A Tool for Fiscal Policy Planning in a Medium-Term Fiscal Framework

The FMM-MTFF Model

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Abstract[*]

This paper describes the FMM-MTFF model, a dynamic stochastic general equilibrium model developed by the Fiscal Management Division (FMM) of the Inter-American Development Bank (IDB) to support the implementation of a medium-term fiscal framework (MTFF) in emerging market and developing economies. Relative to existing models, the present model incorporates several non-standard features. First, fiscal policy is defined in terms of multi-year fiscal plans, instead of restricting attention to univariate, single-period fiscal shocks. Second, the model does not impose the straightjacket of a standard fiscal rule. Under a standard fiscal feedback rule, fiscal policy is countercyclical and sustainable by design and any fiscal challenge is mechanically addressed. Third, the model is calibrated to match a three-sector, stylized version of a country's input-output (I-O) table, which provides a consistent framework on industry output, intermediate input flows, and final demand use data. Fourth, the model embeds a more realistic GDP measurement framework, one that is consistent with what national account compilers do. The model uses a chain-linking method to aggregate real GDP. The model is calibrated to Colombian and Peruvian data to illustrate the use of the model as a tool to quantify the scale of the fiscal challenges, provide consistent medium-term macro fiscal projections, and assess the quantitative implications of past reforms and alternative fiscal policy plans on the economies, that is, the typical questions of interest to an MTFF.

JEL Codes: E62, F41, H68, Q33
Keywords: commodity boom, developing countries, fiscal policy, fiscal policy plans, FMM-MTFF, medium-term fiscal framework, natural resources

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1. Introduction

The importance of taking a medium-term approach to fiscal management is generally accepted by specialists and practitioners in the public finance field. Policymakers design fiscal consolidation packages which are typically multi-year programs. The appropriate pace of consolidation efforts (e.g., shock therapy or gradual adjustment) as well as the choice of policy instruments require an understanding of the short- and medium-term consequences of alternative fiscal programs. The challenge of future spending pressures (e.g., pensions and health spending) may require reforms to entitlement programs that take years to roll out and deliver any real impact. All these examples underscore the importance of fiscal policy planning and the need to frame it in a medium-term context.

This paper presents the FMM-MTFF model, a dynamic stochastic general equilibrium (DSGE) model developed by the Fiscal Management Division (FMM) of the Inter-American Development Bank (IDB) to support the implementation of a medium-term fiscal framework (MTFF) in emerging market and developing economies. An MTFF is the basic building block of a medium-term expenditure framework (MTEF), the budget institution designed to strengthen the link among policy, planning, and budgeting over a multi-year horizon.

An MTEF is intended to progressively achieve the following: (i) aggregate fiscal discipline, (ii) allocative efficiency, and (iii) technical efficiency (World Bank, 2013). In the base of the hierarchy of frameworks is the MTFF, which provides a statement of fiscal policy strategies and objectives and sets the quantitative basis of the fiscal plan through a set of medium-term macroeconomic and fiscal targets and projections. The focus of the MTFF is on continued maintenance of aggregate fiscal discipline and debt sustainability. The next level is the medium-term budgetary framework (MTBF), which builds on an MTFF to allocate resources among individual spending agencies based on the nation's strategic priorities and constrained by the overall resource envelope. At the apex of the hierarchy is the medium-term performance framework (MTPF), which, supported by an MTBF, focuses on performance measurement and evaluation of spending programs to enhance technical efficiency in the use of budgeted resources.

The Latin American and Caribbean (LAC) region underwent a wave of MTFF adoptions during the 2000s, and few countries have been able to move beyond the basic MTFF (World Bank, 2013; Kaufmann, Sanginés, and García, 2015). However, the adoption and strengthening of this budget institution has been losing momentum across the region. Indeed, only 10 out of the 32 LAC countries published MTFF documents in 2017 (see Table 1). More importantly, without exception, the analytical tool used by all 10 countries in the making of MTFF documents is a version of the debt sustainability framework (DSF) developed by the International Monetary Fund (IMF) and World Bank. This rests on a debt accumulation identity from which the path of government debt is obtained by assuming exogenous trajectories for the economic variables determining fiscal deficits. As the assumed path of the economy is independent from fiscal policy, decision makers can speculate about the fiscal transmission mechanism relating policy and macroeconomic performance and overlook possible tradeoffs and risks involved in alternative fiscal plans. This approach to fiscal policy analysis does not extend a country's understanding of its fiscal position nor does it provide an internally consistent quantitative assessment of the scale and scope of the fiscal challenge, the main analytical and quantitative benefits associated with the adoption of an MTFF. As posited by Leeper (2017: 28), "policymakers need *economic analysis* rather than accounting exercises."

Nowadays, DSGE models have become a standard tool for quantitative policy analysis. They should be part of a "suite of models" approach by which policymakers rely on multiple models for multiple purposes to examine the robustness of fiscal projections. Still, Del Negro and Schorfheide (2003) argue that policymakers rarely use DSGE models for quantitative policy assessment. DSGE models make predictions for the behavior of (usually Hodrick-Prescott) filtered series or moments of filtered series, and policies are defined in terms of shocks to a policy rule or innovations to an exogenously given filtered process. None of these magnitudes are readily translated by practitioners into actual observable data or implications for specific problems and decisions. Practitioners may also be interested in specific scenarios, due to information available at the point that the decision is to be made or market knowledge, rather than the forecast distribution of outcomes produced by DSGE models. DSGE models typically characterize fiscal policy as shocks, and the response of the economy is assessed with tools designed to investigate the interaction between variables (impulse-response functions, expenditure and revenue fiscal multipliers, forecast error variance decompositions, historical decomposition of times series).[1] A more suitable experiment for a policymaker interested in building up a medium-term macro fiscal scenario is the simulation of the response of the economy not to a fiscal shock but to a fiscal plan,

[1] The IMF has played an important role in extending the DSGE modeling framework to low-income countries (LICs) to address quantitative fiscal issues such as the effect of scaling up public investment on growth or investment surges in resource-abundant economies. By 2015, IMF's DSGE models had been applied to at least 20 LICs (Development Committee, 2015). The number of country models has been increasing since then. Some examples are: Berg et al. (2012); Buffie et al. (2012); Mu (2012); Clark and Rosales (2013); Melina and Xiong (2013); Adam and Bevan (2014); Issoufou et al. (2014); Melina, Yang, and Zanna (2014); Shen, Yang, and Zanna (2015); and Atolia et al. (2017). IMF's DSGE models share with most of the existing literature the reliance on impulse-response functions for policy evaluation.

a multi-year program along which the government implements or foresees to implement a sequence of policy actions.

The FMM-MTFF model has been designed to be readily usable by a set of diverse countries as a laboratory for the analysis of fiscal plans, to reduce adoption and learning costs for policy analysts, and to capture relevant dynamics of the economies. All these motivate crucial modeling choices described in the next section, some of which can be considered the model's weak points. Among the weak points, it is worth mentioning the purely real environment of the model economy and the loose strategy adopted for the calibration of parameters whose values are not very well established empirically. The FMM-MTFF model represents a small non-monetary open economy designed to analyze medium-term fiscal plans. The omission of a monetary sector and nominal rigidities is a reasonable starting point given the focus on a medium-term fiscal perspective. The model incorporates standard features of existing models, typically introduced to generate plausible aggregate dynamics in highly volatile small open economies. Sources of inertia include real frictions such as investment (Christiano, Eichenbaum, and Evans, 2005) and labor (Sargent, 1978) adjustment costs, as well as consumption adjustment costs (habit formation, Abel, 1990) and non-Ricardian households (Galí, López-Salido, and Vallés, 2004, 2007). The model also features intratemporal non-separability (GHH preferences, Greenwood, Hercowitz, and Huffman, 1988), financial frictions (debt-elastic interest rate premium, Schmitt-Grohé and Uribe, 2003) and a congestion externality for public infrastructure (Glomm and Ravikumar, 1994, 1997), though this latter feature is relatively less common.

Relative to existing models, the FMM-MTFF model incorporates some non-standard elements. First, fiscal policy is defined in terms of multi-year fiscal plans whose magnitude and timing are like those observed in the data or planned by policymakers, instead of restricting attention to univariate, single-period fiscal shocks.

Table 1. MTFFs, MTBFs, and MTPFs in Latin America and the Caribbean

	World Bank (2013)			World Bank (2017a)	Kaufmann et al. (2015)			Ministries of Finance Official Websites		
	MTFF	MTBF	MTPF	MTEF	MTFF	MTBF	MTPF	2017 MTFF document?	Forecasting Horizon (years) (including year 2018)	MTFF Document Contents
Argentina		•		•	•	•	•	✘		
Bahamas					•			✘		
Barbados	•				•			2010	4	a) fiscal projections/macroeconomic assumptions/DSA
Belize								✘		
Bolivia								✘		
Brazil	•			•	•			✘		
Chile	•				•			✔	3	a) fiscal projections/macroeconomic assumptions/DSA c) fiscal cost of new laws passed by Congress/tax expenditures
Colombia	•	•		•	•	•		✔	11	a) fiscal projections/macroeconomic assumptions/DSA/pensional debt b) fiscal risks: macroeconomic/guarantees/legal cases/natural disasters/PPPs c) fiscal cost of new laws passed by Congress/tax expenditures
Costa Rica				•	•			✔	4	a) fiscal projections/macroeconomic assumptions/DSA
Dominican Republic					•			✘		
Ecuador				•	•			✘		
El Salvador				•	•			✔	10	a) fiscal projections/macroeconomic assumptions/DSA/pension system b) fiscal risks: subnational governments/natural disasters c) tax expenditures/tax evasion
Guatemala				•	•	•	•	2013	3	a) fiscal projections/macroeconomic assumptions/DSA
Guyana					•	•		✘		
Haiti								✘		
Honduras	•			•	•	•		✔	4	a) fiscal projections/macroeconomic assumptions/DSA c) tax expenditures
Jamaica					•	•		✘		
Mexico	•			•	•	•	•	✔	5	a) fiscal projections/macroeconomic assumptions/pensions b) fiscal risks: macro/guarantees/state-owned banks/deposit insurance/natural disasters
Nicaragua	•	•		•	•	•	•	✔	3	a) fiscal projections/macroeconomic assumptions/DSA
Panama					•			✔	5	a) fiscal projections
Paraguay	•			•	•	•	•	✔	3	a) fiscal projections/macroeconomic assumptions/DSA c) tax expenditures
Peru	•			•	•			✔	4	a) fiscal projections/macroeconomic assumptions/DSA/pension and health expenditures b) fiscal risks: financial sector/legal cases/subnationals/public enterprises/non-financial private sector/natural disasters/PPPs c) tax expenditures
Suriname					•	•		✘		
Trinidad and Tobago					•	•		✘		
Uruguay	•			•				✘		
Venezuela, RB	•							✘		
TOTAL	11			13	21			10		

Sources: Kaufmann et al. (2015), PEMPAL (2013), World Bank (2013), and Ministry of Finance websites.
DSA = debt sustainability analysis

Second, the model does not impose the straightjacket of a standard fiscal rule. In a DSGE model, fiscal policy behavior is endogenized by introducing feedback rules, processes that describe how policy instruments respond to output and government debt. The response to output captures the countercyclical behavior of fiscal authorities. The response to government debt ensures fiscal solvency. A standard rule barely describes the behavior of fiscal authorities in emerging market and developing economies where fiscal policy is notoriously procyclical (Ilzetzi and Végh, 2008). The procyclicality of discretionary fiscal policy is explained by fundamental forces such as the inability to access foreign credit markets during downturns (Gavin and Perotti, 1997) and institutions or political structures that incentivize overspending of public resources in good times (Tornell and Lane, 1999).[2] These fundamental forces do not fade away despite the normative justification of a fiscal rule. Under a standard feedback rule, fiscal policy is countercyclical and sustainable by design and any fiscal challenge is mechanically addressed. However, in real-life situations, policymakers hesitate to act, postpone, or defer policy actions due to political, social, or economic pressures—probably affecting the cost of future policy measures—or follow policies that are unsustainable in the long run. Policymakers may be interested instead in the medium-term general direction in which public finances and the economy are heading if a given fiscal plan is devised. The FMM-MTFF model assesses a wider range of policy options by adopting a switching rule where a given fiscal plan is initially implemented and then, at some point in the future, it switches to a standard fiscal rule to ensure long-run dynamic stability.

Third, the FMM-MTFF model is calibrated to match a three-sector stylized version of a country's input-output (I-O) table, which provides a consistent framework on industry output, intermediate input flows, and final demand use data. A multisector economy model, with natural resource exports, is developed to better reflect the production structure of economies and to allow for an additional transmission mechanism on top of relative prices and factor supplies. Finally, to improve the quantitative performance of the model, a fourth key element is to embed into the model a more realistic GDP measurement framework that is consistent with what national account compilers do. The standard practice is to define real GDP as the value of all final demand components divided by the GDP deflator (see, for example, Medina and Soto, 2016), in which case a commodity price shock can have a direct real effect. However, a change in world export or import prices is treated as a price phenomenon in national income and product accounts

[2] Calderón and Schmidt-Hebbel (2008)'s findings support the hypothesis that both political distortions and market failures may explain the procyclical bias of fiscal policies in developing countries.

(NIPA). The model incorporates the two approaches used by NIPA accountants to estimate a volume index of GDP: fixed base year and annual chaining.

The use of the model developed in this paper is illustrated with the experiences of Colombia and Peru. Both countries have long ago adopted and consolidated a medium-term fiscal framework with forecasting horizons of four years for Peru and ten for Colombia. The ministries of finance, as required by their fiscal responsibility laws, must provide publicly available forecasts for major fiscal aggregates and for a few macroeconomic variables (basically GDP growth rate). The IMF, as part of its responsibility for surveillance under Article IV of the IMF's Articles of Agreement, also makes available its five-year forecasts. All institutional projections are updated annually on a rolling basis. The proposed policy simulations focus on the comparison of the different projection results and illustrate the use of the model as a tool to quantify the scale of the fiscal challenges, to provide consistent medium-term projections, and to assess the quantitative implications of past reforms and alternative fiscal policy plans on the economies over the short and medium term. In contrast to MTFFs and IMF staff reports, the model provides the evolution of several macroeconomic aggregates whose endogenous behavior may provide additional information to policymakers about the tradeoffs they face.

The rest of the paper is organized as follows. Section 2 describes the main features of the model. Section 3 describes the calibration strategy. Section 4 illustrates how the model can be used to extend the horizon of fiscal policy assessments by simulating alternative fiscal plans, and Section 5 concludes.

2. The FMM-MTFF Model

The model represents a small open economy that has three interdependent sectors producing: traded goods, nontraded goods, and a natural resource–based commodity.[3] Each industry employs capital and labor and purchases intermediate inputs from other producing sectors and overseas. The production structure of the economy and the quantitative interdependence between interrelated economic activities replicate a three-sector stylized version of a country's input-output (I-O) table. The economy is inhabited by a continuum of heterogeneous families (Ricardian and non-Ricardian), as in Galí, López-Salido, and Vallés (2004, 2007), by firms operating in perfect competition in factor and goods markets, and by a government.

[3] Throughout the paper the term "oil" will be used as a shorthand for the natural resource sector.

Consistent with the objective of studying the design, behavior, and impact of general fiscal plans over the medium term, the government performs several actions. The government levies taxes on labor income, capital income, consumption expenditures, imports, and domestic products/production; receives dividends or royalties from the natural resource–based industry; spends on nontraded goods; invests in infrastructure capital; and issues debt in domestic and external capital markets. Public capital is subject to congestion, as in Glomm and Ravikumar (1994, 1997), though there is no endogenous growth. Public capital is introduced as an external input to the production function, but its contribution to productivity is subject to congestion in the sense that a firm's activity congests the facilities available to other firms. Fiscal plans are specified in terms of trajectories for some exogenous fiscal variables, to be specified later on in this section, and the model simulates the response of other endogenous fiscal aggregates and the adjustment that occurs to the economy.

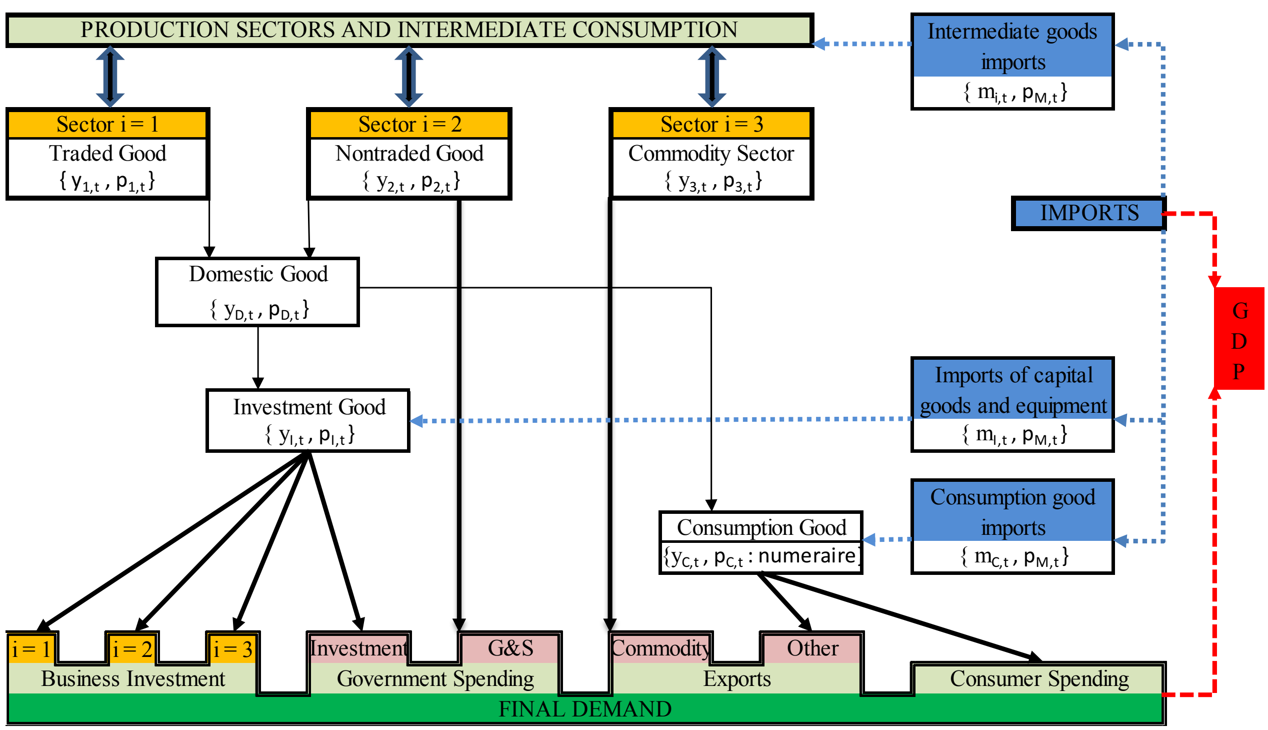
To save on notation, the model economy is directly presented in its stationary intensive form (i.e., the model's variables are expressed in terms of stationary variables). Given appropriate restrictions on preferences and technologies and after removing the built-in random walk property of the equilibrium dynamics proper of a linearized small open economy, the transformed economy exhibits a well-defined steady state balanced growth equilibrium. Real per capita flow variables are detrended by dividing by γ^t, the deterministic component of productivity with γ representing the gross rate of growth of labor-augmenting technical progress, which in turn determines the long-term growth of output per capita. Real per capita stock variables accumulated until the end of period t are detrended by γ^{t+1}. There is no need to detrend per capita labor input. After the solution to the stationary model is constructed, nonstationary per capita aggregates can be recovered by using the inverse transformation and other aggregates, with counterparts in NIPA or other sources, can be obtained by multiplying through by the working age population (ages 15–64).[4] Domestic prices are normalized by the price of the numeraire, the composite consumption basket. The real exchange rate s_t is defined as the price of one unit of the foreign consumption basket in units of the domestic one; the world relative prices of oil $p^*_{OIL,t}$ and imports $p^*_{M,t}$ are defined as the corresponding world prices relative to the foreign consumption price and their domestic counterparts are given by $p_{3,t} = s_t p^*_{OIL,t}$ and $p_{M,t} = s_t p^*_{M,t}$, respectively.

[4] For example, Figures 8 and 9 report model simulations for undetrended real wages and for undetrended and aggregate GDP and total hours.

2.1 The Production Side

Figure 1 provides a graphical representation of the production structure. The figure shows the flow of transformation processes through which inputs are converted into outputs along the various stages of production, from primary production in the top tier of the figure to final use in the bottom part. In the upper layer of activities, gross output is produced by the three industries and all industry-to-industry intermediate transactions, including the purchases of raw materials and intermediate inputs abroad, are carried out. The commodity sector output is partly sold locally to intermediate users in the (small) amount dictated by the input-output table but, in fact, most of it is exported at the world price. The nontraded sector good is in part directly sold for government consumption, also consistent with the I-O matrix, and the rest is combined with (non-resource) traded goods to produce the domestic good. Then, the composite domestic good is used, along with either imports of capital goods and equipment or consumption goods, as input in the production of an investment good and a consumption good. The investment good is accumulated into sector-specific stocks of private capital and productivity-enhancing public infrastructure. The composite numeraire good is consumed domestically or exported.

Figure 1. Production Structure and Flow of Goods in the Economy



Note: G&S = goods and services.

2.1.1. Sectoral Industries

There are three production sectors □, $i = \{1,2,3\}$ in the economy where $y_{i,t}$ represents sectoral gross output and $p_{i,t}$ is the corresponding domestic relative price.Sector $i = 1$ is the (non-resource) traded sector, sector $i = 2$ is the nontraded good–producing sector, and $i = 3$ represents the natural resource sector. Each sector uses a Leontief technology:

$$y_{i,t} = min\left[\mathbb{A}_t^i K_{G,t-1}^{\theta} k_{i,t-1}^{\alpha_i}\left(n_{i,t}\right)^{1-\alpha_i}, \frac{x_{1,i,t}}{\chi_{1i}}, \frac{x_{2,i,t}}{\chi_{2i}}, \frac{x_{3,i,t}}{\chi_{3i}}, \frac{m_{i,t}}{\chi_{Mi}}\right] \qquad \forall i \qquad [1]$$

$$K_{G,t} = \frac{k_{G,t}}{\left(K_{i,t}\right)^{\phi}} \qquad [2]$$

$$\mathbb{A}_t^i = A_t A_t^i A_i \qquad [3]$$

The production function is separable in value added and intermediate inputs and the value added component is a function of the end-of-period $t-1$ private capital stock $\left(k_{i,t-1}\right)$, labor $\left(n_{i,t}\right)$, and the level of productivity. $\alpha_i \in (0,1)$ is the private capital share in sector i value added. A_t stands for an aggregate technology shock around the long-run deterministic level of productivity that affects all sectors simultaneously, and A_t^i represents an idiosyncratic technology shock to sector i. $x_{j,i,t}$ is the amount of input from sector j, $j = \{1,2,3\}$ needed to produce the output of sector i, and parameters $\chi_{j,i}$ represent technological input requirements (i.e., the quantity of input required per unit of gross output i). Similarly, $m_{i,t}$ represents the demand of imported intermediate goods needed to produce sector i gross output and $\chi_{M,i}$ is the corresponding input coefficient.

Government-supplied infrastructure, $k_{G,t}$, enhances private sector productivity where $\theta \in (0,1)$ is the output elasticity of public capital. Public capital is subject to congestion (Glomm and Ravikumar, 1994, 1997) with ϕ, $\phi \in [0,1]$ governing the degree of congestion. This production externality depends not on the level of infrastructure capital but on the ratio of infrastructure to adjusted private capital, the "effective" stock of infrastructure, $K_{G,t}$.

When $\phi = 0$, infrastructure capital services qualify as a pure public good (non-excludable and non-rival). When $\phi > 0$, congestion effects come into play. An increase in public infrastructure boosts private sector productivity, but this effect fades away as the stock of infrastructure decreases relative to the level of private sector activity, proxied here by the stock of private capital installed. In the extreme case of $\phi = 1$, capital infrastructure services can be regarded as a pure private good (rival and excludable). Uppercase letters have been used in the definition of the effective stock of infrastructure (equation [2]) to denote the fact that firms take aggregate sectoral capital stocks $(K_{i,t})$ as a given (i.e., they do not internalize the effect of their investment decisions on congestion). A_i is a scale parameter and the condition $\alpha_i + (1-\square)\theta < 1$ is imposed to rule out endogenous growth. The representative firm in sector i chooses capital $(k_{i,t})$, labor services $(n_{i,t})$, domestic intermediate inputs $(x_{1,i,t}, x_{2,i,t}, x_{3,i,t})$, and imported intermediate inputs $(m_{i,t})$ to maximize the discounted sum of profits, taking prices, and taxes as given,

$$\max \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \Lambda_t \Big[(1-\tau_{i,t}) p_{i,t} y_{i,t} - u_{i,t} k_{i,t-1} - w_{i,t} n_{i,t} - \sum_{j=1}^{3} p_{j,t} x_{j,i,t} - p_{M,t} m_{i,t} - p_{i,t} \frac{\eta_{n_i}}{2} \left(\frac{n_{i,t}}{n_{i,t-1}} - 1 \right)^2 n_{i,t} \Big] \quad \forall i \qquad [4]$$

and subject to the fixed proportions technological constraint [1]-[3]. $\beta^t \Lambda_t$ is the stochastic discount factor, Λ_t is the Lagrange multiplier associated with the budget constraint in the Ricardian household problem (aggregate version of equation [29]), and $\mathbb{E}_0$ is the conditional expectations operator. $\beta \in (0,1)$ is the subjective discount factor and $\tau_{i,t}$ is the effective tax rate on sector i output. $w_{\square,t}$ is the wage rate and $u_{i,t}$ is the rental rate on sectoral capital, both measured in terms of the numeraire. Net returns to capital are equalized across sectors, but wage equalization in the steady state is not imposed by the model. The last term in [4] introduces quadratic labor adjustment costs (Sargent, 1978). The rapid adjustment of employment has a cost in terms of forgone output where η_{n_i} governs the magnitude of output loss in sector i. The first order conditions of the firm's problem in sector i, $i = \{1,2,3\}$, are given by:

$$u_{i,t} = \left((1-\tau_{i,t}) p_{i,t} - \sum_{j=1}^{3} \chi_{ji} p_{j,t} - \chi_{Mi}\, p_{M,t} \right) \alpha_i \frac{y_{i,t}}{k_{i,t-1}} \qquad [5]$$

$$w_{i,t} = \left((1-\tau_{i,t}) p_{i,t} - \sum_{j=1}^{3} \chi_{ji} p_{j,t} - \chi_{Mi} p_{M,t} \right) (1-\alpha_i) \frac{y_{i,t}}{n_{i,t}} - p_{i,t} \left(\frac{\eta_{n_i}}{2} \left(\frac{n_{i,t}}{n_{i,t-1}} - 1 \right)^2 - \eta_{n_i} \left(\frac{n_{i,t}}{n_{i,t-1}} - 1 \right) \left(\frac{n_{i,t}}{n_{i,t-1}} \right) \right) + \beta \eta_{n_i} \mathbb{E}_t \left\{ p_{i,t+1} \frac{\Lambda_{t+1}}{\Lambda_{\square}} \left(\frac{n_{i,t+1}}{n_{i,t}} - 1 \right) \left(\frac{n_{i,t+1}}{n_{i,t}} \right)^2 \right\} \qquad [6]$$

$$y_{i,t} = \mathbb{A}_t^i K_{G,t-1}^{\theta} k_{i,t-1}^{\alpha_i} (n_{i,t})^{1-\alpha_i} \qquad [7]$$

$$x_{1,i,t} = \chi_{1i} y_{i,t} \qquad [8]$$

$$x_{2,i,t} = \chi_{2i} y_{i,t} \qquad [9]$$

$$x_{3,i,t} = \chi_{3i} y_{i,t} \qquad [10]$$

$$m_{i,t} = \chi_{Mi} y_{i,t} \qquad [11]$$

By the law of one price, the commodity sector price is tied to its price abroad: $p_{3,t} = s_t p^*_{OIL,t}$. Similarly, the price of the imported intermediate good is equal to its corresponding world price measured in the home numeraire unit: $p_{M,t} = s_t p^*_{M,t}$. In the model, agents can import consumption, capital, and intermediate goods. For simplicity, they are assumed to share the same price.

2.1.2. The Domestic Good

The domestic good $(y_{D,t})$ is produced by combining (non-resource) traded goods $(z_{1,t})$ and nontraded goods $(z_{2,t})$ according to a constant elasticity of substitution (CES) technology with substitution parameter ω_D, $\omega_D > 0$:

$$y_{D,t} = A_D \left[(\mu_D)^{\frac{1}{\omega_D}} (z_{1,t})^{\frac{\omega_D - 1}{\omega_D}} + (1 - \mu_D)^{\frac{1}{\omega_D}} (z_{2,t})^{\frac{\omega_D - 1}{\omega_D}} \right]^{\frac{\omega_D}{\omega_D - 1}} \quad [12]$$

A_D is a scale parameter and μ_D is the share of traded goods in the domestic good basket. The producer firm of the domestic good chooses $z_{1,t}$ and $z_{2,t}$ to maximize profits:

$$\max_{\{z_{1,t}, z_{2,t}\}} p_{D,t} y_{D,t} - p_{1,t} z_{1,t} - p_{2,t} z_{2,t} \quad [13]$$

subject to [12] and where $p_{D,t}$ is the price of the domestic good. The optimal demand functions for traded and nontraded good inputs are given by:

$$z_{1,t} = \mu_D {A_D}^{\omega_D - 1} \left(\frac{p_{1,t}}{p_{D,t}} \right)^{-\omega_D} y_{D,t} \quad [14]$$

$$z_{2,t} = (1 - \mu_D) {A_D}^{\omega_D - 1} \left(\frac{p_{2,t}}{p_{D,t}} \right)^{-\omega_D} y_{D,t} \quad [15]$$

And the corresponding price index is given by the zero-profit condition:

$$p_{D,t} = {A_D}^{-1} \left(\mu_D (p_{1,t})^{1-\omega_D} + (1 - \mu_D)(p_{2,t})^{1-\omega_D} \right)^{\frac{1}{1-\omega_D}} \quad [16]$$

2.1.3. Investment and Consumption Good Producers

The investment good $y_{I,t}$ and the consumption good $y_{C,t}$ are produced by combining domestic goods and imports in an Armington aggregator production function with constant elasticity of substitution parameters ω_I and ω_C. The firm producing the investment good solves the following static optimization problem:

*

$$\max_{\{d_{I,t}, m_{I,t}\}} p_{I,t} y_{I,t} - p_{D,t} d_{I,t} - p_{M,t} m_{I,t} \quad [17]$$

subject to:

$$y_{I,t} = A_I \left[(\mu_I)^{\frac{1}{\omega_I}} (d_{I,t})^{\frac{\omega_I - 1}{\omega_I}} + (1 - \mu_I)^{\frac{1}{\omega_I}} (m_{I,t})^{\frac{\omega_I - 1}{\omega_I}} \right]^{\frac{\omega_I}{\omega_I - 1}} \quad [18]$$

while the problem of the consumption good–producing firm can be written as:

$$\max_{\{d_{C,t}, m_{C,t}\}} y_{C,t} - p_{D,t} d_{C,t} - \left(1 + \tau_{C,t}^{M}\right) p_{M,t} m_{C,t} \quad [19]$$

subject to:

$$y_{C,t} = A_C \left[(\mu_C)^{\frac{1}{\omega_C}} (d_{C,t})^{\frac{\omega_C - 1}{\omega_C}} + (1 - \mu_C)^{\frac{1}{\omega_C}} (m_{C,t})^{\frac{\omega_C - 1}{\omega_C}} \right]^{\frac{\omega_C}{\omega_C - 1}} \quad [20]$$

The domestic good is wholly consumed in the production of the investment good in the amount $d_{I,t}$ and in the production of the consumption good in the amount $d_{C,t}$. Imports of capital goods, machinery, and equipment $(m_{I,t})$ are inputs into the production of the composite investment good and imports of consumption goods $(m_{C,t})$ are used by domestic firms to produce the composite consumption good. Consumption good imports are subject to tariff duties, consistent with I-O accounts, where $\tau_{C,t}^{M}$ is the exogenous effective tariff rate. In addition to standard input demand and investment price functions (see Annex 1 for a full list of the model equations), the numeraire good satisfies the following price condition, $\forall t$:

$$1 = A_C{}^{-1} \left[\mu_C \left(p_{D,t}\right)^{1-\omega_C} + (1 - \mu_C) \left(\left(1 + \tau_{C,t}^{M}\right) p_{M,t} \right)^{1-\omega_C} \right]^{\frac{1}{1-\omega_C}} \quad [21]$$

2.2 Households

The total number of households is normalized to 1. Following Galí, López-Salido, and Vallés (2004, 2007), a fraction $1 - \lambda$ of households are Ricardian households, which have access to capital markets and adjust savings optimally in response to changing economic conditions, and a fraction λ are non-Ricardian households, households which do not have access to a savings vehicle and just consume their labor income. Ricardian households, denoted by superscript ^{O}, have access to domestic and international bond markets to save and borrow, have access to a physical capital market, and optimize over their lifetimes.

The restricted type of households is denoted by superscript ^{NO}. The representative non-Ricardian household only earns wage income from supplying labor services $\left(h_{i,t}^{NO}\right)$ to each producing sector i and does not pay taxes. Non-Ricardian consumption c_t^{NO} is determined by the budget constraint:

$$c_t^{NO} = \sum_{i=1}^{3} w_{i,t} h_{i,t}^{NO} + tt_t^{NO} \quad [22]$$

where tt_t^{NO} denotes government transfers. The non-Ricardian household's period utility function is given by:

$$u\left(c_t^{NO}, h_i^{NO}\right) = \frac{1}{1-\sigma}\left(c_t^{NO} - \frac{1}{1+\varepsilon}\sum_{i=1}^{3} \psi_i^{NO}\left(h_{i,t}^{NO}\right)^{1+\varepsilon}\right)^{1-\sigma} \qquad [23]$$

where ε is the inverse of the Frisch elasticity of labor supply, σ is the inverse of the intertemporal elasticity of substitution, and $\psi_i^{NO} \in (0,\infty)$ measures the degree of disutility from working in sector i. These are GHH preferences (Greenwood, Hercowitz, and Huffman, 1988), which imply that labor supply depends only on the real wage, precluding wealth effects. However, the lack of an income effect is inconsistent with balanced growth. To ensure consistency, the disutility of labor has been assumed to grow with technological progress (Jaimovich and Rebelo, 2009) preferences. The optimal labor supply decision associated with the utility maximization problem subject to the budget constraint [22] is:

$$\psi_i^{NO}\left(h_{i,t}^{NO}\right)^{\varepsilon} = w_{i,t} \qquad \forall i \qquad [24]$$

The representative Ricardian household maximizes its expected lifetime utility function

$$\max \mathbb{E}_0\left\{\sum_{t=0}^{\infty} \beta^t \frac{1}{1-\sigma}\left(c_t^O - \varsigma C_{t-1}^O - \frac{1}{1+\varepsilon}\sum_{i=1}^{3} \psi_i^O\left(h_{i,t}^O\right)^{1+\varepsilon}\right)^{1-\sigma}\right\} \qquad [25]$$

These preferences featuring external habit persistence, where the period utility depends on the quasi-difference of consumption, are used to introduce an internal persistence mechanism in the consumption process (Abel, 1990). ς denotes the intensity of the habit formation. Again, the notation (uppercase letters) highlights the fact that habit formation is an externality and that the household does not internalize the effect of its consumption-saving decisions on the habit level.

The Ricardian household's optimization problem is subject to the following budget constraint and the laws of motion for physical capital accumulation in the three production sectors:

$$\begin{aligned}\gamma b_t^O + \left(R_{t-1}^W + \eta_{D^*}\left(e^{D_t^* - \partial} - 1\right)\right) s_t d_{t-1}^{*O} + \left(1+\tau_{C,t}\right)c_t^O + p_{I,t}\left(i_{1,t}^O + i_{2,t}^O + i_{3,t}^O\right) \\ = \left(1-\tau_{N,t}\right)\left(\sum\nolimits_i w_{i,t} h_{i,t}^O\right) + \left(1-\tau_{K,t}\right)\left(\sum\nolimits_i u_{i,t} k_{i,t-1}^O\right) \\ + \tau_{K,t}\left(\sum\nolimits_i \delta_i k_{i,t-1}^O\right) - v_t^O + \pi_t^O + tt_t^O + \gamma s_t d_t^{*O} + R_{t-1} b_{t-1}^O\end{aligned} \qquad [26]$$

$$\gamma k_{i,t}^O = (1-\delta_i)k_{i,t-1}^O + i_{i,t}^O - \frac{\eta_i}{2}\left(\frac{i_{i,t}^O}{i_{i,t-1}^O} - 1\right)^2 i_{i,t}^O \qquad \forall i \qquad [27]$$

The intertemporal optimizing household spends on consumption (c_t^O) and investment $\left(i_{i,t}^O, \forall i\right)$ goods; earns real wage income $\left(w_{i,t} h_{i,t}^O, \forall i\right)$, capital income $\left(u_{i,t} k_{i,t-1}^O, \forall i\right)$, and interest income on government bond holdings (b_t^O) paying a gross real interest rate R_t; and receives lump-sum transfers from the government (tt_t^O) and profit distributions in the form of real dividend payments (π_t^O) as owner of all firms of the economy. It also transfers part of the commodity sector capital income to the government (v_t^O) in the form of royalties and pays taxes on labor income $\left(\tau_{N,t}\right)$, capital income net of depreciation $\left(\tau_{K,t}\right)$, and consumption expenditures $\left(\tau_{C,t}\right)$. The household can borrow (d_t^{*O}) from international capital markets by paying a gross interest rate: $R_{t-1}^W + \eta_{D^*}\left(e^{D_t^* - \partial} - 1\right)$. The first component of this expression is the government's gross borrowing rate in international markets, to be specified further below, and the second component is a debt elastic interest rate premium. This premium is taken exogenously by the representative agents and is assumed to be an increasing function of the aggregate stock of private external debt (D_t^*) relative to its steady state level ∂. This interest rate specification is used to induce stationarity in small open economy models (Schmitt-Grohé and Uribe, 2003).

$p_{I,t}$ is the relative price of the investment good in terms of consumption and the investment good accumulates into sector-specific capital stocks [27]. The capital accumulation process in each sector is also subject to investment adjustment costs (Christiano, Eichenbaum, and Evans, 2005). δ_i is the depreciation rate of capital installed in sector i and η_i is an adjustment cost parameter.

The choice variables in the Ricardian household optimization problem are c_t^O, b_t^O, d_t^{*O}, $h_{i,t}^O$, $k_{i,t}^O$, and $i_{i,t}^O$. Then, the optimal path satisfies the following conditions:

$$\psi_i^O (h_{i,t}^O)^\varepsilon = \left(\frac{1-\tau_{W,t}}{1+\tau_{C,t}}\right) w_{i,t} \qquad \forall i \tag{28}$$

$$\Lambda_t = (1+\tau_{C,t})^{-1}\left(c_t^O - \varsigma C_{t-1}^O - \frac{\sum_{j=1}^3 \psi_j^O \left(h_{j,t}^O\right)^{1+\varepsilon}}{1+\varepsilon}\right)^{-\sigma} \tag{29}$$

$$\gamma \Omega_{i,t} = \beta \mathbb{E}_t\left\{\left(1-\tau_{K,t+1}\right) u_{i,t+1}\Lambda_{t+1} + \tau_{K,t+1}\delta_i \Lambda_{t+1} + (1-\delta_i)\Omega_{i,t+1}\right\} \qquad \forall i \tag{30}$$

$$\begin{aligned} p_{I,t}\Lambda_t - \Omega_{i,t}\left(1 - \frac{\eta_i}{2}\left(\frac{i_{i,t}^O}{i_{i,t-1}^O} - 1\right)^2 - \eta_i\left(\frac{i_{i,t}^O}{i_{i,t-1}^O} - 1\right)\left(\frac{i_{\square,t}^O}{i_{i,t-1}^O}\right)\right) \\ = \beta \eta_i \mathbb{E}_t\left\{\left(\frac{i_{i,t+1}^O}{i_{i,t}^O} - 1\right)\left(\frac{i_{i,t+1}^O}{i_{i,t}^O}\right)^2 \Omega_{i,t+1}\right\} \qquad \forall i \end{aligned} \tag{31}$$

$$s_t \Lambda_t = \frac{\beta}{\gamma}\left(R_t^W + \eta_{D^*}\left(e^{D_t^* - \partial} - 1\right)\right) \mathbb{E}_t\{s_{t+1}\Lambda_{t+1}\} \tag{32}$$

$$\Lambda_t = \frac{\beta}{\gamma}\mathbb{E}_t\{R_t \Lambda_{t+1}\} \tag{33}$$

Λ_t and $\Omega_{i,t}$, $i \in \{1,2,3\}$, are Lagrangian multipliers associated with the budget constraint and the capital accumulation equations, respectively.

Finally, aggregating across households yields total household consumption and sectoral investment demands, labor supply and total transfers:

$$c_t = \lambda c_t^{NO} + (1-\lambda) c_t^{O} \quad [34]$$

$$i_{i,t} = (1-\lambda) i_{i,t}^{O} \qquad \forall i \quad [35]$$

$$h_{i,t} = \lambda h_{i,t}^{NO} + (1-\lambda) h_{i,t}^{O} \qquad \forall i \quad [36]$$

$$t_t = t_t^{NO} + t_t^{O} = \lambda tt_t^{NO} + (1-\lambda) tt_t^{O} \quad [37]$$

$$v_t = (1-\lambda) v_t^{O} \quad [38]$$

as well as aggregate assets and liabilities holdings

$$k_{i,t} = (1-\lambda) k_{i,t}^{O} \qquad \forall i \quad [39]$$

$$b_t = (1-\lambda) b_t^{O} \quad [40]$$

$$d_t^{*} = (1-\lambda) d_t^{*O} \quad [41]$$

All transfers (t_t^{NO}, t_t^{O}, v_t), expressed as a percent of GDP, and tax rates follow exogenous processes.

2.3 Government

Each period, the government invests $p_{I,t} i_{G,t}$ where $i_{G,t}$ is the real amount of gross public investment in infrastructure. Government capital spending satisfies the following condition:

$$p_{I,t} i_{G,t} = g_{I,t} p_{GDP,t} GDP_t^{R} \quad [42]$$

where $g_{I,t}$ (with an obvious empirical counterpart) is assumed to follow an exogenous process. $p_{GDP,t}$ is the GDP price index in terms of the consumption basket and GDP_t^{R} is the GDP quantity index. Infrastructure capital investment accumulates following a standard law of motion with adjustment costs, where η_G is the adjustment cost parameter and δ_G is the depreciation rate:

$$\gamma k_{G,t} = (1-\delta_G) k_{G,t-1} + i_{G,t} - \frac{\eta_G}{2}\left(\frac{i_{G,t}}{i_{G,t-1}} - 1\right)^2 i_{G,t} \quad [43]$$

In a similar vein, government spending on nontraded goods and services is governed by the variable $g_{C,t}$, which relates the value of government spending $(p_{2,t} g_t)$ and GDP $(p_{GDP,t} GDP_t^{R})$

$$p_{2,t} g_t = g_{C,t} p_{GDP,t} GDP_t^{R} \quad [44]$$

Government expenditures and revenues must satisfy the current period budget constraint. The government budget constraint is written as follows:

$$\begin{gathered} f_t = p_{2,t} g_t + p_{I,t} i_{G,t} + t_t + R_{t-1}^{W} s_t d_{G,t-1}^{*} + R_{t-1} b_{t-1} - \tau_{C,t}(1-\lambda) c_t^{O} - \\ \tau_{W,t}\left(\textstyle\sum_i w_{i,t}(1-\lambda) h_{i,t}^{O}\right) - \tau_{K,t}\left(\textstyle\sum_i (u_{i,t} - \delta_i) k_{i,t-1}\right) - \tau_{C,t}^{M} p_{M,t} m_{C,t} - \\ \textstyle\sum_i \tau_{i,t} p_{i,t} y_{i,t} - v_t \end{gathered} \quad [45]$$

$$\gamma b_t = g_{F,t} f_t \qquad [46]$$

$$\gamma s_t d^*_{G,t} = \left(1 - g_{F,t}\right) f_t \qquad [47]$$

Abusing notation slightly while understanding all magnitudes as aggregate variables, equation [45] defines the amount of government financing needs (f_t) as the difference between total outlays, including amortization payments to cover interests and principal repayment, and total receipts. Equations [46] and [47] indicate that a fraction $g_{F,t}$ of the total financing needs is funded by issuing end-of-period domestic government bonds and the rest, $1 - g_{F,t}$, by borrowing abroad $\left(d^*_{G,t}\right)$, where $g_{F,t}$ is determined exogenously.

The government's cost of borrowing from abroad (R^W_t) consists of an exogenously determined risk-free international interest rate factor (R^*_t) and an endogenous risk premium ρ_t

$$R^W_t = R^*_t(1 + \rho_t) \qquad [48]$$

The interest rate premium depends on the ratio of external government debt to GDP, where ð is the corresponding steady-state ratio:

$$\rho_t = A_\rho \, exp\left[\pi\left(\frac{s_t d^*_{G,t-1}}{p_{GDP,t} GDP^R_t} - ð\right)\right] \qquad [49]$$

A_ρ is a scale parameter governing the steady-state level of the sovereign spread and π measures its responsiveness to foreign government debt.

2.4 Exports, Imports, and Terms of Trade

The country's export basket comprises natural resources $\left(e^{oil}_t\right)$ and a composite consumption good $\left(e_{C,t}\right)$. The whole production of the natural resource–based commodity is exported once domestic intermediate needs are satisfied. The (semi-small open) economy faces a downward-sloping demand function for its non-resource export good. The export demand is defined as in Kollmann (2002):

$$e_{C,t} = A_X \left(\frac{1}{s_t}\right)^{\varpi_P} (y^*_t)^{\varpi_Y} \qquad [50]$$

where ϖ_P is the (absolute value of the) price elasticity, ϖ_Y is the elasticity with respect to foreign real GDP, which is assumed to capture aggregate demand developments across the country's trading partners, and A_X is a scale parameter. y^*_t is assumed to follow an exogenous process. Aggregate export price $\left(p_{E,t}\right)$ and quantity (e_t) indexes are constructed as Cobb-Douglas aggregates of natural resource and non-resource export prices and bundles:

$$p_{E,t} = A_E{}^{-1}\left(\frac{1}{\mu_E}\right)^{\mu_E}\left(\frac{1}{1-\mu_E}\right)^{1-\mu_E}\left(s_t p^*_{OIL,t}\right)^{1-\mu_E} \quad [51]$$

$$e_t = A_E\left(e_{C,t}\right)^{\mu_E}\left(e_{OIL,t}\right)^{1-\mu_E} \quad [52]$$

where μ_E is the expenditure share on non-resource goods in the base period.

Trade balance (tb_t), total imports (m_t), and terms of trade (tot_t) are defined as follows:

$$tb_t = p_{E,t}e_t - p_{M,t}m_t \quad [53]$$

$$m_t = \sum_{i=1}^{3} m_{i,t} + m_{C,t} + m_{I,t} \quad [54]$$

$$tot_t = \frac{p_{E,t}}{p_{M,t}} \quad [55]$$

2.5 National Income Accounting

A change in world export or import prices is treated as a price phenomenon in national income and product accounting. However, the standard practice is to define real GDP as the value of all final demand components divided by the GDP deflator (see, for example, Medina and Soto, 2016), in which case a commodity price shock can have a direct real effect. The effect of import and export prices on income and welfare is better captured by the notion of GDI (gross domestic income). Even though real GDP is a misleading indicator that underestimates the benefits arising from trading gains, the objective here is to embed into the model a GDP measurement framework that is consistent with what national account compilers do.

There are two approaches for estimating a volume index of GDP: fixed base year and annual chaining.[5] Both approaches will be used in the empirical applications. As a Laspeyres-type volume index, the first method—the “fixed weight” measure of real GDP—values all quantities through time at the set of prices prevailing in the base year:

$$GDP^R_t = c_t + \left(i_{1,t} + i_{2,t} + i_{3,t} + i_{G,t}\right) + g_t + e_t - m_t \quad [56]$$

All prices are set to unity in the base year. The use of a reference price structure gives rise to the so-called “Gerschenkron effect” where a change in the base year may alter the estimated growth rates of the volume index. In the second approach, when indexes are chained, the previous year is used as the base year. The GDP volume index level evolves according to the following law of motion:

[5] The IMF’s WEO database (IMF, 2017b), in Table G of the Statistical Appendix, provides a list of countries using the chain-weighted method. The Country Information page also includes this information.

$$GDP_t^R = \left(\frac{\gamma_{GDP,t}}{\gamma}\right) GDP_{t-1}^R \qquad [57]$$

Setting an arbitrary base year, for which nominal and real GDP are equal, the volume index is constructed by chaining together successive links $(\gamma_{GDP,t})$ from the base year:

$$\gamma_{GDP,t} = \gamma \left(\frac{c_t + p_{I,t-1}(\sum i_{i,t} + i_{G,t}) + p_{2,t-1} g_t + p_{E,t-1} e_t - p_{M,t-1} m_t}{c_{t-1} + p_{I,t-1}(\sum i_{i,t-1} + i_{G,t-1}) + p_{2,t-1} g_{t-1} + p_{E,t-1} e_{t-1} - p_{M,t-1} m_{t-1}}\right) \qquad [58]$$

The link of the chain is the gross rate of real GDP growth in each period. This is a chained Laspeyres volume index[6] (period $t-1$ prices used as weights). The GDP deflator in terms of the numeraire satisfies:

$$p_{GDP,t} GDP_t^R = c_t + p_{I,t}\left(\sum_i i_{i,t} + i_{G,t}\right) + \square_{2,t} g_t + p_{E,t} e_t - p_{M,t} m_t \qquad [59]$$

implying a chained Paasche price index. Note, however, that the proposed accounting framework is not entirely consistent with national accounting practices. GDP at market prices includes indirect taxes and their effect is not being considered by equation [58].

2.6 Market Equilibrium Conditions and Dynamic Stability

In equilibrium all markets in the economy clear in each period. At the sectoral level, supply and demand for labor services satisfy:

$$n_{i,t} = h_{i,t} \qquad \forall i \qquad [60]$$

The market clearing conditions for the various good markets are:

$$y_{C,t} = c_t + e_{C,t} + \tau_{C,t}^M p_{M,t} m_{C,t} \qquad [61]$$

$$y_{I,t} = \sum_{i=1}^3 i_{i,t} + i_{G,t} \qquad [62]$$

$$y_{1,t} = x_{1,1,t} + x_{1,2,t} + \square_{1,3,t} + z_{1,t} \qquad [63]$$

$$y_{2,t} = x_{2,1,t} + x_{2,2,t} + x_{2,3,t} + z_{2,t} + g_t \qquad [64]$$

$$y_{3,t} = x_{3,1,t} + x_{3,2,t} + x_{3,3,t} + e_{OIL,t} \qquad [65]$$

$$y_{D,t} = d_{C,t} + d_{I,t} \qquad [66]$$

By combining household budget constraints, the government budget constraint, profit expressions for all producing firms, and market clearing conditions, the resulting national resource constraint is given by the balance of payments condition

$$\gamma(d_t^* + d_{G,t}^*)s_t = R_{t-1}^W(d_{t-1}^* + d_{G,t-1}^*)s_t + \eta_{D^*}\left(e^{D_{t-1}^* - \partial} - 1\right)s_t d_{t-1}^* + p_{M,t} m_t - p_{E,t} e_t \qquad [67]$$

[6] The Bureau of Economic Analysis in the United States uses a chain-type Fisher index, a geometric mean of the chained Laspeyres index and a Paasche index.

Finally, to ensure dynamic stability of public debt in the long run, a switching rule is implicitly adopted:

$$g_{C,t} = \mathbb{I}_t g_{C,t}^{ex\square} + (1 - \mathbb{I}_t) g_{C,t-1} \, exp\left[-\iota\left(\frac{s_t d_{G,t-1}^* + b_{G,t-1}}{p_{GDP,t} PIB_t^R} - \zeta\right)\right] \quad [68]$$

where $\mathbb{I}_t$ is an indicator function that takes on the value of 1 for at least the simulation period and 0 otherwise. Over the simulation period, the path of government expenditures, relative to GDP, is given by an exogenous trajectory $g_{C,t}^{exo}$ and then it switches to a regular fiscal rule that stabilizes the government debt dynamics around a certain long-run level of debt (ζ). In principle, feedback fiscal rules can be defined for other instruments: lump-sum transfers, government investment, or taxes.

Annex 1 contains the full list of the model equations written in terms of detrended variables for which a deterministic steady state can be computed.

3. Calibration and Steady State

Once the model equations are fully specified and functional forms parameterized, parameter values are then assigned and deterministic steady states computed for Colombia (COL) and Peru (PER). The calibration strategy is the same in each country case. The two economies are assumed to be at their initial steady state in 2010. The steady state value of any variable is represented hereafter by dropping the time index. Annex 2 shows the system of steady state constraints imposed by the deterministic stationary state of the economy. One period in the model is taken to be one year.

Two information sets are used to completely calibrate the model economies. The first is the country's stylized version of the 2010 input-output (I-O) table (Table 2) constructed from tables compiled by the OECD (2017). (Detrended per capita) GDP at market prices has been normalized to 100 in the 2010 I-O table so that matrix entries can be interpreted as percentages of GDP. The second is Table 3, which provides additional aggregate targets (as of 2010) and parameter estimates taken directly from applied econometric studies or other sources. The remaining parameters values are calibrated such that the system of steady state relations is satisfied.

The 34 industries of the original I-O table are consolidated into three sectors: the traded sector, which consists largely of all manufacturing industries and agriculture; the nontraded sector, which groups all service industries; and the natural resource sector, or Mining and Quarrying in the

OECD nomenclature. Several adjustments were made to the original matrix. Without affecting the magnitude of GDP at market prices, taxes (less subsidies) on production are recorded as taxes (less subsidies) on products, rendering the definitions of gross value added at factor cost and at basic prices identical. Thus, value added reflects solely the contribution of capital and labor to production and the corresponding aggregate factor shares are obtained from NIPA data (aggregate labor share COL: 0.617 and PER: 0.572). Labor income is defined as the sum of compensation of employees and mixed income. Moreover, some entries of the I-O matrix, generally small, were set to zero to make the I-O matrix consistent with the model specification. Such is the case of government spending on traded goods (COL and PER: ~0% of GDP), the final use of the natural resource as an investment good (COL: 0.09% and PER: 0.06% of GDP), nontraded sector exports (COL: 0.67% and PER: 1.05% of GDP), taxes on export goods (COL: 0.05% and PER: 0.08% of GDP), taxes on government spending (COL and PER: ~0% of GDP), and re-exports of imported goods (COL: 0.24% and PER: ~0% of GDP). These adjustments give rise to an unbalanced I-O matrix, a problem handled by using the RAS balancing method (Stone, 1961; Stone and Brown, 1962). Rebalanced I-O matrices are displayed in Table 2.

Table 2. Stylized 2010 Input-Output Tables

COLOMBIA: 2010 INPUT-OUTPUT TABLE

	SECTOR 1: TRADED SECTOR	SECTOR 2: NONTRADED SECTOR	SECTOR 3: MINING SECTOR	TOTAL	HOUSEHOLD CONSUMPTION	GENERAL GOVERNMENT CONSUMPTION	GROSS CAPITAL FORMATION	EXPORTS	TOTAL USES
Traded sector	21.2	8.9	0.4	30.6	26.4	0.0	3.1	9.1	69.2
Nontraded sector	7.4	15.1	0.2	22.7	27.1	16.6	13.8	0.0	80.2
Mining sector	2.5	0.8	0.7	4.1	0.0	0.0	0.0	6.0	10.1
Domestic intermediate inputs	31.2	24.8	1.3	57.3	53.5	16.6	16.9	15.1	159.5
Imports	5.3	2.9	0.1	8.3	4.7	0.0	3.2	0.0	16.2
Customs and import duties	0.0	0.0	0.0	0.0	0.9	0.0	0.0	0.0	0.9
Indirect taxes	0.9	0.7	0.0	1.6	5.3	0.0	0.0	0.0	6.8
Total intermediate and final expenditure	37.3	28.4	1.5	67.2	64.3	16.6	20.2	15.1	183.4
Labor compensation and mixed income	18.5	36.9	1.5	57.0					
Gross operating surplus	13.4	14.9	7.0	35.3					
Value added at basic prices	31.9	51.8	8.5	92.3					
GROSS OUTPUT	69.2	80.2	10.1	159.5					

GDP INCOME APPROACH		GDP EXPENDITURE APPROACH	
(+) Value added	92.3	(+) Consumption	64.3
(+) Indirect Taxes	7.7	(+) Government	16.6
(=) GDP	100.0	(+) GFKF	20.2
		(+) Exports	15.1
		(-) Imports	-16.2
		(=) GDP	100.0

PERU: 2010 INPUT-OUTPUT TABLE

	SECTOR 1: TRADED SECTOR	SECTOR 2: NONTRADED SECTOR	SECTOR 3: MINING SECTOR	TOTAL	HOUSEHOLD CONSUMPTION	GENERAL GOVERNMENT CONSUMPTION	GROSS CAPITAL FORMATION	EXPORTS	TOTAL USES
Traded sector	26.7	12.4	2.5	41.7	24.4	0.0	5.8	13.6	85.5
Nontraded sector	7.8	10.0	1.1	18.9	23.9	10.5	14.2	0.0	67.5
Mining sector	5.4	0.3	1.2	6.9	0.0	0.0	0.0	11.3	18.3
Domestic intermediate inputs	39.9	22.7	4.9	67.5	48.3	10.5	20.0	25.0	171.3
Imports	7.1	2.9	0.9	11.0	6.1	0.0	3.8	0.0	20.8
Customs and import duties	0.0	0.0	0.0	0.0	0.2	0.0	0.0	0.0	0.2
Indirect taxes	0.7	0.4	0.1	1.3	6.9	0.0	0.0	0.0	8.2
Total intermediate and final expenditure	47.7	26.1	5.9	79.7	61.5	10.5	23.8	25.0	200.6
Labor compensation and mixed income	16.6	31.7	4.0	52.3					
Gross operating surplus	21.2	9.7	8.4	39.3					
Value added at basic prices	37.8	41.4	12.4	91.6					
GROSS OUTPUT	85.5	67.5	18.3	171.3					

GDP INCOME APPROACH		GDP EXPENDITURE APPROACH	
(+) Value added	91.6	(+) Consumption	61.5
(+) Indirect Taxes	8.4	(+) Government	10.5
(=) GDP	100.0	(+) GFKF	23.8
		(+) Exports	25.0
		(-) Imports	-20.8
		(=) GDP	100.0

Source: Author's calculations based on OECD (2017).

Table 3. Calibration Targets and Basic Parameter Values for Colombia and Peru

PARAMETERS	Colombia	Peru	DESCRIPTION	SOURCE
γ	1.016	1.027	Gross rate of growth of per capita GDP	IMF (2017b) and World Bank (2017b)
δ_G	0.035		Infrastructure depreciation rate	IMF (2015a)
ε	0.455		(Inverse) Frisch elasticity of labor supply	Mendoza (1991)
θ	0.1		Output elasticity of public capital	Calderón et al. (2014) and Bom and Ligthart (2008)
λ	0.65		Fraction of liquidity-constrained households	López and Ortega (1998)
π	2.324		Risk premium coefficient with respect to external (public) debt to output ratio	Martínez, Terceño, and Teruel (2013)
ϖ_P	0.79		(Absolute value of) long-run price elasticity of export demand	Medina and Soto (2007)
ϖ_Y	1		Long-run income elasticity of export demand	
ρ	0.07		Country premium	Schmitt-Grohé and Uribe (2016), Uribe and Yue (2006)
σ	2		(Inverse) Intertemporal elasticity of substitution	Mendoza (1991)
ς	0.2		External habit persistence parameter	Uribe and Yue (2006)
ϕ	0.12		Congestion parameter	Rioja (2004)
$\omega_C = \omega_I$	1.5		Armington elasticity of susbstitution between domestic and imported goods	Backus, Kehoe, and Kydland (1994)
ω_D	0.75		Elasticity of substitution between tradables and nontradables	Ostry and Reinhart (1992)
ω_E	1		Elasticity of substitution in export aggregator	
VARIABLES/AGGREGATES				
b	23.1	11.7	Gross public internal debt (% GDP)	Banco de la República de Colombia (2018) and IMF (2014)
$d^* = \partial$	8.8	14.7	Private external debt (% GDP)	Banco de la República de Colombia (2018) and IMF (2014)
d^*_G	10.5	12.7	Gross public external debt (% GDP)	Banco de la República de Colombia (2018) and IMF (2014)
R	1.11		Domestic real interest rate (1+rate)	Schmitt-Grohé and Uribe (2016), Uribe and Yue (2006)
υ	0.608	1.066	Non-tax revenue from oil sector (% GDP)	Ministerio de Hacienda y Crédito Público (2017) and Ministerio de Economía y Finanzas (2017)
	0.854	0.223	Customs and import duties (% GDP)	OECD/ECLAC/CIAT/IDB (2017)
	5.262	6.917	Value added tax (% GDP)	OECD/ECLAC/CIAT/IDB (2017)
	4.8	6.5	Taxes on income and profits (% GDP)	OECD/ECLAC/CIAT/IDB (2017)
	0.951	1.551	Taxes on income and profits paid by individuals (% GDP)	OECD/ECLAC/CIAT/IDB (2017)
	1.017	1.017	Gross rate of growth of working-age population	World Bank (2017b)

Listed below are the calibration strategies followed for different sets of parameters or variables.

- *Steady state values of endogenous and exogenous variables*. Domestic GDP at market prices is chosen such that $GDP^R = 100$ and all relative prices are normalized to equal unity in the initial steady state:

$$w_1 = w_2 = w_3 = p_1 = p_2 = p_3 = p_{GDP} = p_I = p_E = p_D = p_M = tot = s = 1$$

By employing these assumptions together with Tables 1 and 2, steady state values for all national accounting aggregates and public and private debt levels are computed. Based on the average empirical estimates reported by López and Ortega (1998), the fraction of liquidity-constrained households ($\lambda = 0.65$) is used to estimate Ricardian (c^O) and non-Ricardian (c^{NO}) consumption aggregates. By setting wages to unity, indexes of labor input at aggregate and sectoral levels can be retrieved directly from the I-O table. Ricardian and non-Ricardian households are assumed to work for the same amount of time in the initial steady state.

Regarding the 21 exogenous variables, their steady state values are computed as follows. Steady state technology levels are normalized to unity ($A = A^1 = A^2 = A^3 = 1$), as well as exogenous relative prices ($p^*_{OIL} = p^*_M = 1$). The scale variable in the export demand equation is normalized at $y^* = 100$. The international gross interest rate R^* is set to 1.0374, which is implied by the domestic real interest rate ($R = R^W = 1.11$) and country risk premium $(1 + \rho = 1.07)$ used by Schmitt-Grohé and Uribe (2016) and Uribe and Yue (2006). The nominal ratios of government consumption spending to GDP (g_C) and government investment to GDP (g_I) are obtained from I-O tables and NIPA data, respectively. The share of domestic government debt (g_F) is set to 0.687 for COL and 0.48 for PER, based on information on public debt stocks by currency of denomination in year 2010 (see Table 3).

Effective tax rates are computed by the quotient of the specific tax revenue and the corresponding tax base. Consistent with revenue data (Tables 2 and 3), the following tax rates are calibrated:

	τ_1	τ_2	τ_3	τ_C	τ_C^M	τ_W	τ_K
COL	0.0126	0.0084	0.0039	0.2486	0.1818	0.0477	0.1089
PER	0.0084	0.0066	0.0051	0.3395	0.0369	0.0847	0.1260

Finally, lump-sum transfers are set to zero ($t^{NO} = t^O = 0$) and the percentage of GDP of non-tax revenues from natural resources is calibrated at $\upsilon = 0.609$ for COL and $\upsilon = 1.066$ for PER, based on data reported for year 2010 in the *Marco Fiscal de Mediano Plazo 2017* (Ministerio de Hacienda y Crédito Público, 2017) and the *Marco Macroeconómico Multianual 2018–2021* (Ministerio de Economía y Finanzas, 2017), respectively.

Gross growth rate factors are set at $\gamma = 1.016$ for COL and $\gamma = 1.027$ for PER, reflecting the 1990–2010 average per capita growth rates. GDP in constant local currency and population aged 15–64 figures are obtained from the IMF's World Economic Outlook (WEO) database (IMF, 2017b) and World Bank's World Development Indicator datasets (World Bank, 2017), respectively. The stock of public infrastructure capital (k_G) is estimated from the equilibrium capital accumulation equation in steady state (Annex 2, equation [A37]). Here, the depreciation rate of public capital is assumed to be $\delta_G = 0.035$, following the IMF (2015a)'s estimate for middle-income countries. Private investment is defined as the difference between total investment (from I-O table) and public investment (from national accounts). Total private capital stock is obtained from the aggregate version of equilibrium conditions [A23] and [A26] and aggregate gross operating surplus (I-O table). The sectoral distribution of the private capital stock is assumed to match the sectoral distribution of gross operating surplus, entailing identical rates of return on private physical capital $(u = u_i, \forall i)$ and depreciation rates $(\delta = \delta_i, \forall i)$ across sectors in the initial steady state. I estimated $\delta = 0.096$ for COL and $\delta = 0.060$ for PER.

- *Production technologies*. There is a fixed proportions technology to produce each sector gross output whose arguments are value added and intermediate goods. In calibrating the value added component, the elasticity of output with respect to infrastructure capital is set at $\theta = 0.10$ following Calderón, Moral-Benito, and Servén (2014) and Bom and Ligthart (2008). The congestion parameter ϕ is fixed at 0.12 as suggested by Rioja (2004). Sectoral capital shares in value added (α_i) are estimated using I-O data on:

$$\frac{\alpha_i}{(1-\alpha_i)} = \frac{gross\ operating\ surplus\ in\ sector\ i}{labor\ compensation\ and\ mixed\ income\ in\ sector\ i}$$

Scale parameters A_i, $i = \{1,2,3\}$ are set to match sector sizes in the steady state and do not influence the log-linearized representation of the model. Regarding the intermediate goods component, first order conditions [A7] to [A9] define technical coefficients in terms of I-O data on intermediate input use, as the ratio of input of sector j to sector i output:

$$\chi_{ji} = \frac{x_{ji}}{y_i} = \frac{intermediate\ flow_{j,i}}{gross\ output_i}$$

Similarly, Leontief import coefficients χ_{Mi} are estimated from sectoral import demand functions [A10]. Table 4 presents the calibrated parameter values.

The production functions for the domestic, investment, and consumption goods are specified as CES or Armington CES composites of two inputs. The Armington elasticity of substitution between domestic and imported goods $(\omega_C = \omega_I)$ is set to 1.5 following Backus, Kehoe, and Kydland (1994). As in Ostry and Reinhart (1992) the elasticity of substitution between traded and nontraded goods is set at $\omega_D = 0.75$. Share (μ_D, μ_I, μ_C) and shift (A_D, A_I, A_C) parameters are calibrated from I-O data allocations and intratemporal equilibrium conditions [A12]-[A20] evaluated at the steady state.

- *Preferences*. Parameter β, the subjective time discount factor, is set to 0.915 to ensure consistency with the real interest rate and growth rate factors as expressed by condition [A29]. The curvature parameter in the utility function is set to $\sigma = 2$ according to

Mendoza (1991). ε is calibrated at 0.455 (Mendoza, 1991), which implies a Frisch elasticity of 2.2. The external habit formation parameter ς is calibrated to 0.20 following Uribe and Yue (2006). The disutility weight parameters ψ_i^O and ψ_i^{NO} are chosen to match sectoral hours worked by Ricardian and non-Ricardian households according to conditions [A24] and [A22], respectively.

- *Stochastic structure*. All exogenous variables, expressed either as absolute deviations or log-deviations from trend,[7] are assumed to follow simple stationary AR(1) processes, with shocks drawn from independent normal distributions. Of particular relevance for the simulations conducted in the next section is the autocorrelation parameter associated with the natural resource commodity price process. Fernández, Schmitt-Grohé, and Uribe (2017) compute a serial correlation coefficient of 0.47 for the world real price of fuel and 0.52 for metals. Accordingly, the AR(1) parameter, $\rho_{p^*_{OIL}}$, is calibrated at 0.47 for COL and PER.

[7] The following exogenous variables are assumed to follow AR(1) processes defined in terms of absolute deviations: $\tau_{1,t}$, $\tau_{2,t}$, $\tau_{3,t}$, $\tau_{C,t}$, $\tau_{C,t}^M$, $\tau_{W,t}$, $\tau_{K,t}$, t_t^{NO}, t_t^O, υ_t, $g_{C,t}$, $g_{I,t}$, and $g_{F,t}$.

Table 4. Calibrated Parameter Values

PARAMETER	Colombia	Peru	PARAMETER	Colombia	Peru
γ	1.016	1.027	μ_C	0.912	0.907
σ	2	2	μ_D	0.486	0.535
ε	0.455	0.455	μ_E	0.602	0.546
ς	0.2	0.2	μ_I	0.84	0.839
θ	0.1	0.1	A_1	1.644	1.18
ϕ	0.12	0.12	A_2	1.324	1.268
λ	0.65	0.65	A_3	0.391	0.59
δ_G	0.035	0.035	A_ρ	0.07	0.07
π	2.324	2.324	A_C	1.014	1.003
ϖ_P	-0.79	-0.79	A_D	1	1
ϖ_Y	1	1	A_E	1.959	1.992
ω_C	1.5	1.5	A_I	1	1
ω_D	0.75	0.75	A_X	0.091	0.136
ω_E	1	1	χ_{11}	0.307	0.313
ω_I	1.5	1.5	χ_{12}	0.111	0.184
β	0.915	0.925	χ_{13}	0.044	0.139
α_1	0.42	0.56	χ_{21}	0.107	0.091
α_2	0.288	0.234	χ_{22}	0.189	0.148
α_3	0.821	0.679	χ_{23}	0.018	0.061
δ_1	0.096	0.06	χ_{31}	0.037	0.063
δ_2	0.096	0.06	χ_{32}	0.01	0.005
δ_3	0.096	0.06	χ_{33}	0.073	0.068
ψ_1^{NO}	0.265	0.278	χ_{M1}	0.076	0.083
ψ_2^{NO}	0.194	0.207	χ_{M2}	0.036	0.043
ψ_3^{NO}	0.825	0.535	χ_{M3}	0.013	0.052
ψ_1^{O}	0.202	0.191	$\eth$	0.105	0.127
ψ_2^{O}	0.148	0.142	∂	8.8	14.7
ψ_3^{O}	0.629	0.365	η_{D^*}	0.0001	0.0001

Based on frequently used calibrations in the literature, productivity shocks are assumed to be highly persistent ($\rho_A, \rho_{A^1}, \rho_{A^2}, \rho_{A^3} = 0.95$). The persistence parameter for the aggregate foreign output, ρ_{y^*}, is set at 0.75 following Aguiar and Gopinath (2007), who estimate it at 0.75 for the average of developed countries and at 0.76 for the average of emerging market economies. The persistence parameter for the foreign interest rate ρ_{R^*} is set at 0.81 adopting Neumeyer and Perri (2005)'s regression estimate. The lack of reliable data and the lack of empirical estimates are a major hindrance to calibrate other shock processes. It is well known that models like the one specified here lack an effective, endogenous, internal propagation mechanism to transform temporary shocks into highly persistent output responses. Aggregate output essentially inherits the persistence of the exogenous processes. To avoid failing in this regard, the persistence parameter for the remaining exogenous variables is set at 0.50, the persistence parameter for per capita GDP estimated by Agénor, McDermott, and Prasad (1999).

The methodological approach followed in this paper has been heavily influenced by the Great Depressions literature (Kehoe and Prescott, 2007), which conducts scenario analysis by plugging in to the model (typically a perfect foresight neoclassical growth model) the evolution of exogenous variables (generally total factor productivity), as measured in the data, to simulate the response of the endogenous variables (generally GDP). Consistent with this literature, parameter calibration matches steady state features established from data that exclude the period under study.

4. Policy Experiments

The nonlinear rational expectations model is log-linearized around its deterministic steady state and solved using standard numerical techniques. The solution consists of a set of linear difference equations relating the current endogenous variables to the state vector—a vector containing the exogenous variables and some lagged endogenous variables. The economy is driven by a $21x1$ vector of forcing variables (listed in Table 5), which includes world business cycle and price shocks, domestic technology shocks, and fiscal policy shocks.

The simulations conducted in this section seek to illustrate the use of the model as a tool to quantify the scale of the fiscal challenges, to provide consistent medium-term macroeconomic and fiscal projections, and to assess the quantitative implications of past reforms and alternative fiscal policy plans on the economy over the short and medium term. Simulations are sequentially run in the sense that the setup of one simulation serves as a starting point for the next. The operation of the model is illustrated with the experiences of Colombia and Peru.

Table 5. List of Exogenous Variables in the Model

	Variable
A_t	TFP around long-term trend
A_t^1	Sectoral TFP on top of aggregate TFP
A_t^2	Sectoral TFP on top of aggregate TFP
A_t^3	Sectoral TFP on top of aggregate TFP
$p_{OIL,t}^*$	World relative price of oil
$p_{M,t}^*$	World relative price of imports
y_t^*	World GDP index
R_t^*	Gross world real interest rate
$g_{C,t}^{exo}$	Government current expenditures (% of GDP)
$g_{I,t}^{exo}$	Government capital spending (% of GDP)
$g_{F,t}$	Share of domestic currency public debt in total public debt
$\tau_{1,t}$	Tax rate on sector 1 gross output
$\tau_{2,t}$	Tax rate on sector 2 gross output
$\tau_{3,t}$	Tax rate on sector 3 gross output
$\tau_{C,t}$	Value added tax rate
$\tau_{C,t}^M$	Import tariff rate
$\tau_{W,t}$	Labor income tax rate
$\tau_{K,t}$	Corporate income tax rate
t_t^{NO}	Government transfers to non-Ricardian households (% of GDP)
t_t^O	Government transfers to Ricardian households (% of GDP)
υ_t	Government non-tax revenue from commodity sector (% of GDP)

Note: TFP = total factor productivity.

Both economies are highly dependent on commodities with substantial mineral or hydrocarbon sectors, and both were hit hard by the most recent collapse of commodity prices. The commodity price bust began in mid-2011 and has been the main driving force behind the business cycle. After more than half a decade of persistently low commodity prices, both countries are still struggling to restore growth and rebalance it toward non–natural resource sectors. Government budgets have been adversely impacted as commodity-related revenues and economic activity promptly reflected the effect of weak commodity markets. Before considering policy responses and the effects of subsequent fiscal consolidation efforts, the first simulation is intended to simulate the impact on public finances of the most recent collapse in commodity prices. The simulated macro fiscal aggregates are obtained by feeding into the model a commodity price trajectory that replicates its actual realization from 2010 through 2016 and its expected path afterwards, as forecast by the World Bank.

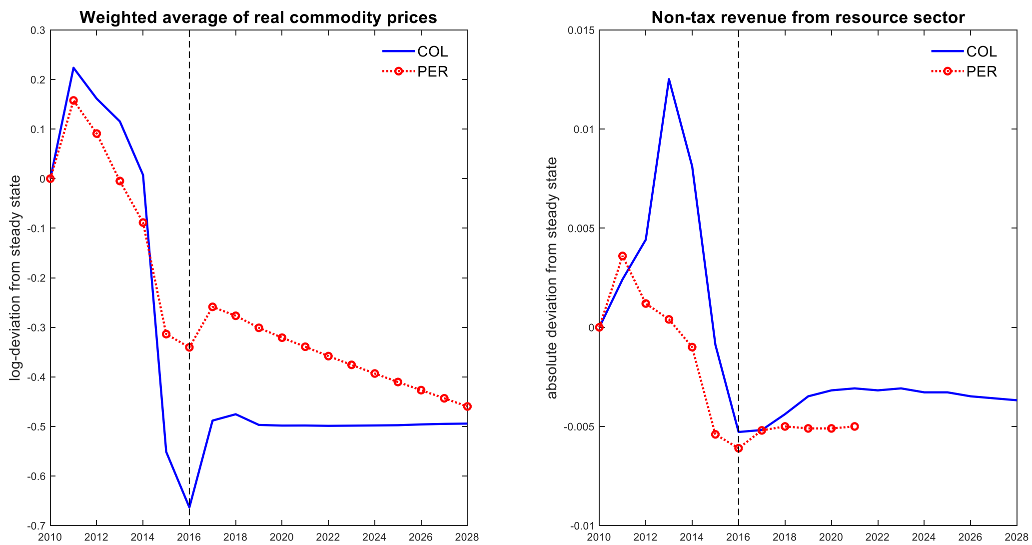
Besides calibration differences pointed out in the preceding section, the two countries differ in two additional respects. First, fiscal targets, ceilings, and projections specified in the MTFF, as well as in the fiscal rule and in other budget institutions, are designed to cover the general government sector in Peru, while in Colombia coverage is circumscribed to central government operations. Consequently, some models' definitions of fiscal aggregates are adjusted in the Colombian case to better approximate results for the central government level. Secondly, Peru uses the traditional Laspeyres fixed base year while Colombia uses a Laspeyres chain-weighted procedure to measure GDP growth. The model's real GDP accounting is adjusted accordingly (see section 2.5).

4.1 Simulation 1: Fiscal Impact of the Recent Boom-Bust Commodity Price Cycle

Strictly speaking, a commodity price shock in the model is described by two related exogenous processes: the world relative price of the commodity good, $p^{*}_{OIL,t}$, and the government non-tax revenue receipts from the commodity sector as a percentage of GDP, υ_t. For Peru, the commodity price is calculated as the weighted average of cooper, tin, iron ore, gold, silver, lead, zinc, crude oil, and natural gas prices, weighted with the average export shares for 2010–2016. The Colombia weighted average includes prices of coal, crude oil, and ferronickel. Average commodity prices are deflated by the U.S. Consumer Price Index (CPI) and normalized to unity in 2010. The source for commodity price data, both historical (2010–2016) and forecasts (2017–2028), is the World Bank's *Commodity Markets Outlook* (World Bank, 2018); for the U.S. CPI, the source is the IMF's World Economic Outlook (WEO) database (IMF, 2017b) from 2010 to 2022, with the data extrapolated up to 2028 using the inflation rate implicit in WEO's 2022 CPI forecast. Commodity export volumes are obtained from the central banks' official websites (Banco de la República de Colombia, 2018; Banco Central de Reserva del Perú, 2018). υ_t is taken directly from Colombia's *Marco Fiscal de Mediano Plazo* (MFMP) (Ministerio de Hacienda y Crédito Público, 2017) and Peru's *Marco Macroeconómico Multianual* (MMM) *2018–2021* (Ministerio de Economía y Finanzas, 2017). In the case of Peru, this variable mainly comprises mineral, oil, and gas royalties levied by the general government on mining concessions, while in the case of Colombia it corresponds to dividend payments to the central government from ECOPETROL, the large majority state-owned oil company.

Figure 2 depicts the behavior of the commodity price index in log-deviations from the steady state and non-tax revenues from natural resource extraction (relative to GDP) in absolute deviation from the 2010 steady state ratio. Despite differences in composition of the countries' commodity production baskets, the recent boom-and-bust cycle is highly correlated. However, the boom-bust pattern seems to be more severe in the case of Colombia. The timing of non-tax revenues is also somewhat different. In Peru, royalty collection started falling in tandem with mineral prices since 2011, while in Colombia, the distribution of ECOPETROL dividends peaked in 2013 and fell quite dramatically with the oil price collapse of 2015.

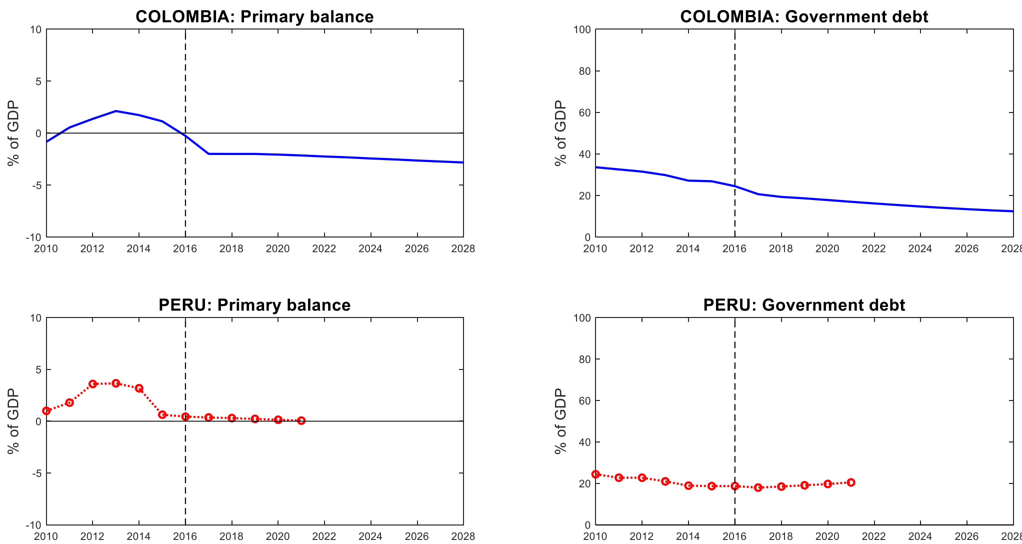
Figure 2. Commodity Shock Processes: Non-Tax Revenues and Commodity Prices



In this experiment, the response of policymakers has been muted. Discretionary government spending is frozen at 2010 levels as a share of GDP. On the other hand, nondiscretionary fiscal policy plays a limited role in the stabilization of the business cycle through the automatic stabilizing action of constant tax rates. Transfer payments to households, another automatic stabilizer, are set to zero in all simulations.

Figure 3 shows the fiscal effect of the commodity price cycle. If prices and non-tax receipts had not changed, the government primary balance and debt outstanding would have remained constant at their 2010 steady state levels. In contrast, under the stated experimental conditions, the totality of the revenue windfall at the end the upward phase of the commodity cycle is saved as well as the tax revenues stemming from the operation of automatic stabilizers in a booming economy, yielding a stream of positive primary balances and declining debt-GDP ratios. The accumulated fiscal buffer would have been sizable enough to weather the subsequent bust phase without threatening the solid fiscal stance. In the following simulations, policy responses are introduced into the picture.

Figure 3. Automatic Fiscal Response to the Boom-Bust Cycle



4.2 Simulation 2: Policy Response (Part I)—Discretionary Government Spending

In the model, discretionary government spending is captured by two processes: $g_{C,t} = g_{C,t}^{exo}$ and $g_{I,t} = g_{I,t}^{exo}$, the shares in GDP of government current and capital expenditures, respectively. As noted before, expenditure aggregates comprise expenditures made by the central government in the case of Colombia and by the general government in Peru. The data sources are the MFMP (Ministerio de Hacienda y Crédito Público, 2017) and MMM 2018–2021 (Ministerio de Economía y Finanzas, 2017). The time series are constructed by splicing together post-2016 forecasts published in MTFF documents with observed data from 2010 to 2016, and all expressed as absolute deviations from the corresponding initial steady states.

Figure 4 depicts a similar pattern of procyclical policy response in both countries along the commodity price cycle. Expenditures exhibit a hump-shaped response and the timing of peaks coincides in both countries, taking place in 2014 for current expenditures and in 2013 for capital spending. The 2010-to-peak increase in current expenditures is close to 2.5 percent of GDP and around 1 percent of GDP in capital investment. An important difference in the behavior of expenditure processes lies in the episode of fiscal retrenchment during the bust phase. While Peru managed to fully undo the increase in expenditures over the course of two years, Colombian discretionary expenditures have been only partially reversed, exhibiting higher persistence. Relative to current policies, defined by the latest observed data (2016), over the post-2016 period the Colombian MTFF foresees severe cutbacks of 1.3 percent of GDP in current expenditures and 0.87 percent of GDP in public investment.[8] In contrast, Peru's MTFF foresees a recovery in capital outlays financed by reducing non-investment spending.

Figure 4. Fiscal Policy Response: Discretionary Expenditures

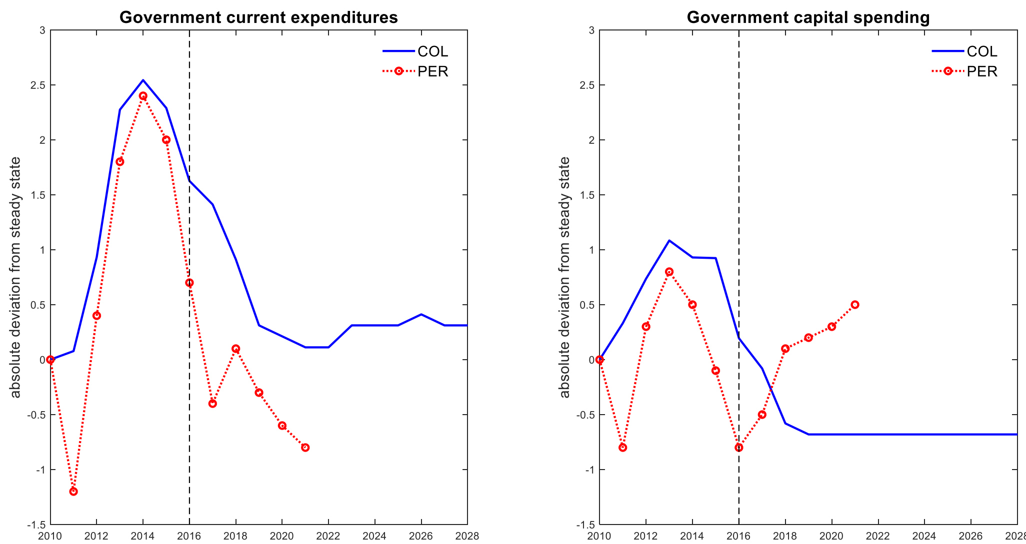
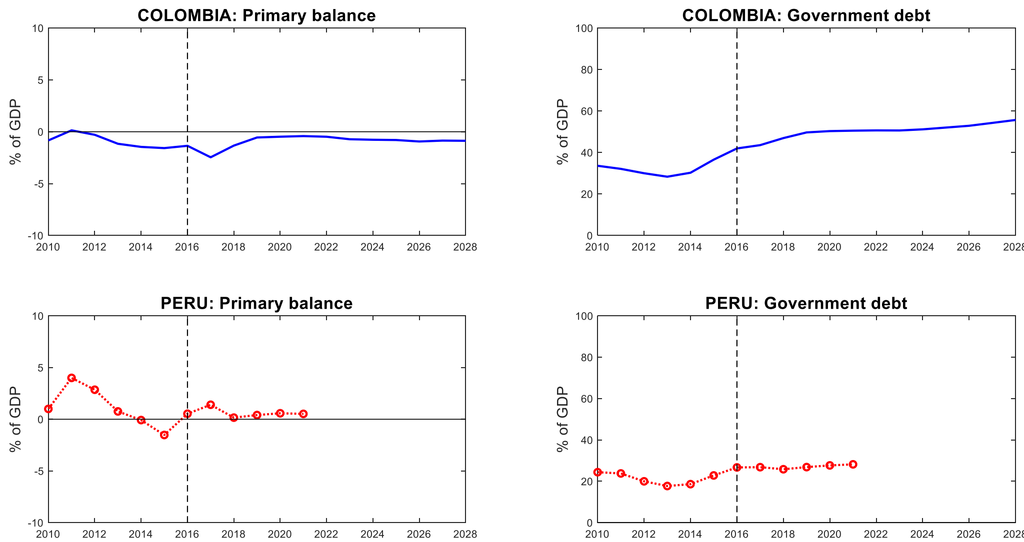


Figure 5 reports the estimated impacts on selected fiscal aggregates when the economies are hit by commodity shocks $\{p^*_{OIL}, v_t\}$ and discretionary expenditure responses $\{g_{C,t}, g_{I,t}\}$. Here it is important to bear in mind that simulation results not only depend on exogenous variables but also on the starting fiscal position and growth prospects. In this regard, Peru is better positioned: the calibrated steady state growth rate is higher, and the initial primary deficit and public debt are lower than in Colombia.

[8] The implications of a no-policy-change scenario after 2016 are assessed in Simulation 4.

Figure 5. Fiscal Effects of Discretionary Expenditure Response to Commodity Cycle



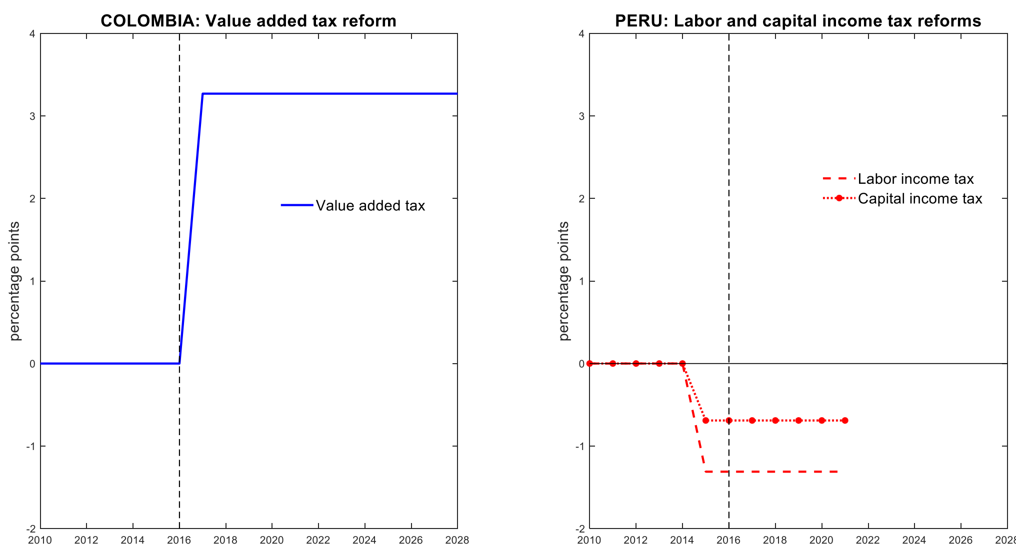
Despite the apparently similar magnitude and profile of the driving forces, the two economies obey different debt dynamics. After a slight increase from 2013, Peru's net government debt swiftly stabilized at a relatively low level, implying that the devised MTFF expenditure plan is consistent with debt sustainability under the prevailing conditions of inherited debt, tax collection, and macroeconomic outlook. Under the prevailing conditions in Colombia, government debt trends upwards over the simulation period, leading to concerns over the risk of stabilizing it at relatively high levels. As a result, Colombian authorities used discretionary tax policy to flatten the debt dynamics.

4.3 Simulation 3: Policy Response (Part II)—Discretionary Tax Reform

Tax reform is introduced into the model by setting up the time path of changes for the exogenous tax vector $\{\tau_{1,t}, \tau_{2,t}, \tau_{3,t}, \tau_{C,t}, \tau_{C,t}^{M}, \tau_{W,t}, \tau_{K,t}\}$ relative to its initial steady state. Peru enacted a countercyclical income tax reform that reduced government revenues from 2015 onwards. The IMF estimates a revenue fall of approximately 0.51 percent of GDP, using a static scoring methodology (IMF, 2015b). Nearly half of it (0.27% of GDP) is explained by a fall in the corporate income tax and, the rest, by an individual income tax cut. As shown in the left-hand panel of Figure 6, these changes translate into a reduction of 0.69 and 1.31 percentage points of the effective corporate income tax rate, $\tau_{K,t}$, and the personal income tax rate, $\tau_{W,t}$, respectively. On December 2016, a tax reform was enacted through special powers granted by Congress to legislate on tax matters. The reform package, not considered in Simulation 3, is estimated to yield additional revenues for 0.3 percent of GDP in 2017 and 0.1 percent of GDP in 2018 (Banco Central de Reserva del Perú, 2017). As a result, Simulation 3 overestimates the effective reduction in taxes.

On the contrary, Colombia increased the standard VAT rate by 3 percentage points from 2017 onwards. It adds 0.80 percent of GDP in central government revenue, according to the IMF's "static scoring" estimate (IMF, 2017a). In terms of the model's variables, the effective tax rate τ_C increases permanently by 3.27 percentage points (see right-hand panel of Figure 6). Though it is the most important revenue-generating tax change introduced into the tax code since the oil price downturn started, it is indeed a partial and incomplete representation of the tax policy implemented. Enacted legislation has also reduced upcoming taxes. The corporate income tax rate was reduced from 40 percent in 2016[9] to 33 percent as of 2019 and the net wealth tax has been scrapped for corporations starting in 2018 and for individuals as of 2019. The value added tax paid on capital good purchases was made fully deductible from the income tax liability since 2017, reducing the overall tax bill. In the ensuing simulation, tax cut and expiring tax provisions are ignored, or equivalently, they are assumed to be replaced by the same sources of revenue. In the case of Colombia, Simulation 3 clearly overestimates the actual effort of the authorities to raise additional revenue.

Figure 6. Fiscal Policy Response: Discretionary Tax Reform

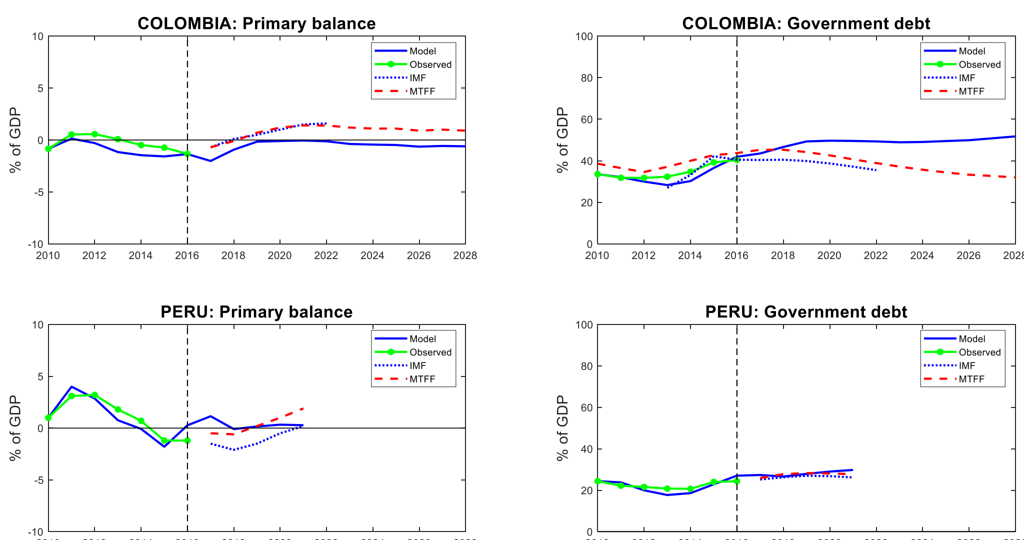


[9] The mentioned CIT rate includes income tax (25%), CREE tax (9%), and CREE surtax (6%) rates. CREE is the Spanish acronym for the "Fairness Tax."

This simulation estimates the dynamic response of an economy that has experienced a series of commodity shocks $\{p^*_{OIL}, \upsilon_t\}$, discretionary expenditure shocks $\{g_{C,t}, g_{I,t}\}$, and discretionary tax shocks (COL: $\{\tau_{C,t}\}$ and PER: $\{\tau_{K,t}, \tau_{W,t}\}$) similar in magnitude and timing to those observed, estimated, or projected in the data. Results are illustrated in Figure 7 (COL and PER: public finances), Figure 8 (COL: other macroeconomic aggregates), and Figure 9 (PER: other macroeconomic aggregates). In addition to model predictions, the figures also depict historical data up to 2016, and MTFF and IMF Article IV projections for the few selected macro-fiscal variables they report on.

The tax hike in Colombia enables the economy to reduce the pace of government bond issuance by improving primary balances (Figure 7), but still, the debt ratio trend seems to level off at a level higher than before the collapse of commodity prices. This prediction stands in stark contrast to projections of MFMP (Ministerio de Hacienda y Crédito Público, 2017) and IMF (2017a), for which the debt ratio is expected to revert to its pre-shock level. The difference is not explained by expenditure-related differences (not shown in figures) because primary expenditure paths are practically identical, but it comes down to differences in projected revenues. Both the MFMP (Ministerio de Hacienda y Crédito Público, 2017) and IMF (2017a) assume higher growth rates for the economy (see Figure 8), yielding positive and sustained primary surpluses after 2018/2019 and for the remaining forecasting period.

Figure 7. Fiscal Effects of Policy Responses and Commodity Cycle



Besides major fiscal aggregates, neither MFMP (Ministerio de Hacienda y Crédito Público, 2017) nor IMF (2017a) provide informative inputs to decision makers about the underlying behavior of the economy. For them, the growth rate of the economy is an assumption and the path of the economy is independent of fiscal developments. Figure 8 shows the evolution of some selected macroeconomic aggregates whose endogenous behavior may shed some additional light on the tradeoffs that policymakers face. The model does not do a good job at matching bilateral real exchange rate data. It considerably underestimates the real depreciation that occurred in the first years following the price shock. Along other dimensions the model performs remarkably well. Just a handful of exogenous variables provide a good account of the behavior of variables such as trade balance, terms of trade, real wage, and hours worked.

As expected, in the case of Peru, the reduction in taxes does have very small effects and the fiscal and macroeconomic panoramas do not change much relative to the preceding simulation of no tax policy reform. The model, the MTFF, and the IMF Article IV staff report provided very close fiscal projections (Figure 7). Regarding other macroeconomic aggregates (Figure 9), with only six exogenous processes, the model can match quite well the path of their observed counterparts since 2010. The only considerable difference is in aggregate consumption, where it simply reflects the different definitions of consumption used in the model and in the national income accounts (at market prices).

Figure 8. Colombia: Behavior of Macroeconomic Aggregates

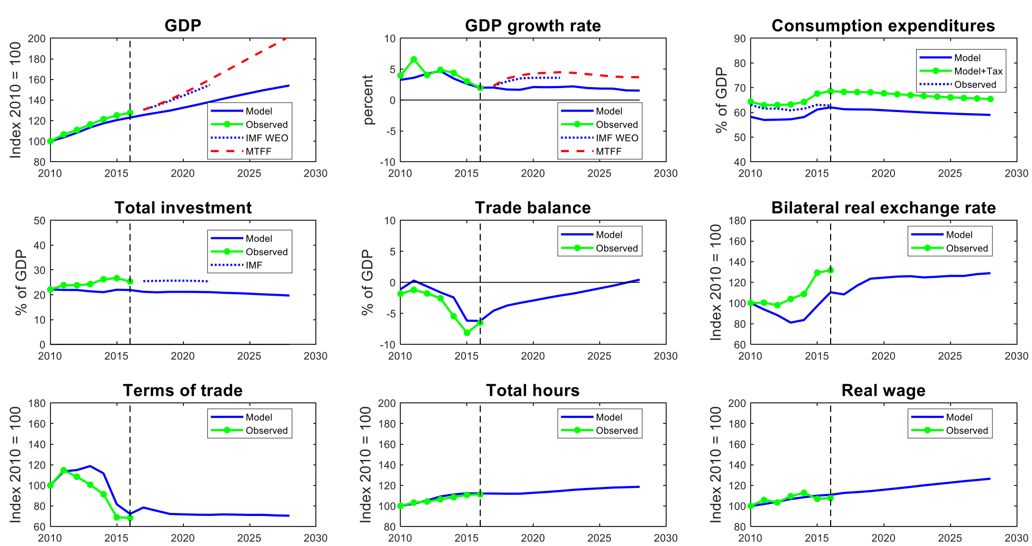
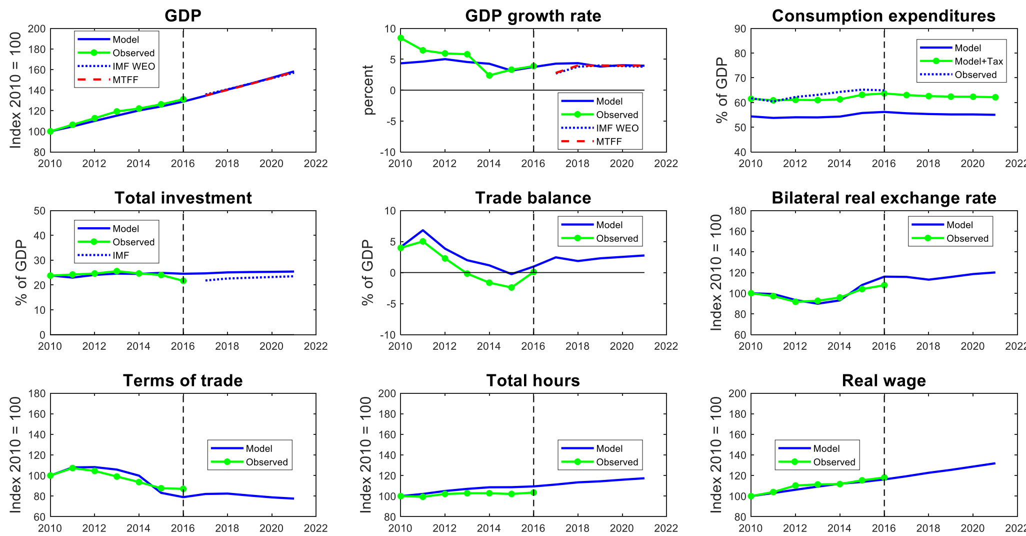


Figure 9. Peru: Behavior of Macroeconomic Aggregates



4.4 Simulation 4: No-Policy-Change Scenario

The different projections for Colombia (model, MTFF document, and IMF Article IV report) use roughly the same primary expenditure plan. It is the path designed in the last MFMP (Ministerio de Hacienda y Crédito Público, 2017) and, as previously highlighted, this fiscal plan foresees substantial spending cuts over the post-2016 period. The natural question that arises is how the MFMP (Ministerio de Hacienda y Crédito Público, 2017) and IMF (2017a) result—that the government debt ratio will revert to its pre-shock level—depends on the assumed expenditure path. How will the assumed spending plan limit the freedom of movement of future governments?

The no-policy-change scenario assumes that the latest observed policy (2016), i.e., $\{g_{C,2016}^{exo}, g_{I,2016}^{exo}\}$, is maintained during the whole forecasting horizon. Because no mandatory future expenditure cuts have been passed through Congress, the last observed policy is an indication of the fiscal consolidation effort effectively carried out, in contrast to a simply assumed scenario. Figure 10 shows the time path of the alternative expenditure plan.

Figure 11 shows model simulation results under the no-policy-change scenario. MTFF planned expenditure cuts are required to stabilize the debt ratio and prevent fiscal vulnerabilities and risks from increasing. Future governments will be pressured to act: if the currently policy is maintained, the fiscal deficit will erode, worsening the debt dynamics.

Figure 10. Colombia: No-Policy-Change Scenario

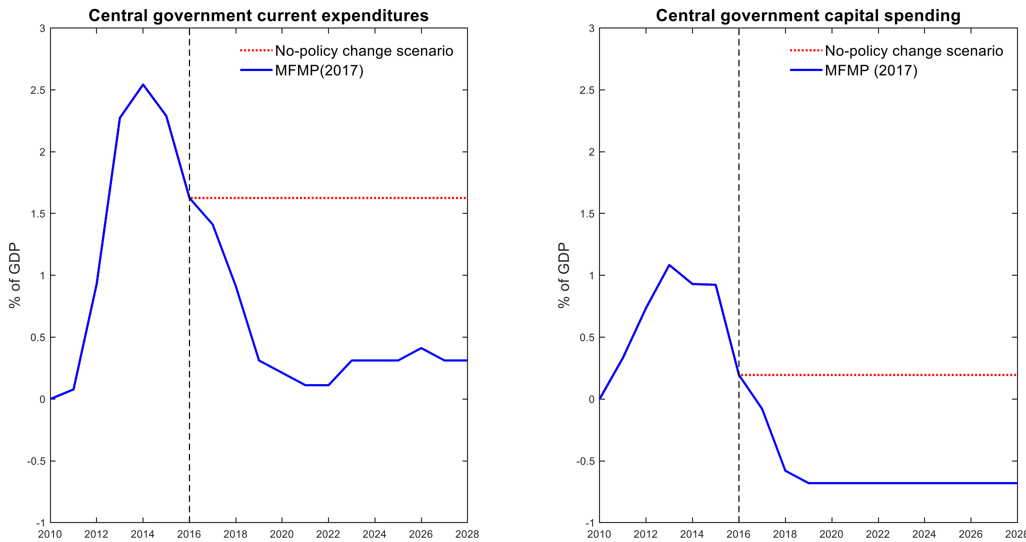
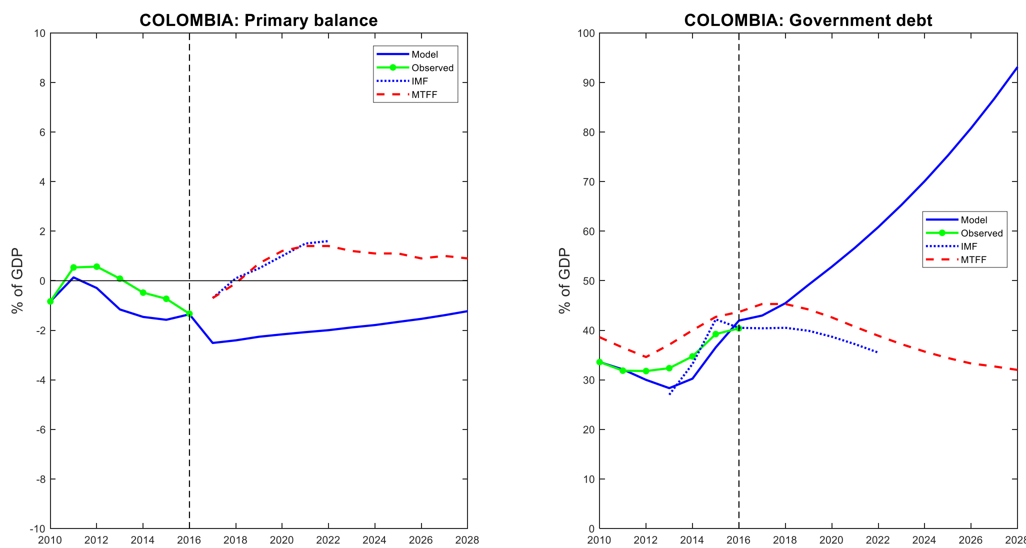


Figure 11. Colombia: Public Finances Under the No-Policy-Change Scenario



5. Conclusions

In contrast to central banks and international organizations, policymakers at the ministry of finance and practitioners, in general, in emerging market and developing economies have limited resources and a shortage of trained personnel to improve or develop the type of analytical tools usable for extending the horizon of fiscal policy analyses. The IDB's Fiscal Division has supported the development of such a tool to assist the LAC region in building a sensible and logically consistent MTFF. An important consideration in building the tool is to offer a reasonable alternative to the widely used standardized debt sustainability framework jointly developed by the IMF and World Bank. The model was designed to reflect three strategic priorities: to be readily usable by a set of diverse countries as a laboratory for the analysis of fiscal plans, to reduce adoption and learning costs for policy analysts, and to capture relevant dynamics of the economies. All these motivate crucial modeling choices, some of which can be considered as the model's weak points. As a result, the model can be refined along several dimensions by relaxing some simplifying assumptions. Among the weak points, it is worth mentioning the purely real environment of the model economy and the loose strategy adopted for the calibration of parameters whose values are not very well established empirically.

This paper has outlined a calibrated model of a small open economy to serve as a simulation tool to provide a medium-term perspective to fiscal policy assessments. The FMM-MTFF is a dynamic general equilibrium model that is fairly in line with other existing models. However, some prominent features differentiate this model from the existing ones. First, fiscal policy is defined in terms of multi-year fiscal plans whose magnitude and timing are like those observed, estimated, or projected in the data, instead of restricting attention to univariate fiscal shocks. Second, the model does not impose the straightjacket of a standard fiscal rule. Under a standard fiscal rule, fiscal policy is countercyclical and sustainable by design and any fiscal challenge is mechanically addressed. Third, the model is calibrated to match a three-sector stylized version of a country's input-output (I-O) table, which provides a consistent framework on industry output, intermediate input flows, and final demand use data. Four, the model embeds a more realistic GDP measurement framework that is consistent with what national account compilers do. The model uses a chain index method to aggregate real GDP.

The recent experiences of Colombia and Peru are examined to illustrate the use of the model as a tool to quantify the scale of the fiscal challenges, to provide consistent medium-term projections, and to assess the quantitative implications of past reforms and alternative fiscal policy plans on the economies. This type of improved understanding of the macro fiscal conditions will be helpful in the preparation of a more sensible MTFF.

References

Abel, A. 1990. Asset Prices under Habit Formation and Catching Up with the Joneses. *American Economic Review* 80(2): 38–42.

Adam, C., and D. Bevan. 2014. Public Investment, Public Finance, and Growth: The Impact of Distortionary Taxation, Recurrent Costs, and Incomplete Appropriability. IMF Working Paper WP/14/73. Washington, D.C.: International Monetary Fund.

Agénor, R., J. McDermott, and E. Prasad. 1999. Macroeconomic Fluctuations in Developing Countries: Some Stylized Facts. IMF Working Paper WP/99/35. Washington, D.C.: International Monetary Fund.

Aguiar, M., and G. Gopinath. 2007. Emerging Market Business Cycles: The Cycle Is the Trend. *Journal of Political Economy* 115: 69–102.

Atolia, M., B. Li, R. Marto, and G. Melina. 2017. Investing in Public Infrastructure: Roads or Schools? IMF Working Paper WP/17/105. Washington, D.C.: International Monetary Fund.

Backus, D., P. Kehoe, and F. Kydland. 1994. Dynamics of the Trade Balance and the Terms of Trade: The J-Curve? *American Economic Review* 84(1): 84–103.

Banco Central de Reserva del Perú. 2017. Panorama Actual y Proyecciones Macroeconómicas 2017–2018. Reporte de Inflación, March. Lima: Banco Central de Reserva del Perú.

———. 2018. Website. http://www.bcrp.gob.pe/.

Banco de la República de Colombia. 2018. Website. http://www.banrep.gov.co/.

Berg, A., R. Portillo, S. Yang, and F. Zanna. 2012. Public Investment in Resource-Abundant Developing Countries. IMF Working Paper WP/12/274. Washington, D.C.: International Monetary Fund.

Bom, P., and J. Ligthart. 2008. How Productive Is Public Capital? A Meta-Analysis. CESIfo Working Paper No. 2266. Tilburg, Netherlands: Tilburg University.

Buffie, E., A. Berg, C. Pattillo, R. Portillo, and F. Zanna. 2012. Public Investment, Growth, and Debt Sustainability: Putting Together the Pieces. IMF Working Paper WP/12/144. Washington, D.C.: International Monetary Fund.

Calderón, C., E. Moral-Benito, and L. Servén. 2014. Is Infrastructure Capital Productive? A Dynamic Heterogenous Approach. *Journal of Applied Econometrics* 30(2): 177–98.

Calderón, C., and K. Schmidt-Hebbel. 2008. Business Cycles and Fiscal Policies: The Role of Institutions and Financial Markets. Working Paper No. 481. Santiago: Central Bank of Chile.

Christiano, L., M. Eichenbaum, and C. Evans. 2005. Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy. *Journal of Political Economy* 113(1): 1–45.

Clark, W., and M. Rosales. 2013. The Investment-Financing-Growth Nexus: The Case of Liberia. IMF Working Paper WP/13/237. Washington, D.C.: International Monetary Fund.

Del Negro, M., and S. Schorfheide. 2003. Take Your Model Bowling: Forecasting with General Equilibrium Models. *Economic Review* 88(4): 35–50.

Development Committee. 2015. The Role of the IMF in Supporting the Implementation of the Post-2015 Development Agenda. IMF DC2015-0005, September. Washington, D.C.: International Monetary Fund.

Fernández, A., S. Schmitt-Grohé, and M. Uribe. 2017. World Shocks, World Prices, and Business Cycles: An Empirical Investigation. *Journal of International Economics* 108: S2–S14.

Galí, J., D. López-Salido, and J. Vallés. 2004. Rule-of-Thumb Consumers and the Design of Interest Rate Rules. *Journal of Money, Credit, and Banking* 36(4): 739–63.

———. 2007. Understanding the Effects of Government Spending on Consumption. *Journal of the European Economic Association* 5(1): 227–70.

Gavin, M., and R. Perotti. 1997. Fiscal Policy in Latin America. In B. Bernanke and J. Rotemberg (eds.), *NBER Macroeconomics Annual*. Cambridge, MA: MIT Press.

Glomm, G., and B. Ravikumar. 1994. Public Investment in Infrastructure in a Simple Growth Model. *Journal of Economic Dynamics and Control* 18(6): 1173–88.

———. 1997. Productive Government Expenditures and Long-Run Growth. *Journal of Economic Dynamics and Control* 21(1): 183–204.

Greenwood, J., Z. Hercowitz, and W. Huffman. 1988. Investment, Capacity Utilization, and the Real Business Cycle. *American Economic Review* 78(3): 402–17.

Ilzetzki, E., and C. Végh. 2008. Procyclical Fiscal Policy in Developing Countries: Truth or Fiction? NBER Working Paper No. 14191. Cambridge, MA: National Bureau of Economic Research.

International Monetary Fund (IMF). 2014. *Peru. Staff Report for the 2013 Article IV Consultation.* Washington, D.C.: International Monetary Fund.

———. 2015a. *Making Public Investment More Efficient.* Washington, D.C.: International Monetary Fund.

———. 2015b. *Peru: Selected Issues.* Washington, D.C.: International Monetary Fund.

———. 2017a. *Colombia. 2017 Article IV Consultation.* Washington, D.C.: International Monetary Fund.

———. 2017b. World Economic Outlook (WEO) database. Available at https://www.imf.org/external/pubs/ft/weo/2017/02/weodata/index.aspx.

Issoufou, S., E. Buffie, M. Diop, and K. Thiaw. 2014. Efficient Energy Investment and Fiscal Adjustment in Senegal. IMF Working Paper WP/14/44. Washington, D.C.: International Monetary Fund.

Jaimovich, N., and S. Rebelo. 2009. Can News about the Future Drive the Business Cycle? *American Economic Review* 99(4): 1097–1118.

Kaufmann, J., M. Sanginés, and M. García. 2015. Construyendo Gobiernos Efectivos. Logros y Retos de la Gestión Pública para Resultados en América Latina y el Caribe. Washington, D.C.: Inter-American Development Bank.

Kehoe, T., and E. Prescott (eds.). 2007. *Great Depressions of the Twentieth Century.* Minneapolis, MN: Federal Reserve Bank of Minneapolis.

Kollmann, R. 2002. Monetary Policy Rules in the Open Economy: Effects on Welfare and Business Cycles. *Journal of Monetary Economics* 49(5): 989–1015.

Leeper, E. 2017. Fiscal Analysis Is Darned Hard. In L. Ódor, *Rethinking Fiscal Policy after the Crisis.* Cambridge, MA: Cambridge University Press.

López, A., and R. Ortega. 1998. Private Saving in Colombia. IMF Working Paper WP/98/171. Washington, D.C.: International Monetary Fund.

Martínez, L., A. Terceño, and M. Teruel. 2013. Sovereign Bonds Spreads Determinants in Latin American Countries: Before and During the XXI Financial Crisis. *Emerging Markets Review* 17: 60–75.

Medina, J., and C. Soto. 2007. The Chilean Business Cycles through the Lens of a Stochastic General Equilibrium Model. Working Paper No. 457. Santiago: Central Bank of Chile.

———. 2016. Commodity Prices and Fiscal Policy in a Commodity Exporting Country. *Economic Modelling* 59(C): 335–51.

Melina, G., and Y. Xiong. 2013. Natural Gas, Public Investment, and Debt Sustainability in Mozambique. IMF Working Paper WP/13/261. Washington, D.C.: International Monetary Fund.

Melina, G., S. Yang, and F. Zanna. 2014. Debt Sustainability, Public Investment, and Natural Resources in Developing Countries: The DIGNAR Model. IMF Working Paper WP/14/50. Washington, D.C.: International Monetary Fund.

Mendoza, E. 1991. Real Business Cycles in a Small Open Economy. *American Economic Review* 81(4): 797–818.

Ministerio de Economía y Finanzas. 2017. *Marco Macroeconómico Multianual 2018–2021.* Lima: Ministerio de Economía y Finanzas.

Ministerio de Hacienda y Crédito Público. 2017. *Marco Fiscal de Mediano Plazo 2017.* Bogotá: Ministerio de Hacienda y Crédito Público.

Mu, Y. 2012. Achieving an Appropriate Balance among Public Investment, Growth, and Debt Sustainability in Cape Verde. IMF Working Paper WP/12/280. Washington, D.C.: International Monetary Fund.

Neumeyer, P., and F. Perri. 2005. Business Cycles in Emerging Economies: The Role of Interest Rates. *Journal of Monetary Economics* 52(2): 345–80.

OECD. 2017. Input-Output Tables (IOTs). http://www.oecd.org/trade/input-outputtables.htm.

OECD/ECLAC/CIAT/IDB. 2017. *Revenue Statistics in Latin America and the Caribbean 1990–2015.* Paris: OECD.

Ostry, J., and C. Reinhart. 1992. Private Saving and Terms of Trade Shocks: Evidence from Developing Countries. IMF Staff Papers, pp. 495–517.

Rioja, F. 2004. Infrastructure and Sectoral Output along the Road to Development. *International Economic Journal* 18(1): 49–64.

Sargent, T. 1978. Estimation of Dynamic Labor Demand Schedules under Rational Expectations. *Journal of Political Economy* 86(6): 1009–44.

Schmitt-Grohé, S., and M. Uribe. 2003. Closing Small Open Economy Models. *Journal of International Economics* 61(1): 163–85.

———. 2016. How Important Are Terms of Trade Shocks? NBER Working Paper No. 21253. Cambridge, MA: National Bureau of Economic Research.

Shen, W., S. Yang, and F. Zanna. 2015. Government Spending Effects in Low-Income Countries. IMF Working Paper WP/15/286. Washington, D.C.: International Monetary Fund.

Stone, R. 1961. Input-Output and National Accounts. Paris: Organization for European Economic Cooperation.

Stone, R., and A. Brown. 1962. A Computable Model of Economic Growth (A Programme for Growth, Vol. I). London: Chapman and Hall.

Tornell, A., and P. Lane. 1999. The Voracity Effect. *American Economic Review* 89(1): 22–46.

Uribe, M., and V. Yue. 2006. Country Spreads and Emerging Countries: Who Drives Whom? *Journal of International Economics* 69(1): 6–36.

World Bank. 2013. Beyond the Annual Budget: Global Experience with Medium Term Expenditure Frameworks. Washington, D.C.: World Bank.

———. 2017a. FMIS and Open Budget Data Global Dataset World Development Indicator database. https://datacatalog.worldbank.org/dataset/fmis-and-open-budget-data-global-dataset

———. 2017b. World Development Indicator database. https://data.worldbank.org/products/wdi

———. 2018. Commodity Markets Outlook. Washington, D.C.: World Bank. Available at http://www.worldbank.org/en/research/commodity-markets.

Annex 1: The FMM-MTFF Model Equation Listing

Sectoral firms, $\forall i,\ i = \{1,2,3\}$:

$$u_{i,t} = \left(\left(1-\tau_{i,t}\right)p_{i,t} - \sum_{j=1}^{3}\chi_{ji}p_{j,t} - \chi_{Mi}\,p_{M,t}\right)\alpha_i \frac{y_{i,t}}{k_{i,t-1}} \qquad [1]\text{-}[3]$$

$$\begin{aligned} w_{i,t} = {} & \left(\left(1-\tau_{i,t}\right)p_{i,t} - \sum_{j=1}^{3}\chi_{ji}p_{j,t} - \chi_{Mi}\,p_{M,t}\right)(1-\alpha_i)\frac{y_{i,t}}{n_{i,t}} \\ & - p_{i,t}\left(\frac{\eta_{n_i}}{2}\left(\frac{n_{i,t}}{n_{i,t-1}}-1\right)^2 - \eta_{n_i}\left(\frac{n_{i,t}}{n_{i,t-1}}-1\right)\left(\frac{n_{i,t}}{n_{i,t-1}}\right)\right) \\ & + \beta\eta_{n_i}\mathbb{E}_t\left\{p_{i,t+1}\frac{\Lambda_{t+1}}{\Lambda_t}\left(\frac{n_{i,t+1}}{n_{i,t}}-1\right)\left(\frac{n_{i,t+1}}{n_{i,t}}\right)^2\right\} \end{aligned} \qquad [4]\text{-}[6]$$

$$y_{i,t} = \mathbb{A}_t^i K_{G,t-1}^{\theta} k_{i,t-1}^{\alpha_i}\left(n_{i,t}\right)^{1-\alpha_i} \qquad [7]\text{-}[9]$$

$$x_{1,i,t} = \chi_{1i}y_{i,t} \qquad [10]\text{-}[12]$$

$$x_{2,i,t} = \chi_{2i}y_{i,t} \qquad [13]\text{-}[15]$$

$$x_{3,i,t} = \chi_{3i}y_{i,t} \qquad [16]\text{-}[18]$$

$$m_{i,t} = \chi_{Mi}y_{i,t} \qquad [19]\text{-}[21]$$

$$p_{3,t} = s_t p_{OIL,t}^{*} \qquad [22]$$

$$p_{M,t} = s_t p_{M,t}^{*} \qquad [23]$$

$$\mathbb{A}_t^i = A_t^i A_t A_i \qquad [24]\text{-}[26]$$

$$K_{G,t} = \frac{k_{G,t}}{\left(K_{i,t}\right)^{\phi}} \qquad [27]\text{-}[29]$$

Domestic good

$$z_{1,t} = \mu_D A_D{}^{\omega_D-1}\left(\frac{p_{1,t}}{p_{D,t}}\right)^{-\omega_D} y_{D,t} \qquad [30]$$

$$z_{2,t} = (1-\mu_D)A_D{}^{\omega_D-1}\left(\frac{p_{2,t}}{p_{D,t}}\right)^{-\omega_D} y_{D,t} \qquad [31]$$

$$p_{D,t} = A_D{}^{-1}\left(\mu_D\left(p_{1,t}\right)^{1-\omega_D} + (1-\mu_D)\left(p_{2,t}\right)^{1-\omega_D}\right)^{\frac{1}{1-\omega_D}} \qquad [32]$$

Investment good

$$d_{I,t} = \mu_I A_I{}^{\omega_I-1}\left(\frac{p_{D,t}}{p_{I,t}}\right)^{-\omega_I} y_{I,t} \qquad [33]$$

$$m_{I,t} = (1-\mu_I)A_I{}^{\omega_I-1}\left(\frac{p_{M,t}}{p_{I,t}}\right)^{-\omega_I} y_{I,t} \qquad [34]$$

$$p_{I,t} = {A_I}^{-1}\left(\mu_I\left(p_{D,t}\right)^{1-\omega_I} + (1-\mu_I)\left(p_{M,t}\right)^{1-\omega_I}\right)^{\frac{1}{1-\omega_I}} \qquad [35]$$

Consumption good

$$d_{C,t} = \mu_C {A_C}^{\omega_C - 1}\left(p_{D,t}\right)^{-\omega_C} y_{C,t} \qquad [36]$$

$$m_{C,t} = (1-\mu_C){A_C}^{\omega_C - 1}\left(\left(1+\tau^M_{C,t}\right)p_{M,t}\right)^{-\omega_C} y_{C,t} \qquad [37]$$

$$1 = {A_C}^{-1}\left(\mu_C\left(p_{D,t}\right)^{1-\omega_C} + (1-\mu_C)\left(\left(1+\square^M_{C,t}\right)p_{M,t}\right)^{1-\omega_C}\right)^{\frac{1}{1-\omega_C}} \qquad [38]$$

Non-Ricardian households

$$c_t^{NO} = \sum_{i=1}^{3} w_{i,t} h_{i,t}^{NO} + tt_t^{NO} \qquad [39]$$

$$\psi_i^{NO}\left(h_{i,t}^{NO}\right)^{\varepsilon} = w_{i,t} \qquad \forall i \qquad [40]\text{-}[42]$$

Ricardian households

$$\gamma k_{i,t}^O = (1-\delta_i)k_{i,t-1}^O + i_{i,t}^O - \frac{\eta_i}{2}\left(\frac{i_{i,t}^O}{i_{i,t-1}^O} - 1\right)^2 i_{i,t}^O \qquad \forall i \qquad [43]\text{-}[45]$$

$$\psi_i^O(h_{i,t}^O)^{\varepsilon} = \left(\frac{1-\tau_{W,t}}{1+\tau_{C,t}}\right) w_{i,t} \qquad \forall i \qquad [46]\text{-}[48]$$

$$\Lambda_t = (1+\tau_{C,t})^{-1}\left(c_t^O - \varsigma C_{t-1}^O - \frac{\sum_{j=1}^{3}\psi_j^O\left(h_{j,t}^O\right)^{1+\varepsilon}}{1+\varepsilon}\right)^{-\sigma} \qquad [49]$$

$$\gamma\Omega_{i,t} = \beta\mathbb{E}_t\left\{(1-\tau_{K,t+1})u_{i,t+1}\Lambda_{t+1} + \tau_{K,t+1}\delta_i\Lambda_{t+1} + (1-\delta_i)\Omega_{i,t+1}\right\} \qquad \forall i \qquad [50]\text{-}[52]$$

$$\begin{aligned} p_{I,t}\Lambda_t - \Omega_{i,t}&\left(1 - \frac{\square_i}{2}\left(\frac{i_{i,t}^O}{i_{i,t-1}^O} - 1\right)^2 - \eta_i\left(\frac{i_{i,t}^O}{i_{i,t-1}^O} - 1\right)\left(\frac{i_{i,t}^O}{i_{i,t-1}^O}\right)\right) \\ &= \beta\eta_i\mathbb{E}_t\left\{\left(\frac{i_{i,t+1}^O}{i_{i,t}^O} - 1\right)\left(\frac{i_{i,t+1}^O}{i_{i,t}^O}\right)^2 \Omega_{i,t+1}\right\} \qquad \forall i \end{aligned} \qquad [53]\text{-}[55]$$

$$s_t\Lambda_t = \frac{\beta}{\gamma}\left(R_t^W + \eta_{D^*}\left(e^{D_t^* - \partial} - 1\right)\right)\mathbb{E}_t\{s_{t+1}\Lambda_{t+1}\} \qquad [56]$$

$$\gamma\Lambda_t = \beta\mathbb{E}_t\{R_t\Lambda_{t+1}\} \qquad [57]$$

Private sector aggregation

$$c_t = \lambda c_t^{NO} + (1-\lambda)c_t^O \qquad [58]$$

$$i_{i,t} = (1-\lambda)i_{i,t}^O \qquad \forall i \qquad [59]\text{-}[61]$$

$$h_{i,t} = \lambda h_{i,t}^{NO} + (1-\lambda)h_{i,t}^O \qquad \forall i \qquad [62]\text{-}[64]$$

$$k_{i,t} = (1-\lambda)k_{i,t}^O \qquad \forall i \qquad [65]\text{-}[67]$$

$$d_t^* = (1-\lambda)d_t^{*O} \quad [68]$$

$$t_t = t_t^{NO} + t_t^O = \lambda tt_t^{NO} + (1-\lambda)tt_t^O \quad [69]$$

Government

$$p_{I,t}i_{G,t} = g_{I,t}GDP_t^N \quad [70]$$

$$\gamma K_{G,t} = (1-\delta_G)K_{G,t-1} + i_{G,t} - \frac{\eta_G}{2}\left(\frac{i_{G,t}}{i_{G,t-1}} - 1\right)^2 i_{G,t} \quad [71]$$

$$p_{2,t}g_t = g_{C,t}GDP_t^N \quad [72]$$

$$\gamma b_t = g_{F,t}f_t \quad [73]$$

$$\gamma s_t d_{G,t}^* = \left(1-g_{F,t}\right)f_t \quad [74]$$

$$f_t = p_{2,t}g_t + p_{I,t}i_{G,t} + R_{t-1}^W s_t d_{G,t-1}^* + R_{t-1}b_{t-1} + t_t - v_t u_{3,t}k_{3,t-1} - \tau_{W,t}(1-\lambda)\left(\sum_i w_{i,t}h_{i,t}^O\right) - \tau_{K,t}\left(\sum_i\left(u_{i,t}-\delta_i\right)k_{i,t-1}\right) - \tau_{C,t}(1-\lambda)c_t^O - \tau_{C,t}^M p_{M,t}m_{C,t} - \sum_i \tau_{i,t}p_{i,t}y_{i,t} \quad [75]$$

Exports and imports

$$e_{C,t} = A_X\left(\frac{1}{s_t}\right)^{\varpi_P}(y_t^*)^{\varpi_Y} \quad [76]$$

$$e_t = A_E\left(e_{C,t}\right)^{\mu_E}\left(e_{OIL,t}\right)^{1-\mu_E} \quad [77]$$

$$p_{E,t} = A_E{}^{-1}\left(\frac{1}{\mu_E}\right)^{\mu_E}\left(\frac{1}{1-\mu_E}\right)^{1-\mu_E}\left(s_t p_{OIL,t}^*\right)^{1-\mu_E} \quad [78]$$

$$m_t = \sum_{i=1}^3 m_{i,t} + m_{C,t} + m_{I,t} \quad [79]$$

$$tb_t = p_{E,t}e_t - p_{M,t}m_t \quad [80]$$

$$tot_t = \frac{p_{E,t}}{p_{M,t}} \quad [81]$$

Interest rate and risk premium

$$R_t^W = R_t^*(1+\rho_t) \quad [82]$$

$$\rho_t = A_\rho\, exp\left[\pi\left(\frac{s_t d_{G,t-1}^*}{GDP_t^N} - \eth\right)\right] \quad [83]$$

Nominal and real GDP

$$GDP_t^N = c_t + p_{I,t}\left(\sum_{\square=1}^3 i_{i,t} + i_{G,t}\right) + p_{2,t}g_t + p_{E,t}e_t - p_{M,t}m_t \quad [84]$$

$$GDP_t^R = \left(\frac{\gamma_{GDP,t}}{\gamma}\right)GDP_{t-1}^R \quad [85]$$

$$\gamma_{GDP,t} = \gamma\left(\frac{c_t + p_{I,t-1}\left(\sum i_{i,t} + i_{G,t}\right) + p_{2,t-1}g_t + p_{E,t-1}e_\square - p_{M,t-1}m_t}{c_{t-1} + p_{I,t-1}\left(\sum i_{i,t-1} + i_{G,t-1}\right) + p_{2,t-1}g_{t-1} + P_{E,t-1}e_{t-1} - p_{M,t-1}m_{t-1}}\right) \quad [86]$$

$$p_{GDP,t} = \frac{GDP_t^N}{GDP_t^R} \quad [87]$$

Market clearing conditions

$$n_{i,t} = h_{i,t} \qquad \forall i \quad [88]\text{-}[90]$$

$$y_{C,t} = c_t + e_{C,t} + \tau_{C,t}^M p_{M,t} m_{C,t} \quad [91]$$

$$y_{I,t} = \sum_{i=1}^{3} i_{i,t} + i_{G,t} \quad [92]$$

$$y_{1,t} = x_{1,1,t} + x_{1,2,t} + x_{1,3,t} + z_{1,t} \quad [93]$$

$$y_{2,t} = x_{2,1,t} + x_{2,2,t} + x_{2,3,t} + z_{2,t} + g_t \quad [94]$$

$$y_{3,t} = x_{3,1,t} + x_{3,2,t} + x_{3,3,t} + e_{OIL,t} \quad [95]$$

$$y_{D,t} = d_{C,t} + d_{I,t} \quad [96]$$

$$\gamma\left(d_t^* + d_{G,t}^*\right)s_t = R_{t-1}^W\left(d_{t-1}^* + d_{G,t-1}^*\right)s_t + \eta_{D^*}\left(e^{D_{t-1}^* - \partial} - 1\right)s_t d_{t-1}^* + p_{M,t} m_t - p_{E,t} e_t \quad [97]$$

Annex 2: Steady State System

The system of equations in Annex 1 is expressed in terms of the variables' long-run values and expectations of operators removed. Variables without time subscripts represent steady state values.

$$w_1 = w_2 = w_3 = p_1 = p_2 = p_3 = p_{GDP} = p_I = p_E = p_D = p_M = tot = s = p^*_{OIL} = p^*_M = 1 \quad \text{[A1]}$$

$$A = A^1 = A^2 = A^3 = 1 \quad \text{[A2]}$$

$$GDP^R = GDP^N = y^* = 100 \quad \text{[A3]}$$

Sectoral firms, $\forall i$, $i = \{1,2,3\}$:

$$u_i = (1 - \tau_i - \chi_{1i} - \chi_{2i} - \chi_{3i} - \chi_{Mi})\alpha_i \frac{y_i}{k_i} \quad \text{[A4]}$$

$$w_i = (1 - \tau_i - \chi_{1i} - \chi_{2i} - \chi_{3i} - \chi_{Mi})(1 - \alpha_i) \frac{y_i}{n_i} \quad \text{[A5]}$$

$$y_i = A_i K_i^\theta k_i^{\alpha_i} (n_i)^{1-\alpha_i} \quad \text{[A6]}$$

$$x_{1,i} = \chi_{1i} y_i \quad \text{[A7]}$$

$$x_{2,i} = \chi_{2i} y_i \quad \text{[A8]}$$

$$x_{3,i} = \chi_{3i} y_i \quad \text{[A9]}$$

$$m_i = \chi_{Mi} y_i \quad \text{[A10]}$$

$$K_i = \frac{k_G}{(k_i)^\phi} \quad \text{[A11]}$$

Domestic good

$$z_1 = \mu_D {A_D}^{\omega_D - 1} y_D \quad \text{[A12]}$$

$$z_2 = (1 - \mu_D) {A_D}^{\omega_D - 1} y_D \quad \text{[A13]}$$

$$A_D = 1 \quad \text{[A14]}$$

Investment good

$$d_I = \mu_I {A_I}^{\omega_I - 1} y_I \quad \text{[A15]}$$

$$m_I = (1 - \mu_I) {A_I}^{\omega_I - 1} y_I \quad \text{[A16]}$$

$$A_I = 1 \quad \text{[A17]}$$

Consumption good

$$d_C = \mu_C {A_C}^{\omega_C - 1} y_C \quad \text{[A18]}$$

$$m_C = (1 - \mu_C) {A_C}^{\omega_C - 1} (1 + \tau_C^M)^{-\omega_C} y_C \quad \text{[A19]}$$

$$A_C = (\mu_C + (1 - \mu_C)(1 + \tau_C^M)^{1-\omega_C})^{\frac{1}{1-\omega_C}} \quad \text{[A20]}$$

Non-Ricardian households

$$c^{NO} = \sum_{i=1}^{3} w_i h_i^{NO} + tt^{NO} \qquad \text{[A21]}$$

$$\psi_i^{NO}\left(h_i^{NO}\right)^{\varepsilon} = 1 \qquad \forall i \qquad \text{[A22]}$$

Ricardian households

$$(\gamma - 1 + \delta_i)k_i^O = i_i^O \qquad \forall i \qquad \text{[A23]}$$

$$\psi_i^O (h_i^O)^{\varepsilon} = \left(\frac{1-\tau_W}{1+\tau_C}\right) \qquad \forall i \qquad \text{[A24]}$$

$$\Lambda = (1+\tau_C)^{-1}\left((1-\varsigma)c^O - \frac{\sum_{j=1}^{3}\psi_j^O\left(h_j^O\right)^{1+\varepsilon}}{1+\varepsilon}\right)^{-\sigma} \qquad \text{[A25]}$$

$$\frac{\gamma}{\beta} - 1 = (1-\tau_K)(u_i - \delta_i) \qquad \forall i \qquad \text{[A26]}$$

$$\Omega_i = \Lambda \qquad \forall i \qquad \text{[A27]}$$

$$\frac{\gamma}{\beta} = R^W \qquad \text{[A28]}$$

$$R^W = R \qquad \text{[A29]}$$

Private sector aggregation

$$c = \lambda c^{NO} + (1-\lambda)c^O \qquad \text{[A30]}$$

$$i_i = (1-\lambda)i_i^O \qquad \forall i \qquad \text{[A31]}$$

$$h_i = \lambda h_i^{NO} + (1-\lambda)h_i^O \qquad \forall i \qquad \text{[A32]}$$

$$k_i = (1-\lambda)k_i^O \qquad \forall i \qquad \text{[A33]}$$

$$d^* = (1-\lambda)d^{*O} \qquad \text{[A34]}$$

$$t = t^{NO} + t^O = \lambda tt^{NO} + (1-\lambda)tt^O \qquad \text{[A35]}$$

Government

$$i_G = g_I GDP^N \qquad \text{[A36]}$$

$$(\gamma - 1 + \delta_G)k_G = i_G \qquad \text{[A37]}$$

$$g = g_C GDP^N \qquad \text{[A38]}$$

$$\gamma b = g_F f \qquad \text{[A39]}$$

$$\gamma d_G^* = (1-g_F)f \qquad \text{[A40]}$$

$$f = g + i_G + R^W d_G^* + Rb + t - vu_3k_3 - \tau_W(1-\lambda)\left(\sum_i h_i^O\right) - \sum_i \tau_i y_i - \tau_K\left(\sum_i (u_i - \delta_i)k_i\right) - \tau_C(1-\lambda)c^O - \tau_C^M m_C \qquad \text{[A41]}$$

Exports and imports

$$e_C = A_X (y^*)^{\varpi_Y} \qquad \text{[A42]}$$

$$e = A_E (e_C)^{\mu_E} (e_{OIL})^{1-\mu_E} \quad \text{[A43]}$$

$$A_E = \left(\frac{1}{\mu_E}\right)^{\mu_E} \left(\frac{1}{1-\mu_E}\right)^{1-\mu_E} \quad \text{[A44]}$$

$$m = \sum_{i=1}^{3} m_i + m_C + m_I \quad \text{[A45]}$$

$$tb = e - m \quad \text{[A46]}$$

Interest rate and risk premium

$$R^W = R^*(1+\rho) \quad \text{[A47]}$$

$$\rho = A_\rho \quad \text{[A48]}$$

Nominal and real GDP

$$GDP^N = c + \left(\sum_{i=1}^{3} i_i + i_G\right) + g + e - m \quad \text{[A49]}$$

$$\gamma_{GDP} = \gamma \quad \text{[A50]}$$

$$GDP^N = GDP^R \quad \text{[A51]}$$

Market clearing conditions

$$n_i = h_i \qquad \forall i \quad \text{[A52]}$$

$$y_C = c + e_C + \tau_C^M m_C \quad \text{[A53]}$$

$$y_I = \sum_{i=1}^{3} i_i + i_G \quad \text{[A54]}$$

$$y_1 = x_{1,1} + x_{1,2} + x_{1,3} + z_1 \quad \text{[A55]}$$

$$y_2 = x_{2,1} + x_{2,2} + x_{2,3} + z_2 + g \quad \text{[A56]}$$

$$y_3 = x_{3,1} + x_{3,2} + x_{3,3} + e_{OIL} \quad \text{[A57]}$$

$$y_D = d_C + d_I \quad \text{[A58]}$$

$$(\gamma - R^W)(d^* + d_G^*) = m - e \quad \text{[A59]}$$