# On the Implications of Taxation for Investment, Savings and Growth:

## Evidence from Brazil, Chile and Mexico

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## Abstract[1]

This paper explores the qualitative and quantitative implications of taxation for growth and savings in three Latin American countries: Brazil, Chile and Mexico, studying a small open economy in the context of an endogenous growth model where the domestic interest rate depends on the level of domestic debt. The model's parameters are calibrated to the Brazilian, Chilean and Mexican economies. The findings suggest that, in order to implement the optimal tax regime, Brazil must tax capital at a considerably lower rate than at present. Consumption should be heavily taxed in Brazil and Mexico and optimal labor taxes should be lower than actual taxes in Brazil and Chile. However, while sub-optimal taxes seem to imply lower long-run growth in these three countries, low saving rates do not seem to be a direct consequence of sub-optimal taxation.

**JEL Classification:** E61, E62, H21

**Keywords:** Optimal fiscal policy, endogenous economic growth, savings.

# 1 Introduction

Despite successful stabilization programs and reforms during the last decades, Latin America saving rates have remained relatively stagnant, especially in comparison with the East Asian "miracle" economies. As Figure 1 illustrates, national saving in Latin America has averaged, during the past decade, less than 20 percent of GDP, in comparison with over 30 percent in six rapidly growing East Asian economies. Each economy of Latin America and the Caribbean (LAC) had a saving rate substantially below that recorded in the Asian "miracle" economies, and in several LAC economies saving rates were only about one-third of the Asian "miracle" average. This is far too striking. Does this indicate some sort of misallocation? If so, what are their main determinants of these differences?

**Figure 1. National Savings as a Percentage of GDP in East Asia and LAC Countries**
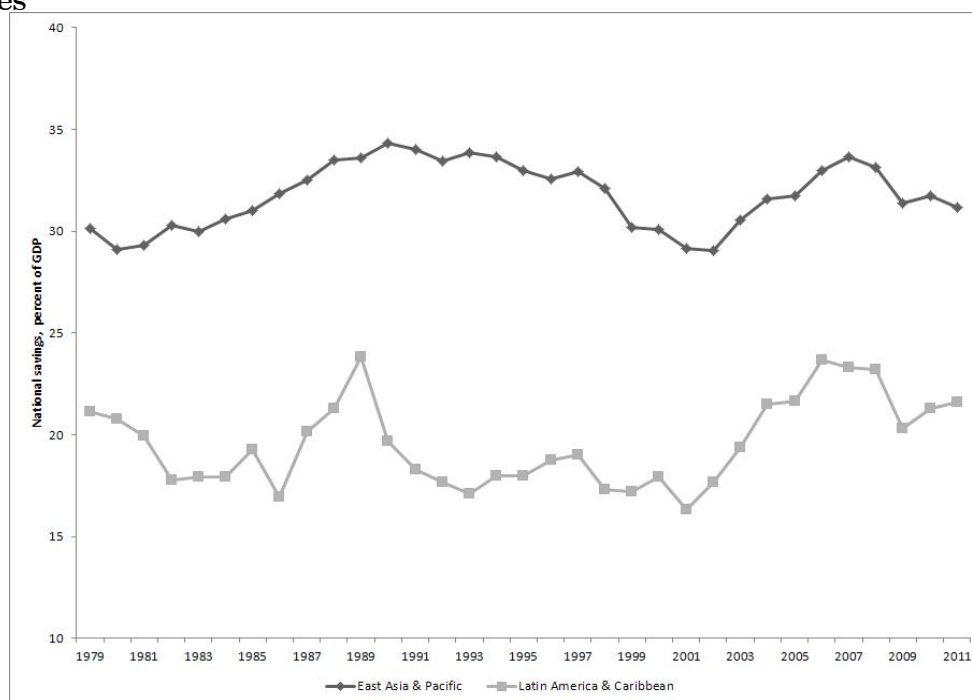


*Source:* Authors' calculations using the World Bank WDI database.

These questions lead to key policy issues, as one of the dominant views in the literature highlights that Latin America's low rate of saving condemns the region to an inefficient allocation of resources that delivers low investment and consequently low sustainable growth rates (see Gavin, Hausmann and Talvi, 1997).

As saving rates are just endogenous variables reacting optimally to incentives, policies should concentrate on removing impediments to growth rather than trying to establish programs aimed directly at promoting saving that are likely to be of dubious effectiveness and may involve economic inefficiencies. As a matter of fact, policy should aim to establish an environment conducive to high and sustainable growth, trusting saving to follow in response to the incentives that such an environment provides. While policies aimed at increasing saving may exhibit a substantial overlap with those aimed at removing impediments to growth, the shift of emphasis in the policy objective from saving to growth is non-trivial. Figure 2 below shows that taxes in Latin America are substantially higher than taxes in East Asia and the Pacific. Average tax revenues in Latin America have been almost 13 percent of GDP between 1991 and 2007, while in East Asia and the Pacific this figure amounts to only 9 percent. Consequently, we study the role played by fiscal policies, more precisely taxes, on economic growth and their implications for domestic savings (see Attanasio and Wakefield, 2010). More precisely, this paper aims to answer the following questions: What are the implications of sub-optimal taxation for sustainable growth? How much of the relatively low levels of saving rates is a consequence of these sub-optimal policies? The answers to these questions are not straightforward, as many efficiency-raising, growth-promoting policies are likely to have an adverse effect on savings that, although temporary, may last for many years (see Gavin, Hausmann and Talvi, 1997).

In order to answer these questions, we study the effect of optimal taxation on growth and savings under full commitment to finance an exogenous path of public expenditures in a small open economy in the context of an endogenous growth model. The model economy for analyzing these issues includes some non-standard assumptions to capture particular features of Latin American developing countries. The informal sector in these countries produces between 25 to 76 percent of gross domestic product or GDP (Schneider and Enste, 2000). Therefore, the design of public policies must especially consider this peculiar feature: a large share of labor market relationships cannot be monitored by governments. So, in the model economy there are two sectors, tradable and non-tradable, that can hire labor in the formal or in the informal labor market. There is a neoclassical technology to produce commodities and non-tradable goods that displays constant returns to scale. Additionally, the domestic interest rate has an extra component determined by the level of domestic debt. We quantify the behavior of this economy along the competitive equilibrium balanced growth path to understand how changes in taxes affect variables in the long run. Then, using the characterization of the competitive equilibrium, we study the design of optimal tax policy. That is, we solve a dynamic optimal taxation problem to provide a quantitative analysis in a calibrated economy in which we first we characterize and compute the optimal allocations,

**Figure 2.  Tax Revenue as a Percentage of GDP in East Asia and LAC Countries, 1991-2010**
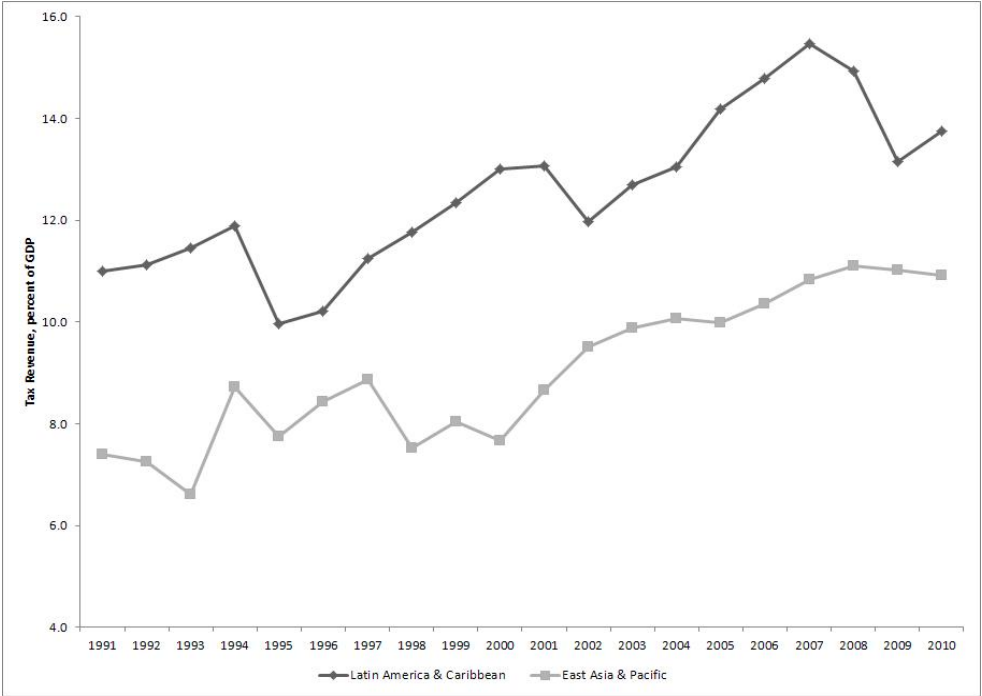


*Source:*  Authors' calculations using the World Bank WDI database.

that are decentralized as competitive equilibrium, and the corresponding optimal taxes, and then we compute the competitive equilibrium allocations stemming from actual (potentially suboptimal) tax systems and compare these economies. In this way it is possible to quantify the negative impact on welfare and sustainable levels of growth, as well as consequent implications for domestic savings. As shown by Espino and González-Rozada (2013) in a similar setting, the negative impact might not be trivial, and therefore its quantification might be of interest on several fronts.[2]

The quantitative results obtained outline not only the optimal design of fiscal policies but also the challenges governments face in implementing them. The findings point out the costs in terms of growth, savings and welfare of deviating from these optimal policies in three Latin American countries: Brazil, Chile and Mexico. In order to do so, we compare this optimal design with non-optimal tax schemes, including the status quo and a counterfactual tax scheme, given by the tax structure of one of the East Asia "miracle" countries: Thailand.

The rest of the paper is organized as follows. Section 2 presents a review of the related literature. Section 3 provides a detailed description of the model, a small open economy with endogenous growth, and two sectors, tradable and non-tradable, that can hire not only formal but also informal labor. Commodities and non-tradable goods are produced with a constant returns to scale neoclassical technology, and the domestic interest rate depends on the level of domestic debt. Section 4 formalizes the competitive equilibrium, characterizes the corresponding balanced growth path and provides detailed computed examples to study the impact of taxes on alternative equilibrium variables. The evidence found in this section suggests that the introduction of an informal sector into the economy has a negative impact on the long-run growth rate but does not affect private savings in the three countries analyzed. Increasing labor taxes induces a reduction in the long-run growth rate, and an increase in the capital tax rate produces a fall in private savings along the balanced growth path. Additionally, and as expected, increasing labor taxes reduces the time devoted to work in the formal sector and increases the time allocated to work in the informal sector of the economy. However, the reduction in working time in the formal sector of the three countries is larger than the increase in working time in the informal sector, resulting in a decline in total time allocated to work. Increasing consumption taxes has similar effects, as does increasing labor taxes. That is, an increment in the consumption tax rate slows down the economy but does not affect private savings along the balanced growth path. Finally, increasing government expenditures induces an increase in both the growth rate and private savings in the three

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[2]The solutions that we obtain are time-inconsistent, a common characteristic of models of this type. This is not unreasonable, since this is a normative analysis. These models do not seek to develop testable implications but rather to provide quantitative guidelines for optimal decision-making by governments, which is the main purpose of this study.

countries. Section 5 introduces the concept of optimal fiscal policy, describes in detail the Ramsey allocation problem and provides a detailed characterization of the corresponding balanced growth path. The main objective of this section is twofold: first, to measure how much the tax rates observed in Brazil, Chile and Mexico should change to decentralize the Ramsey allocation problem, that is, to switch to the optimal tax policy and second, to assess the costs in terms of growth and savings of using the actual tax scheme instead of the tax structure of one of the East Asia "miracle" economies: Thailand. The empirical evidence in this section indicates that sub-optimal taxes seem to imply lower long-run growth in the three Latin American countries. However, it seems that the low levels of saving rates are not a direct consequence of sub-optimal taxation, as optimal taxes imply a case, Brazil, where private savings are lower than the actual situation; a case, Chile, where optimality induces larger private savings and a case, Mexico, where the optimal tax structure does not affect private savings. In the three countries, optimal taxes reallocate labor from the informal to the formal sector. The policy recommendations stemming from Section 6 suggest that in Brazil, capital, which, based on standard neoclassical growth models, should not be taxed, has to be taxed, albeit at a considerably lower rate than the rate observed. Consumption should be heavily taxed in Brazil and Mexico, and optimal labor taxes should be lower than actual taxes in Brazil and Chile. Finally, Section 7 concludes the paper.

## 2    Literature Review

There is a vast theoretical literature that studies optimal fiscal policy in some version of the neoclassical growth model.[3] Chamley (1986) showed that the long-run tax rate on capital should be zero. This finding was extended to an endogenous growth model by Lucas (1990) and Jones, Manuelli and Rossi (1993). The basic intuition behind this result is that a capital income tax distorts the investment decision and therefore should, in the long run, be replaced entirely by an income tax. This is an important result since the optimal tax structure that it describes is significantly different from what is observed in practice. As such, the model on which it is based requires further consideration. In particular, a situation in which the zero tax will not apply is the following. Correia (1996) studies a small open economy and assumes that there are one or more factors of production that the government cannot tax (or cannot tax optimally). Then the tax on capital income will be dependent on the relationship between capital and non-taxable factors.

Given these theoretical results, actual tax systems are apparently far from these prescriptions. This raises the possibility that reforms in these systems can raise the rate of

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[3]A comprehensive survey of this area can be found in Chari, Christiano and Kehoe (1998).

growth and the level of welfare. This leads to a purely quantitative question of whether a policy reform can be justified that considers a budget-balanced replacement of the capital tax by taxes on consumption or labor.

The first major contribution in this respect was by Lucas (1990), who analyzed an endogenous growth model with investment in human capital driving growth in a representative agent setting that eliminates distributional issues to focus entirely upon efficiency. Using data from the U.S. economy, he measures what would have happened if the tax on capital had been set to zero in 1985, with revenue-neutrality ensured by increasing the tax on labor. The policy change results in a significant level effect but an insignificant growth effect. These findings can be explained as follows. Since time is the only input into the production of human capital, the cost (and return) is just the forgone wage. This leaves the human capital choice unaffected by taxation and, since it is this that drives growth, there is no growth effect. The level effect arises simply because of the replacement of a distortionary tax by a non-distortionary one.

The analysis of Lucas is extended by King and Rebelo (1990), who consider both an open economy and a closed economy. The model differs from Lucas's through having physical capital as an input into the production of human capital. In addition, King and Rebelo also permit depreciation of both capital inputs. In their benchmark case, where the share of physical capital in human capital production is a third, increases in the capital tax and the labor tax from 20 percent to 30 percent reduce the growth rate by 1.52 percent from its initial level of 1.02 to –0.5 percent. The level effect is a 62.7 percent decrease in welfare. A 10 percent increase in the capital tax alone reduces growth to 0.5 per cent. When the share of physical capital in human capital production is decreased to 0.20, 0.52 percent falls to 0.11 percent. In the open economy version of the model, which is characterized by an interest rate fixed at the world level, the fall in growth is even greater: a 10 percent increase in the capital tax reduces growth by 8.6 percent.

Jones, Manuelli and Rossi (1993) provide the most general and ambitious quantitative exercises in a setting that combines elements of those of Lucas and of King and Rebelo (in particular, human capital requires time and goods to be produced). Where it differs significantly from the Lucas model is in the parameters of utility.

Lucas' intertemporal marginal rate of substitution, $1/\sigma$, is 0.5 and the elasticity of labor supply, $\alpha$ in their notation, is 0.5. In contrast, Jones et al. calibrate $\alpha$ with the data and so, when $\sigma = 2$, the corresponding $\alpha$ is 4.99; i.e., labor supply is much more elastic, implying in turn that taxation will have a greater distortionary effect. For $\sigma = 2$, Jones et al. find that the elimination of all taxes (so distortions are completely removed) raises the growth rate from 2 percent to 5 percent with a welfare gain of 15 percent (e.g., 1.15 is the

factor by which the consumption path must be raised in order to bring utility under the current system up to the level attained in the Ramsey solution). For lower values of $\sigma$, and hence greater values of $\alpha$, the effect is even more dramatic.[4]

Summarizing these contributions, Lucas finds no growth effect but a significant level effect. In contrast, King and Rebelo and Jones et al. find very strong growth and level effects. King and Rebelo use a much lower share of human capital in their own production than Lucas and a depreciation rate of 10 percent. For human capital especially, this rate would seem excessive. For Jones et al., it is the higher degree of elasticity of labor supply that leads to the divergence with Lucas.[5]

National saving matters for growth as high rates of saving are highly correlated with high rates of growth. Different studies of the determinants of saving rates (see Gavin, Hausmann and Talvi, 1997 and Gutiérrez, 2007, and the references therein) suggest that the long-run behavior of savings is closely related to both the rate of growth and per capita income levels. However, close correlation does not translate into causation, and this is a crucial element in the design of policies. Starting with Carroll and Weil (1993), the literature has studied the relationship between saving and economic growth in samples covering a large number of countries over several decades, and it has found that past growth predicts future saving rates, while past saving rates do not predict future growth. Therefore, the fact that increased growth tends to precede increased saving rates suggests that saving may be, to an important extent, caused by economic growth. The pattern of strong increases in saving rates after an acceleration of growth is illustrated by the experience of the Asian "miracle" economies. Gavin, Hausmann and Talvi (1997) shows that saving rates increased substantially in these economies, while they had remained stagnant in Latin America since the early 1970s. The Asian economies have displayed during recent years a much higher rate of saving than LAC countries. But this is a relatively recent phenomenon, resulting from a long and gradual increase in Asian saving from rates that were, in the 1970s, generally below those recorded in Latin America. Only in the late 1970s and early 1980s, after the acceleration of growth in Asia, did Asian saving rates rise consistently and substantially above Latin American rates. In Latin America, Chile is an example where strong increases in saving followed the acceleration of growth. Chile's economic recovery began in 1984, when domestic saving was still quite depressed, and the economy was as a result heavily reliant upon capital inflows to finance domestic investment. It was only in the late 1980s, after

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[4]If $\sigma = 1.1$ while the corresponding $\alpha$ is calibrated to be 7.09, the increase in growth rates is about 8 percent. The reason for this increase in growth can be seen in the response of labor supply to the tax changes.

[5]The role played by each ingredient to explain the divergence between the results is studied in Stokey and Rebelo (1995), who use a model that encompasses the previous three.

several years of sustained economic growth, that Chilean saving rates approached the high levels now observed. See Gutiérrez (2007) and Attanasio and Székely (2000) for detailed discussions.

The most related work is Espino and González-Rozada (2013), which not only encompasses Jones, Manuelli and Rossi (1993) and Correia (1996) but also adds some key ingredients to study developing economies. The authors explore the qualitative and quantitative implications of optimal taxation in a developing economy when economic growth is endogenously determined. This class of economies differentiates from developed economies in two aspects: i) the informal sector is quantitatively significant and ii) tax-collecting technologies are more rudimentary. The model is calibrated to the Chilean economy and their results suggest that capital should still be taxed but considerably less than actual taxes (that is, 10.78 percent versus 18.5 percent). Labor should be subsidized (to stimulate accumulation of human capital), while consumption taxes should be increased by approximately 50 percent (from 19 percent to 28 percent). As expected, the better collecting technologies, the higher the corresponding taxes. In this context, the resulting growth rate increases only slightly along the balanced growth path.

In this paper we study the effect of optimal taxation on growth and savings under full commitment to finance an exogenous path of public expenditures in a small open economy in the context of an endogenous growth model in three Latin American countries: Brazil, Chile and Mexico. The next section provides a detailed description of the theoretical model.

# 3    A Theoretical Framework

In this section we describe the physical setting, the asset market structure and the role played by the government.

## 3.1    Technology and Households

There are two sectors that produce goods in this economy. The first is an intermediate commodity (denoted $T$) that is internationally traded, and the second is a non-tradable consumption good ($N$). Let $P_t^N$ denote the prices of non-tradable goods in terms of the commodity.

There is a neoclassical technology to produce the commodity that displays constant returns to scale.

Commodities are produced using effective units of labor and domestic capital. This technology is represented by

$$Y_t^T = F^T(K_t^T, L_t^T) = A_T \ (K_t^T)^{\alpha_T} (L_t^T)^{(1-\alpha_T)}$$

$$L_t^T = \left( \beta \ \left( L_t^{T,F} \right)^{\frac{\eta-1}{\eta}} + (1-\beta) \ \left( L_t^{T,I} \right)^{\frac{\eta-1}{\eta}} \right)^{\frac{\eta}{\eta-1}}$$

where $A_T$ is a technology parameter, $\alpha_T \in (0,1), \beta \in (0.5, 1]$ and $\eta \geq 0$. The distribution parameter $\beta$ reflects intensity in units of effective formal labor while $\alpha_T$ is the participation of capital. $\eta$ is the elasticity of substitution between effective informal, $(L_t^I)$, and effective formal, $(L_t^F)$, labor while the elasticity of substitution between capital and composite labor is 1. Note that when there is strict complementarity $(\eta < 1)$, other things equal, a rise in $L_t^F$ leads in equilibrium to an increase in informal labor, and conversely, when there is strict substitutability $(\eta > 1)$, a rise in $L_t^F$ induces a decrease in informal labor. If $\eta = 1$, this technology reduces to the standard Cobb-Douglas production function.

Non-tradable goods are also produced using effective units of labor and domestic capital. The corresponding technology is represented by

$$Y_t^N = F^N(K_t^N, L_t^N) = A_N \ (K_t^N)^{\alpha_N} (L_t^N)^{(1-\alpha_N)}$$

$$L_t^N = \left( \beta \ \left( L_t^{N,F} \right)^{\frac{\eta-1}{\eta}} + (1-\beta) \ \left( L_t^{N,I} \right)^{\frac{\eta-1}{\eta}} \right)^{\frac{\eta}{\eta-1}}$$

Parameters have the same interpretation. This technology also displays constant returns to scale. Total factor productivity parameters, namely $A_T$ and $A_N$, are constant.

Observe that firms can hire labor either in the formal labor market or in the informal labor market. Informality translates into less productivity, $(\beta > 0.5)$, and labor hired in the informal market does not pay labor taxes. At date $t$, firms face competitive factor markets. They pay wages $w_t^F$ and $w_t^I$ per unit of effective formal and informal labor, respectively, in units of commodity goods. The rental price of capital at date $t$, also in units of the commodity, is denoted $r_t$. We model informality under the assumption that there is a representative firm that operates in both labor markets, creating both formal and informal jobs. This assumption is made to simplify the analysis, but broader interpretations are possible. To see this, notice first that constant returns to scale and competitive factor markets imply that the equilibrium number of firms is undetermined. There is an alternative interpretation which can be summarized as follows. In each sector there are two types of representative firms. One type of firms hires capital and formal labor and uses the technology corresponding to more productive labor. The other type, on the other hand, hires capital and informal labor while using the less productive technology. We emphasize that we do not

aim to explain the emergence of informal labor markets, but we take their existence as given to study the implications of taxation.

Agents value non-tradable consumption and leisure. At date $t$, let $C_t$ and $x_t$ denote private consumption of the non-tradable good and leisure, respectively.[6] Representative household preferences are described by time-separable, discounted utility where $\{C_t, x_t\}_{t=0}^{\infty}$ is valued

$$\sum_{t=0}^{\infty} \rho^t \frac{\left( C_t \ (x_t)^{\theta} \right)^{1-\sigma}}{1-\sigma},$$

where $\rho \in (0,1)$, $\sigma > 0$ and $\theta \geq 0$. $\rho$ is the discount rate and $\sigma$ is the intertemporal elasticity of substitution.[7]

The representative household is endowed with a unit of time every period which must be allocated across three types of activities and leisure. Effective units of formal and informal labor are

$$L_t^F = u_t \ H_t$$

$$L_t^I = v_t \ H_t$$

where $u_t$ and $v_t$ is the date$-t$ fraction of time working in the formal and informal sector, respectively. $H_t$ is the date$-t$ stock of human capital that evolves according to

$$H_{t+1} = A^H \ e_t \ H_t + (1 - \delta_H) \ H_t$$

where $e_t H_t$ is interpreted as the effective units of labor allocated to the human capital sector at date $t$, $A^H > 0$ is a human capital technology parameter and $\delta_H \in (0,1)$ denotes human capital depreciation rate. The effective amount of leisure that the representative household consumes is given by $x_t = (1 - du_t - v_t - e_t)$. We assume that $d > 1$ as an artificial device so that formal wages are lower than net formal wages.

### 3.2   Factor Mobility, Asset Market Structure and the Government

Let $G_t$ denote public consumption of the non-tradable good at date $t$. We consider a benevolent government that provides public goods, $G_t$, financed levying linear taxes on labor,

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[6]Leisure in this model should be interpreted in a broad sense, that is, including any non-market activity like home goods production.

[7]It is important to mention that, similar to Lucas (1990), if agents do not value leisure ($\theta = 0$) then taxes have no impact on growth rates.

capital and consumption as well as issuing debt. We assume that $G_t = gY_t^N$ for all $t$, where $g \in (0,1)$ is the ratio of government spending to non-tradable output.

There is no international labor mobility and physical capital is domestic. Production of new capital is domestic, but investment is imported from abroad at the (exogenous) price $P_t$. Let $K_t$ denote the domestic stock of capital at date $t$. The linear technology to produce new capital is standard and renders the law of motion for capital given by

$$K_{t+1} = I_t + (1 - \delta_K) \, K_t$$

where $\delta_K \in (0,1)$ denotes the depreciation rate $I_t$ denotes investment at date $t$.

The government can levy a tax of $\tau_t^k \in [0, \overline{\tau}_K]$ on the the net return on capital, $(r_t - \delta_K) \, K_t$, where $r_t$ denotes the domestic rental price of capital before taxes.[8] Think of $\tau_t^k$ as a tax on corporate profits that is levied on firms operating in the country. The government can also tax consumption at the rate $\tau_t^c$ and the formal sector at the rate $\tau_t^w$. As stressed above, the informal sector does not pay taxes.

There is a one-period bond to trade internationally at the price $q_t = 1/R_t$, where $R_t$ denotes the gross interest rate, which will be determined endogenously. The government and households have access to the credit market. Let $B_t^p$ and $B_t^g$ denote private and government debt holdings, respectively and $B_t = B_t^p + B_t^g$. The government's budget constraint is

$$(1 + R_t) \, B_t^g + P_t^N G_t = \tau_t^c \, P_t^N C_t + \tau_t^w \, w_t^F \, L_t^F + \tau_t^k r_t \, K_t - \tau_t^k \delta_K \, K_t + B_{t+1}^g$$

where $\left\{ B_{t+1}^g \right\}_{t=0}^{\infty}$ is further restricted by a no-Ponzi condition specified later. We denote $\pi = \left\{ \tau_t^c, \tau_t^w, \tau_t^k, G_t, B_{t+1}^g \right\}_{t=0}^{\infty}$ as a *fiscal policy*.

The domestic interest rate depends positively on the domestic debt-tradable output ratio as follows

$$R_t = R \left( \frac{B_t}{Y_t^T} \right)$$

where $R' > 0$.

Domestic agents take this rate as given; i.e., they do not internalize the impact of alternative debt choices.

The market clearing conditions for the market of non-tradable goods, domestic capital and domestic formal and informal labor are

$$C_t + G_t = Y_t^N$$

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[8]Following the convention in the literature we assume that return on capital after depreciation is taxed.

$$K_t = K_t^T + K_t^N$$

$$u_t\ H_t = L_t^{T,F} + L_t^{N,F}$$

$$v_t\ H_t = L_t^{T,I} + L_t^{N,I}$$

for all $t$.

## 4    Competitive Equilibrium Analysis

In this section, we formalize the corresponding competitive equilibrium concept (Subsection 4.1), and then we quantify the behavior of the economy along the balanced growth path (BGP, Subsection 4.2). The goal of this section is twofold. First, we find it useful to understand how changes in taxes affect the variables in the long run. Second, we use the characterization of the competitive equilibrium to study the design of the optimal tax policy discussed in Section 5.

### *4.1    Fiscal Policy and Competitive Equilibrium*

Given a fiscal policy $\pi = \left\{\tau_t^c, \tau_t^w, \tau_t^k, \tau_t^R, \tau_t^L, G_t\right\}_{t=0}^{\infty}$ and prices $\left\{r_t, w_t^F, w_t^I, P_t, P_t^N\right\}_{t=0}^{\infty}$, the representative household solves

$$\max_{\{C_t, x_t, u_t, v_t, e_t, H_{t+1}, K_{t+1}, B_{t+1}^p\}} \sum_{t=0}^{\infty} \rho^t \frac{\left(C_t\ (x_t)^\theta\right)^{1-\sigma}}{1-\sigma},$$

subject to

$$(1+\tau_t^c)P_t^N C_t + P_t I_t + (1+R_t)\ B_t^p \tag{1}$$
$$= \ \left((1-\tau_t^w)\ w_t^F\ u_t\ H_t + w_t^I\ v_t\ H_t\right) + \left((1-\tau_t^k)r_t + \tau_t^k \delta_K\right)\ K_t + B_{t+1}^p$$

$$H_{t+1} = A^H\ e_t\ H_t + (1-\delta_H)\ H_t \tag{2}$$

$$K_{t+1} = I_t + (1-\delta_K)\ K_t \tag{3}$$

$$\lim_{T\to\infty} \prod_{j=0}^{T} \frac{B_T^p}{(1+R_j)} \leq 0 \tag{4}$$

where $(K_0, H_0, B_0^p = 0)$ are given and $x_t = (1 - du_t - v_t - e_t)$. Notice that consumption as well as labor and capital taxes are paid by households. In equilibrium, it is indistinct if factor taxes are paid either by firms or workers.

Firms in the tradable and non-tradable sectors take prices as given and, respectively, solve the static problems

$$\max_{\left(L_t^{T,F}, L_t^{T,I}, K_t^T\right) \geq 0} \left\{ F^T(K_t^T, L_t^{T,F}, L_t^{T,I}) - w_t^F \ L_t^{T,F} - w_t^I \ L_t^{T,I} - r_t \ K_t^T \right\}$$

and

$$\max_{\left(L_t^{N,F}, L_t^{N,I}, K_t^N\right) \geq 0} \left\{ P_t^N \ F^N(K_t^N, L_t^{N,F}, L_t^{N,I}) - w_t^F \ L_t^{N,F} - w_t^I \ L_t^{N,I} - r_t \ K_t^N \right\}$$

Given a fiscal policy $\pi$ and a price system $\left\{ R_t, w_t^F, w_t^I, r_t, P_t^N, P_t \right\}_{t=0}^{\infty}$, denote

$$\{ C_t(\pi), x_t(\pi), u_t(\pi), v_t(\pi), e_t(\pi) K_{t+1}(\pi), H_{t+1}(\pi), B_{t+1}^p(\pi);$$
$$L_t^{T,F}(\pi), L_t^{T,I}(\pi), K_t^T(\pi), L_t^{N,F}(\pi), L_t^{N,I}(\pi), K_t^T, K_t^N(\pi) \}_{t=0}^{\infty}$$

as the corresponding solutions to the representative household's problem and the firms' problem in the tradable and non-tradable sector.

We say that a fiscal policy is *feasible* if

$$(1 + R_t) \ B_t^g(\pi) + P^N G_t \ = \ \tau_t^c \ C_t(\pi) + \tau_t^w \ w_t^F \ u_t(\pi) \ H_t(\pi) + B_{t+1}^g(\pi)$$
$$+ \tau_t^k \ r_t \ K_t(\pi) - \tau_t^k \delta_K \ K_t(\pi),$$

for all $t$; that is, a fiscal policy is feasible if it satisfies the government budget constraint evaluated at the allocation that is a solution. We restrict ourselves to feasible fiscal policies without any further reference.

**Definition 1.** *Given a fiscal policy* $\pi = \left\{ \tau_t^c, \tau_t^w, \tau_t^k, G_t, B_{t+1}^g \right\}_{t=0}^{\infty}$ *and investment prices* $\{P_t\}_{t=0}^{\infty}$, *a competitive equilibrium (CE) is an allocation* $\{ C_t, x_t, u_t, v_t, K_{t+1}, H_{t+1}, B_{t+1}^p, L_t^{T,F}, L_t^{T,I}, K_t^T, L_t^{N,F}, L_t^{N,I}, K_t^T, K_t^N \}_{t=0}^{\infty}$ *and a price system* $\left\{ R_t, w_t^F, w_t^I, r_t, P^N_t \right\}_{t=0}^{\infty}$, *such that the following conditions are satisfied:*

*CE.1. Given* $\pi$ *and* $\left\{ P_t, R_t, w_t^F, w_t^I, r_t \right\}_{t=0}^{\infty}$, *the allocation* $\{ C_t(\pi), x_t(\pi), u_t(\pi), v_t(\pi), e_t(\pi) K_{t+1}(\pi), H_{t+1}(\pi), B_{t+1}^p(\pi) \}_{t=0}^{\infty}$ *solves the representative household's problem.*

*CE.2. Given* $\pi$ *and* $\left\{ P_t, R_t, w_t^F, w_t^I, r_t \right\}_{t=0}^{\infty}$, $\{ L_t^{T,F}(\pi), L_t^{T,I}(\pi), K_t^T(\pi), L_t^{N,F}(\pi), L_t^{N,I}(\pi), K_t^T, K_t^N(\pi) \}_{t=0}^{\infty}$ *solves the firms' static problems in the tradable and non-tradable sector, respectively.*

*CE.3. Fiscal policy* $\pi = \left\{ \tau_t^c, \tau_t^w, \tau_t^k, G_t, B_{t+1}^g \right\}_{t=0}^{\infty}$ *is feasible.*

*CE.4. There is consistency of the domestic interest rate; that is, for all t*

$$R_t = R\left(B_t/Y^T\right).$$

Notice that, as we couple the government's and the domestic agent's budget constraints, we obtain the last equilibrium condition

$$\left(B_t^p + B_t^g\right)\ (1 + R_t) + P_t\ I_t = F^T(K_t^T, L_t^T)\ + \left(B_{t+1}^p + B_{t+1}^g\right)$$

### 4.2 Balanced Growth Competitive Equilibrium: Quantitative Implications

We are particularly interested in studying the balanced growth path in this setting that displays some novel features. First, the growth rate is endogenously determined by the fact that the interest rate depends on a measure of relative indebtedness. This friction will be critical for closing the model for the developing economies we study. Second, the degree of informality in the economy is determined endogenously and it depends, among other things, on the design of the fiscal policy.

In what follows, we analyze the quantitative implications of alternative tax structures for three Latin American countries, Brazil, Chile and Mexico. We are particularly interested in studying the BGP.

#### 4.2.1 Only Formal Sector

The first exercise consists of removing the informal sector of the economy ($\beta = 1$) to see how $\tau_K$ and $\tau_w$ affect the BGP. Figure 3 shows how capital and labor taxes affect the economy's growth rate in this scenario.

As Figure 3 illustrates, in the three countries, increasing the labor tax rates reduces the growth rate along the BGP. Without capital and labor taxes the growth rate is around 3.7 percent in Chile, 4.3 percent in Mexico and 5.4 percent in Brazil. Increasing the labor tax rate by 20 percent reduces growth around half a percentage point in the three countries. This is expected, since distortionary tax rates should slow down the economy. The effect of increasing the capital tax rate on growth goes in the same direction as increasing labor taxes, but the magnitude is much lower, almost imperceptible in the figure. When looking at private savings the picture is the opposite.

Figure 4 shows the impact of capital and labor taxes on what we called private savings as a percentage of total income. Private savings are defined as total income, tradable and non-tradable, minus non-tradable consumption as a fraction of total income. That is, in this

**Figure 3. Growth Rate along the BGP, Formal Sector Only (in percent)**
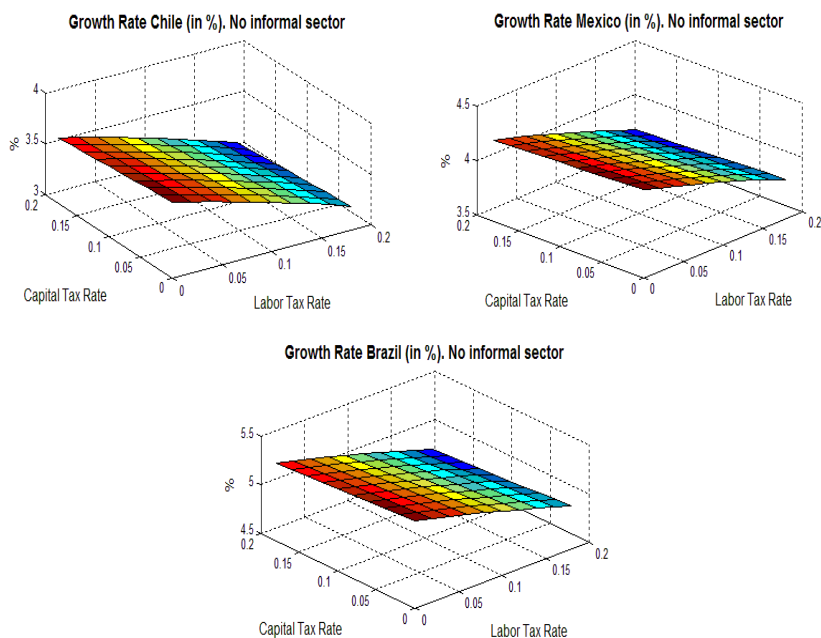


*Source:* Authors' estimations.

definition private savings include investment and bond savings. As is clear from the figure, while increasing capital taxes reduces private savings in the three countries, an increase in the labor tax rate does not seems to affect savings along the BGP. Increasing the capital tax rate to around 20 percent reduces private savings from around 35 to 33 percent in Chile, from 29 to 26 percent in Mexico and from 48 to 45 percent in Brazil. Consumption taxes play a similar role as labor taxes. Increasing the consumption tax rate induces a reduction in growth along the BGP and does not affect private savings in the three countries, as can be seen in Figures 5 and 6.

Figures 7 and 8 show the impact of changes in capital and labor taxes on the dis-aggregated components of private savings (investment and bond savings). As can be seen in Figure 7, increasing the capital tax rate induces a fall in investment measured as a per-centage of total income in the three countries. This effect is larger for Chile and Brazil, where an increase of capital tax rate from zero to 0.18 reduces investment by around 16 percent along the BGP. Labor taxes do not seem to affect investment in any of the three countries. Figure 8 shows that, when there is no informal sector, there is no bond savings in the three countries, and a rise in the capital tax rate induces an increase in debt along the BGP. Again, Chile and Brazil are the countries where the impact on debt of increasing

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**Figure 4. Private Savings along the BGP, Formal Sector Only (in % of Total Income)**



Private Savings Chile (% of Total Income). No informal sector

Private Savings Mexico (% of Total Income). No informal sector

Private Savings Brazil (% of Total Income). No informal sector
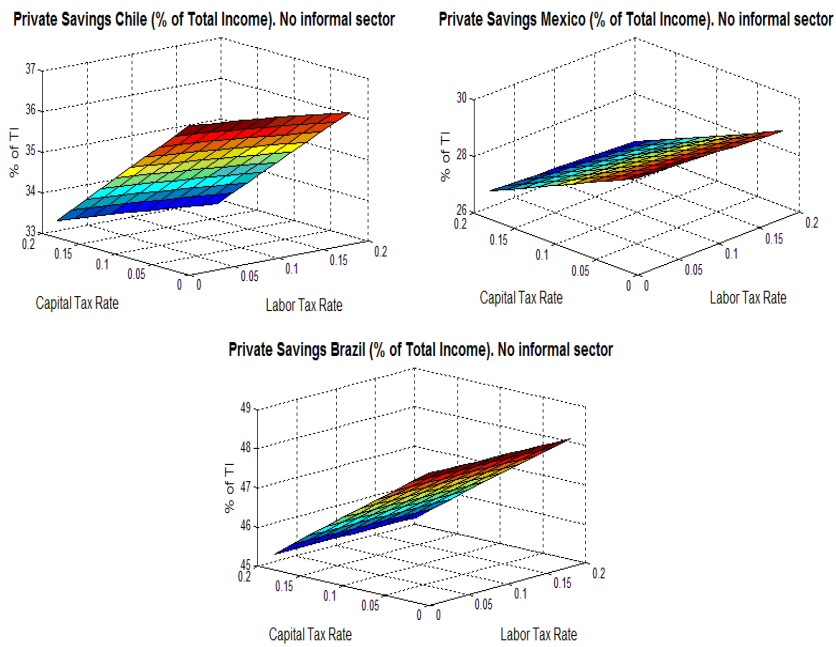
*Source:* Authors' estimations.

**Figure 5. Growth Rate along the BGP, Formal Sector Only (in percent)**
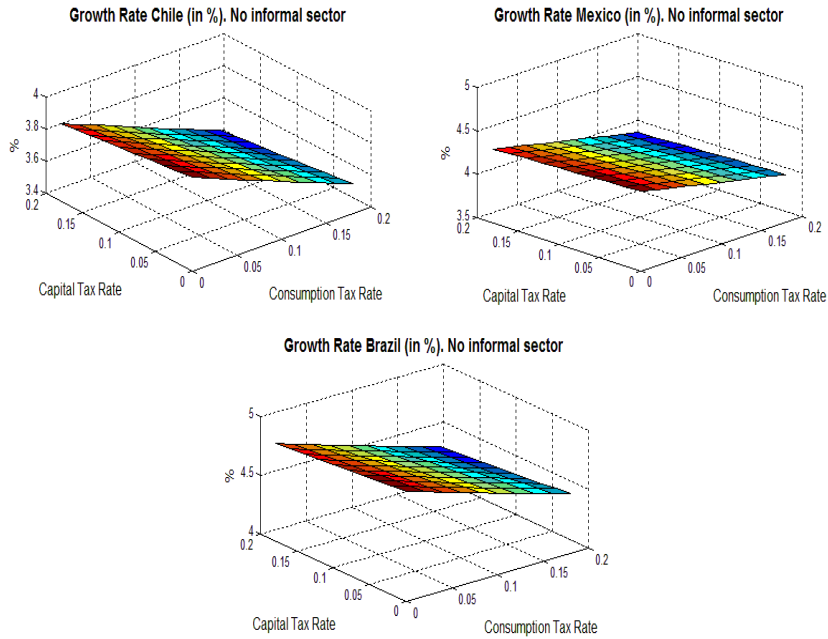


*Source:* Authors' estimations.

capital taxes is larger. In both countries, increasing the capital tax rate from zero to 0.18 induces an increment in debt along the BGP of approximately 20 percent. The impact of increasing consumption taxes on investment and bond savings is similar to the impact of increasing labor taxes and so they are not shown here.

Figure 9 shows the impact of an increase in government expenditure (defined as a percentage of the non-tradable output) over growth along the BGP. As can be seen in the figure, as government increases its expenditures there is an increase in growth in the three countries analyzed here. The growth rate when there is no capital tax and the government expenditure is zero is around 2.6 percent in Chile, 3.7 percent in Mexico and 3 percent in Brazil. Increasing government expenditures by 20 percent induces an increase in growth of around 19 percent in Chile, 11 percent in Mexico and 17 percent in Brazil.

Increasing government expenditures also induces an increase in private savings along the BGP in the three countries as illustrated in Figure 10.

Figure 11 shows the effect of increasing capital and labor taxes on the time devoted to work in the formal sector of the economy. As labor taxes increase, the time devoted to work decreases, while an increase in the capital tax rate does not seem to affect the time devoted to work, in the three countries.

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**Figure 6. Private Savings along the BGP, Formal Sector Only (in % of Total Income)**



Private Savings Chile (% of Total Income). No informal sector

Private Savings Mexico (% of Total Income). No informal sector

Private Savings Brazil (% of Total Income). No informal sector
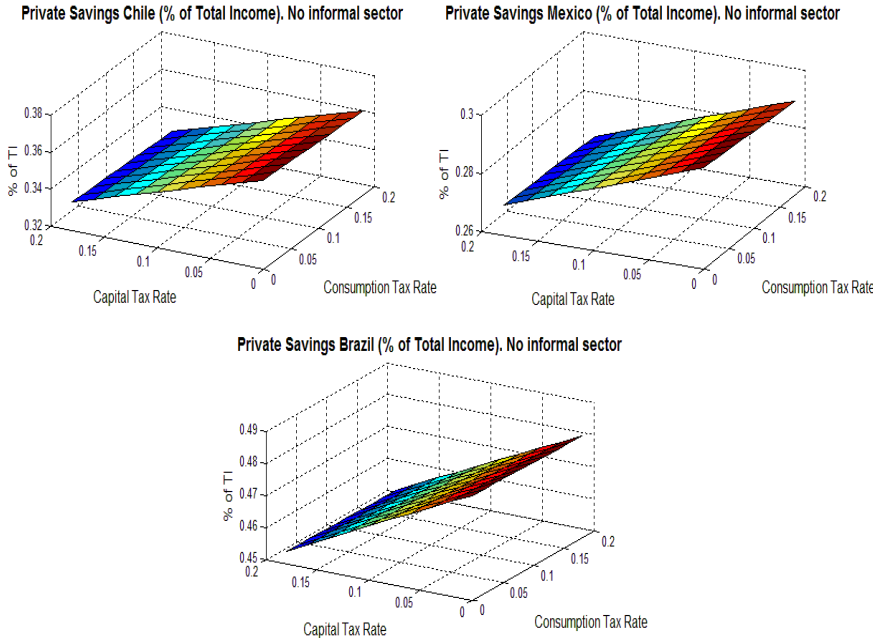
*Source:* Authors' estimations.

**Figure 7. Investment along the BGP, Formal Sector Only (in % of Total Income)**
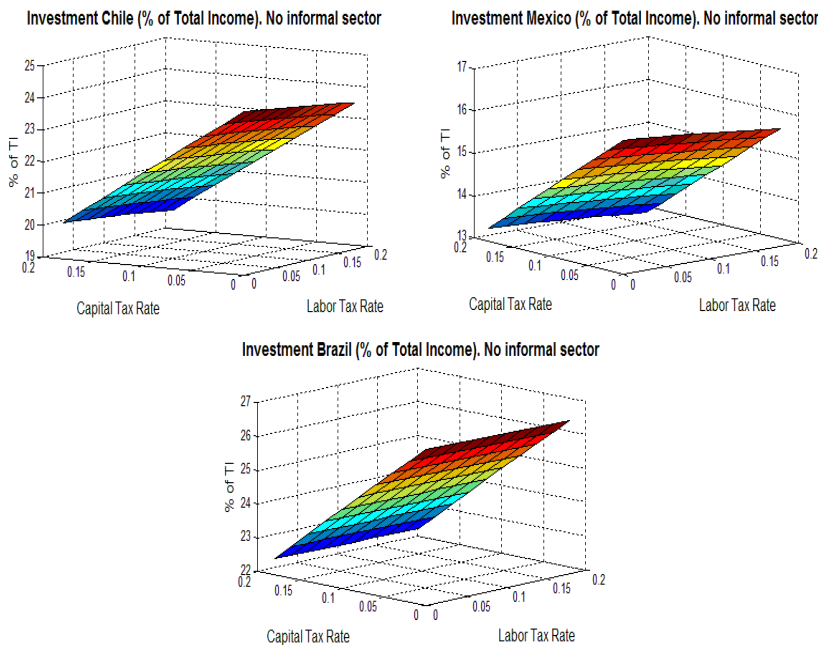
Overall, the intuition behind these results is that increasing labor taxes leads to a decrease in the time devoted to human capital accumulation. The reduction in the time devoted to human capital accumulation induces a reduction in the growth rate along the BGP. A similar effect can be obtained when increasing consumption taxes. The same mechanism operates with the capital tax rate, although much more smoothly. Increasing capital taxes induces a mild reduction in the time devoted to accumulate human capital, and this latter reduction implies a slight fall (almost imperceptible in the figures) in the long-run growth rate. In the case of labor and consumption taxes, increases in both tax rates reduce the time devoted to work in the formal sector of the economy, implying a non-trivial increase in the time devoted to non-market activities, i.e., leisure. It is important to interpret leisure in a broad sense, as modeled, and thus it must include non-market production goods. In other words, an agent who devotes less time accumulating human capital is not necessarily at home doing nothing; rather, he or she could be engaged in non-market activities (i.e., producing goods). Increasing capital taxes also has a positive effect on time devoted to leisure. Since increasing labor taxes increase the time allocated to non-market activities, reducing the time devoted to accumulating human capital and the total time allocated to work there is an intra-temporal substitution between work and leisure and private consumption does not

**Figure 8. Bond Savings along the BGP, Formal Sector Only (in % of Total Income)**



Bond Savings Chile (% of Total Income). No informal sector

Bond Savings Mexico (% of Total Income). No informal sector

Bond Savings Brazil (% of Total Income). No informal sector
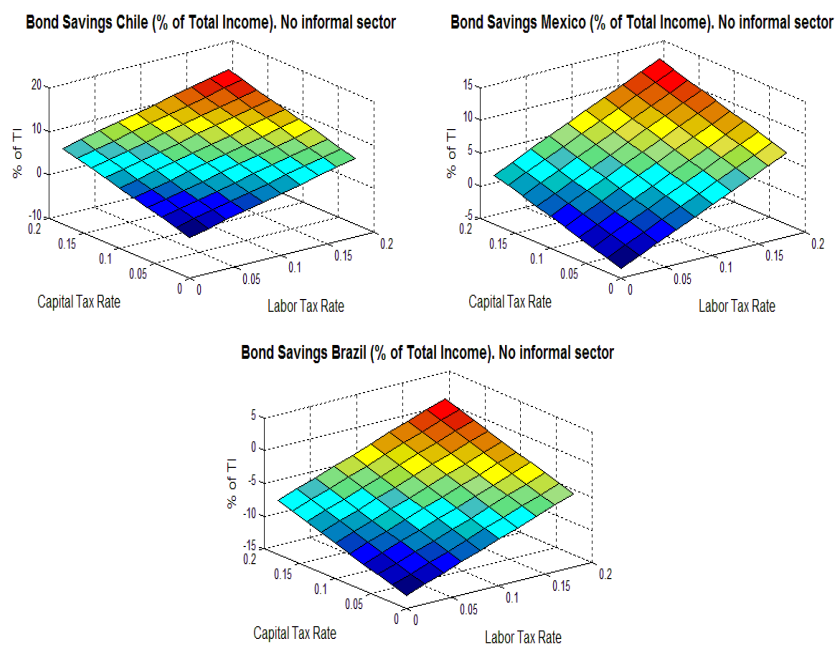
*Source:* Authors' estimations.

**Figure 9. Growth Rate along the BGP, Formal Sector Only (in percent)**
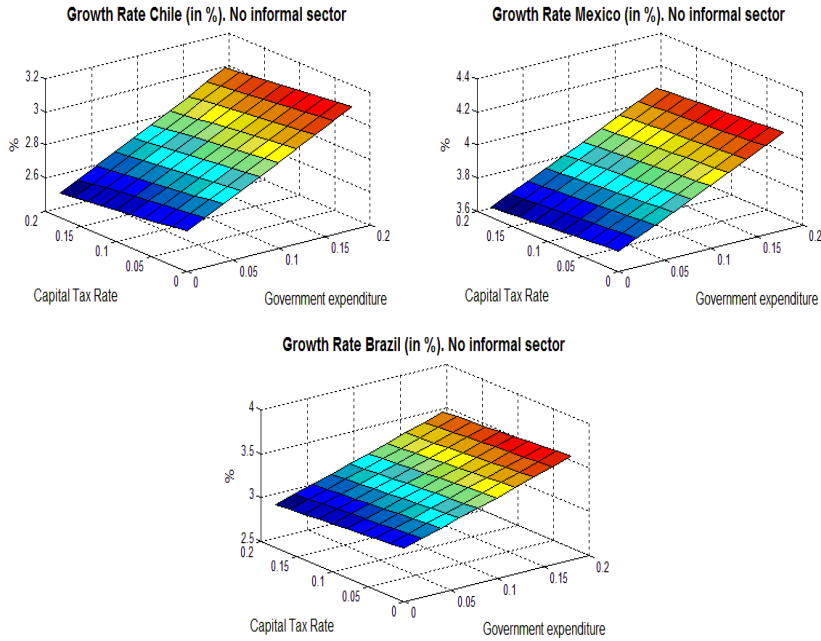
change or changes slightly. At the same time, the increase in domestic capital compensates the fall in units of labor and the product does not change or changes slightly. Both effects, no impact on consumption and no impact on production, imply that there is no change in private savings.

### 4.2.2 *Formal and Informal Sector*

The second exercise introduces the informal sector into the economy. Here, we extend our analysis to study the impact of capital and labor taxes as well as the impact of capital and consumption taxes.

Figure 12 shows the impact of changes in the capital and labor tax rates on the growth rate along the BGP in an economy with both formal and informal sectors. As the figure shows, an increase in the labor tax rate has a negative effect on the growth rate, while increases in the capital tax rate does not seem to affect growth. Overall, the average growth rate in an economy with an informal sector is lower than the average growth rate without one. A comparison of Figures 3 and 12 suggests that the introduction of an informal sector into the economy diminishes growth; in particular, even if there is no change in the tax rates,

**Figure 10. Private Savings along the BGP, Formal Sector Only (in % of Total Income)**



Private Savings Chile (% of Total Income). No informal sector

Private Savings Mexico (% of Total Income). No informal sector

Private Savings Brazil (% of Total Income). No informal sector
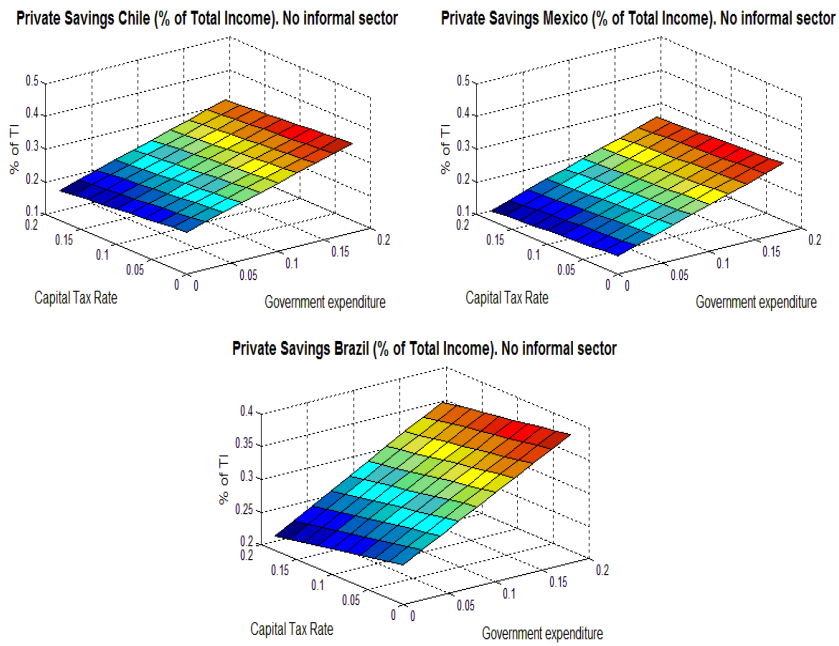
*Source:* Authors' estimations.

**Figure 11. Total Time Devoted to Work, Formal Sector Only (in percent)**



Total time devoted to work (in %) Chile

Total time devoted to work (in %) Mexico

Total time devoted to work (in %) Brazil
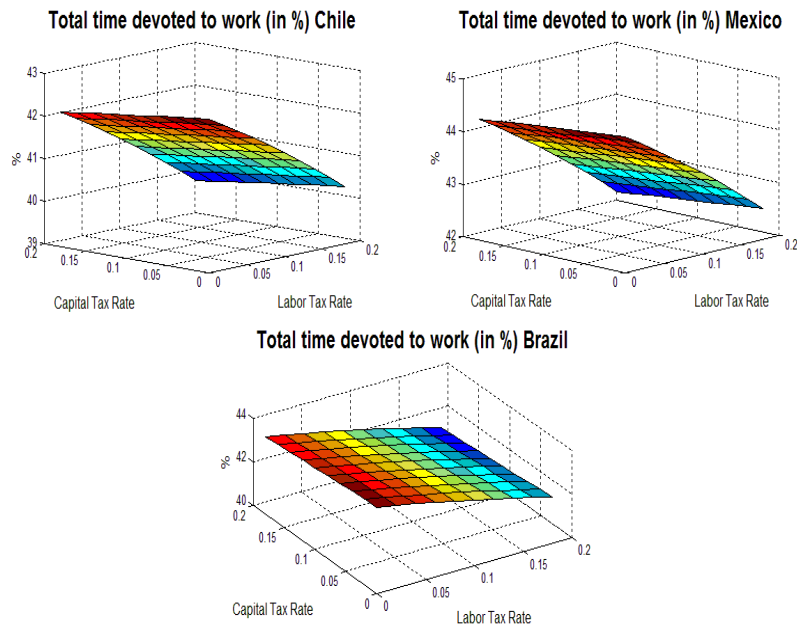
*Source:* Authors' estimations.

**Figure 12. Growth Rate along the BGP, Formal and Informal Sectors**



Growth Rate Chile (in %)

Growth Rate Mexico (in %)

Growth Rate Brazil (in %)
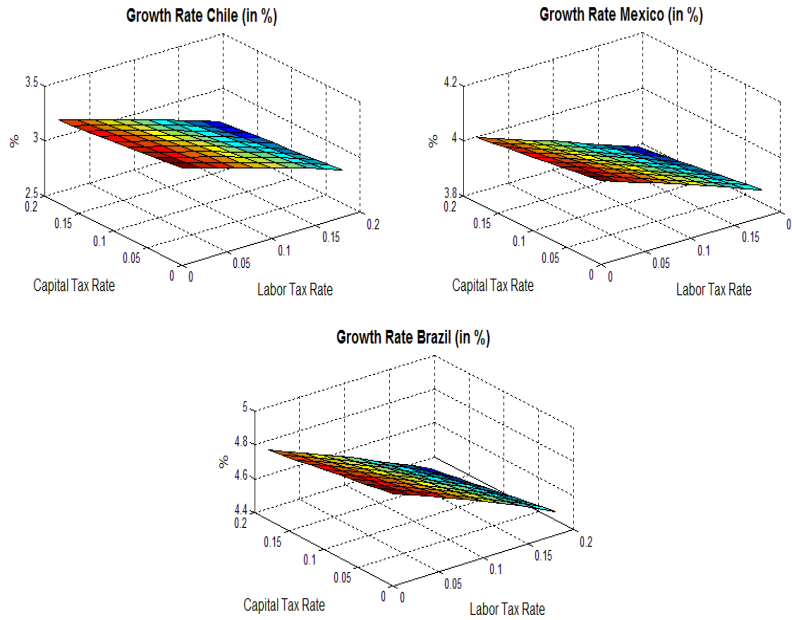
*Source:*  Authors' estimations.

the growth rate decreases from 3.7 to 3.4 percent in Chile, from 4.3 to 4.1 percent in Mexico and from 5.4 to 4.9 percent in Brazil.

Figure 13 presents the effects of changes in the capital and labor tax rates on private savings along the BGP when the economy has an informal sector. The figure shows that when the capital tax rate increases private savings fall along the BGP. When comparing with 4 one notices that introducing the informal sector seems not to affect private savings.

Figures 14 and 15 show the impact of capital and labor taxes on investment and bond savings. The introduction of an informal sector into the economy does not affect investment, but it does have a large effect on bond savings. With only a formal sector and neither capital nor labor taxes there is no bond savings in the three countries, while the introduction of informality induces positive bond savings in Chile and Mexico and a reduction in debt in Brazil. As can be observed in Figure 14, an increase in the capital tax rate reduces investment along the BGP in the three countries. The magnitude of this reduction is very similar to the case with no informal sector. Figure 15 shows the impact of capital and labor taxes on bond savings. It is clear from the figure that increasing the capital and labor tax rates induces a rise in bond savings along the BGP in the three countries analyzed here. In particular, raising capital taxes from zero to 0.18 induces an increase in bond savings from 6 to 14 percent in

**Figure 13. Private Savings along the BGP, Formal and Informal Sectors**



Private Savings Chile (% of Total Income)

Private Savings Mexico (% of Total Income)

Private Savings Brazil (% of Total Income)
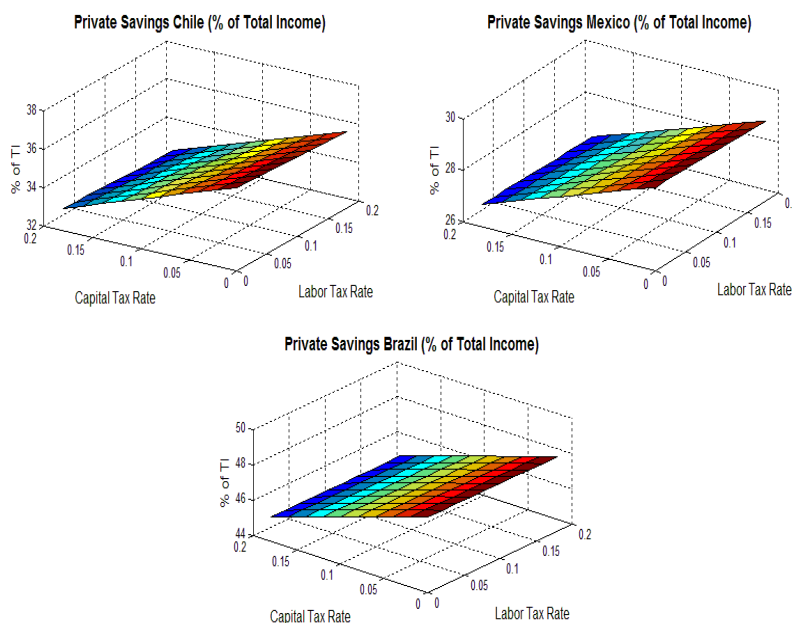
*Source:* Authors' estimations.

Chile, from 2 to 8 percent in Mexico and from -4 to almost 3 percent in Brazil. Increasing consumption taxes has the same effect on investment and bond savings as increasing labor taxes.[9]

Figure 16 shows the impacts of capital and consumption taxes on the growth rate along the BGP. Consumption taxes negatively affect the long-run growth rate, due to additional distortions. When both taxes are zero, the growth rate is 3.5 percent in Chile, 4.2 percent in Mexico and 4.5 percent in Brazil. If the consumption tax rate is increased to 20 percent, the growth rate is only 3.1 percent in Chile, 3.9 percent in Mexico and 4.1 percent in Brazil. Again, this is expected, since distortionary taxes tend to slow down the economy.

The next two figures show the impact of increasing government expenditures on the growth rate and the impact of private savings on the BGP. Figures 17 and 18 suggest that increasing government expenditures induces an increase in both the growth rate and private savings in alll three countries.

Figures 19, 20 and 21 illustrate for the three countries that, when the labor tax rate increases, time devoted to working in the formal sector decreases, while time devoted to working in the informal sector increases, which are the expected results of this model.

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[9]Figures for the impact of consumption taxes are available from the authors upon request.

**Figure 14. Investment along the BGP, Formal and Informal Sectors**



Investment Chile (% of Total Income)

Investment Mexico (% of Total Income)

Investment Brazil (% of Total Income)
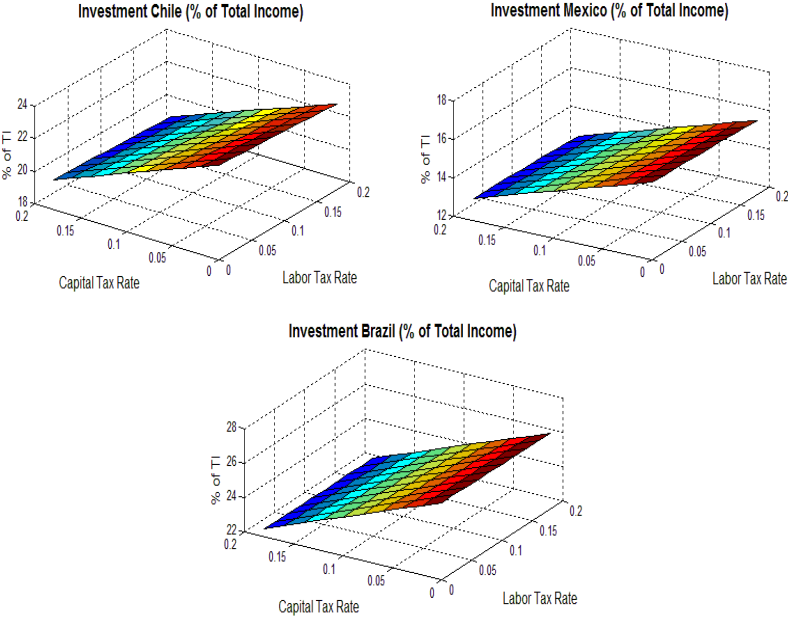
*Source:*   Authors' estimations.

**Figure 15. BondSavings along the BGP, Formal and Informal Sectors**



Bond Savings Chile (% of Total Income)

Bond Savings Mexico (% of Total Income)

Bond Savings Brazil (% of Total Income)
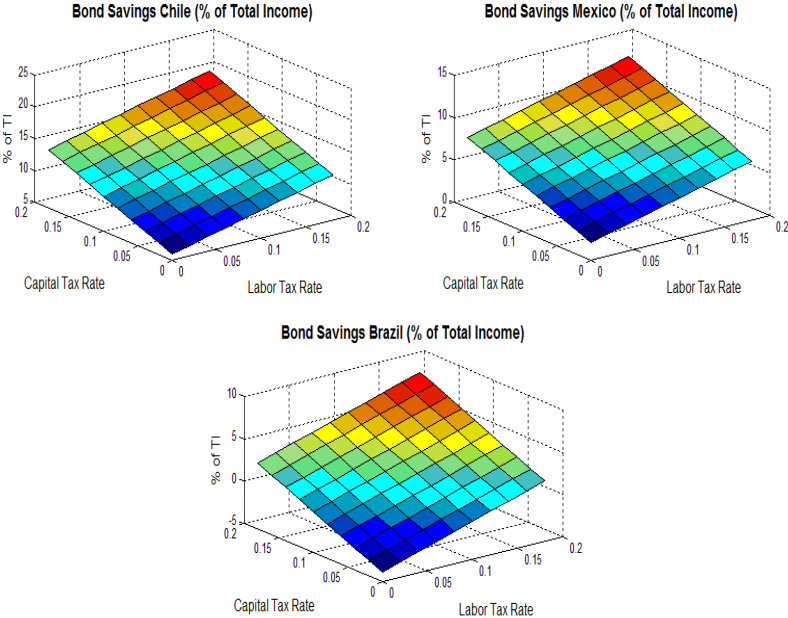
*Source:*   Authors' estimations.

**Figure 16. Impact of Capital and Consumption Taxes on the Growth Rate along the BGP (in percent)**
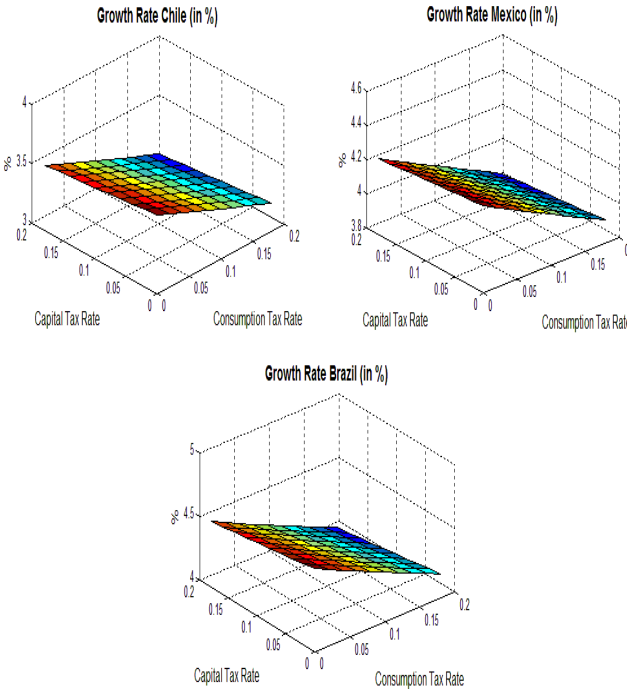


*Source:* Authors' estimations.

**Figure 17. Growth Rate along the BGP (in percent)**



Growth Rate Chile (in %)

Growth Rate Mexico (in %)

Growth Rate Brazil (in %)
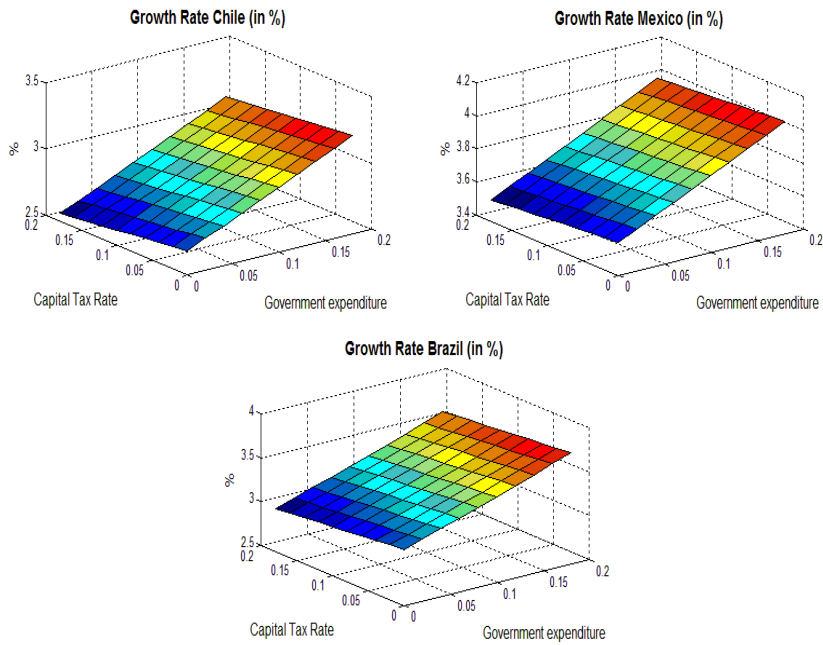
*Source:*   Authors' estimations.

**Figure 18. Private Savings along the BGP (in % of Total Income)**



Private Savings Chile (% of Total Income)

Private Savings Mexico (% of Total Income)

Private Savings Brazil (% of Total Income)
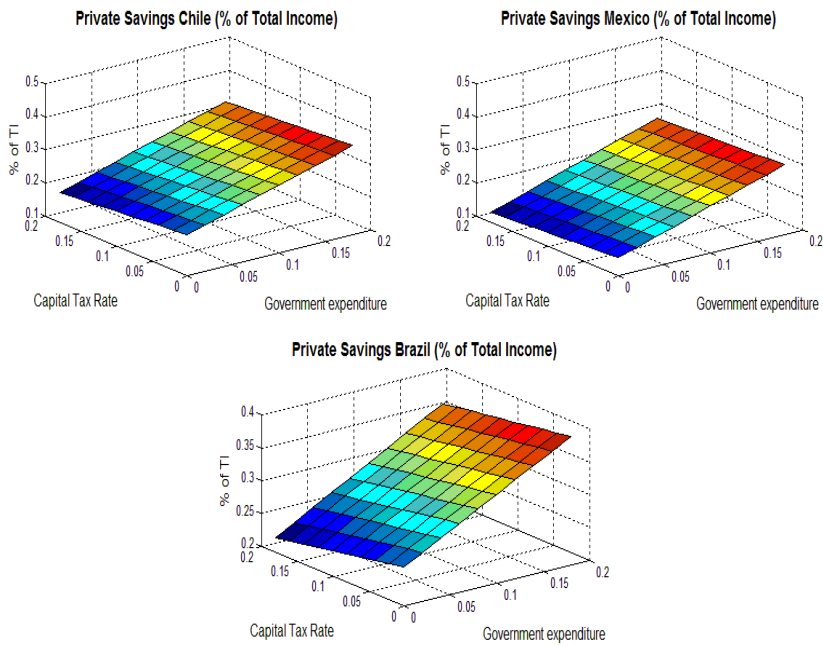
*Source:*   Authors' estimations.

**Figure 19. Brazil: Time Devoted to Work, Formal and Informal Sectors (in percent)**



Total time devoted to work (in %)

Time devoted to work in the Formal sector (in %)

Time devoted to work in the Informal sector (in %)
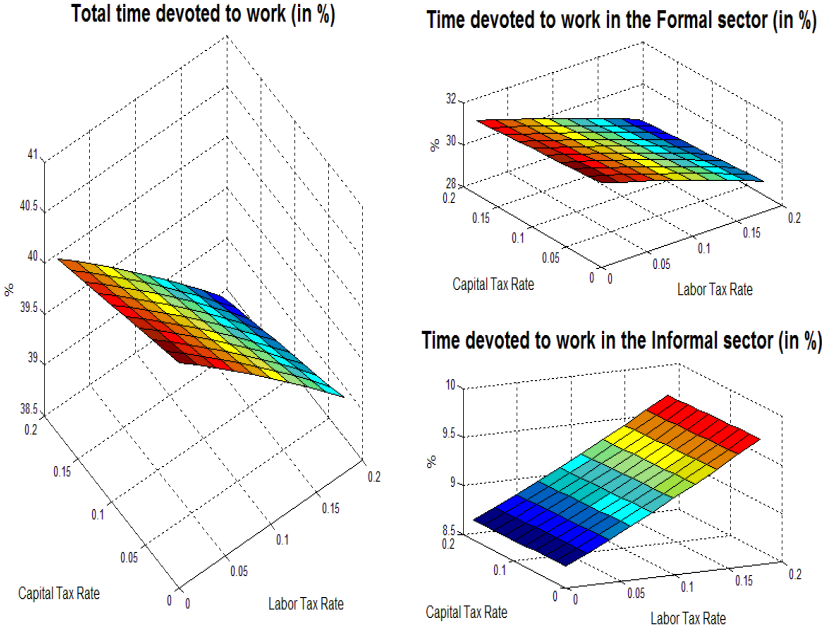
*Source:* Authors' estimations.

However, the figure also suggests that the total time devoted to work decreases with this tax rate, implying that the disincentive to work in the formal sector is, on average, greater than the incentive to work in the informal sector. Increasing the capital tax rate does not have an impact on the time devoted to work in both sectors. As before, the effect of the consumption tax rate is very similar to the labor tax rate, that is, increasing the consumption tax rate reduces the time devoted to working in the formal sector and increases the time devoted to work in the informal sector of the economy. The introduction of informality reduces the total time devoted to work compared with the economy without an informal sector.

**Figure 20. Chile: Time Devoted to Work, Formal and Informal Sectors (in percent)**



Total time devoted to work (in %)

Time devoted to work in the Formal sector (in %)

Time devoted to work in the Informal sector (in %)
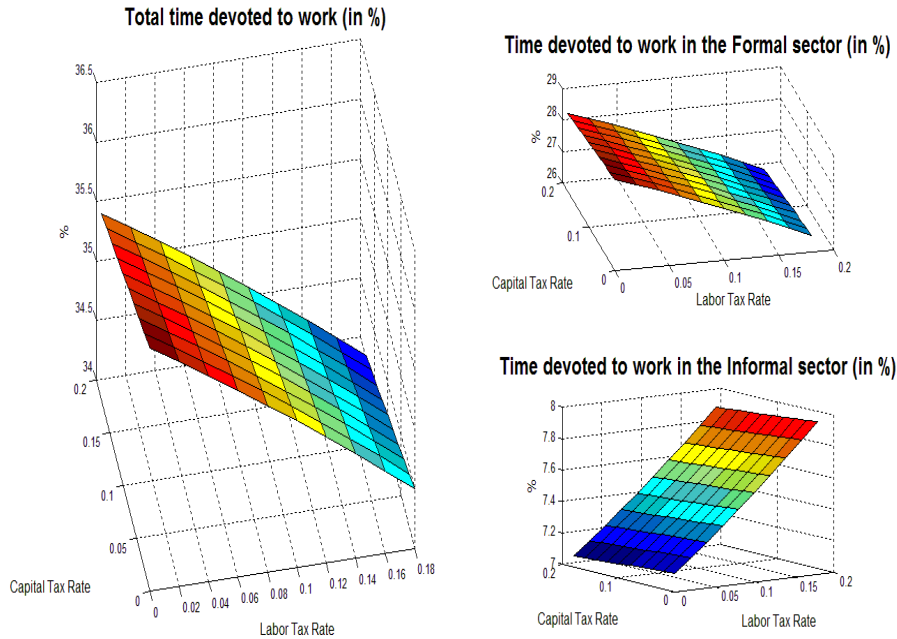
*Source:* Authors' estimations.

Overall, the evidence found in this section suggests that the introduction of an informal sector into the economy and increasing labor and consumption taxes have a negative impact on the long-run growth rate. This last effect is expected, since distortionary taxes should slow down the economy. Increasing capital taxes reduces private savings in the three countries analyzed here. Additionally, and again as expected, an increase in labor taxes reduces the time devoted to work in the formal sector and increases the time devoted to work in the informal sector. However, the reduction in the formal sector is greater than the increase in the informal sector, resulting in a decline in total time allocated to work.

An increase in capital taxes seems not affect the time devoted to work both in the formal and informal sectors. A rise in consumption tax rates reduces the time devoted to work in both sectors, formal and informal. Therefore, increasing labor or consumption taxes increases the time devoted to leisure and diminishes the time devoted to human capital accumulation, and this mechanism induces a fall in the growth rate along the BGP.

In the next section, we study the behavior of this type of economy along the BGP when the government sets optimal tax rates.

**Figure 21. Mexico: Time Devoted to Work, Formal and Informal Sectors (in percent)**
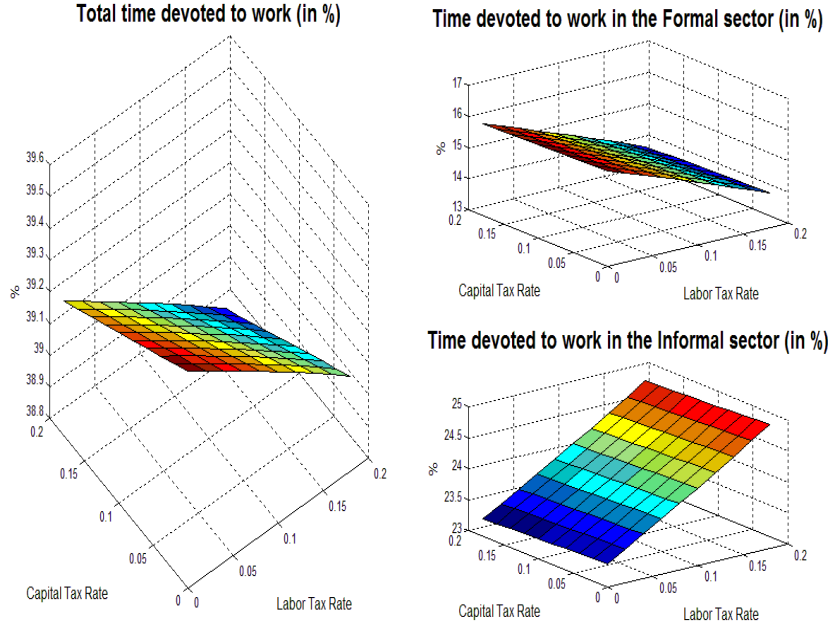
## 5 Optimal Fiscal Policies

This section describes the *Ramsey approach to optimal taxation* in regard to the problem faced by a benevolent government that chooses optimal taxes and transfers given that only distortionary tax instruments are available. The government sets taxes that it has available so that, within the set of competitive equilibria, the utility of the representative agent is maximized. In other words, the government choose the optimal tax scheme that ensures financing an exogenous path of public expenditures and, at the same time, maximizes the representative agent's utility. More precisely, the formal definition is the following.

**Definition 2.** *Given a fiscal policy* $\pi = \left\{ \tau_t^c, \tau_t^w, \tau_t^k, G_t, B_{t+1}^g \right\}_{t=0}^{\infty}$, *let*
$\{ C(\pi), x(\pi), u(\pi), v(\pi), K(\pi), H(\pi), B^p(\pi), L^{T,F}(\pi), L^{T,I}(\pi), K^T(\pi), L^{N,F}(\pi), L^{N,I}(\pi), K^T(\pi), K^N(\pi) \}$
*be the corresponding competitive equilibrium allocation. The* optimal fiscal policy $\pi^*$ *is defined as the solution to*

$$\max_{\pi} \sum_{t=0}^{\infty} \rho^t \frac{\left( C_t(\pi) \ (x_t(\pi))^{\theta} \right)^{1-\sigma}}{1-\sigma},$$

Solving this problem directly might be a nontrivial task. There are two common approaches to solving Ramsey problems. The first is the primal approach, which characterizes

a set of allocations that can be implemented as a competitive equilibrium with taxes. By implementation we mean the following: for a set of taxes, find a set of (consumption and labor) allocations and equilibrium prices such that these allocations are a competitive equilibrium given taxes. Conversely, a set of (consumption and labor) allocations is implementable if it is possible to find taxes and equilibrium prices such that these allocations are a competitive equilibrium given these prices and taxes. Implementation often makes it possible to simplify a Ramsey problem by reformulating a problem of finding optimal taxes as the problem of finding implementable allocations. This reformulation of the problem is referred to as a primal approach to Ramsey taxation, and its application in this setting is discussed in detail in the Technical Appendix.

## 5.1 *Optimal Fiscal Policies: Quantitative Implications*

In this section we compute the optimal tax structure and compare its implications on growth and labor market participation with the CE allocation stemming from the actual tax systems. In addition, we perform a counterfactual exercise in which we quantify the implications of imposing the tax in one of the fast-growing Asian economies (Thailand in our examples). Tables 1, 2 and 3 report the results for Brazil, Chile and Mexico, respectively. The first row in each table shows the result of the competitive equilibrium when the economy has a formal and informal sector. The second row in each table reports the optimal tax structure computed from the Ramsey problem, and the last row reports a counterfactual exercise imposing on the economy the actual tax structure of Thailand and computing the competitive equilibrium. As mentioned, Appendix A details the calibration.

Consider first the case of the Brazilian economy. Table 1 shows that optimality dictates that labor and capital should be taxed substantially less than observed while consumption should be heavily taxed instead. The impact on growth is large, an increase of more than 30 percent, while there is a non-trivial reallocation of labor from the informal sector to the formal sector. On the other hand, optimal taxation reduces the savings rate around 4 percent with respect to the competitive equilibrium. If the actual tax system were replaced with the tax structure of a fast-growing East Asian economy the growth rate along the BGP would increase by almost 20 percent, while the savings rate should also increase by 3 percent.

Table 2 shows the results for Chile. Actual taxes imply a growth rate along the BGP of 3 percent and private savings of around 33 percent of total income. The second row of the table shows the optimal taxes. As can be seen, the government should tax consumption less and tax capital and labor substantially less to obtain an increase of 30 percent in the

## Table 1. Brazil: quantitative implications

|  | $\tau^c$ | $\tau^w$ | $\tau^k$ | $\gamma$ | $u$ | $v$ | Savings |
|---|---|---|---|---|---|---|---|
| *Competitive equilibrium* | 17.00 | 30.00 | 15.00 | 4.00 | 26.98 | 10.62 | 45.79 |
| *Ramsey allocation* | 22.27 | 16.00 | 4.09 | 5.28 | 34.07 | 7.71 | 43.95 |
| *Comp. equil. (Thailand)* | 16.60 | 0.62 | 0.71 | 4.77 | 31.02 | 9.06 | 47.13 |

*Source:* Authors elaboration.

growth rate and 6 percent in private savings along the BGP. Optimal taxes also suggest a reallocation of labor from the informal to the formal sector of the economy. Tax structure in Thailand is similar to the optimal case, where capital, consumption and labor taxes are lower than actual taxes in Chile. This situation produces higher growth and private savings along the BGP than the competitive equilibrium.

## Table 2. Chile: quantitative implications

|  | $\tau^c$ | $\tau^w$ | $\tau^k$ | $\gamma$ | $u$ | $v$ | Savings |
|---|---|---|---|---|---|---|---|
| *Competitive equilibrium* | 19.00 | 7.00 | 18.50 | 3.00 | 27.63 | 7.37 | 32.97 |
| *Ramsey allocation* | 17.36 | 1.78 | 0.24 | 3.90 | 31.43 | 6.50 | 34.98 |
| *Comp. equil. (Thailand)* | 16.60 | 0.62 | 0.71 | 3.24 | 28.16 | 7.41 | 35.05 |

*Source:* Authors elaboration.

Finally, Table 3 shows the results for Mexico. In this case optimality suggests taxing consumption more heavily and taxing capital substantially less to obtain a slightly higher growth rate. The table also shows that optimal tax structure does not affect competitive equilibrium private savings. In contrast with the other two countries, in Mexico labor in the informal sector is larger than in the formal one. Optimal taxes imply a little reallocation of labor from the informal to the formal sector. Thailand taxes labor substantially less than the actual and optimal situations and taxes consumption more than the actual situation but less than the optimal case. This counterfactual situation produces values for the growth rate, private savings and allocation of labor in the formal and informal sector very similar to the competitive equilibrium situation.

## Table 3. Mexico: quantitative implications

|  | $\tau^c$ | $\tau^w$ | $\tau^k$ | $\gamma$ | $u$ | $v$ | Savings |
|---|---|---|---|---|---|---|---|
| *Competitive equilibrium* | 13.90 | 7.90 | 8.50 | 4.00 | 15.18 | 24.02 | 28.08 |
| *Ramsey allocation* | 21.93 | 7.10 | 0.28 | 4.16 | 16.20 | 23.40 | 28.10 |
| *Comp. equil. (Thailand)* | 16.60 | 0.62 | 0.71 | 3.96 | 15.29 | 23.74 | 28.24 |

*Source:* Authors elaboration.

Overall, the evidence found in this section suggests that sub-optimal taxes imply lower long-run growth in the three countries analyzed here. However, it seems that the low levels of saving rates are not a direct consequence of sub-optimal taxation, as optimal taxes imply a case, Brazil, where private savings are lower than the actual situation; a case, Chile, where optimality induces larger private savings and a case, Mexico, where the optimal tax structure does not affect private savings. In the three countries, optimal taxes reallocate labor from the informal to the formal sector.

## 6 Policy Recommendations

The findings in the last sections provide some policy implications for Brazil, Chile and Mexico. First, we measured how much the actual tax structure should be modified to reach the optimal tax scheme and second, we estimated the impact of these changes along the BGP on the long-run growth rate, private savings and the allocation of time between the formal and informal labor market.

In the case of Brazil, implementing the optimal tax regime would imply a significant change in the actual tax rates. Labor taxes should be reduced almost 47 percent and consumption taxes increased more than 70 percent, while the capital tax rate should decrease more than 30 percent. This last finding is important because, unlike conclusions in previous studies, capital should be taxed in the long run and, as a matter of fact, at a relatively high rate. The optimal tax system translates into a significant reallocation of time between formal and informal work. Time devoted to work in the informal sector declines around 26 percent while time devoted to work in the formal sector increases by around the same magnitude when implementing the optimal tax structure. Notice that in the case of Brazil, optimality will imply a higher growth rate but lower private savings along the BGP.

Comparing optimal tax rates to those observed in the Chilean economy suggest, similar to the Brazilian case, that the tax rate changes needed to decentralize the Ramsey allocations are significant. In this case, capital should be taxed minimally as the optimal tax rate on capital is almost zero ,and there labor should be taxed substantially less as the optimal tax rate on labor is only 1.78 percent compared to the actual 7.00 percent. Implementing the optimal tax regime will induce an increase in both the growth rate and private savings along the BGP. Optimal taxation will additionally reallocate labor from the informal to the formal sector as in the case of Brazil. However, these changes are not as large as in Brazil.

Optimality in Mexico suggests that, in comparison to those actual taxes observed, consumption should be more heavily taxed and capital should be taxed substantially less.

Implementing the optimal tax regime would imply a 4 percent higher long-run growth rate but no significant changes in private savings along the BGP. Reallocation of labor between the formal and informal sector is of small magnitude. Some of these results are in line with the fiscal reform proposed by Antón, Hernández and Levy (2012) in order to mitigate the harmful effects of informality on the labor market. These authors' reforms would shift taxation to cover social insurance from labor to consumption, eliminating labor taxes and setting a uniform value added tax rate of 16 percent. The quantitative exercise presented here suggests shifting taxation towards consumption while lowering capital taxes.

## 7   Conclusions

This paper has made progress in characterizing competitive equilibrium and optimal fiscal policies in the context of a small open economy with the following characteristics: the interest rate is endogenously determined and some workers can be hired in the informal market. We have addressed two questions in this setting. The first is Ramsey's (1927) normative question: What choice of tax rates will maximize consumer utility, consistent with given government consumption and with market determination of quantities and prices? The second is positive and quantitative: How much difference does it make?

Our quantitative exercises show that, from a baseline economy, the inclusion of an informal sector reduces the growth rate over the BGP but does not affect private savings in the three countries analyzed here. Increasing labor and consumption taxes also reduces the long-run growth rate. Increasing capital taxes reduces private savings in Brazil, Chile and Mexico. Additionally, and as expected, an increase in labor taxes reduces the time devoted to work in the formal sector and increases the time devoted to work in the informal sector. However, the reduction in the formal sector is greater than the increase in the informal sector, resulting in a decline in total time allocated to work.

Optimal taxes (stemming from the Ramsey allocation) suggest that the tax rate changes needed to implement this optimal tax structure are significant. Sub-optimal taxes imply lower long-run growth in the three countries. However, it seems that the low levels of saving rates actually observed are not a direct consequence of sub-optimal taxation as optimal taxes imply a case, Brazil, where private savings are lower than the actual situation; a case, Chile, where optimality induces larger private savings and a case, Mexico, where the optimal tax structure does not affect private savings. Finally, in the three countries, optimal taxes reallocate labor from the informal to the formal sector.

# 8    Appendix A: Calibration

Before presenting the parameters values, we describe the ones used. The relative productivity of tradable to non-tradable sector was obtained from Soto and Valdés (1998) for Chile, and from Urrutia and Meza (2010) for Mexico. $A_T$ was chosen so that $\frac{A_N}{A_T} = \frac{1}{P_N} \frac{Y^T/Y}{Y^N/Y} \frac{L^{N,F}/L}{L^{T,F}/L} \frac{1-\alpha_T}{1-\alpha_N}$ for Brazil ($P_N = \frac{p_N}{p_T}$ was obtained from Carrera and Restout (2008), as an average of the real exchange rate from 1970-2000). The value of $A_N$ was just a normalization for all countries. We chose $A_H$ to match an annual growth rate of 3 percent for Chile, 4 percent for Mexico and 4 percent for Brazil when we included an informal sector. $\alpha_T$ and $\alpha_N$ were obtained from Urrutia and Meza (2010) for Mexico. For Brazil and Chile, they were calculated with data from national accounts. To estimate $d$, we considered the following relationship: $d = \frac{w^F}{w^I}(1 - \tau^w)$, where the ratio of formal to informal wages were obtained from Frankema (2010) for Brazil and Mexico, and from Sánchez and Álvarez (2010) for Chile. The value of $\beta$ satisfies: $\beta \left[ (1 - \alpha_T)\frac{Y^T}{Y} + (1 - \alpha_N)\frac{Y^N}{Y} \right] = \frac{w^F L^F}{Y}$, where $w^F L^F$ represents formal labor's share of income. For $g$, we used the average of the government spending to non-tradable GDP ratio, from 1960 to 2000 for Chile, 1961 to 2012 for Mexico, and 1960 to 2012 for Brazil. As for $\delta_K$, we calculated it using the gross and net capital stock series presented by Pérez Toledo (2003) in the case of Chile, and considered estimates by Loría and de Jesús (2006) for Mexico, and Morandi and Reis (2004) for Brazil. The value of $P$ was normalized to one.

The variables $\phi$ and $\bar{b}$ belong to the particular specification that was used of the function $R(.)$, taking the form:

$$ R\left(\frac{B^p + B^g}{Y^T}\right) = R^* + \phi \left[ e^{\left\{ \bar{b} + \frac{B^p + B^g}{Y^T} \right\}} - 1 \right] $$

where $R^*$ is the international interest rate. For $\bar{b}$, we used the average of the Net International Investment Position to tradable GDP ratio, from 1997 to 2008 in the case of Chile, 1998 to 2008 in the case of Mexico, and 2005 to 2012 for Brazil. For $R^*$, we used the average one year treasury bill rate, as a measure of the international interest rate that the economies faced on an annual basis. We chose $\rho$ to validate the statement $\rho(1 + R^*) = 1$, which is standard in literature on small open economies. We took the value of $\sigma$ from Arrau (1990) and chose the value of $\eta$ to be 1. We follow Lucas (1990) to make $\theta = 0.5$. Therefore, the values of $\sigma$ and $\theta$ calibrated jointly determine the corresponding Frish elasticities of labor supply in the range of 1.31 for Mexico to 1.56 for Chile, which are in line with the roughly 1.3 value estimated by Keane (2009) in the classic life-cycle model augmented to include human capital accumulation.

Taxes on consumption, capital and labor were obtained from Antón (2005) for Mexico; from Gandullia, Iacobone and Thomas (2012) for Brazil; and from OECD data for Chile.

The following table summarizes the calibration done for Chile, Mexico and Brazil:

**Table 4.  Calibrated Parameters**

|  | Chile | Mexico | Brazil |
|---|---|---|---|
| $A_T$ | 0.3 | 0.516 | 0.36 |
| $A_N$ | 1 | 1 | 1 |
| $A_H$ | 0.158 | 0.16 | 0.18 |
| $\alpha_T$ | 0.6 | 0.48 | 0.6 |
| $\alpha_N$ | 0.5 | 0.35 | 0.45 |
| $\beta$ | 0.85 | 0.512 | 0.83 |
| $\delta_K$ | 0.02 | 0.012 | 0.04 |
| $\delta_H$ | 0 | 0 | 0 |
| $g$ | 0.17 | 0.157 | 0.295 |
| $\rho$ | 0.96 | 0.96 | 0.96 |
| $\sigma$ | 1.6 | 1.6 | 1.6 |
| $\eta$ | 1 | 1 | 1 |
| $R^*$ | 0.04 | 0.04 | 0.04 |
| $\phi$ | 1 | 1 | 1 |
| $\bar{b}$ | 0.33 | -0.0116 | 0.025 |
| $P$ | 1 | 1 | 1 |
| $\tau^c$ | 0.19 | 0.139 | 0.17 |
| $\tau^w$ | 0.07 | 0.079 | 0.3 |
| $\tau^k$ | 0.185 | 0.085 | 0.15 |
| $d$ | 1.17 | 1.26 | 1.044 |
| $\theta$ | 0.5 | 0.5 | 0.5 |

*Source:*  Authors elaboration.

# 9 Appendix B: Technical Details

This section provides all the technical details omitted in the text.

## 9.1 Competitive Equilibrium: Characterization

The following conditions (5)-(18) are necessary and sufficient to characterize the representative agent's problem. Let $(\rho^t \lambda_t, \rho^t \mu_t)$ be the Lagrange multiplier corresponding to (1) and (2), respectively, normalized to date 0.

$$(C_t) : \rho^t \left[ C_t \ x_t^\theta \right]^{-\sigma} x_t^\theta = \rho^t \ U_C(C_t, x_t) = (1 + \tau_t^c) \rho^t \lambda_t P_t^N \tag{5}$$

$$(x_t) : \rho^t \left[ C_t x_t^\theta \right]^{-\sigma} C_t \ \theta \ x_t^{\theta-1} = \rho^t \ U_x(C_t, x_t) = \rho^t \mu_t \ A^H \ H_t \tag{6}$$

$$(u_t) : \rho^t \left[ C_t x_t^\theta \right]^{-\sigma} C_t \ \theta \ x_t^{\theta-1} = \rho^t \ U_x(C_t, x_t) = \frac{1}{d} \rho^t \lambda_t \ (1 - \tau_t^w) \ H_t \ w_t^F, \tag{7}$$

$$(v_t) : \rho^t \left[ C_t x_t^\theta \right]^{-\sigma} C_t \ \theta \ x_t^{\theta-1} = \rho^t \ U_x(C_t, x_t) = \rho^t \lambda_t \ H_t \ w_t^I, \tag{8}$$

$$(K_{t+1}) : \rho^t \lambda_t P_t = \rho^{t+1} \lambda_{t+1} P_{t+1} \left( (1 - \delta_K) + \left( (1 - \tau_t^k) r_{t+1} + \tau_t^k \delta_K \right) \frac{1}{P_{t+1}} \right) \tag{9}$$

$$(H_{t+1}) : \rho^t \mu_t = \rho^{t+1} \mu_{t+1} \ \left( A^H \ e_{t+1} + (1 - \delta_H) \right) + \rho^{t+1} \lambda_{t+1} \ (d \ u_{t+1} + v_{t+1}) \ w_{t+1}^I \tag{10}$$

$$\left( B_{t+1}^p \right) : \rho^t \lambda_t = \rho^{t+1} \lambda_{t+1} \ (1 + R_{t+1}) \tag{11}$$

$$(1 + \tau_t^C) P_t^N C_t + P_t I_t + (1 + R_t) B_t^p$$
$$= \ \left( (1 - \tau_t^w) \ w_t^F \ u_t \ H_t + w_t^I \ v_t \ H_t \right) + \left( (1 - \tau_t^k) r_t + \tau_t^k \delta \right) \ K_t + B_{t+1}^p \tag{12}$$

$$H_{t+1} = A^H \ e_t \ H_t + (1 - \delta_H) \ H_t, \tag{13}$$

$$K_{t+1} = I_t + (1 - \delta_K) \ K_t \tag{14}$$

$$x_t = (1 - d u_t - v_t - e_t) \tag{15}$$

$$\left( TCB_{t+1}^p \right) : \lim_{T \to \infty} \rho^T \lambda_T B_{T+1}^p = 0, \tag{16}$$

$$(TCK_{t+1}) : \lim_{T \to \infty} \rho^T \lambda_T K_{T+1} = 0, \tag{17}$$

$$(TCH_{t+1}) : \lim_{T \to \infty} \mu_T H_{T+1} = 0 \tag{18}$$

Additionally, the following conditions (19)-(22) characterize the solutions to firms' problem in the tradable and non-tradable sectors, respectively:

$$r_t = \frac{\partial F^T(K_t^T, L_t^T)}{\partial K_t^T} \tag{19}$$

$$w_t^F = \frac{\partial F^T(K_t^T, L_t^T)}{\partial L_t^T} \frac{\partial L_t^T}{\partial L_t^{T,F}}$$

$$w_t^I = \frac{\partial F^T(K_t^T, L_t^T)}{\partial L_t^T} \frac{\partial L_t^T}{\partial L_t^{T,I}}$$

$$r_t = P_t^N \frac{\partial F^N(K_t^N, L_t^N)}{\partial K_t^N} \tag{20}$$

$$w_t^F = P_t^N \frac{\partial F^N(K_t^N, L_t^N)}{\partial L_t^N} \frac{\partial L_t^N}{\partial L_t^{N,F}} \tag{21}$$

$$w_t^I = P_t^N \frac{\partial F^N(K_t^N, L_t^N)}{\partial L_t^N} \frac{\partial L_t^N}{\partial L_t^{N,I}} \tag{22}$$

where, for $j = T, N$

$$F^j(K_t^j, L_t^j) = A_j \left(K_t^j\right)^{\alpha_j} \left(L_t^j\right)^{(1-\alpha_j)},$$

$$L_t^j = \left(\beta \left(L_t^{j,F}\right)^{\frac{\eta-1}{\eta}} + (1-\beta) \left(L_t^{j,I}\right)^{\frac{\eta-1}{\eta}}\right)^{\frac{\eta}{\eta-1}}.$$

Market clearing conditions impose

$$C_t + G_t = F^N(K_t^N, L_t^N) \tag{23}$$

$$K_t = K_t^T + K_t^N \tag{24}$$

$$u_t\, H_t = L_t^{T,F} + L_t^{N,F} \tag{25}$$

$$v_t\, H_t = L_t^{T,I} + L_t^{N,I} \tag{26}$$

for all $t$.

Notice that conditions (7) and (8) imply that

$$(1 - \tau_t^w)\, w_t^F = w_t^I\, d,$$

Finally, as we couple the government's and the domestic agent's budget constraints,[10] we obtain the last equilibrium condition

$$(B_t^p + B_t^g) \ (1 + R_t) + P_t \ I_t = F^T(K_t^T, L_t^T) \ + \left(B_{t+1}^p + B_{t+1}^g\right)$$

### *9.2  Competitive Equilibrium: Balanced Growth Analysis*

For any variable $Z$, define $z = \frac{Z}{H}$ (i.e., lower case letters denote the corresponding variables in terms of $H$.). Also, let $\tilde{\lambda} = \frac{\lambda}{\left[C \ x^\theta\right]^{-\sigma} x^\theta}$ and $\tilde{\mu} = \frac{\mu}{\left[C \ x^\theta\right]^{-\sigma} x^\theta}$. The following system of equations characterizes the balanced growth path (BGP) of the economy,

$$1 = (1 + \tau^c) \ \tilde{\lambda} \ P^N \tag{27}$$

$$c \ \theta \ x^{-1} = \tilde{\mu} \ A^H \tag{28}$$

$$\tilde{\mu} \ A^H = \frac{1}{d} \ \tilde{\lambda} \ (1 - \tau^w) \ w^F \ , \tag{29}$$

$$(1 - \tau^w) \ w^F = w^I \ d, \tag{30}$$

$$\left(1 + R\left(\frac{b^p + b^g}{k}\right)\right) = \left((1 - \delta_K) + \left((1 - \tau^k) \ r \ + \tau^k \ \delta_K\right) \frac{1}{P}\right) \tag{31}$$

$$\frac{\tilde{\mu}}{\tilde{\lambda}} \left(1 + R\left(\frac{b^p + b^g}{A_T \ (k^T)^{\alpha_T} \ (l^T)^{(1-\alpha_T)}}\right)\right) = \frac{\tilde{\mu}}{\tilde{\lambda}} \ \gamma + (d \ u + v) \ w_I \tag{32}$$

$$\gamma^\sigma = \rho \left(1 + R\left(\frac{b^p + b^g}{A_T \ (k^T)^{\alpha_T} \ (l^T)^{(1-\alpha_T)}}\right)\right) \tag{33}$$

---

[10]When we couple the constraints, we obtain

$$\begin{aligned} & P_t^N C_t + P_t I_t + P^N G_t + (1 + R_t)(B_t^p + B_t^g) \\ = \ & w_t^F u_t H_t + w_t^I v_t H_t + r_t K_t + (B_{t+1}^p + B_{t+1}^g) \end{aligned}$$

Since both technologies display constant returns of scale

$$w_t^F u_t H_t + w_t^I v_t H_t + r_t K_t = P_t^N Y_t^N + Y_t^T$$

With the market-clearing condition for the non-traded goods, we can derive the following expression

$$\begin{aligned} & P_t I_t + (1 + R_t)(B_t^p + B_t^g) \\ = \ & Y_t^T + (B_{t+1}^p + B_{t+1}^g) \end{aligned}$$

$$
\begin{aligned}
(1 + \tau_C) P^N \; c + P \; i \\
= \; (1 - \tau_W) \; w^F \; u + w^I \; v + ((1 - \tau_K) \; r \; + \tau_K \; \delta_K) k - \left( 1 + R \left( \frac{b^p + b^g}{A_T \; (k^T)^{\alpha_T} \; (l^T)^{(1-\alpha_T)}} \right) - \gamma \right) b^p
\end{aligned}
\tag{34}
$$

$$
\gamma = A^H \; e + (1 - \delta_H),
\tag{35}
$$

$$
i = (\gamma + \delta_K - 1) \; k,
\tag{36}
$$

$$
x = (1 - du - v - e)
\tag{37}
$$

$$
r = A_T \; \alpha_T \; \left( k^T \right)^{\alpha_T - 1} \left( l^T \right)^{(1-\alpha_T)}
\tag{38}
$$

$$
r = P^N A_N \; \alpha_N \; \left( k^N \right)^{\alpha_N - 1} \left( l^N \right)^{(1-\alpha_N)}
$$

$$
w^F = A_T \; (1 - \alpha_T) \; \left( k^T \right)^{\alpha_T} \left( l^T \right)^{-\alpha_T} \; \beta \; \left( \beta + (1 - \beta) \; \left( \frac{l^{T,I}}{l^{T,F}} \right)^{\frac{\eta-1}{\eta}} \right)^{\frac{1}{\eta-1}}
\tag{39}
$$

$$
w^F = P^N A_N \; (1 - \alpha_N) \; \left( k^N \right)^{\alpha_N} \left( l^N \right)^{-\alpha_N} \; \beta \; \left( \beta + (1 - \beta) \; \left( \frac{l^{N,I}}{l^{N,F}} \right)^{\frac{\eta-1}{\eta}} \right)^{\frac{1}{\eta-1}}
$$

$$
w^I = A_T \; (1 - \alpha_T) \; \left( k^T \right)^{\alpha_T} \left( l^T \right)^{-\alpha_T} \; (1 - \beta) \; \left( (1 - \beta) + \beta \; \left( \frac{l^{T,F}}{l^{T,I}} \right)^{\frac{\eta-1}{\eta}} \right)^{\frac{1}{\eta-1}}
\tag{40}
$$

$$
w^I = P^N A_N \; (1 - \alpha_N) \; \left( k^N \right)^{\alpha_N} \left( l^N \right)^{-\alpha_N} \; (1 - \beta) \; \left( (1 - \beta) + \beta \; \left( \frac{l^{N,F}}{l^{N,I}} \right)^{\frac{\eta-1}{\eta}} \right)^{\frac{1}{\eta-1}}
$$

$$
l^T = \left( \beta \; \left( l^{T,F} \right)^{\frac{\eta-1}{\eta}} + (1 - \beta) \; \left( l^{T,I} \right)^{\frac{\eta-1}{\eta}} \right)^{\frac{\eta}{\eta-1}}.
\tag{41}
$$

$$
l^N = \left( \beta \; \left( l^{N,F} \right)^{\frac{\eta-1}{\eta}} + (1 - \beta) \; \left( l^{N,I} \right)^{\frac{\eta-1}{\eta}} \right)^{\frac{\eta}{\eta-1}}.
\tag{42}
$$

$$
c = (1 - g) \; A_N \; \left( k^N \right)^{\alpha_N} \left( l^N \right)^{(1-\alpha_N)}
\tag{43}
$$

$$
k = k^T + k^N
\tag{44}
$$

$$
u = l^{T,F} + l^{N,F}
\tag{45}
$$

$$
v = l^{T,I} + l^{N,I}
\tag{46}
$$

$$
(b^p + b^g) \; \left( 1 + R \left( \frac{b^p + b^g}{A_T \; (k^T)^{\alpha_T} \; (l^T)^{(1-\alpha_T)}} \right) \right) + P \; i = A_T \; \left( k^T \right)^{\alpha_T} \left( l^T \right)^{(1-\alpha_T)} \; + (b^p + b^g) \; \gamma
\tag{47}
$$

This (nonlinear) system of 24 equations has the following 24 unknowns:

$$\left(c, P^N, \tilde{\lambda}, \tilde{\mu}, (w^F, w^I, r), \gamma, (x, u, v, e), (b^p, b^g), (k, i), (k^T, k^N, l^T, l^N, l^{T,I}, l^{T,F}, l^{N,I}, l^{N,F})\right).$$

## 9.3 Optimal Fiscal Policy and the Ramsey Problem

Our framework builds on the primal approach to optimal taxation. (See, for example, Atkinson and Stiglitz, 1980, Lucas and Stokey, 1983, and Chari, Christiano and Kehoe, 1991.) This approach characterizes the set of allocations that can be implemented as a competitive equilibrium with distorting taxes by two sets of conditions: resource constraints and implementability constraints. The implementability constraint is the consumer budget constraint in which the consumer's and the firm's first order conditions are used to substitute out for prices and policies. Thus both constraints depend only on allocations. This characterization implies that optimal allocations are solutions to a programming problem. We refer to this optimal tax problem as the Ramsey problem and to the solutions and the associated policies as the Ramsey allocations and the Ramsey plan, respectively.

We need to emphasize that we focus on economies in which the government effectively has access to a commitment technology. As is well known, without such a technology there are time inconsistency problems, so the equilibrium outcomes with commitment are not necessarily sustainable without commitment.[11] We interpret our analysis as the natural starting point to quantify the upper bound on the effects of inefficient tax systems.

## 9.4 The Ramsey Problem

We proceed as in Espino and González-Rozada (2013) (see also Ljungqvist and Sargent, 2004, Section 15.15) to apply the *primal approach*. First we multiply the budget constraint (12) by $\rho^t \lambda_t$ and add them up to date $T$ to get

$$
\sum_{t=0}^{T} (1 + \tau_t^c) \rho^t \lambda_t P_t^N \, C_t + \sum_{t=0}^{T} \rho^t \lambda_t P_t I_t - \sum_{t=0}^{T} \left( (1 - \tau_t^k) r_t + \tau_t^k \delta_K \right) \, \rho^t \lambda_t \, K_t \tag{48}
$$
$$
= \sum_{t=0}^{T} \rho^t \lambda_t \left( (1 - \tau_t^w) \, w_t^F \, u_t + w_t^I \, v_t \right) \, H_t - \sum_{t=0}^{T} (1 + R_t) \, \rho^t \lambda_t B_t^p + \sum_{t=0}^{T} \rho^t \lambda_t B_{t+1}^p
$$

---

[11]Economies with commitment technologies can be interpreted in two ways. One is that the government can simply commit to its future actions by, say, restrictions in its constitution. The other is that the government has no access to a commitment technology, but the commitment outcomes are sustained by reputational mechanisms.

Notice that conditions (5) (9) and (11) and the fact that (10) implies that

$$\rho^{t+1}\lambda_{t+1}\left[(1-\tau_{t+1}^w)\,w_{t+1}^F\,u_{t+1}+v_{t+1}w_{t+1}^I\right]\,H_{t+1} \tag{49}$$

$$=\;\rho^{t+1}\lambda_{t+1}(du_{t+1}+v_{t+1})w_{t+1}^I\,H_{t+1} \tag{50}$$

$$=\;\rho^t\mu_t\,H_{t+1}-\rho^{t+1}\mu_{t+1}\left(A^H\,e_{t+1}+(1-\delta_H)\right)\,H_{t+1}$$

$$=\;\rho^t\mu_t\,H_{t+1}-\rho^{t+1}\mu_{t+1}\,H_{t+2}.$$

make it possible to write condition (48) as follows

$$\sum_{t=0}^{T}\rho^t U_C(C_t,x_t)\,C_t+\rho^T\lambda_T P_T K_{T+1}-\left((1-\tau_K)r_0+\tau_K\delta_K\right)\,\lambda_0\,K_0 \tag{51}$$

$$=\;\mu_0\,H_1-\rho^T\mu_T\,H_{T+1}.-(1+R_0)\,\lambda_0 B_0^p+\rho^T\lambda_T B_{T+1}^p$$

where $U_C(C_t,x_t)=\left(C_t\,x_t^\theta\right)^{-\sigma}x_t^\theta$ for all $t$.

Hence, as one lets $T\to\infty$, the tranversality conditions (16)-(18) make (52) reduce to

$$\sum_{t=0}^{\infty}\rho^t U_C(C_t,x_t)\,C_t \tag{52}$$

$$=\;U_C(C_0,x_0)\,C_0\,\theta\,x_0^{-1}\,\frac{1}{A^H}(A^H\,e_0+(1-\delta_H))+\left(\frac{(1-\tau_0^K)r_0+\tau_K\delta_K}{(1+\tau_0^C)P_0^N}\right)\,K_0\,U_C(C_0,x_0)$$

$$\equiv\;W_0$$

since we assumed that $B_0^p=0$ as a normalization.

Let $\phi$ be the Lagrange multiplier corresponding to the incentive constraint (52) and define

$$V(C_t,x_t;\phi)\;\equiv\;U(C_t,x_t)+\phi\,U_C(C_t,x_t)\,C_t$$

$$=\;(1+(1-\sigma)\,\phi)\,\frac{\left(C_t\,(x_t)^\theta\right)^{1-\sigma}}{1-\sigma}.$$

From conditions (8), (6), (10) and (2), we derive an additional constraint

$$\left[A^H e_t+(1-\delta_H)\right]\,U_x(C_t,x_t)=\rho\,U_x(C_{t+1},x_{t+1})\,\left[(1-\delta_H)+A^H(1-x_{t+1})\right]$$

The Ramsey problem for this economy is

$$\max_{(C_t,x_t,u_t,v_t,H_{t+1},B_{t+1},L_t^{T,F},L_t^{T,I},K_t^T,L_t^{N,F},L_t^{N,I},K_t^N)} \sum_{t=0}^{\infty} \rho^t \ V(C_t, x_t; \phi) \ - \ \phi \ W_0,$$

subject to

$$\left[A^H(1 - du_t - v_t - x_t) + (1 - \delta_H)\right] \ U_x(C_t, x_t) \tag{53}$$
$$= \ \rho \ U_x(C_{t+1}, x_{t+1}) \ \left[(1 - \delta_H) + A^H(1 - x_{t+1})\right]$$

$$H_{t+1} = A^H \left(1 - du_t - v_t - x_t\right) H_t + (1 - \delta_H)H_t \tag{54}$$

$$C_t = (1 - g) \ A_N \ \left(K_t^N\right)^{\alpha_N} \left(L_t^N\right)^{(1-\alpha_N)}) \tag{55}$$

where

$$L_t^N = \left(\beta \ \left(L_t^{N,F}\right)^{\frac{\eta-1}{\eta}} + (1 - \beta) \ \left(L_t^{N,I}\right)^{\frac{\eta-1}{\eta}}\right)^{\frac{\eta}{\eta-1}}.$$

$$B_t \left(1 + R_t\right) + P_t \ I_t = A_T \ \left(K_t^T\right)^{\alpha_T} \left(L_t^T\right)^{(1-\alpha_T)} + \left(B_{t+1}^p + B_{t+1}^g\right), \tag{56}$$

where

$$L_t^T = \left(\beta \ \left(L_t^{T,F}\right)^{\frac{\eta-1}{\eta}} + (1 - \beta) \ \left(L_t^{T,I}\right)^{\frac{\eta-1}{\eta}}\right)^{\frac{\eta}{\eta-1}}.$$

$$K_{t+1}^T + K_{t+1}^N = I + (1 - \delta_K)(K_t^T + K_t^N), \tag{57}$$

$$u_t \ H_t = L_t^{T,F} + L_t^{N,F} \tag{58}$$

$$v_t \ H_t = L_t^{T,I} + L_t^{N,I} \tag{59}$$

Notice that (56) and (57) reduces to

$$B_t \left(1 + R_t\right) + P_t \ \left[K_{t+1}^T + K_{t+1}^N - (1 - \delta_K)(K_t^T + K_t^N)\right] = A_T \ \left(K_t^T\right)^{\alpha_T} \left(L_t^T\right)^{(1-\alpha_T)} + \left(B_{t+1}^p + B_{t+1}^g\right), \tag{60}$$

We denote the corresponding (date $t$) Lagrange multipliers by $\rho^t \chi_t^n$ for $n = 1, ..., 6$. The necessary first order conditions that characterize a Ramsey allocation are given by

$$C_t \ : \ \rho^t \chi_t^3 = \rho^t \ V_C(C_t, x_t; \phi)$$
$$+ \ \rho^t U_{xC}(C_t, x_t) \left[\chi_t^1 \left(A^H e_t + (1 - \delta_H)\right) - \chi_{t-1}^1 \left(A^H(1 - x_t) + (1 - \delta_H)\right)\right] \tag{61}$$

$$x_t \quad : \quad \rho^t V_x(C_t, x_t; \phi) = \rho^t \chi_t^2 A^H H_t$$
$$+ \quad \rho^t \, U_{xx}(C_t, x_t) \left[ \chi_{t-1}^1 \, \left( A^H(1-x_t) + (1-\delta_H) \right) - \chi_t^1 \, \left( A^H e_t + (1-\delta_H) \right) \right]$$
$$+ \quad \rho^t \, U_x(C_t, x_t) \, A_H \left( \chi_t^1 - \chi_{t-1}^1 \right) \tag{62}$$

$$u_t : \rho^t \, \chi_t^5 \, H_t = \rho^t \, \chi_t^2 \, A^H \, H_t \, d + \rho^t \, U_x(C_t, x_t) \, dA^H \, \chi_t^1,$$

$$v_t : \rho^t \, \chi_t^6 \, H_t = \rho^t \, \chi_t^2 \, A^H \, H_t \, + \rho^t \, U_x(C_t, x_t) \, A^H \chi_t^1,$$

$$K_{t+1}^T : \rho^t \, \chi_t^4 \, P_t = \rho^{t+1} \, \chi_{t+1}^4 \, \left( A_T \, \alpha_T \left( K_{t+1}^T \right)^{\alpha_T - 1} \left( L_{t+1}^T \right)^{(1-\alpha_T)} + P_{t+1} \, (1-\delta_K) \right),$$

$$K_{t+1}^N \quad : \quad \rho^t \, \chi_t^4 \, P_t = \rho^{t+1} \, \chi_{t+1}^3 (1-g) \, A_N \, \alpha_N \left( K_{t+1}^N \right)^{\alpha_N - 1} \left( L_{t+1}^N \right)^{(1-\alpha_N)}$$
$$+ \quad \rho^{t+1} \, \chi_{t+1}^4 \, P_{t+1} \, (1-\delta_K), \tag{63}$$

$$H_{t+1} \quad : \quad \rho^t \, \chi_t^2 = \rho^{t+1} \, \chi_{t+1}^2 \left( A^H \, (1 - du_{t+1} - v_{t+1} - x_{t+1}) + (1-\delta_H) \right)$$
$$+ \quad \rho^{t+1} \left( \chi_{t+1}^5 u_{t+1} + \chi_{t+1}^6 v_{t+1} \right), \tag{64}$$

$$B_{t+1} : \rho^t \chi_t^4 = \rho^{t+1} \, \chi_{t+1}^4 \left( 1 + R \left( \frac{B_{t+1}}{Y_{t+1}^T} \right) \right)$$

$$L_t^{T,F} : \rho^t \chi_t^5 = \rho^t \chi_t^4 A_T \, (1-\alpha_T) \left( K_t^T \right)^{\alpha_T} \left( L_t^T \right)^{(-\alpha_T)} \frac{\partial L_t^T}{\partial L_t^{T,F}}$$

$$L_t^{T,F} : \rho^t \chi_t^6 = \rho^t \chi_t^4 A_T \, (1-\alpha_T) \left( K_t^T \right)^{\alpha_T} \left( L_t^T \right)^{(-\alpha_T)} \frac{\partial L_t^T}{\partial L_t^{T,I}}$$

$$L_t^{N,F} : \rho^t \chi_t^5 = \rho^t \chi_t^3 (1-g) A_N \, (1-\alpha_N) \left( K_t^N \right)^{\alpha_N} \left( L_t^N \right)^{(-\alpha_N)} \frac{\partial L_t^N}{\partial L_t^{N,F}}$$

$$L_t^{N,I} : \rho^t \chi_t^6 = \rho^t \chi_t^3 (1-g) A_N \, (1-\alpha_N) \left( K_t^N \right)^{\alpha_N} \left( L_t^N \right)^{(-\alpha_N)} \frac{\partial L_t^N}{\partial L_t^{N,I}}$$

$$\left[ A^H e_t + (1-\delta_H) \right] \, U_x(C_t, x_t) = \rho \, U_x(C_{t+1}, x_{t+1}) \, \left[ A^H(1-x_{t+1}) + (1-\delta_H) \right] \tag{65}$$

$$H_{t+1} = A^H \, (1 - du_t - v_t - x_t) \, H_t + (1-\delta_H) H_t \tag{66}$$

$$C_t = (1-g) \, A_N \, \left( K_t^N \right)^{\alpha_N} \left( L_t^N \right)^{(1-\alpha_N)} \tag{67}$$

$$B_t \, (1 + R_t) + P_t \, I_t = A_T \, \left( K_t^T \right)^{\alpha_T} \left( L_t^T \right)^{(1-\alpha_T)} + \left( B_{t+1}^p + B_{t+1}^g \right), \tag{68}$$

$$K_{t+1}^T + K_{t+1}^N = I_t + (1-\delta_K)(K_t^T + K_t^N), \tag{68}$$

$$u_t \, H_t = L_t^{T,F} + L_t^{N,F} \tag{69}$$

$$v_t \, H_t = L_t^{T,I} + L_t^{N,I} \tag{70}$$

where for $j = T, N$

$$e_t = (1 - du_t - v_t - x_t)$$

$$L_t^j = \left( \beta \, \left( L_t^{j,F} \right)^{\frac{\eta-1}{\eta}} + (1-\beta) \, \left( L_t^{j,I} \right)^{\frac{\eta-1}{\eta}} \right)^{\frac{\eta}{\eta-1}}.$$

$$\frac{\partial L_t^j}{\partial L_t^{j,F}} = \beta \, \left( \beta + (1-\beta) \, \left( \frac{L_t^{j,I}}{L_t^{j,F}} \right)^{\frac{\eta-1}{\eta}} \right)^{\frac{1}{\eta-1}}$$

$$\frac{\partial L_t^j}{\partial L_t^{j,I}} = (1-\beta) \, \left( (1-\beta) + \beta \, \left( \frac{L_t^{j,F}}{L_t^{j,I}} \right)^{\frac{\eta-1}{\eta}} \right)^{\frac{1}{\eta-1}}$$

## 9.5   Balanced Growth Analysis: Ramsey Allocation

Notice that

$$V_C(C, x; \phi) = (1 + (1-\sigma)\,\phi) \, U_C(C, x) = (1 + (1-\sigma)\,\phi) \, C^{-\sigma} \, x^{\theta(1-\sigma)},$$
$$U_x(C_t, x_t) = U_C(C_t, x_t) \, \theta \, C \, x^{-1},$$
$$U_{xC}(C_t, x_t) = U_C(C_t, x_t) \, \theta \, (1-\sigma)x^{-1},$$
$$U_{xx}(C_t, x_t) = U_C(C_t, x_t) \, \theta(\theta(1-\sigma) - 1) \, x^{-2} \, C.$$

Along the balanced growth path (BGP), the following (normalized) variables are constant, namely, $\tilde{\chi}^1 = \chi_t^1$ and $\tilde{\chi}^n = \frac{\chi_t^n}{U_C(C_t, x_t)}$ for $n = 2, ..., 6$. The following conditions characterize the BGP of a Ramsey allocation

$$\tilde{\chi}^3 = (1 + (1-\sigma)\,\phi) - \theta \, (1-\sigma) \, x^{-1} \, \tilde{\chi}^1 A^H(1 - x - e),$$

$$(1 + (1-\sigma)\,\phi) \, \theta \, c \, x^{-1} = A^H \left[ \tilde{\chi}^2 + c\tilde{\chi}^1 \, \theta(\theta(1-\sigma) - 1) \, x^{-2} \, (1 - x - e) \right],$$

$$\tilde{\chi}^5 = \tilde{\chi}^2 \, A^H \, d + \tilde{\chi}^1 d \, A^H \, c \, x^{-1}\theta, \tag{71}$$

$$\tilde{\chi}^6 = \tilde{\chi}^2 \, A^H + \tilde{\chi}^1 A^H \, c \, x^{-1}\theta, \tag{72}$$

$$P = \rho \, \gamma^{-\sigma} \left( A_T \, \alpha_T \, \left(k^T\right)^{\alpha_T - 1} \left(l^T\right)^{(1-\alpha_T)} + P \, (1 - \delta_K) \right), \tag{73}$$

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$$P = \rho \, \gamma^{-\sigma} \left( \frac{\tilde{\chi}^3}{\tilde{\chi}^4} \, (1-g) A_N \, \alpha_N \left(k^N\right)^{\alpha_N - 1} \left(l^N\right)^{(1-\alpha_N)} + P\,(1-\delta_K) \right), \tag{74}$$

$$\tilde{\chi}^2 = \rho \, \gamma^{-\sigma} \left( \tilde{\chi}^2 \gamma + \left( \tilde{\chi}^5 u + \tilde{\chi}^6 v \right) \right), \tag{75}$$

$$1 = \rho \, \gamma^{-\sigma} \left( 1 + R \left( \frac{b}{A_T \, (k^T)^{\alpha_T} (l^T)^{(1-\alpha_T)}} \right) \right) \tag{76}$$

$$\tilde{\chi}^5 = \tilde{\chi}^4 \, A_T \, (1-\alpha_T) \left(k^T\right)^{\alpha_T} \left(l^T\right)^{(-\alpha_T)} \beta \left( \beta + (1-\beta) \left( \frac{l^{T,I}}{l^{T,F}} \right)^{\frac{\eta-1}{\eta}} \right)^{\frac{1}{\eta-1}} \tag{77}$$

$$\tilde{\chi}^6 = \tilde{\chi}^4 A_T \, (1-\alpha_T) \left(k^T\right)^{\alpha_T} \left(l^T\right)^{(-\alpha_T)} (1-\beta) \left( (1-\beta) + \beta \left( \frac{l^{T,F}}{l^{T,I}} \right)^{\frac{\eta-1}{\eta}} \right)^{\frac{1}{\eta-1}} \tag{78}$$

$$\tilde{\chi}^5 = \tilde{\chi}^3 (1-g) A_N \, (1-\alpha_N) \left(k^N\right)^{\alpha_N} \left(l^N\right)^{(-\alpha_N)} \beta \left( \beta + (1-\beta) \left( \frac{l^{N,I}}{l^{N,F}} \right)^{\frac{\eta-1}{\eta}} \right)^{\frac{1}{\eta-1}} \tag{79}$$

$$\tilde{\chi}^6 = \tilde{\chi}^3 (1-g) A_N \, (1-\alpha_N) \left(k^N\right)^{\alpha_N} \left(l^N\right)^{(-\alpha_N)} (1-\beta) \left( (1-\beta) + \beta \left( \frac{l^{N,F}}{l^{N,I}} \right)^{\frac{\eta-1}{\eta}} \right)^{\frac{1}{\eta-1}} \tag{80}$$

$$1 = \rho \gamma^{-\sigma} \left[ A^H (1-x) + (1-\delta_H) \right] \tag{81}$$

$$\gamma = A^H \, (1 - du - v - x) + (1 - \delta_H) \tag{82}$$

$$c = (1-g) \, A_N \, \left(k^N\right)^{\alpha_N} \left(l^N\right)^{(1-\alpha_N)} \tag{83}$$

$$b \left( 1 + R \left( \frac{b}{A_T \, (k^T)^{\alpha_T} (l^T)^{(1-\alpha_T)}} \right) - \gamma \right) + P \, (k^T + k^N)(\gamma + \delta_K - 1) = A_T \, \left(k^T\right)^{\alpha_T} \left(l^T\right)^{(1-\alpha_T)} \tag{84}$$

$$u = l^{T,F} + l^{N,F} \tag{85}$$

$$v = l^{T,I} + l^{N,I} \tag{86}$$

where for $j = T, N$

$$l^j = \left( \beta \, \left(l^{j,F}\right)^{\frac{\eta-1}{\eta}} + (1-\beta) \, \left(l^{j,I}\right)^{\frac{\eta-1}{\eta}} \right)^{\frac{\eta}{\eta-1}}.$$

The optimal taxes can be obtained from the balanced growth conditions of the competitive equilibrium.

$$\tilde{\mu} = \frac{c\theta}{A_H \ x}$$

$$r = A_T \ \alpha_T (k^T)^{\alpha_T - 1} (l^{T,F})^{\beta(1-\alpha_T)} (l^{T,I})^{(1-\alpha_T)(1-\beta)}$$

$$w_F = A_T \ \beta \ (1-\alpha_T)(k^T)^{\alpha_T} \ (l^{T,F})^{\beta(1-\alpha_T)-1} (l^{T,I})^{(1-\alpha_T)(1-\beta)}$$

$$w_I = A_T \ (1-\beta) \ (1-\alpha_T)(k^T)^{\alpha_T} \ (l^{T,F})^{\beta(1-\alpha_T)} (l^{T,I})^{(1-\alpha_T)(1-\beta)-1}$$

$$\tilde{\lambda} = \frac{\tilde{\mu} \ dA_H}{(1-\tau^w)w_F}$$

$$Y^N = A_N \ (k^N)^{\alpha_N} (l^{N,F})^{\beta(1-\alpha_N)} (l^{N,I})^{(1-\alpha_N)(1-\beta)}$$

$$P^N = \frac{w_F \ l^{N,F} + w_I \ l^{N,I} + rk^N}{Y^N}$$

$$\tau^w = 1 - d\frac{w_I}{w_F}$$

$$\tau^k = \frac{r - p(R - \delta_K)}{r - \delta_K}$$

$$\tau^c = \frac{1}{\tilde{\lambda}P^N} - 1$$

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