A Simple Stylized Long-Run Growth Model for Haiti

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Martín Cicowiez and Agustín Filippo*

1. Introduction

The modeling of the growth prospects of an economy like Haiti's requires a multidimensional approach. The long run growth dynamics of developed and some high-income developing economies can be captured by Solow or Ramsey-Cass-Koopmans type models. However, low or very low level of development economies such as the Haitian economy present multiple problematic dimensions or issues that cannot be captured within a single type of modeling exercise.

In principle, we can identify three main sets of issues. First, internal and international migration issues, which can be addressed with a model of the Harris-Todaro family, something that was done in the work of Katz (2016). Second, intersectoral coordination and structural change issues that can be addressed with a Computable General Equilibrium model such as the one developed by Cicowiez and Filippo (2018). Third, issues of intertemporal coordination between population growth, TFP (Total Factor Productivity), capital accumulation, foreign debt, output and consumption, something that is done in a very stylized first approximation in the present work with a model of the Ramsey-Cass-Koopmans type.[1] This class of models are the workhorse of most contemporary work in modeling the long and very long term growth of countries (Barro and Sala-i-Martin (2004) and Acemoglu (2009)).

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[1] It is interesting to note that most of the empirical work using this framework sets out closed economy steady-state models, thus missing some very basic features of developing countries such as Haiti: they are small open economies; have quite limited absorptive capacity for new capital; can be credit constrained in international financial markets; and, last but not least, they are usually far from the steady state. Thus, transitional dynamics starting from actual initial conditions matters, and matters a lot (Mercado and Cicowiez, 2013).

The objective of the model and simulations presented here is to provide a basic, rough and stylized framework for counterfactual exercises in order to obtain orders of magnitude of the dynamic growth potential of the main macroeconomic aggregates of Haiti. By the above, in no way the model aims to fully capture the dynamics of development of the Haitian economy. It should be seen as a complementary model to those developed by Katz (2016) and Cicowiez and Filippo (2018).

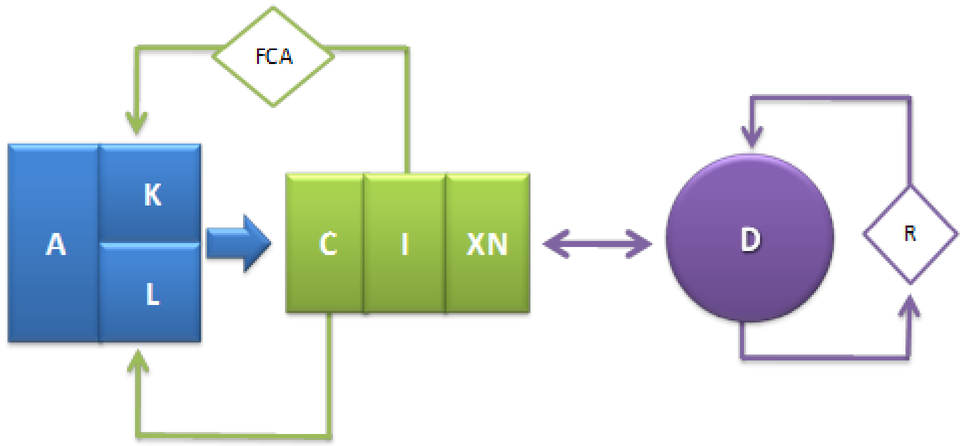
This model is a streamlined and consistent framework to consider questions such as the following. What very long run growth rates of output and consumption could be achieved by the Haitian economy based on different levels of population and total factor productivity (TFP) growth? What would be the consequent requirements in terms of capital accumulation and foreign debt? What impact would have on those dynamics changes in financing conditions or international aid, rationalized as real or virtual changes in the international interest rate? How would be, from a very stylized point of view, the transitional dynamics of the main macroeconomic stocks and flows starting from the current situation until achieving steady-state levels of very long rung growth?

2. Model and Data

2.1. Model

In Figure 2.1 we show the main features of our growth model; it shows that the stock of factors of production (capital (*K*), labor (*L*) and technology (*A*)) generates a flow of output. In turn, part of this output is consumed (*C*) by the workforce, and the part that is not consumed (i.e., saved) is invested in physical capital (*I*). As will be explained, investment is mediated by an absorptive capacity function (*G*), which determines the proportion of investment that can be transformed effectively in increases in the stock of physical capital. The expansion of the stock of physical capital helps to increase output in the next period, and so on. In an open economy, a share of output takes the form of net exports (*XN*) (i.e., the difference between exports and imports), and its sign means either an increase or a decrease in foreign debt (*D*), whose dynamics also depends on the international interest rate (*R*).

Figure 2.1: the growth model in a snapshot



Source: Authors' elaboration.

In mathematical terms, our model can be presented as follows.

Production Function

Output Y_t is produced using physical capital K_t and labor L_t as inputs, given the stock of technology A_t (equation (1)). The production function is Cobb-Douglass with constant returns to scale (i.e., factor shares add up to one), and technical change increases the efficiency of labor (i.e., it is labor augmenting, or Harrod-neutral).[2] Mathematically,

(1) $$Y = K_t^{\alpha}(A_t L_t)^{1-\alpha}$$

where

Y_t = output

K_t = physical capital

L_t = labor

[2] These assumptions are widely used in growth studies, since they play a crucial role in generating a growth behavior consistent with the famous Kaldor's stylized facts: investment to capital ratio, capital to output ratio, rate of return of capital, and shares of capital and labor are all constant; also, capital-labor ratio, output-labor ratio, and real wage all grow at a constant rate.

A_t = stock of technology

α = capital share

Physical Capital Accumulation

The accumulation of physical capital is given by equation (2). In general, it is not feasible to increase the capital stock in large proportions within a given period of time, particularly in developing countries. Thus, from a modeling perspective, it is necessary to constrain how much investment can be transformed into effective additions to the capital stock within a single time period. To that end, we implement the concave absorptive capacity function shown in equation (3).[3] Figure 2.1 shows some examples; the forty-five degree line represents the case of perfect absorption, while the other two lines show functions with different asymptotic value parameters (μ=0.5 and μ=1). In the figure, we can see that, when μ=0.5, increases of the physical capital stock beyond 50% within a year will likely be impossible, no matter how much investment is made since the absorptive capacity of the economy would be saturated.

(2) $$\dot{K}_t = G_t - \delta K_t$$

(3) $$G_t = \mu K_t \left(1 - \left(1 + \frac{\varepsilon I_t}{\mu K_t}\right)^{-\frac{1}{\varepsilon}}\right)$$

where

δ = rate of depreciation of the physical capital stock

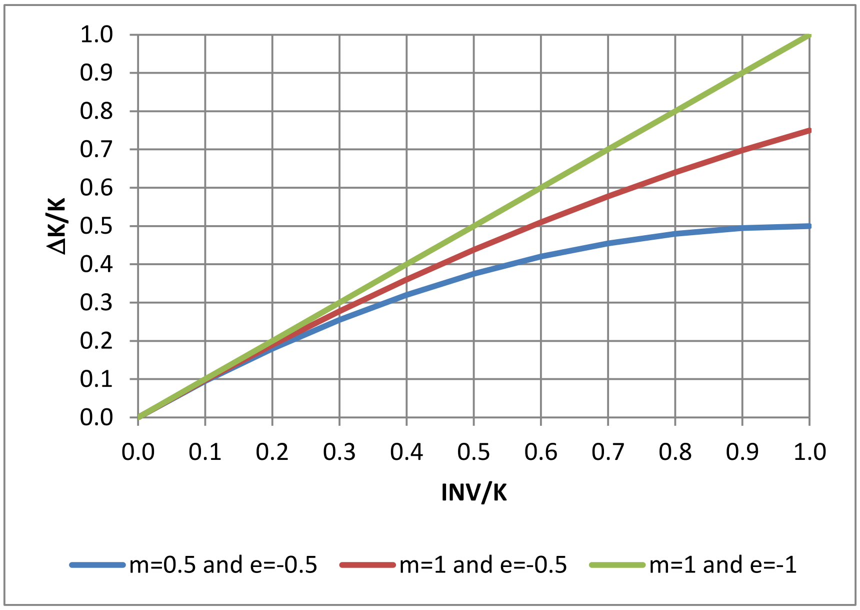
G_t = absorptive capacity

μ; $\mu \geq 0$= parameter that controls the asymptotic value of G_t

ε; $-1 \leq \varepsilon \leq 1$ = parameter that controls the curvature absorptive capacity function

[3] This function was first introduced by Kendrick and Taylor in their pioneering dynamic multi-sectoral growth model (Kendrick and Taylor, 1970; Mercado et al., 2003); for a discussion and its parametrization see Mercado and Cicowiez (2013).

Figure 2.2: absorptive capacity function



Source: Authors' elaboration.

Foreign Debt Accumulation and Foreign Debt Constraint

The foreign debt D_t stock evolves according to equation (4), where the term rD_t represents interest payments.[4] However, given our model parameterization (specifically, see the rate of time preference and the elasticity of intertemporal substitution in Table 1 below), assuming that Haiti has an unrestricted access to foreign borrowing would imply that the debt stock D_t grows indefinitely. Consequently, we impose an upper bound to the debt-to-output ratio, an indicator commonly used to characterize the debt burden of a given country (see equation (5)). In our model, a large inflow of foreign grants can be rationalized as foreign borrowing at (very) low interest rate. In other words, although Haiti receives significant amounts of foreign funds regardless of its country risk premium, we assume that lenders do impose rationing by quantity.

[4] Strictly speaking, D_t should account for the resident's stock of net assets. However, here it is interpreted in a more restricted way as the country's foreign debt.

Technically, the model will display two different intertemporal dynamics, depending on whether the foreign debt constraint is binding or not.[5]

(4) $\dot{D}_t = rD_t - NX_t$

(5) $\frac{D_t}{Y_t} \leq \chi$

where

r = international interest rate

NX_t = net exports

χ = upper bound to the debt-to-output ratio

Intertemporal Welfare

Thus far, we presented the production and accumulation equations that characterize the dynamics of the economy. Now, we need an optimization criterion to consider the intra- and inter-temporal tradeoffs implicit in the many possibilities of allocation of resources in this economy. To that end, and following the standard procedure in growth models, we set as the optimization criterion the maximization of an additively separable inter-temporal welfare function W of the form shown in equation (6).[6]

(6) $W = \int_{t=0}^{\infty} \frac{\left(\frac{C_t}{L_t}\right)^{1-\theta} - 1}{1-\theta} \; e^{nt} \; e^{-\rho t} \; dt$

where

ρ = rate of time preference

n = labor force/population growth rate

$1/\theta$ = elasticity of intertemporal substitution

[5] See Mercado and Cicowiez (2013) for details and mathematical derivations.

[6] Thus, utility derives from consumption through a constant elasticity of substitution function. This functional form, together with the Cobb-Douglass form for the production function, ensures that the “canonical” form of the Ramsey-Cass-Koopmans model has a steady state.

Resource Constraint

Finally, a resource constraint establishes that within each time period output hast to be equal to the sum of consumption, investment, and net exports (equation (7))

(7) $Y_t = C_t + I_t + NX_t$

2.2. Data

Ideally, the parameters of growth models as the one presented here should be obtained from the simultaneous econometric estimation of the system of differential equations that characterize its dynamics. However, this is rarely possible, in particular for the case of Haiti, since we do not have consistent and long enough time series data. Alternatively, we could use mixed methods that combine the estimation of some parameters for which there is insufficient information; the imposition of other parametric values taken from the macro or microeconomic literature either of the country or from other countries considered comparable; and calibration of the remaining parameters so that help to fit certain states of the model (e.g., steady state or "saddle path") or contribute to generate reasonable dynamic paths. This is what we do in this study.

In order to operationalize our growth model for Haiti, we need a consistent dataset similar to – but much smaller than -- the one described in Cicowiez and Filippo (2018). Besides, we need estimates for the parameters that describe the inter-temporal welfare function.[7] In what follows, we discuss each of the required data elements and its source. Table 2.1 shows all model parameters and initial conditions.

[7] Naturally, these two parameters (i.e., rate of time preference and intertemporal elasticity of substitution) were not needed to calibrate the recursive dynamic CGE implemented in Cicowiez and Filippo (2018).

Table 2.1: Haiti growth model parameterization

Symbol	Description	Value	Source
θ	inverse elast of intertemporal subst	2.299	Reinhart et al. (1996)
ρ	rate of time preference	0.030	literature review
α	capital share	0.444	Social Accounting Matrix in Cicowiez (2016)
n	labor force/population growth rate	0.010	UN (2015) for period 2015-2050
g	total factor productivity growth rate	0.005	Katz (2016) based on TFP growth rate during the 70s
δ	capital depreciation rate	0.050	literature review
μ	asymptotic value absorptive capacity fn	0.5	literature review
ε	curvature absorptive capacity fn	1	literature review
Y_0	GDP in 2013 (mill gourdes)	364,526	National Accounts
K_0	capital stock in 2013 (GDP share)	2.5	SK considering invest ineff and natural disasters
L_0	labor force in 2013 (# persons)	4,489,196	World Bank WDI
D_0	foreign debt stock in 2013 (GDP share)	18.4	World Bank WDI

Source: Authors' elaboration.

For the production function in equation (1), we need (a) labor and capital share parameters, (b) labor and (initial) capital stocks, and (c) TFP growth rate. For labor and capital shares, they can be directly estimated from National Accounts data, under the assumption that the social marginal products can be measured by observed factor prices. In fact, these shares are reported in a section of the national income and product accounts (NIPA) often referred to as the "functional distribution of income". In our case, we computed labor and capital shares from the Haiti 2013 Social Accounting Matrix (SAM) described in Cicowiez and Filippo (2018), built using the supply and use tables for the same year as its main source of data. It is worth mentioning that our estimate for α takes into account the presence of a large number of non-salaried workers (i.e., "mixed income" within the NIPA), particularly in the agricultural sector. Overall, our 44.4 percent capital share is consistent with those reported by Gollin (2002) for a set of developing countries.

For capital stock and TFP growth, we used the results from the growth accounting exercise for Haiti conducted by Katz (2016). Specifically, he implemented the perpetual inventory method (PMI) using NIPA data on investments as its main source of data. In addition, he considered the impact of natural disasters and inefficiencies in the accumulation process when implementing the PIM for Haiti. It is interesting to note that inefficiencies in the capital accumulation process are captured in our long-term growth model through the absorptive capacity function described in equation (3) above. In short, for K_0 we used the capital-to-GDP ratio of 2.5 estimated by Katz (2016). For TFP growth, the same author estimated a TFP growth rate of one percent during the 70s. However, for periods other than the 70s, the estimated TFP growth rate is negative. Thus, our base case assumes a TFP growth rate of 0.5 percent. Finally, the size of the labor stock was estimating using census (population size) and household survey data (participation rate).

For the intertemporal welfare function in equation (6), we need estimates for the elasticity of intertemporal substitution (EIS) and the rate of time preference. The EIS reflects the sensitivity of consumption (and therefore savings) to changes in intertemporal prices (i.e., the consumption interest rates), with higher values indicating greater sensitivity. In Ogaki et al. (1996), the EIS is estimated for 85 countries, including Haiti. However, given the uncertainty associated with this specific parameter, in Appendix A we conduct a sensitivity analysis considering 1.757 and 3.322 as lower and upper bounds for θ, respectively. In turn, the rate of time preference or, equivalently, the discount factor, describes the preference for present consumption over future consumption. In this case, we do not have estimates for Haiti. Thus, based on a literature review, we assign a value of 0.03 to ρ. In addition, we conduct a piecemeal sensitivity analysis by also considering 0.02 and 0.04 (see Appendix A).

3. Illustrative Simulations

In what follows, we present some illustrative simulations, in line with the main questions regarding the very long-run growth dynamics raised in the Introduction to this work. Each simulation includes 150 periods, which can be interpreted as annuals. From a mathematical point of view, our intertemporal model is a system of differential equations that presents what is known as a "two point boundary value problem"; i.e., their numerical resolution requires the simultaneous imposition of initial and terminal conditions. For this particular model, initial and terminal conditions sufficient to solve it are given by the values 5.1 and 10 of the capital stock in efficiency units, respectively.[8] In gourdes of 2013, the initial value for the capital stock is the one reported in Table 2.1, based on estimates from Katz (2016). In turn, the terminal value is obtained from analytically solving the steady-state of the model with optimal control techniques. This results in the following equations from which the terminal value of the capital stock can be obtained, where *y* is income, *k* is the capital stock, g is the function of absorption capacity (equation 3) and where *q* is "Tobin's q", all expressed in efficiency units, and where the rest are parameters defined above.

(8) $$r = \frac{1}{q}\frac{\partial y}{\partial k} + \frac{\partial g}{\partial k} - \delta$$

(9) $$\rho + \lambda\theta = \frac{1}{q}\frac{\partial y}{\partial k}\,[1 + \chi\,(\rho + \lambda\theta - r)] + \frac{\partial g}{\partial k} - \delta$$

Equation 8 applies when the foreign debt constraint is not binding, while equation 9 applies when the constraint is binding.[9]

Now, we turn to assessing long-run growth scenarios for Haiti. First, we present a base case, parameterized according to the data in Table 2.1, with a TFP growth rate equal to half percentage point, a population growth rate equal to one percent, and international interest rate equal to three percent. In the simulations, under these assumptions, the economy converges to its steady-state in about 90 periods.

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[8] In efficiency units, given our assumption of Harrod-neutral technical change, each variable X_t is expressed as $x_t = \frac{X_t}{A_t L_t}$, where A_t and L_t are the efficiency parameter in the production function and the labor force, respectively. Also, note that the steady state conditions that are used as terminal conditions for the simulations were analytically derived from the first order conditions of the model. In doing so, we considered the corresponding transversality conditions assuming that (a) $r > n + \lambda$ to avoid an unbounded welfare intertemporal integral, and (b) $r \leq \rho + \lambda\theta$ to avoid an accumulation of foreign assets that would violate our assumption that Haiti is a small open economy.

[9] For details on the derivation and interpretation of these equations see Mercado and Cicowiez (2013).

In addition, the steady state growth rate and capital-to-output ratio are 1.9 percent and 3.8, respectively. Thus, we see that the starting capital-to-output ratio of 2.5 is well below its steady-state value of 3.8.

Next, we assess the following three non-base scenarios:

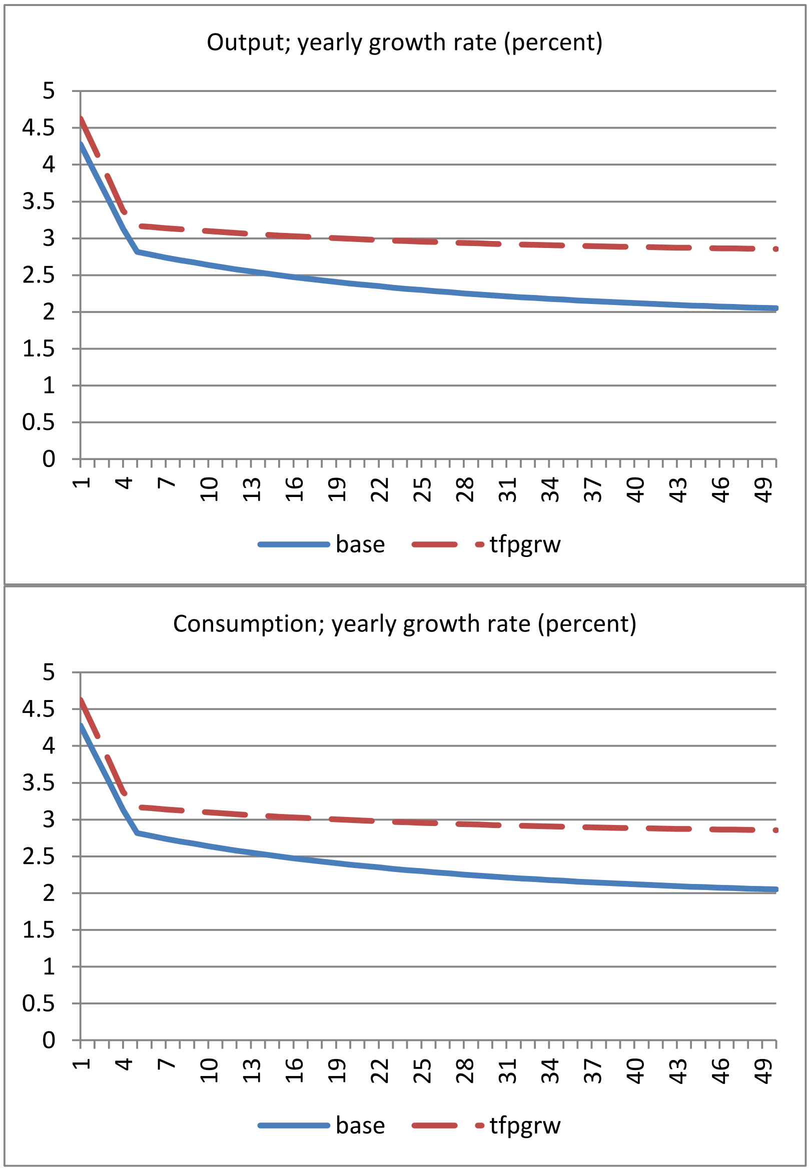
- tfpgrw = increase in TFP growth rate to 1.5 percent, half percentage point higher than in the base;
- debconst = decrease in the foreign financing constraint, from 60 to 30 percent of output;
- abscap = improvement in absorptive capacity, by increasing the value of the mu parameter to 1.

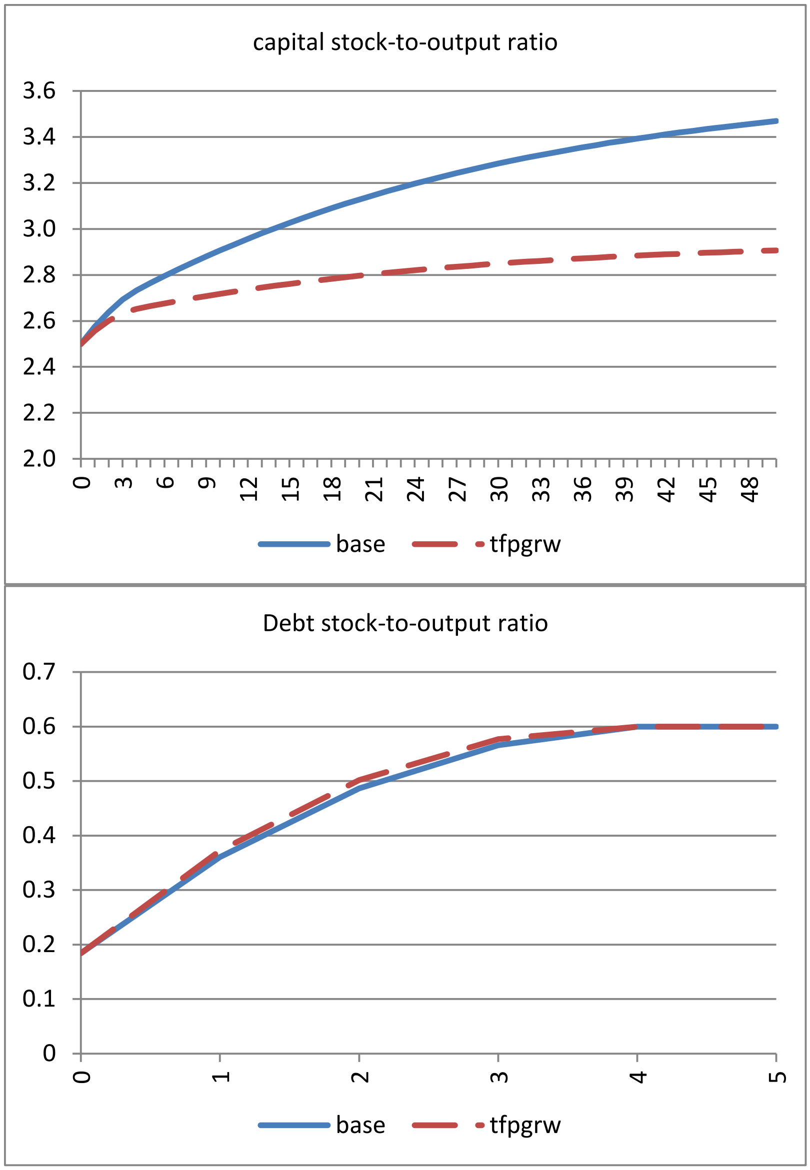
In order to obtain our results, we run the model for 150 periods. However, in what follows we report the results for the first 50 periods, which are enough to show the transitional dynamics from the initial conditions shown in Table 2.1 to the steady state. As expected, we see that the economy converges asymptotically to the steady state. The transition is monotonic. The growth rate is positive and decreases over time towards zero if k<kss. Similarly, the rate of per-capita consumption growth $\frac{c_{t+1}}{c_t}$ (i.e., in efficiency units) is positive and decreasing over time and converges monotonically to zero. Alternatively, the steady state growth rate of per capita output is equal to growth rate of technological change. For each non-base simulation, we report figures for the yearly growth rate for output consumption and the capital-to-output and debt-to-output ratios.

As already said, our non-base TFP growth rate is higher than the one estimated for Haiti in the recent past. However, it is still far from those reached by other developing countries such as, for example, Bolivia (3.8% during 2001-2010), Dominican Republic (2.1% during 2001-2010), and Honduras (2.1% during 2001-2010) (Andrade Araujo et al., 2014). In the literature, through various methods and datasets, several determinants that have an impact on TFP growth are identified. Of these, education, health, infrastructure, institutions, openness, competition, financial development and the business environment appear to be the most important. However, the said literature only establishes statistical associations and provides no causal direction. Thus, any policy discussion can only be indicative rather than directive. Nonetheless, these determinants suggest areas for policymaking.

In Figure 3.1 (see panels a and b), we can see that the increase in TFP growth implies an increasing divergence between the base and non-base levels of output and consumption (i.e., the growth rate is higher under tfpgrw). For example, when TFP growth increases from 0.5 (base scenario) to one percent (tfpgrw scenario), after 35 periods this results into a difference of about 21 percent in output and consumption per capita. Alternatively, in the base and tfpgrw scenarios, it would take Haiti about 185 and 102 periods to reach the current level of Dominican Republic GDP per capita, respectively. (In 2013, the per capita GDP at PPP of Haiti and Dominican Republic were $1,684 and $12,325, respectively.) In terms of the capital-to-output ratio, we see an inverse relationship with TFP growth. Of course, this reflects the well-established result that, the higher the TFP growth, the lower the savings and investment effort required to attain a given level of welfare. In panel (d) of Figure 3.1, we show the dynamics of the debt stock-to-output ratio. In all cases, we see that the credit constraint is reached relatively quickly; this result reflects that the effective rate of time discount of Haiti is greater than the interest rate. In other words, Haiti is a relatively “impatient” country.

Figure 3.1: results for the TFP growth scenario

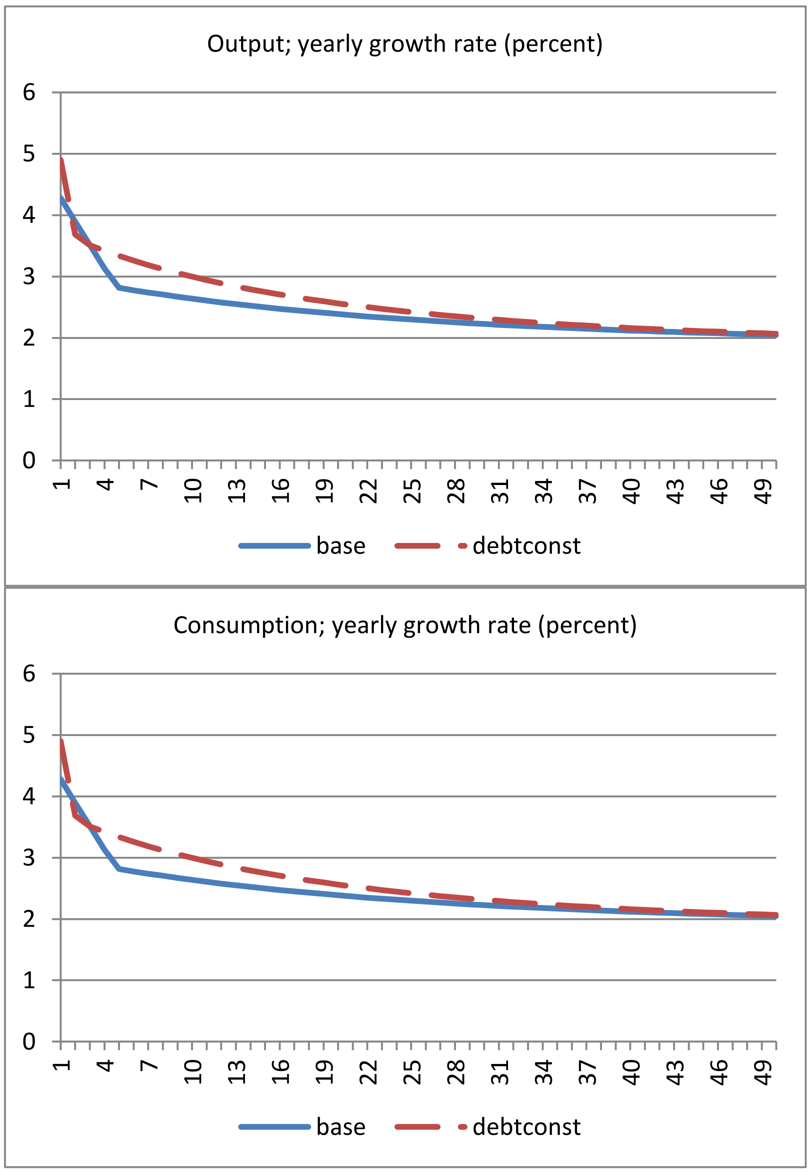


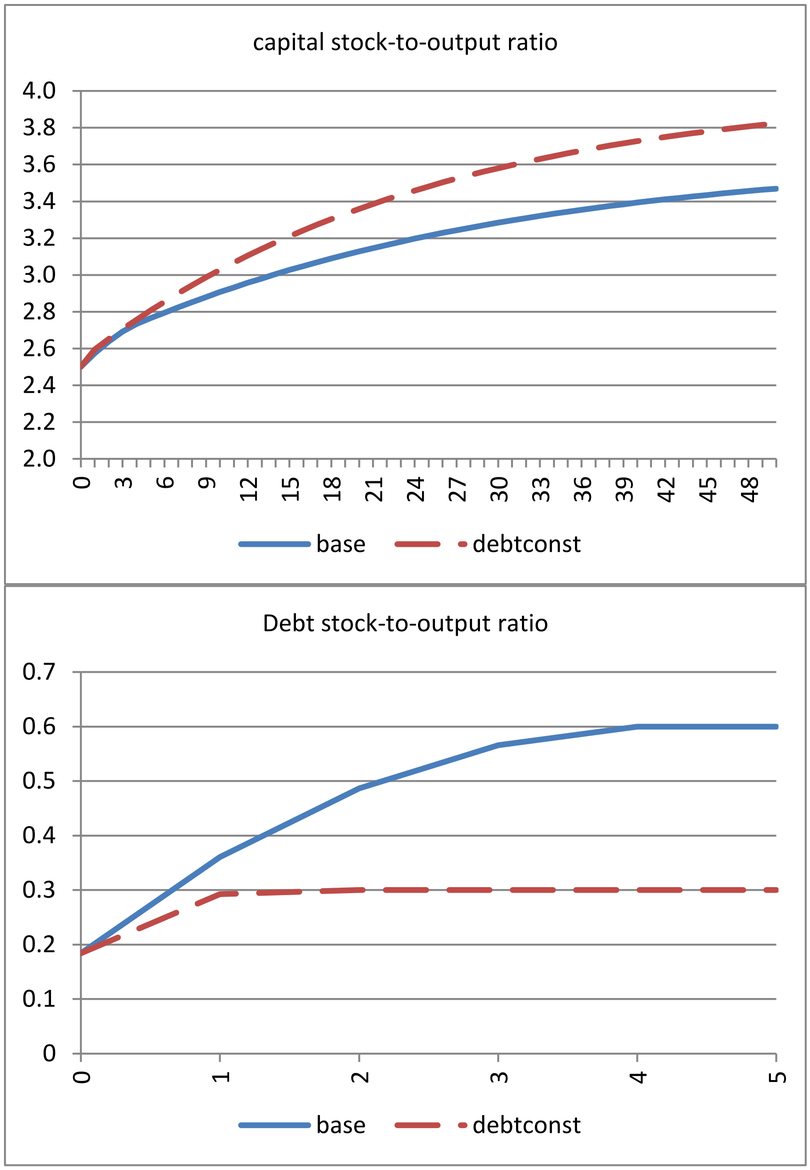


Source: Authors' elaboration.

In Figure 3.2 we show the results of tightening the constraint on the debt stock-to-output ratio, from 60 to 30 percent. In principle, this scenario could be interpreted, in a rough manner, as decrease in foreign aid and/or international remittances from migrants. In fact, according to recent projections, it is expected that foreign aid to Haiti will decrease during 2016-2020 relative to the amounts registered during 2010-2015 (see Filippo (2017)). As a result of the shock, the effort required in terms of domestic savings and investment increases, which is reflected in a higher capital-to-output ratio (see panel c). Also, the growth rates of income and consumption are a bit higher during the transitional dynamics. In fact, since it is an "impatient" country, while the constraint is not binding, it will borrow more from abroad in order to have a high level of consumption early on.

Figure 3.2: results for the debt constraint scenario

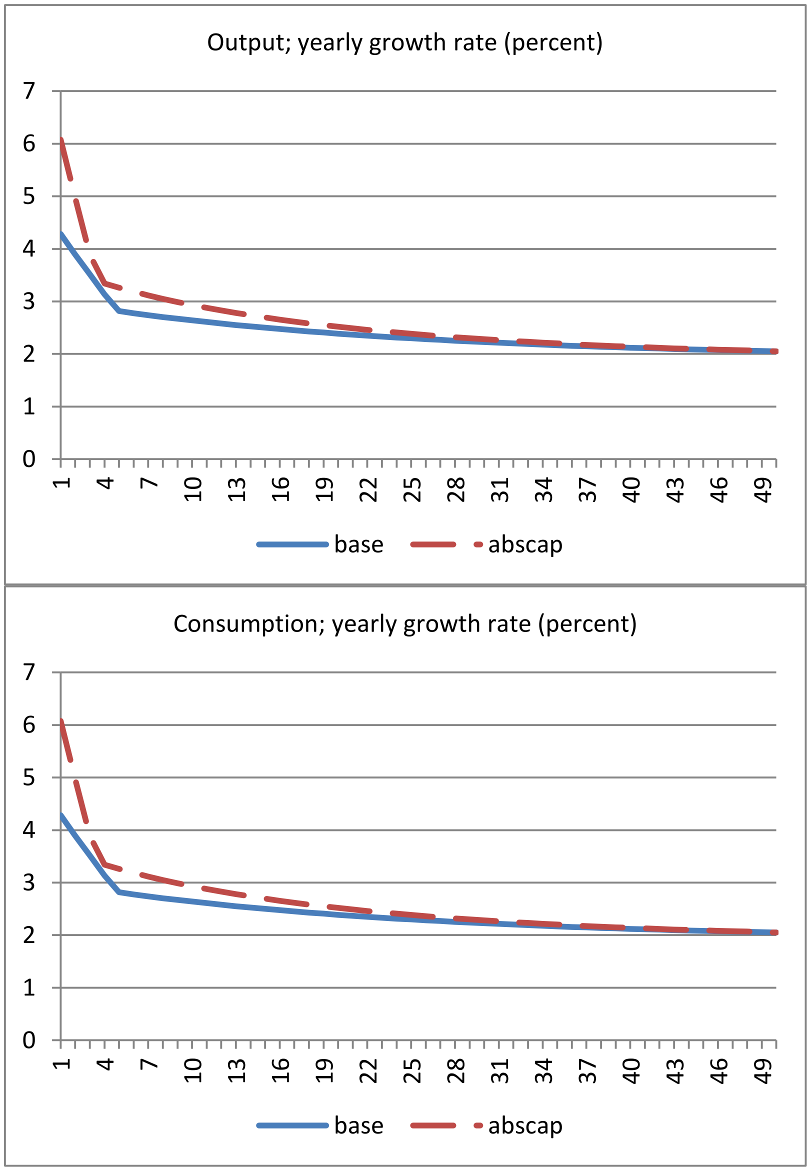


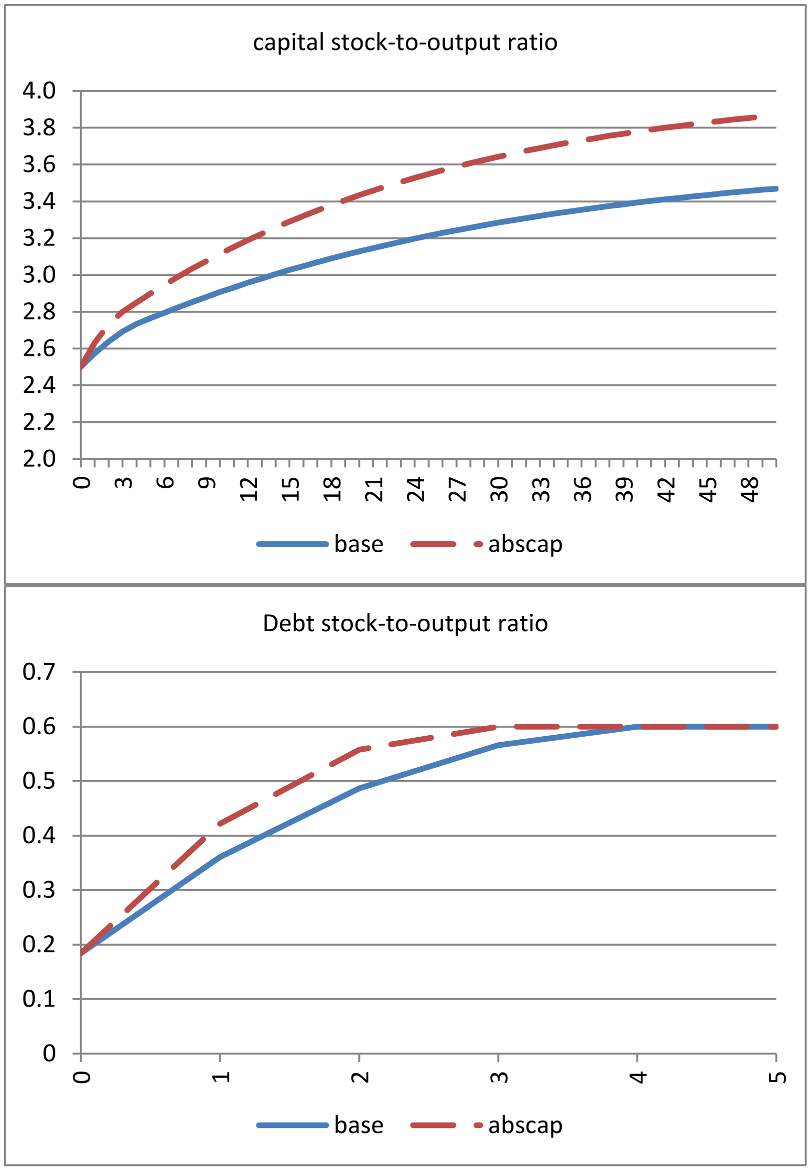


Source: Authors' elaboration.

Lastly, we examine the effect of a substantial improvement in the absorptive capacity of capital, which could be interpreted as a significant improvement in physical and institutional infrastructure or management capacity (see Figure 3.3). In this case, the transitional dynamics show a higher growth rate. In other words, with an improvement in the absorptive capacity as defined above, Haiti can accumulate capital faster and thus reaches its steady-state in a shorter period of time, and with a higher capital-to-output ratio.

Figure 3.3: results for the absorptive capacity scenario





Source: Authors' elaboration.

4. Final Remarks

In this study, we presented a simple and highly stylized model to perform basic counterfactual exercises on the very long-term growth of the Haitian economy, which should be seen as a complementary model to the ones by Katz (2016) and Cicowiez and Filippo (2018). With a relatively simple structure, the model allows to perform a series of counterfactual exercises in an orderly and consistent manner. Model simulations can accommodate changes in the rate of population growth and in the TFP, in the international interest rate and in the absorptive capacity of the capital, as well as other possible parametric changes affecting the intertemporal discount rate, the intertemporal elasticity of substitution or the participation of labor and capital income. We have presented some possible exercises, but a number of extensions or combinations among them would not be difficult to implement.

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Appendix A: Sensitivity Analysis

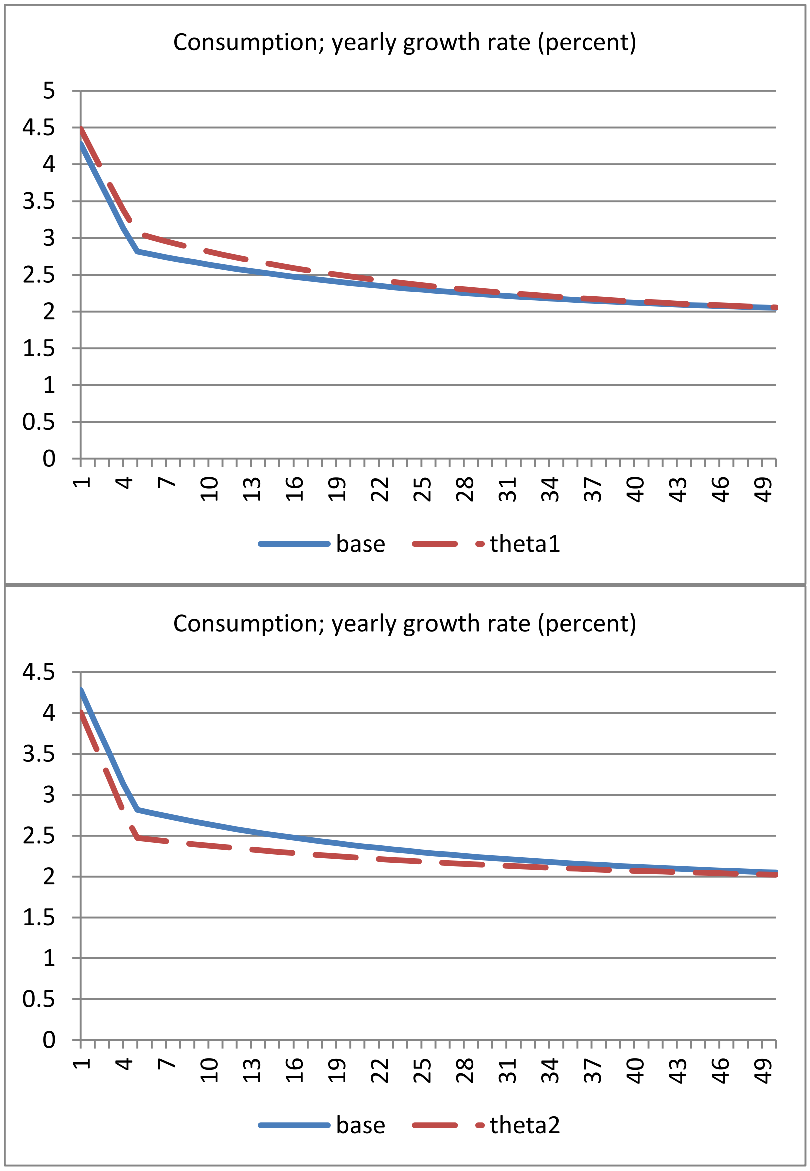
Certainly, the results from our long-term growth model for Haiti are a function of (i) the functional forms used to model production and consumption decisions, (ii) the base year data used for model calibration (i.e., the share parameters and stocks in Table 1), and (iii) the values assigned to the model elasticities and or, more generally, to the model's free parameters. In other words, the preference parameters used in this study implicitly carry an estimation error, as in any similar model. Consequently, we have performed a piecemeal sensitivity analysis of the results with respect to two key parameters: the inverse elasticity of intertemporal substitution (theta), and the rate of time preference (rho). Specifically, we have run the base simulation under the following sets of assumption:

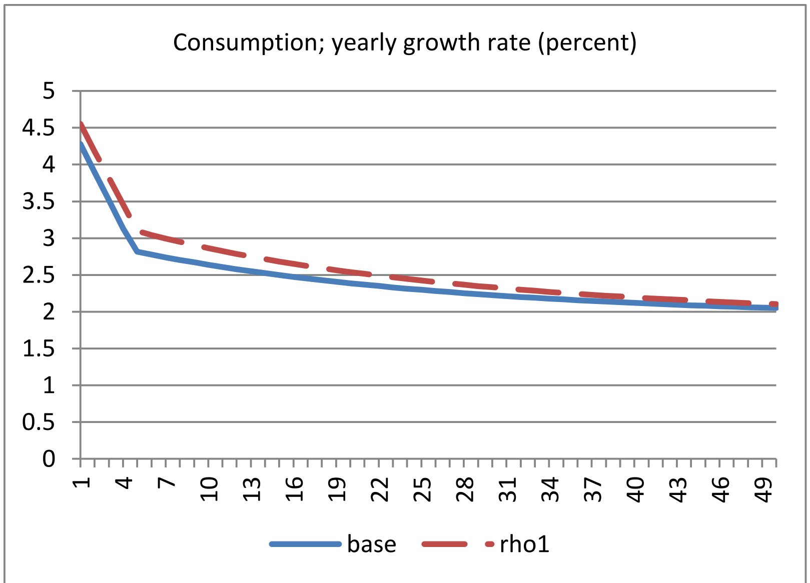
Table A.1: sensitivity analysis; alternative scenarios

Parameter	base	case 1	case 2	case 3	case 4
θ	2.299	1.757	3.322	2.299	2.299
ρ	0.030	0.030	0.030	0.010	0.030

For each case, in Figure A.2 and Figure A.3 we show the yearly growth rate of consumption and the path for the capital-to-output ratio, respectively.

Figure A.2: yearly growth rate of consumption (percent); sensitivity analysis




Consumption; yearly growth rate (percent)
5
4.5
4
3.5
3
2.5
2
1.5
1
0.5
0
1
4
7
10
13
16
19
22
25
28
31
34
37
40
43
46
49
base
rho1

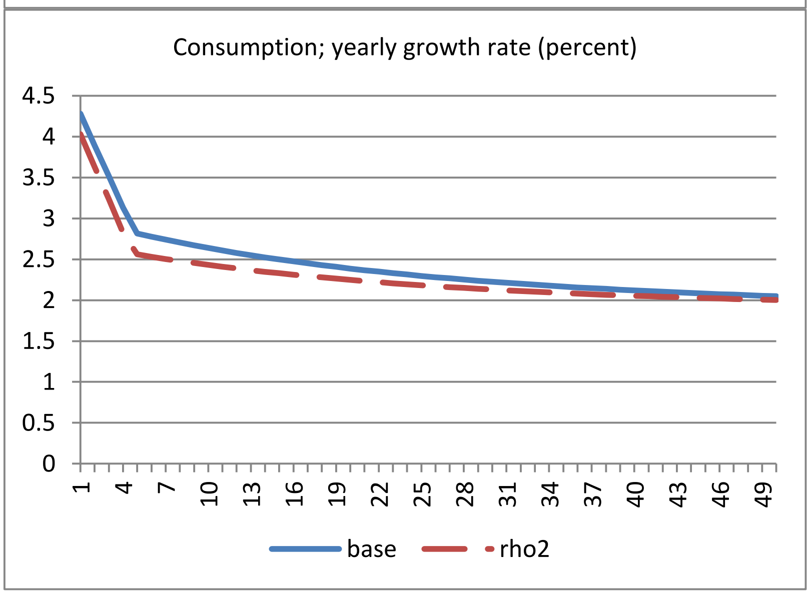
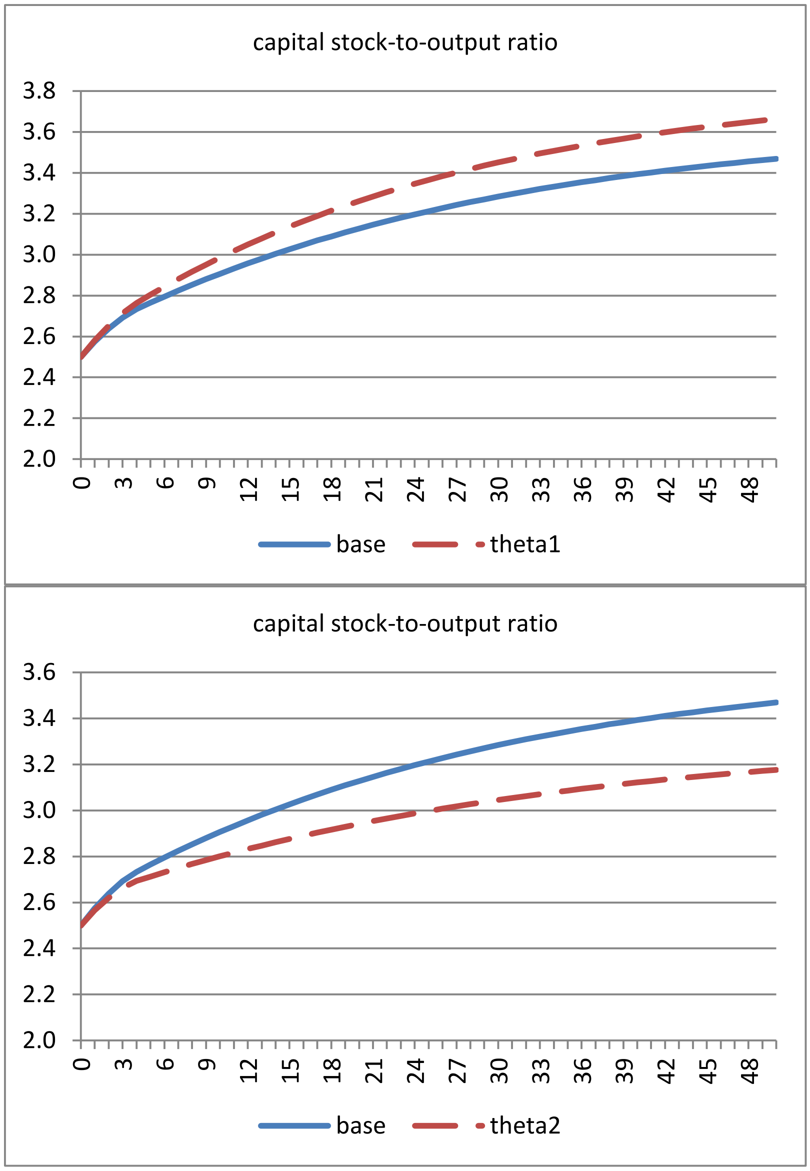
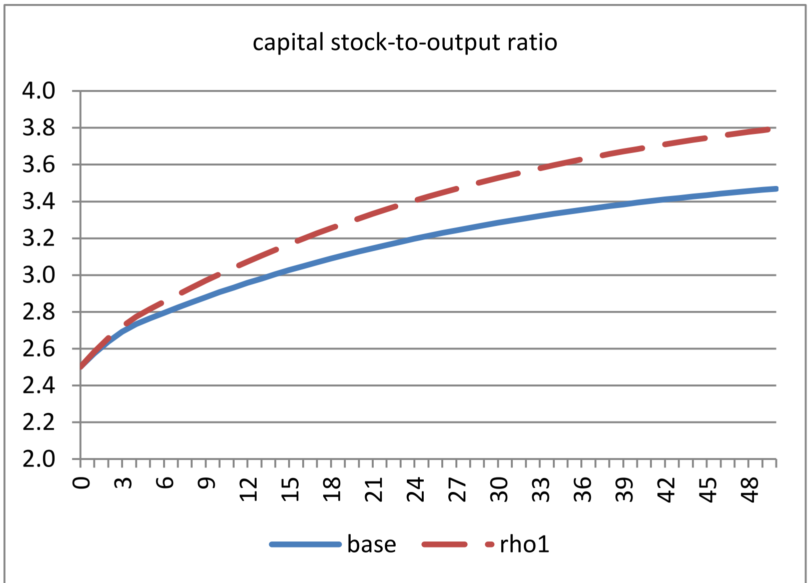
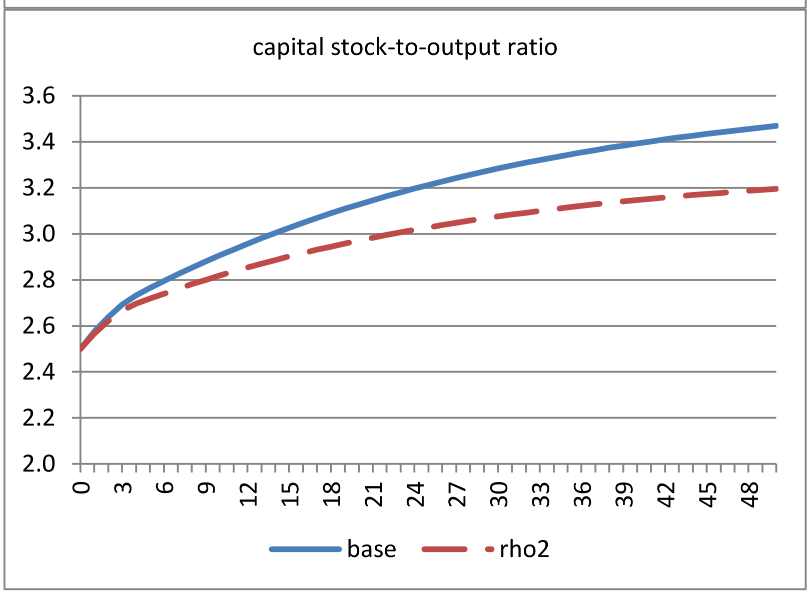

Consumption; yearly growth rate (percent)
4.5
4
3.5
3
2.5
2
1.5
1
0.5
0
1
4
7
10
13
16
19
22
25
28
31
34
37
40
43
46
49
base
rho2

Figure A.3: capital-to-output ratio; sensitivity analysis




capital stock-to-output ratio
4.0
3.8
3.6
3.4
3.2
3.0
2.8
2.6
2.4
2.2
2.0
0
3
6
9
12
15
18
21
24
27
30
33
36
39
42
45
48
base
rho1


capital stock-to-output ratio
3.6
3.4
3.2
3.0
2.8
2.6
2.4
2.2
2.0
0
3
6
9
12
15
18
21
24
27
30
33
36
39
42
45
48
base
rho2

Appendix B: The Complete Model in Efficiency Units

In this appendix, we show all model's variables and equations transformed into their intensive form[10], and we eliminate time subscripts to save notation.

B.1) Max $W = \int_{t=0}^{\infty} \frac{c^{1-\theta}-1}{1-\theta} e^{-vt}\ dt$

subject to the accumulation equations

B.2) $\dot{k} = g_k - \gamma_k k$ B.3) $\dot{h} = g_h - \gamma_h h$ B.4) $\dot{d} = \varphi d - nx$

and the resource and foreign debt constraints

B.5) $y = c + i_k + i_h + nx$

B.6) $\frac{d}{y} \leq \chi$

given the production function

B.7) $y = k^{\alpha} h^{\beta}$

and the concave absorptive capacity functions

B.8) $g_k = i_k \left(1 + \frac{1}{m_k} \frac{i_k}{k}\right)^{-1}$ B.9) $g_h = i_h \left(1 + \frac{1}{m_h} \frac{i_h}{h}\right)^{-1}$

and where

[10] Given the assumption of Harrod-neutral technical change, each variable X_t is transformed such that

$$x_t = \frac{X_t}{A_t L_t}$$

where A_t and L_t are the efficiency and the stock of labor respectively. By the same token, each variable $\dot{X}_t$ becomes

$$\dot{x}_t + x_t\,(n + \lambda)$$

where n is the population growth rate and λ is the growth rate of the efficiency of labor. Finally, the expression

$$\left(\frac{C_t}{L_t}\right)^{1-\theta}$$

becomes $c_t^{1-\theta} A_0^{1-\theta} e^{\,\lambda\,(1-\theta)t}$, where A_0 is not relevant since it's a constant.

B.10) $v = \rho - n - (1-\theta)\lambda$

B.11) $\gamma_k = \delta_k + n + \lambda$

B.12) $\gamma_h = \delta_h + n + \lambda$

B.13) $\varphi = r - n - \lambda$

with initial conditions

B.14) $k_0 = \bar{k}$

B.15) $h_0 = \bar{h}$

B.16) $d_0 = \bar{d}$

and transversality conditions

B.17) $\lim_{t\to\infty} \mu_1\, k\;\; e^{-vt} = 0$

B.18) $\lim_{t\to\infty} \mu_2\, h\;\; e^{-vt} = 0$

B.19) $\lim_{t\to\infty} \mu_3\, d\;\; e^{-vt} = 0$

and where, from B.7, B.8 and B.9, we have the following derivatives:

B.20) $\frac{\partial y}{\partial k} = \alpha k^{\alpha-1} h^{\beta}$

B.21) $\frac{\partial y}{\partial h} = \beta h^{\beta-1} k^{\alpha}$

B.22) $\frac{\partial g_k}{\partial i_k} = \left(1 + \frac{1}{m_k}\frac{i_k}{k}\right)^{-2}$

B.23) $\frac{\partial g_h}{\partial i_h} = \left(1 + \frac{1}{m_h}\frac{i_h}{h}\right)^{-2}$

3.24) $\frac{\partial g_k}{\partial k} = \frac{1}{m_k}\left(\frac{i_k}{k}\right)^2 \left(1 + \frac{1}{m_k}\frac{i_k}{k}\right)^{-2}$

3.25) $\frac{\partial g_h}{\partial h} = \frac{1}{m_h}\left(\frac{i_h}{h}\right)^2 \left(1 + \frac{1}{m_h}\frac{i_h}{h}\right)^{-2}$

In addition, given the model calibration, we assume that condition $r < n + \lambda$ applies, otherwise the intertemporal welfare integral will be unbounded; and condition $r \leq \rho + \lambda\theta$ applies also, otherwise the country would eventually accumulate enough assets to violate the small economy assumption.