presents key long-run moments for the decentralized, social planner, and hedge economies. We also present the corresponding moments for hedge economies where lower fractions of the commodity are hedged (40%, 60%, and 80%). We see that the social planner is very effective in terms of reducing the volatility of the economy, expressed as a nearly 53% reduction in the current account-to-GDP volatility relative to the decentralized equilibrium, and a Sudden Stop likelihood of 1.3% (1.8 percentage points lower than the decentralized equilibrium). The social planner achieves this by actively managing household borrowing in the economy, which is reflected by a sizable debt tax, 4.42%, and a tax that is active (positive) 93% of the time. The welfare gain is roughly 0.02% of a permanent increase in the composite good $x$.

The bottom panel shows selected financial crisis moments. We see that the social planner is not only very effective in reducing volatility in the long run, but also during financial crises. GDP and tradable consumption declines as well as current account reversals are orders of magnitude milder in the planner's economy.

The third column in Table 2 presents the hedge economy moments. In terms of long-run moments, we see that the hedge economy averages similar debt-to-GDP ratios to the decentralized and social planner economies, but it is less volatile as shown by the lower current account volatility. In terms of the likelihood of a Sudden Stop we do not observe important differences between the hedge economy and the decentralized equilibrium. We do observe large welfare gains that

[8] We drop the first 1,000 simulations in order to remove any dependence from initial conditions.

Table 2: Long-Run Moments

| Long-run Moments | DE | SP | HE | | | |
|---|---|---|---|---|---|---|
| | | | $\phi = 1$ | $\phi = 0.4$ | $\phi = 0.6$ | $\phi = 0.8$ |
| $\mathbb{E}[b/Y]$ % | -31.47 | -31.37 | -31.99 | -31.96 | -31.99 | -32.00 |
| $\sigma(CA/Y)$ % | 2.70 | 1.26 | 2.30 | 2.46 | 2.39 | 2.34 |
| $\mathbb{E}\left[c^T/(c^T + p^N c^N)\right]$ % | 29.79 | 29.78 | 29.81 | 29.80 | 29.80 | 29.81 |
| $\mathbb{E}\left[g/(y^T + p^N y^N)\right]$ % | 1.19 | 1.19 | 1.40 | 1.27 | 1.32 | 1.36 |
| $\sigma\left(g/(y^T + p^N y^N)\right)$ % | 0.45 | 0.45 | 0.31 | 0.38 | 0.35 | 0.33 |
| Prob. of Crisis % | 3.12 | 1.30 | 3.24 | 3.17 | 3.19 | 3.22 |
| Pr.$(\mu_t > 0)$ % | 6.61 | 1.90 | 6.14 | 6.46 | 6.51 | 6.45 |
| $\mathbb{E}[\tau_t^b]$ % | n.a. | 4.42 | n.a. | n.a. | n.a. | n.a. |
| $\sigma(\tau_t^b)$ % | n.a. | 4.01 | n.a. | n.a. | n.a. | n.a. |
| Pr.$(\tau_t^b > 0)$ % | n.a. | 92.71 | n.a. | n.a. | n.a. | n.a. |
| Pr. Active Hedge % | n.a. | n.a. | 67.24 | 67.24 | 67.24 | 67.24 |
| Welfare Gain % | n.a. | 0.018 | 21.10 | 10.70 | 14.66 | 18.06 |
| Financial Crisis Moments | | | | | | |
| $\Delta GDP$ % | -27.52 | -13.73 | -25.82 | -26.70 | -26.32 | -26.01 |
| $\Delta c^T$ % | -31.74 | -14.76 | -29.43 | -30.57 | -30.08 | -29.69 |
| $CA/Y$ % | 12.02 | 3.74 | 10.26 | 11.11 | 10.76 | 10.47 |

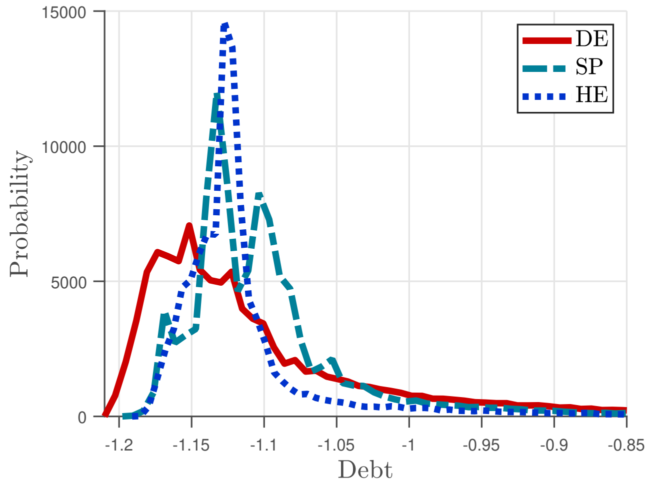
*Notes:* Regimes DE, SP and HE, correspond to the decentralized, social planner's, and hedge economies, respectively. A Sudden Stop is defined as a period in which the constraint binds and the current account-to-GDP ratio exceeds by more than two standard deviations its long-run average. Average drops during Sudden Stop states are expressed in percent of long-run means with the exception of the current account-to-GDP ratio, which is expressed in percentage point differences. Welfare gains are computed as compensating variations of the composite good $x$ across states that equate welfare in the corresponding economy with the decentralized equilibrium. We use the ergodic distribution of the decentralized economy to compute the long-run average welfare gain.

come from the ability to hedge against the downside risk in commodity prices. Why is this the case? The hedge economy is able to achieve an average level of public expenditure larger than the decentralized and social planner economies (16% larger). Moreover, this expenditure is not only larger but substantially less volatile than in the economies with no option contracts (30% less). Since households have no possibility of transforming tradable or nontradable goods into public goods, the hedge generates large welfare gains. Additionally, given that in the hedge economy the government benefits from the upside and covers against the downside in commodity prices, the average public good consumption is larger.

The fourth, fifth and sixth columns show patterns similar to those seen in the benchmark hedge economy case. Specifically, hedging leads to less volatility in the current account and slightly larger debt positions relative to the decentralized economy. Welfare gains are substantial, where higher fractions of hedging lead to larger gains.

Figure 1 presents the ergodic distributions of bond holdings for the three economies. We see that, although the three economies experience similar levels of debt relative to their GDPs, we do see that the distributions vary importantly. As expected, the social planner reduces the mass of maximum debt holdings relative to the no-intervention case, but also limits the scenarios where debt drops abruptly (financial crises). In the hedge economy case, we see that debt is less spread across the state space, relative to the decentralized equilibrium. The hedge contract stabilizes debt, but is larger relative to the centralized equilibrium.

Figure 1: Ergodic Distribution - Bond Holdings




The question that remains is whether insuring a lower fraction of the endowment can generate different welfare gains. We explore this alternative. Specifically, we compute welfare gains (relative to the decentralized economy) when varying $\phi$, the fraction of the commodity endowment insured by the hedge contract. Note that when $\phi = 0$ we are back in the decentralized scenario, and when $\phi = 1$ we are in the hedge economy case described above. Figure 2 presents the results of this exercise.

Figure 2: Welfare Gains & Share of Hedged Commodity Revenues

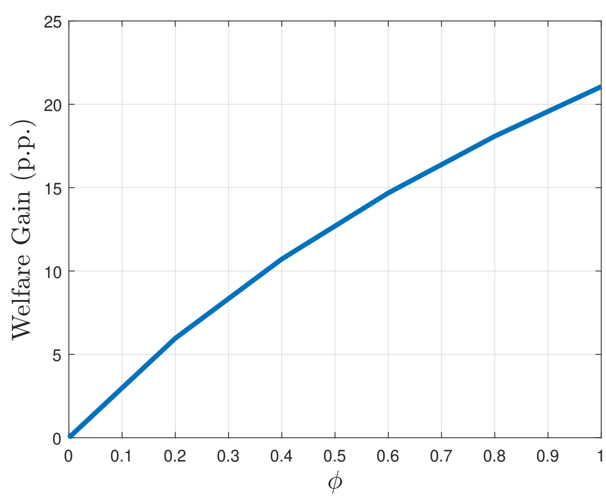



Figure 2 suggests that welfare gains are *always* positive, regardless of the fraction of the commodity endowment insured. However, welfare gains differ in magnitude. The relationship is monotonic, where larger fractions of insurance lead to larger welfare gains. This result is intuitive, as purchasing the option contract leads to better consumption-smoothing patterns and allows, to some extent, to transform tradable goods into the public good.

### 3.3 Sudden Stops Dynamics

In this section, we study how the dynamics of Sudden Stops differ across the different economies. For this, we perform an event study around Sudden Stops.[9] In particular, we simulate our model for 100,000 periods and identify those where a Sudden Stop occurred. Figure 3 presents the average paths of tradable consumption, the current account-to-GDP ratio, GDP and sequence of shocks that precedes and follows a Sudden Stop, for the three economies studied above.[10]

[9] We define a Sudden Stop is defined as a period in which the constraint binds and the current account-to-GDP ratio exceeds by more than two standard deviations its long-run average.

[10] For comparison purposes we use the Sudden Stops endogenously generated in the decentralized economy and we use the policy functions of the social planner and hedge economies to simulate the outcome dynamics around these events.

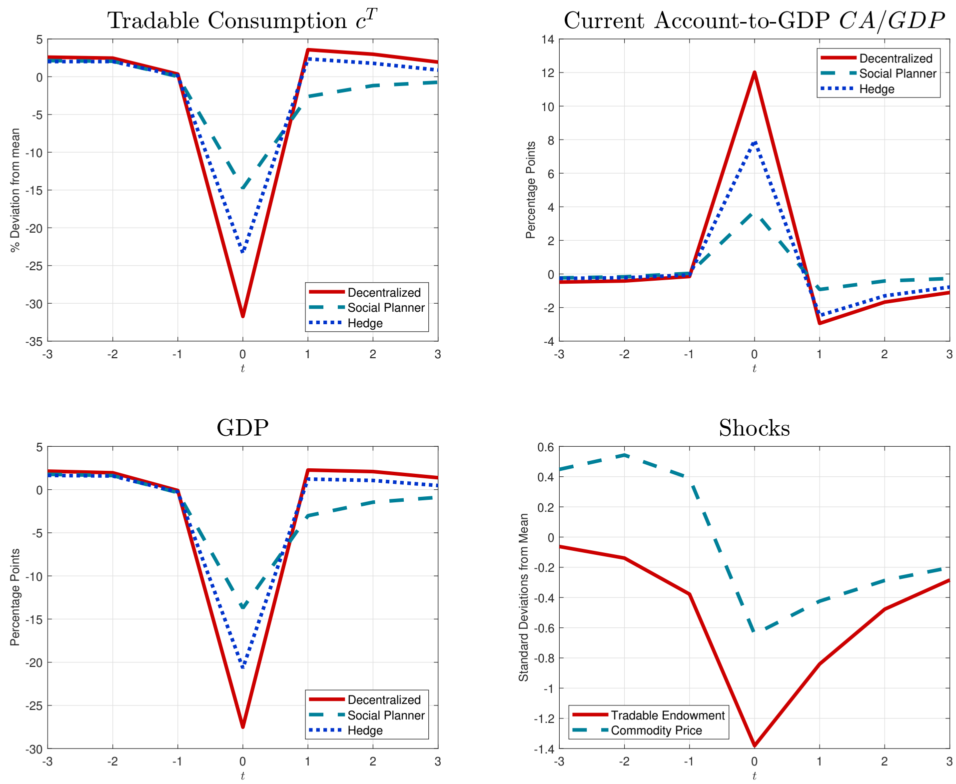



Figure 3: Sudden Stops Dynamics

We see in Figure 3 that Sudden Stops in the decentralized economy are the most severe. Specifically, in the decentralized economy the collapses in tradable consumption and GDP are the largest. A similar pattern is observed for the current account-to-GDP ratio, where the most aggressive reversals are again observed in the decentralized equilibrium. The bottom right panel presents the dynamics of the commodity price shock and the tradable endowment. Both have been standardized so that they represent standard deviations from their corresponding long-run means. We see that the average Sudden Stop occurs when both the commodity price and tradable endowment experience sharp and sudden declines.

The social planner is the most successful in terms of managing Sudden Stops, followed by the hedge economy. The hedge, although not reducing the likelihood of financial crises, does provide a cushion during these events.

## 3.4 Simple Hedging Rules

In addition to our previous analysis, we study simple hedging rules. Specifically, we allow the fraction of hedged commodity revenues to vary across time and to be a function of the commodity price or a country's net foreign assets.

We explore two rules, which are specified below:

$$\phi_t^a = \max\{\min(\{\mathbb{1}\{\ln p_t - \mu_p) > 0\} \cdot \zeta_a, 1\}, 0\} \tag{35}$$

$$\phi_t^b = \max\left\{ \min\left\{ \zeta_b \frac{b_t}{\bar{b}}, 1 \right\}, 0 \right\}. \tag{36}$$

The first rule, $\phi_t^a$, considers hedging only when the commodity price is above its long-run average. The parameter $\zeta_a$ measures the intensity of the hedging (i.e. fraction). Regarding the second rule, we assume that the fraction of hedged commodity revenues is a function of how indebted the economy is. In particular, the fraction hedged is increasing in the size of debt, specially when it is above a level $\bar{b}$. Lastly, as in the first rule, $\zeta_b$ measures the intensity of the hedging.

Since $\zeta_a$ and $\zeta_b$ are not known ex-ante, we perform a grid search analysis to find the values that maximize welfare gains with respect to the decentralized scenario. Figure 4 presents the results of the grid search analysis.

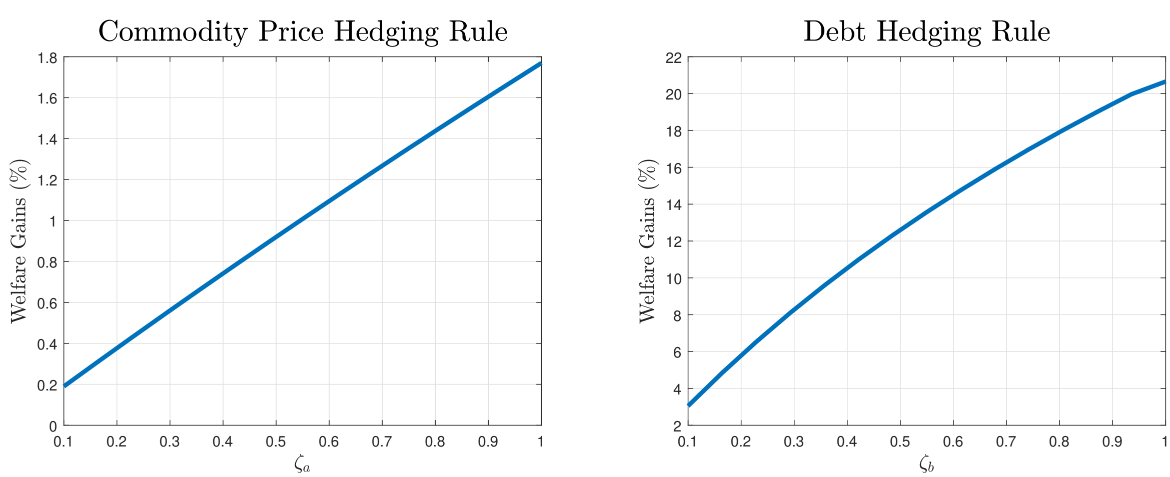



Figure 4: Simple Hedging Rules & Welfare Gains

Figure 4 shows that welfare gains under the commodity price hedging rule are important, but orders of magnitude lower than in the baseline scenario. More importantly, we see that when $\zeta_a = 1$ welfare gains are maximized. This is equivalent to fully hedging whenever the (log) commodity price exceeds its (log) long-run average. As in the baseline hedging scenario, the rule is pushing for full hedging of commodity revenues.

The right panel of Figure 4 presents welfare gains for the debt rule as a function of $\zeta_b$.[11] Under this rule welfare gains are substantial. Additionally, we see that, as with the previous rule, welfare gains are maximized when $\zeta_b = 1$, which is equivalent to setting full hedge of commodity revenues. Table 3 presents long-run and financial crisis moments for the welfare-maximizing simple rules.

Under the commodity price rule we see that the economy tends to be less volatile than the decentralized case (lower current account standard deviation), while experiencing similar levels of debt and probabilities of crisis. Sudden Stops are slightly milder, and welfare gains are important (1.8 percent). Under the simple debt rule the economy experiences *less* current account volatility relative to the benchmark hedge case, while having similar levels of debt and likelihood of Sudden Stops. Financial crises are slightly milder, and welfare gains are almost the same as those in the benchmark scenario.

[11] We set $\bar{b}$ to be equal to the long-run average of the decentralized equilibrium. Our results did not change importantly when considering different thresholds.

Table 3: Long-Run Moments

| Long-run Moments | Commodity Price Rule | Debt Rule |
| --- | --- | --- |
| $\mathbb{E}[b/Y]$ % | -31.32 | -31.96 |
| $\sigma(CA/Y)$ % | 2.63 | 2.28 |
| $\mathbb{E}\left[c^T/(c^T + p^N c^N)\right]$ % | 29.81 | 29.81 |
| $\mathbb{E}\left[g/(y^T + p^N y^N)\right]$ % | 1.23 | 1.40 |
| $\sigma\left(g/(y^T + p^N y^N)\right)$ % | 0.45 | 0.31 |
| Prob. of Crisis % | 3.18 | 3.23 |
| Pr.($\mu_t > 0$) % | 6.35 | 6.05 |
| Pr. Active Hedge % | 67.14 | 100.00 |
| Welfare Gain % | 1.77 | 20.69 |
| Financial Crisis Moments | | |
| $\Delta GDP$ % | -26.71 | -25.77 |
| $\Delta c^T$ % | -30.70 | -29.37 |
| $CA/Y$ % | 11.41 | 10.23 |

**Notes:** A Sudden Stop is defined as a period in which the constraint binds and the current account-to-GDP ratio exceeds by more than two standard deviations its long-run average. Average drops during Sudden Stop states are expressed in percent of long-run means with the exception of the current account-to-GDP ratio, which is expressed in percentage point differences. Welfare gains are computed as compensating variations of the composite good $x$ across states that equate welfare in the corresponding economy with the decentralized equilibrium. We use the ergodic distribution of the decentralized economy to compute the long-run average welfare gain.

## 3.5 Model Extensions

We explore 2 extensions of our model to assess the role that hedging plays in the economy under the presence of additional sources of macroeconomic fluctuations. In our first extension, we relax the assumption that foreign borrowing costs are independent of commodity prices. Specifically, we assume that borrowing costs and commodity prices are correlated. In our second extension, we allow for the presence of stochastic volatility in commodity prices.

Appendix A presents the main results of our analysis. In the first extension, we find that borrowing costs, measured by the Colombian EMBI spread, are negatively correlated with oil prices. When incorporating commodity price-dependent financing costs into the model, we observe that long-run moments remain largely unchanged compared to the baseline economy. However, the welfare gains from hedging are slightly higher, suggesting that hedging becomes more beneficial when countries face fluctuations in their financing costs.

In our second extension, we introduce stochastic volatility in two different ways. In the first exercise, we assume that stochastic volatility only introduces a time-varying variance for the commodity price. In the second case, we assume that, additionally, stochastic volatility affects borrowing costs, to better reflect the empirical fact that larger world uncertainty raises financing costs of households in the economy. We find that the presence of stochastic volatility in the commodity price reduces the effectiveness of hedging by nearly 50% relative to the benchmark economy, as it introduces more dispersion in prices and hence in allocations. Nevertheless, hedging via a *put* option is still generating sizable welfare gains.

# 4 Optimal Policy Under Hedging

So far we have studied optimal macroprudential policy when the social planner was not using hedging options ($\phi = 0$). This particular scenario is interesting since we see that hedging can have nontrivial effects on consumption, borrowing and Sudden Stops probabilities (Table 2). In this section, we study how a social planner's optimal allocations under hedging differ from those of the decentralized economy under hedging.

The problem that the social planner faces is similar to the one described in section 2.3, except for the presence of hedging. We assume that the fraction of revenues that are hedged is given for the planner.

Table 4 shows patterns similar to the ones observed in Table 2, where the social planner is very effective in terms of the reducing volatility in the economy (lower current account-to-GDP volatility) and likelihood of Sudden Stops. Additionally, we see that the size of the overborrowing externality under the presence of (full) hedging of commodity revenues is relatively *larger* than when there is no hedge. This is illustrated by the size of the average optimal tax, which is 4.67%, 25 basis points larger than the one in the absence of hedging. Also, under hedging, the macro-prudential tax is active in nearly the entirety of the simulation (around 99% of the time), which is also larger than the scenario with no hedging.

Table 4: Long-Run Moments

| Long-run Moments | HE | HE-SP |
|---|---|---|
| $\mathbb{E}[b/Y]$ % | -31.99 | -31.71 |
| $\sigma(CA/Y)$ % | 2.30 | 1.02 |
| $\mathbb{E}\left[c^T/(c^T + p^N c^N)\right]$ % | 29.81 | 29.78 |
| $\mathbb{E}\left[g/(y^T + p^N y^N)\right]$ % | 1.40 | 1.40 |
| $\sigma\left(g/(y^T + p^N y^N)\right)$ % | 0.31 | 0.30 |
| Prob. of Crisis % | 3.24 | 1.28 |
| Pr.$(\mu_t > 0)$ % | 6.14 | 1.72 |
| $\mathbb{E}[\tau_t^b]$ % | n.a. | 4.67 |
| $\sigma(\tau_t^b)$ % | n.a. | 3.50 |
| Pr.$(\tau_t^b > 0)$ % | n.a. | 98.62 |
| Pr. Active Hedge % | 67.24 | 67.24 |
| Welfare Gain % | n.a. | 0.022 |
| **Financial Crisis Moments** | | |
| $\Delta GDP$ % | -25.82 | -20.22 |
| $\Delta c^T$ % | -29.43 | -22.07 |
| $CA/Y$ % | 10.26 | 6.18 |

**Notes:** Regimes HE, HE-SP correspond to the hedge and social planner (hedge) economies, respectively. A Sudden Stop is defined as a period in which the constraint binds and the current account-to-GDP ratio exceeds by more than two standard deviations its long-run average. Average drops during Sudden Stop states are expressed in percent of long-run means with the exception of the current account-to-GDP ratio, which is expressed in percentage point differences. Welfare gains are computed as compensating variations of the composite good $x$ across states that equate welfare in the corresponding economy with the decentralized (with hedge) equilibrium. We use the ergodic distribution of the decentralized economy to compute the long-run average welfare gain.

The fact that the overborrowing externality is stronger when the economy has access to insurance instruments suggests that households have now *weaker* precautionary motives. The intuition behind this is straightforward: hedging reduces the volatility of public goods, which stabilizes the marginal utility of tradable consumption, since private and public consumption goods are complements. Given this, households have now less motives to save for states of the world where large consumption adjustments would need to be made. Thus, Weaker precautionary motives causes additional vulnerability to the economy when the government is hedging against downside risk in commodity revenues. Thus, the costs of hedging are not only the resources given up in order to purchase the call option contract, but also more financial vulnerability of the economy. This is an important result since it implies that optimal macroprudential policy and hedging are not independent from each other, and that an interaction of the two can bring welfare gains to the economy.

How does the fraction of hedged commodity revenues and the optimal macroprudential tax interact with each other? We answer this question in Figure 5. In this figure, we present the optimal macroprudential tax that decentralizes the social planner's allocations, for different shares of hedged commodity revenues, as well as its corresponding welfare gains. Three patterns arise. First, the optimal tax is, in general, increasing in the share of hedged commodity revenues. This is consistent with a stronger overborrowing externality fueled by weaker precautionary motives due to the lower downside risk in public goods consumption. The planner tackles the overborrowing externality in a more aggressive way, explaining the increase in the optimal tax. Second, the optimal tax is non-monotonic in the share of hedged commodity revenues for certain levels of the latter variable. In particular, we see that when the share of hedged revenues is larger than 60% the optimal tax is decreasing in this dimension, but still larger relative to the no hedge scenario. Lastly, we see that welfare gains associated to the optimal tax are always increasing in the share of hedged commodity revenues. In fact, welfare gains are roughly 25% larger when the country is fully hedged relative to when it has no hedged at all, implying that stronger intervention from macroprudential policy as the size of the hedge increases is desirable. Moreover, these results suggest that the economy would experience the largest welfare gains when it is fully hedged and when it faces a relatively large a debt tax (or capital control), indicating a strong complementarity between the roles of hedging and macroprudential policy.

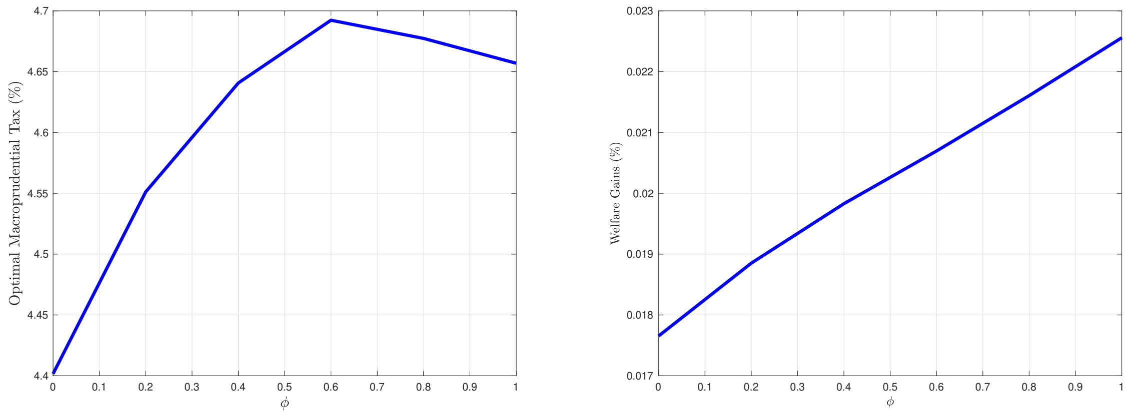



Figure 5: Optimal Macroprudential Tax & Welfare Gains

As we have seen, the optimal macroprudential tax can be very effective but at the same time very nonlinear, state dependent, and hence hard to implement. Due to this, we consider simple tax rules as an alternative, and we assess how efficient they are relative to the optimal benchmark. We consider a simple rule, which is to set a constant macroprudential tax, regardless of the state of the economy. We compute the optimal constant tax (welfare-maximizing) for different shares of hedged commodity revenues. Figure 6 presents two sets of results. The left panel shows the optimal constant tax by shares of hedged revenues. The right panel shows welfare gains for different levels of constant tax rates, by shares of hedged commodities.

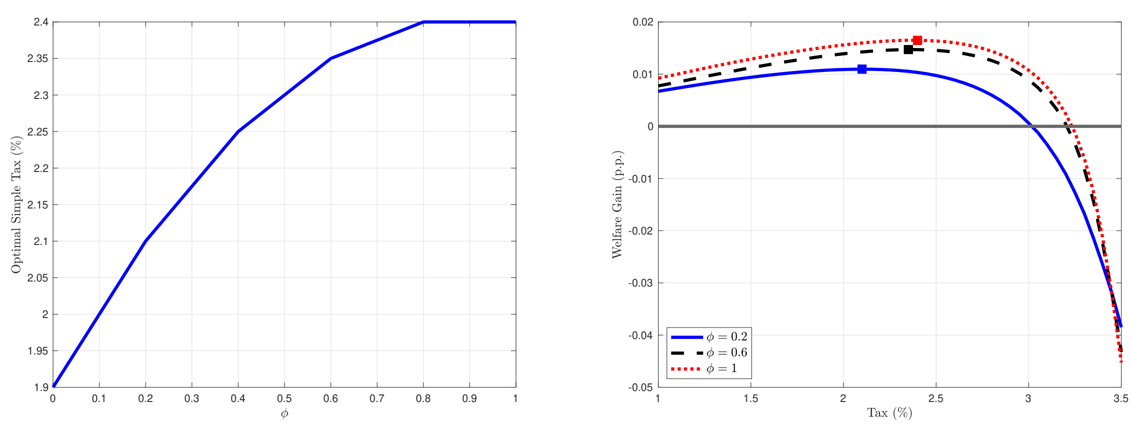



Figure 6: Simple Tax Rules & Welfare Gains

The left panel of Figure 6 presents a pattern similar to the one observed in Figure 5: the optimal constant tax is, in general, increasing in the share of hedged commodity revenues, and is also non-monotonic. However, there are two important differences with respect to the optimal tax case. First, the *level* of the optimal constant tax is roughly *half* of the optimal tax. This is intuitive, since the constant tax is active as long as the borrowing constraint binds, and it does not vary across states of the world. Second, the non-monotonicity of the tax occurs at hight levels of the share of hedged commodity revenues (nearly 80% onward).

The right panel of Figure 6 illustrates welfare gains for different constant tax rules, given shares of hedged commodity revenues. Each curve represents welfare gains for different levels of the debt tax, where the welfare-maximizing constant tax is marked by a square of the corresponding color. Three results are worth highlighting. First, welfare gains are increasing in the share of hedged commodity revenues, as in Figure 5. Second, welfare gains are lower with the constant optimal tax relative to the optimal tax. However, the effectiveness of the simple rule can be quite substantial. For example, when the government fully hedges its commodity revenues, the welfare gains of the simple tax account for nearly 75% of the welfare gains of the optimal tax (this goes down to roughly 55% when $\phi = 0$). Thus, a simple constant tax rule can potentially be very successful in generating substantial welfare gains.[12] Third, we see that the range for which a constant tax generates positive welfare gains increases with $\phi$. This has important policy implications. To illustrate this, suppose the case where a macroprudential authority does not know exactly the level of the optimal constant tax. If the authority were to set the tax incorrectly, it is less likely to generate welfare losses the larger the share of hedged commodity revenues. This last result shows that there is an additional benefit to the complementarity between hedging and macroprudential policy, as welfare losses are less likely to occur if capital controls were not perfectly implemented.

[12]The success of the constant tax rule is also high relative to results found in related literature (Mendoza and Rojas (2019); Rojas and Saffie (2022)).

# 5 Conclusions

In this paper, we examine the role of hedging as a macroprudential tool in an economy subject to commodity price fluctuations, where agents have access to *put* options. We compare three settings: (i) a decentralized economy with neither hedging nor macroprudential policy, (ii) a hedging economy where commodity revenues are hedged, and (iii) a social planner's economy that incorporates both hedging and macroprudential policy.

In our quantitative analysis, we calibrate the decentralized model to match features of the Colombian economy, and numerically solve the three economies described above. The main results regarding optimal policy are in line with those of the Sudden Stops literature: the social planner is very effective at the moment of tackling the pecuniary externality as it reduces the likelihood and severity of financial crises.

When assessing the effectiveness of a hedge contract we find that it does not significantly alter long-run moments such as debt-to-GDP, or the likelihood of experiencing Sudden Stops. However, welfare gains are substantial. The hedge contract covers the downside risk, which reduces the volatility of public expenditure and raises its long-run average. The latter occurs because the representative household uses tradable resources as insurance against declines in the public good, which cannot be produced by it. Hence, by using the option contract as a mechanism to transform tradable goods into the public good, the household is able to experience large gains in welfare.

Lastly, we examine the interaction between optimal macroprudential policy and hedging. Interestingly, we find that they are *complements* rather than *substitutes*. The intuition is straightforward: hedging reduces the downside risk of large fluctuations in public good provision, which weakens households' precautionary motives. While this enhances stability by lowering the variance of public good provision, it also encourages households to take on more risk. As a result, optimal macroprudential policy tends to be more aggressive as the share of commodity revenues increases. Our findings suggest that while hedging has beneficial macroeconomic effects, it does not eliminate the need for macroprudential policy.

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# Appendix

# A Model Extensions

In this section we consider two extensions of the baseline model. In the first one, we assume that the financing cost of households in the economy is a function of commodity price fluctuations. The purpose of this exercise is to study the role of hedging in more realistic contexts, where financing conditions faced by an emerging economy respond to commodity price cycles.

The second extension considers the possibility that commodity prices now experience stochastic volatility. This responds to empirical observations, where the price of many commodities tends to experience time-varying volatility. Studying the role of hedging under these new circumstances is hence relevant from the point of view of policymakers. A variant of this extension is also considered, where stochastic volatility of commodity prices is allowed to have a direct effect on borrowing conditions and on the price of a hedge contract.

## A.1 Correlation Between Financing Costs and Commodity Prices

The main departure from the baseline model is that now the interest rate faced by the economy is a function of commodity price deviations from their long-run mean. In particular, we assume that the interest rate faced by households in period $t$ follows the following process:

$$r_t = \bar{r} + \text{spread}_t, \tag{37}$$

$$\text{spread}_t = \varsigma + \zeta \cdot \tilde{p}_t \tag{38}$$

where $\bar{r}$ denotes the long-run world interest rate, $\varsigma$ denotes the average spread of the economy, and $\tilde{p}_t$ denotes the cyclical component (HP-filtered) of the log oil price.[13] We use EMBI spreads

[13]Since the new interest rate faced by the economy is changed relative to the baseline scenario, we also consider the same interest rate process for the pricing of options made by financial intermediaries.

from 2010 to 2019 for Colombia as well as the BRENT price for oil, and we regress the above equation. We find that $\varsigma = 0.02$ (average spread of 2%) and $\zeta = -0.09$.[14] These results suggest that higher oil prices are correlated with lower spreads.

The rest of the parameters of our calibration remain constant. Table 5 presents key long-run and financial crises moments for the decentralized, social planner's, and hedge economies.

Table 5: Long-Run Moments

| Long-run Moments | DE | SP | HE |
|---|---|---|---|
| $\mathbb{E}[b/Y]$ % | -31.44 | -31.34 | -32.01 |
| $\sigma(CA/Y)$ % | 2.72 | 1.38 | 2.30 |
| $\mathbb{E}\left[c^T/(c^T + p^N c^N)\right]$ % | 29.79 | 29.78 | 29.81 |
| $\mathbb{E}\left[g/(y^T + p^N y^N)\right]$ % | 1.18 | 1.18 | 1.40 |
| $\sigma\left(g/(y^T + p^N y^N)\right)$ % | 0.45 | 0.45 | 0.30 |
| Prob. of Crisis % | 3.12 | 1.29 | 3.26 |
| Pr.$(\mu_t > 0)$ % | 6.59 | 1.86 | 6.16 |
| $\mathbb{E}[\tau_t^b]$ % | n.a. | 4.44 | n.a. |
| $\sigma(\tau_t^b)$ % | n.a. | 4.06 | n.a. |
| Pr.$(\tau_t^b > 0)$ % | n.a. | 92.13 | n.a. |
| Pr. Active Hedge % | n.a. | n.a. | 67.24 |
| Welfare Gain % | n.a. | 0.013 | 21.02 |
| **Financial Crisis Moments** | | | |
| $\Delta GDP$ % | -27.70 | -13.88 | -25.85 |
| $\Delta c^T$ % | -31.96 | -14.95 | -29.46 |
| $CA/Y$ % | 12.09 | 3.76 | 10.22 |

**Notes:** Regimes DE, SP and HE, correspond to the decentralized, social planner's, and hedge economies, respectively. A Sudden Stop is defined as a period in which the constraint binds and the current account-to-GDP ratio exceeds by more than two standard deviations its long-run average. Average drops during Sudden Stop states are expressed in percent of long-run means with the exception of the current account-to-GDP ratio, which is expressed in percentage point differences. Welfare gains are computed as compensating variations of the composite good $x$ across states that equate welfare in the corresponding economy with the decentralized equilibrium. We use the ergodic distribution of the decentralized economy to compute the long-run average welfare gain.

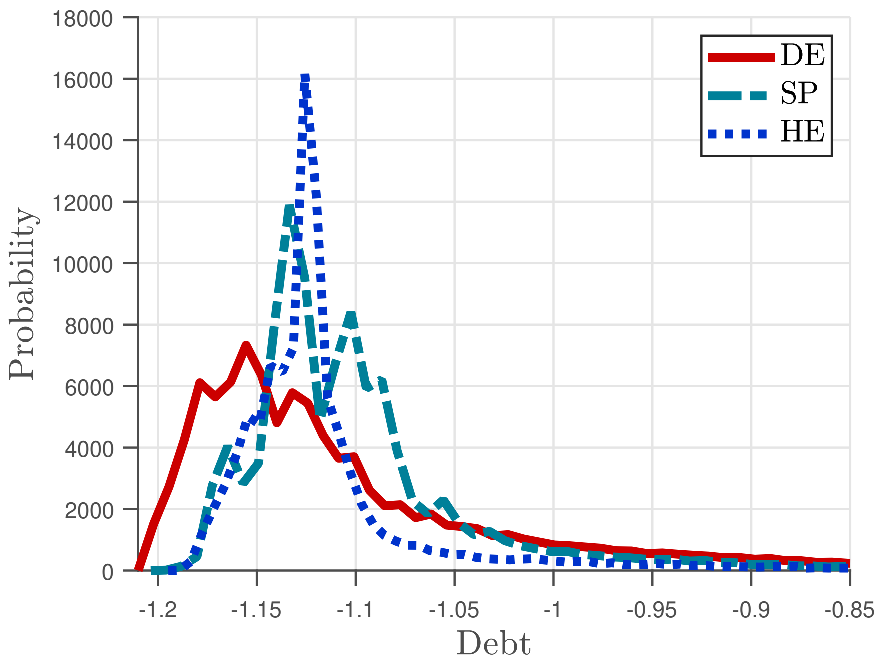
We see that overall, including a financing cost that is inversely related to the commodity price leads to relatively small differences with respect the baseline scenario. We do observe that the hedge still produces large welfare gains, although slightly lower. Given the larger volatility in the economy caused by the movement in spreads, we also observe that the current account is more

[14] The t-statistic for the constant in the regression is 23.58, while for the slope is -3.81.

volatile. Lastly, we see that the average optimal debt tax set by the planner is slightly higher, which is an indication of its desire to curb more aggressively the pecuniary externality. In terms of the severity of financial crises, we observe that during these episodes consumption and GDP contractions and current account reversals are slightly more pronounced.

Lastly, we illustrate in Figure 7 bond holdings ergodic distributions in this setup. Similar patterns to those in the baseline case are observed.

Figure 7: Ergodic Distribution - Bond Holdings




## A.2 Stochastic Volatility in Commodity Prices

The model is largely unaltered, except for the stochastic process for the commodity price. In particular, we assume that the stochastic process of the price $p_t$ follows the following structure:

$$\begin{aligned} p_t &= (1-\rho_p)\mu_p + \rho_p p_{t-1} + \exp(\sigma_t^p)\varepsilon_t, \quad \varepsilon_t \sim N(0,1) \\ \sigma_t^p &= (1-\varrho)\mu_\sigma + \varrho\sigma_{t-1}^p + \varsigma_t, \quad \varsigma_t \sim N(0,\vartheta^2). \end{aligned} \tag{39}$$

We assume that the standard deviation of the commodity price $\sigma_t^p$ takes two values, $\sigma_L^p$ and $\sigma_H^p$, and that these regimes switch according to a transition probability matrix

$$\Pi^p = \begin{bmatrix} \pi_{LL} & 1-\pi_{LL} \\ 1-\pi_{HH} & \pi_{HH} \end{bmatrix}. \tag{40}$$

The 2-state ergodic distribution implied by this transition matrix is such that $\sigma_p = \Pi_L \sigma_L^p + \Pi_H \sigma_H^p$, where $\sigma_p$ corresponds to the parameter calibration in the previous section. This is, the standard deviation is now stochastic its long-run average is still the same one as in the baseline economy.

We recalibrate the model to match features of the Colombian economy. Table 6 presents the calibration for this extension, only for parameters that were recalibrated or are key for the new setup.

Table 6: Calibration for the Colombian Economy - Stochastic Volatility Model

| Parameter | Value | Reference |
|---|---|---|
| $\zeta$ | 0.332 | Sudden Stop probability |
| $\rho_p$ | 0.664 | Crude Oil WTI |
| $\mu_p$ | 1 | Normalization |
| $\pi_{LL}$ | 0.882 | Low Variance Regime, Crude Oil WTI |
| $\pi_{HH}$ | 0.818 | High Variance Regime Crude Oil WTI |
| $\sigma_L^p$ | 0.250 | Low Standard Deviation Regime |
| $\sigma_H^p$ | 0.380 | High Standard Deviation Regime |

We define periods of high and low volatility in the commodity price as periods where the 10-year standard deviation exceeds the unconditional standard deviation. Similarly, periods of low volatility are those where the metric is below the long-run standard deviation. Given that we have two regimes, we can directly estimate the Markov transition probability matrix. We choose the grid points so that the implied long-run average volatility is consistent with the one in the data. With this in hand we now have a state space that is summarize by the vector $\mathbf{s} = (y^T, p, \sigma^p, b)$, where the first three variables correspond to exogenous states and the last to the endogenous state.

We then proceed to solve the model. The solution method is very similar to the one employed for the baseline framework, with the exception that now we have an additional exogenous state. Table 7 presents key long-run moments for the three economies we studied in the previous section.

Table 7: Long-Run Moments - Stochastic Volatility Model

| Long-run Moments | DE | SP | HE |
|---|---|---|---|
| $\mathbb{E}[b/Y]$ % | -31.89 | -31.71 | $-32.10$ |
| $\sigma(CA/Y)$ % | 3.04 | 1.55 | 2.42 |
| $\mathbb{E}\left[c^T/(c^T+p^Nc^N)\right]$ % | 29.78 | 29.77 | 29.79 |
| $\mathbb{E}\left[g/(y^T+p^Ny^N)\right]$ % | 1.24 | 1.24 | 1.37 |
| $\sigma\left(g/(y^T+p^Ny^N)\right)$ % | 0.31 | 0.31 | 0.22 |
| Prob. of Crisis % | 3.05 | 1.46 | 3.63 |
| Pr.($\mu_t > 0$) % | 5.19 | 3.21 | 4.57 |
| $\mathbb{E}[\tau_t^b]$ % | n.a. | 4.65 | n.a. |
| $\sigma(\tau_t^b)$ % | n.a. | 2.88 | n.a. |
| Pr.($\tau_t^b > 0$) % | n.a. | 97.02 | n.a. |
| Pr. Active Hedge % | n.a. | n.a. | 67.24 |
| Welfare Gain % | n.a. | 0.020 | 10.86 |
| Financial Crisis Moments | | | |
| $\Delta GDP$ % | -31.68 | -18.02 | -26.45 |
| $\Delta c^T$ % | -36.57 | -19.52 | -29.75 |
| $CA/Y$ % | 13.81 | 4.80 | 9.78 |

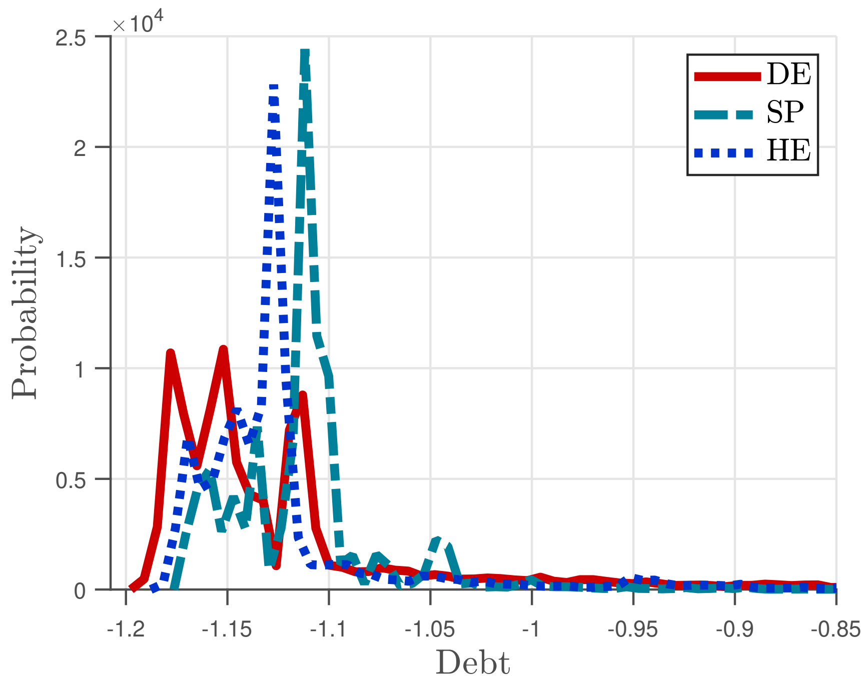
**Notes:** Regimes DE, SP and HE, correspond to the decentralized, social planner's, and hedge economies, respectively. A Sudden Stop is defined as a period in which the constraint binds and the current account-to-GDP ratio exceeds by more than two standard deviations its long-run average. Average drops during Sudden Stop states are expressed in percent of long-run means with the exception of the current account-to-GDP ratio, which is expressed in percentage point differences. Welfare gains are computed as compensating variations of the composite good $x$ across states that equate welfare in the corresponding economy with the decentralized equilibrium. We use the ergodic distribution of the decentralized economy to compute the long-run average welfare gain.

We observe that under stochastic volatility the three economies present similar features to the baseline scenario. However, we do observe higher current account volatility and slightly higher Sudden Stops likelihood. This is consistent with facing more risk coming from periods of high volatility in commodity prices. Notice that under the presence of stochastic volatility we now have that the overborrowing externality is stronger, reflected by a larger optimal tax welfare gains of the social planner.

As in the previous section, we observe that welfare gains stemming from the use of the option contract continue to be sizable. Welfare gains in the hedge economy are now half of what they were in the baseline scenario, which is consistent with worse consumption-smoothing possibilities.

Figure 8 presents the ergodic distribution of the decentralized, social planner, and hedge economies. As in Section 5, we see that there is overborrowing in the decentralized equilibrium relative to the social planner's equilibrium. Additionally, when facing hedging against commodity price fluctuations, households tend to borrow less (although the overborrowing externality is still present), but still larger amounts relative to the planner's economy..

Figure 8: Ergodic Distribution - Bond Holdings




Overall, we see that under stochastic volatility the main patterns observed in Section 5 are still present. Crucially, we observe that hedging can provide large welfare gains to households in the economy when in presence of stochastic volatility in commodity prices.

## A.3 Stochastic Volatility in Commodity Prices - Alternative Formulation

In this subsection we consider an alternative pricing function for interest rates. Specifically, we assume that the interest rate faced by households in the economy depends on commodity prices, which are subject to stochastic volatility. The effective interest rate faced by households in the economy is the reciprocal of

$$q_t = \mathbb{E}_t \exp(-(R_{t+1} + \iota \sigma^p_{t+1})). \tag{41}$$

This formulation is similar to the one employed in Gornemann et al. (2024). Here, the volatility of the commodity price increases the financial cost faced by households in the economy. The rest of the model remains identical to the one described in Section A.2. We calibrate $\iota$ so that the average spread of the economy is 2% in the long-run, in line with the EMBI spread observed for Colombia. We set $\iota = 0.064$.

Table 8 presents the long-run moments for this alternative model. Overall, we see that the long-run moments of the model do not differ substantially from the one described in Section A.2, implying that the alternative formulation of the discount factor of financial intermediaries does not introduce noticeable changes in the behavior of households.

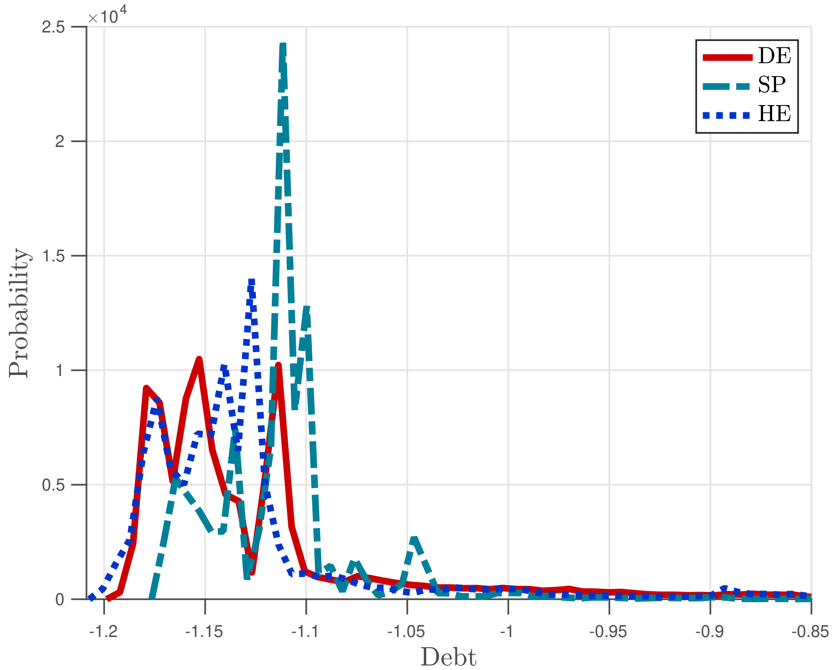
Table 8: Long-Run Moments - Alternative Stochastic Volatility Model

| Long-run Moments | DE | SP | HE |
|---|---|---|---|
| $\mathbb{E}[b/Y]$ % | -31.89 | -31.71 | -31.66 |
| $\sigma(CA/Y)$ % | 3.04 | 1.55 | 3.24 |
| $\mathbb{E}\left[c^T/(c^T+p^N c^N)\right]$ % | 29.78 | 29.77 | 29.71 |
| $\mathbb{E}\left[g/(y^T+p^N y^N)\right]$ % | 1.24 | 1.24 | 1.35 |
| $\sigma\left(g/(y^T+p^N y^N)\right)$ % | 0.31 | 0.31 | 0.23 |
| Prob. of Crisis % | 3.02 | 1.47 | 3.82 |
| $\Pr.(\mu_t > 0)$ % | 5.18 | 2.68 | 5.67 |
| $\mathbb{E}[\tau_t^b]$ % | n.a. | 4.73 | n.a. |
| $\sigma(\tau_t^b)$ % | n.a. | 2.89 | n.a. |
| $\Pr.(\tau_t^b > 0)$ % | n.a. | 96.51 | n.a. |
| Pr. Active Hedge % | n.a. | n.a. | 67.24 |
| Welfare Gain % | n.a. | 0.020 | 11.07 |
| **Financial Crisis Moments** | | | |
| $\Delta GDP$ % | -31.78 | -18.04 | -31.39 |
| $\Delta c^T$ % | -36.69 | -19.54 | -35.88 |
| $CA/Y$ % | 13.88 | 4.80 | 13.23 |

**Notes:** Regimes DE, SP and HE, correspond to the decentralized, social planner's, and hedge economies, respectively. A Sudden Stop is defined as a period in which the constraint binds and the current account-to-GDP ratio exceeds by more than two standard deviations its long-run average. Average drops during Sudden Stop states are expressed in percent of long-run means with the exception of the current account-to-GDP ratio, which is expressed in percentage point differences. Welfare gains are computed as compensating variations of the composite good $x$ across states that equate welfare in the corresponding economy with the decentralized equilibrium. We use the ergodic distribution of the decentralized economy to compute the long-run average welfare gain.

We next present the ergodic distributions of bond holdings for the three economies. Figure 9 presents the results. Overall, we see that there is not a large difference with respect to our setup with the less complex discount factor of financial intermediaries.

Figure 9: Ergodic Distribution - Bond Holdings



# B Solution Algorithm

## B.1 Decentralized Economy

The equilibrium conditions of our model are:

$$p_t^N = \frac{1-\omega_p}{\omega_p}\left(\frac{c_t^T}{c_t^N}\right)^{1+\eta_p} \tag{42}$$

$$\frac{b_{t+1}}{1+r_t} \geq -\zeta(y_t^T + p_t^N y_t^N + (1-\varphi)p\overline{y}) \tag{43}$$

$$c_t^T + \frac{b_{t+1}}{1+r_t} = y_t^T + b_t + (1-\varphi)p\overline{y} \tag{44}$$

$$c_t^N = y_t^N = \bar{L}^\alpha \tag{45}$$

$$u_T(t) = \beta R_t \mathbb{E}_t[u_T(t+1)] + \mu_t \tag{46}$$

where,

$$u_T(t) = \omega_x \omega_p x_t^{1+\eta_x-\sigma} c_t^{\eta_p-\eta_x}(c_t^T)^{-\eta_p-1}.$$

We start with a conjecture for the debt holdings policy function, $b'$, defined over the state space $(y^T, p, b)$.[15] The steps of the solution algorithm are the following:

1. Start iteration $j$ with a guess for $b_j'(y^T, p, b)$. Using this guess construct:

$$c_j^T(y^T, p, b) = y^T + b - \frac{b_j'(y^T, p, b)}{1+r} + (1-\varphi)p\overline{y} \tag{47}$$

$$c_j^N(y^T, p, b) = \bar{L}^\alpha \tag{48}$$

$$x_j(y^T, p, b) = \left[\omega_x(c_j(y^T, p, b))^{-\eta_x} + (1-\omega_x)(g_j(y^T, p, b))^{-\eta_x}\right]^{-\frac{1}{\eta_x}} \tag{49}$$

[15] Note that this guess corresponds to a matrix with dimensions $N_y \times N_p \times N_b$, where $N_y$, $N_p$ and $N_b$ correspond to the number of elements in the grid of the tradable endowment, commodity price, and debt, respectively. In the numerical solution we set $N_y = 7$, $N_p = 7$ and $N_b = 120$.

$$c_j(y^T,p,b)=\left[\omega_p(c_j^T(y^T,p,b))^{-\eta_p}+(1-\omega_p)(c_j^N(y^T,p,b))^{-\eta_p}\right]^{-\frac{1}{\eta_p}} \tag{50}$$

$$g_j(y^T,p,b)=\varphi p\bar{y} \tag{51}$$

$$p_j^N(y^T,b)=\frac{1-\omega_p}{\omega_p}\left(\frac{c_j^T(y^T,p,b)}{c_j^N(y^T,p,b)}\right)^{1+\eta_p}. \tag{52}$$

Lastly, compute the discounted expected marginal utility

$$\beta R^*\mathbb{E}_{y^{T\prime},p'|y^T,p}\left[u_{T,j}(y^{T\prime},p',b_j'(y^T,p,b))\right], \tag{53}$$

where $u_{T,j}(y^T,p,b)=u_T(c_j(y^T,p,b),x_j(y^T,p,b))$.

2. Assume the borrowing constraint binds. Note that when the constraint binds we have that tradable consumption is $c_{j+1}^T(y^T,p,b)=(y^T+(1-\varphi)p\overline{y})(1+\zeta)+b+\zeta p_j^N(y^T,p,b)\bar{L}^\alpha$ and the composite goods are $c_{j+1}(y^T,p,b)=\left[\omega_p(c_{j+1}^T(y^T,p,b))^{-\eta_p}+(1-\omega_p)(c_{j+1}^N(y^T,p,b))^{-\eta_p}\right]^{-\frac{1}{\eta_p}}$ and $x_{j+1}(y^T,p,b)=\left[\omega_x(c_{j+1}(y^T,p,b))^{-\eta_x}+(1-\omega_x)(\varphi p\bar{y})^{-\eta_x}\right]^{-\frac{1}{\eta_x}}$. We check whether this assumption holds by calculating the residual of the Euler equation:

$$\mathcal{R}(y^T,p,b)=u_{T,j+1}(y^T,p,b)-\beta R^*\mathbb{E}_{y^{T\prime},p'|y^T,p}\left[u_{T,j}(y^{T\prime},p',b_j'(y^T,p,b))\right]. \tag{54}$$

If $\mathcal{R}(y^T,p,b)>0$, we keep the values for $c_{j+1}^T(y^T,p,b)$, $c_{j+1}(y^T,p,b)$ and $x_{j+1}(y^T,p,b)$. Otherwise, the constraint does not bind for that point of the state space and we discard $c_{j+1}^T(y^T,p,b)$, $c_{j+1}(y^T,p,b)$ and $x_{j+1}(y^T,p,b)$. We then numerically solve for the value of $c_{j+1}^T(y^T,p,b)$ that satisfies

$$\begin{aligned}u_T(c(c_{j+1}^T(y^T,p,b),c_{j+1}^N(y^T,p,b)),x(c_{j+1}^T(y^T,p,b),c_{j+1}^N(y^T,p,b),\varphi p\bar{y}))\\=\beta R\mathbb{E}_{y^{T\prime},p'|y^T,p}\left[u_{T,j}(y^{T\prime},p',b_j'(y^T,p,b))\right].\end{aligned} \tag{55}$$

We construct then $c_{j+1}(y^T,p,b)=\left[\omega_p(c_{j+1}^T(y^T,p,b))^{-\eta_p}+(1-\omega_p)(c_{j+1}^N(y^T,p,b))^{-\eta_p}\right]^{-\frac{1}{\eta_p}}$ and $x_{j+1}(y^T,p,b)=\left[\omega_x(c_{j+1}(y^T,p,b))^{-\eta_x}+(1-\omega_x)(\varphi p\bar{y})^{-\eta_x}\right]^{-\frac{1}{\eta_x}}$.

3. Using the resource constraint of tradables obtain the updated conjecture for debt $b'_{j+1}(y^T, p, b) = (1+r)(y^T + b - c^T_{j+1}(y^T, p, b) + (1-\varphi)p\overline{y})$. We also construct an update for the relative price of nontradables $p^N_{j+1}(y^T, p, b) = \frac{1-\omega_p}{\omega_p}\left(\frac{c^T_{j+1}(y^T,p,b)}{c^N_{j+1}(y^T,p,b)}\right)^{1+\eta_p}$.

4. Check for convergence. If $\|b'_{j+1}(y^T, p, b) - b'_j(y^T, p, b)\| < \epsilon$, then the problem is solved. Otherwise, discard $b'_j$ and use $b'_{j+1}$ as the new guess for the problem (go back to step 1).

## B.2 Social Planner's Economy

The equilibrium conditions of our model are:

$$p^N_t = \frac{1-\omega_p}{\omega_p}\left(\frac{c^T_t}{c^N_t}\right)^{1+\eta_p} \tag{56}$$

$$\frac{b_{t+1}}{1+r_t} \geq -\zeta(y^T_t + p^N_t y^N_t + (1-\varphi)p_t\overline{y}) \tag{57}$$

$$c^T_t + \frac{b_{t+1}}{1+r_t} = y^T_t + b_t + (1-\varphi)p\overline{y} \tag{58}$$

$$c^N_t = y^N_t = \bar{L}^\alpha \tag{59}$$

$$u_T(t) + \mu_t\zeta(1+\eta_p)\left(\frac{1-\omega_p}{\omega_p}\right)\left(\frac{c^T_t}{\bar{L}^\alpha}\right)^{\eta_p} = \mu_t + \beta R_t \mathbb{E}_t\left[u_T(t+1) + \mu_{t+1}\zeta(1+\eta_p)\left(\frac{1-\omega_p}{\omega_p}\right)\left(\frac{c^T_{t+1}}{\bar{L}^\alpha}\right)^{\eta_p}\right] \tag{60}$$

where,

$$u_T(t) = \omega_x\omega_p x_t^{1+\eta_x-\sigma} c_t^{\eta_p-\eta_x}(c^T_t)^{-\eta_p-1}.$$

We start with a conjecture for the debt holdings policy function, $b'$, and Lagrange multiplier $\mu$, defined over the state space $(y^T, p, b)$.[16] We use the solutions to the decentralized problem as guesses for these policy functions. The steps of the solution algorithm are the following:

[16]Note that these guesses correspond to a matrix with dimensions $N_y \times N_p \times N_b$, where $N_y$, $N_p$ and $N_b$ correspond to the number of elements in the grid of the tradable endowment, commodity price, and debt, respectively.

1. Start iteration $j$ with a guess for $b'_j(y^T, p, b)$. Using this guess construct:

$$c_j^T(y^T, p, b) = y^T + b - \frac{b'_j(y^T, p, b)}{1+r} + (1-\varphi)p\overline{y} \tag{61}$$

$$c_j^N(y^T, p, b) = \bar{L}^\alpha \tag{62}$$

$$x_j(y^T, p, b) = \left[\omega_x (c_j(y^T, p, b))^{-\eta_x} + (1-\omega_x)(g_j(y^T, p, b))^{-\eta_x}\right]^{-\frac{1}{\eta_x}} \tag{63}$$

$$c_j(y^T, p, b) = \left[\omega_p (c_j^T(y^T, p, b))^{-\eta_p} + (1-\omega_p)(c_j^N(y^T, p, b))^{-\eta_p}\right]^{-\frac{1}{\eta_p}} \tag{64}$$

$$g_j(y^T, p, b) = \varphi p \bar{y} \tag{65}$$

$$p_j^N(y^T, b) = \frac{1-\omega_p}{\omega_p}\left(\frac{c_j^T(y^T, p, b)}{c_j^N(y^T, p, b)}\right)^{1+\eta_p}. \tag{66}$$

Lastly, compute the discounted expected marginal utility

$$\beta R^* \mathbb{E}_{y^{T\prime}, p' | y^T, p}\left[u_{T,j}(y^{T\prime}, p', b'_j(y^T, p, b)) + \zeta \Omega_j(y^{T\prime}, p', b'_j(y^T, p, b))\right], \tag{67}$$

where $u_{T,j}(y^T, p, b) = u_T(c_j(y^T, p, b), x_j(y^T, p, b))$ and $\Omega_j(y^T, p, b) = \mu_j(y^T, p, b)(1+\eta_p)\left(\frac{1-\omega_p}{\omega_p}\right)\left(\frac{c_j^T(y^T,p,b)}{\bar{L}^\alpha}\right)^{\eta_p}$.

2. Assume the borrowing constraint binds. Note that when the constraint binds we have that tradable consumption is $c_{j+1}^T(y^T, p, b) = (y^T + (1-\varphi)p\overline{y})(1+\zeta) + b + \zeta p_j^N(y^T, p, b)\bar{L}^\alpha$ and the composite goods are $c_{j+1}(y^T, p, b) = \left[\omega_p (c_{j+1}^T(y^T, p, b))^{-\eta_p} + (1-\omega_p)(c_{j+1}^N(y^T, p, b))^{-\eta_p}\right]^{-\frac{1}{\eta_p}}$ and $x_{j+1}(y^T, p, b) = \left[\omega_x (c_{j+1}(y^T, p, b))^{-\eta_x} + (1-\omega_x)(\varphi p \bar{y})^{-\eta_x}\right]^{-\frac{1}{\eta_x}}$. We check whether this assumption holds by calculating the residual of the Euler equation:

$$\mathcal{R}(y^T, p, b) = u_{T,j+1}(y^T, p, b) - \beta R^* \mathbb{E}_{y^{T\prime}, p' | y^T, p}\left[u_{T,j}(y^{T\prime}, p', b'_j(y^T, p, b)) + \zeta \Omega_j(y^{T\prime}, p', b'_j(y^T, p, b))\right]. \tag{68}$$

If $\mathcal{R}(y^T, p, b) > 0$, we keep the values for $c_{j+1}^T(y^T, p, b)$, $c_{j+1}(y^T, p, b)$ and $x_{j+1}(y^T, p, b)$. We can construct an updated guess for the Lagrange multiplier, which is given by

$$\mu_{j+1}(y^T,p,b)=\frac{u_{T,j+1}(y^T,p,b)-\beta R^*\mathbb{E}_{y^{T\prime},p'|y^T,p}\left[u_{T,j}(y^{T\prime},p',b'_j(y^T,p,b))+\zeta\Omega_j(y^{T\prime},p',b'_j(y^T,p,b))\right]}{1-\zeta(1+\eta_p)\left(\frac{1-\omega_p}{\omega_p}\right)\left(\frac{c^T_{j+1}(y^T,p,b)}{\bar{L}^\alpha}\right)^{\eta_p}} \tag{69}$$

If $\mathcal{R}(y^T,p,b)\leq 0$, the constraint does not bind for that point of the state space and we discard $c^T_{j+1}(y^T,p,b)$, $c_{j+1}(y^T,p,b)$ and $x_{j+1}(y^T,p,b)$. Note that $\mu_{j+1}(y^T,p,b)=0$. We then numerically solve for the value of $c^T_{j+1}(y^T,p,b)$ that satisfies

$$\begin{aligned}&u_T(c(c^T_{j+1}(y^T,p,b),c^N_{j+1}(y^T,p,b)),x(c^T_{j+1}(y^T,p,b),c^N_{j+1}(y^T,p,b),\varphi p\bar{y}))\\&=\beta R^*\mathbb{E}_{y^{T\prime},p'|y^T,p}\left[u_{T,j}(y^{T\prime},p',b'_j(y^T,p,b))+\zeta\Omega_j(y^{T\prime},p',b'_j(y^T,p,b))\right].\end{aligned} \tag{70}$$

We construct then $c_{j+1}(y^T,p,b)=\left[\omega_p(c^T_{j+1}(y^T,p,b))^{-\eta_p}+(1-\omega_p)(c^N_{j+1}(y^T,p,b))^{-\eta_p}\right]^{-\frac{1}{\eta_p}}$ and $x_{j+1}(y^T,p,b)=\left[\omega_x(c_{j+1}(y^T,p,b))^{-\eta_x}+(1-\omega_x)(\varphi p\bar{y})^{-\eta_x}\right]^{-\frac{1}{\eta_x}}$.

3. Using the resource constraint of tradables obtain the updated conjecture for debt $b'_{j+1}(y^T,p,b)=(1+r)(y^T+b-c^T_{j+1}(y^T,p,b)+(1-\varphi)p\overline{y})$. We also construct an update for the relative price of nontradables $p^N_{j+1}(y^T,p,b)=\frac{1-\omega_p}{\omega_p}\left(\frac{c^T_{j+1}(y^T,p,b)}{c^N_{j+1}(y^T,p,b)}\right)^{1+\eta_p}$.

4. Check for convergence. If $\|b'_{j+1}(y^T,p,b)-b'_j(y^T,p,b)\|<\epsilon$, then the problem is solved. Otherwise, discard $b'_j$ and use $b'_{j+1}$ as the new guess for the problem (go back to step 1).

## B.3 Hedge Economy

The equilibrium conditions of our model are:

$$p^N_t=\frac{1-\omega_p}{\omega_p}\left(\frac{c^T_t}{c^N_t}\right)^{1+\eta_p} \tag{71}$$

$$\frac{b_{t+1}}{1+r_t}\geq-\zeta(y^T_t+p^N_t y^N_t+T_t) \tag{72}$$

$$c_t^T + \frac{b_{t+1}}{1+r_t} = y_t^T + b_t - q_t\phi\bar{y} + (1-\varphi)(\phi\max\{\theta, p_t\} + (1-\phi)p_t\bar{y}) \tag{73}$$

$$c_t^N = y_t^N = \bar{L}^\alpha \tag{74}$$

$$u_T(t) = \beta R_t \mathbb{E}_t[u_T(t+1)] + \mu_t \tag{75}$$

where,

$$u_T(t) = \omega_x\omega_p x_t^{1+\eta_x-\sigma} c_t^{\eta_p-\eta_x}(c_t^T)^{-\eta_p-1}.$$

We start with a conjecture for the debt holdings policy function, $b'$, defined over the state space $(y^T, p, b)$.[17] We also construct the price of the hedge contract outside the loop, since it will not change across iterations. We define the price as

$$q(y^T, p) = \frac{1}{1+r}\mathbb{E}_{y^{T\prime},p'|y^T,p}\left[\max\{\theta - p', 0\}\right]. \tag{76}$$

The steps of the solution algorithm are the following:

1. Start iteration $j$ with a guess for $b_j'(y^T, p, b)$. Using this guess construct:

$$c_j^T(y^T, p, b) = y^T + b - \frac{b_j'(y^T, p, b)}{1+r} - q(y^T, p)\phi\bar{y} + (1-\varphi)(\phi\max\{\theta, p\} + (1-\phi)p\bar{y}) \tag{77}$$

$$c_j^N(y^T, p, b) = \bar{L}^\alpha \tag{78}$$

$$x_j(y^T, p, b) = \left[\omega_x(c_j(y^T, p, b))^{-\eta_x} + (1-\omega_x)(g_j(y^T, p, b))^{-\eta_x}\right]^{-\frac{1}{\eta_x}} \tag{79}$$

$$c_j(y^T, p, b) = \left[\omega_p(c_j^T(y^T, p, b))^{-\eta_p} + (1-\omega_p)(c_j^N(y^T, p, b))^{-\eta_p}\right]^{-\frac{1}{\eta_p}} \tag{80}$$

$$g_j(y^T, p, b) = \varphi(\phi\max\{\theta, p\}\bar{y} + (1-\phi)p\bar{y}) \tag{81}$$

$$p_j^N(y^T, b) = \frac{1-\omega_p}{\omega_p}\left(\frac{c_j^T(y^T, p, b)}{c_j^N(y^T, p, b)}\right)^{1+\eta_p}. \tag{82}$$

[17]Note that this guess corresponds to a matrix with dimensions $N_y \times N_p \times N_b$, where $N_y$, $N_p$ and $N_b$ correspond to the number of elements in the grid of the tradable endowment, commodity price, and debt, respectively.

Lastly, compute the discounted expected marginal utility

$$\beta R^{*}\mathbb{E}_{y^{T\prime},p'|y^{T},p}\left[u_{T,j}(y^{T\prime},p',b_j'(y^{T},p,b))\right], \tag{83}$$

where $u_{T,j}(y^T,p,b)=u_T(c_j(y^T,p,b),x_j(y^T,p,b))$.

2. Assume the borrowing constraint binds. Note that when the constraint binds we have that tradable consumption is $c^T_{j+1}(y^T,p,b)=(y^T-q(y^T,p)\phi\bar{y}+(1-\varphi)(\phi\max\{\theta,p\}+(1-\phi)p_t\bar{y}))(1+\zeta)+b+\zeta p^N_j(y^T,p,b)\bar{L}^{\alpha}$ and the composite goods are $c_{j+1}(y^T,p,b)=\left[\omega_p(c^T_{j+1}(y^T,p,b))^{-\eta_p}+(1-\omega_p)(c^N_{j+1}(y^T,p,b))^{-\eta_p}\right]^{-\frac{1}{\eta_p}}$ as well as $x_{j+1}(y^T,p,b)=\left[\omega_x(c_{j+1}(y^T,p,b))^{-\eta_x}+(1-\omega_x)(\varphi(\phi\max\{\theta,p\}+(1-\phi)p\bar{y}))^{-\eta_x}\right]^{-\frac{1}{\eta_x}}$. We check whether this assumption holds by calculating the residual of the Euler equation:

$$\mathcal{R}(y^T,p,b)=u_{T,j+1}(y^T,p,b)-\beta R^{*}\mathbb{E}_{y^{T\prime},p'|y^{T},p}\left[u_{T,j}(y^{T\prime},p',b_j'(y^{T},p,b))\right]. \tag{84}$$

If $\mathcal{R}(y^T,p,b)>0$, we keep the values for $c^T_{j+1}(y^T,p,b)$, $c_{j+1}(y^T,p,b)$ and $x_{j+1}(y^T,p,b)$. Otherwise, the constraint does not bind for that point of the state space and we discard $c^T_{j+1}(y^T,p,b)$, $c_{j+1}(y^T,p,b)$ and $x_{j+1}(y^T,p,b)$. We then numerically solve for the value of $c^T_{j+1}(y^T,p,b)$ that satisfies

$$\begin{aligned}u_T(c(c^T_{j+1}(y^T,p,b),c^N_{j+1}(y^T,p,b)),x(c^T_{j+1}(y^T,p,b),c^N_{j+1}(y^T,p,b),\varphi(\phi\max\{\theta,p_t\}+(1-\phi)p\bar{y})))\\ =\beta R\mathbb{E}_{y^{T\prime},p'|y^{T},p}\left[u_{T,j}(y^{T\prime},p',b_j'(y^{T},p,b))\right].\end{aligned} \tag{85}$$

We construct then $c_{j+1}(y^T,p,b)=\left[\omega_p(c^T_{j+1}(y^T,p,b))^{-\eta_p}+(1-\omega_p)(c^N_{j+1}(y^T,p,b))^{-\eta_p}\right]^{-\frac{1}{\eta_p}}$ and $x_{j+1}(y^T,p,b)=\left[\omega_x(c_{j+1}(y^T,p,b))^{-\eta_x}+(1-\omega_x)(\varphi(\phi\max\{\theta,p\}+(1-\phi)p\bar{y})^{-\eta_x}\right]^{-\frac{1}{\eta_x}}$.

3. Using the resource constraint of tradables obtain the updated conjecture for debt $b'_{j+1}(y^T,p,b)=(1+r)(y^T+b-q(y^T,p)\phi\bar{y}+\varphi(\phi\max\{\theta,p\}+(1-\phi)p\bar{y})-c^T_{j+1}(y^T,p,b))$. We also construct

an update for the relative price of nontradables $p^N_{j+1}(y^T, p, b) = \frac{1-\omega_p}{\omega_p}\left(\frac{c^T_{j+1}(y^T,p,b)}{c^N_{j+1}(y^T,p,b)}\right)^{1+\eta_p}$.

4. Check for convergence. If $\|b'_{j+1}(y^T, p, b) - b'_j(y^T, p, b)\| < \epsilon$, then the problem is solved. Otherwise, discard $b'_j$ and use $b'_{j+1}$ as the new guess for the problem (go back to step 1).