

Theoretical Background on External Sustainability Assessments

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Abstract*

This paper explains in detail the external sustainability assessment approach given by the stock-flow relationship between the net external positions, non-income current account plus net capital transfers, and real exchange rate. This approach consists of determining the non-income current account over GDP that would stabilize a benchmark net foreign asset (NFA) position over the medium term, and a “gap” by comparing the NFA-stabilizing non-income current account over GDP with the non-income current account expected to prevail over the medium term. Additionally, the paper addresses the role of the returns differential in explaining the channels through which the external sustainability adjustment may occur (i.e., trade and financial channels).

JEL classifications: E01, F31, F32, F37

Keywords: Net foreign assets positions, Trade balance, Non-Income current account balance, External sustainability template, Return differentials between assets and liabilities

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1. Introduction

External sustainability assessment comes in many forms, such as medium-term balance of payment projections, external debt scenario analysis, assessing reserves adequacy, external vulnerability, and intertemporal solvency constraint analysis, among others.

Our analysis addresses the stock-flow relationship between the sustainability of a country's net external positions, non-income current account balance plus net capital transfers (henceforth called non-income current account balance), and real exchange rate. This approach consists of determining the non-income current account balance over GDP that would stabilize a benchmark net foreign asset (NFA) position over the medium term, which may be arbitrarily chosen by taking the latest year with available data. Then, we calculate a "gap" by comparing the NFA-stabilizing non-income current account balance over GDP with its value expected to prevail over the medium term, assuming no real exchange rate misalignments. Finally, we can calculate the real effective exchange rate misalignment to stabilize the net foreign assets positions over GDP over the medium-term, on the basis of considering a Semi-Elasticity of Trade balance¹ over GDP to Real Exchange Rate (REER).

The external sustainability approach becomes a benchmark against other sophisticated econometric approaches, because it relies on a few assumptions regarding the long-term real GDP growth rate, inflation rate, and real rates of return on external assets and liabilities. This framework raises important questions about the implications of the external sustainability assessments. For example, can a net debtor country run a trade balance deficit and ensure intertemporal sustainability? In addition, are the current account and trade imbalances sustainable? These issues may be addressed by asking whether the continuation of the current policy would require a drastic policy shift (i.e., a huge real exchange rate adjustment to stabilize the net external positions). If such a drastic shift were not implemented, then the external imbalances may be unsustainable.

In this paper, we tackle that the returns differential assumption between real rates of return on foreign assets and liabilities is a key factor in addressing which channels the

¹ We have assumed that the Semi-Elasticities of Net Current Transfers and Net Capital Transfers over GDP to REER are zero.

external sustainability adjustment may occur through (i.e., trade and financial channels). Lee et al. (2008), Gourinchas and Rey (2007), and Lane and Milesi-Ferretti (2002) documented the important role of the returns differential on the External Sustainability Assessment. Thus, when the rate of return on assets equals the rate of return on liabilities,² the external sustainability adjustment can only occur through the trade channel,³ where the net debtor countries must run trade surpluses (and creditor countries deficits) to stabilize external positions over GDP. Conversely, when the rate of return on assets differs from the rate of return on liabilities, the external sustainability adjustment may occur through both the trade and financial adjustment channels.⁴

Under the assumption of returns differential, there are two possible scenarios. In the first, the expected rates of return on foreign assets are more favorable than the rates of return on foreign liabilities (i.e., positive returns differential), then the requirement of trade surpluses may be lightened or even entirely offset. This could imply that a net debtor country may run a perpetual trade balance deficit to stabilize the net external positions and, in the second scenario, if the returns differential were negative, the large net debtor countries⁵ would undergo greater adjustment of future trade flows (i.e., requiring larger relative price adjustments) to stabilize net foreign assets than implied if the rate of returns differential were positive. We will elaborate more on the returns differential discussion in Section 2.

Due to the key role of the Financial Adjustment Channel in the External Sustainability Assessment, we have constructed a comprehensive historical database of nominal and real rates of return (yields plus capital gains and losses) on assets and liabilities for Latin America and The Caribbean countries, using the Balance of Payments (BOP) database⁶ of interest earnings and payments on external holdings and capital flows published by the International Monetary Fund (IMF) and the updated Lane and Milesi-

² When the rate of returns on external assets and liabilities are the same, the external sustainability analysis reduces to the standard debt accumulation analysis.

³ Trade Channel is driven by Trade Balance plus services

⁴ Financial Adjustment Channel provides evidence on the importance of valuation effects on the dynamics of Net Foreign Assets, where rates of return inclusive of capital gains are allowed to vary over time.

⁵ For more details see Appendix, Figures B2-A to B2-E show that all Latin American and the Caribbean Countries, with the exception of Argentina in the latest years, are net debtor countries.

⁶ This approach undertakes substantial measurement error problems, which will be addressed later in Section 2.

Ferretti (2007) dataset ranging from 1970-2013.⁷ This way, we can broadly study the pattern of BOP derived-returns differential over time in our sample. (For further details, see Section 3.2).

The relevance of the Financial Adjustment Channel was broadly addressed in Lane and Milesi-Ferretti (2002, 2005), who questioned the traditional view of net external wealth as the accumulation of current account balance. They highlighted the empirical significance of asset price and exchange rate movements in the dynamics of NFA. More recently, a growing literature that incorporates portfolio choice and asset pricing in open economy macro models has explored the effects of valuation effects on the NFA positions in a general equilibrium setting (see Evans and Hnatkovska, 2007, and Devereux and Sutherland, 2008).

Following, we enumerate the main differences between our approach and the developed one in Lee et al. (2008), which is currently carried out by the IMF and frequently reported in the Article IV Consultation-Staff Reports. Lee et al. (2008) addresses the total current account balance that stabilizes the net external positions and ignores net capital transfers, and valuation changes, such as net capital gains and losses. Carrying out the external sustainability assessment based on the current account balance as a whole entails several caveats since it only yields dividends earned and paid on cross-border portfolio equity holdings. However, most of the returns on portfolio equity and foreign direct investment (FDI) occur through net capital gains and losses, which are not captured in the current account balance and represent a significant share of GDP.

Considering the aforementioned caveats, our approach also addresses the factors that accounts for the returns differential; incorporates the returns differential into the external sustainability assessment; considers the changes in the capital value of stocks (capital gains or losses), the net current transfers (which includes remittances) and the net capital transfers (including grants), which can be an important financing source for tourism-dependent economies with large amounts of FDI inflows.

The remainder of this document is organized as follows. Section 2 discusses the theoretical background for the external sustainability approach, and Section 3 details the

⁷ Although Lane and Milesi-Ferretti (2007) have published updates to their database up to 2011 so far, we have this database available up to 2013 as a courtesy of the authors.

choice of the benchmark values for NFA over GDP and rates of return on foreign assets and liabilities and proposes alternative measures. Section 4 provides data sources for the external sustainability assessment.

2. Theoretical Background

2.1 Net Foreign Assets Dynamics

This section describes the external sustainability approach,⁸ which relies on an intertemporal budget constraint given by equation (1) that relates net foreign assets, trade balance, rates of return on foreign assets and liabilities:

$$NFA_t - NFA_{t-1} = CA_t + KG_t + KT_t \quad (1)$$

where NFA_t is the net foreign asset position, CA_t denotes the current account balance, KG_t denotes net capital gains or losses arising from valuation changes, and KT_t includes capital account transfers (including grants)⁹ and errors and omissions, which can drive a wedge between the current account balance and net financial flows.

In equation (2), we decompose the current account into the trade balance in goods and services (TB_t), net current transfers (NTR_t) and the investment income balance components denoted as $(i_t^{fa}FA_{t-1} - i_t^{fl}FL_{t-1})$, where FA_t and FL_t denote the nominal assets and liabilities stocks, respectively.

Additionally, i_t^{fa} and i_t^{fl} denote nominal yields on foreign assets and liabilities held in period $t - 1$, defined as: $i_t^{fa} = \frac{IC_t}{FA_{t-1}}$ and $i_t^{fl} = \frac{ID_t}{FL_{t-1}}$, respectively, where investment income receipts (income credit) in U.S. dollars in year t is denoted as IC_t , and investment income payments (income debits) in U.S. dollars in year t is denoted as ID_t .

$$CA_t = TB_t + NTR_t + (i_t^{fa}FA_{t-1} - i_t^{fl}FL_{t-1}) \quad (2)$$

By combining equations (1) and (2), we obtain the following equation:

$$NFA_t - NFA_{t-1} = TB_t + NTR_t + (i_t^{fa}FA_{t-1} - i_t^{fl}FL_{t-1}) + KG_t + KT_t \quad (3)$$

⁸ For more detail on the External Sustainability Approach, see Lee et al. (2008).

⁹ Net Total Capital Transfers account for grants and debt forgiveness and they are particularly representative for highly indebted poor countries.

We can express equation (3) as ratios to local GDP denominated in US\$ with lowercase letters:

$$nfa_t - nfa_{t-1} = tb_t + ntr_t + \frac{(i_t^{fa}FA_{t-1} - i_t^{fl}FL_{t-1})}{GDP_t} + \frac{KG_t}{GDP_t} + \frac{1-(1+g_t^n)}{(1+g_t^n)}nfa_{t-1} + kt_t \quad (4)$$

where g_t^n is the growth rate of nominal GDP in US\$, π_t is the U.S. inflation rate, and the term kt_t includes the ratio of net capital transfers and errors and omissions to GDP. The third term on the right hand-side of equation (4) captures the effect of nominal returns on external assets and liabilities on the dynamics of external positions, since the subcomponents of the current account balance are not immune to the composition of net foreign assets and its returns. Now we redefine the nominal net capital gains or losses as:

$$KG_t = (kg_t^{fa}FA_{t-1} - kg_t^{fl}FL_{t-1})$$

where kg_t^{fa} (kg_t^{fl}) represent nominal capital gains on assets (liabilities)¹⁰ and they are defined by the difference between the change in the stock of assets (liabilities) between t and $t - 1$ and its underlying capital outflows (inflows)¹¹ during t denoted as a_t^f (l_t^f), divided by the initial stock of assets (liabilities), as it may be seen below:

$$kg_t^{fa} = \frac{FA_t - FA_{t-1} - a_t^f}{FA_{t-1}} \quad kg_t^{fl} = \frac{FL_t - FL_{t-1} - l_t^f}{FL_{t-1}}$$

Next, we may re-write equation (4) as follows:

$$nfa_t - nfa_{t-1} = tb_t + ntr_t + \frac{((i_t^{fa} + kg_t^{fa})fa_{t-1} - (i_t^{fl} + kg_t^{fl})fl_{t-1})}{(1+g_t^n)} + \frac{1-(1+g_t^n)}{(1+g_t^n)}nfa_{t-1} + kt_t \quad (5)$$

After straightforward calculations, equation (5) may be expressed in terms of real rate of return on foreign assets (r_t^{fa}) and liabilities (r_t^{fl}) in U.S. dollars as follows:

$$nfa_t - nfa_{t-1} = tb_t + ntr_t + kt_t + \left(\frac{((1+\pi_t)(1+r_t^{fl}) - (1+g_t^n))}{1+g_t^n} \right) nfa_{t-1} + \frac{(1+\pi_t)(r_t^{fa} - r_t^{fl})}{1+g_t^n} fa_{t-1} \quad (6)$$

¹⁰ For further explanation about the capital gains or losses calculation, see Section 2.2.

¹¹ Capital Inflows enters with positive sign and Capital Outflows with negative sign.

where total real rates of return on assets (r_t^{fa}) and liabilities (r_t^{fl}), inclusive of capital gains, may be written as in the following equations:

$$r_t^{fi} = \left(\frac{1+i_t^{fi}}{1+\pi_t} - 1 \right) + \left(\frac{1+kg_t^{fi}}{1+\pi_t} - 1 \right) + \frac{\pi_t}{1+\pi_t} \quad i = \{a, l\}$$

It is important to note that by construction, the real total rate of return is not equal to the sum of the real yield and the real rate of capital gains, but it is necessary to add the term $\left(\frac{\pi_t}{1+\pi_t} \right)$.

In order for equation (6) to capture the effect of changes in the real exchange rate (ΔX_t) on the net foreign assets path we express the growth rate of nominal GDP in US\$ (g_t^n) in terms of the growth rate of real GDP in local currency units (g_t^{LCU}), the annual change of real exchange rate (Δe_t), and the domestic inflation rate denoted as (π_t^{DOM}), defined by the equation below:

$$(1 + g_t^n) = \frac{(1 + g_t^{LCU})(1 + \pi_t^{DOM})}{(1 + \Delta e_t)}$$

Additionally, we define the rate of change of the real exchange rate over time for the local currency unit versus the dollar as (ΔX_t), as follows:

$$(1 + \Delta X_t) = \frac{(1 + \Delta e_t)(1 + \pi_t)}{(1 + \pi_t^{DOM})}$$

Therefore, we can rewrite equation (6) in terms of the growth rate of real GDP in local currency units (LCU) and the annual change of real exchange rate, by using the latter equation as follows:

$$\begin{aligned} nfa_t - nfa_{t-1} = & tb_t + ntr_t + kt_t + \left(\frac{(r_t^{fl} - g_t^{LCU})(1 + \Delta X_t)}{(1 + g_t^{LCU})} + \Delta X_t \right) nfa_{t-1} \\ & + \frac{(r_t^{fa} - r_t^{fl})(1 + \Delta X_t)}{(1 + g_t^{LCU})} f a_{t-1} \quad (6') \end{aligned}$$

Equation (6') delivers three key takeaways. First, the trade balance is just one component in determining the dynamics of the net foreign asset position. Second, the fourth term on the right-hand side is familiar from the standard debt accumulation equation. Third, according to the fifth term on the right-hand side, when rates of return on

assets and liabilities differ (i.e., returns differential), the *gross* scale of the international balance sheet matters.

2.2 NFA-Stabilizing Non-Income Current Account over GDP at a Benchmark Level

The External Sustainability approach based on equation (6') calculates the non-income current account plus net capital transfers over GDP ($nica^s$) that stabilizes the NFA over GDP at a benchmark level,¹² denoted as nfa^s , as follows:

$$nica^s = tb^s + ntr^s + kt^s = - \left(\frac{(r_s^{fl} - g_s^{LCU})(1 + \Delta X_s)}{(1 + g_s^{LCU})} + \Delta X_s \right) nfa^s - \frac{(r_s^{fa} - r_s^{fl})(1 + \Delta X_s)}{(1 + g_s^{LCU})} fa^s \quad (7)$$

As noted in Section 2.1, when the returns on external assets and liabilities are the same, equation (7)¹³ reduces to the standard debt accumulation equation. Assuming this case, if the rate of return on liabilities is higher than the real GDP growth rate, a debtor country will need to run a trade surplus to prevent its net external position from deteriorating.

If the expected rates of return on foreign assets were more favorable than those on foreign liabilities, then the requirement of trade surpluses may be lightened or even entirely offset in the absence of the returns differential. This could imply that a net debtor country may run a perpetual trade balance deficit to stabilize the net external positions. Conversely, if the returns differentials were negative, the debtor country would undergo a larger trade balance to stabilize the net foreign assets than in the case of positive returns differentials.

The second step compares the WEO-projected non-income current account balance over GDP (\overline{nica}) (assuming real exchange rates that are constant at their early 2015 levels), with the *NFA-stabilizing* non-income current account balance over GDP ($nica^s$) from equation (7), denoted as R_{adj} . For this purpose, we must set the country's ad-hoc assumptions that bear out the most adequate measure of non-income current account balance over GDP (\overline{nica}) to calculate the required adjustment:

$$R_{adj} = nica^s - \overline{nica} \quad (8)$$

¹² Variables at a benchmark level are denoted with superscript and subscript *s*.

¹³ This equation shows that returns differential has a key role explaining the evolution of net foreign assets when a country has larger stocks of external assets and liabilities.

The measures (\overline{nica}) usually considered in the practice to calculate the required adjustment (R_{adj}) are: i) Historical Medium-Term Non-Income Current Account Balance over GDP, ii) Actual Non-Income Current Account Balance over GDP, and iii) Medium-Term Projection Non-Income Current Account Balance over GDP. This gap suggests the required adjustment that may be needed to bring in line the most adequate measure of non-income current account balance over GDP to its NFA-stabilizing value. Thus, when that required adjustment is positive, the country must run a non-income current account surplus to stabilize the net external positions, and conversely when this measure is negative.

2.3 Real-Exchange Rate Adjustment Effects on the Net Foreign Asset Positions

Krugman (1999), among other authors, notes that the net foreign asset positions require some degree of real exchange rate adjustment to ensure a sustainable debt path over the medium term in a baseline scenario (e.g., real depreciation (appreciation) would deteriorate (strengthen) the ratio of NFA position to GDP measured in U.S. dollars). Likewise, Lane and Milesi-Ferretti (2007) state an *endogeneity* problem in that NFA over GDP is measured in U.S. dollars, where a nominal depreciation causes both the real exchange rate to depreciate and dollar GDP to fall,¹⁴ also known as the valuation effect.

Therefore, without accounting for potential revaluation in the medium-term, the NFA over GDP could require a much larger real exchange adjustment to place the external position on a sustainable path in the baseline scenario. To operationalize a real exchange rate adjustment in the NFA dynamic (equation (9)), we assume that current account variables and GDP growth rates-WEO forecasts are calculated by holding the REER (X_T) constant at time T over the medium-term (baseline scenario).

$$nfa_T = tb_T + ntr_T + kt_T + \left(\frac{(1+r_T^{fl})(1+\Delta\bar{X}_T)}{(1+g_T^{LCU})} \right) nfa_{T-1} + \frac{(r_T^{fa} - r_T^{fl})(1+\Delta\bar{X}_T)}{(1+g_T^{LCU})} fa_{T-1} \quad (9)$$

Moreover, we denote a benchmark REER (X_T^*), as that level of the REER which satisfies that the NFA over GDP at T (which will potentially be revalued when X_T moves

¹⁴ Thus, if a net debt country has net external liabilities primarily denominated in US dollars (i.e., external debt liabilities), the NFA over GDP would *deteriorate*. This effect would be partly offset by the dynamics of FDI liabilities, whose dollar value falls when the nominal exchange rate depreciates.

to X_T^*) is equal to the present value of future trade balance (which will also change in proportion to trade semi-elasticities, when X_T moves to X_T^*), plus net current and capital transfers and external rates of return differential that places the revalued external position on a sustainable path conditional on WEO forecasts, given by equation (10).

$$nfa_T(X_T^*) = -E_t \left\{ \sum_{i=1}^{\infty} \left(\left(\prod_{j=1}^i \frac{(1+g_{T+j}^{LCU})}{(1+r_{T+j}^{fl})(1+\Delta\bar{x}_{T+j})} \right) \left(tb_{T+i} + ntr_{T+i} + kt_{T+i} + \frac{(r_{T+i}^{fa} - r_{T+i}^{fl})(1+\Delta\bar{x}_{T+i})}{(1+g_{T+i}^{LCU})} fa_{T+i-1} \right) \right) \right\} \quad (10)$$

when X_T^* is equal to X_T (the current REER at which WEO forecasts are made), that would imply that the net foreign asset position at T is consistent with inter-temporal solvency, conditional on WEO forecasts of trade balance flows. Conversely, when X_T^* is different from X_T , that would imply that in order to satisfy the inter-temporal solvency condition, the expected trade flows must be adjusted in a way that aligns the current value of NFA over GDP with the present value of future net exports and external rates of return according to equation (10).

In the External Sustainability Assessment framework, we implement an adjustment of the real exchange rate misalignment by assuming a one-time jump in the REER from X_T to its benchmark level X_T^* at time T which induces a gradual adjustment in the expected trade (exports and imports) flows during the forecasting horizon after T , in proportion to trade semi-elasticities. The intuition behind this assumption relies upon the steady-state conditions that a country must meet to ensure external sustainability over the long-run after closing the REER misalignment at the beginning (T) of the forecasting horizon.

For simplicity, in order to proxy the one-time jump in the REER from X_T to a benchmark level X_T^* at T , we may take the real exchange rate misalignment at T from the latest IMF's Article IV document, if available. However, we could also calculate the benchmark REER X_T^* at time T , by computing the REER which solves equation (10) conditional on the expected trade flows, GDP growth rates and rate returns in a long period of time. Additionally, the exchange rate adjustment could be computed as the required REER to close the gap between the observed non-interest current account deficit

and the required current account balance that stabilizes the net foreign assets position given by equation (9).

In order to address the impact of an one-time jump adjustment in the real exchange misalignment at T on the expected trade balance in goods and services over the medium-term horizon, we follow Tokarick (2010),¹⁵ who discusses in detail the implicit shortcomings and advantages of assuming a trade semi-elasticities partial equilibrium setup. Thus, it may be assumed that the relationship between real exchange rate marginal movements and trade balance over GDP (tb) in a small country depends on estimated trade semi-elasticities, as follows:

$$\frac{dtb}{dX/X} = \delta_{tb, reer} = (1 + \varepsilon^X \phi^E) S_X - (1 + \eta^M \phi^M) S_M > 0$$

where S_X and S_M are the shares of exports and imports in GDP respectively; $\varepsilon^X > 0$ and $\eta^M < 0$ are the estimated export supply and import demand elasticities taken from Tokarick (2010) and defined with respect to changes in the real exchange rate (X), respectively. Lastly ϕ^E and ϕ^M are the pass-through coefficients for export and import prices, respectively and lie between zero and one.¹⁶ There is no consensus in the literature on values for these parameters. Frankel, Parsley, and Wei (2005) estimate that for developing countries and emerging markets, the pass-through coefficient is in the range of 0.66 to 0.77.

Therefore, equation (9) may be re-written in terms of the adjusted expected trade balance in goods and services (tb_T^*) over the medium-term horizon, measuring the impact of the one-time jump in the real exchange rate adjustment at T , as follows:

$$nfa_T = tb_T^* + ntr_T + kt_T + \left(\frac{(1+r_T^{fl})(1+\Delta\tilde{x}_T)}{(1+g_T^{LCU})} \right) nfa_{T-1} + \frac{(r_T^{fa} - r_T^{fl})(1+\Delta\tilde{x}_T)}{(1+g_T^{LCU})} fa_{T-1} \quad (11)$$

¹⁵ Box 1 tackles the conceptual issues about the underlying assumptions to estimate the Semi-Elasticity of Trade Balance over GDP to REER ($\delta_{tb, reer}$).

¹⁶ Pass-through equal to 1 entails that pass through is complete and changes in foreign prices are fully reflected in domestic prices.

where tb_T^* , the adjusted trade balance over GDP¹⁷ after the one-time REER adjustment is defined as follows:

$$tb_j^* = \begin{cases} (1 + \delta_{tb, reer}(X_T - X_T^*)/5)tb_j & \text{if } tb_T < 0 \\ (1 - \delta_{tb, reer}(X_T - X_T^*)/5)tb_j & \text{if } tb_T > 0 \end{cases} \text{ for } j = \{T + 1 \dots T + 4\}$$

Lastly, considering that under the context of WEO forecasts, the REER is assumed constant at time T over the forecast horizon, the change in the REER does not reflect to have impact on the expected rates of return. However, expected returns could respond to the change in the REER through its impact on expected yields (e.g., if future interest payments are denominated in foreign currency), but reflecting this impact on future returns would require detailed information about the foreign currency composition of foreign assets and liabilities. Therefore, due to data shortcomings, we only assume that the one-time jump in REER does have an impact on expected trade balance flows.

3. Further Assumptions for Assessing the External Sustainability Approach

3.1 Benchmark Level for Net Foreign Assets Position

The choice of an adequate benchmark level for the net foreign asset position is a key point in the external sustainability assessment. Typically, the benchmark level used for net foreign assets is the latest year with available data. Even though that benchmark has little normative content, it provides a useful perspective on whether projected trade balance at a current exchange rate is expected to affect the net external asset position over the medium term.

In Appendix B, Figures B1-A and B1-B plot benchmark net foreign assets over GDP in 2013 derived from an updated version of the Lane and Milesi-Ferretti (2007) dataset ranging from 1970-2013¹⁸ for Latin American and the Caribbean countries. Next, we discuss three alternatives for choosing benchmark levels for the net foreign assets that

¹⁷ For example, if the real exchange rate of a country at T (\tilde{X}_T) were initially overvalued (undervalued), the country would require to adjust the trade flows with a gradual depreciation (appreciation) that will improve (deteriorate) the trade balance over the medium term.

¹⁸ Although Lane and Milesi-Ferretti (2007) have published updates to their database up to 2011 so far, we have this database available up to 2013 as a courtesy of the authors.

might be used in the external sustainability assessment and a note on how they should be calibrated.

1. For a select group of economies with extremely high net external liabilities and low net external liabilities, or that are exporters of non-renewable resources, the benchmark is modified on the basis of regional averages or other average criteria.
2. NFA-benchmark levels can be estimated on the basis of cross-section and time-series evidence, relating the net external positions to several determinants such as fiscal policy, demographics, exchange rate regime, and trade openness over GDP, oil and non-oil trade balance, economic growth, economic crises, domestic credit growth, capital controls, and financial center dummies. By using this approach, we could assess an oil price shock's implications for external positions.¹⁹
3. Catão and Milesi-Ferretti (2014) address the existence of a systematic link between NFA and external crises. Therefore, it would be very useful to estimate a benchmark value for NFA based on the threshold for NFA over GDP that raises the conditional probability of external crises. They establish whether external liabilities beyond a certain point appear to be risky with tangible implications for fiscal and macro prudential policies.
4. One aspect to be considered in addressing the benchmark level of net foreign assets is to what extent a country's external positions may be affected by large shifts (positive or negative) in oil prices (or in general commodity prices). For instance, an increase (decrease) in oil prices should be reflected in a temporary accumulation of net foreign assets through current account surpluses (deficits) forecasts for oil exporter countries, and conversely for oil importer countries.

¹⁹ In general, we should expect some degree of endogenous offset in the oil trade balance driven by oil price shifts. That means if a country's oil trade surplus (deficit) narrows in response to a huge oil price drop, then its non-oil current account will, to some extent, tend to move in the opposite direction.

Box. 1. Semi-Elasticity of Trade Balance over GDP to REER

Background

Following Tokarick (2010), the generalized way to define the Semi Elasticity of the Trade Balance over GDP to REER is given by:

$$\delta_{tb, reer} = \left(\frac{-\eta^X(1 + \varepsilon^X \phi^E)}{\eta^X - \varepsilon^X \phi^E} \right) \frac{X_t}{GDP_t} + \left(\frac{\varepsilon^M(1 - \phi^M \eta^M)}{\eta^M \phi^M + \varepsilon^M} \right) \frac{M_t}{GDP_t}$$

where $\eta^X < 0$ and $\eta^M > 0$ denote export and import demand elasticities, respectively. Likewise ε^X and ε^M denote export and import supply elasticities, respectively. ϕ^E , ϕ^M measure the extent of pass-through into exports and import prices. Those elasticities are defined with respect to changes in REER.

Lee et al. (2008) use the following definition for the Semi-Elasticity of Trade Balance over GDP to REER:

$$\delta_{tb, reer} = \eta^X \frac{X_t}{GDP_t} + (1 - \eta^M) \frac{M_t}{GDP_t}$$

This equation implies a number of strong assumptions described as follows:

- The CGER-IMF estimated semi-export supply and import demand semi-elasticities of -0.71 and 0.92, respectively.
- ϕ^E and ϕ^M are equal to 1, i.e., complete pass-through, or in other words, changes in foreign prices and REER are fully reflected in domestic prices.
- Export and Import Supply Elasticities are assumed to be perfectly elastic.

Challenges in Estimating Semi-Trade Elasticities

- Identification of shocks, endogeneity of REER
- Trade Responses to Changes in REER have evolved significantly over time
- Estimates calculated over long periods of time are potentially unreliable
- Frankel, Parsley and Wei (2005) estimate that for developing countries and emerging markets, the pass-through coefficients are in the range of 0.66 to 0.77.

3.2 BOP-Derived Total Rates of Return

In this sub-section, we attempt to describe the calculation of BOP-derived rates of return according to Lane and Milesi-Ferretti (2002) and also propose some market-index alternatives to the BOP-derived measures, which are not exempt from pitfalls, as discussed afterwards.²⁰

²⁰ In the Appendix, Figures B4-A to B4-E plot real rates of return and yields on foreign assets and Figures B5-A to B5-E on liabilities based on the IFS and updated versions of the Lane and Milesi-Ferretti (2007) databases for Latin American and the Caribbean countries over 1980-2014.

According to equation (9), we must calculate real rates and yields of return on foreign assets and liabilities (and, when possible, also on their subcomponents) by using IMF's *BOP* statistics database (investment income receipts and payments on external holding), and IMF's International Financial Statistics (IFS) database (foreign assets and liabilities, and capital flows). In order to collect more historical international investment positions and capital flows data, we may also use the updated Lane and Milesi-Ferretti (2007) publicly available dataset ranging from 1970-2013.

In order to calculate the total real rates of return on foreign assets and liabilities, we firstly define the US\$ aggregated real yield on assets, $i_t^{R,fa}$ (liabilities, $i_t^{R,fl}$), as follows:

$$i_t^{R,fa} = \frac{i_t^{fa}}{1 + \pi_t} = \frac{IC_t/CPI_t}{FA_{t-1}/CPI_{t-1}} \quad i_t^{R,fl} = \frac{i_t^{fl}}{1 + \pi_t} = \frac{ID_t/CPI_t}{FL_{t-1}/CPI_{t-1}}$$

Later we must calculate the US\$ aggregated real capital gains or losses on assets, $kg_t^{R,fa}$ (liabilities, $kg_t^{R,fl}$), as follows:

$$kg_t^{R,fa} = \frac{kg_t^{fa}}{1 + \pi_t} - \frac{\pi_t}{1 + \pi_t} = \frac{(FA_t/CPI_t - FA_{t-1}/CPI_{t-1} - a_t^f/CPI_t)}{FA_{t-1}/CPI_{t-1}}$$

$$kg_t^{R,fl} = \frac{kg_t^{fl}}{1 + \pi_t} - \frac{\pi_t}{1 + \pi_t} = \frac{(FL_t/CPI_t - FL_{t-1}/CPI_{t-1} - l_t^f/CPI_t)}{FL_{t-1}/CPI_{t-1}}$$

where total real rates of return on assets (r_t^{fa}) and liabilities (r_t^{fl}), inclusive of capital gains may be written as in the following equations:

$$r_t^{fi} = i_t^{R,fi} + kg_t^{R,fi} = \underbrace{\frac{i_t^{fi}}{1 + \pi_t}}_{\text{Real Yield}} + \underbrace{\frac{kg_t^{fi}}{1 + \pi_t} - \frac{\pi_t}{1 + \pi_t}}_{\text{Real Capital Gains/Losses}} \quad i = \{a, l\}$$

We could also calculate weighted averages²¹ of the yields and returns on four subcategories of the financial account (i.e., portfolio equity, FDI, external debt and other assets and liabilities), with weights given by the portfolio composition of the total stock

²¹ In Section 4, we calculate aggregates and disaggregated measures of the real rates of return on assets and liabilities.

of assets and liabilities. The importance of this alternative approach relies on smoothing out the aggregate yields and returns.²²

The data difficulties in undertaking this scenario are well documented in Lane and Milesi-Ferretti (2002). They address that the measurement error problems for balance of payment-derived yields and rates of return can arise from several sources, which are briefly described²³ as follows:

- Reclassification of external assets and liabilities over time.
- Recording of interest receipts and payments in balance of payments accounts. For instance, several countries' investment income payments on foreign direct investment (i.e., remitted dividends and interest plus reinvested earnings) appear to be underestimated, since a number of developing countries do not collect information on reinvested FDI earnings. Reinvested FDI earnings enter the Balance of Payments twice: once as investment income payments (Current Account) and second as a capital inflow (Capital Account) of the new foreign direct investment. Brazil, Bolivia, Colombia, Costa Rica, El Salvador, Honduras, Jamaica and Mexico have long time series on reinvested earnings.
- Most countries record FDI stocks at book values (e.g., assets and liabilities from accounted balance sheet) while a few use market values (e.g. outstanding assets is income accrued but not received, and outstanding liabilities is liability accrued but not yet paid). It is not clear whether this leads to an under or overestimation of the rate of return on foreign direct investment. It will be (under) overestimated to the extent that the book value of the stock of investment is (greater) less than its true value at current market prices. Consequently, taking market interest rates as a proxy of rate of return on FDI may be misleading.

²² For more details, see Section 4.

²³ For more detailed information about the measurement problems related to the construction of balance of payment yields and rate of returns, see Lane and Milesi-Ferretti (2002).

3.3 Market Indices Rates of Return on Assets and Liabilities

Due to the pitfalls in constructing a benchmark the BOP-derived nominal yields and rates of return on assets and liabilities, we propose two alternative scenarios to measure the rates of return on assets and liabilities based on market indices in aggregated terms as in weighted averages based on four subcategories of the financial account (i.e., portfolio equity, FDI, external debt and other assets and liabilities).

3.3.1 Scenario One: Aggregate Indices

In this scenario, the nominal rate of return on assets may be proxied by the *US Treasury Bond Long-Term Real Yield*, and the nominal rate of return on liabilities by:

1. The J.P. Morgan Emerging Market Bond Index (EMBI), which is a set of three bond indices to track bonds in emerging markets,
2. J.P. Morgan's Corporate Emerging Market Bond Index (CEMBI), which is a global, liquid corporate emerging markets benchmark that tracks U.S.-denominated corporate bonds issued by emerging markets entities,
3. *Morgan Stanley Capital International* (MSCI) stock price indices, available for Brazil, Chile, Colombia, Mexico and Peru.

3.3.2 Scenario Two: Weighted Average Indices

According to this scenario, we may calculate weighted averages of the returns based on four subcategories (i.e. portfolio equity, FDI, external and other assets and liabilities). The nominal rate of return on assets for each subcategory may be proxied in the ways listed below.

1. *Portfolio Equity Assets*. Returns may be calculated as the weighted average of dollar returns on either Morgan Stanley Capital International (MSCI) stock price indices or EMBI yield or other representative bond index for the major countries of nonresidents issuers with weights derived from the geographical allocation of foreign portfolio equity assets as reported in the IMF's Coordinated

Portfolio Investment Survey (CPIS), which is available for all countries spanning 2010-2012.

2. *FDI assets*. The same returns are used as for portfolio equity assets, but using the weights derived from the geographical allocation of FDI assets, as reported in the IMF's Coordinated Direct Investment Survey (CDIS).
3. *External Debt Assets*. The returns may be calculated as a weighted average of the long-term interest rates on government bonds, with weights derived from the geographical allocation of long-term external debt assets reported in the IMF's CPIS.
4. *Other Assets*. The returns may be calculated as a weighted average of the dollar short-term interest rates, on government bonds with weights derived from the geographical allocation of short-term external debt assets reported in the IMF's CPIS.

Likewise, the nominal rates of return on portfolio equity and FDI liabilities may be estimated as the nominal rates of return on MSCI or EMBI indices in U.S. dollars. For nominal rates of return on external debt and other liabilities, we may use national long- and short-term interest rates on government bonds in U.S. dollars.

3.4 Factors Affecting Rates of Return Differential

Based on our calculations derived from the BOP and the updated Lane and Milesi-Ferretti (2007) databases, we have found mixed historical evidence of negative and positive returns differentials in the Latin American and the Caribbean countries (i.e., a country may show either positive or negative returns differential across different time periods).

In the Appendix, Figures B3-A to B3-E plot the stylized facts of the real rate of return differential between assets and liabilities for Latin American and the Caribbean countries from the IFS and the updated Lane and Milesi-Ferretti (2007) databases. Thus, in order to shed light on the mixed evidence, we proceed to explain which factors can explain differences in the rates of return between external assets and liabilities.

As pointed out by Lane and Milesi-Ferretti (2005), with large gross external positions, even a small returns differential has significant implications for the net international investment position, so given the relevance of the returns differentials on the external sustainability analysis, we briefly lay out the determinants that play an important role in explaining them such as: the composition and magnitude of the external portfolio of assets and liabilities, business cycles, exchange rates, and asset prices movements.

First, the composition of the external portfolio matters in determining the sign and size of returns differential. In the Appendix, Figures B2-A to B2-E plot the stylized facts of the composition of the net foreign assets positions, gross assets and liabilities for Latin American and the Caribbean countries from the IFS and the updated Lane and Milesi-Ferretti (2007) databases.

Generally, equity-type (less conservative) instruments yield higher returns over the medium term than debt (more conservative) instruments. Thus, countries with a larger share of equity-type assets (FDI and portfolio equity) in total assets than of debt-type liabilities in total liabilities could have a positive returns differential. Conversely, if the weight of equity instruments in the liabilities portfolio is less than the weight of debt instruments in the assets portfolio, countries could have a negative returns differential.

Second, business cycle is another potential factor that accounts for the returns differentials, but needs more empirical analysis to be considered. Hamada, Kashyap and Weinstein (2010) study the cyclicity of international capital flows in Japan and show, in relation to the returns differentials, that the correlation between the output growth rate and the returns differentials varies over different sub-periods, given that the rate of return on assets is less pro-cyclical than the rate of return on liabilities. Therefore, they observe a very negative returns differential during a boom or recovery period and a positive one in recession.

Third, stock price movements also account for addressing returns differentials. Thus, higher (lower) price increases in foreign stock markets relative to the domestic stock market would lead to a positive (negative) returns differential. Finally, movements in exchange rates also play a key role in explaining the sign and size of returns differentials. However, there is a lack of information concerning the currency

composition of external assets and liabilities from the IMF's BOP database. This prevents us from going into detail about this aspect.

4. Case Study: External Sustainability Assessment in Latin America

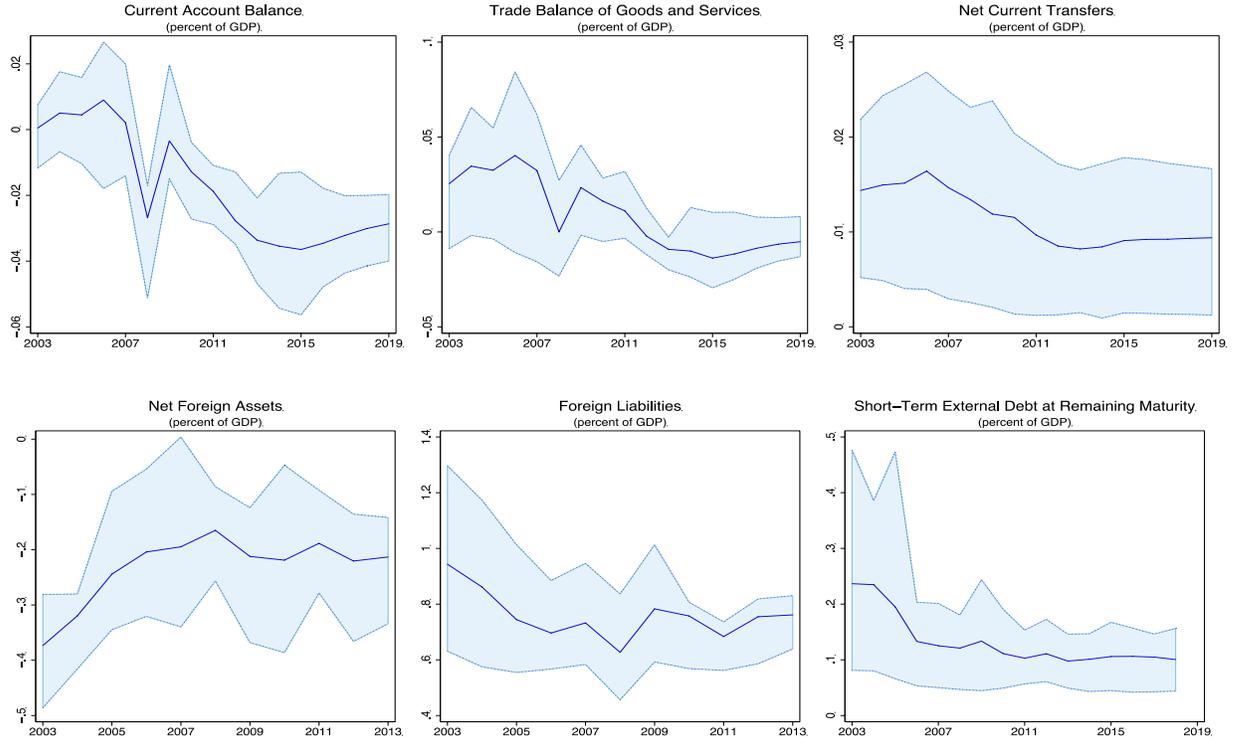
Over the last decade Latin America experienced a notable strengthening of external sector variables (Figure 3), which ended with the global financial crisis in 2009. Recently, the oil price negative shock impacted in different degrees the forecast of the external positions of the oil exporters' and importers' countries in the region.

Since Latin America is exposed to external sustainability risks stemming from different sources, it is important to carry out an external sustainability analysis in some Latin American countries based on the intertemporal budget constraint given by equation (6'). Based on our approach, we address the external sustainability analysis (ESA) developing three methodologies: Standard Approach, Endogenous Net Foreign Assets Dynamics Approach, and Fan-Charts Approach.

We have elaborated a country case analysis based on seven Latin American countries, henceforth called LAT7,²⁴ using data from the IMF WEO October 2015 database; for the net foreign assets positions, we have used an updated version of the Lane and Milesi-Ferretti (2007) database, and as proxy for the rates of return on assets and liabilities, we have used the US Treasury Real Annualized Yield and EMBI yield, respectively. The historical analysis covers the period 1995-2014, and the forecast horizon corresponds to 2015-2020. Next, we describe each of the approaches in more detail.

²⁴ We carried out the exercise for Argentina, Brazil, Chile, Colombia, Mexico, Peru, and Uruguay.

Figure 3. Latin America: External Indicator, 2002-2014
(Simple average and 20th and 80th percentiles)



4.1 Standard Approach

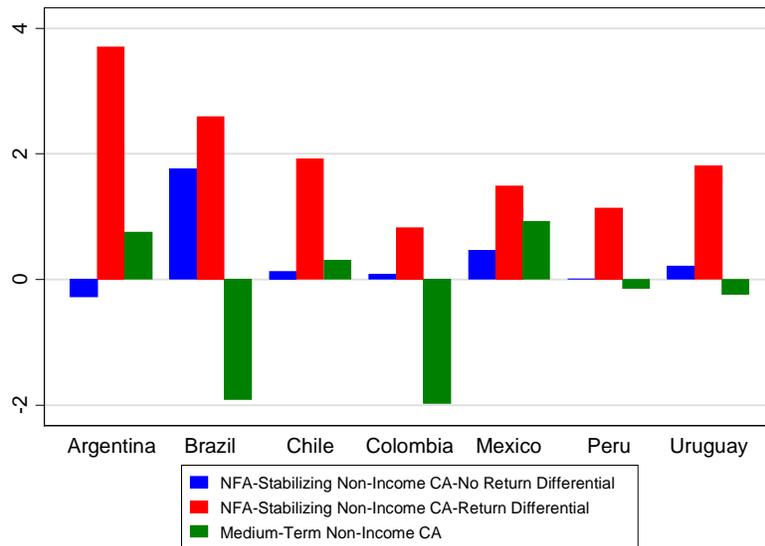
The Standard Approach consists of determining the non-income current account balance plus net capital transfers (denoted as NICA) over GDP that would stabilize a benchmark net foreign asset (NFA) position over the medium-term given by equation (7). The benchmark NFA position was arbitrarily chosen with the latest year with available data (for more information see Section 2.4.). Then, we calculate a “gap” by comparing the NFA-stabilizing NICA over GDP with its value expected to prevail over the medium-term, assuming no real exchange rate misalignments, given by equation (9).

In order to address the Standard Approach, we set the long-run real GDP growth rate in LCU (\tilde{g}_t^{LCU}), and long-run real rates of return on assets (\tilde{r}_t^{fa}) and liabilities (\tilde{r}_t^{fl}), as five-year average rates²⁵ of the historical time series. The selected benchmark level for

²⁵ Even though the five-year average percent change rate is an arbitrary measure and represents more the medium term than the long-term perspective. That window has been assumed mainly due to the lack of a long time series in some countries of the Latin American and the Caribbean region, but it might be changed without loss of generality.

net foreign assets is set as the value in 2014 (the latest year available). Figure 4 depicts the NFA-Stabilizing and Medium-Term NICA over GDP,²⁶ considering two possible scenarios, with negative return differentials and no return differentials, where the gap between these two measures stand for the required adjustment needed to bring in line the medium-term NICA over GDP to its NFA-Stabilizing value, given by equation (9).

Figure 4. NFA-Stabilizing and Medium-Term Non-Income Current Account Balance
(Percent of GDP, Value in 2014)



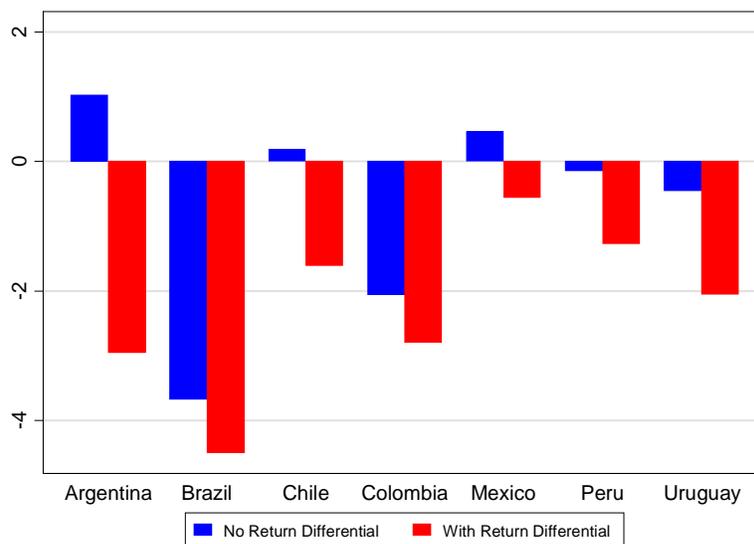
From equation (9), it is straightforward to infer when negative return differential has been assumed (as in this exercise), as no return differential brings in a negative bias in the calculation of the NFA-stabilizing NICA over GDP. As previously explained in Section 3.3, the magnitude and sign of return differentials depend on the composition and magnitude of the external portfolio of assets and liabilities, business cycles, exchange rates, and asset prices movements. So, assuming negative return differential in Latin America makes much sense, since the external portfolio of liabilities is on average predominantly composed of FDI and External Debt, which have higher returns than the

²⁶ This assumption is consistent with the External Sustainability Approach carried out by the IMF in the Article IV Staff Report, as explained previously.

external portfolio of assets which are predominantly composed by international reserves with a lower return.

According to Figure 5, assuming no return differentials would entail that the LAT7 countries would turn out more sustainable over the medium term than when negative return differential is assumed. So the assumption of return differential is a very important aspect to be considered in the analysis, since we could mistakenly infer that a country seems to be sustainable over the medium-term, when not necessarily is. The required adjustment of the LAT7 countries stands in a range between -2 percent and -4 percent. With the exception of Argentina, all LAT7 countries, except for Argentina, need to implement a NICA deficit (percentage of GDP) to stabilize the NFA positions over the medium term. Assuming a negative return differential, the countries with higher required adjustments are Brazil (-4.5 percent), Chile (-2 percent), Argentina, Colombia and Uruguay (-2 percent). The countries with lower required adjustments (less than -2 percent) are Mexico and Peru.

Figure 5. Required Non-Income CA Adjustment to Stabilize NFA positions
(Percent of GDP, value in 2014)



On the other hand, by assuming no return differential, the countries with the higher required adjustments are Brazil (-4.5 percent), Colombia (-3 percent), and with lower required adjustment, Chile, Mexico, Peru, Uruguay (close to -1 percent), and

Argentina (a surplus of 1 percent). The expected acceleration of the economy on the back of a recovery of commodity prices should reverse the weakened performance of the NICA over the medium-term.

4.2 Endogenous Net Foreign Assets Dynamics Approach

The Endogenous Net Foreign Assets Dynamics Approach depicts a scenario analysis of the future path of the NFA over GDP (given by equation (9)), subject to temporary standard shocks to each underlying variable of interest, i.e., growth rate of real GDP in LCU, real rates of return on assets and liabilities (including capital gains, if applicable), net current and capital transfers, change in real exchange rate, and foreign assets over GDP. As previously discussed in Section 2.3, the way to implement a real exchange adjustment is by assuming a one-time jump in the REER from X_T to its benchmark level X_T^* at T taken from the latest IMF's Article IV at time T . This one-time jump at time T induces a gradual adjustment in the expected trade (exports and imports) flows over a five-year period during the forecasting horizon after T , in proportion to trade semi-elasticities.

The initial real exchange rate misalignment at time T was chosen as the actual exchange rate minus a benchmark REER estimated and reported in the latest published IMF's Article IV Staff Consultation. For the Brazilian case, according to the latest IMF staff report, the real exchange rate was on average some 5-15 percent below the benchmark level implied by fundamentals in 2015. Subsequently, the observed appreciation of the real exchange rate as of June 2016 suggests that the real exchange rate remains overvalued with respect to the level implied by the estimated equilibrium real exchange rate.

In the Case Study Appendix, Figure A depicts the simulated NFA over GDP path for Brazil over the medium-term forecasting horizon by considering five-period permanent negative shocks of one standard deviation to the variables given by equation (10). Subplots 1 and 3 show i) an NFA over GDP simulation without considering the adjustment of the REER misalignment, which means that it is assumed to remain over the medium-term (aka baseline scenario); ii) an NFA over GDP simulation considering a one-time jump adjustment of the REER misalignment, which, as explained before, also

induces a gradual adjustment in the expected trade (exports and imports) flows over a five-year period during the forecasting horizon after T , in proportion to trade semi-elasticities (Tokarick, 2010); and iii) shocked scenarios to growth rate of real GDP in LCU, real rates of return on liabilities, and trade balance after closing the REER misalignment, assuming non-return and return differential, respectively. Likewise, subplots 2 and 4 show an NFA over GDP simulation but without considering the adjustment of the REER misalignment.

4.3 Fan-Chart Forecasts Approach

The Fan-Chart Approach lays out that the expected future dynamic path of the NFA over GDP is uncertain and subject to temporary shocks. Fan-Chart Approach depicts the probability distribution of the NFA over GDP under a medium-term adjustment scenario by simulating ($n \times 1$) shocks to each of its underlying determinants, where (n) is a large number. One deterministic and three statistical methodologies have been performed in the template to implement the Fan Chart Approach: i) External Forecasts, ii) VAR Approach, and iii) External Forecasts with Correlated Errors. Arizala et al. (2008) explain these methodologies in detail. Additionally a brief description of each methodology is laid out in Box 2. Following, we estimate the standard and VAR approaches for Brazil.

4.3.1 External Forecasts

The External Forecasts approach examines the volatility around the baseline scenario using the *historical volatility* of the determinants of equation (9). As a result, it measures a confidence interval for the NFA-to-GDP ratio distribution. Figure 6a shows that the expected path of the NFA-to-GDP is slightly upward sloping but at a gradual pace. Additionally, Figure 6b shows a value at risk analysis widely used to measure the probability that the NFA-to-GDP ratio falls under a selected threshold for every time t . NFA is more likely to remain below the minus 40 percent of GDP ratio in 2019, with probability 1 of reaching minus 30 percent of GDP.

Figure 6a. Brazil: External Projection, Net Foreign Assets Position (Percent of GDP)

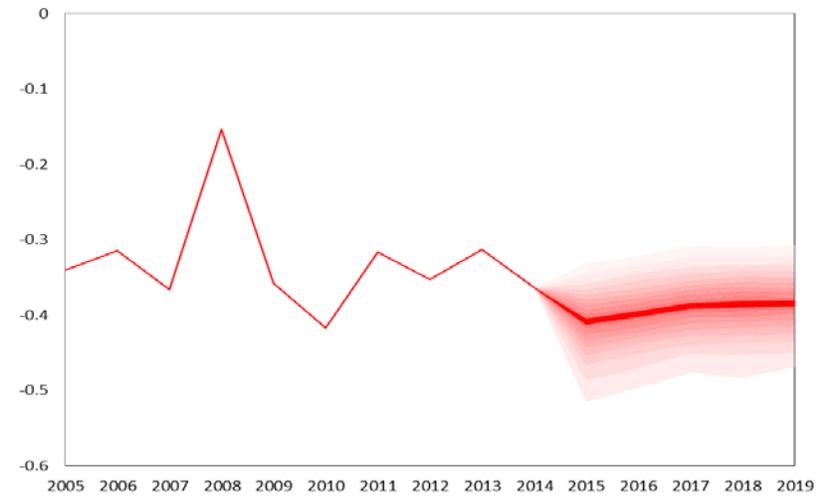


Figure 6b. Brazil: External Projection, Net Foreign Assets Position (Percent of GDP)

Threshold of the NFA over GDP limit (X)	-40%
Prob(NFA/ GDP) < X	0.43

Intervals	Sensitivity Analysis of NFA over GDP			
	<-0.4	<-0.45	<-0.5	<-0.55
2015	0.55	0.22	0.06	0.02
2016	0.48	0.17	0.04	0.01
2017	0.41	0.15	0.04	0.01
2018	0.36	0.11	0.03	0.01
2019	0.36	0.12	0.03	0.01
Total	0.43	0.15	0.04	0.01

4.3.1 VAR Approach

For the VAR Approach, we estimate the relationship between the determinants of the NFA-to GDP ratio, by applying a vector autoregressive (VAR) model of order 1, denoted as VAR(1). The fan chart forecasts may be used as a broad benchmark, or “confidence interval” of the NFA-to GDP ratio distribution. For illustrative purposes we plot the fan charts starting from 2005. We use the VAR(1) estimates (coefficients and variance-covariance matrix) to predict the “deep” determinants trajectories, assuming the common distribution and co-movements among the variables included in equation (9) such as: Trade Balance over GDP, Net Current and Capital Transfers over GDP, Real GDP

Growth Rate in LCU, Real Rates of Return on Assets and Liabilities, Annual Change of Real Exchange Rate, and Foreign Assets over GDP.

Figure 7a. Brazil: VAR Approach, Net Foreign Assets Position (Percent of GDP)

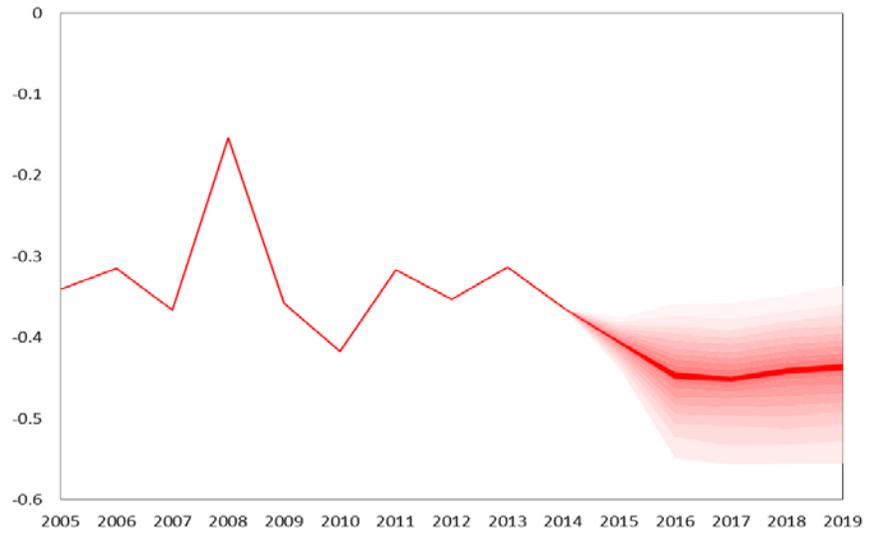


Figure 7b. Brazil: VAR Approach, Net Foreign Assets Position (Percent of GDP)

Threshold of the NFA over GDP limit (X)	-40%
Prob(NFA/ GDP) < X	0.84

Intervals	Sensitivity Analysis of NFA over GDP			
	<-0.4	<-0.45	<-0.5	<-0.55
2015	0.63	0.06	0.00	0.00
2016	0.95	0.83	0.60	0.36
2017	0.93	0.77	0.54	0.33
2018	0.88	0.70	0.47	0.28
2019	0.80	0.59	0.38	0.23
Total	0.84	0.59	0.40	0.24

Box. 2. Fan Charts

The Fan-Chart tool incorporated in this template relies on historical annual data for: Trade Balance over GDP, Net Current and Capital Transfers over GDP, Long-Term Growth Rate of Real GDP, Real Rate of Return on Assets and Liabilities (inclusive Capital Gains), Annual Change of Real Exchange Rate, and Foreign Assets over GDP.

Historical data of each of the underlying variables of the NFA over GDP (as discussed above) are used (in different ways) to generate the sample means and variance-covariance matrix that define the joint normal distribution for these variables.

Draws for each of those determinant variables from the joint normal distribution are used to generate shocks that are added to the baseline projections for each of the variables and fed into the NFA over GDP given by equation (9) to calculate a distribution of projected NFA over GDP.

By construction, based on the joint normal distribution of shocks, we generate a symmetrical fan chart. Therefore, it would be interesting to relax the normality assumption and address different effects on upside and downside risks.

The *fan shape* of the distribution over the projection period is due to the increased uncertainty over time. Accordingly, if simulations every year lead to adverse shocks (e.g., low real GDP growth), then the NFA over GDP will increase and evolve along a path in the upper bound of the fan chart distribution.

Conversely, if every year simulations lead to positive shocks (e.g., high real GDP growth), then the NFA over GDP will decline and evolve along a path in the lower bound of the fan chart distribution.

One deterministic and three statistical methodologies have been performed to implement this approach: a) External Forecasts, b) VAR Approach, c) External Forecasts with Correlated Errors and d) Weighted Projections. These methodologies are explained in detail in Arizala et al. (2008).

External Forecasts

This approach is carried out by generating shocks $iid e_t \sim N(0, \sigma_i^2)$ for each of the underlying variables of the NFA over GDP to then simulate $N = 1000$ repetitions of x_τ^s of (τ) dimension, based on the following equation:

$$x_\tau^s = x_\tau^{ext} + e_t$$

where $\tau \in [t + 1, T]$, $\tau = 5$ corresponds to the longest forecast horizon assumed by the IMF, x_τ^{ext} corresponds to the assumed external projection of each underlying variable and σ_i^2 corresponds to its historical variance.

Afterwards, all x_τ^s are fed into the NFA over GDP given by equation (9) to calculate a distribution of projected NFA over GDP.

Box 2. Fan Charts (continued)

VAR Approach

This approach considers the following autoregressive model:

$$y_t = \alpha_0 + \alpha_1 y_{t-1} + e_t$$

where y_t is a vector of k dimension that comprises each of the k underlying x_τ^{VAR} variables of the NFA over GDP identity given by equation (4), the residual matrix e_t is normally distributed following $N(0, \Omega)$, where Ω represents the variance and covariance of the autoregressive model. In order to simulate the correlated k underlying variables paths (following the latter equation), we must use the Cholesky decomposition Ψ of the variance and covariance matrix Ω defined as:

$$\eta_t = \Psi v_t$$

with v_t normally distributed with mean 0 and variance 1. Once the k paths of dimension (τ, N) have been simulated, they are fed into the N

FA over GDP given by equation (4) to calculate a distribution of projected NFA over GDP.

External Forecasts with Correlated Errors Approach

This approach follows the same *External Forecast approach*, but instead assuming correlated errors coming from the VAR Approach in the NFA over GDP simulation.

Weighted Projections

This is a weighted combination between External Forecasts with correlated errors and VAR Approach, following the equation below:

$$x_\tau^S = \beta x_\tau^{ext} + (1 - \beta) x_\tau^{VAR} + \eta_t$$

where x_τ^{VAR} corresponds to the simulation of each k underlying variable coming from the VAR model, and $\beta \in [0, 1]$. When $\beta = 1$, the latter equation collapses to the case of *External Forecasts with correlated errors* and when $\beta = 0$, it corresponds to the *VAR Approach*.

5. Data Sources for the External Sustainability Assessment

Name of the Variables	Definition	Sources
U.S. Consumer Price Index	U.S. Consumer Price Index end of per. base year = 1	WEO or Official Sources
Nominal GDP, in USD	Nominal Gross Domestic Product, in USD	
Real GDP Growth Rate in LCU (%)	Real GDP Annual Change, in LCU (in percent)	
Real Effective Exchange Rate (INS/IMF) (+) Overvaluation, (-) Undervaluation)	Real Effective Exchange Rate	INS/IFS
Domestic Consumer Price Index	Domestic Consumer Price Index end of per. base year = 1	WEO or Official Sources
Net Foreign Assets	Net Foreign Assets, (Current US\$)	International Investment Position (IIP) Data from IMF or Official Sources and updates of Lane and Milesi-Ferretti (2007).
External Assets	Foreign External Assets, (Current US\$)	
External Liabilities	Foreign External Liabilities, (Current US\$)	
Current Account Balance	Current Account Balance in USD	Balance of Payments, WEO, IFS, or Official Sources
Net Current Transfers	Net Current Transfers in USD, Current Account Balance	
Trade Balance (goods plus services)	Trade Balance (goods plus services) in USD, Current Account Balance	
Income Balance	Income Balance in USD, Current Account Balance	
Exports Goods and Services	Exports Goods and Services in USD, Current Account Balance	
Imports Goods and Services	Imports Goods and Services in USD, Current Account Balance	
Net Capital Transfers	Net Capital Transfers (Capital Account, e.g. Debt Forgiveness, Grants) in USD, Capital Account Balance	BOP/IFS or Official Sources
Portfolio Equity Assets Stock	Portfolio Equity Assets Stock, Foreign Assets	International Investment Position (IIP) Data from IMF, Official Sources and updates of Lane and Milesi-Ferretti (2007).
FDI Assets Stock	Foreign Direct Investment Assets Stock, Foreign Assets	
External Debt Assets Stock	External Debt (Portfolio Debt + Other Investment) Assets Stock, Foreign Assets	
Financial Derivatives Assets Stock	Financial Derivatives Stock, Foreign Assets	
Reserves minus gold	Reserves minus gold, Foreign Assets	
Portfolio Equity Liabilities Stock	Portfolio Equity Liabilities Stock, Foreign Liabilities	
FDI Liabilities Stock	Foreign Direct Investment Liabilities Stock, Foreign Liabilities	
External Debt Liabilities Stock	External Debt (Portfolio Debt + Other Investment) Liabilities Stock, Foreign Liabilities	
Financial Derivatives Liabilities Stock	Financial Derivatives Liabilities Stock, Foreign Liabilities	
Investment Income Receipts (Income Credit)	Investment Income Receipts (Income Credit). International Investment Position (IMF) Data	International Financial Statistics (IFS-IMF) or Official Sources

Data Sources, continued

Name of the Variables	Definition	Sources
Investment Income Payments (Income Debit)	Investment Income Payments (Income Debit). International Investment Position (IMF) Data	
Capital Outflows	International Capital Flows on Assets, Financial Account Balance	Balance of Payment (BOP) Data from IMF, Official Sources and updates of Lane and Milesi-Ferretti (2007).
Capital Inflows	International Capital Flows on Liabilities, Financial Account Balance	
US Treasury Bond Long-Run Real Yield	US Treasury Bond Long-Run Real Annualized Yield	Bloomberg. For other alternatives, see Box 2 of Castillo (2016).
US Treasury Bond Long-Run Real Yield plus EMBI Yield	US Treasury Bond Long-Run Real Yield plus EMBI Yield	
BOP Real Rate of Return on Assets	IIP and BOP derived Real Rate of Return on Assets (Real Capital Gains inclusive)	Calculation and Sources explained in detail in Section 3 of Castillo (2016).
BOP Real Rate of Return on Liabilities	IIP and BOP derived Real Rate of Return on Liabilities (Real Capital Gains inclusive)	

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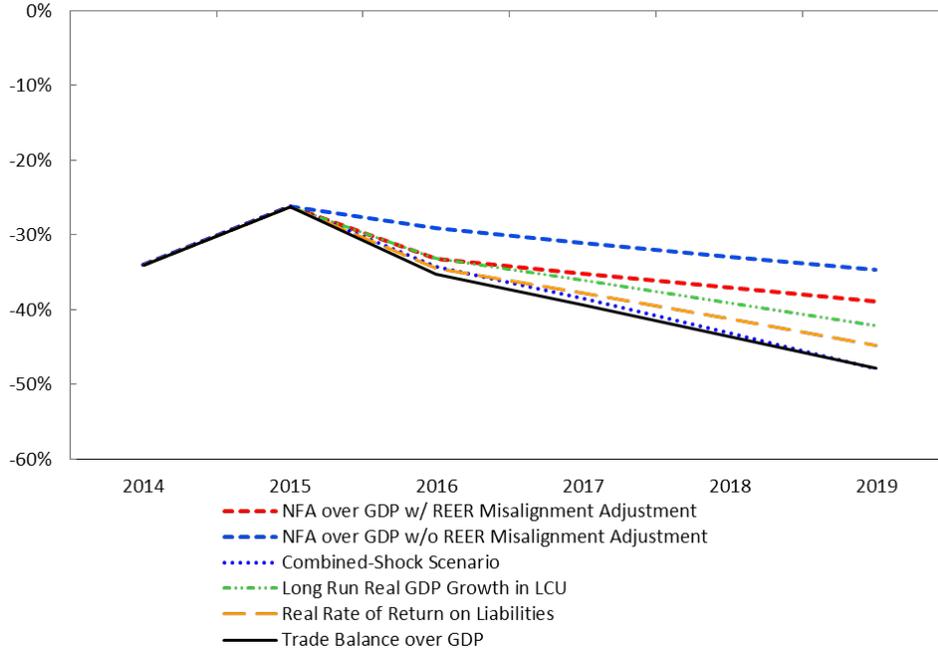
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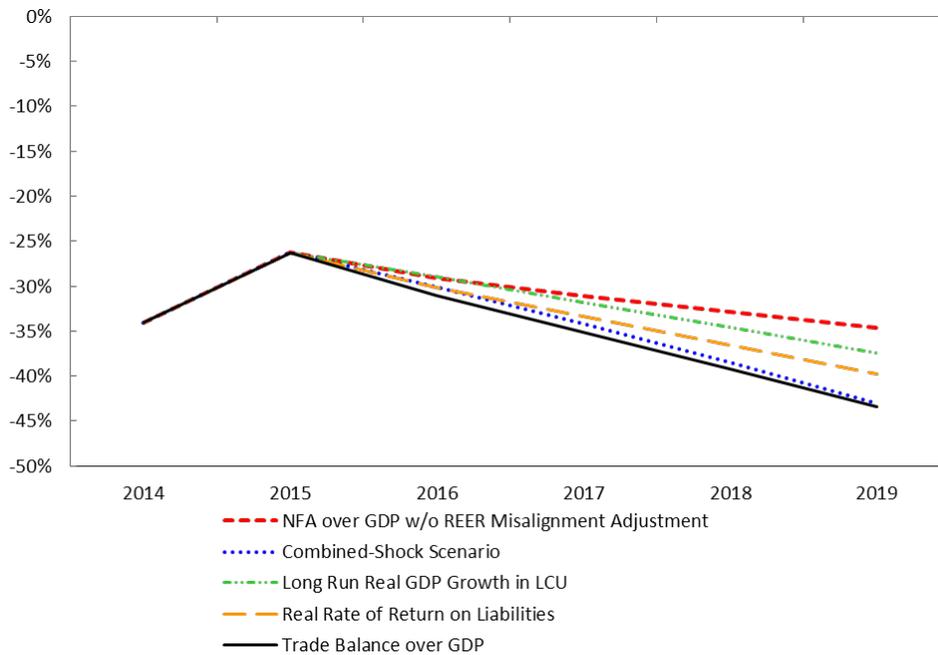
Appendix A: Case Study

Figure A1. Brazil: Projected-Simulation of NFA over GDP

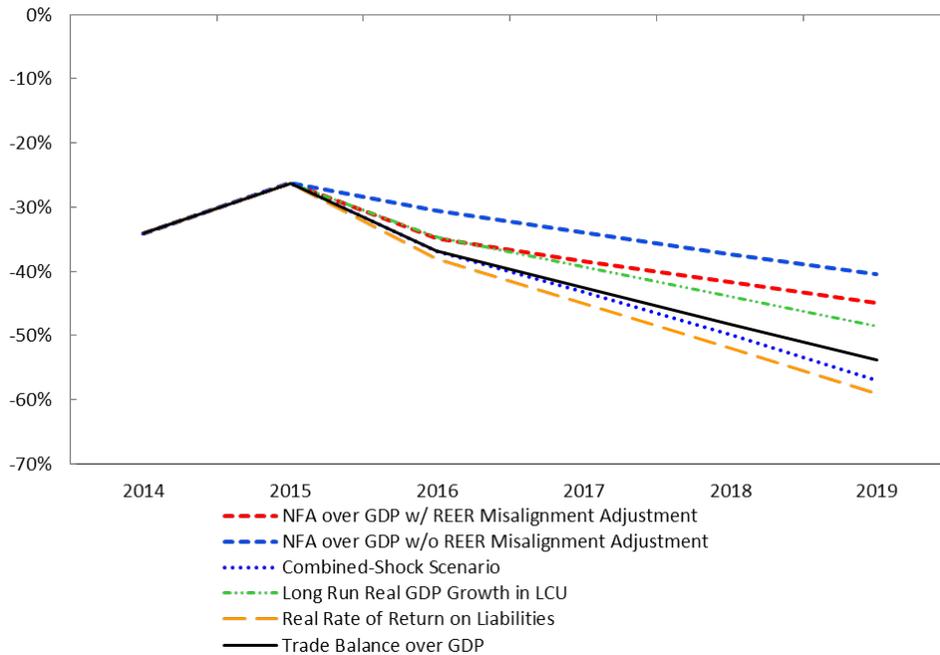
1. No return differential, assuming a one-time adjustment of the REER misalignment (-15%, where (-) means overvaluation), with shock



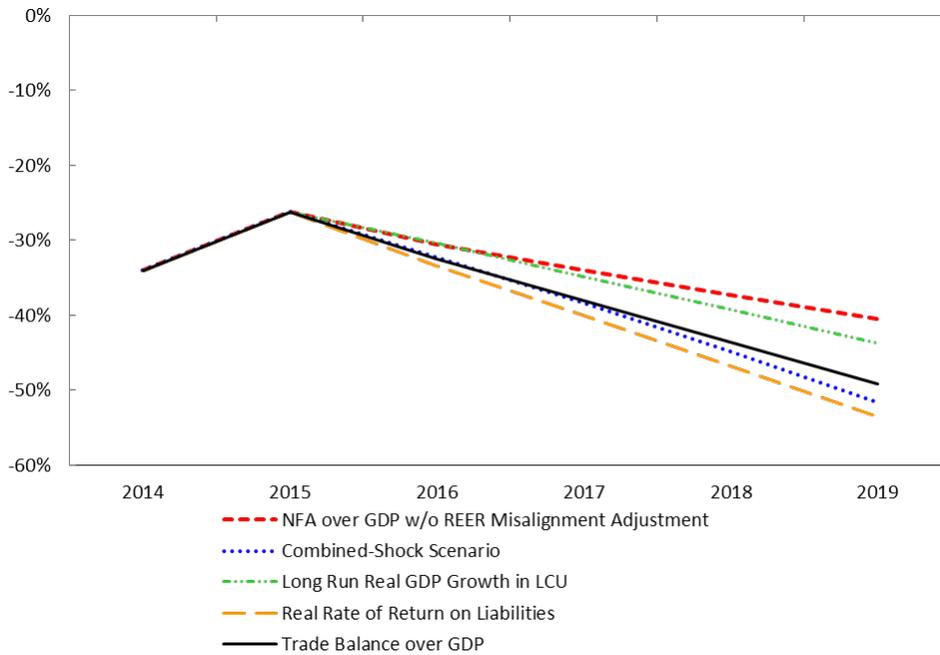
2. No return differential, assuming no adjustment of the REER misalignment



3. Return differential, assuming a one-time adjustment of the REER misalignment (-15%, where (-) means overvaluation)



4. Return differential, assuming no adjustment of the REER misalignment



Appendix B. Additional Figures

Figure B1-A. Net Foreign Assets over GDP (%) in 2013
(Under 100% of GDP in absolute value, MF-Lane Database)

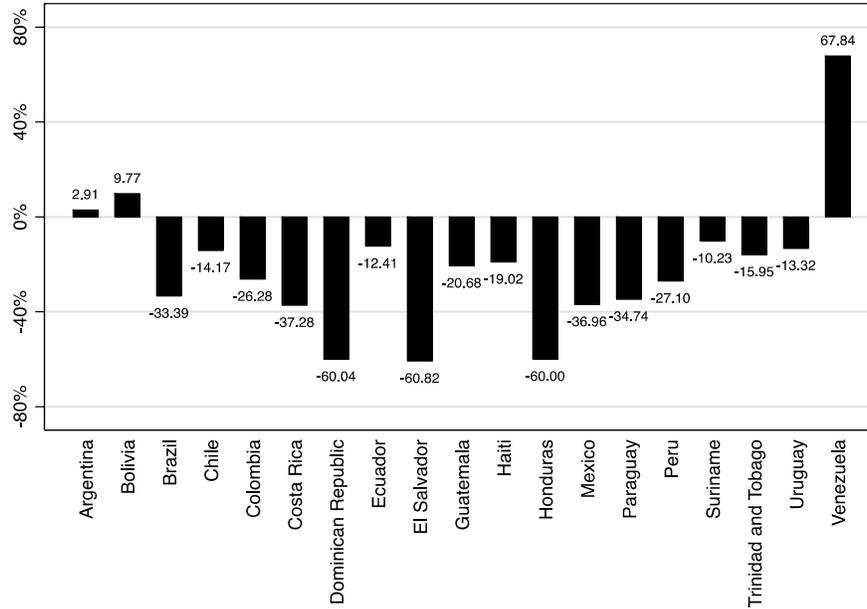


Figure B1-B. Net Foreign Assets over GDP (%) in 2013
(Exceeding 100% of GDP in absolute value, MF-Lane Database)

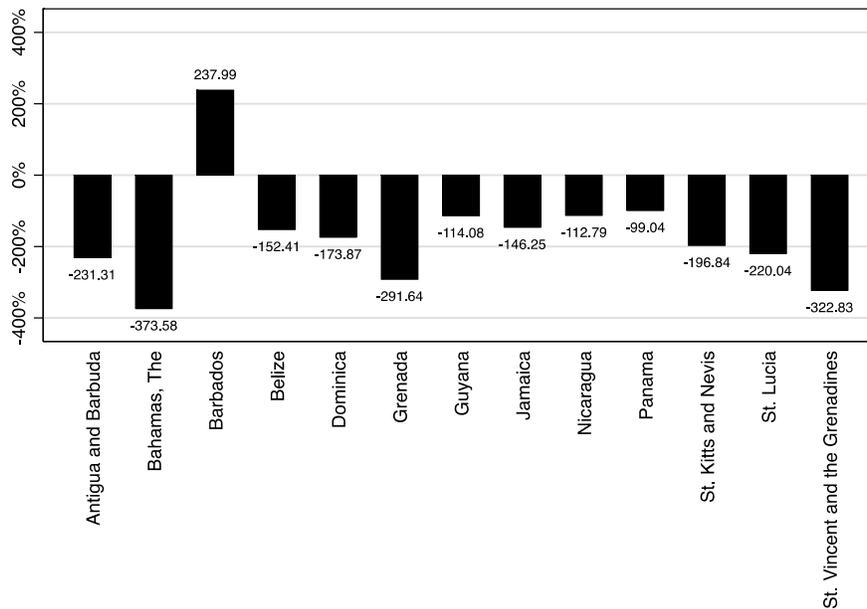


Figure B2-A. Composition of Net Foreign Asset Position in Billions USD

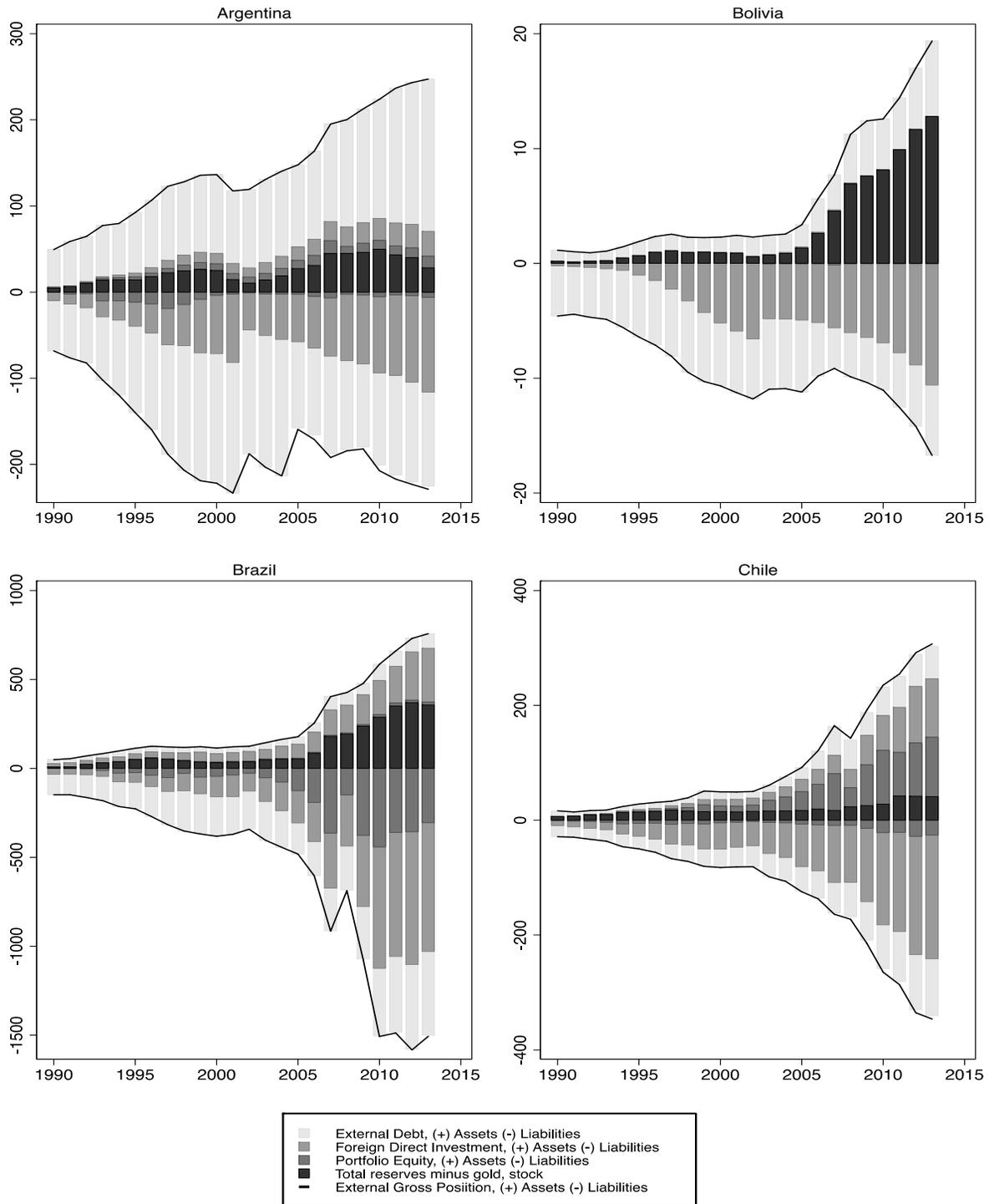


Figure B2-B. Composition of Net Foreign Asset Position in Billions USD

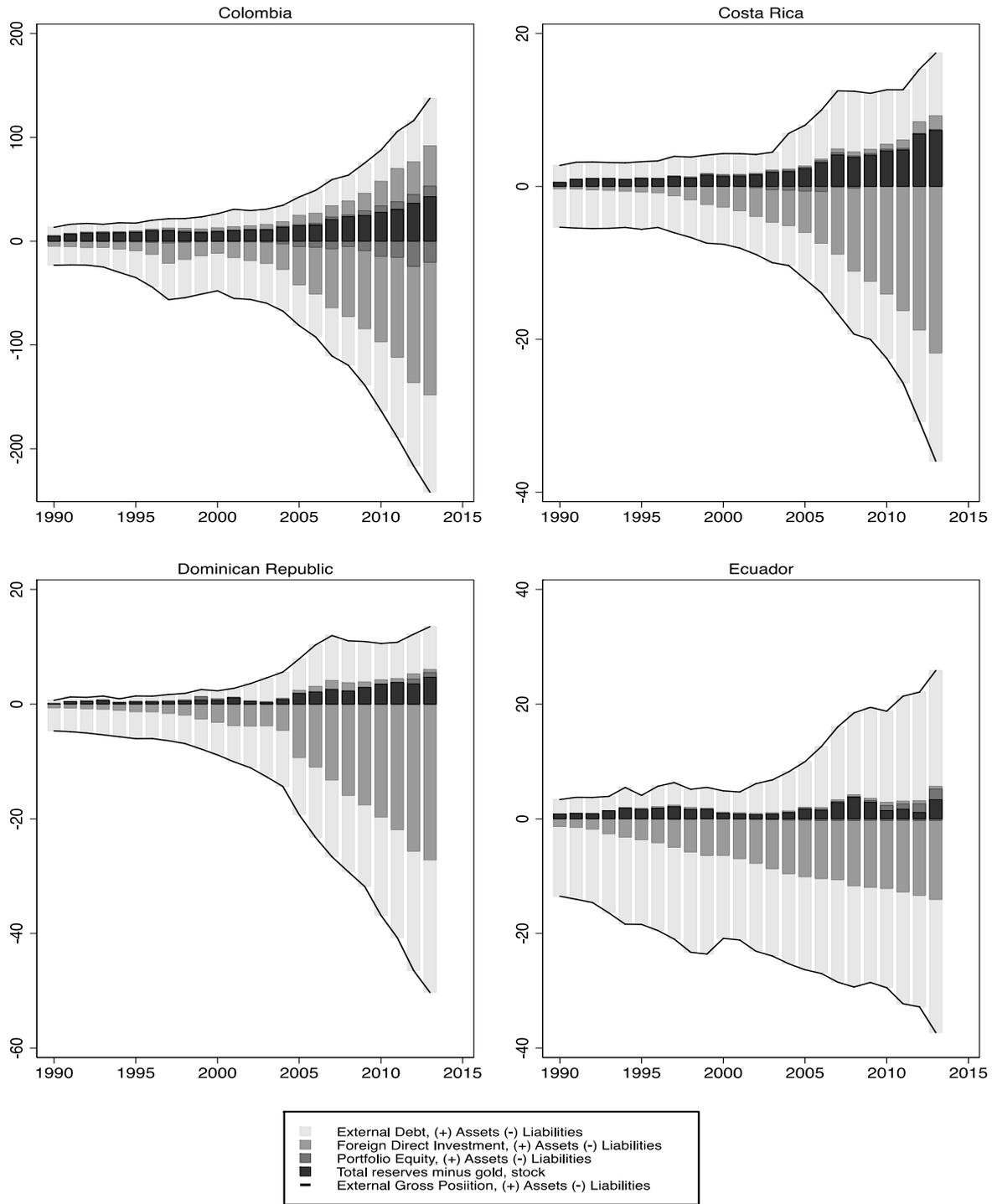


Figure B2-C. Composition of Net Foreign Asset Position in Billions USD

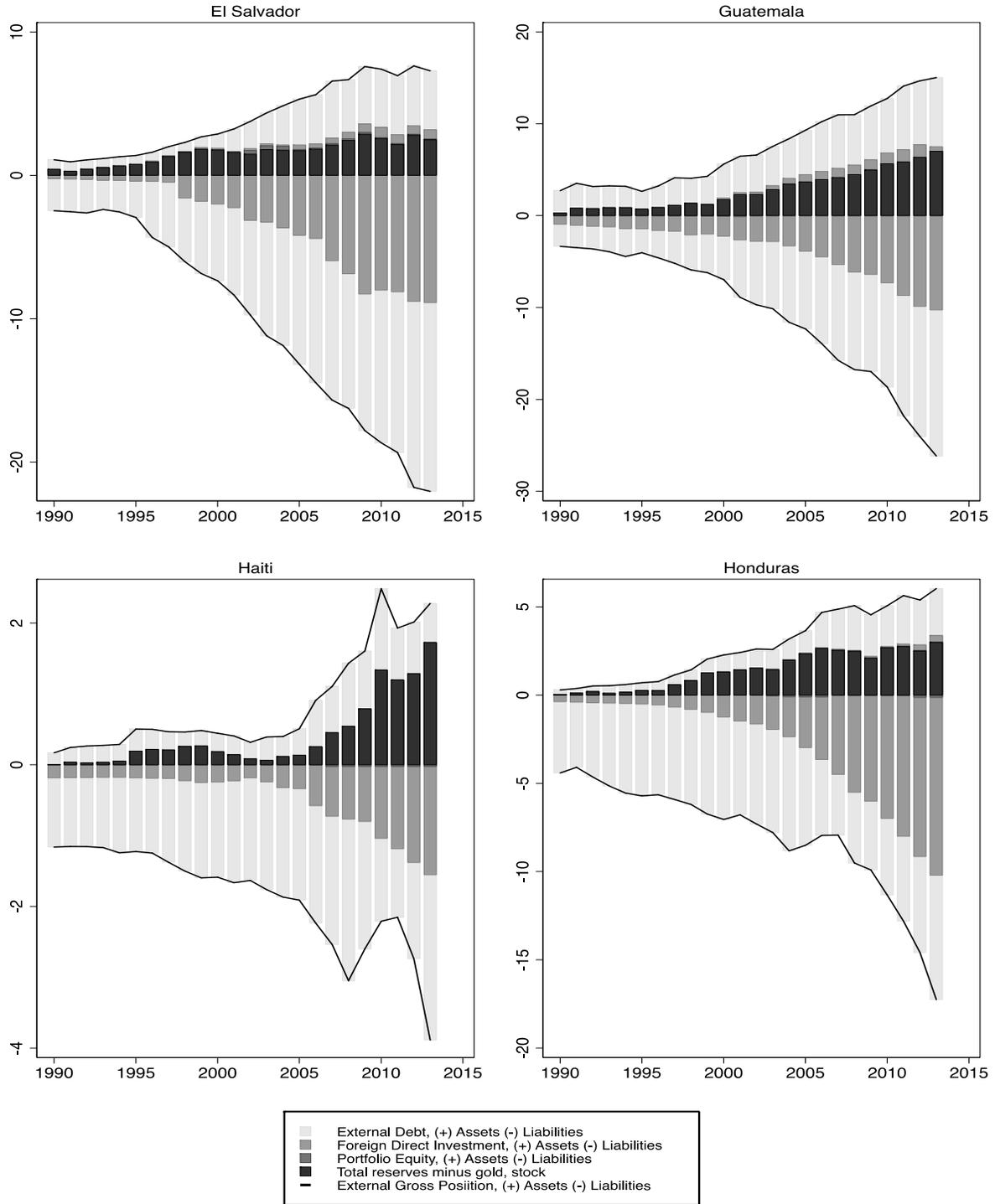


Figure B2-D. Composition of Net Foreign Asset Position in Billions USD

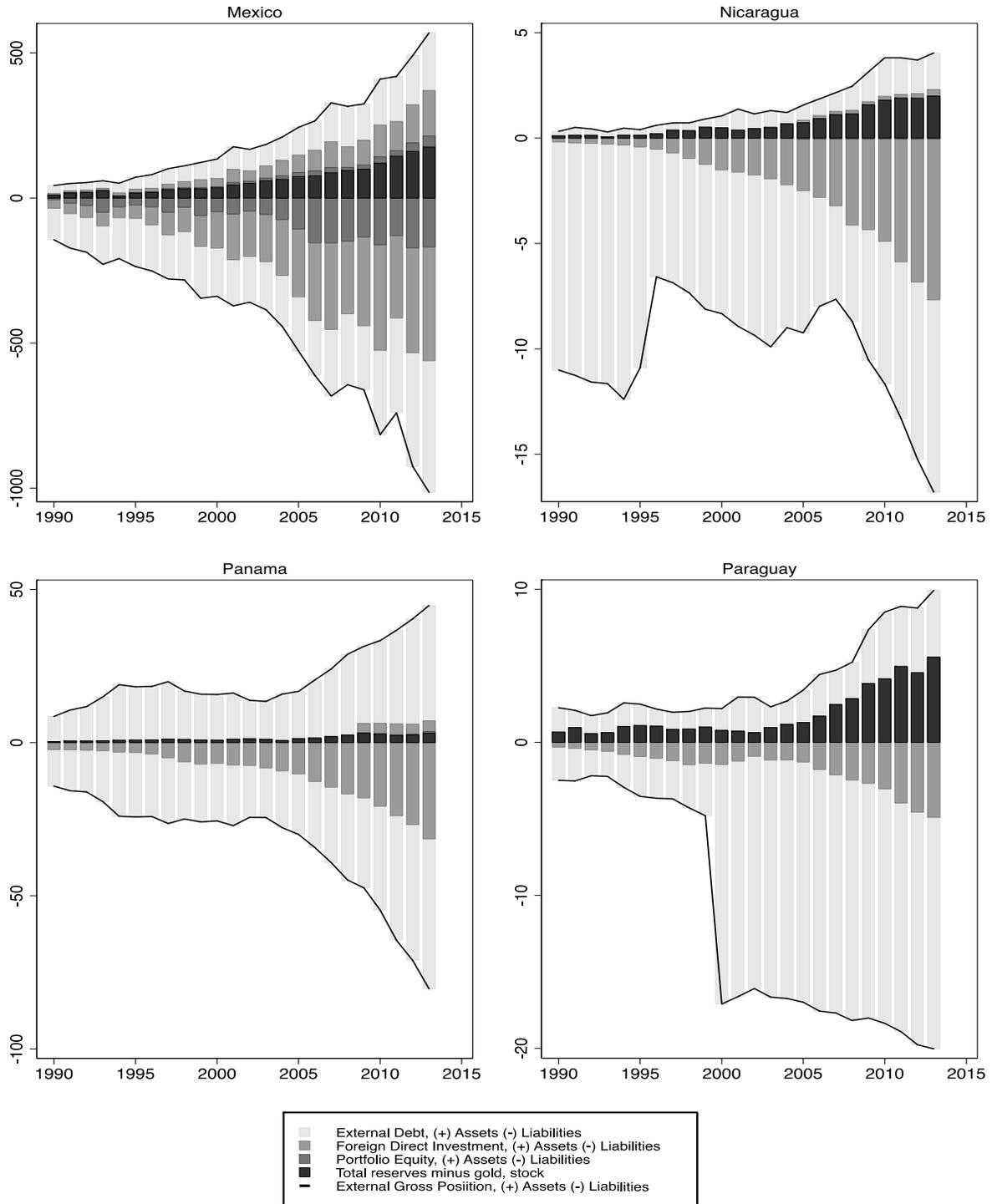


Figure B2-E. Composition of Net Foreign Asset Position in Billions USD

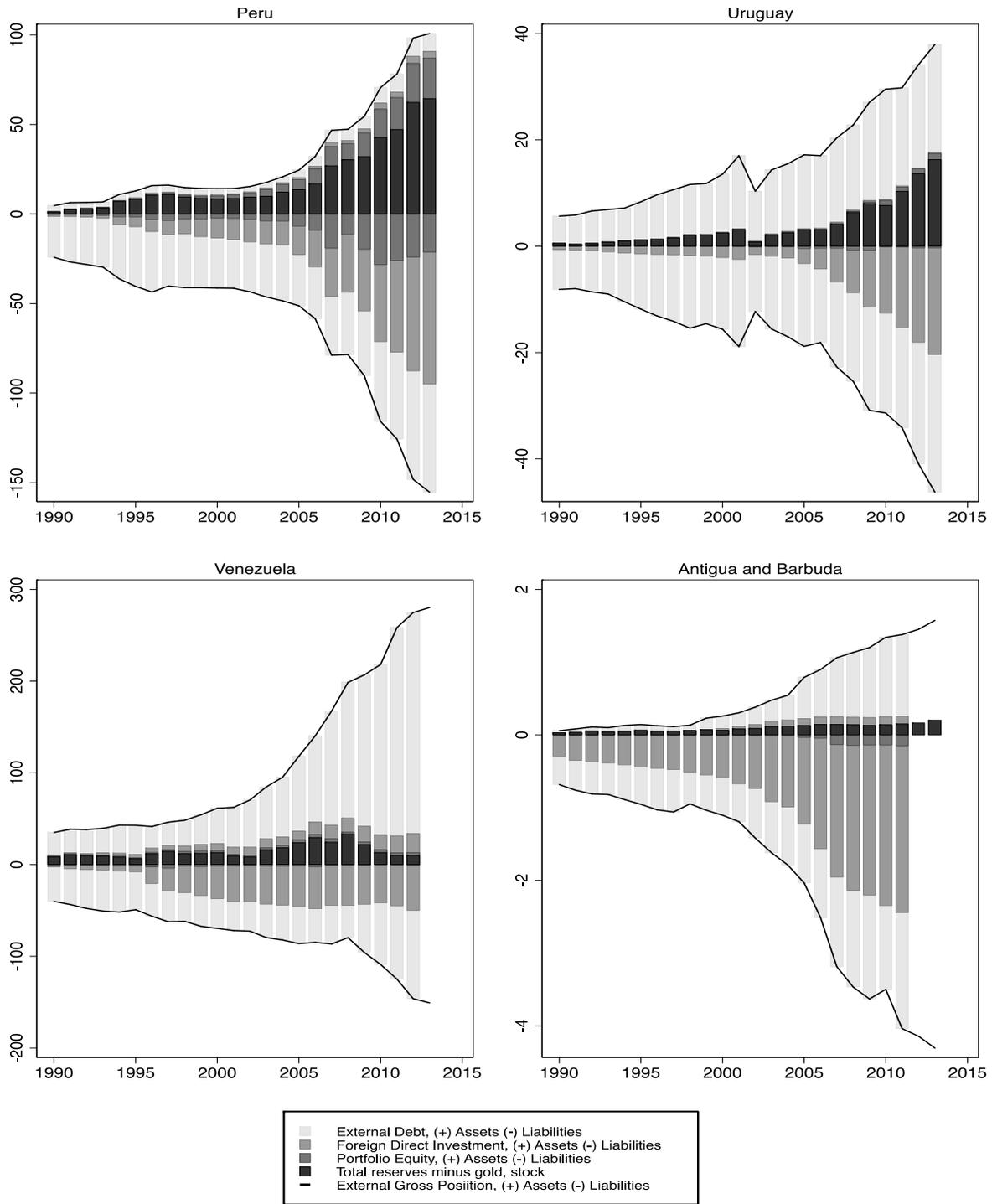


Figure B2-F. Composition of Net Foreign Asset Position in Billions USD

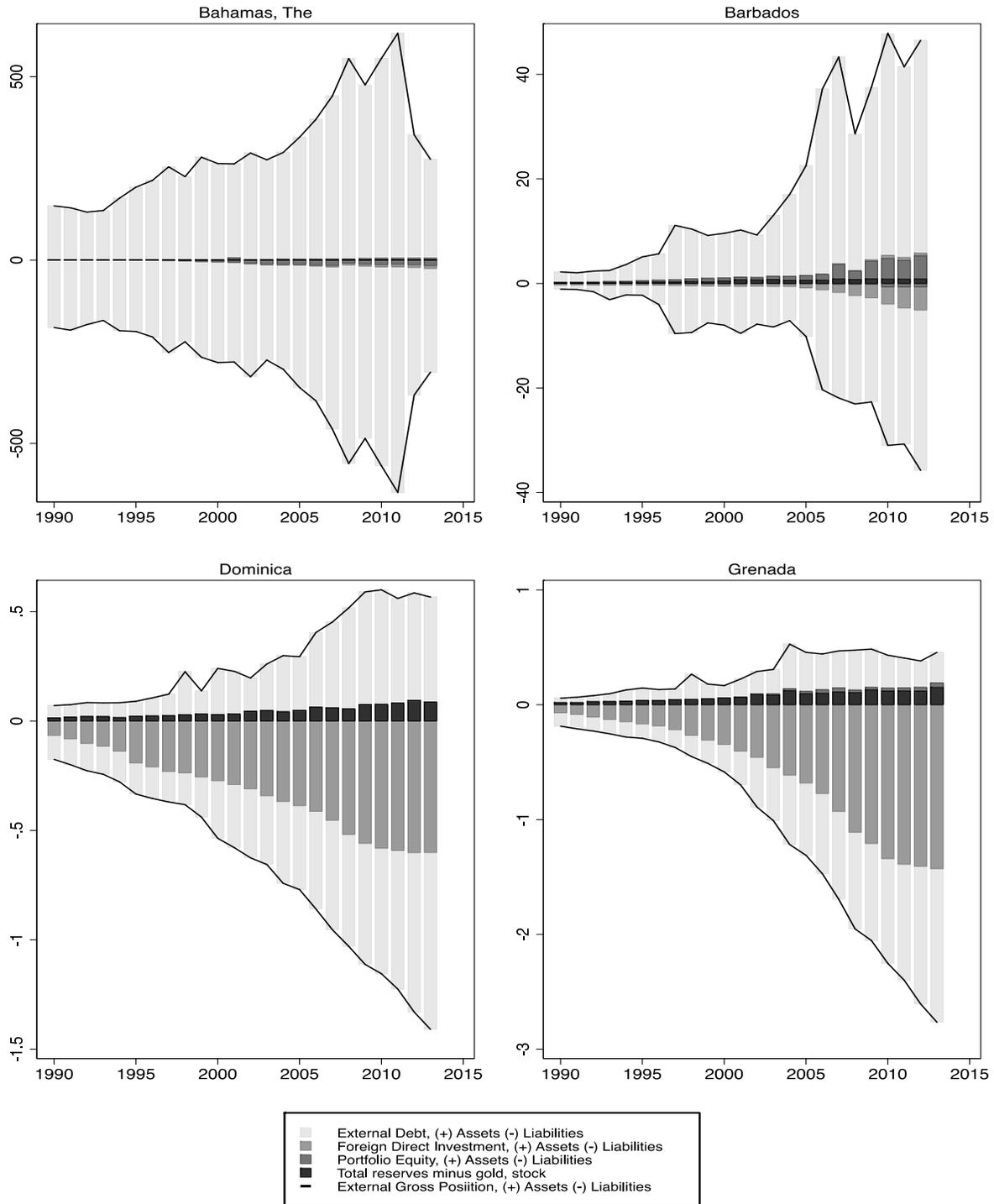


Figure B2-G. Composition of Net Foreign Asset Position in Billions USD

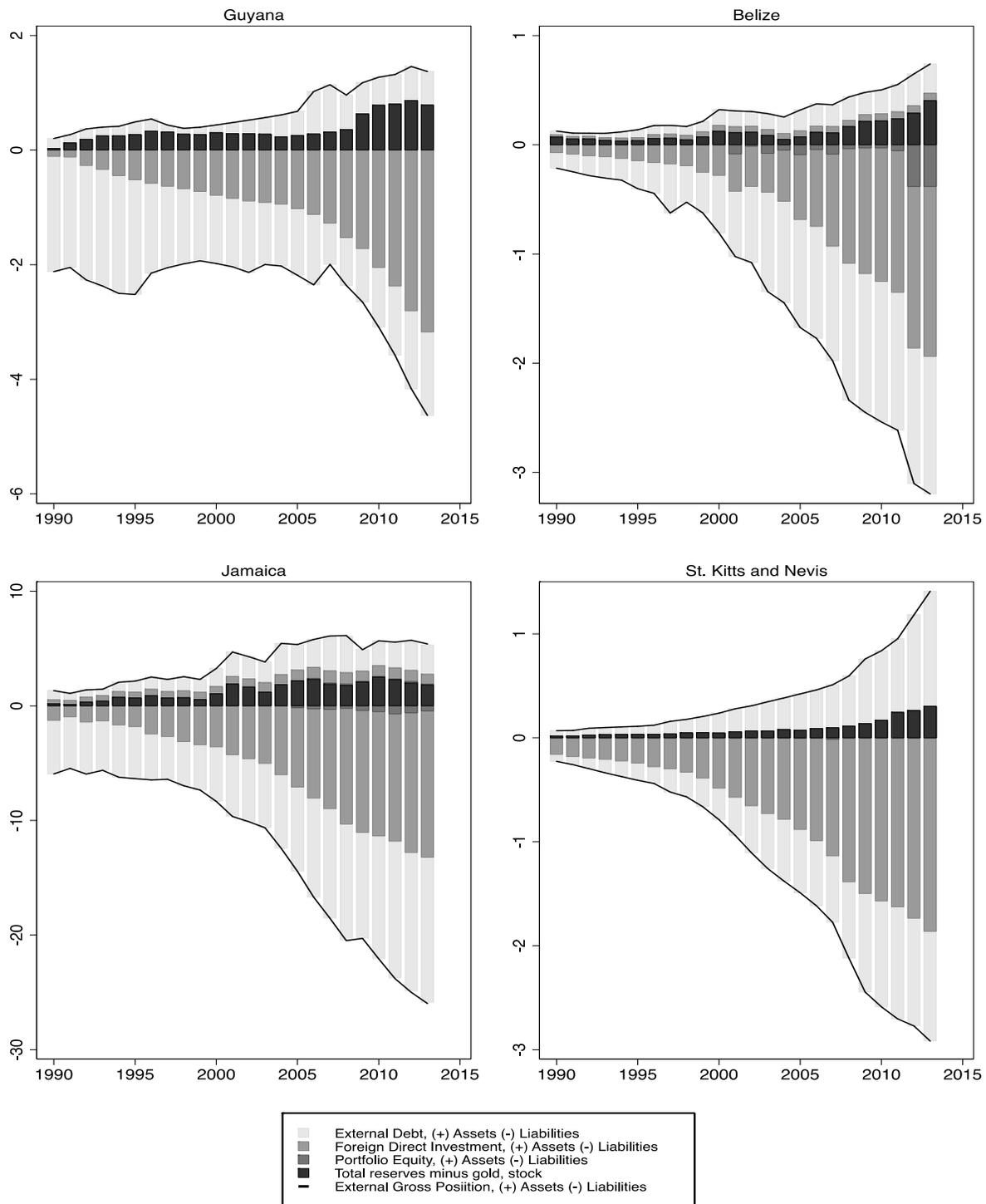


Figure B2-H. Composition of Net Foreign Asset Position in Billions USD

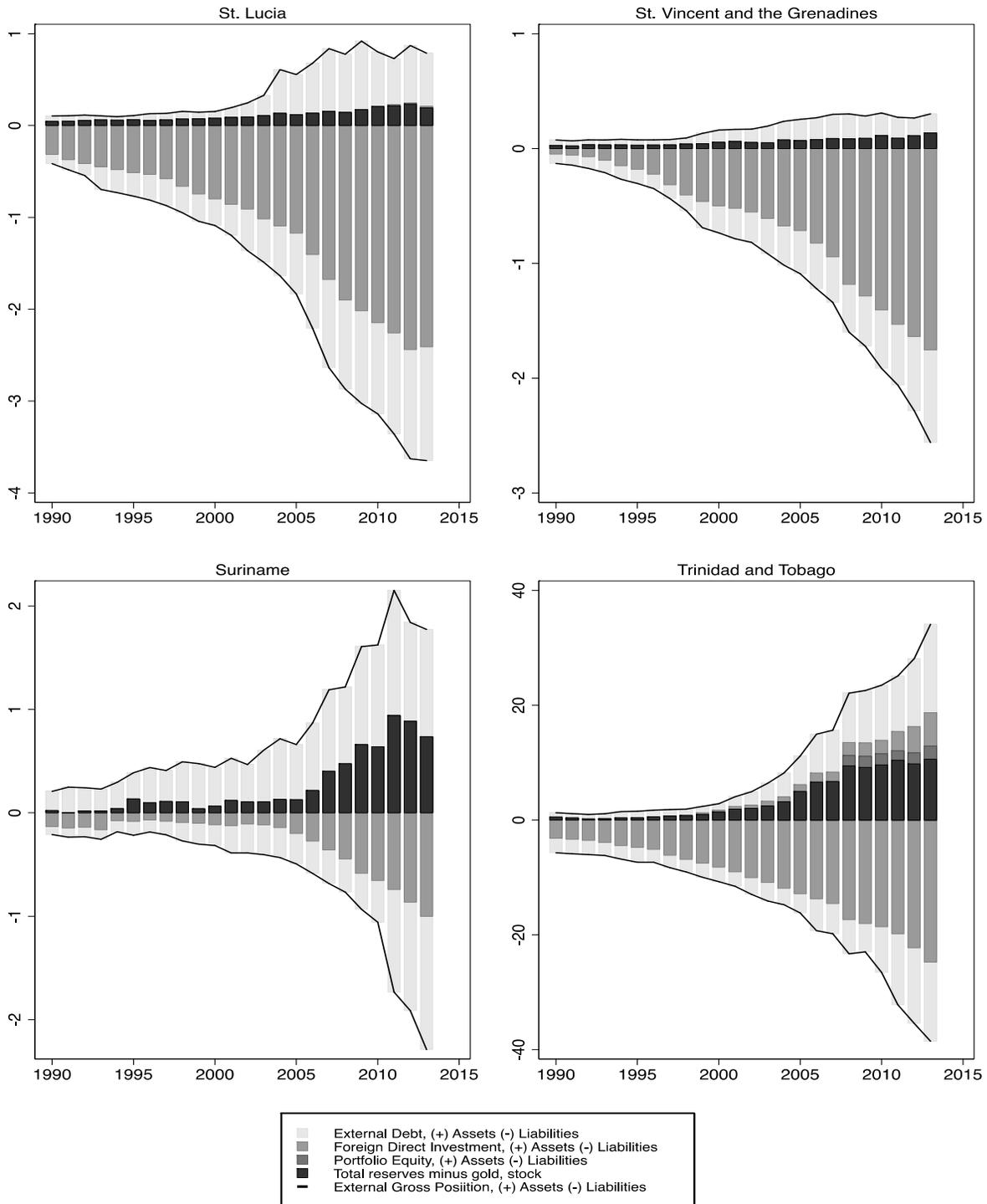


Figure B3-A. BOP-derived Real Rates of Returns Differential



Figure B3-B. BOP-derived Real Rates of Returns Differential

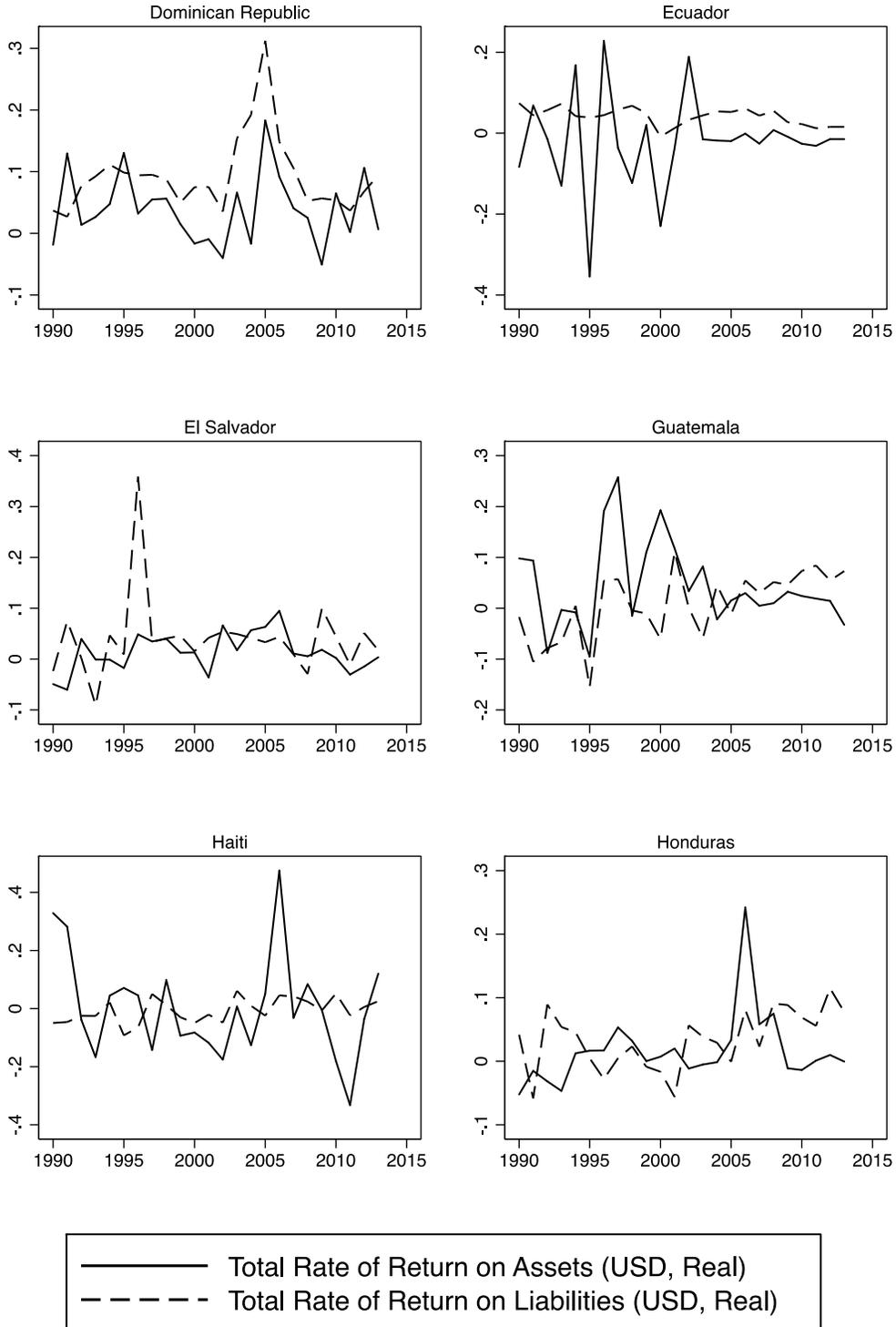


Figure B3-C. BOP-derived Real Rates of Returns Differential

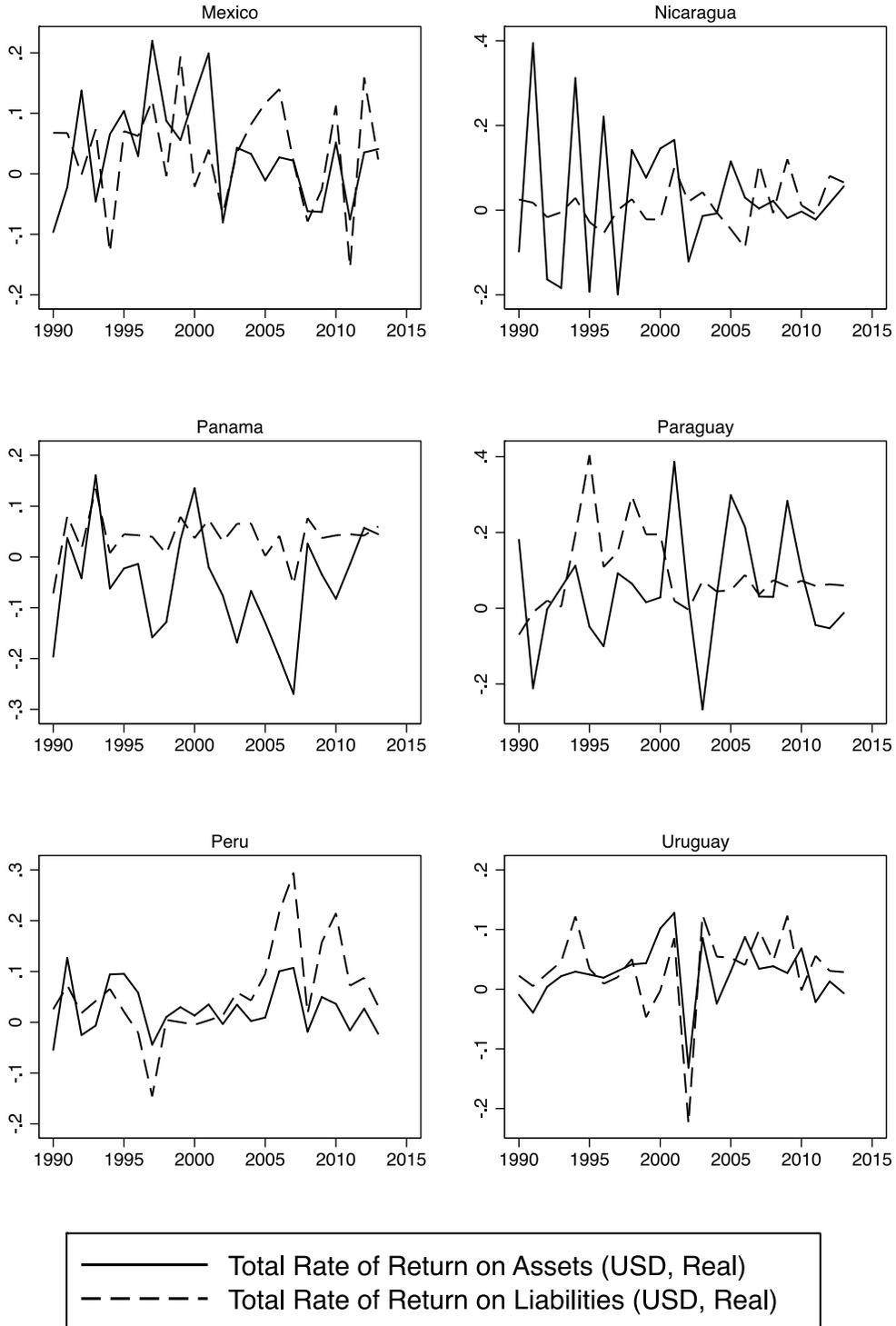


Figure B3-D. BOP-derived Real Rates of Returns Differential

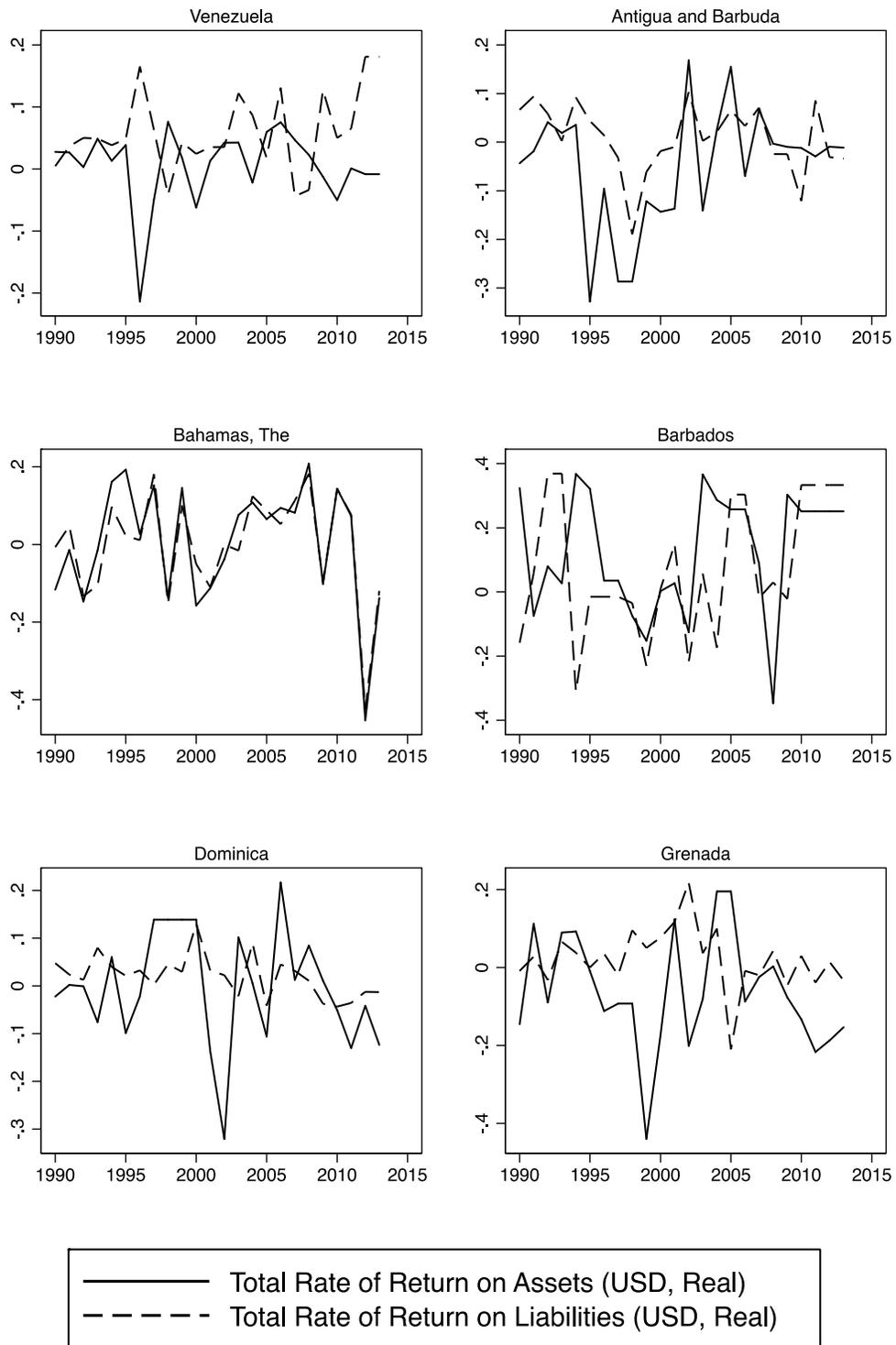


Figure B3-E. BOP-derived Real Rates of Returns Differential

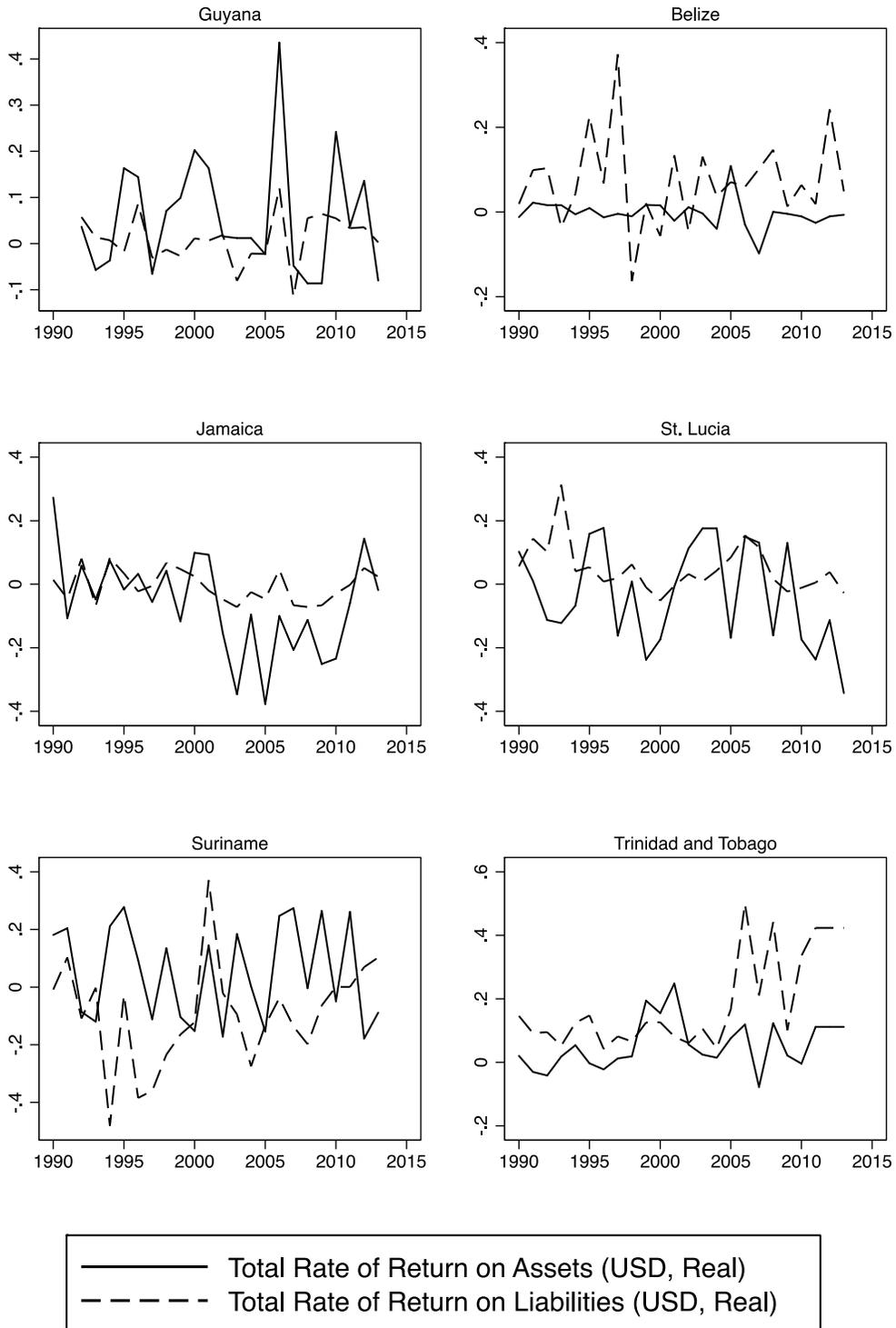


Figure B4-A. BOP-derived Real Rates of Return and Yields on Assets

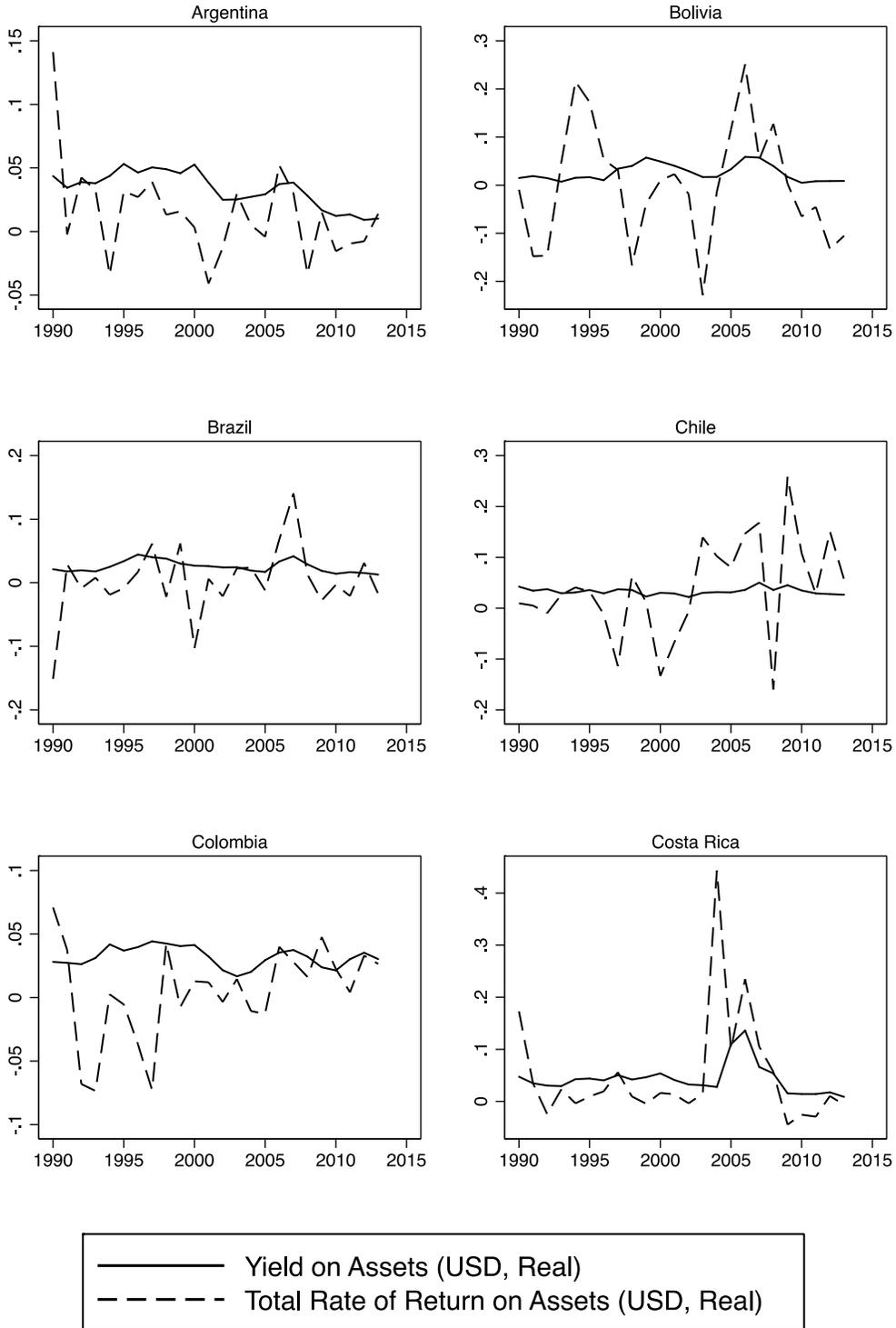


Figure B4-B. BOP-derived Real Rates of Return and Yields on Assets

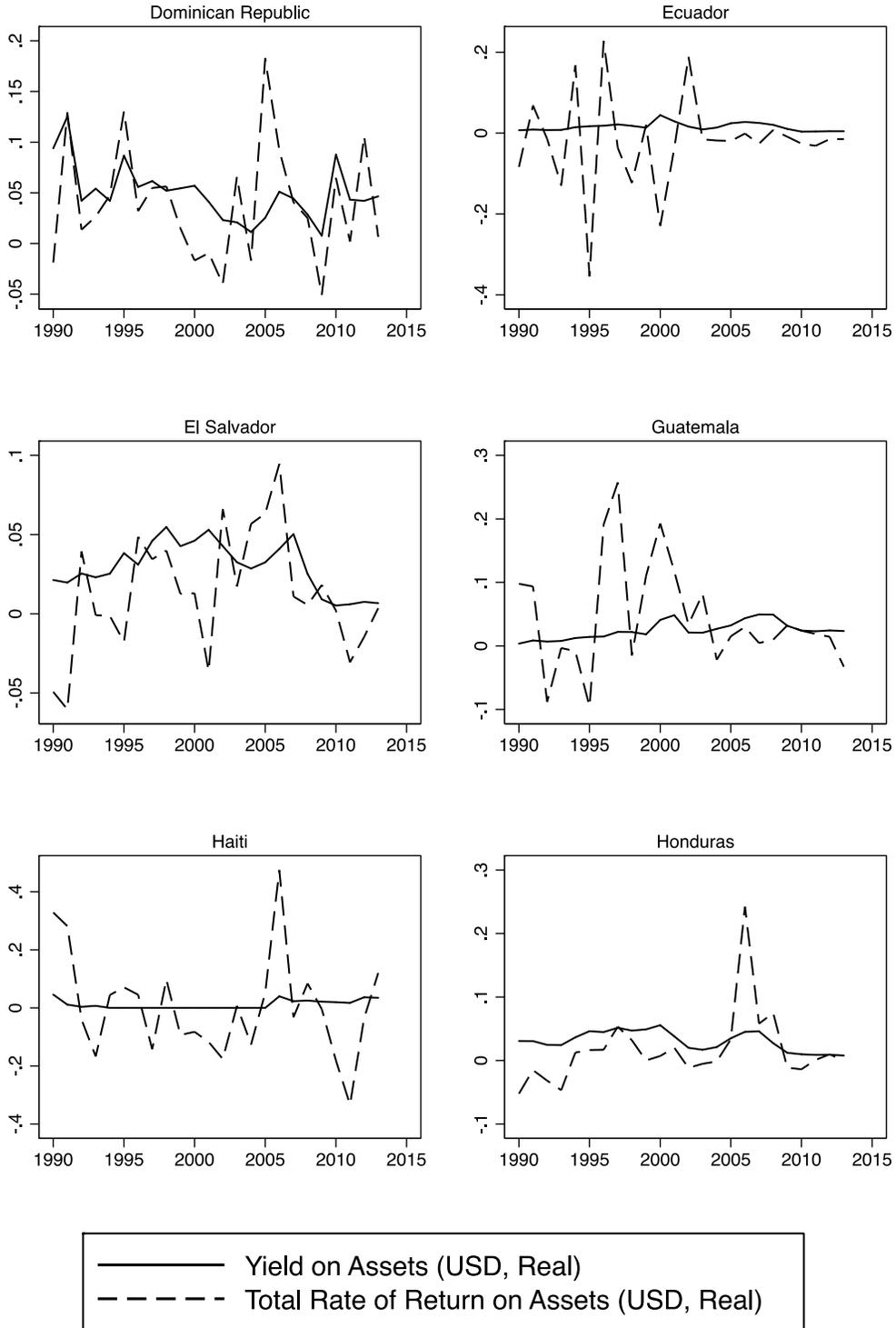


Figure B4-C. BOP-derived Real Rates of Return and Yields on Assets

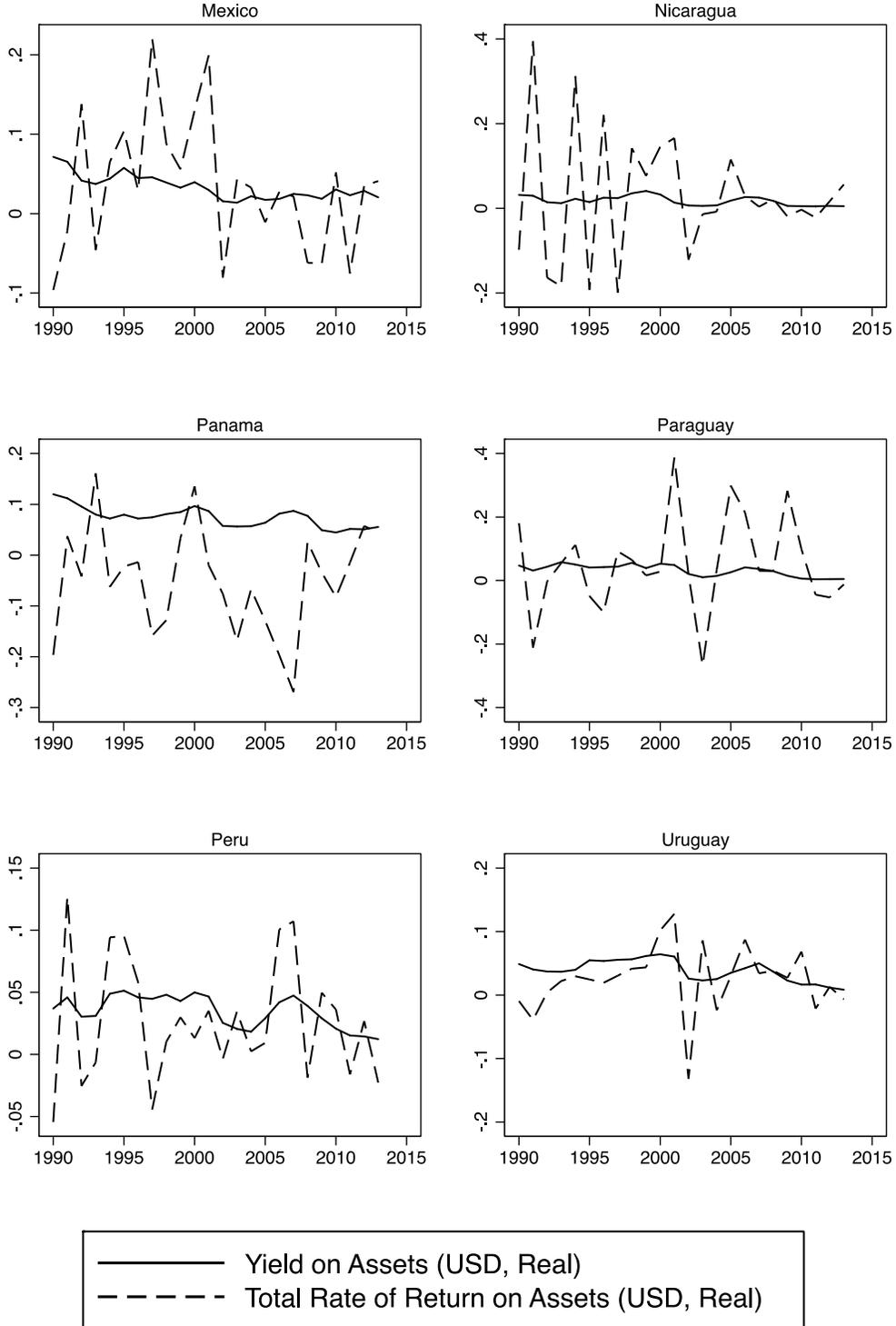


Figure B4-D. BOP-derived Real Rates of Return and Yields on Assets

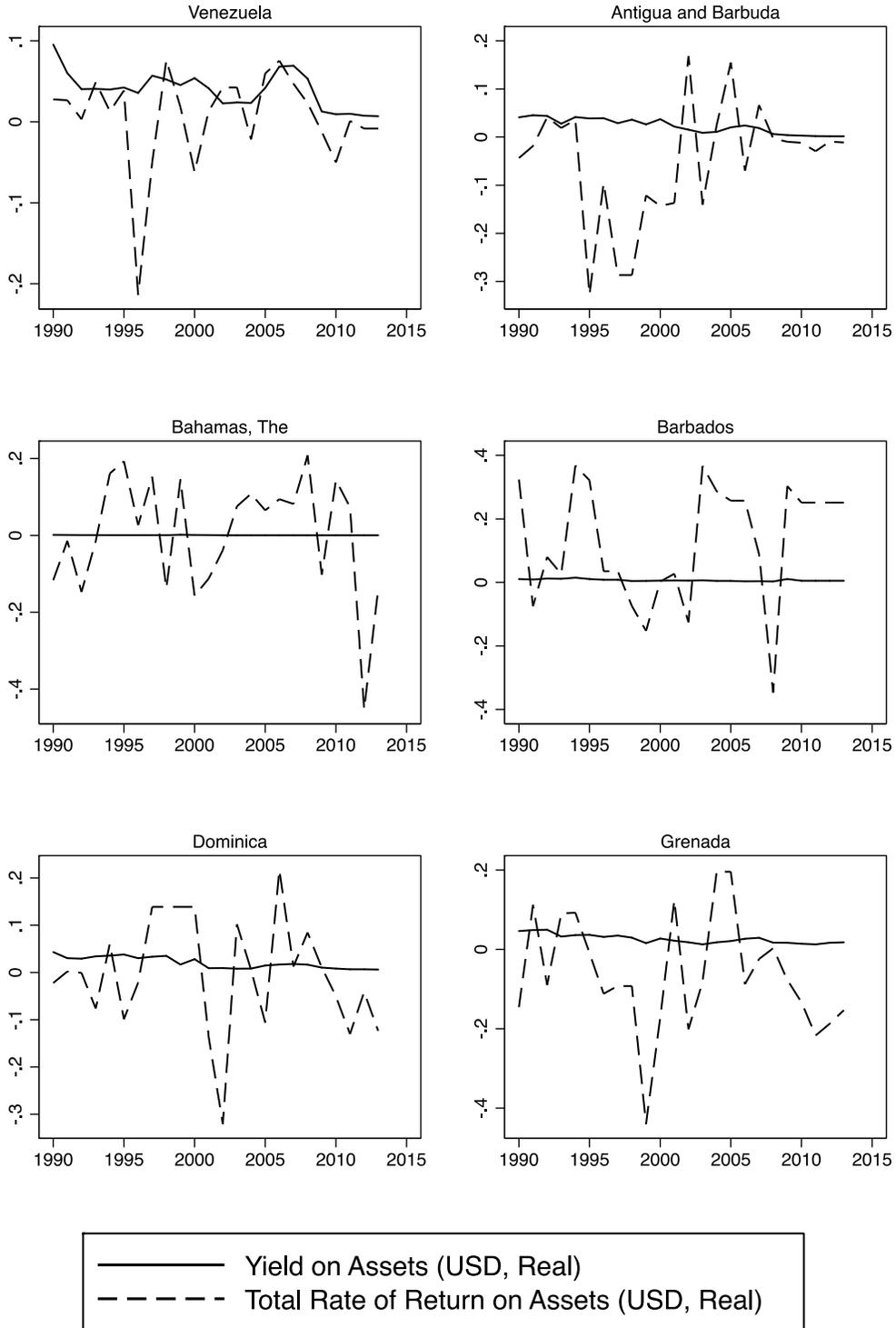


Figure B4-E. BOP-derived Real Rates of Return and Yields on Assets

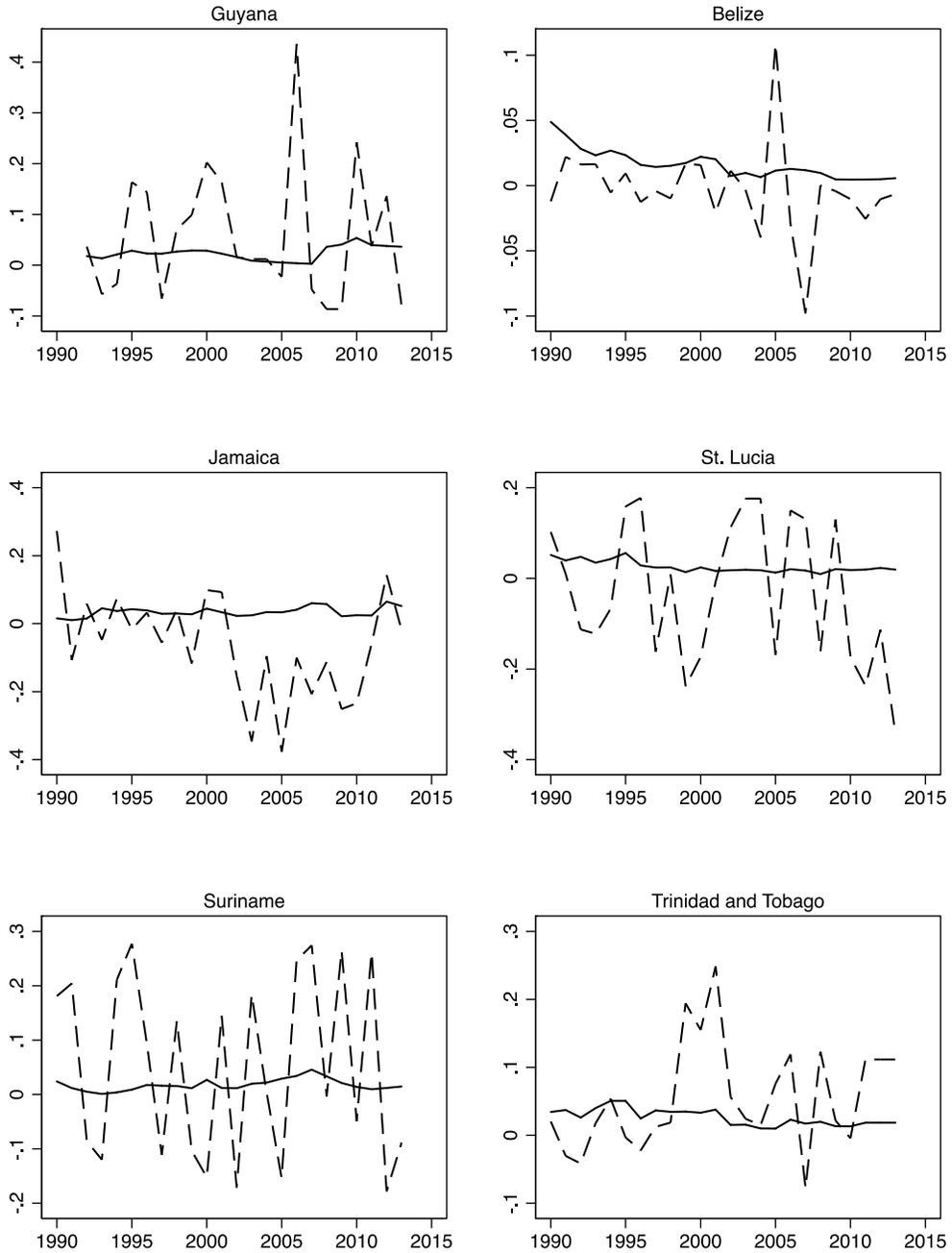


Figure B5-A. BOP-derived Real Rates of Return and Yields on Liabilities

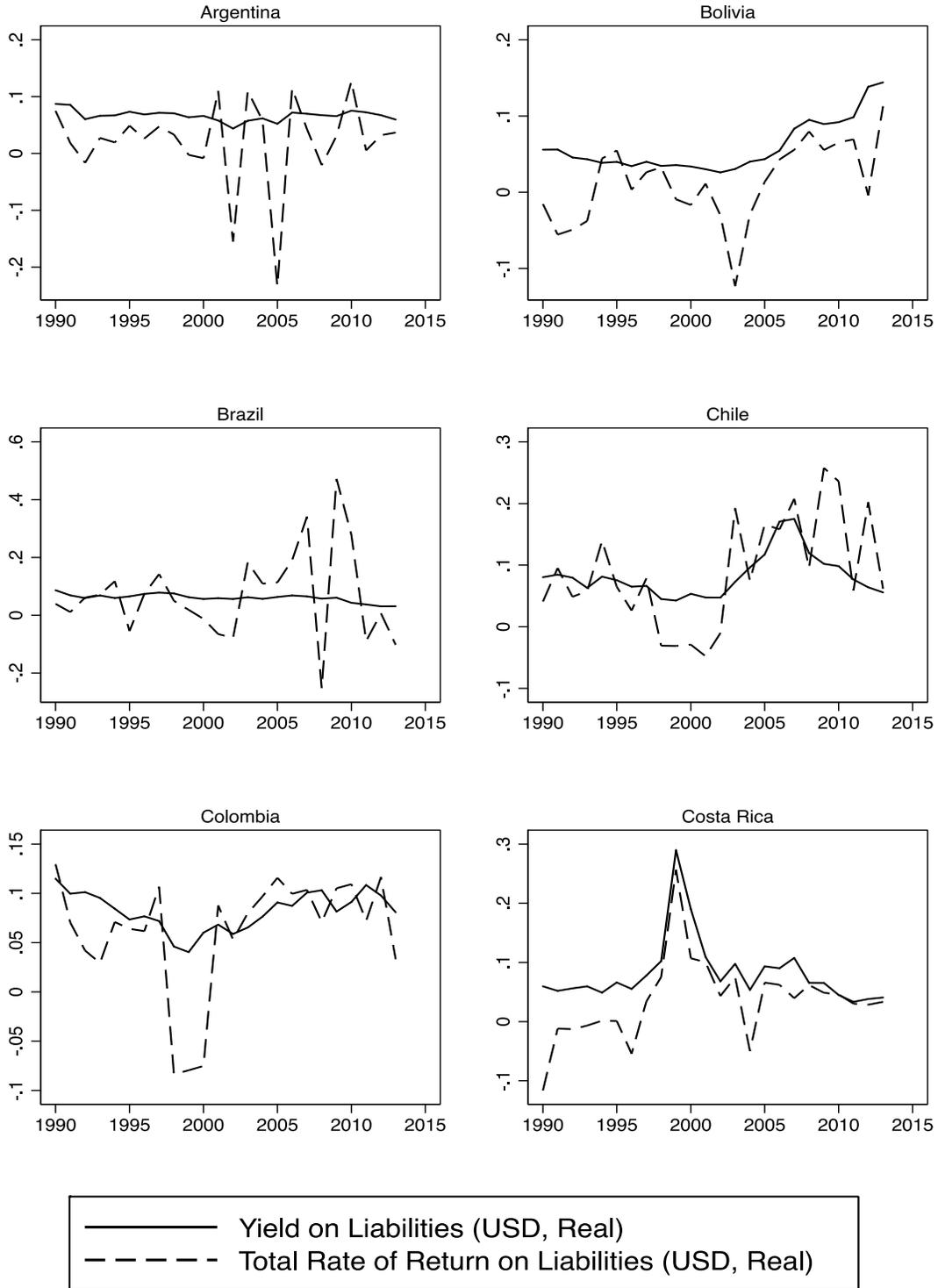


Figure B5-B. BOP-derived Real Rates of Return and Yields on Liabilities

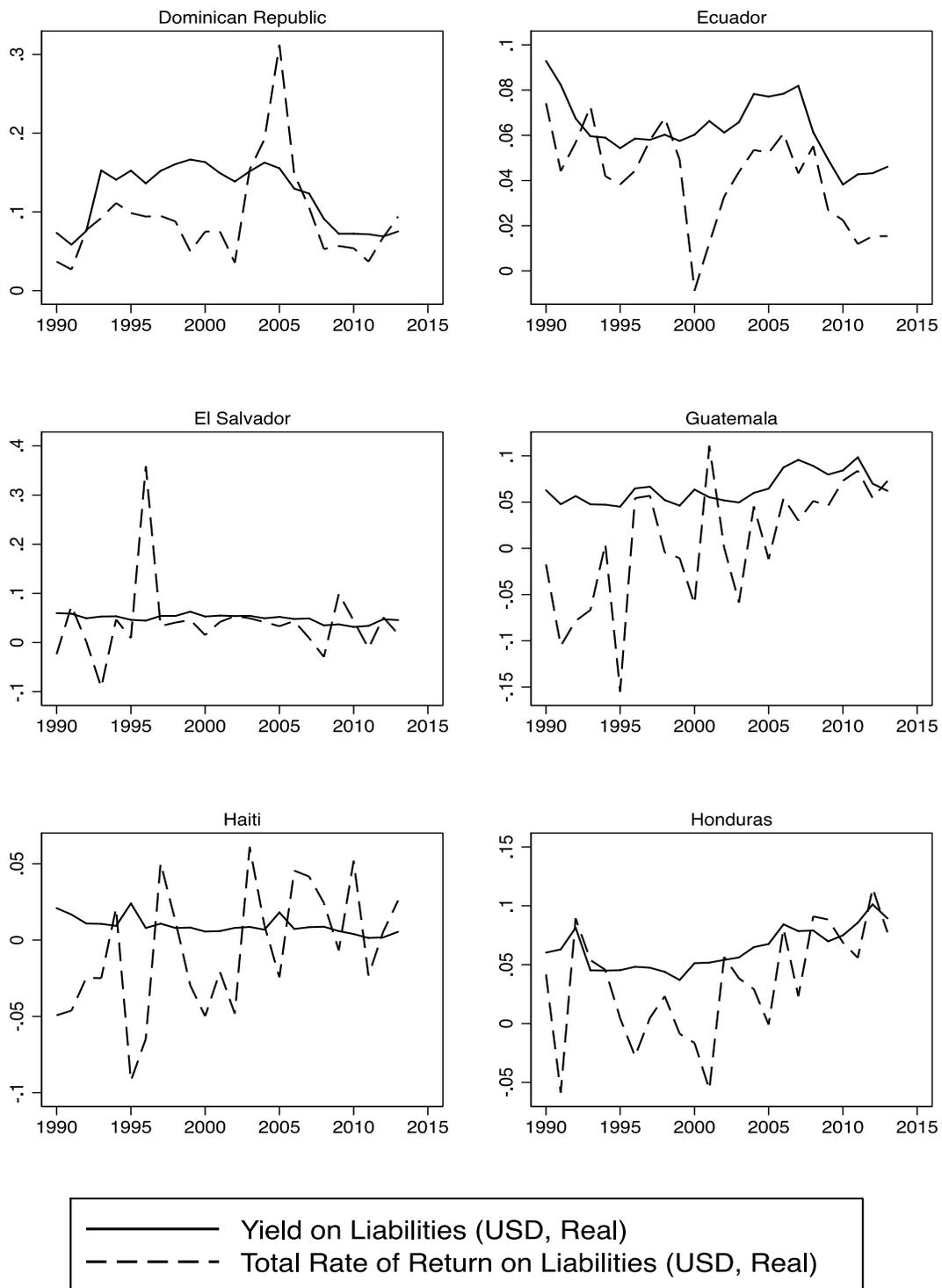
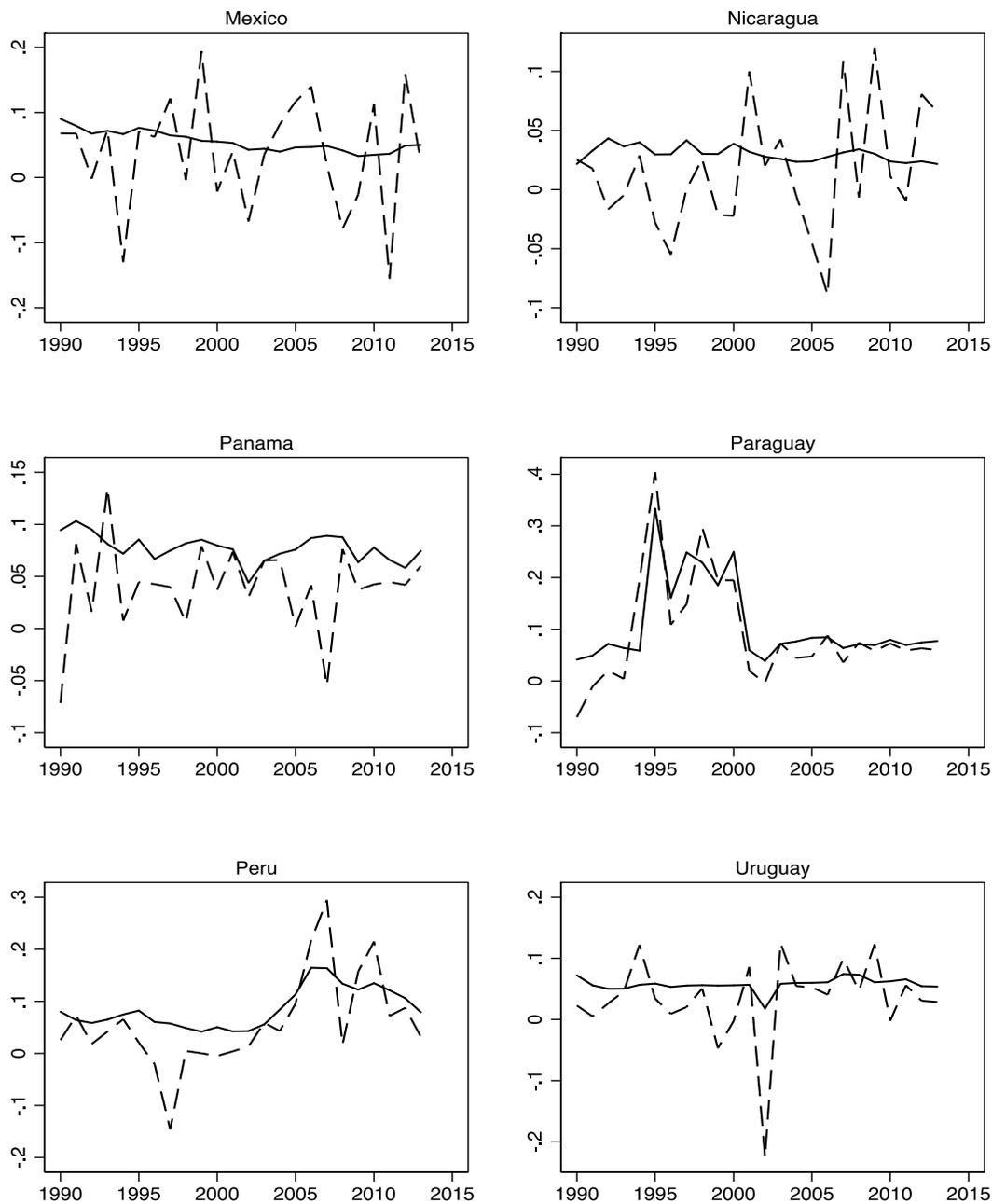
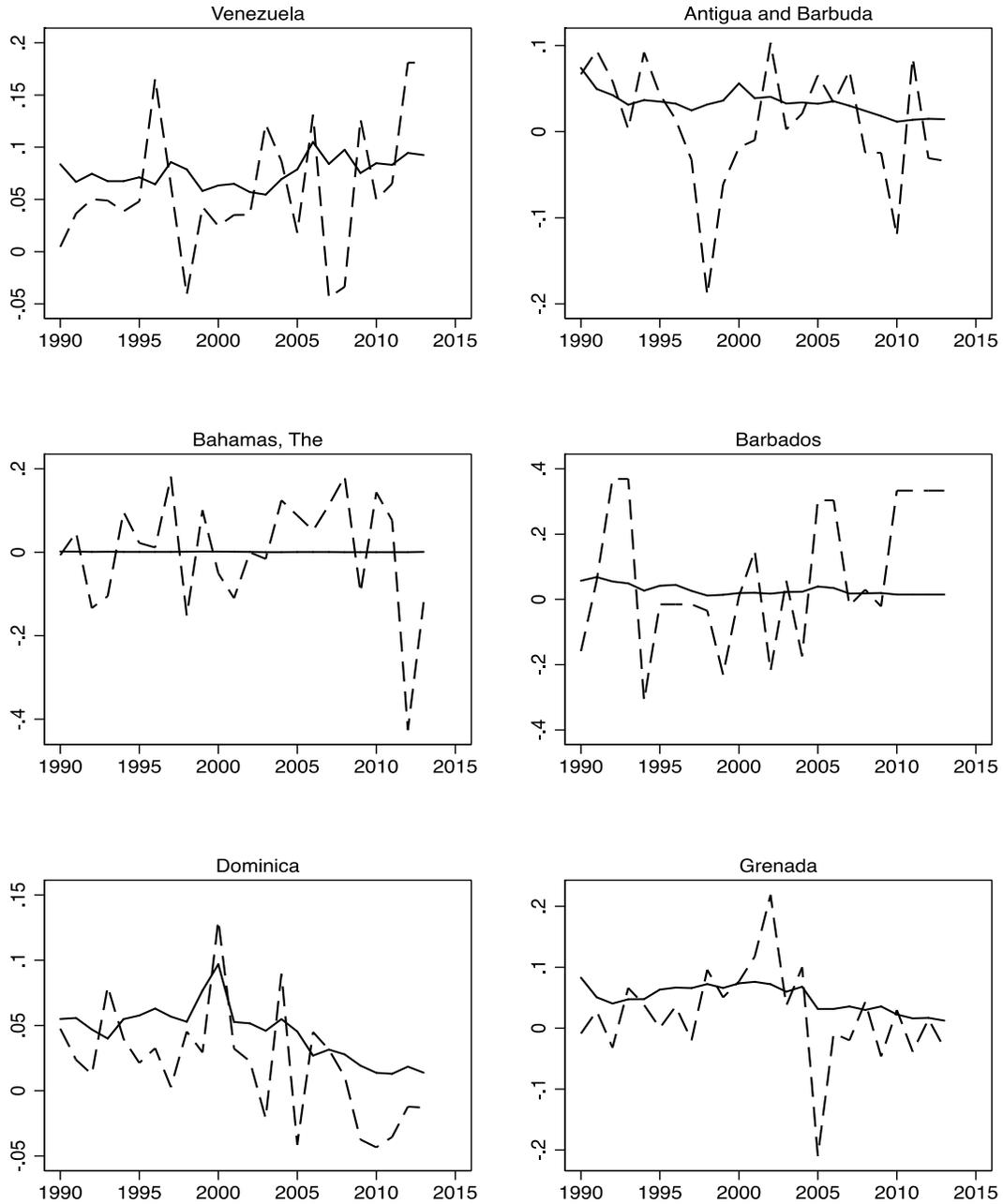


Figure B5-C. BOP-derived Real Rates of Return and Yields on Liabilities



—	Yield on Liabilities (USD, Real)
- - -	Total Rate of Return on Liabilities (USD, Real)

Figure B5-D. BOP-derived Real Rates of Return and Yields on Liabilities



—	Yield on Liabilities (USD, Real)
- - -	Total Rate of Return on Liabilities (USD, Real)

Figure B5-E. BOP-derived Real Rates of Return and Yields on Liabilities

