A Note on Obtaining Welfare Bounds in Referendum Contingent Valuation Studies

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I. INTRODUCTION

In a recent article published in *Land Economics*, Boman, Bostedt and Kriström (1999, hereafter BBK) show how nonparametric lower bound, intermediate and upper bound measures of average welfare change can be extracted from discrete-response contingent valuation survey data, building on the work of Kriström (1990) and Haab and McConnell (1997). The article appears to offer a promising route to getting estimates of mean willingness to pay (WTP) along with their respective variances without having to estimate parametric choice models, thereby providing a computationally simple way to capture both the methodological uncertainty about the choice of a central tendency measure and the statistical uncertainty associated with any particular measure. Having an accurate estimate of the variance of mean WTP allows one to make confidence interval statements about mean WTP. When measures of mean WTP and its variance are extracted from stated preference data and used in the cost-benefit analysis of proposed projects or policies, the extent of statistical uncertainty measured by the variance of mean WTP can be incorporated in a Monte-Carlo risk analysis to produce an empirical distribution of net benefits rather than a single point estimate (See Vaughan *et al.* 1999, 2000a, 2000b). In short, the methods proposed by BBK potentially have a number of useful applications.

While BBK try to put all three measures on a consistent symbolic footing, there appear to be minor errors in their notation for the means. Unfortunately, and more importantly, their variance formulas (BBK p. 289) are conceptually incorrect because they treat the bid, not the cell proportions, as random variables. In fact the bids are constants because they are pre-selected by the designer of the referendum contingent valuation (CV) survey that the respondents must react to by accepting or rejecting the offer. Those reactions, not the bids, are random variables. Moreover, the nonparametric expected value formulas are just a linear function of those random variables, but the BBK variance formulas (provided for reference in Appendix Table A1) were apparently not derived explicitly with that fundamental property in mind. Instead, an inappropriate textbook formula for the variance of the mean was forced to stand in.[1]

Below, the nonparametric lower bound mean and variance formulas of Haab and McConnell (1997) are explained in simple terms and generalized to cover the intermediate and upper bound measures suggested by BBK. Discrepancies between the competing formulas for the

[1] We originally discovered this discrepancy by comparing the variances of the lower bound (Turnbull) means produced using the Haab and McConnell formula and the BBK formula. The variance from the latter was roughly double the former using referendum CV survey data collected in Brazil (discussed in Vaughan *et al.* 1999). Since subsequent sections of this paper demonstrate that the BBK variance formulas are incorrect, we do not discuss them further. It should be noted, however, that the BBK mean formulas contain notational idiosyncracies that make it difficult to replicate the nonparametric means reported by BBK in their Table 1, p. 291. Specifically, the summation should cover the range i = 0 to k and not i = 0 to k-1 as presented by the authors.

variance of the mean are illustrated using the Wolf Survey data from Table A.1 in BBK. The extended versions of the Haab and McConnell variance formulas are then empirically verified by Monte-Carlo simulation, and conclusions are drawn.

II. MECHANICS OF THE NONPARAMETRIC MEAN AND VARIANCE CALCULATIONS

There are (at least) three nonparametric estimators of mean WTP that can be obtained from referendum CV survey data; a lower bound measure that understates (the Turnbull mean of Haab and McConnell, 1997 which is conceptually equivalent to the Laspeyres mean of BBK); an intermediate measure (Kriström's mean, see Kriström 1990 and BBK) and an upper bound measure that overstates (BBK's Paasche mean). The logic behind all three nonparametric estimators is the same. The proportion of "No" answers at each bid level b_j in a referendum CV survey provides a discrete stepwise approximation to the cumulative distribution function F(b). The mean E(b) of a continuous random variable b with a cumulative distribution function F(b) and probability density function f(b) – which is the first derivative of F(b) w.r.t. b – is given by:[2]

$$(1)\ E(b) = \int_{0}^{\infty} b\ f(b)\ db$$

where the lower limit of the integral is set to zero under the assumption that willingness to pay for a utility-improving intervention should not be negative.

The problem is to use a discrete approximation to compute a nonparametric mean:

$$(2)\ E(b) = E(WTP) \approx \sum_{J} b_j\ f(b_j)$$

where the range of b is from zero to some upper limit b_{max} that forces F(b) close to 1.0 because the bid is so high that almost all respondents would be unwilling to pay that amount for the postulated environmental improvement.

The fundamental theorem of the calculus tells us that the area under a curve f(b) between the limits b_1 and b_2 is (i) the sum of a number of infinitesimally small subdivisions in b of length n; (ii) the definite integral of f(b) between the limits; or the difference between the integral F(b) evaluated at b_1 and b_2:

$$(3)\ \lim \sum_{i=1}^{n} f(b_i)\ \Delta b_i = \int_{b_1}^{b_2} f(b)\ db = F(b_2) - F(b_1)$$

[2] To obtain the mean from the survival function, 1-F(x), t he same reasoning developed below also applies.

We know the value of the cumulative distribution F(b) for a set of bids b from the bid group proportions. Therefore, the bid or b range can be split into intervals and the means from each small interval summed to get the grand mean. That is, the contribution to the overall mean from the approximate mean *within* any bid group interval is the product of some b within the interval (i.e. the lower limit, b_1, the upper limit, b_2, or some value of b in between (which Kriström's method sets at the group mid-point) times the probability that b lies between b_1 and b_2:

$$\text{(4) E}(b) \text{ in interval } b_2 - b_1 = \int_{b_1}^{b_2} b \, f(b)\, db = b[F(b_2) - F(b_1)] \text{ for } (b_1 \preceq b \preceq b_2)$$

Since any b inside a given interval will do, b can be measured as the weighted combination of the interval's upper and lower bounds such that $b = \kappa b_1 + (1\text{-}\kappa)b_2$ where $0 \leq \kappa \leq 1$. The grand mean across all bid groups is just the sum of the interval sub-means so repeatedly applying (4) above gives a general expression for the nonparametric mean, as a numerical analyst might derive it:

$$\begin{aligned}\text{(5) E}(b) = \text{E(WTP)} \approx\ & [\kappa b_1 + (1\text{-}\kappa)b_2] \bullet [F(b_2) - F(b_1)] \\ & + [\kappa b_2 + (1\text{-}\kappa)b_3] \bullet [F(b_3) - F(b_2)] \\ & + \ldots + [\kappa b_m + (1\text{-}\kappa)b_{m+1}] \bullet [F(b_{m+1}) - F(b_m)]\end{aligned}$$

where $b_1 = 0$ and b_{m+1} equals a large positive number b_{max} when bounding from above at average income or some assumed fraction of average income.

If κ equals 1, Eq. (5) is Haab and McConnell's lower bound Turnbull mean[3] which sets each b_j to the lower bound of the bid interval (i.e. the first interval runs from zero to the lowest bid offered so $b_{j=1}$ is set to 0, etc.). The intermediate and upper bound means are obtained analogously by simply redefining the point of evaluation, b, in each bid interval. Kriström's intermediate mean sets κ to ½ (the mid point of the interval) while the BBK upper bound mean sets κ to 0. In the second column of Table 1 all three nonparametric mean measures are recast in a generalization of Haab and McConnell's notation.

Providing a simple explanation of the correct expression for the variance of the mean is slightly more difficult, and comes from the general formula for the variance of a linear combination of random variables (see Box, Hunter and Hunter 1978, p. 87; Burrington and May, p. 27, Rohatgi 1984, p. 258). Suppose an outcome y is a linear function of the $j = 1,\ldots,m+1$ random variables $x_1, x_2, \ldots, x_m$ so $y = a_1 x_1 + a_2 x_2 + \ldots + a_m x_{m+1}$ where the a_j are positive constants. The mean E(y) of the variable y is a linear function of the respective means $\mu_1,\ldots,\mu_{m+1}$ of the x_j, so:

[3] Notice that the (unobserved) value of b_{m+1}, which represents the bid driving the probability of acceptance to zero and the probability of rejection to one does not figure in Haab and McConnell's (1997) lower bound calculation, but must be assumed to implement the intermediate and upper bound estimates of Kriström (1990) and BBK.

$$(6)\ E(y) = \sum_{j=1}^{m+1} a_j \mu_j$$

Eq. (6) says the same thing as Eq. (5), if the constants $[\kappa b_j + (1-\kappa) b_{j+1}]$ are represented by a_j and the probability density in each bid interval, $[F(b_j) - F(b_{j-1})] = p_j$, by the μ_j .

TABLE 1
GENERALIZED HAAB AND McCONNELL FORMULAE FOR THE NONPARAMETRIC MEANS AND THEIR VARIANCES

Measure	**Mean**[a]	**Variance of Mean**
Lower Bound	$\sum_{j=1}^{M+1} b_{j-1}\, p_j$	$\sum_{j=1}^{M+1} (b_{j-1})^2[V(F_j) + V(F_{j-1})] - 2 \sum_{j=1}^{M} b_j\, b_{j-1}\, V(F_j)$
Inter-mediate	$\sum_{j=1}^{M+1} [\kappa\, b_{j-1} + (1-\kappa)\, b_j]\, p_j$	$\sum_{j=1}^{M+1} [\kappa\, b_{j-1} + (1-\kappa) b_j]^2\, [V(F_j) + V(F_{j-1})] - 2 \sum_{j=1}^{M} [\kappa\, b_{j-1} + (1-\kappa) b_j] \bullet [\kappa\, b_j + (1-\kappa)\, b_{j+1}] V(F_j)$
Upper Bound	$\sum_{j=1}^{M+1} b_j\, p_j$	$\sum_{j=1}^{M+1} (b_j)^2[V(F_j) + V(F_{j-1})] - 2 \sum_{j=1}^{M} b_j\, b_{j+1}\, V(F_j)$

Notes:

a. The probability density in bid group j, p_j , equals the difference between the estimates of cumulative density in the current and preceding bid groups, $F_j - F_{j-1}$, where, letting N_j represent the number of "No" responses and Y_j the "yes" responses in group j, $F_j = N_j / (N_j + Y_j)$. There are j = 1...M distinct bids specified in the survey. The bid j = M+1 is the ultimate bid level that the researcher must *assume*. It presumably drives F_j to 1.0.

b. The variance of each proportion F_j is equal to $[F_j \bullet (1 - F_j)] \div (N_j + Y_j)]$.

Writing the variances of the μ_j subgroup expectations as σ_j^2 / n_j , where there are n_j observations in each subgroup, the overall variance of E(y) is, in general:

$$(7)\ \text{Var}\,[E(y)] = \sum_{j=1}^{m+1} a_j^2 (\sigma_j^2 / n_j) + 2 \sum\sum_{i<j} a_i\, a_j\, (\sigma_i / n_i^{1/2})\, (\sigma_j / n_j^{1/2})\, \rho_{ij}$$

where ρ_{ij} is the correlation between i and j and the double sum $\sum\sum_{i<j}$ denotes the sum over all i and j for $i < j$.

The covariance between all μ pairs is the term $(\sigma_i / n_i^{1/2})\ (\sigma_j / n_j^{1/2})\ \rho_{ij}$. When there is dependency between all pairs of μ_i and μ_j the covariance matrix is block triangular and when the variables are statistically independent all of the ρ_{ij} are zero and the second term on the r.h.s of Eq.(7) vanishes.

In the specific case of the variance of the nonparametric means, Haab and McConnell (1997, Eq. 10, p.258) prove that the covariance between any nonconsecutive $p_{i<j}$ is zero,

and equal to the *negative* of the variance of F_j for consecutive pairs $i = j-1; j$. Then, the nonzero elements in the covariance matrix reduce to a vector, and Eq. (7) reduces to:

$$(8)\quad \text{Var}\,[E(y)] = \sum_{j=1}^{m+1} a_j^2 (\sigma_j^2 / n_j) - 2 \sum_{j=1}^{m} a_i\, a_j\, (\sigma_i / n_i^{1/2})\, (\sigma_j / n_j^{1/2})\, \rho_{ij}$$

where the summation on the second term on the r.h.s. of (8) runs from j = 1,...m because the variance of the final (m+1th) term is zero.[4] As before, represent constants $[\kappa b_j + (1-\kappa)b_j]$ by a_j . Following Haab and McConnell (1997), the variance of the probability density in each bid interval, $V(p_j)$ is $V[F(b_j)] + V[F(b_{j-1})]$ or $\{[F(b_j)(1 - F(b_j))] / n_j\} + \{[F(b_{j-1})(1 - F(b_{j-1}))] / n_j\}$; represent that variance by the (σ_j^2 / n_j) appearing in the first term on the r.h.s. of (8). As noted, the covariance of p_i , p_j is $-V(F(b_j)$ or $-\{[F(b_j)(1 - F(b_j))] / n_j\}$; represent its absolute value in the second term on the r.h.s. of Eq.(8) by $(\sigma_i / n_i^{1/2})\, (\sigma_j / n_j^{1/2})\, \rho_{ij}$. After making the substitutions we end up with an expression for the variance of the mean from Haab and McConnell (1997, Eq. 12, p. 259), that has been generalized to apply to the BBK intermediate and upper bound means:

$$(9)\ \text{Var}(E(WTP)) = \sum_{j=1}^{M+1} [\kappa\, b_{j-1} + (1-\kappa) b_j]^2 \bullet [V(F_j) + V(F_{j-1})]$$

$$- 2 \sum_{j=1}^{M} [\kappa\, b_{j-1} + (1-\kappa) b_j] \bullet [\kappa\, b_j + (1-\kappa)\, b_{j+1}] V(F_j)$$

The last column of Table 1shows the specifics for the lower bound, intermediate and upper bound means.

III. EMPIRICAL VERIFICATION

Using the generalized Haab and McConnell notation presented in Table 1 above, we set up a spreadsheet calculation of the means. First we calculated the means from the data presented in the article on a contingent valuation survey on the benefits of preserving a viable Swedish wolf population. The results, provided in the Appendix (Tables A2, A4, and A6) matched the means reported by BBK, calculated from their formulas (see Appendix Table A1). We then generated 100,000 lower bound, intermediate and upper bound means via Monte Carlo simulation in Crystal Ball letting each cell proportion at each trial involve a draw from a binomial distribution with parameters defined as the number of observations in each bid cell and the probability of refusal (the setup for the lower bound mean simulation is shown in Table A8; the other two proceed analogously). By construction, the variances of the empirically generated distributions of 100,000 sample means are completely

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[4] That is, the cumulative distribution at $F(b_{m+1})$ equals 1.0 so $\text{Var}(F(b_{m+1})) = \{[F(b_{m+1})] \bullet [1 - F(b_{m+1})]\} / n_{m+1}$. The denominator is not observed but the numerator is $(1 \bullet 0) = 0$.

independent of either the BBK or the extended Haab and McConnell variance formulas, thereby providing a way to discriminate between them.[5]

The results in Table 2 confirm the correctness of the generalized Haab and McConnell variance formula. The difference between its variance estimates and the empirical estimates is less than 0.1%. In contrast, the difference between the variance estimates from the BBK formula and the empirical estimates is large and grows as the bid chosen to delimit the intervals shifts from the lower to the upper bound. Therefore, the coefficient of variation is stable with the Haab and McConnell formulas and the empirical simulation, but declines as the bid intervals move from lower toward upper bounds under the BBK formulas.[6]

TABLE 2.
VERIFICATION OF MEAN AND VARIANCE FORMULAE USING THE BBK WOLF DATA

Measure	**Means** [SEK]			**Variances of the Means**		
	Reported by BBK[a]	Calculated Using Generalized Haab & McConnell Formulae[b]	Crystal Ball/Excel Monte-Carlo Simulation, 100,000 Trials[c]	Reported by BBK[a]	Calculated Using Generalized Haab & McConnell Formulae[b]	Crystal Ball/Excel Monte-Carlo Simulation, 100,000 Trials[c]
Lower Bound	540.2	540.2	540.2	3093.5	3438.4	3442.4
Intermediate	897.9	898.7	898.7	3535.1	5006.2	4998.7
Upper Bound	1209.4	1211.2	1211.2	2726.8	8620.5	8612.8

Notes:

a. BBK, Table 1, p. 291.

b. Application of our formulas in Table 1 to the BBK data in their Table A1, p. 293. See Appendix Tables A2 - A7 below for the calculations.

c. Uses repeated draws from the binomial distribution in each bid group, b_j to generate the number of "No" answers and hence F_j and p_j. See the simulation setup in Appendix Table A8. Monotonicity was not imposed. The program *Crystal Ball* by Decisioneering Inc., Denver Colorado was used for the simulation.

[5] The empirical Monte Carlo variance estimate comes from applying the conventional textbook variance formula to the sample of 100,000 sample means.

[6] BBK also report on a Monte Carlo exercise, but if they in fact applied their formula for the variance of the mean to 1000 simulated survey data sets they could not have discovered the error.

IV. CONCLUDING REMARKS

By choosing the value of the weighting factor κ, the Haab and McConnell (1997) formulas for the lower bound (Turnbull) nonparametric mean and its variance can be safely generalized to cover alternative measures, such as the intermediate mean proposed by Kriström (1990) the BBK upper bound mean, or any other plausible combination of upper and lower bound values. For instance, if one believes that the true but unobserved continuous relationship between the marginal willingness to pay for a fixed amount of a public good and the probability of payment is everywhere convex to the origin (i.e. Figure 1 in BBK), both the intermediate mean of Kriström and the upper bound mean of BBK will overstate benefits, so κ should probably be ≥ 0.5.

The errors depicted in the variance calculation may seem trivial. But in some applications, the variance is as relevant as the mean. For instance, when WTP estimates from CV surveys are used in a risk analysis of the distribution of the net benefits of an investment or a policy proposal, the distribution of the expected willingness to pay will have an effect on the distribution of the net present value outcome. In such cases, the likelihood of project success in the face of uncertainty about benefits can be quite different depending on how the variance of mean WTP was estimated.

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APPENDIX

TABLE A.1
BOMAN ET AL. FORMULAE FOR MEAN AND VARIANCE

Measure	Mean	Variance of Mean
Lower Bound	$\hat{\mu}_L = \sum_{i=0}^{k-1} \hat{\pi}_{i+1}(\lvert A_{i+1}\rvert - \lvert A_i\rvert)$	$v\hat{a}r(\hat{\mu}_L) = \frac{\sum_{i=0}^{k-1} (A_{i+1} - \hat{\mu}_L)^2(\hat{p}_i - \hat{p}_{i+1})}{n}$
Intermediate	$\hat{\mu}_I = \sum_{i=0}^{k-1} \frac{1}{2}(A_i + A_{i+1})(p_i - \hat{p}_{i+1})$	$v\hat{a}r(\hat{\mu}_I) = \frac{\sum_{i=0}^{k-1} (A_{i+1} - \hat{\mu}_I)^2(\hat{p}_i - \hat{p}_{i+1})}{n}$
Upper Bound	$\hat{\mu}_P = \sum_{i=0}^{k-1} \hat{\pi}_i(\lvert A_{i+1}\rvert - \lvert A_i\rvert)$	$v\hat{a}r(\hat{\mu}_P) = \frac{\sum_{i=0}^{k-1} (A_{i+1} - \hat{\mu}_P)^2(\hat{p}_i - \hat{p}_{i+1})}{n}$

Notes:

In the notation of BBK, responses are obtained from m_1, m_2, ...m_{k-1} individuals observing h_i people in each group accepting the bid amount A_i. The total number of observations across all m in the k-1 bid groups is n. A_i represents the bid level, k-1 are the i subsamples (i = 1, 2,...., k-1), and $\hat{\pi}$ in the mean formulae and $\hat{p}_i$ in the variance formulae represent the estimated proportions or the probability of acceptance. That is $p_i = h_i / m_i$.

TABLE A2.
NONPARAMETRIC LOWER BOUND MEAN, BBK WOLF DATA

Bid Group j	Bid b_j	Total # of "No" Answers	Total # of Cases	Cumulative Distribution CDF=F_j= $NO_j/TOTAL_j$	Probability Density PDF=P_j= F_j-F_{j-1}	Estimate of E(WTP): $b_{j-1} \bullet p_j$
j=0	0	na	none	0.000	na	na
j=1	20	68	248	0.274	0.274	0.00
j=2	100	104	252	0.412	0.138	2.77
j=3	500	182	247	0.736	0.324	32.41
j=4	1500	194	237	0.818	0.081	40.86
j=M=5	5000	224	237	0.945	0.126	189.87
j=M+1=6	>5000	na	none	1.000	0.054	274.26
	Column Sum:	772	1221		1.000	540.18
					Average WTP:	**SEK 540.18**

TABLE A3.
VARIANCE AND STANDARD ERROR OF THE LOWER BOUND MEAN

Bid Group j	$V(F_j)$	b_{j-1}^2	$b_j \bullet b_{j-1}$	$V(F_j) + V(F_{j-1})$	Term#1: $b_{j-1}^2 \bullet [V(F_j) + V(F_{j-1})]$	Term #2: $-2b_j \bullet b_{j-1} \bullet V(F_j)$
j=0	0.000000	…	…	…	…	…
j=1	0.000802	0	0	0.000802	0.00	0.00
j=2	0.000962	400	2000	0.001764	0.71	-3.85
j=3	0.000785	10000	50000	0.001747	17.47	-78.50
j=4	0.000627	250000	750000	0.001412	352.92	-939.98
j=M=5	0.000219	2250000	7500000	0.000845	1902.15	-3281.24
j=M+1=6	0.000000	25000000	…	0.000219	5468.73	0.00
				Column Sum	7741.97	-4303.56
					Variance of the Mean=Term1+Term2	3438.41
					Standard Error of the Mean, SE	**58.63**

TABLE A4.
NONPARAMETRIC INTERMEDIATE MEAN, BBK WOLF DATA

Bid Group j	Bid Mid Point b_j	Total # of "No" Answers	Total # of Cases	Cumulative Distribution CDF=F_j= $NO_j/TOTAL_j$	Probability Density PDF=P_j= F_j-F_{j-1}	Estimate of E(WTP): $b_{j\ mid} \bullet p_j$
j=0	0	na	none	0.000	na	na
j=1	-58.8	68	248	0.274	0.274	-16.12
j=2	60	104	252	0.412	0.138	8.31
j=3	300	182	247	0.736	0.324	97.24
j=4	1000	194	237	0.818	0.081	81.72
j=M=5	3250	224	237	0.945	0.126	411.39
j=M+1=6	5763.9	na	none	1.000	0.054	316.16
Column Sum:		772	1221		1.000	898.71
					Average WTP:	**SEK 898.71**

Note: The range in bids used by BBK runs from -137.6 to 6527.8 so the bid mid point at j=1 is the average of -137.6 and the first actual bid in the survey, 20. Likewise, the bid mid point at j=6 is the average of the last actual bid in the survey, 5,000, and 6527.8.

TABLE A5.
VARIANCE AND STANDARD ERROR OF THE INTERMEDIATE MEAN

Bid Group j	$V(F_j)$	b_{midj}^2	$b_{midj} \bullet b_{midj+1}$	$V(F_j) + V(F_{j-1})$	Term#1: $b_{midj}^2 \bullet [V(F_j) + V(F_{j-1})]$	Term #2: $-2b_{midj} \bullet b_{midj+1} \bullet V(F_j)$
j=0	0.000000	…	…	…	…	…
j=1	0.000802	3457	-3528	0.000802	2.77	5.66
j=2	0.000962	3600	18000	0.001764	6.35	-34.63
j=3	0.000785	90000	300000	0.001747	157.22	-471.03
j=4	0.000627	1000000	3250000	0.001412	1411.69	-4073.23
j=M=5	0.000219	10562500	18732675	0.000845	8929.53	-8195.51
j=M+1=6	0.000000	33222543	…	0.000219	7267.40	0.00
				Column Sum	17774.97	-12768.72
					Variance of the Mean=Term1+Term2	**5006.24**
					Standard Error of the Mean, SE	**70.75**

TABLE A6.
NONPARAMETRIC UPPER BOUND MEAN, BBK WOLF DATA

Bid Group j	Bid b_j	Total # of "No" Answers	Total # of Cases	Cumulative Distribution CDF=F_j= $NO_j/TOTAL_j$	Probability Density PDF=P_j= F_j-F_{j-1}	Estimate of E(WTP): $b_j \bullet p_j$
j=0	0	na	none	0.000	na	na
j=1	20	68	248	0.274	0.274	5.48
j=2	100	104	252	0.412	0.138	13.85
j=3	500	182	247	0.736	0.324	162.07
j=4	1500	194	237	0.818	0.081	122.58
j=M=5	5000	224	237	0.945	0.126	632.91
j=M+1=6	5000	na	none	1.000	0.054	274.26
	Col. Sum:	772	1221		1.000	1211.16
					Average WTP:	**SEK1211.16**

Note: BBK set the assumed bid at j=6 as equal to the last observed bid at j=5. This is inconsistent with the assumption for the intermediate mean calculation in Table A4 above, and the fact that at a bid of 5000, F(b_j) is not equal to 1.0.

TABLE A7.
VARIANCE AND STANDARD ERROR OF THE UPPER BOUND MEAN

Bid Group j	$V(F_j)$	b_j^2	$b_j \bullet b_{j+1}$	$V(F_j) + V(F_{j-1})$	Term#1: $b_j^2 \bullet [V(F_j) + V(F_{j-1})]$	Term #2: $-2b_j * b_{j+1} * V(F_j)$
j=0	0.000000	…	…	…	…	…
j=1	0.000802	400	2000	0.000802	0.32	-3.21
j=2	0.000962	10000	50000	0.001764	17.64	-96.18
j=3	0.000785	250000	750000	0.001747	436.72	-1177.57
j=4	0.000627	2250000	7500000	0.001412	3176.31	-9399.75
j=M=5	0.000219	25000000	25000000	0.000845	21134.98	-10937.46
j=M+1=6	0.000000	25000000	0	0.000219	5468.73	0.00
				Column Sum	30234.70	-21614.17
					Variance of the Mean=Term1+Term2	8620.53
					Standard Error of the Mean, SE	**92.84**

TABLE A8.
CRYSTAL BALL MONTE CARLO SPREADSHEET SETUP FOR THE LOWER BOUND MEAN

Column / Row	A	B	C	D	E	F
1	b_j	**Total # of "No" Answers**[a]	**Total # of Cases**	$F_j = No_j/Total_j$	$P_j = F_j - F_{j-1}$	**Mean**
2	0	na	none	0.00	na	
3	20	Binomial (n=248, θ = 68/248)	248	B3/C3	D3-D2	E3•A2
4	100	Binomial (n=252, θ =104/252)	252	B4/C4	D4-D3	E4•A3
5	500	Binomial (n=247, θ =182/247)	247	B5/C5	D5-D4	E5•A4
6	1500	Binomial (n=237, θ =194/237)	237	B6/C6	D6-D5	E6•A5
7	5000	Binomial (n=237, θ =224/237)	237	B7/C7	D7-D6	E7•A6
8	5000	na	none	1.00	D8-D7	E8•A7
				Column Sum, Forecast Mean[b]		= SUM (F3:F8)

Notes:
[a/]Crystal Ball Assumption Cells
[b/]Crystal Ball Forecast Cell