



Analysis of Public Spending

An Evaluation Methodology for Measuring the Efficiency of Brazilian State Spending on Education

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Abstract*

This study uses Data Envelopment Analysis (DEA) to evaluate the efficiency of state public education systems in Brazil. State government spending on education and state gross domestic product are interpreted as input variables, with the latter being non-discretionary. Output variables are based on the number of state public school students taking the ENEM (*Exame Nacional do Ensino Médio*, a national exam taken at the end of high school studies) and the *Prova Brasil* (a national exam taken at the end of the 5th and 9th grades) and their scores on these exams. Variables were not scaled per capita in order to allow us to draw conclusions about economies of scale. The main result was a negative correlation between per capita state spending on education and the relative efficiency of the state educational system, indicating that there is a limit to per capita educational spending beyond which technical efficiency in educational programs falls significantly.

JEL Codes: H52, H75

Keywords: education, efficiency, state governments

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Executive Summary

This article uses the Data Envelopment Analysis (DEA) methodology to assess the efficiency of state education networks. To this end, various analyses were performed, including: the calculation of state efficiency scores, analysis of the weights determined, evaluation of the change in productivity over time, and formulation of state goals for the standard exams adopted by the Ministry of Education.

Result variables associated with the performance of students in state schools on the *Prova Brasil* (5th and 9th grades) and the ENEM were used. The input variables used were state spending on education and the state gross domestic product (GDP), with the latter modeled as a non-discretionary variable. Variables were not scaled per capita in order to allow us to draw conclusions about economies of scale.

The following are highlights of the principal findings:

- There is a negative correlation between a state's relative efficiency and i) the level of per capita spending on education and ii) the state per capita GDP, indicating that problems with state educational systems go beyond lack of resources and extend to management problems, since more resources are wasted in states that have relatively more funds.
- With respect to productivity, we were able to verify that some states lost ground over time. This phenomenon may be due to the lack of institutionalization and to instability in educational management.
- The goals established using the DEA methodology reflect the fact that more resources are squandered in relatively richer states. Thus, these states would need to improve their results more than proportionally in order to reach goals for overall levels of efficiency.
- The division of Brazilian states into four groups allows us to prioritize actions for each situation. To start with, the group of relatively rich states with high efficiency scores would not be a priority for public policy action, despite recognizing that these efficiency scores are relative and, in absolute terms, they may be far below worldwide parameters.
- Relatively rich states with low efficiency scores would require programs to improve educational system management, while the relatively poor states with above-average scores would improve their results with the injection of new funds into their school systems.
- Finally, for relatively poor states with low relative efficiency scores, the most appropriate approach would be a combination of management programs together with a conditional increase in funding.

Introduction

The Brazilian economy's recent loss of vitality seems to show that, unless structural reforms are undertaken, the country's growth will stall in the near future. This will lead to stagnation in tax collection by both the federal and other levels of government. On the other hand, the social demands for more government resources still seem far from being met.

The reconciliation of these two conflicting trends hinges on improving the quality and efficiency of public spending. Improved quality would increase the positive impacts of spending, while increased efficiency would provide capacity for expansion of services provided to the population in a manner much less costly to the taxpayer.

In Brazil, improving the efficiency of public spending is a federative issue, since the states and municipalities, along with the federal government, share responsibility for providing public services.

Roughly, states are responsible for about 30 percent of public spending in Brazil, and are thus an important channel for the provision of public goods and services.

There is, however, a long way to go before the efficiency of public spending in Brazil can be improved overall, especially at the state level. The first step is to estimate the magnitude of waste of public resources. Without having an idea of how much more is spent than necessary, it is difficult to establish plausible objectives to reduce this waste.

In recent studies on the efficiency of public spending, techniques usually employed in analyzing the efficiency of productive units in general have been used. According to this approach, the government is a producer of goods and services and its productivity may be evaluated and compared with other productive units.

The techniques used to evaluate government performance vary according to the defined objectives. For example, the DEA method is appropriate in situations in which the supply of multiple products must be evaluated. This nonparametric method is very flexible and does not impose standards on the units assessed; the performance of the units of the sample serve as their own evaluation criteria.

Recent literature provides examples of application of the DEA method to evaluate government performance in delivering public services. Afonso, Schuknecht, and Tanzi (2006) used the method to evaluate the performance of national governments in OECD countries, using public spending as a proportion of GNP (gross national product) as the input variable and some indicators, such as level of corruption, unemployment, GDP growth and others, as representative of government output. Afonso, Romero, and Monsalve (2013) used a similar methodology to study the performance of Latin American governments.

The DEA method has also been used to compare the efficiency of subnational governments. For example, Afonso and Fernandes (2006) compared the performance of local governments in Portugal (more specifically, in the Lisbon region), while Gasparini and Melo (2004) investigated the optimal level of transfers to Brazilian municipalities

based on the amount that these governments would need to close their respective fiscal gaps if they made efficient use of available resources.

Others have studied inefficiencies in the provision of public services by category. For example, Sola and Prior (2001) and Butler and Li (2005) studied the efficiency of public provision of health services. In Brazil, both Marinho (2003) and Souza, Rocha, and Nishijima (2010) focused their research on the efficiency of municipal hospitals, the former studying the efficient provision of public hospital services in the state of Rio de Janeiro and the latter studying hospitals in the state of São Paulo.

The DAE method has also been widely used to study the efficiency of educational service providers in relation to primary education (Anderson, Walberg, and Weinstein [1998], Mancebon and Molinero [2000], Cherchyea et al. [2010]), secondary education (Barbetta and Turati [2003], Alexander and Jaforullah [2004]), and even higher education (Thursby and Kemp [2002] and Chapple et al. [2005]).

In Brazil, educational efficiency has been analyzed using the DEA method, as well as more descriptive methods. For example, Brunet, Bertê, and Borges (2008), using the Normal Cumulative Distribution Function Standard Score method, developed many educational indicators and then ranked states and municipalities according to them.

Nicolella and Fernandes (2012) used the DEA methodology and data on the educational parameters of Brazilian municipalities to create potential and effective education indicators by state.

This study aims to evaluate the potential financial, economic, and social impacts if Brazilian states were to spend educational budgetary resources efficiently. Efficiency here is defined as the level of production and the corresponding expenses of the states with the best performance.

In addition, the DEA analysis will allow identification of the education subcategories most likely to improve their efficiency, as well as establishing goals to improve state school student performance on standard examinations. Finally, the use of biannual education data will allow us to analyze the evolution of state productivity in this sector.

This paper also intends to make a contribution that is, to the best of our knowledge, unprecedented, because it directly analyzes state public schools and treats each state as a productive unit for educational services. Thus, unlike what is usually done, we seek to evaluate the efficiency of the state educational system as a whole, based on the individual results of students enrolled in the state's schools, and on state spending on education.

The Brazilian Educational System and the Federation

The Brazilian educational system is structured according to the Educational Directives and Foundations Act (LDB, No. 9394/96), according to which education develops in

four stages: pre-school, elementary, secondary and higher education, with basic education consisting of the first three phases.¹

Educational institutions that operate within the system can be private or public. Among the former, there are for-profit, religious, and cooperative educational institutions. In the public sphere, schools may be associated with the three levels of government: federal, state, and municipal.²

Although the LDB defines the regulatory powers of each level of government—under which the federal government is responsible for both higher education and the private network, state governments are responsible for secondary schools, and municipal governments are responsible for elementary schools—this structure does not apply to the operation of educational institutions themselves. Thus, the three spheres of government overlap in the provision of educational services, and the three levels of government have primary, secondary, and higher education institutions.

Funding for the system is governed by the Federal Constitution. It stipulates that at least 18 percent of federal revenue and 25 percent of state and municipal revenue be used for education.³

The federal government coordinates the public education funding system through the Fund for Maintenance and Development of Basic Education and Valorization of Teaching Professionals (FUNDEB), which was created by constitutional amendment in 2006 and regulated in 2007. This fund replaced the Fund for Maintenance and Development of Elementary Education and Valorization of Teachers (FUNDEF), which was in force between 1998 and 2006. The difference between the two funds was that FUNDEF, unlike its successor, did not apply to secondary education.

The operation of FUNDEB is based on the contribution of 25 percent of state and local revenues. The federal government contributes a sum corresponding to 10 percent of the funds contributed by states and municipalities.

The distribution of FUNDEB funds follows two basic criteria:

- i. The number of students enrolled in public schools
- ii. Guarantee of a minimum amount per student in each state

Thus, it seeks to minimally equalize resources for every public school student in the country.

¹ In addition to the regular system described above, there are other educational services, such as the education of youth and adults, vocational education and special education.

² Federal District institutions are included in the state category.

³ Federal Constitution Art. 212.

Table 1
Resources Contributed to FUNDEB
Nominal Amounts and Annual Growth Rates
Brazil: 2007 – 2012

Year	Amount (Billion R\$)	Annual growth (%)
2007	25.099	
2008	33.366	32.9
2009	37.733	13.1
2010	44.021	16.7
2011	53.013	20.4
2012	57.773	9.0

Source: Secretary of the Treasury.

Prepared by the author.

The amounts contributed to the fund have grown substantially over the past few years, as can be seen in table 1. This growth is mainly due to the greater amount of taxes collected by states and municipalities during the period under investigation and to increases in the percentage allocated to these federative entities, as well as to the federal government.

On state budget balance sheets, FUNDEB resources are entered as credit transfers from the federal government and as debits in various ways, such as personnel, investments, etc. However, state balance sheets also list expenses by budget category, and thus identify spending on education.

Importance of the Categories and Subcategories Studied in State Public Spending

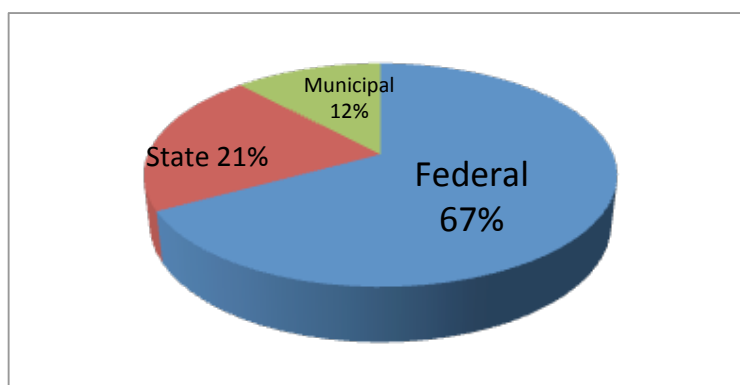
In 2009,⁴ the consolidated expenditures of the Brazilian public sector reached about R\$2.114 trillion. Of this total, the state governments accounted for approximately R\$450 billion, or 21 percent of consolidated expenditures (see Figure 1). For a more precise idea of what that amount means, note that it was equivalent to 13.7 percent of Brazil's GDP that year.

Of this total spent by states, four budget areas can be seen as the principal expense categories: education, health, public safety and transportation.⁵ As can be seen in Figure 2, education alone represents 29 percent of the state budget in 2011, and was the largest expense category. Therefore, in a way it is natural that we begin our study of the comparative efficiency of state public spending with this category.

⁴ This was the last year for which municipal spending data were available.

⁵ The concept of budget expense excludes financial expenses incurred by state governments, as well as expenses categorized as "administration" and "social security," since the latter refers exclusively to benefits paid to state employees.

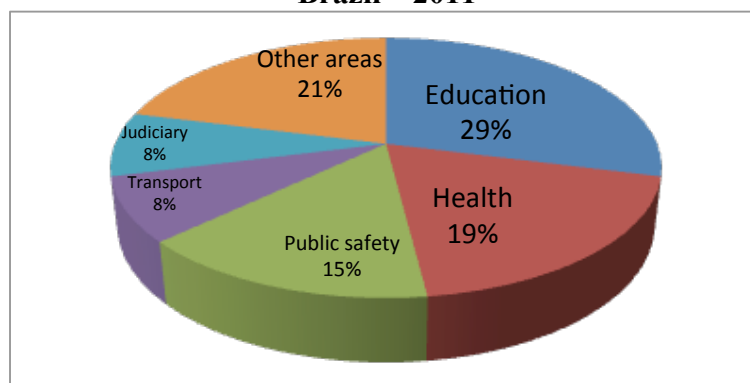
Figure 1
Distribution of Consolidated Spending of the Brazilian Public Sector by
Government Level
Brazil – 2009



Source: Secretary of the Treasury.

Of the other budget categories, only health, public safety, transport, and the judiciary surpass 5 percent of the total and could be the object of future studies.

Figure 2
Distribution of Consolidated Budget Spending of Brazilian States
Brazil – 2011



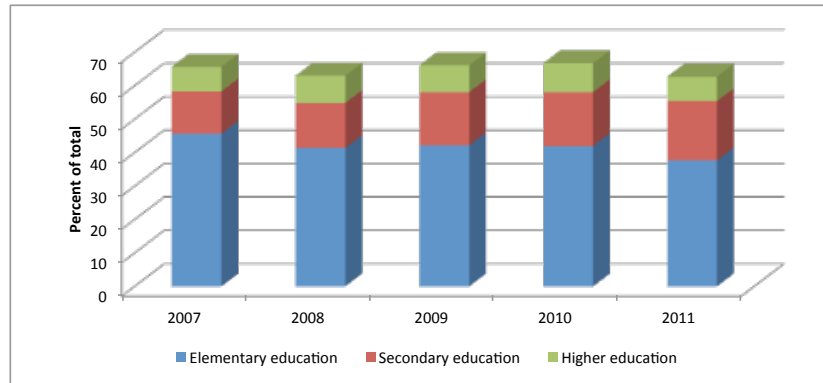
Source: Secretary of the Treasury.

In order to select the variables relevant to the analysis of the efficiency of the educational sector on which this study focuses, an assessment of state activities in this sector is necessary. To this end, spending in each of the sector's sub-activities must be examined, in order to then define quantitative variables that could reflect this spending.

In 2011, Brazilian states spent a total of R\$90.1 billion. Of this amount, R\$33.7 billion, or 37.7 percent, were spent on elementary education, which is the most costly sub-category of the states, contrary to what is commonly believed.

Total state spending on the secondary education sub-category reached R\$16.0 billion in 2011 (17.7 percent of the total), while higher education represented 7.3 percent of spending, totaling R\$6.6 billion that year. Of the remaining education sub-categories, none reached 3 percent of total spending on this category. Figure 3 presents the evolution of state spending by sub-category.

Figure 3
Evolution of Aggregate State Spending on Education for Selected Sub-Categories
Brazil — 2007 to 2011



Source: Secretary of the Treasury.

Figure 3 shows that the elementary education and secondary education sub-categories accounted for more than 50 percent of state education spending. This means that the selection of variables for measuring the efficiency of public spending in this area should reflect this sub-category composition.

DEA Methodology

The DEA methodology was first proposed by Farrel (1957), but only became popular in the literature after Charnes, Cooper, and Rhodes (1978) used it to evaluate concrete problems. These first implemented models, known now as CCR models, assume constant returns to scale. This limitation, however, was overcome by the work of Banker, Charnes, and Cooper (1984), which extends the original model for the case of variable returns to scale (BCC model).

Basic Concepts

The principle behind the DEA methodology is the physical definition of efficiency, according to which efficiency is calculated as the ratio between input used and products produced. Thus, the higher a unit's production for a given input, or alternatively the smaller the amount of input used for a given quantity of product, the greater the efficiency of the unit.

The problem that then arises is that this definition cannot be directly applied in the case of multiple inputs and/or products. In this case, one must assign weights to the quantities produced and inputs used. In cases where both inputs and products can be clearly priced, this limitation is easily overcome by using prices as weights in the evaluation.

However, it is often difficult or impossible to assign prices. In this case, the assignment of weights to products and inputs should be based on some arbitrary criteria established by the evaluator. The great merit of the DEA methodology is that it does not force the evaluator to establish arbitrary criteria: the weights are defined by the available data.

The idea is that the weights are chosen in the most favorable way for each unit, following certain consistency rules.

Calculation of Efficiency

Suppose that there are I decision units, that transform N different types of inputs into M different types of products. Then, an efficiency measurement could be calculated as the sequential solution of the following i ($i = 1, 2, \dots, I$) equations:

$$\left\{ \begin{array}{l} \text{Max}_{\{v_i, \mu_i\}}: \theta_i = \frac{\sum_{n=1}^N v_{i,n} y_{i,n}}{\sum_{m=1}^M \mu_{i,m} x_{i,m}} \\ \text{s.t.: } \frac{\sum_{n=1}^N v_{i,n} y_{j,n}}{\sum_{m=1}^M \mu_{i,m} x_{j,m}} \leq 1, (j \neq i) \\ v_i, \mu_i \neq 0 \end{array} \right. \quad (NLP_i),$$

where θ_i is the efficiency measure calculated by dividing the weighted sum of the N products produced by unit i ($y_{i,n}$) by the weighted sum of the M inputs used ($x_{i,m}$). The weights of the products, v_i , and the inputs, μ_i , are chosen to maximize this ratio. They are, however, subject to certain consistency conditions. The first is that the ratio θ_i be greater than or equal to one. This is not exactly a restriction, but rather a normalization condition. It would not matter if, instead of 1, the number 1,000 had been chosen as the limit for maximum efficiency.⁶

The other $(I - 1)$ restrictions prevent the weights chosen from leading to efficiency levels greater than one when applied to other units. These consistency constraints are imposed to ensure that the weights chosen are compatible not only with the unit in question, but also with all other units studied.

This problem can be solved for each unit involved in the evaluation. Units that achieve the maximum allowed value (i.e., 1) are considered efficient. Unit inefficiency is measured using values less than 1.

There are, however, two problems with this approach. The first is due to the infinite number of solutions to the NLP_i problem and the second to the fact that NLP_i is a nonlinear programming problem, which can make it computationally complex. Fortunately, NLP_i may be replaced with a linear problem with the same solutions. Moreover, there is a solution belonging to the (infinite) subset of solutions that

⁶In this case, modification of the other restrictions would also be necessary.

represents a correct interpretation of the problem and is obtained from imposing an additional normalizing constraint. This result, obtained by Cooper, Seiford, and Tone (2006), simplifies the NLP_i problem to the following linear formulation (in vector form):

$$\left\{ \begin{array}{l} \text{Min}_{\{\phi_i, \lambda_i\}} : \phi_i x_i \\ \text{s.t.} : \lambda_i y_i = 1 \\ \phi_i X - \lambda_i Y \geq 0 \\ \phi_i, \lambda_i \geq 0 \end{array} \right. \quad (\text{CCR}_i)$$

In this notation x_i is the vector representing the inputs used by unit i and y_i is the vector representing the products generated by that unit. X and Y are matrices whose rows are the input and output vectors of each of the units involved in the evaluation.⁷

This model focuses on products, as it specifies how much the production of each could be proportionally increased, given the quantities of inputs used by the various units.

Returns to Scale

Another limitation of this formulation is that it establishes constant returns to scale for the units being assessed. However, in most cases one would expect losses or gains in scale, depending on the activity in which each unit is involved. To overcome this limitation, a new version of the problem was formulated, called the BCC model, which takes variable returns to scale into account. Thus, the BCC model adds a new choice variable, ϕ_0 , which is added to the objective function. This variable can be positive or negative, and captures the possible gains (or losses) in scale of each unit. It is also chosen in the most favorable way for each unit.

Mathematically, this model can be described as follows:

$$\left\{ \begin{array}{l} \text{Min}_{\{\phi_i, \lambda_i\}} : \phi_i x_i - \phi_0 \\ \text{s.t.} : \lambda_i y_i = 1 \\ \phi_i X - \lambda_i Y - \phi_0 e \geq 0 \\ \phi_i, \lambda_i \geq 0 \end{array} \right. \quad (\text{BCC}_i)$$

In this case, e is the unit vector with dimension I .

An interesting result that arises from comparing the solutions obtained from the BCC and CCR models is the inefficiency due to the scale of production. One can show that the value of the objective function π_i will always be less than or equal to the value of the objective function θ_i [see Cooper, Seiford, and Tone (2006), p. 88]. Thus, the ratio π_i/θ_i will show the proportion of inefficiency arising from the unit's production scale. This analysis allows us, on the one hand, to determine how much inefficiency is due to the

⁷ Therefore, X is an $I \times M$ matrix and Y is an $I \times N$ matrix.

size of the unit, and how much is due to administration and management problems. On the other hand, it allows us to draw conclusions about the optimum size of the units.

Analysis of the Weights

The analysis of the weights assigned to each input and product using the DEA methodology can be informative, allowing us to detect the strengths and weaknesses of each unit. This is because, since the DEA method chooses the most favorable weights for each unit, the selection of a low weight for, say, product 1 means that the unit is comparatively more productive in the generation of other products.

Therefore, analysis of the weights may indicate priority areas of action, especially in cases where the generation of a more homogeneous overall product is desirable.

Variations in Productivity

The DEA models can also be used to measure the differences in productivity of each unit during two different periods. In order to do this, a data set containing information on the units during each of these periods is necessary.

Suppose that X^s and Y^s are matrices whose columns contain, respectively, the records of the inputs used and products generated by the various units during the (first) period s , and X^t and Y^t are the vectors corresponding to the (second) period t . One can calculate the set of most favorable weights for each unit for each period using the DEA model with constant returns to scale (CCR).

Thus, ϕ_i^s and λ_i^s are the maximizing weights for unit i during period s and ϕ_i^t and λ_i^t are the corresponding weights for period t . If x_i^s , x_i^t , y_i^s and y_i^t express the quantities of inputs used and products generated during each period, then the measure of unit productivity variation will be given by the following multiplier:

$$mvp = \sqrt{\frac{\lambda_i^s y_i^t}{\phi_i^s x_i^t} \times \frac{\lambda_i^t y_i^s}{\phi_i^t x_i^s}} \quad (1)$$

The idea here is to apply the maximizing weights for the unit in period s to the quantities in period t and vice versa, in order to then obtain a geometric mean of the quantities generated. If there has been an increase in productivity, mvp will be greater than 1, whereas if productivity has suffered, mvp will be less than 1. Thus, an mvp of 1.035 would mean an increase in productivity of 3.5 percent, while a value of 0.970 would mean a decrease in productivity of 3 percent.

Goal Setting

The DEA model also lends itself very well to setting targets for improved efficiency for the units studied. The simplest way to define goals is to measure the level of efficiency of each unit and establish a specific product vector as a goal. This goal vector is proportional to the observed vector and makes the unit 100 percent efficient. Here we describe the formulation of goals for products. Goals for inputs could be defined similarly. In this case, instead of an expansion of the products, we would seek a

proportionate reduction in the amount of inputs used in order to make the unit fully efficient.

However, this type of goal definition could cause serious problems when the whole system does not work properly. The DEA methodology establishes a ranking of relative efficiencies. Thus, if all units in the sample have low productivity, units with the least unfavorable performance will be used as the basis to define maximum efficiency. This does not mean, obviously, that these units cannot or should not improve their productivity, just that the others are even worse.

An alternative way to define goals is to set an overall efficiency goal for the system as a whole and establish objectives to increase the productivity of each unit, consistent with the overall goal.⁸ The individual incremental goals should take into consideration the size of each unit, in terms of production.

Another criterion suggested here is that the increases stipulated for each unit be inversely proportional to their calculated efficiency scores. So, a less efficient unit would not have to catch up to the most efficient units immediately, but it would need to have more significant improvements in order to reduce the gap.

In practice, the calculation of these individual goals requires the efficiency vector previously calculated using the DEA method, eff , which represents the participation of each unit in generating the product, wp ($\sum_i wp_i = 1$) and an overall goal, tgt .

From these parameters one can calculate a vector of multipliers, $mult$, which will contain the improvement goal for each unit, in percentage terms, that will allow the system as a whole to reach the global target (tgt), and the improvement of each unit will be inversely proportional to its level of efficiency at the moment. Mathematically:

$$mult = \frac{tgt}{eff \times wp} \quad (2)$$

From this multiplier, one can obtain the degree of improvement required of each unit, in accordance with the following formula:

$$improv = 1 + mult \times eff \quad (3)$$

And finally, to calculate the targets for each unit, simply multiply each element of the vector $improv$ by the corresponding row of the matrix Y in order to obtain the projected values for each product.

$$metas = improv \times Y \quad (4)$$

The vector $metas$ is dependent on the overall goal, tgt , which is a number expressing how the system as a whole must evolve.

⁸ This methodology would be consistent, for example, with a national goal to improve teaching based on international standards.

Choice of Variables for the Efficiency Analysis

The DEA methodology is quite sensitive to the problem of a limited number of degrees of freedom, defined as the difference between the number of records and the number of variables. Given the state-level nature of the study, the number of annual observations is reduced to 27. So, in order to ensure the effective use of DEA under these circumstances, an alternative is to minimize the number of variables, maximizing the degrees of freedom.

Thus, the variables selected should address the problem in the most representative way possible, so that we can reconcile the need to use fewer variables with the need to represent the main aspects of the problem.

In the education sector, there are variables that express the results of government actions. They are the standardized tests that the Ministry of Education administers periodically and that allow comparison of effectiveness and impact.

Education Variables

For the educational system efficiency analysis, three result variables, one input variable, and one non-discretionary variable were chosen. The input variable chosen was State Education Spending (DEE), which is composed of state spending for the education category less spending in the higher education sub-category.

Expenses related to higher education were excluded because, unlike for primary and secondary education, there are no consistent outcome variables for assessing university performance in Brazil.

On the other hand, expenditure on professional education, education for youth and adults, and special education are included in the DEE variable, since many of the graduates of these programs take the ENEM (*Exame Nacional do Ensino Médio*, a national exam taken at the end of high school studies) and the *Prova Brasil* (a national exam taken at the end of 5th and 9th grades). Spending on pre-school education was also included, since this program represents the educational base and certainly impacts students' future performance.

State expenditures were corrected for inflation using the implicit GDP price deflator. The deflator was obtained from the site of the Brazilian Institute of Geography and Statistics (IBGE),⁹ whereas state spending figures were obtained from the site of the Secretary of the Treasury (STN).¹⁰

The outcome variables selected relate to the scores of state high school students on the *Prova Brasil* and on the ENEM.

The *Prova Brasil*, officially named the National Assessment of Educational Achievement (ANRESC), is administered to all students in the 5th and 9th grades in

⁹ <http://www.ibge.gov.br/home/download/estatistica.shtm>.

¹⁰ http://www3.tesouro.fazenda.gov.br/series_temporais/principal.aspx?subtema=15#ancora_consulta.

state, municipal and federal public schools in which there are at least 20 students enrolled for that grade.

For the purposes of this study, scores will be calculated for each state separately for students in the 5th and 9th grades, giving rise to two outcome variables, PB5 and PB9. In order to analyze the efficiency of scale in the provision of educational services, the state average was not used for these variables, but rather the sum of the individual scores of the students enrolled in state public schools. As there are two exams, Portuguese and mathematics, the sum of the scores was calculated based on the simple average of the individual scores on the two exams.

The more students who take the test and the higher their grades, the greater these variables; thus, in terms of efficiency, there is a positive relationship between expenditure on education, DEE, and the quantity and quality of the results obtained.

The output variable obtained from the ENEM (ENM) follows the same principle: the individual scores of students from state schools are added together. The summation was also based on the individual average of each student on the various exam sections.¹¹

For ENEM scores, this variable calculation strategy may be even more appropriate, since it is a competitive exam that serves as an acceptance criterion for many institutions of higher education, as well as to obtain university scholarships from the federal government.

The data obtained covered two years, 2009 and 2011, and all states plus the federal district. The choice of dates was due to the periodicity of the *Prova Brasil*, which is conducted every two years. The PB5, PB9, and ENM variables were compiled from data obtained from INEP's site.¹²

Finally, a socio-economic variable was used to scale the performance of the states. This variable is justified by the fact that there is a high correlation between family socio-economic status and students' scholastic achievement.

The variable chosen for this task was the state's GDP, obtained from the IBGE site, and deflated by the implicit GDP deflator. There was a problem with the state GDP: the lack of data for 2011. The solution to this limitation was to use the values for the preceding year, that is, while all other variables refer to the years 2009 and 2011, the state GDP figures refer to 2008 and 2010.

Results

As mentioned above, the input variables chosen for the analysis of state efficiency in the education sector were total state spending on education (2011), and the state's GDP (2010), which functioned as non-discretionary variable.

¹¹ The state averages used in this study, both on the Prova Brasil and on the ENEM, differ slightly from the aggregation used by INEP (National Institute for Educational Studies and Research), since our figures include the scores of students from all state schools, not just those from schools with more than 50 students.

¹² <http://portal.inep.gov.br/basica-levantamentos-acessar>.

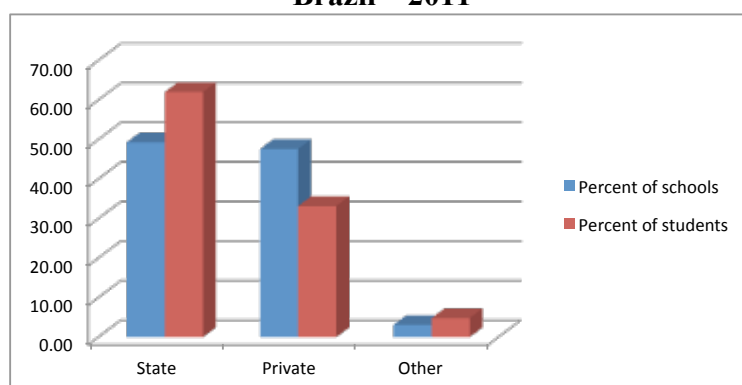
In relation to products, three variables were used: the sum of the scores of students enrolled at state secondary schools on the ENEM exam (2011), the sum of the scores of 5th grade students enrolled in state primary schools on the *Prova Brasil* (2011) and the sum of the scores of 9th grade students enrolled in state secondary schools on the *Prova Brasil* (2011).

Statistics on the Performance of State Schools

Figure 4 shows the percentage of schools and students that took the ENEM, by type of school. The blue columns represent the percentage of schools that participated, while the red columns correspond to the percentage of students who participated.

The “other” columns represent federal and municipal schools, which are not very representative. The number of state schools is practically the same as the number of private schools, with a slightly greater number of the former.

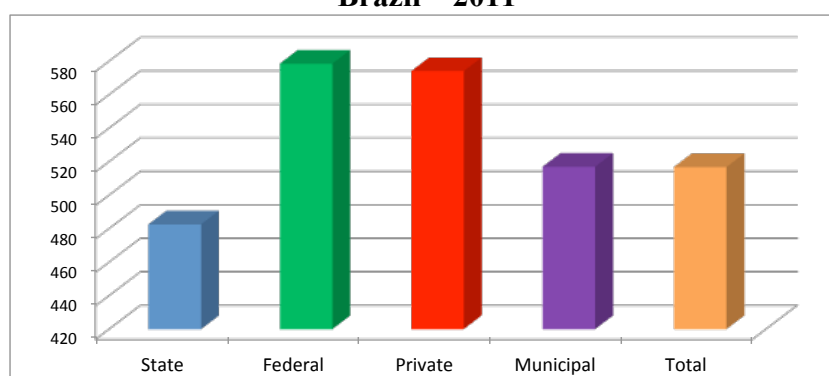
Figure 4
Percent of Schools and Students that Took the ENEM by Type of School
Brazil – 2011



Source: INEP

When one looks, however, at the number of students, it is remarkable how the state systems stand out. The direct implication is that state schools enroll a larger number of students per educational institution than schools in the private network. This is probably reflected in test scores, as shown in Figure 5.

Figure 5
Average Scores of Students on the ENEM by Type of School
Brazil – 2011

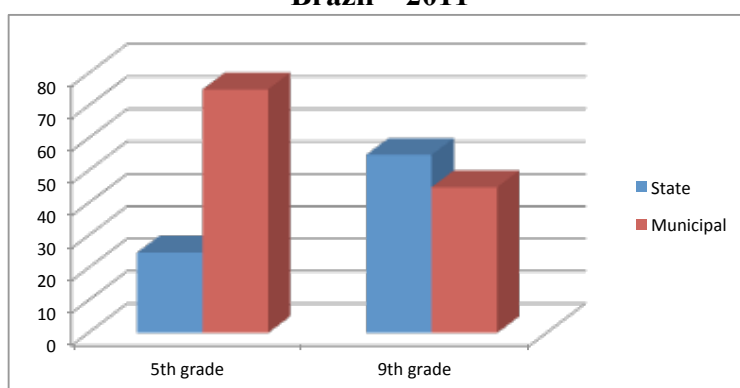


Source: INEP.

Here one can see that state schools are the worst performers. Another interesting fact observed in the figure is the average performance of federal institutions, which surpasses even the average of private schools. This strong performance might be explained by the link that most of these federal schools have with federal universities.

With respect to the *Prova Brasil*, note that only public institutions participate. In this case, mostly state and municipal school students take it, as can be seen in Figure 6. This fact may also lead to the impression that state schools do better on the *Prova Brasil* than on the ENEM. Although this is partly true, the fact that private schools do not participate in the *Prova Brasil* results in lower standards on this exam.

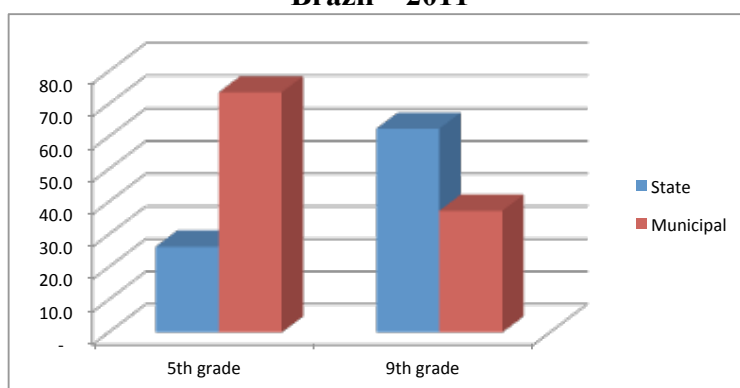
Figure 6
Percentage of Schools Taking the *Prova Brasil* by Type of School and Grade
Brazil – 2011



Source: INEP.

Figure 6 does not present the results for the federal schools participating in the *Prova Brasil* because there are very few. In this figure one can see that municipal school students predominate on the 5th grade test, whereas state school students predominate on the 9th grade test.

Figure 7
Percentage of Students Taking the *Prova Brasil* by Type of School and Grade
Brazil – 2011



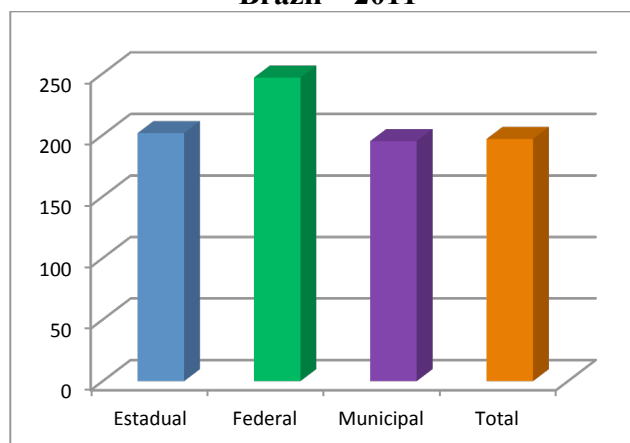
Source: INEP.

Figure 7 tells a similar story about the percentage of students taking the exam. However, when observed closely and contrasted with Figure 6, one can see that the number of state school students taking the exam in the 9th grade is greater than the

number of municipal school students, once again indicating the relative overcrowding of state schools.

Despite this fact, the performance of state schools is near the average on both the 5th grade exam (Figure 8) and the 9th grade exam.

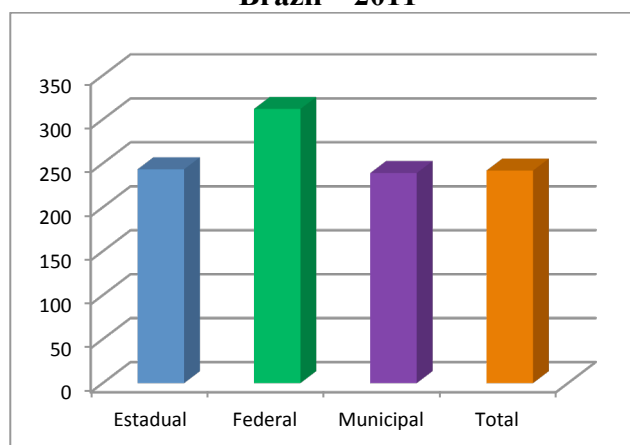
Figure 8
Average Student Scores on the 5th grade *Prova Brasil* by Type of School
Brazil – 2011



Source: INEP.

In the two cases, once again, the federal schools stood out, and once again this superior performance can be explained by their connection to federal institutions of higher education. There are serious doubts, however, about whether this model could be expanded, since its strong point seems to be operating on a small scale.

Figure 9
Average Student Scores on the 9th grade *Prova Brasil* by Type of School
Brazil – 2011



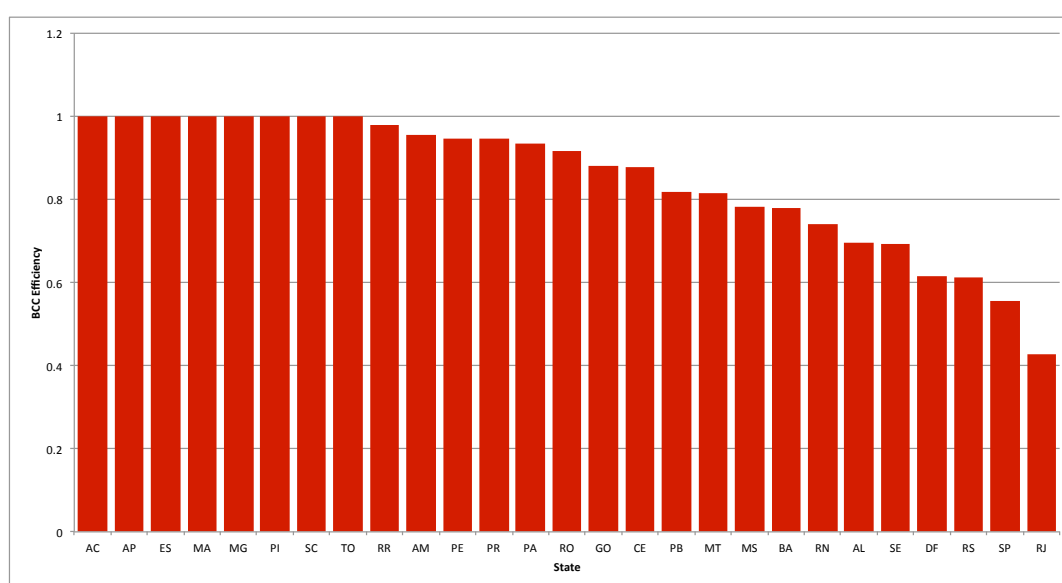
Source: INEP.

State Educational Efficiency

Figure 10 presents state educational system efficiency scores. Some comments can aid the interpretation of this figure. The first is that scores vary from zero to one, and the higher the score, the more efficient the educational system of the state in question compared to other states.

Another important aspect to highlight is that if one state has a higher efficiency score than another, this does not mean that the educational system of the former is better than that of the latter, only that the former is obtaining better results in relation to how much it spends on education.

Figure 10
Efficiency Scores of Brazilian States in Education:
Model without Returns to Scale from the Product Perspective



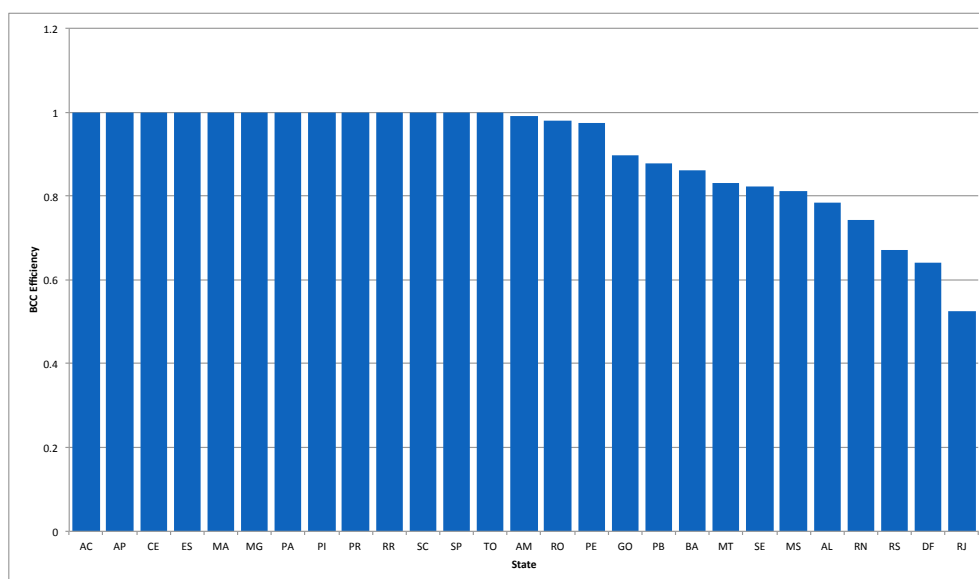
Source: Authors' elaboration.

In Figure 10, the states with a score of 1 are the most efficient compared to the other states in terms of obtaining the best results given the resources being allocated to education. These scores change when the model with variable returns to scale is introduced, but some results remain the same.

As would be expected, the introduction of variable returns to scale allows more states to be considered relatively efficient. However, some states that already had low scores in the model without returns to scale, including the Federal District, Rio de Janeiro, and Paraná, still had low scores when the variable returns to scale were inserted into the model.¹³

¹³ See CCR and BCC efficiency scores, by state, in the table in the appendix.

Figure 11
Efficiency Scores of Brazilian States in Education:
Model with Returns to Scale from the Product Perspective

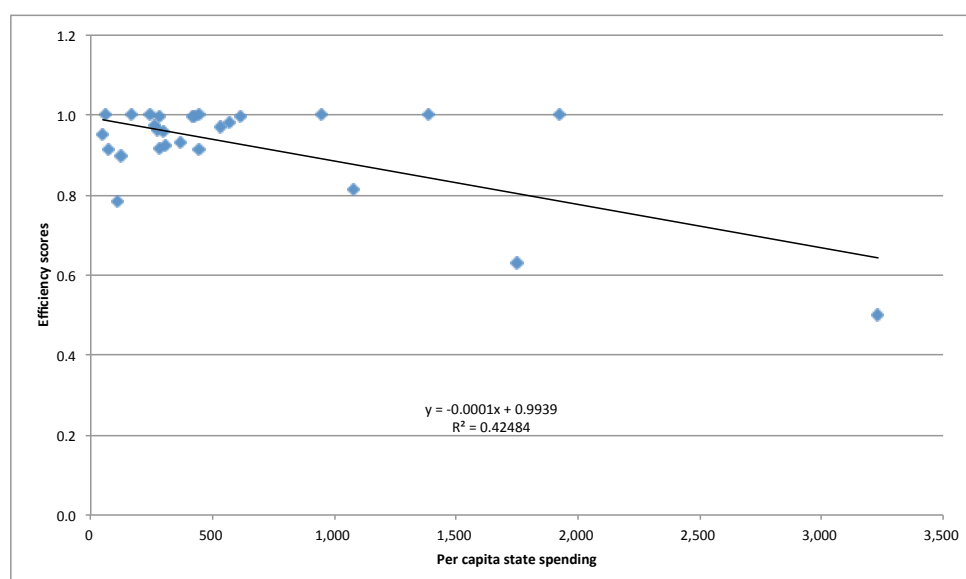


Source: Authors' elaboration.

What appears to be occurring here is a negative correlation between per capita state spending and the relative efficiency of the state educational system. In other words, greater per capita spending appears to be correlated with poorer resource management.

In fact, this phenomenon can be inferred from Figure 12, in which the per capita spending of Brazilian states is plotted with respect to the efficiency scores of their educational systems.

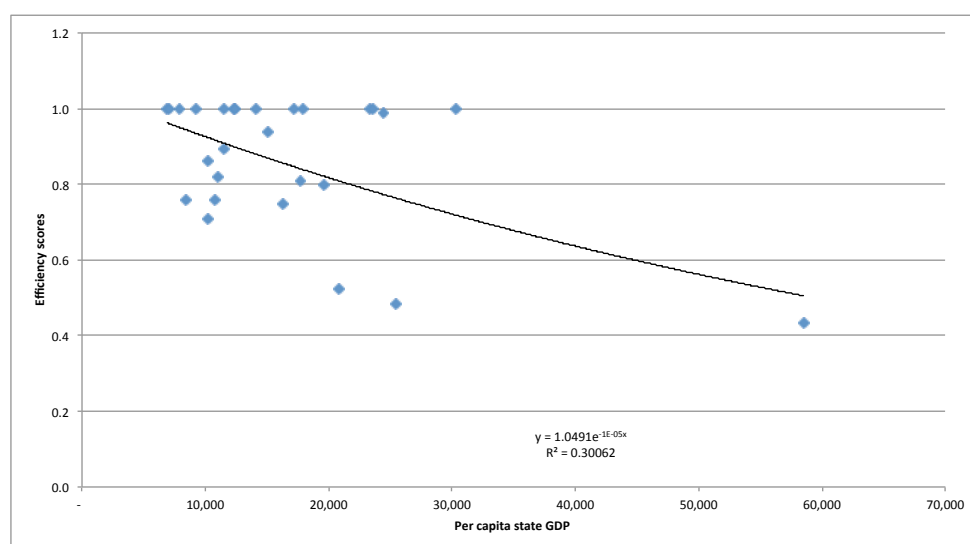
Figure 12
Correlation between the Efficiency Scores of Brazilian States in Education
and their Per Capita Spending on Education



Source: STN.

This negative correlation is also seen when efficiency scores are plotted against the state per capita GDP, as in Figure 13.

Figure 13
Correlation between the Efficiency Scores of Brazilian States in Education and
their Per Capita GDP



Source: STN.

This correlation coefficient is -0.495 for the relation between CCR efficiency and state per capita GDP, and -0.456 for the relation between CCR efficiency and spending on education.

These results indicate a chronic scale problem. There seems to be great difficulty, above a certain spending level, in converting greater expenditures into proportionately better results.

The expected policy prescription, based on these results, would be to create educational resource allocation mechanisms with more beneficial results than simply spending more money. These mechanisms could include planning redistribution of FUNDEB resources in order to benefit the poorest states.

Analysis of Product Weights

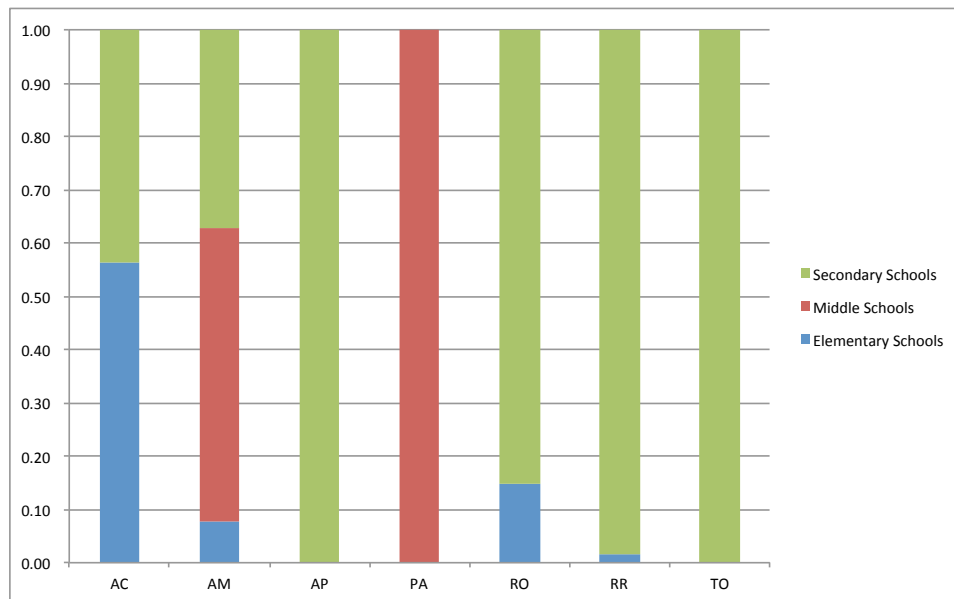
The analysis of the weights that the DEA method attributes to each product, by state, allows assessment of the weaknesses of each state educational systems. The fact that the methodology assigns low or zero weights for some of the products studied means that those states' products are relatively deficient.

Since, in education, there is no point in having one good sub-category while the others are poor, the presence of low or zero scores indicate the products (elementary, middle school or secondary education) that need more attention.

Note that normalized weights must be analyzed so that the differences in the measurement scales of the products do not influence the results. Figure 14 shows the normalized weights for the states in the North of Brazil.

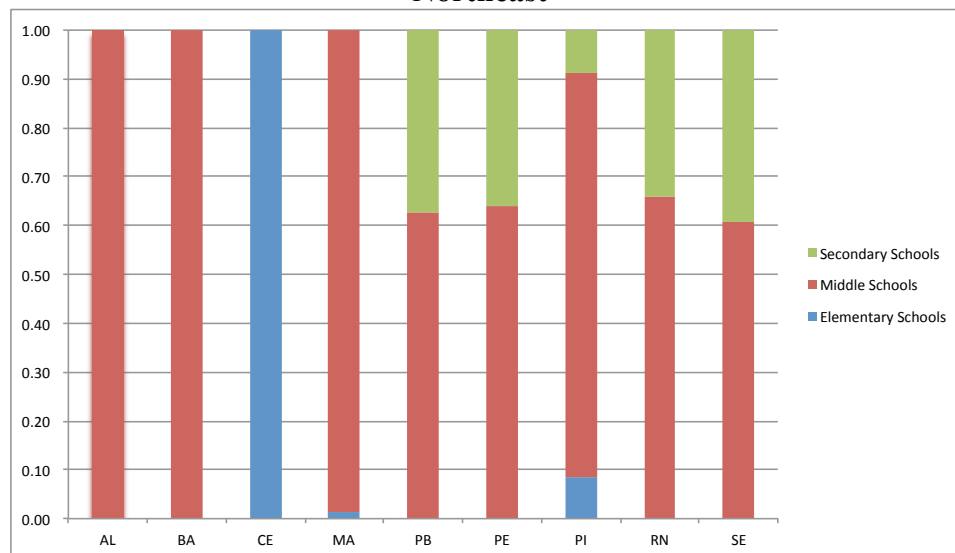
The pattern shows that most of the states in this region emphasize secondary school and, to a certain extent, neglect the other levels. The exception is Amazonas, whose educational structure shows greater balance between the product weights, indicating that none of the levels is relatively undeveloped. Figures 15 to 18 present the corresponding weights for states in the other regions.

Figure 14
Weights of Educational Products Analyzed, by State
North



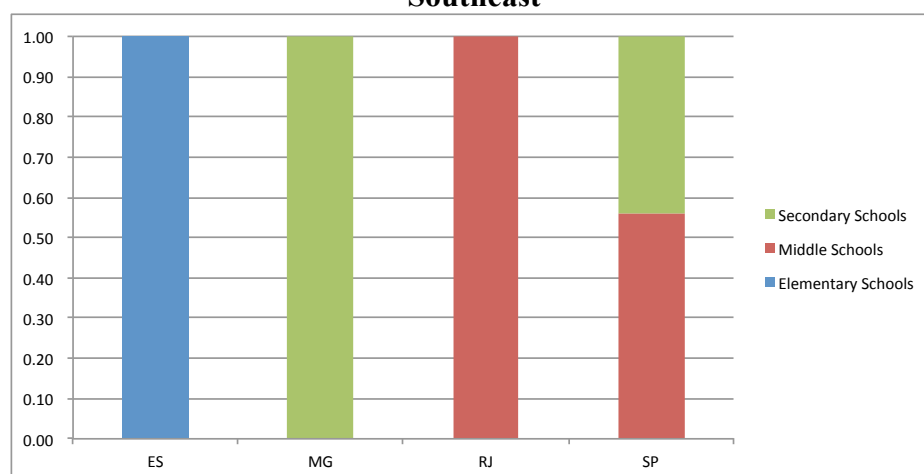
Source: Authors' elaboration.

Figure 15
Weights of Educational Products Analyzed, by State
Northeast



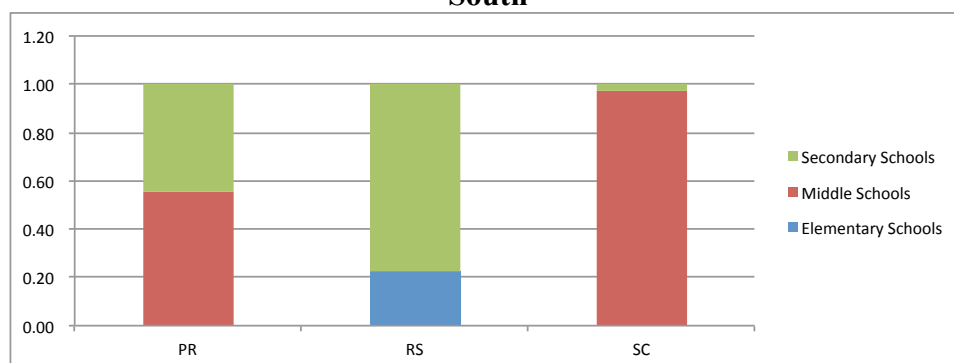
Source: Authors' elaboration.

Figure 16
Weights of Educational Products Analyzed, by State
Southeast



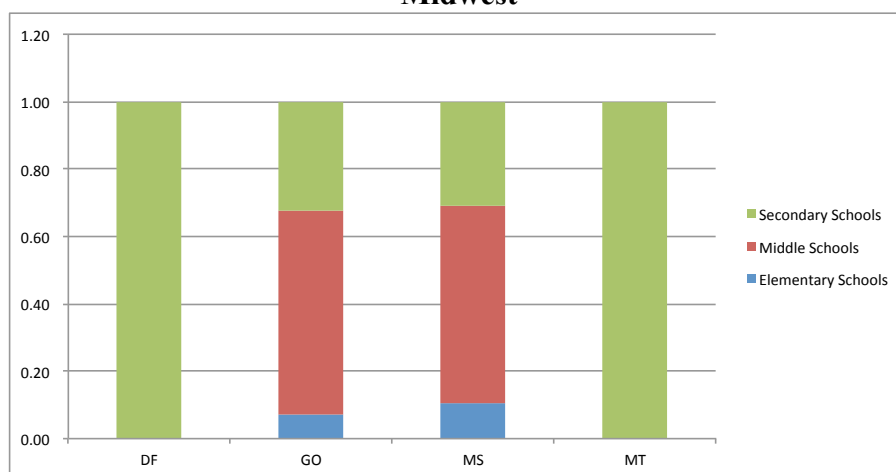
Source: Authors' elaboration.

Figure 17
Weights of Educational Products Analyzed, by State
South



Source: Authors' elaboration.

Figure 18
Weights of Educational Products Analyzed, by State
Midwest



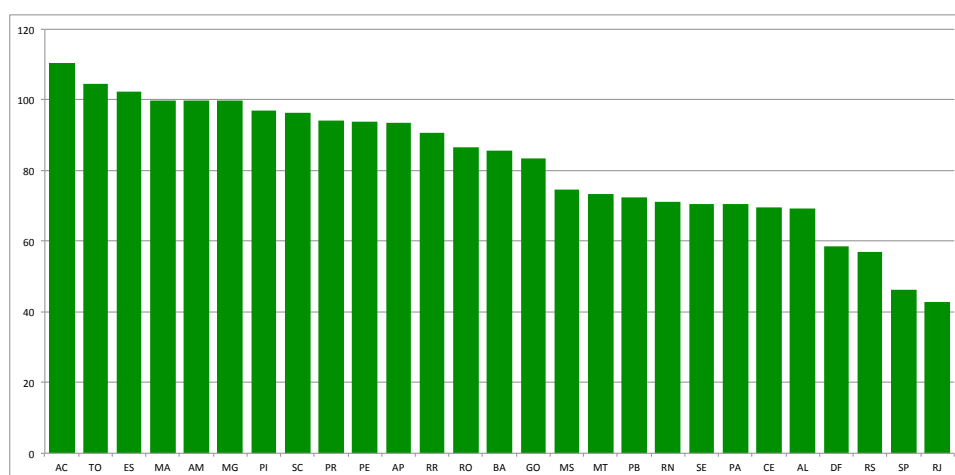
Source: Authors' elaboration.

Measurement of Improvements in State Productivity

The analysis carried out in the section Calculation of Efficiency was repeated for the 2009 data. Then, the CCR weight vectors for each state were calculated for 2009, in the same way as for 2011.

The next step was to apply formula (1), with $s = 2009$ and $t = 2011$. The increase in productivity is shown in Figure 19. A dotted line passes through the index 100. States with an index over 100 had gains in productivity, while those with indices below 100 suffered losses of productivity between 2009 and 2011.

Figure 19
Educational Productivity Gains of Brazilian States
2009 to 2011



Source: Authors' elaboration.

As can be seen in the figure, only three states increased their productivity: Acre (increased 10.3 percent), Tocantins (4.5 percent), and Espírito Santo (2.3 percent), while three others (Maranhão, Amazonas, and Minas Gerais) maintained a reasonably constant productivity. The other 21 states suffered losses in productivity, with the four richest at the bottom of this ranking.¹⁴

It is important to note that an increase in productivity can result from various factors, including the generation of more products. Since the products in this study are dictated by variables that take into account both the average score of students on various standardized tests and the number of students who take these tests, states that seek greater productivity can either improve teaching or have more students in the region take these exams.

Another relevant fact derived from Figure 19 is that increases in productivity are negatively correlated with the level of state spending on education (-0.475). This observation reinforces the argument that improving educational management and the

¹⁴ Complete information on state indices of productivity is provided in the table in Appendix I.

redistribution of educational resources from the richer states to the poorer ones could have powerful effects on the system as a whole.

Formulation of State Goals

As described above, the DEA methodology can be used to formulate state goals. In this section, simulations of goals customized for each state were performed after setting overall goals for the Brazilian state educational system.

These simulations allow us to compare the average scores actually obtained by students in state schools with those that would be necessary to reach the stipulated overall goal. However, first the methodology described in the section Goal Setting must be applied. Note that the methodology used to reach the goals respects two basic criteria:

1. Each state should aim for score increases on the three exams inversely proportional to its CCR efficiency, calculated using the DEA methodology;
2. The weighted average¹⁵ of the increase in state averages should equal the pre-established overall goal.

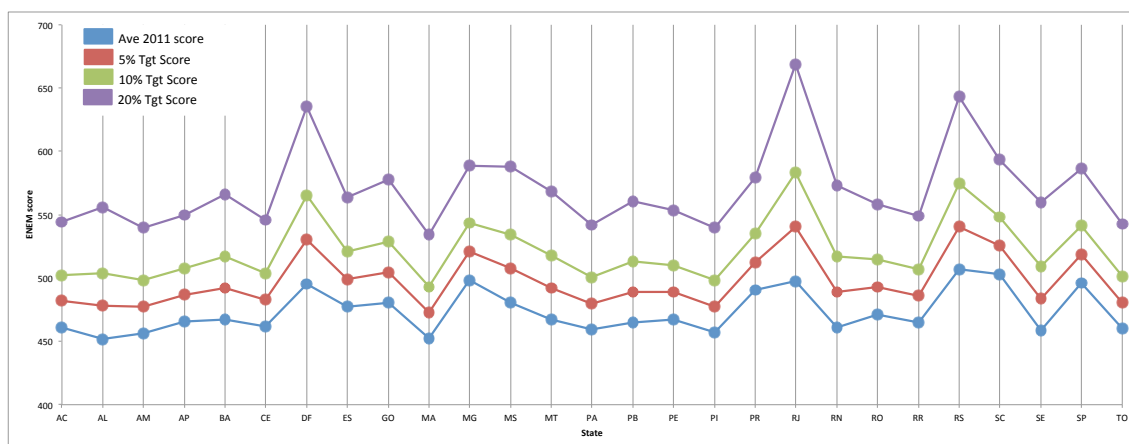
The first criterion ensures that the worst-off states, in terms of efficiency, are compelled to improve the most, while the second requires just consistency.

Figure 20 shows the situation of the states in 2011 with respect to ENEM scores, as well as the scores that would represent weighted increases of 5, 10 and 20 percent in the overall ENEM score. Thus, looking at the situation in Acre (AC) as an example, one can see (blue line) that the average ENEM score of students in that state in 2011 was 460.9.¹⁶ Therefore, its goal, to reach the overall ENEM goal of 20 percent improvement, would be 544.3. Note that this would mean an increase of less than 20 percent in the average score of students in Acre (18.1 percent, to be exact). This is because the level of efficiency in Acre was above the national average in 2011, and therefore its improvement could be less than proportional to the overall percent improvement.

¹⁵ In this analysis, the weighting variable used was the number of children taking the Prova Brasil in 2011.

¹⁶ The scores and goals, both for the ENEM and for the two *Prova Brasil* levels, can be found in the tables in the appendix.

Figure 20
Average ENEM Scores by State in 2011 and Goals for an Overall Improvement of 5, 10, and 20 Percent



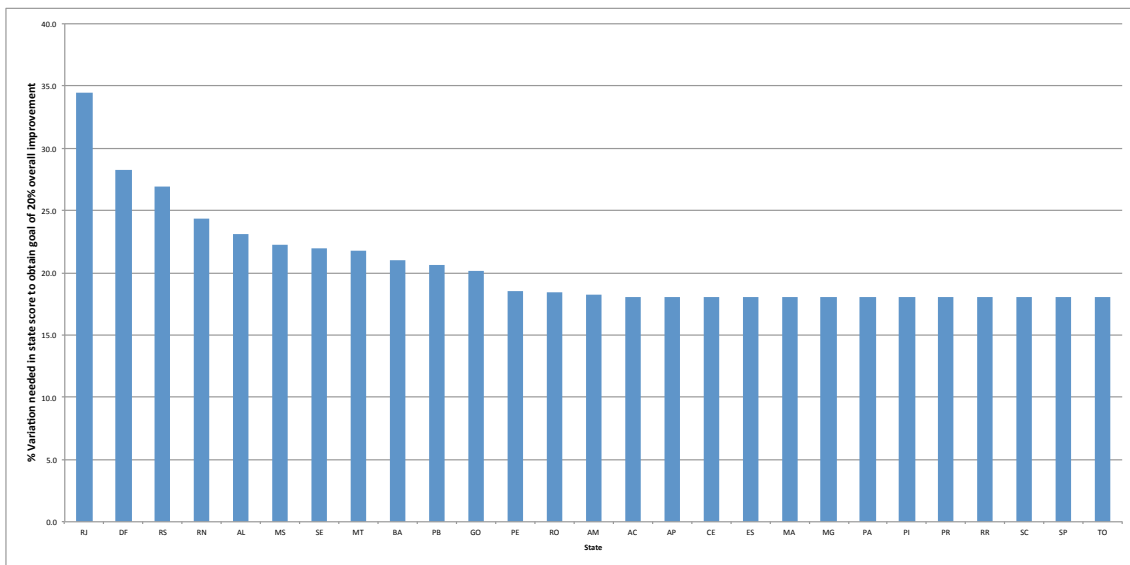
Source: Authors' elaboration.

As can be seen in Figure 20, the scores of all states would need to improve in order to reach the goals. But some states, especially the richest, would have to improve more than others due to their measured high level of inefficiency. The improvement in efficiency required of each state in order to reach a 20 percent overall improvement is shown in Figure 21.

States such as Rio de Janeiro, the Federal District, and Rio Grande do Sul would need to obtain significantly better than average improvement in their ENEM scores (34.4 percent, 28.2 percent and 26.9 percent, respectively), while others would need to improve their averages by less than 20 percent in order to reach the goal (see the position of the states in the figure, from Pernambuco to the right).

Note that this does not mean that the former group has lower averages than those in the second group; rather, given the available resources, the scores of the former are relatively further from their potential.

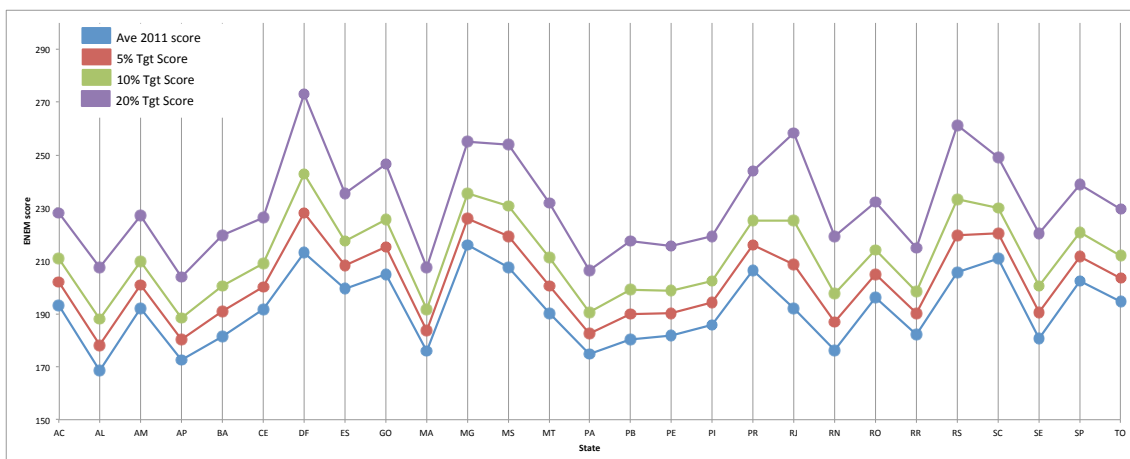
Figure 21
Target Grade for each State to Obtain a General 20 Percent Improvement



Source: Authors' elaboration.

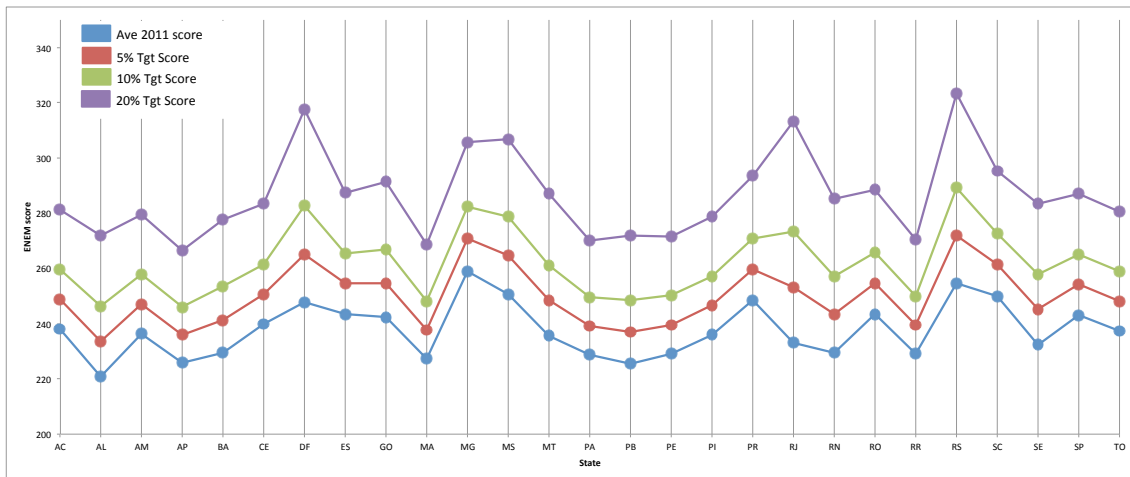
Figures 22 and 23 show the average state scores on the 5th and 9th grade *Prova Brasil*, respectively, as well as the desired scores to obtain national score increases of 5, 10, and 20 percent.

Figure 22
Average State Scores on the 5th grade *Prova Brasil* in 2011 and Goals for an Overall Improvement of 5, 10, and 20 Percent



Source: Authors' elaboration.

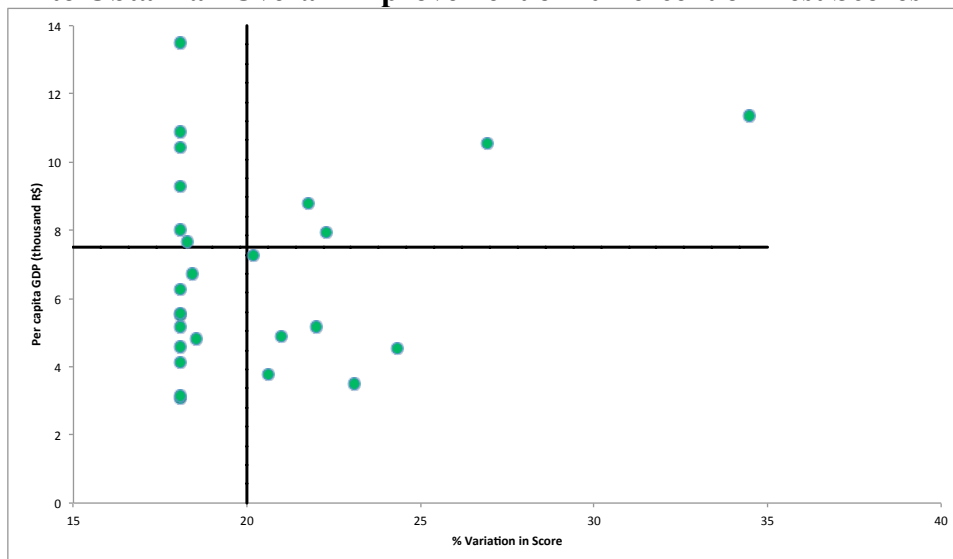
Figure 23
Average State Scores on the 9th grade *Prova Brasil* in 2011
and Goals for an Overall Improvement of 5, 10, and 20 Percent



Source: Authors' elaboration.

Given per capita income and the improvement in average exam scores needed to reach the overall goal, these two variables can be plotted and the resulting graph can be divided into four quadrants, as in Figure 24.

Figure 24
Relationship between the State Per Capita GDP and the Need for Improvement
to Obtain an Overall Improvement of 20 Percent on Test Scores



Source: Authors' elaboration.

Based on the division into quadrants, the state public education systems can be divided into four categories. The first contains the states located in quadrant I, (DF, 17 MS, MT, RJ, RS). These states had an above-average per capita GDP (over R\$7,500)¹⁸ and need to improve average scores more than the desired overall improvement of 20 percent. In these states, in particular, an increase in expenditure on education would be a waste, since the additional resources already available to these states have not resulted in sufficiently greater scores.

Quadrant II is composed of states with a below-average per capita GDP and which need to improve average scores more than the average (AL, BA, GO, PB, RN and SE). These cases are the most problematic, since educational management is relatively poor and there are not enough financial resources to improve educational conditions. Here both greater funding (federal or redistributed) and programs to improve educational management would be necessary.

Quadrant III contains relatively poor states, in per capita GDP terms, but which require improvements in test scores below the overall goal of 20 percent. This quadrant includes AC, AP, CE, MA, PA, PE, PI, RO, RR, and TO. The political prescription in this situation would involve greater investment of financial resources in education, since the quality of management in these states is relatively superior.

Finally, in quadrant IV, which encompasses AM, ES, MG, PR, SC, and SP, the situation is more comfortable, as they have an above-average per capita state GDP and the quality of their educational management is also relatively high.

It is important to highlight that the quadrant analysis above is relative, in terms of both the management of the state educational process and the resources needed to improve teaching conditions. This approach allows us to visualize priorities, as one cannot deny that even the richer states must spend more on education, nor that even those with better quality management could improve their management significantly.

Conclusions

This study uses the DEA method to evaluate the efficiency of state public education systems in Brazil. State government spending on education was used as an input variable and state gross domestic product was interpreted as a non-discretionary input variable.

Output variables are based on the number of state public school students taking the ENEM and the *Prova Brasil* and their scores on these exams. The latter is taken at two different times, during the 5th grade and during the 9th grade.

To calculate the product variables, the scores of the students taking the three exams were summed for each state school system. This sought to capture not only the

¹⁷ The data point corresponding to the Federal District does not appear on the graph due to the scale.

¹⁸ Figures from 2010.

qualitative aspect (scores), but also the quantitative aspect, namely the number of students taking the test. Thus, higher scores and more students taking the exam increase the product variables. This approach allows direct assessment of the economies of scale in education as a product, which would be impossible through traditional per capita treatment, given that, in that case, all states would be measured on the same scale.

The principal finding of the study was the detection of a per capita limit on educational spending beyond which the technical efficiency of educational production fell substantially. This result suggests that improvements in the management and redistribution of existing resources should precede the expenditure of more resources on education. However hard-hearted this statement may seem, the efficiency scores indicate a highly positive correlation between per capita spending on education and the inefficiency of educational systems, meaning that additional resources would result in even greater waste if management problems are not fixed beforehand.

Bibliography

- Afonso, A. and S. Fernandes. 2006. "Local Government Spending Efficiency: DEA Evidence for Lisbon Region." *Regional Studies*, 40: 39–53.
- Afonso, A., L. Schuknecht, and V. Tanzi. 2006. "Public Sector Efficiency: Evidence for New EU Member States and Emerging Markets." European Central Bank Working Paper. No. 581. Frankfurt, Germany: European Central Bank.
- Afonso, A., A. Romero, and E. Monsalve. 2013. "Public Sector Efficiency: Evidence for Latin America." IDB Discussion Paper. IDB-DP-279. Washington, DC: Inter-American Development Bank.
- Alexander, W. R. J. and M. Jaforullah. 2004. "Explaining Efficiency Differences of New Zealand Secondary Schools." *Economics Discussion Papers N. 0403*, 1–34. Dunedin, New Zealand: University of Otago.
- Anderson, L., H. J. Walberg, and T. Weinstein. 1998. "Efficiency and Effectiveness Analysis of Chicago Public Elementary Schools: 1989, 1991, 1993." *Educational Administration Quarterly* 34(4): 484–504.
- Banker, R. D., A. Charnes, and W. W. Cooper. 1984. "Some Models for Estimating Technical and Scale Inefficiencies in Data Envelopment Analysis." *Management Science*, 30(9): 1078–92.
- Barbetta, G. P. and G. Turati. 2003. "Efficiency of Junior High Schools and the Role of Proprietary Structure." *Annals of Public and Cooperative Economics* 74(4), 529–42.
- Brunet, J. F. G., A. M. A. Bertê, and C. B. Borges. 2008. "Qualidade do gasto público em educação nas redes públicas estaduais e municipais." Brasília, Brazil: ESAF. Monografia agraciada com menção honrosa no XIII Prêmio Tesouro Nacional – 2008. Qualidade do Gasto Público.
- Butler, T. W. and L. Li. 2005. "The Utility of Returns to Scale in DEA Programming: An Analysis of Michigan Rural Hospitals." *European Journal of Operational Research* 161(2): 469–78.
- Caballero, R., T. Galache, T. Gomez, J. Molina, and A. Torrico. 2004. "Budgetary Allocations and Efficiency in the Human Resources Policy of a University Following Multiple Criteria." *Economics of Education Review* 23: 67.
- Chakraborty, K., B. Biswas, and W. C. Lewis. 2001. "Measurement of Technical Efficiency in Public Education: A Stochastic and Nonstochastic Production Function Approach." *Southern Economic Journal* 67: 889–905.
- Chapple, W., A. Lockett, D. Siegel, and M. Wright. 2005. "Assessing the Relative Performance of UK University Technology Transfer Offices: Parametric and Non-parametric Evidence." *Research Policy*, 34(3): 369–84.
- Charnes, A., W. W. Cooper, and E. Rhodes. 1978. "Measuring the Efficiency of Decision Makers Units." *European Journal of Operational Research*, 2(6): 429–44.
- Cherchyea, L., K. De Witte, E. Ooghe, and I. Nicaise. 2010. "Efficiency and Equity in Private and Public Education: A Nonparametric Comparison." *European Journal of Operational Research*, 202: 563–73.

- Coeli, T. J., D. S. P. Rao, C. J. O'Donnell, and G. E. Battese. 2005. *An Introduction to Efficiency and Productivity Analysis*. Second edition. New York, NY: Springer.
- Cooper, W. W., L. M. Seiford, and K. Tone. 2006. *Introduction to Data Envelopment Analysis and Its Uses*. New York, NY: Springer.
- Farrel, M. J. 1957. "The Measurement of Productive Efficiency." *Journal of the Royal Statistical Society, Series A, CXX*, Part 3: 253–90.
- Gasparini, C. E. and C. S. L. Melo. 2004. "Equidade e eficiência municipal: uma avaliação do Fundo de Participação dos Municípios (FPM)." Brasília: Editora Universidade de Brasília.
- Gupta, S. and M. Verhoeven. 2001. "The Efficiency of Government Expenditures: Experiences from Africa." *Journal of Policy Modelling*, 23: 433–67.
- Mancebon, M. J. and C. M. Molinero. 2000. "Performance in Primary Schools." *Journal of the Operational Research Society*, 51: 843–60.
- Marinho, A. 2003. "Avaliação da eficiência técnica nos serviços de saúde nos municípios do estado do Rio de Janeiro." *Revista Brasileira de Economia*, 57: 515–34.
- Ministério da Educação. 2008. "FUNDEB – Manual de Orientação." Brasília, DF.
- Musgrave R. A. 1997. "Reconsidering the Fiscal Role of the Government." *American Economic Review*, 87: 156–59.
- Nicolella, A. and R. Fernandes. 2012. "Indicador de gestão para a educação básica. Apoio ao desenho do indicador." Washington, DC: Inter-American Development Bank.
- Rocha, F. F., A. C. Zoghbi, and E. Mattos. 2013. "Education Production Efficiency: Evidence from Brazilian Universities." *Economic Modelling*, 31: 94–103.
- Sola, M. and D. Prior. 2001. "Measuring Productivity and Quality Changes Using Data Envelopment Analysis: An Application to Catalan Hospitals." *Financial Accountability and Management*, 17: 219–34.
- Souza, I. V., F. F. Rocha, and M. Nishijima 2010. "Eficiência do setor hospitalar nos municípios paulistas." *Economia Aplicada*, 14: 51–66.
- Tanzi, V. and L. Schuknech. 1977. "Reconsidering the Fiscal Role of the Government: The International Perspective." *American Economic Review*, 87: 164–68.
- Thursby, J. G. and S. Kemp. 2002. "Growth and Productive Efficiency of University Intellectual Property Licensing." *Research Policy*, 31(1): 109–124.

Appendix I
Efficiency Scores, Efficiency of Scale, and Variation
in Educational Productivity of Brazilian States

State	CCR efficiency 2011 (%)	BCC efficiency 2011 (%)	Efficiency of scale 2011 (%)	Variation in productivity 2009-11 (%)
AC	100.0	100.0	100.0	10.3
AL	69.4	78.3	88.7	-30.7
AM	95.4	99.0	96.4	-0.2
AP	100.0	100.0	100.0	-6.3
BA	78.0	86.2	90.6	-14.3
CE	87.9	100.0	87.9	-30.4
DF	61.7	64.1	96.2	-41.3
ES	100.0	100.0	100.0	2.3
GO	88.2	89.6	98.5	-16.7
MA	100.0	100.0	100.0	-0.1
MG	100.0	100.0	100.0	-0.3
MS	78.2	81.1	96.4	-25.3
MT	81.5	83.1	98.1	-26.7
PA	93.3	100.0	93.3	-29.6
PB	81.8	87.7	93.2	-27.5
PE	94.7	97.5	97.1	-6.2
PI	100.0	100.0	100.0	-3.0
PR	94.6	100.0	94.6	-5.9
RJ	42.8	52.5	81.6	-57.1
RN	74.1	74.3	99.6	-29.0
RO	91.6	98.0	93.5	-13.4
RR	98.0	100.0	98.0	-9.4
RS	61.4	67.2	91.3	-43.1
SC	100.0	100.0	100.0	-3.6
SE	69.3	82.3	84.3	-29.6
SP	55.5	100.0	55.5	-53.8
TO	100.0	100.0	100.0	4.5

Source: Authors' elaboration.

Appendix II
Average ENEM Scores and Goals for Various Degrees of Improvement

State	Average 2011 score	Score for 5% improvement	Score for 10% improvement	Score for 20% improvement
AC	460.9	481.8	502.6	544.3
AL	451.9	478.0	504.1	556.3
AM	456.5	477.4	498.2	539.9
AP	465.5	486.5	507.5	549.6
BA	467.6	492.2	516.7	565.8
CE	462.1	483.0	503.9	545.7
DF	495.5	530.5	565.4	635.4
ES	477.6	499.2	520.7	563.9
GO	480.6	504.9	529.1	577.6
MA	452.4	472.8	493.3	534.2
MG	498.5	521.0	543.6	588.6
MS	480.7	507.5	534.3	587.9
MT	466.9	492.3	517.7	568.5
PA	459.1	479.9	500.7	542.2
PB	465.1	489.1	513.1	561.0
PE	467.1	488.7	510.4	553.7
PI	456.8	477.4	498.1	539.4
PR	490.3	512.5	534.6	579.0
RJ	497.3	540.1	582.9	668.6
RN	461.2	489.2	517.3	573.4
RO	471.4	493.2	514.9	558.4
RR	465.1	486.1	507.2	549.2
RS	506.6	540.7	574.8	643.0
SC	502.8	525.5	548.2	593.7
SE	458.8	484.0	509.2	559.6
SP	496.2	518.7	541.1	586.0
TO	459.9	480.7	501.5	543.1

Source: Authors' elaboration.

Appendix III
Average Scores on the 5th grade *Prova Brasil* and
Goals for Various Degrees of Improvement

State	Average 2011 score	Score for 5% improvement	Score for 10% improvement	Score for 20% improvement
AC	193.4	202.1	210.9	228.4
AL	168.6	178.3	188.0	207.5
AM	192.1	200.9	209.7	227.2
AP	172.7	180.5	188.3	203.9
BA	181.6	191.1	200.6	219.7
CE	191.6	200.3	209.0	226.3
DF	213.0	228.0	243.0	273.1
ES	199.4	208.4	217.4	235.5
GO	205.1	215.4	225.8	246.5
MA	175.9	183.8	191.8	207.7
MG	216.1	225.9	235.6	255.2
MS	207.8	219.3	230.9	254.1
MT	190.4	200.7	211.1	231.8
PA	174.8	182.7	190.6	206.4
PB	180.5	189.8	199.1	217.7
PE	181.9	190.3	198.7	215.6
PI	185.8	194.2	202.6	219.4
PR	206.6	215.9	225.2	243.9
RJ	192.2	208.7	225.3	258.3
RN	176.3	187.1	197.8	219.2
RO	196.1	205.2	214.2	232.3
RR	182.0	190.2	198.5	214.9
RS	205.8	219.7	233.5	261.2
SC	211.0	220.5	230.1	249.1
SE	180.7	190.6	200.5	220.4
SP	202.4	211.6	220.7	239.0
TO	194.6	203.4	212.2	229.8

Source: Authors' elaboration.

Appendix IV
Average Scores on the 9th grade *Prova Brasil* and
Goals for Various Degrees of Improvement

State	Average 2011 score	Score for 5% improvement	Score for 10% improvement	Score for 20% improvement
AC	238.2	249.0	259.7	281.3
AL	220.8	233.6	246.3	271.8
AM	236.4	247.2	258.0	279.6
AP	225.7	235.9	246.1	266.5
BA	229.4	241.4	253.4	277.5
CE	239.9	250.7	261.6	283.3
DF	247.8	265.2	282.7	317.7
ES	243.5	254.5	265.5	287.5
GO	242.6	254.8	267.0	291.5
MA	227.4	237.7	248.0	268.6
MG	259.0	270.7	282.4	305.8
MS	250.8	264.7	278.7	306.6
MT	235.6	248.4	261.2	286.8
PA	228.8	239.1	249.5	270.2
PB	225.4	237.0	248.6	271.8
PE	229.0	239.6	250.2	271.4
PI	236.0	246.6	257.3	278.6
PR	248.4	259.6	270.9	293.3
RJ	233.0	253.1	273.2	313.3
RN	229.4	243.4	257.3	285.2
RO	243.5	254.7	265.9	288.4
RR	229.2	239.5	249.9	270.6
RS	254.8	272.0	289.1	323.4
SC	250.0	261.4	272.7	295.3
SE	232.4	245.2	257.9	283.5
SP	243.2	254.2	265.2	287.2
TO	237.5	248.2	258.9	280.4

Source: Authors' elaboration.