The Optimal Sample Size for Contingent Valuation Surveys:

Applications to Project Analysis

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ABSTRACT

Fundamentally, this paper is about the value of information.

Whenever a cost-benefit analysis has to be undertaken using benefits that are estimated from household survey data the size of the survey sample must be specified. The most obvious case is the valuation of environmental amenity improvements through contingent valuation (CV) surveys of willingness to pay. One of the first questions that has to be answered in the survey design process is “How many subjects should be interviewed?” The answer can have significant implications for the cost of project preparation, since in Latin America and the Caribbean costs per interview can range from US$20 to US$100.

Traditionally, the sample size question has been answered in an unsatisfactory way by either dividing an exogenously fixed survey budget by the cost per interview or by employing some variant of a standard statistical tolerance interval formula. Neither of these approaches can balance the gains to additional sampling effort against the extra interviewing costs. One might just as well use a dart board or pick a random number between 100 and 10,000.

The answer is not to be found in the environmental economics literature. But, it can be developed by adapting a Bayesian decision analysis approach from business statistics. The paper explains and illustrates, with a worked example, the rationale for and mechanics of a sequential Bayesian optimization technique, which is only applicable when there is some monetary payoff to alternative courses of action that can be linked to the sample data. In this sense, unlike pure valuation studies that are unconnected to a policy decision, investigators who use contingent valuation results directly in cost-benefit analysis have a hidden advantage that can be exploited to optimize the sample size. The advantage lies in the link between willingness to pay and the decision variable, the net present value of the prospective investment.

The core objective of the paper is practical. Readers without a statistical background can easily implement the method. Annexes are provided to show how, with just six key pieces of information, anyone can solve the optimal sample size problem in a spreadsheet. An automated spreadsheet algorithm is available from the authors on request. To run the program all the user has to do is enter the key data and then activate a macro that automatically computes the optimum number of additional observations needed to augment any initial “small” survey sample.

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THE INVESTMENT PROJECT ANALYSIS CONTEXT

Many multilateral financial institutions require that investment project proposals be screened using economic analysis, preferably of the cost-benefit (CB) rather than the cost-effectiveness variety. In recent years, contingent valuation (CV) has become the preferred route in some of these institutions for estimating the benefits yielded by investments aimed at improving ambient air or water quality (Ardila *et al.* 1998). An issue that is repeatedly raised by those responsible for actually doing the analysis (staff economists and their counterparts in the executing agencies of prospective borrowing countries) is "How many subjects should be interviewed in a CV survey to produce a reliable estimate of per capita or per household benefits?"

It would be hard to disagree in principle with Mitchell and Carson's (1989, p. 223) admonition that "No matter how realistic the scenario, the data collected from even the best contingent valuation survey instruments are useless unless sample statistics of sufficient quality for policy purposes can be obtained." In practice this message is often lost in decision-making, even when investments of a quarter of a billion dollars or more are at stake. In testimony to its obscurity, no guidelines on sample size selection exist in the economic analysis protocols of the Inter-American Development Bank, and our conversations with economists at sister lending institutions suggest that they are perplexed as well.[1]

In these days of low overheads and limited budgets in public sector and international institutions involved in grant or loan financing of projects and programs, management is justifiably reluctant to overspend on project preparation. One high level IDB executive with considerable expertise in economic analysis remarked to the authors that he suspected, without looking deeper into the question, that CV samples in the area of 250 interviews would probably be sufficient for a reliable CB project appraisal. The literature contains little that refutes this conventional wisdom.

Standard Sub-optimal Approaches

In their seminal book on CV, Mitchell and Carson (1989) suggest that, based on a simple statistical tolerance formula, sample sizes between 200 and 2500 are probably appropriate (Chapter 10, footnote 13, p. 225), assuming a best guess of 2.0 for the coefficient of variation which drives the calculation. The academic literature on CV has little more useful to say. Instead, and perhaps quite realistically, it devotes a great deal of effort to algorithms for figuring out how many bid groups should be used in a referendum CV survey, and how many interviews should be allocated to each, taking the total sample size as exogenously set by the *deus ex machina* of research funding limits (Cooper 1993, p. 28).

Given the paucity of guidance, determining an appropriate sample size is often handled loosely by everyday practitioners in government and international agencies, whose work is often underfunded and fast-tracked.

[1] In the only known internal IDB source discussing sample size, Vaughan (1994) looks at the size of a sample of projects needed to confirm the overall economic viability of so-called "multiple works programs" which are designed to finance a group of similar works, of which only a subset is exposed to ex-ante economic analysis. The issue of CV sample size is not addressed therein. At a training workshop on CV presented by the authors at the World Bank in June 1999, the sample size question was raised by the audience, but we were unable to provide a wholly satisfactory answer beyond intuitively suggesting that samples between 500 and 1000 usable observations probably would be sufficient.

Budget considerations play a predominant role. But, does it make any sense to divide a predetermined budget amount (net of fixed costs) by the cost per interview to get to a CV survey sample size? What determines the level of the budget for analysis and CV surveying? Yet, is it any more legitimate to challenge any budget ceiling on the basis of a statistical tolerance formula that says "We need N interviews to produce a sample estimate of mean willingness to pay (WTP) that comes within ± X percent of the true population mean with (1-α) percent confidence?" There is no particularly strong rationale for choosing a percentage difference of, say, 5% rather than 10% between the true population willingness to pay and the sample mean, or any reason other than custom for selecting a significance level, α, of 1% rather than 5%.

Arguments for augmented survey budgets along these lines are unlikely to be persuasive to those who must allocate limited resources. Without a criterion that balances the value of additional information provided by larger samples with the cost of collecting them, the only recourse is to fall back on rough statistical rules of thumb (Mitchell and Carson 1989) or do one's best with whatever funds are made available.

Seeking Optimality

The immediate objective of much CV work is valuation per se, that is, estimation of the value of natural assets like protected areas, or the benefits of ambient quality improvement. While these CV values may serve as inputs into the assessment of the prospective monetary gains and losses of alternative investment decisions or courses of environmental policy action, that subsequent decision analysis is often someone else's responsibility once the job of the CV experts is done. When the CV exercise is effectively sealed off from the policy or investment decision step, CV researchers working in isolation have no recourse but to satisfice when it comes to sample size, choosing a size N that somehow is defensible or appears reasonable. However, investigators using contingent valuation results directly in cost-benefit analysis have a hidden advantage that can be exploited to reach a much more persuasive conclusion about the optimal CV sample size. This paper explains how the value of additional sample information can be quantified (either approximately or precisely) and balanced against the cost of obtaining it. That value is closely related to the way the distribution of the investment's expected net present value reacts to the size of a CV sample.

The optimal sample size approaches explained herein have their origins in Schlaifer's (1959, 1961) Bayesian decision analysis approach, which is discussed in an accessible way by a number of standard texts on the use of statistics in business decision-making (Bonini *et al.*1997; Jedamus and Frame 1969; Pfaffenberger and Patterson 1987; Lapin 1994; Winkler 1972). For a rapid Bayesian analysis, an approximate normal probability distribution for NPV can be obtained by constructing a simple a linear relation between it and sample estimates of the mean WTP and its standard error. Larger CV samples reduce the uncertainty about willingness to pay (reduce the standard error of mean WTP) and about NPV as well, where the compression in variance (uncertainty) with increasing sample size is transmitted through the linear relationship. By monetizing variance reduction in this way, the marginal costs of expanding sample sizes can be compared to the marginal benefits of the additional information they contain to reach an optimal sample size decision.

Alternatively, the linearity assumption linking NPV to mean WTP may not hold, and NPV may not be normally distributed even if WTP is, because other random influences on NPV skew the distribution of the outcome. A more precise Monte Carlo risk analysis in the Bayesian mold can be employed to characterize the way the empirically generated probability distribution of NPV reacts to better information on WTP to verify the approximate optimum based on the normality assumption.

Optimal Results Without Pain

Anyone can implement the method developed in the remainder of this paper using a spreadsheet algorithm in Quattro Pro that is available from the authors on request.[2] The optimization routine presumes that an initial small survey sample has already been taken, and asks whether it would be optimal to add to it in a second round of sampling. To run the program all the user has to do is click on an "Optimizer Macro" button to compute the optimum number of additional observations needed to augment an initial "small" survey, if any. The data entry and output results forms are shown in Table 1. Annex 3 to this paper contains the full set of spreadsheet instructions.

Table 1. Quattro Pro Macro: Data Input Form and Optimal Results Summary Output

DATA ENTRY			INSTRUCTIONS:
STEP I. ENTER THE INITIAL SMALL SAMPLE DATA	**Units**	**Data Entry**	ENTER DATA IN BOX AT LEFT
Size of Initial "Small" Sample?	# of Cases	250	
Sample Mean Willingness to Pay?	$/Household/Unit Time	$7.47	
Variance of Sample Mean?	$/Household/Unit Time	0.70	
Sampling Cost per Household Interview?	$/Case	$89.00	
STEP II. SPECIFY THE LINEAR CVPI FUNCTION RELATING NPV TO WTP (NPV = α + β •MEAN WTP)			
Intercept (α) ?	$ Total Discounted Costs [Enter as Negative #]	-$594,653,984.00	
Slope (β) ?	# of Beneficiaries [Total Discounted]	100,988,487	
			AND THEN
RESULTS			
STANDARD ERRORS OF NPV AWAY FROM NPV = 0		**1.89**	
SHOULD A SECOND SAMPLE BE TAKEN TO AUGMENT THE INITIAL SAMPLE?		***Probably Yes***	
IF "Yes" CLICK ON THE BUTTON AT THE RIGHT TO RUN THE OPTIMIZER MACRO		OPTIMIZER MACRO	HIT THE OPTIMIZER BUTTON
Approximate Sample Size (Used as a Starting Value for Optimization)		2,793	
EXACT OPTIMUM		**2,378**	PROGRAM RETURNS THE OPTIMUM
Note: This routine assumes the analyst has no prior knowledge about average WTP or its variance beyond what the initial "small" sample reveals. Neither the authors nor the Inter-American Development Bank warranty this program or the methods it employs.			

Only six pieces of input information are needed: (1) the size of the initial small CV survey sample, (2) the expected value (mean) of willingness to pay (WTP) extracted from that sample, (3) the variance of mean WTP, (4) the average (equals marginal) cost of collecting a single survey observation, (5) the intercept of

[2] Contact William J. Vaughan by e-mail at williamv@iadb.org and ask for the "Sample Size Template". Indicate whether you will be using Quattro Pro versions 6, 7 & 8 or Quattro Pro Version 9. The template is not available in Microsoft Excel.

a linear function (called the CVPI function) relating NPV to WTP and (6) the slope of the linear function. All but the last two are obvious. The intercept and slope of the CVPI function are also easy to get, without doing a complex cost-benefit analysis beforehand. The intercept is just the discounted sum of project investment and operating costs (net of any non-CV benefits, if they exist). The slope is just the discounted sum of the number of beneficiaries to whom mean WTP benefits from the CV survey are attributed. Subsequent sections develop the rationale for the proposed method and explain each step in detail.

Anticipating the Optimal Results for a Case Study Example

It is always difficult and perhaps even dangerous to make generalizations about optimal sample size. The best sample size is always case specific because it depends on the cost and benefit flows of the prospective investment under consideration. Yet there is a general pattern: the better the project appears ex-ante the smaller the CV sample size needed to obtain the benefit estimates that justify it. Given the investment cost and return data for the worked example used later, the total sample size required depends critically on just two pieces of information: the standardized distance of expected NPV from zero and the unit variable cost of collecting a single WTP survey response. In our experience in Latin America and the Caribbean, sampling costs will be high if an interview costs around US$100, and more typical if interviews cost about US$20 to US$35 each.

The remainder of this paper uses actual data from a case study to illustrate the general principle that the optimal sample size for CV surveys is a decreasing function of how good the project appears to be ex-ante. The results are based on the benefits and costs of a proposed project to clean up the Tietê River in São Paulo, Brazil (for details about the project see Russell *et al.* forthcoming and Vaughan *et al.* 1999, 2000a, 2000b, 2000c).[3]

Figure 1. **Sample Size Depends on Project Prospects**

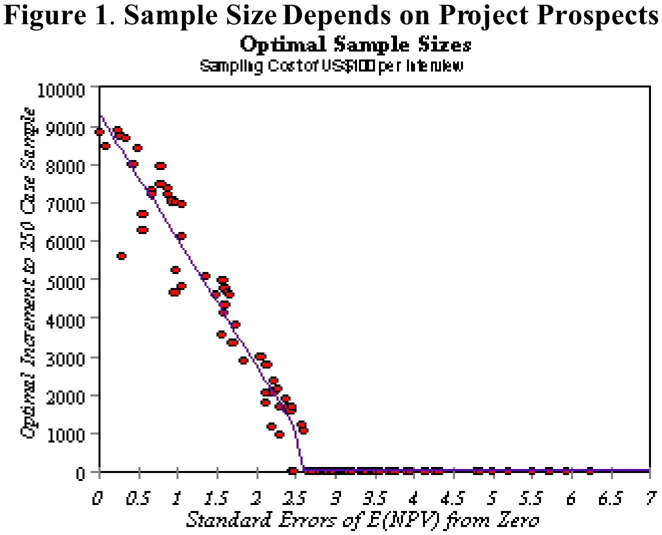

Figure 1 shows the results of calculating the optimal sample size for 100 randomly drawn values of mean WTP and its standard error, assuming a high cost of sampling of R$89 (US$100) per interview. The figure shows that investments with a high likelihood of success (expected NPV more than two and one half standard errors from zero because the mean WTP is high and the variance of the mean is low) require very little sampling effort beyond taking a small sample of 250 or so observations.

An ex-ante speculation about the chances that a project will succeed measures the degree of

[3] The figure assumes an initial small sample of 250 cases has already been taken and shows how many cases should be added to it. To generate the figure, the baseline deterministic cost and energy benefit flows from the case study example discussed in Vaughan *et al.* (1999, 2000a, 2000b,and 2000c) were combined with mean WTP estimates that were randomly drawn from a uniform distribution covering R$5.89 to R$13.00, which spans most of the range of the estimates discussed in Vaughan *et al.* 1999 and Vaughan *et al.* 2000a, 2000b, and 2000c. The variance of each mean was derived from a random draw of values of the coefficient of variation between 0.75 and 6.0, following Mitchell and Carson (1989, p. 225). No prior information about the mean WTP or its variance was incorporated, in contrast with Table 1, which includes some runs with tight priors.

confidence. Extremely promising projects have large positive expected net present values (NPVs), while marginal projects have NPVs near zero. By dividing the expected net discounted benefits of any project by the standard error of the mean NPV any investment can be put on a commensurate scale that has a ready interpretation from basic statistics. Assuming approximate normality in the distribution around the estimate of expected NPV, projects whose standardized NPV is more than 2.33 standard normal deviates from zero have a 99% chance of being successful, projects that are 1.28 deviates above zero have a 90% chance of success, and so on down the line. An investment whose standardized NPV is expected to be only slightly above the critical value of zero will fail half the time so it only has a 50 percent chance of success.[4]

A compact summary of the results of a large number of other Bayesian optimization exercises not shown in Figure 1 appears in Table 2.

Table 2. Range of Nearly Optimal Total Sample Sizes from the Tietê Case Study Data

Chances of Project Success / Interview Cost in Latin America	**Over 99%** [>2.33 S.E]	**90% to 99%** [1.28 to 2.33 S.E.]	**50% to 90%** [0 to 1.28 S.E.]
High (Brazil: US$ 100 per Interview)	250 to 1150	250 to 3300	1600 to 8600
Typical (US$35 per Interview)	250 to 2000	250 to 5200	3000 to 14400

The optimal sample sizes [5] reported for variations on the case study data alone cannot be generalized to other situations. But exact optimal sample size answers can easily be found for any specific investment project using the optimization algorithm supplied in Annex 3. The interesting characteristic demonstrated by the results from the case study is that no specific sample size, either large or small, is desirable across the gamut of situations.

Before getting involved in the details of the optimization approach, several variants of the standard classical method are illustrated next using contingent valuation survey data that was collected in 1998 to estimate the water pollution control benefits of the Tietê project. The results of exploring several different sets of assumptions with the classical sample size selection method reveal just how little guidance it actually provides. Then, small sample means of WTP and their variances are constructed from the data before moving on to explain and apply the principles of sequential optimal sample size determination using data from a small initial sample as a point of departure.

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[4] To arrive at these percentages, the normal distribution in question must be centered on the expectation of NPV, not on zero.

[5] The minimum sample size is 250 cases because a small initial sample is needed to jump start the optimization routine. The net benefits of increasing the sample size above 250 cases rise initially at a rapid rate and then become almost flat over a wide span (± 20 to 30%) around the optimum. Consequently, the sample sizes in Table 2 are termed near optimal because the additional cases needed beyond the minimum of 250 have been scaled back to 70% of the optimum following Schlaifer (1961, Figure 21.5, p. 337); they are lower bound ranges.

THE STANDARD APPROACH TO SURVEY SAMPLE SIZE DETERMINATION: VARIATIONS ON THE THEME

For its referendum CV questionnaire to estimate willingness to pay for improved water quality in the Tietê River, the original project analysis drew a sample based on household characteristics drawn from a 1996 survey of households in the Saõ Paulo Metropolitan Area (SPMA). The strategy was to represent the population of São Paulo in terms of the factors thought to have a strong influence on willingness to pay. In theory such factors might include the household's income and its perception of odors from the river, environmental awareness, and education. Of these, the Census only had information on income and education, which are highly correlated.

According to the Census, the average household income in the SPMA is R$828/month with a standard deviation of R$702. Using a 95% confidence interval and a 10% sampling error, the original analysts figured that a sample of 276 homes would be required. The necessary sample size was initially calculated based on the amount of tolerable error in the sample estimate of mean income rather than mean WTP (which was unknown), using a standard statistical formula that acknowledges only Type I error (e.g. Paffenberger and Patterson 1987, p. 389).

The standard formula used was:

$$N = [z_{\alpha/2}\ \sigma/E]^2 = [(1.96*702) / 82.8]^2 = 276$$

where:

N	=	desired sample size
z	=	the 95% confidence interval statistic (1.96) at significance level α = 5%, 2 sided test.
σ	=	standard deviation of income (R$702).
E	=	acceptable error (R$82.80) in the sample estimate of the population mean WTP obtained as one-tenth of census estimate of average household income of R$828 (i.e. a 10% error).

Note that the variable of interest is household willingness to pay (WTP), not income, so the above application of the standard sample size formula only holds if the mean and standard deviation of WTP bear a fixed proportional relationship to the mean and standard deviation of income, which is unlikely.

In a second line of attack, the analyst might try to formulate explicit prior beliefs about the population mean and standard deviation based on historical experience and proceed from there. For instance, assume a simple distribution for willingness to pay, such as the triangular. The mean and standard deviation can easily be obtained from this distribution given a guess about just three values; the minimum, the most likely, and the maximum WTP (Vose 1996, p.88):

Mean, Triangular = (a + b + c) / 3

Variance, Triangular = $(a^2 + b^2 + c^2 - ab - ac - bc) / 18$

where:

a = Minimum; b = Mode; c = Maximum

Establishing the minimum WTP is easy if the investment improves utility or at least does no harm; so it can be safely set to zero. For the maximum, we know from experience that, on average, people are willing to pay about three to four percent of income for sewer connections and that the willingness to pay for ambient water quality is, on average, less than one-third of that (Ardila *et al.*1998). So the maximum of WTP could be set to what households are willing to pay for sewer connections—around 3% of income. If income is approximately right-triangular distributed the maximum income is three times the mean of R$828, or R$2484. Then, the highest individual observation of WTP would be 3% of R$2484, or R$74.52. The modal value is more difficult, but Choe *et al.* (1996) found that willingness to pay for water pollution control in the Philippines was only about 1 % of income, which is consistent with our experience in Latin America (Ardila *et al.* 1998), yielding a most likely value of R$8.28. Then, from the triangular distribution formulas the prior mean WTP becomes R$27.60, the variance R$346.93, and the standard deviation, σ, R$18.63.

As before, if the sample is to come within ± 10% of the mean, E = R$2.76. Then:

$$N= [z_{\alpha/2}\ \sigma/E]^2 = [(1.96*18.63) / 2.76]^2 = 175$$

This recommendation for a very small sample is based on ostensibly reasonable guesses. As we will see later, these prior estimates turn out to be extremely poor when compared to the mean and standard deviation of the actual sample data.

For another variant of the same game, suppose we believe the <u>average</u> WTP per household is 1% of income, or R$8.28 and that the most frequent response (the mode) is zero. This yields a right triangular distribution with a=0 and b=0. The implied maximum WTP, c, is 3 times R$8.28, or R$24.84 and the variance is just $c^2/18$, or R$34.28, yielding σ = R$5.85. Again, if the sample is to come within ± 10% of the mean, E = R$0.83 and another recommendation for a small sample results:

$$N= [z_{\alpha/2}\ \sigma/E]^2 = [(1.96*5.85) / 0.83]^2 = 191$$

Finally, for a fourth route, Mitchell and Carson (1988, p.224) suggest a clever manipulation of the standard formula above that obviates the need to guess about σ or the absolute magnitude of acceptable error in the mean of WTP. Instead, a guess about the ratio of the standard deviation to the population mean (the Coefficient of Variation, V) is required; Mitchell and Carson suggest a value for V of about 2. At the α=5% level, using V=2 and Δ of 10% as an acceptable difference between the true population mean WTP and the sample estimate:

$$N= [\ (z_{\alpha/2}\ V) / \Delta\ ]^2 = [(1.96\bullet 2.0) / 0.10]^2 = 1537$$

The four standard routes to sample size determination illustrated above lead to quite different answers because the first three implicitly assume the value of V is less than 1.0 and therefore recommend small samples (i.e. the V values are R$702/R$828 = 0.85; R$18.63/R$27.60 = 0.68; and R$5.85/R$8.28 = 0.71) rather than the value of 2.0 reflecting Mitchell and Carson’s review of actual contingent valuation surveys in the 1980s, most of which were undertaken in developed countries.

The guessing game played above could go on ad-infinitum without ever producing a firm conclusion about the reliable sample size needed for any particular CV survey. Although it would seem to suggest that, in developing country applications, small samples will suffice, that is an erroneous generalization. The rest of this paper will show that small samples sometimes suffice, but the classical method fails to isolate the circumstances under which it is safe to take a small CV survey sample rather than a big one.

In any event, the first result, 276 households, was not used for the Tietê referendum CV survey. Neither were the second or third of 175 and 191 cases or the highest estimate of 1537 from Mitchell and Carson's route. Instead 600 interviews were actually undertaken for the project analysis, split between two sub-samples to account for the distance effect on WTP (184 households living in districts bordering the polluted river and 416 living farther away and presumably less affected by its noxious odors and health risks). Even though the variable cost of each interview, US$100, was fairly expensive,[6] the available budget permitted an expenditure of US$60,000 to take a larger sample and get more precise results than the lowest estimate of US$17,500 could provide, but not aim for the tighter variances from 1537 interviews that could be purchased for US$153,700. The question is, which of the sample size estimates, in retrospect, comes closer to the optimal size? The answer lies beyond the quick and convenient, but imprecise and arbitrary, classical method.

[6] This value was equal to $114 Brazilian reals in 1998. For the loss-cost exercise it must be put on equal terms with the value of information, which is shadow priced in the project analysis. Multiplying it by the shadow price of non-tradeable inputs of 0.78 gives a variable (equal to marginal) cost per observation of R$88.92. CV surveys in Latin America usually do not cost this much; costs around US$30 per case have typically been quoted (Ardila et. al. 1998).

STARTING TOWARDS AN OPTIMAL SOLUTION: SMALL SAMPLE MEANS AND THEIR STANDARD ERRORS

The next step toward an optimal answer begins with the collection of an initial small CV survey sample (say, 250 cases) and the calculation of the mean WTP and its variance. With this information in hand, the researcher can then proceed to ask whether additional sample information would be desirable, following the optimization procedure explained subsequently.

But first, initial estimates that characterize the distribution of WTP are needed. In the case of WTP, the standard error of the mean can, in principle, be reduced by increasing the sample size. However, estimation of this effect using conventional parametric techniques on referendum CV data is problematic, since analytical formulas are generally lacking.[7] However, the nonparametric estimators (McConnell 1995; Haab and McConnell 1997, Vaughan *et al.* 1999, Vaughan *et al.* 2000a, 2000b, 2000c) directly relate the standard errors of lower bound, intermediate or upper bound mean WTP estimates to sample size, and these formulae can be exploited to help compute the optimal sample size (see Annex 1). Alternatively, an initial open-ended rather than referendum CV survey could be conducted and the mean and its standard error calculated directly from the explicitly stated WTPs of the respondents.

To demonstrate, a balanced random sub-sample of 250 observations was drawn from the actual 600 observation grand sample of Tietê CV survey interviews.[8] We chose a size of 250 in order to have a reasonable minimum number of observations in each of the five bid groups in the referendum. Unlike the grand sample, the small sub-sample was deliberately drawn to be representative of the spatial distribution of the respondents, so the distinction between the WTPs of residents living close to and far from the river can be dropped, which simplifies the sample size selection problem. Table 3 presents the small sample estimates of the nonparametric means and their variances, and Annex 1 provides the details.[9] Our prior

[7] The parametric approach requires that a conditional cumulative density (or survival) function be statistically fit to the data and, subsequently, an expected value extracted using formulae that are functions of the estimated parameters of that assumed density (usually Logistic, see Vaughan *et al.* 1999, 2000a, 2000b, 2000c). Lacking analytical formulas, the mean standard error must be found either via the delta method (a second-order Taylor series approximation of an unknown variance function which itself depends on the standard errors of the survival function parameter estimates) or by bootstrapping.

[8] The actual 600 observation referendum CV sample from our case study was unbalanced because it undersampled households living in districts that are contiguous to the river (31 % in the sample, 61 % from the metropolitan area census). Since households living in districts bordering the river are willing to pay significantly more on average for improved water quality than households in noncontiguous districts (R$6.07 per household per month versus R$4.51) the mean from the grand sample is a biased estimate of the population's average willingness to pay. We corrected for this by randomly drawing 250 observations from the grand sample using the constraint of the census proportions, which meant that 152 of the 184 available households living close to the river were included in the small sample, along with 98 of the 416 families living in more distant districts.

[9] The variance estimates in the table were independently verified by simulation, drawing from separate binomial distributions reflecting the number of "No" answers in each bid group and repeatedly calculating the mean 5000 times. The standard errors of the means matched those from the analytical formulas provided in Annex 1. For the balance of the discussion, the approximately equal allocation of cases across bid levels is taken as given, ignoring the possibilities for variance reduction at

guesses for mean WTP and the population standard deviation in the section on the standard approach did not turn out to be very prescient, although setting the average WTP at 1% of income comes close (R$8.28).

Table 3. Small Sample Nonparametric Means

Estimator	Mean	Variance of Mean	Standard Error of Mean	Population Standard Deviation [a]
Turnbull Lower Bound	5.75	0.45	0.67	10.61
Weighted Turnbull (0.75) and Paasche (0.25)	7.47	0.70	0.84	13.23
Kriström’s Intermediate	9.20	1.02	1.01	15.97
Paasche Upper Bound	12.66	1.88	1.37	21.68

Note:
a. Approximation from the square root of the product of the variance of the mean and the sample size, 250.

To jump the gun a bit, what would the classical method recommend if our guesses for μ and σ were exactly equal to what the actual sample reveals? The Turnbull mean WTP is R$5.75 and the sample estimate of the population standard deviation is R$10.61 (the standard error of the mean multiplied by the square root of 250). Applying the familiar formula, the recommended sample size under these nearly perfect guesses would be large indeed, and very close to what Mitchell and Carson’s approach recommends:

$N= [z_{\alpha/2}\ \sigma/E]^2 = [(1.96*10.61) / 0.58]^2 = 1286$

Similar calculations for the other means in Table 3 also yield sample sizes of around 1200 cases.

Unfortunately, following the recommendation of the classical method, even if it is based on actual sample information, is potentially misleading. The optimal Bayesian decision under the baseline configuration of net project benefits and sampling costs, assuming a mean WTP of R$7.47, recommends a sample of over 2000 observations, as demonstrated below. In short, the standard classical method is no more useful than a dart board. On the other hand, the optimization approach is more useful, but the optimal solution is extremely sensitive to the choice of nonparametric mean, which is ultimately a subjective decision.[10] For the balance of this paper, our preferred mean is the intermediate nonparametric mean composed of a weighted combination of the Turnbull lower bound mean (75% weight) and the Paasche upper bound mean (25% weight).

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any given total sample size that might be achieved by concentrating the bulk of the sample in the region of bid levels where F_j. = 0.5.

[10] See Vaughan *et al.*1999 for a review of parametric versus nonparametric means.

PRELIMINARIES ON PROJECT RISK, THE VALUE OF INFORMATION AND LOSS-COST MINIMIZATION

The analogues of the decision analysis approach to sample design in general (Schlaifer 1959, 1961) and in statistical quality control applications in particular (Vaughan and Russell 1984, Russell, Harrington and Vaughan 1986) provide the keys to unlocking the optimal CV sample size problem. Somewhat loosely stated, the core concept involves finding the sample size that minimizes the sum of sampling costs and expected losses.

The pure form of the optimal sample size approach involves Bayesian decision analysis and expands on the concept of prior information that was employed above in the second and third variants of the standard approach. It combines prior subjective characterizations of the probability distribution of mean willingness to pay with data from an initial "small" sample of, say, 250 cases to decide whether an additional round of sampling should be undertaken and, if so, how many subjects should be interviewed in that second round. Schlaifer (1961) calls this Bayesian "preposterior" decision-making about the desirable sample size because a decision can be reached on the basis of partial information before actually doing any additional sampling.[11]

Expected Gains and Losses

In the terminology of decision analysis, the CB decision is a two-action problem with infinite states of nature. The investment proposal can either be accepted if in expectation it will yield a positive discounted net cash flow above the break-even point of NPV equal to zero, or rejected if it does not. Because the many influences on NPV are random variables, so is NPV. Therefore, at least conceptually, there are an infinite number of possible net cash flow values, each with its own probability of occurrence.

CB risk analysis accommodates the variance in benefits and other variables, so the risk-neutral decision rule (Brent 1996) is clearer than it would be in a deterministic analysis that inconsistently combines extreme values for some variables with various measures of central tendency for others. The rule is to proceed with a capital investment project if the *expected value* of its discounted stream of net benefits, E(NPV), is non-negative; but if the expectation of discounted net benefits is negative the project proposal is economically infeasible. In the probabilistic context of risk analysis, following this expected value decision rule has a quantifiable cost called the *cost of uncertainty*.

The cost of uncertainty is the expected opportunity loss of making the decision determined by the decision rule. That is, if the expectation E(NPV) taken over the entire NPV distribution is non-negative, the

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[11] In Winkler's (1972, p. 297) words, "This type of decision is called a *preposterior decision* because it involves the *potential* posterior distributions following the *proposed* sample." Winkler notes that preposterior analysis can be carried out repeatedly in sequential decision-making. Our proposal involves a two-step sequence of taking an initial "small" sample and then doing a preposterior analysis that looks for the optimal number of surveys to add to the initial sample, which can either turn out to be zero or some positive number. Of course, in some circumstances the initial sample size itself may be suboptimal (too large), but then there will be no need to add to it.

investment will be made. But if some portion of the NPV distribution falls below zero, actual losses in specific instances are still possible. The cost of uncertainty can therefore be measured as the mean of that portion of the NPV distribution truncated from above at zero (the average loss, given that a loss might indeed occur), multiplied by the probability of a negative NPV occurring. If the project is not undertaken because the expected value of NPV is negative, the investment will not be made, thus foregoing any possibility of positive net returns. Symmetrically, the loss in this situation is the mean of that portion of the NPV distribution truncated from below at zero (the average net gain foregone, given that a net gain might occur), multiplied by the probability of a positive NPV occurring.[12] The two opportunity loss situations are pictured below.

If the project is economically feasible its global mean NPV will be non-negative. The project will be undertaken so the region of opportunity loss is from negative infinity (or the minimum possible NPV) to zero:

Case I: Project Feasible: Correct Decision is to Invest

Region of Losses, NPV<0 | **Region of Gains, NPV≥0**

-∞ or Min NPV | **E(NPV) | NPV<0** | **0** | **E(NPV)** | **+∞ or Max NPV**

If the investment's expected NPV is negative it should not be undertaken, thus foregoing some possible gains lying in the region of opportunity loss from zero to plus infinity (or the maximum possible positive NPV):

Case II: Project Infeasible: Correct Decision is Not to Invest

Region of Losses, NPV<0 | **Region of Gains, NPV≥0**

-∞ or Min NPV | **E(NPV)** | **0** | **E(NPV) | NPV ≥ 0** | **+∞ or Max NPV**

Table 4 provides more formal definitions of the decision criterion, the probability of opportunity loss, the truncated mean loss and the cost of uncertainty. The cost of uncertainty ($E_{Loss, I}$ or $E_{Loss, II}$ in the table) is in part a function of the amount of prior subjective and sample information size on hand when the options are weighed to either invest immediately or wait and collect more information At the point after a small initial sample of size N_0 is taken (or even before when a prior guess is formed without any sampling at all) it represents the most the investor would be willing to pay to gather more information and eliminate all uncertainty about the project, which is why it is also called the expected value of perfect information, EVPI.

Additional sampling can never eliminate all uncertainty. But changes in $E_{Loss, I}$ (or $E_{Loss, II}$) with increases in sample size beyond the original small sample N_0 provide a measure of the gross benefit of the second stage of a sequential CV sampling scheme. Incremental CV samples with $\Delta N>0$ reduce the standard error of the

[12] Only if the probability distribution of NPV lies entirely in either the positive or negative domains will there be no cost of uncertainty because you literally can't go wrong; either an unattractive investment is unambiguously bad over the entire range of NPV outcomes or no losses are possible because there is zero probability that NPV will fall below zero. In either of these extreme situations, case-specific sample estimates of willingness to pay may not even be necessary. If extreme upper and lower limits for willingness to pay can be posited a-priori via benefits transfer or other past experience, and the investment either fails the CB test using the highest possible WTP or passes it using the lowest conceivable non-negative WTP value, the investment decision can be made without incurring sampling costs.

CV mean estimate of project benefits (WTP), which transmits into a reduction in both the truncated mean loss in project NPV and the cumulative probability of that loss.

Table 4. Fundamental Definitions

	Case I **Correct Decision: Invest** **E(NPV) ≥ 0**	**Case II** **Correct Decision: Don't Invest** **E(NPV) < 0**
Decision Criterion: Global Mean NPV	$E(NPV) = \int_{-\infty}^{\infty} NPV_i \bullet p_i \, d\,NPV$	
Probability of Opportunity Loss	$F_{Loss\ I} = \int_{-\infty \text{ or Min}}^{0} p_i \, d\,NPV$	$F_{Loss\ II} = \int_{0}^{+\infty \text{ or Max}} p_i \, d\,NPV$
Truncated Mean Loss	$E_{T,I} = E(NPV \mid NPV< 0)$ $= \{\int_{-\infty \text{ or Min}}^{0} NPV_i * p_i \, d\,NPV\}/ F_{Loss\ I}$	$E_{T,II} = E(NPV \mid NPV>0)$ $= \{\int_{0}^{+\infty \text{ or Max}} NPV_i * p_i \, d\,NPV\}/ F_{Loss\ II}$
Cost of Uncertainty or **Expected Value of Perfect Information** or **Expected Loss of a Terminal Action**[a]	$E_{Loss,\ I} = E_{T,I} \bullet F_{Loss\ I}$	$E_{Loss,\ II} = E_{T,II} \bullet F_{Loss\ II}$

a. These terms all appear in the literature and they all mean essentially the same thing. It may seem unnecessarily roundabout to express the cost of uncertainty as the product of a truncated mean and the fraction of the total probability distribution lying in the region of opportunity loss. However, this is necessary given the way the information is produced by the Crystal Ball Monte Carlo simulation routine we used to verify the approximate solutions in the worked examples.
b. The probability of occurrence of the i^{th} NPV is represented as p_i in the table.

Value of Information: Variance Reduction Through Sample Size Increases

Given any initial referendum CV survey's sample size and the prospective investment project's NPV estimates based on the survey's mean WTP, decisionmakers can either finalize the project acceptance/rejection decision or commission further studies to try to reduce the uncertainty about the outcomes.

Only information about the factors that can have a significant impact on the project outcome reduce the cost of uncertainty in a meaningful way; in most cases uncertainty about benefits will be a major influence (Vaughan *et al*. 1999, Vaughan *et al*. 2000a, 2000b, 2000c). The *value of information* is the change in the cost of uncertainty occasioned by gathering additional information. The value of information must be compared with the cost of information. If the value exceeds the cost, it is worth doing additional sampling to gather more information; otherwise the project should be accepted or rejected on the basis of the information on hand.

To sum up in words, the steps to find the optimal sample size via Bayesian decision analysis in a sequential approach are:

(1) Postulate an a-priori guess about the expected value of WTP per household (or per person) and a reasonable opinion about the range in the expected value.

(2) After the survey focus group sessions and the pre-test, draw a small initial referendum CV sample (e.g. N_0 of around 250 observations, say 50 in each of 5 bid groups) and administer the final questionnaire. Calculate a nonparametric sample mean WTP per household, the variance of the sample mean, and the standard error of the sample mean. Approximate the population variance, σ^2, as the product of the initial sample size N_0 and the estimate of the variance of the sample mean.

(3) Do an initial economic project CB analysis to estimate the expected value of discounted net benefits, E(NPV), at baseline conditions. Determine whether the opportunity loss follows Case I (project acceptance) or Case II (project rejection) and locate the region of opportunity loss for NPV. Establish the parameters of a linear relationship between the expected value of opportunity loss in NPV and the expected value of WTP.

(4) Combine the prior guesses from Step 1 with the sample WTP information from Step 2 following a Bayesian formula to develop posterior estimates of the mean and standard error of WTP.

(5) Using the posterior estimates from Step 4 as prior estimates, hypothetically increase the sample size from the base used in Step 2. Repeatedly compute the reduction in the variance of mean WTP that would result posterior to sample augmentation over a range of sample sizes ΔN above the initial base $N = N_0$.

(6) Assume the expected value of NPV is normally distributed. Using the linear relationship between NPV and WTP from Step 3, monetize the reduction in variance in the expected value of NPV losses associated with different degrees of augmentation of the original sample. These reductions in the *expected cost of uncertainty* ($E_{Loss,I}$ or $E_{Loss, II}$ as the case may be) from a second round of sampling represent the expected value of additional sample information, EVSI, or the benefits of sample augmentation.

(7) Over a range of hypothetical ΔNs above zero, numerically compare the *expected value of information* contributed by an additional sample observation (i.e. successive changes in the *cost of uncertainty* obtained in Step 6) to the marginal cost of a sample interview. Find the sample size where the marginal value of information is approximately equal to the cost of an additional referendum CV interview (for simplicity, assumed equal to the variable sampling cost and hence constant). The result is the *optimal (additional) sample size,* ΔN^*. The total sampling effort N_T will thus equal $N_0 + \Delta N^*$. The original small sample will be adequate if ΔN^* equals zero.[13]

The first two steps have already been covered. The next section explains the rest of the above steps in detail. A subsequent section simplifies the procedure by eliminating the need to formulate priors (Step 1). Then,

[13] If a more precise measure of ΔN^* is desired because the normality assumption is in doubt, a full Monte Carlo CB analysis can be undertaken to compute the EVSI empirically and find ΔN^*.

the approach is demonstrated using the case study project data in a worked example, assuming tight, diffuse, and nonexistent prior judgements. The effects of project cost increases on the optimal sample size are explored, and conclusions drawn.

AN OPTIMIZATION METHOD FROM BAYESIAN DECISION ANALYSIS

The Linear Payoff Function

The first key to implementing Schlaifer's (1961) approximate optimization method is the linear payoff function. It describes the relationship between the quantity measured by the sample (mean WTP in this instance) and the payoff decision variable that depends on the sample information, in this case the expected value of NPV. This function is a compact summary of the CB analysis. Net present value is written as the linear relation E(NPV) = $-\alpha + \beta \bullet$E(WTP). If the expected value of WTP from a CV survey is the only source of benefit, the intercept, $-\alpha$, represents the sum of discounted capital and operating costs of the investment. If there are any other sources of benefit (such as our energy generation benefits) they can be netted out of the discounted costs to get the intercept. The slope, β, is the marginal contribution to discounted net benefits of an increase in average WTP per household (undiscounted).[14] It too can be easily calculated by simply taking the present value of the number of beneficiaries to whom the mean WTP is applied over the project's lifetime. For our case study E(NPV in R\$) = $-$594,653,964 + 100,988,487 $\bullet$E(WTP).[15]

Given this linear relationship between discounted profits and household WTP, if the sample mean WTP is normally distributed, the outcome variable, E(NPV) will also be normally distributed with mean E(NPV) = $-\alpha + \beta \bullet$ E(WTP) and variance VAR(NPV) = $\beta^2 \bullet$ VAR[E(WTP)].[16] The break-even value that sets E(NPV) to zero is $\mu_b = \alpha/\beta$, = 594,653,964/100,988,487, or R\$5.89. For any expectation of WTP less than μ_b opportunity losses in NPV will be incurred. From Table 4 it is clear that all of the nonparametric sample means other than the Turnbull lower bound mean WTP are above the break-even value, so the correct decision is to invest.[17] But, the sample mean is a random variable so there is some non-zero probability that it could be below the break even value. For example, the preferred measure, a 75-25 weighted combination of the lower and upper bound means from Table 3, is R\$7.47, and its standard error is 0.84, putting the

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[14] It is assumed that in the CB analysis of the investment, per household (or per capita) WTP benefits are multiplied by the size of the beneficiary population in every year to obtain aggregate gross benefits. The expected signs for α (negative) and β (positive) are assigned.

[15] The intercept for costs was shadow priced. The slope also incorporates a shadow price factor to allow the WTP to be expressed in terms of the original survey responses, without shadow pricing. Because WTP per household is on a monthly basis, population in every year has to be multiplied by a factor of 12 in addition to the shadow price factor.

[16] From the properties of the expectation and variance operators E($\alpha + \beta$X) = $\alpha + \beta$E(X). This says that the expected value of a constant (α), plus another constant (β) times a random variable (letting X represent WTP) is the constant α plus the constant β times the expected value of the random variable. For example, see Paaffenberger and Patterson 1987, p. 208 and Little 1978, Chapter 10 on strictly linear relationships between random variables versus error propagation formulas.

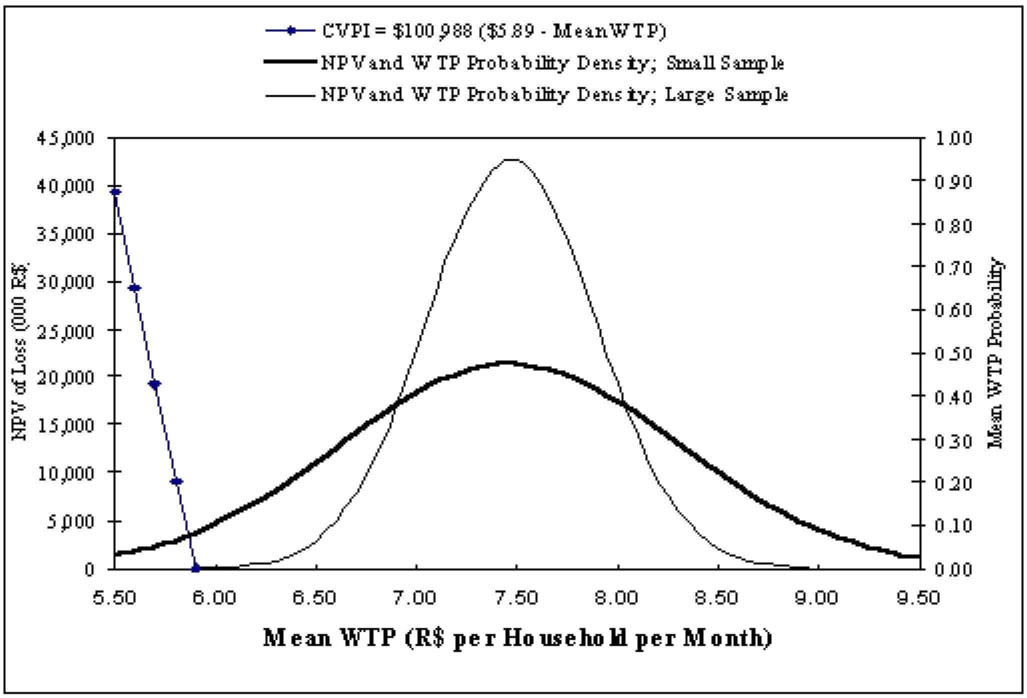
[17] Probabilistic cost-benefit analysis not reported herein strongly suggests that the investment is justified if a reasonable mean benefit just slightly higher than the Turnbull lower bound (Vaughan *et al.,* 2000a) so this paper does not employ the Turnbull mean and its variance in choosing the optimal sample size.

sample mean 1.88 standard errors above the break even value. The Kriström and Paasche means are even more distant from the break even value (R$3.31 and R$6.80 respectively in absolute terms and 3.28 and 4.96 standard errors respectively). Under these means the cumulative probability of a loss is clearly lower than it would be using the 75-25 weighted average to measure WTP and predict NPV.

A Normal Approximation to the Distribution of the Benefits of Additional Sampling

This leads to the second key to Schlaifer's approach. It is that, given a successful project on average, each possible NPV loss has a probability associated with it. Centering the net benefits distribution on the most

Figure 2. Losses and Loss Probabilities for WTP Below Break-Even



likely value of WTP, the observed sample mean, the loss probabilities are defined by the tail portion of a normal density function lying below μ_b. The sum of the products of all the possible expected losses and their associated probabilities reveals the cost of uncertainty. Figure 2 illustrates the superimposition of the linear relation representing the opportunity loss function (also called the conditional[18] value of perfect information or CVPI function) on the normal E(WTP) distribution whose standard error is assumed known from the first small sample.

[18] Opportunity losses are conditional because, after the first sample has been taken and the optimal act is chosen (in the case illustrated, invest because E(NPV)>0) they are conditional on that act. See Paffenberger and Patterson, p. 1069. For a full discussion that may be more accessible than Schlaifer's original book, see Winkler 1972.

The distribution of gains and losses in Figure 2 is centered on the sample mean WTP of R$7.47, with an initial spread given by the mean standard error of the small sample, 0.84. Increasing the sample size decreases the amount of spread in the (assumed) normal density, thus decreasing the expected value of a loss, as demonstrated by the probability density function generated by a larger sample and a smaller standard error (the lightly shaded line in Figure 2). Unlike the the small sample's density, it has an infinitesimal amount of its area to the left of the break-even point of R$5.89.

Losses in NPV are shown as positive in the figure. The horizontal axis intercept of the CVPI function is the break-even value μ_b, given discounted costs equal to $-\alpha$. It can be found by setting E(NPV) to zero and solving the linear function $-\alpha + \beta \bullet \mu_b = 0$ for $\mu_b = \alpha / \beta$. The slope, β, measures the decrease below zero in NPV for any WTP below the break-even value. So, for WTP < μ_b, the CVPI function's dependent variable equals $\beta \bullet [\mu_b - E(WTP)]$ and for WTP≥ μ_b the CVPI function is zero.

All else equal, higher discounted costs (a larger negative α) shift the CVPI function to the right, raise the requisite break-even value and, given the sample mean WTP, put more mass of the tail of the normal probability density under the non-zero part of the CVPI loss line. The expected opportunity loss or cost of uncertainty is the sum of the products of the normal density function to the left of the break-even value and the conditional value of perfect information to the left of the break-even value. Therefore, higher costs raise the cost of uncertainty, given the sample mean estimate of willingness to pay.

Developing the Objective Function

To calculate the required loss integrals, Schlaifer (1959, 1961) normalizes the extent of departure of the break even point from the sample mean WTP and computes a "unit loss integral" from the standard normal distribution (Table IV in Schlaifer 1961). Multiplying the value of the unit loss integral by β representing the marginal contribution of the sample measurement (WTP) to the outcome (NPV) yields the expected loss of an (optimal) terminal action, ELTA. As previously mentioned in Table 4, the ELTA is also called the cost of uncertainty or the expected value of perfect information, EVPI, if the decision to invest is made immediately after taking the first small sample without gathering any additional information. That is:

$$ELTA = EVPI = \beta \bullet \sigma_1(\tilde{\mu}) \bullet L_N(|D|)$$

where:

β Marginal contribution of WTP to NPV, in $.

$\sigma_1(\tilde{\mu})$ Standard error of the mean WTP posterior to taking a first small sample. The posterior can either be the standard error of the mean of that sample in the absence of a subjective prior (or under a very diffuse prior) or the posterior combination of a prior guess and the sample standard error. (See the next section for details). Specifically (Schlaifer, 1961, p.305; Paffenberger and Patterson, 1987, Ch. 23) the information contained in the prior distribution, I_0 , is the reciprocal of the prior variance of the population mean, or $1/\sigma_0^2(\tilde{\mu})$, denoting the prior with a "0" subscript. The information in the first small sample, $I_{\bar{X}}$ is the reciprocal of the variance of the sample mean, or $1/\sigma^2(\bar{x})$. The posterior variance of the mean, $1/I_1$ is the sum $1/(I_0 + I_{\bar{X}})$ and the posterior standard error is the square root of that sum, or:

$$\sigma_1(\tilde{\mu}) = \sqrt{1/I_1} = [1/(I_0 + I_{\bar{X}})]^{1/2} = [1/(1/\sigma_0^2(\tilde{\mu}) + 1/\sigma^2(\bar{x}))]^{1/2}$$

$$= [(\sigma_0^2(\tilde{\mu}) \bullet \sigma^2(\overline{x})) / (\sigma_0^2(\tilde{\mu}) + \sigma^2(\overline{x}))]^{1/2}$$

$|D|$ The absolute value of the standardized difference between the break-even value of WTP, μ_b, and the mean posterior to taking a first small sample, $E_1(\tilde{\mu})$.[19] That is, in Schlaifer's notation:

$$|D| = |\mu_b - E_1(\tilde{\mu})| / \sigma_1(\tilde{\mu}).$$

$E_1(\tilde{\mu})$ The mean posterior to taking a first small sample. It can either be the mean, $\overline{x}$, of that sample in the absence of a subjective prior or the posterior combination of a prior guess about the mean, $E_0(\tilde{\mu})$ and the sample mean. Using I_0 and $I_{\overline{x}}$ from directly above as weights:

$$E_1(\tilde{\mu}) = [I_0 \bullet E_0(\tilde{\mu}) + I_{\overline{x}} \bullet \overline{x}\,] \div [I_0 + I_{\overline{x}}\,].$$

L_N Unit loss integral, or the expected value of the difference between the normalized random variable of interest, x, and D.[20]

By taking a second sample and not acting immediately on the basis of the first small sample, it may be possible to reduce expected losses. The expected value of the new sample information, EVSI, is a function of the monetary value of the reduction in variance due to the second sample, or the reduction in the ELTA. To find the optimal size of a second sample, ΔN^*, the function to be maximized includes the benefit of variance reduction and the sampling costs. The benefits are measured as the expected value of information obtained from a second sample of size $\Delta N > 0$, assuming the population variance of WTP is known or set equal to the variance obtained from the first sample. Analogous to EVPI, the expected value of the sample information, EVSI, is the value of the reduction in losses due to the reduction in variance brought about by taking more observations, ΔN:

$$EVSI = \beta \bullet \sigma(\tilde{E}_1) \bullet L_N(|D_E|)$$

where:

$\sigma(\tilde{E}_1)$ Is the preposterior reduction in the standard error of the mean attributable to taking a second sample of size ΔN. It is calculated as the square root of an information-weighted average of the posterior variance of the mean from above, $\sigma_1^2(\tilde{\mu})$, and the variance of the mean the new sample is presumed to produce, $\sigma^2/\Delta N$. To get $\sigma^2/\Delta N$, assume the population standard deviation (of individual

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[19] For a profitable investment $D < 0$.

[20] While Schlaifer is not too clear, Bonini *et al.* (1997) define L_N as:

$$L_N = \int_{-\infty}^{-D} (-D - x) f_N(x)\, dx = \int_{D}^{\infty} (x-D) f_N(x)\, dx$$

where $f_N(x)$ is the standardized normal density function. The expression on the left applies to profitable investments, the expression on the right to unprofitable ones. The expressions are symmetric; for any $-D$ whose absolute value equals D they both produce the same value of L_N. In words, the expression on the left is the integral of the standard normal variate from negative infinity up to $-D$, which is the standardized offset between the sample mean and the break-even value. Beyond $-D$, the probability of an opportunity loss is zero, so, although the zero probability is ostensibly omitted from the calculation, L_N is not the mean loss of the truncated distribution, but measures the untruncated mean loss of the entire distribution (Jedamus and Frame 1969, p.97).

observations, not the mean) σ, is approximately equal to the standard deviation from the first sample. The value of σ^2 can then be obtained as the product of the size of the first sample, N=250, and the variance of the mean WTP (see Table 3 above), or $\sigma^2 \approx N\,\sigma^2(\bar{x})$. Then, the Bayesian preposterior reduction in the standard error of the mean (Schlaifer 1961, p. 324; Paffenberger and Patterson 1987, p. 1108; Winkler 1972, p.364; Lapin 1994, p. 464) is just the square root of:

$$\sigma^2(\tilde{E}_1) = \sigma_1{}^2(\tilde{\mu}) \left[\frac{\sigma_1{}^2(\tilde{\mu})}{\sigma_1{}^2(\tilde{\mu}) + \sigma^2/\Delta N} \right]$$

$|D_E|$ The absolute value of the standardized difference between the break-even value of WTP, μ_b, and the mean posterior to taking a first small sample, $E_1(\tilde{\mu})$, now using $\sigma(\tilde{E}_1)$ as the preposterior estimate of the standard error of the mean WTP at new sample size ΔN. That is, in Schlaifer's notation:

$$|D_E| = |\mu_b - E_1(\tilde{\mu})| \,/\, \sigma(\tilde{E}_1).$$

The costs of sampling are assumed to be a linear function of ΔN, with fixed costs K_s and unit variable costs k_s. Then, the full loss-cost function to be minimized with respect to ΔN is:

$$£ = \underset{\Delta N}{\text{Min}}: (\text{EVPI} - \text{EVSI}) + (K_s + k_s\,\Delta N)$$

Once N_0 is chosen, EVPI and K_s are constants.[21] Therefore minimization of £ is equivalent to maximizing a concentrated net benefit function £′ where EVSI represents the benefits of taking an additional sample of size ΔN and incurring total variable costs of $k_s\,\Delta N$. The expected net gain from (additional) sampling, ENGS, becomes:

$$£' = \underset{\Delta N}{\text{Max}}: \text{ENGS} = \text{EVSI} - k_s\,\Delta N = \beta \bullet \sigma(\tilde{E}_1) \bullet L_N(|D_E|) - k_s\,\Delta N$$

EVSI is a function of ΔN because $\sigma(\tilde{E}_1)$ and $L_N(|D_E|)$ are nonlinear functions of ΔN. The optimum sample size that maximizes £′ with respect to ΔN has to be found numerically. In some cases, the EVSI function will be less than the variable costs of sampling for all values of ΔN, so no additional sampling effort is warranted. In other cases, the net gain from additional sampling will be positive for ΔN between a new sample size of one and the number of cases where EVSI = $k_s\,\Delta N$, and should be relatively easy to locate. Finally, the net gain from additional sampling may initially be negative and decrease with ΔN (because EVSI < $k_s\,\Delta N$ over this range), and only later exceed variable costs in a narrow region of values for ΔN.[22] Finding the optimum in this case may depend on making a good choice of the starting value for the numerical search. Approximations to aid the search are discussed in Annex 2.

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[21] Throughout we assume the fixed costs of taking a second sample, K_s are zero, because most of these costs (for consulting services, focus groups, questionnaire pretesting and design) would be incurred to obtain the initial sample of 250 cases.

[22] See Schlaifer (1961, pp. 330-331) for a discussion of the behavior of the ENGS function; the explanation is complicated and defies intuitively obvious summary.

SIMPLIFYING THE BAYESIAN DECISION ANALYSIS BY ASSUMING TOTAL IGNORANCE

The Bayesian approach is not difficult to implement. Although it looks complicated, the steps involved are relatively simple.[23] The appearance of complexity is misleading, arising mainly from the need for an elaborate system of notation in order to keep track of the several prior and posterior means and variances involved in the several solution steps.

However, things can be made simpler yet by dropping the requirement that priors be formed. Tight priors are desirable because they reduce the required size of the optimal sample, all else equal. But while many CV studies have been done in developing countries they almost defy easy summarization (Ardila et. al. 1998) so forming reasonable prior beliefs on the basis of fragmented and inconsistent past experience is difficult indeed. In fact, unless priors are reasonably accurate they will not contribute much information on WTP location and spread beyond what an initial survey sample contains, so the influence of relatively diffuse priors on the optimal decision will be trivial. In this common situation, little can be gained from formulating wildly inaccurate prior estimates; all the information content will be in the first small sample, N_0. The simplified sequential approach suggested in this section mirrors those realities.

Modifying the Bayesian Linear Profit and Normal Loss Distribution Method

Recall the fundamental Bayesian relation between the information content of the posterior (denoted with a 1 subscript, or I_1) and the information contained in the prior (denoted with a 0 subscript, or I_0) and the sample ($I_{\overline{X}}$):

$$I_1 = I_0 + I_{\overline{X}}$$

Equivalently, the posterior information content equals the sum of the reciprocals of the prior and sample variances of the mean:

$$1/\sigma_1^2 = 1/\sigma_0^2 + 1/\sigma^2_{\bar{x}}$$

Under total ignorance there is no information content in the prior because the prior variance is extremely large, so the only useful information comes from the sample itself. Then $I_1 = I_{\overline{X}}$; $\sigma_1^2 = \sigma_{\bar{x}}^2$ and $\mu_1 = \overline{x}$. This means that the expression for the expected value of perfect information, EVPI, from above can be rewritten as a function of the standard error of the mean from the first small sample:

$$\text{ELTA} = \text{EVPI} = \beta \bullet \sigma(\overline{x}) \bullet L_N(|D|) = \beta \bullet \sigma/\sqrt{N_0} \bullet L_N(|D|)$$

[23] For a flowchart that is clear and easy to follow see Lapin (1994), Figure 26-15, p. 1046.

where now:

|D| The absolute value of the standardized difference between the break-even value of WTP, μ_b, and the mean from the first small sample, standardizing with the standard error of the sample mean:

$$|D| = |\mu_b - \overline{x}| / \sigma(\overline{x})$$

Under this simplification the prior mean and standard error entering into the preposterior step where the optimal ΔN is sought are just $\overline{x}$ and $\sigma(\overline{x})$. The value of the reduction in opportunity loss brought about by the contribution of any sample size increase of ΔN to variance reduction now becomes a simpler expression. It depends on standard deviation ($s \approx \sigma$) from the first sample, without any adjustment for a subjective prior.

$$EVSI = \beta \bullet \sigma^* \bullet L_N(|D_E|)$$

where:

σ^* the preposterior degree of reduction in the standard error of the mean contributed by a second sample of size ΔN under total ignorance. Here, σ^* is the amount of revision in the standard error of the mean from the prior to the posterior distribution.

$|D_E|$ The absolute value of the standardized difference between the break-even value of WTP, μ_b, and the mean from the first small sample, standardizing with σ^* representing the amount of revision in the standard error of the mean between the first small sample of size N_0 and the ultimate preposterior sample of size $N_0 + \Delta N$.

As before, the trick is to place the posterior variance of the mean from the previous step in the role of prior in this step, so $\sigma(\overline{x})^2$ from the first sample now plays the role of the prior. From the Bayesian rule $I_1 = I_0 + I_{\overline{x}}$ above, the preposterior variance of the mean after the second sample is taken is the sum of the prior variance from the first sample and the variance of the mean from the second sample. With the population standard deviation $\sigma = s = \sigma(\overline{x}) \bullet \sqrt{N_0}$ assumed known, the information content of the posterior is greater than the prior because of the expansion in sample size from N_0 to $N_0 + \Delta N$:

$$1/\sigma_1^2 = 1/\sigma_0^2 + 1/\sigma_{\overline{x}}^2 = 1/(\sigma^2/N_0) + 1/(\sigma^2/\Delta N) = (N_0 + \Delta N) / \sigma^2$$

As intuition would suggest, the posterior variance is the variance of the pooled sample $N_T = N_0 + \Delta N$. Taking reciprocals of the preceding:

$$\sigma_1^2 = \sigma^2 / [N_0 + \Delta N]$$

Then, (Paffenberger and Patterson, 1987 p. 1108; Lapin 1994, pp. 1032-1038; Winkler 1972, pp. 363-364) the shrinkage in the standard error of the mean due to sample size augmentation, σ^* is defined as:

$$\sigma^* = \sqrt{\sigma_0^2 - \sigma_1^2} = \sqrt{[\sigma^2/N_0] - [\sigma^2 / (N_0 + \Delta N)]}$$

Under total ignorance, σ^* is simply a function of the population variance, the initial sample size, and the addition to it. The rest of the optimization proceeds just like the pure Bayesian case.

Verification: Hueristics of a Monte Carlo Approach

Schlaifer (1961, p. 341) comments that even in "violently non-Normal problems" a number of numerical analyses showed that the approximation performs well, but he also cautions that "In problems where a good deal is at stake it will be well to use the Normal optimum only as a starting point and then use exact methods to trace out expected total loss in the neighborhood of this point."

Let us back up to the beginning and suppose the researcher admits to near total ignorance about mean WTP and its variance before an initial sample is actually taken.[24] Once the first sample of size N_0 is in hand, measures of mean WTP and its variance can be calculated. Using Monte Carlo CB analysis, the initial NPV distribution and EVPI can be easily obtained, as can the EVSI for sample size increases above the initial base.

The reduction in the standard error of the mean (σ^*) for a range of values of ΔN near the optimum previously found under the Normal approximation method can be calculated as a function of the population variance, σ, N_0 and ΔN. Repeated Monte Carlo CB simulations can then be run using the shrinkage in the standard error of mean WTP associated with each of several ascending values of ΔN as initial conditions. Without having to invoke the normality assumption, empirical estimates of EVSI and ENGS can be extracted from each simulation and the optimum ΔN^* found by trial and error.

This tedious, time consuming and computationally intensive Monte Carlo process is hardly operational. Monte Carlo CB analysis with the case study data was undertaken to verify the accuracy of Schlaifer's approximate solution. The results confirmed that in practical work Monte Carlo analysis can be safely bypassed by using Schlaifer's linear loss function and normal NPV distribution approximations instead, and maximizing EVSI with respect to ΔN. The next section demonstrates the results of applying these steps to the Tietê investment project data using Schlaifer's approximate solution that assumes linear profits, normal distributions, and a known (or knowable via the first sample) population variance.

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[24] Before consulting the Bayesian decision analysis literature and Schlaifer's optimization method we originally took an intuitive Monte Carlo loss-cost minimization approach that was similar, but not identical, to the Monte Carlo routine discussed here. We are grateful to a reviewer of an early version of this paper who asked for a theoretical justification and generalization of that intuitive brute force method. His comments directed us to the Bayesian decision analysis literature and the approximate solution for optimal sample size assuming linear profits and normal distributions.

A WORKED EXAMPLE OF FINDING THE OPTIMUM SAMPLE SIZE

In review, using actual data involves forming a prior "guesstimate" about the mean WTP and the population variance, drawing an initial "small" sample, combining the sample estimates of mean and variance with the prior estimates to arrive at posterior estimates, and using those estimates to monetize the potential reduction in expected opportunity loss that might be gained by gathering more data and hence decide whether a larger sample would be optimal. By invoking the assumption of total ignorance the sequential optimization approach only requires mean and variance information from a small original sample. If additional sampling would be optimal, the extra observations can be collected in a subsequent round of interviewing.

The case study demonstration follows the structure of the spreadsheet algorithm for finding the optimal sample size provided in Annex 3, which documents all of the calculation steps.[25] The subsequent discussion is based on the weighted 75-25 mean and its standard error, but similar calculations using any of the other means (e.g the Turnbull, Kriström or Paasche means) can easily be done by following the same structure.

The WTP Distribution

Referring to the spreadsheet algorithm for sample size optimization provided in Annex 3, the first two steps have already been touched on. The concise summary below uses Schlaifer's (1959, 1961) notation to make it easier for the reader to consult the original sources and the Annex.

Priors for the Parameters of the WTP Distribution

When example priors were constructed for the classical method above, the triangular distribution was invoked for ease of use. Now, assume instead that the population mean of WTP, μ, is a random variable having prior probabilities that can be obtained from a normal density function.[26] Since normality is the operative assumption, suppose the prior mean $E_0(\tilde{\mu}) = \hat{\mu}$ is 1% of income, or R$ 8.28. A prior measure of the standard deviation of μ is needed to summarize the a-priori variability in possible values of μ.

To guesstimate the variability in mean WTP, μ, in advance of taking any measurements at all, Schlaifer's technique (1961, p. 301) asks the decisionmaker to speculate about what interval around the prior mean would give the guess an even (50-50) chance of being correct. Somewhat arbitrarily choosing an error of R$4.00 on either side of the prior says the true mean is as likely as not to fall between R$4.28 and R$12.28.

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[25] The spreadsheet was successfully benchmarked using the example data in Schlaifer 1961. It was also independently replicated by a colleague to verify the cell formulas. The interested reader can safely duplicate the structure and insert his/her project data to compute an optimal sample size using the Bayesian approach. To get results under total ignorance, a separate spreadsheet is not needed; simply insert a very large number in Row # 10 for the prior standard error of the mean. This will wash out the influence of the prior in all subsequent calculations.

[26] In the treatment of the standard method, we had to form guesses about the mean WTP and the standard deviation of individual observations in the population. Here, we are speculating about the mean of all possible prior means and the spread in that (normal) prior distribution of hypothetical means. This explains the use of the notation $\sigma_0(\tilde{\mu})$ rather than σ_0.

From the standard normal distribution, the standardized value of $[\mu\text{-}E_0(\tilde{\mu})] / \sigma_0(\tilde{\mu})$ that demarcates 25% of the distribution's area is 0.67 so, solving $0.67 = 4.00/\sigma_0(\tilde{\mu})$, the prior for the population standard deviation of μ is 4.00/0.67 = R$5.97. This represents a weak or diffuse prior because the guess about the mean WTP has a relatively broad band of uncertainty and therefore $E_0(\tilde{\mu})$ has very little information content.

Initial Sample Estimates of the Parameters of the WTP Distribution

The expected value of the weighted sample mean is the population mean, μ. That is, $E(\bar{x}) = \mu$ = R$7.47. Recall from Table 3 that the variance and standard deviation of the distribution of sample means at N_0 =250 cases are $\sigma^2(\bar{x}) = s^2/n$ = R$0.70; and $\sigma(\bar{x}) = s/n^{1/2}$ = R$0.84. Finally, the sample standard deviation can be obtained from the sample estimate of the standard error (or deviation) of the mean (Schlaifer 1961, p. 265) and used as if it were the true population standard deviation. Thus $\sigma \approx s$ and $s = \sigma(\bar{x}) \cdot N_0^{1/2}$ = R$ 13.23.

By definition, the mean is asymptotically normally distributed. Therefore, discounted net benefits would be normally distributed if WTP benefits are the only source of gross benefit, if NPV is linearly related to mean WTP, and if costs are either deterministic or normally distributed as well.

Profits from Investment and Sampling Costs

Linear Profit Function

The relation between NPV and WTP can be easily extracted from a deterministic CB spreadsheet model through a simple sensitivity analysis by fitting a linear Ordinary Least Squares model to the NPV data points that result from varying WTP. Or, simpler yet, the shortcuts covered previously to finding the intercept and slope of the CVPI function can be used. In the case of our sample data the fit is perfectly linear. At baseline cost conditions E(NPV) = R$159,730,009 = -594,653,983 + 100,988,485 (WTP). The break-even value of E(WTP) is R$5.89 per household per month. The intercept represents discounted project costs and some market benefits for energy production that were not estimated via CV.[27]

Linear Sampling Cost Function

The sampling cost function is linear, with a marginal (equals variable) cost per observation of R$89 in shadow-priced terms, as required by IDB protocols (Powers 1981). Zero fixed costs for the second round of sampling are assumed.

Results

Under baseline initial conditions, including project costs and the diffuse prior, it is optimal to augment the initial sample size beyond 250 cases. The logit probability formula from Annex 2 indicates that additional sampling should be done, and brute force exploration reveals that additional sampling can produce positive values for EVSI net of variable sampling cost. Numerical optimization using Excel's Solver routine returns

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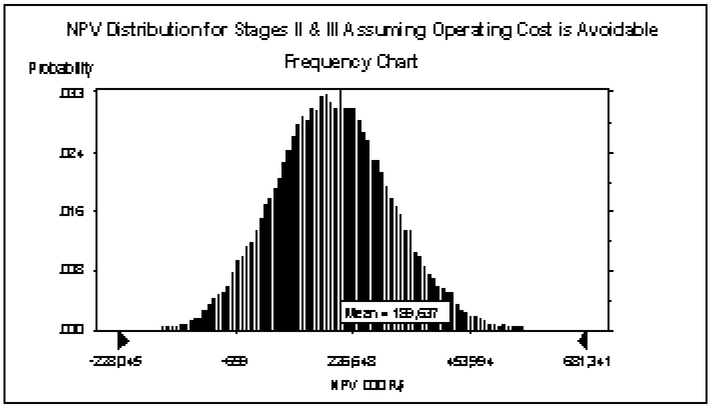
[27] The energy benefit offset to discounted capital and operating costs is R$38,892,992, while at baseline conditions total discounted costs are R$633,546,963. For changes in cost conditions relative to the baseline, the intercept $-\alpha$ in the linear profit function varies according to the linear relationship $-\alpha$ = 38,892,992 − 633,546,963 (Cost Level/Base Cost Level), while the slope remains unchanged. For instance, if costs increase by ten percent, the intercept changes from −594,653,984 at the baseline to −626,331,332. This relationship can be used to explore the effect of decreasing the standardized distance of E(NPV) from zero on the optimal sample size.

a solution of 2243 cases for ΔN^*. The optimal sample size needed en toto is 2,493 cases, which is almost 1,000 cases larger than the largest sample size recommended by the standard method. The explanation for this result, while not intuitively obvious, can be uncovered by looking at the empirical net benefits distribution shown in Figure 3.[28]

Figure 3 Empirical NPV Distribution



The empirical Monte Carlo NPV distribution in the figure is approximately normal because the influence of the normally distributed WTP benefit estimates dominates other non-normal sources of variability in the model. The baseline expected value of NPV is R$190 million and the standard deviation of the empirically generated distribution of mean NPV is R$114 million. Because the grand mean of the distribution of means is 1.67 standard errors above zero[29] the (empirical) probability of project success is 95% so there is a 5% chance of incurring an opportunity loss. While this is a fairly robust investment, recall from Figure I that the standardized distance between E(NPV) and zero has to be above about 2.4 for ΔN to be zero. Therefore, stopping with the initial small sample would have been recommended only if the sample mean WTP were R$7.92 or greater rather than the R$7.49 that was actually observed (e.g. no additional sampling would be recommended if the optimization were based on Kriström's mean of R$9.20).

Another relevant issue is how the optimal sample size would behave if the gap between the mean of the E(NPV) distribution and zero were narrowed rather than widened. The optimal sample size is sensitive to the extent of the displacement of the decision variable, the expected value of NPV, from zero, which is equivalent to the standardized offset of E(WTP) from the break even value μ_b. Looking back at the EVSI formula, as $|D_E|$ increases with a widening gap between the expectation of WTP and the break-even WTP, the more certain the decision maker becomes that the optimal decision under the prior information is correct without additional sampling. As $|D_E|$ increases the expected value of opportunity loss, L_N falls, lowering the EVSI of any particular ΔN (see Lapin 1994; Winkler 1972).

For contrast with improving on the baseline case, looking at the opposite extreme is instructive. Raising discounted costs by 25% (ie. shifting the intercept of the linear net profit function from −R$595 million to

[28] The figure was produced by 50,000 Monte Carlo trials of a risk-based CB analysis using Crystal Ball. No formal test of normality (e.g. the Komolgrov-Smirnof statistic) was undertaken, but the median, R$ 187 million, is very close to the mean, and the measures of skewness (0.12) and kurtosis (2.98) are consistent with approximate normality.

[29] From the linear profit function VAR[E(NPV)] = β^2 • VAR[E(WTP)] or $(100{,}988{,}485)^2$ •0.70 and SE[E(NPV)] is approximately $[\text{VAR}(E(\text{NPV}))]^{1/2}$ = R$84 million. This is lower than the empirical result of R$114 million because it only reflects variation in WTP benefits. Under the linear approximation the distance of NPV from zero in SE units is $160,955,974/R$84,493,829, or 1.9. Equivalently, the sample mean of WTP, R$7.47, is the same number of standard errors away from the break-even value of R$5.89 because (R$7.47 − R$5.89)/0.835 also equals 1.9 with the discrepancy due to rounding. The Monte Carlo mean NPV and its standard error differ from the deterministic linear prediction because they are affected asymmetrically by randomness in cost and timing variables that the deterministic linear function ignores.

−R$753 million; see footnote 27 above) brings the expected value of NPV very close to zero and puts half of the distribution of expected net returns into the negative region. Here, the optimal size of ΔN will be at a maximum. Table 5 shows the elements of a crude trial and error search for an optimum, assuming a diffuse prior for WTP, and Figure 4 shows the optimum graphically.

Figure 4. The Expected Net Gain from Additional Sampling

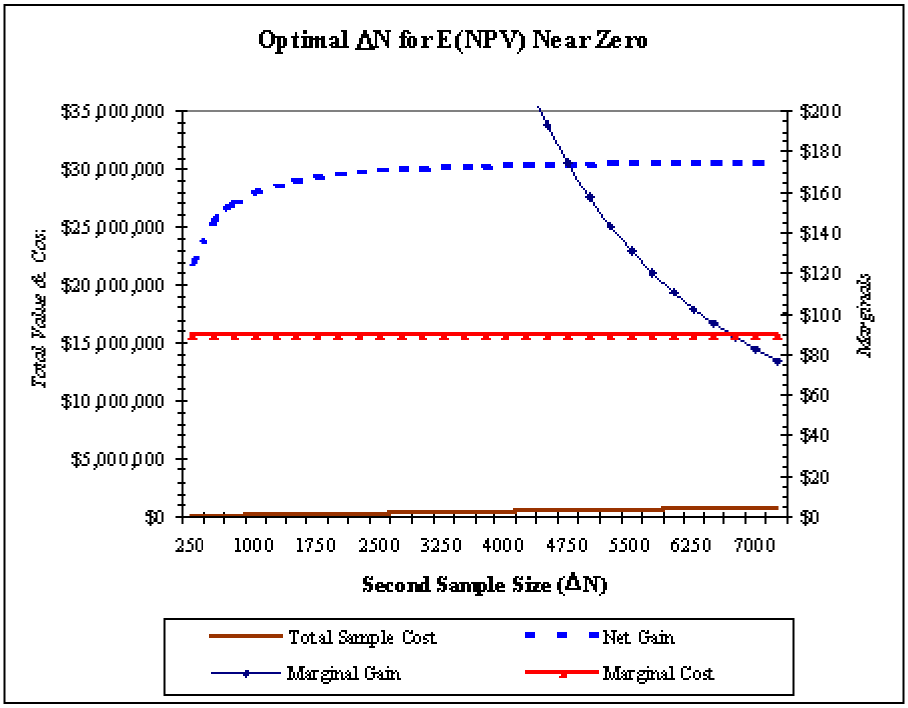


Table 5. Crude Step Search for N* with E(NPV) Near Zero

	Trial Value for ΔN:	5750	**6750 ≈ Optimum**	7750
Change in SE of Mean	σ*	0.809321811	0.811879233	0.81379242
Standardized Distance	D_E	0.0410	0.0409	0.0408
Unit Normal Loss Integral	$L_N(D_E)$	0.378772051	0.378834534	0.378881025
Value of Sample Information	EVSI (N)	$30,957,867	$31,060,815	$31,137,831
Total Sample Cost	$K_S + k_s \cdot$ N	$511,750	$600,750	$689,750
Net Gain	ENGS(N)	$30,446,117	$30,460,065	$30,448,081
Marginal Gain	EVSI (N)/ N	$120	$88	$67
Marginal Cost	k_s	$89	$89	$89

The table and figure show that the response surface is very flat. The approximate optimum is 6750 cases but the gains to additional sampling diminish quickly after about 2000 cases. By inspection of the figure, the net gains from an additional sample of 4000 cases (R$30,303,436) or even less are fairly close to the net gains at the optimum (R$30,460,065). This is consistent with Schlaifer's (1961, Figure 21.5) numerical investigations which showed that moderate departures from the optimum number of cases (± 20% or even ± 30%) are likely to be inconsequential.

To find the effect on the optimal N of the location of the E(NPV) distribution relative to E(NPV) equal to zero, the baseline E(NPV) distribution has to be shifted leftward. To shift the NPV distribution, several exercises were conducted, raising the mean of total project operating and investment costs above the baseline by 5% through 25%, while holding the mean WTP constant at R$7.49.[30] The increase of 25% is the extreme discussed above that brings E(NPV) as close as possible in the risk analysis to the break-even point of zero while remaining barely positive. Table 6 shows the effect that the standardized distance of E(NPV) away from zero has on the optimal incremental sample size ΔN* under diffuse priors, tight priors and total ignorance.

Table 6. Optimal Incremental Sample Sizes, ΔN*, Depending on Priors and Initial EVPI

Costs Relative to Baseline ($\alpha_i \div \alpha_0$)	1.0	1.05	1.10	1.15	1.20	1.25
Small Sample Standard Errors $\sigma(\bar{x})$ of E(NPV) from Zero [a]	1.90	1.52	1.15	0.77	0.40	0.02
High Sampling Cost of R$89 Per Interview						
Tight Prior [b]	0	0	0	1996	4393	6530
Diffuse Prior [b]	2243	3411	4600	5657	6409	6715 [c]
Total Ignorance	2351	3507	4673	5697	6413	6687
Low Sampling Cost of R$30 Per Interview						
Tight Prior [b]	0	0	0	3994	7458	10729
Diffuse Prior [b]	3866	5656	7506	9160	10340	10821
Total Ignorance	4022	5799	7615	9219	10343	10774

Notes:

a. Because the prior mean exceeds the sample mean, the posterior standardized distance exceeds the distance using the sample mean alone..
b. Prior guess of E(WTP) of R$8.28 with a prior ±50% error of R$0.50 for the tight prior and R$4.00 for the diffuse prior.
c. Exact optimum corresponding to the approximate optimum in Table 5 and Figure 4.

[30] Increasing project costs can be thought of as a proxy for decreasing E(WTP) or reducing the standardized distance between E(NPV) and zero at the initial sample size of N_0 = 250 cases.

Figure 1 and the results in Table 6 suggest that, in this case, small samples suffice when sampling costs are high and the mean of the NPV distribution is over about 2.4 standard errors away from the break-even point of E(NPV)=0 because the decision has little downside risk. There is no payoff in taking larger samples to shrink that risk by reducing the variance and further compressing the portion of the NPV distribution lying below zero. However substantial gains to increasing the sample size begin to emerge after the expected value of NPV falls below about 2.4 standard errors from zero. Although the algorithm does not explicitly incorporate Type II error, the fact that the required sample size increases as the gap between E(NPV) and the break-even point narrows provides protection against false acceptance of a mean WTP that justifies the project when in fact the true mean WTP would lead to the opposite conclusion.

Table 6 convincingly shows that good prior estimates of the mean WTP and its spread can significantly reduce the amount of sampling effort needed to reach an optimum CV survey sample size for investment decisions. Unfortunately, given the state of the art, good prior estimates remain unattainable, especially in developing countries.

CONCLUDING OBSERVATIONS

Small CV survey samples are probably adequate for CB analysis when the fixed and variable costs of sampling are relatively high and the investment is extremely robust. If the investment has a probability of failure of less than one percent, it is not necessary to take large samples. In this sense the common perception is correct. If investment proposals are carefully screened and only the very best of them become candidates for final project analysis, massive CV sampling efforts to measure WTP more precisely are not worthwhile. However, investments with infinitesimal risks are rare.

At the other more common extreme, when the investment is borderline because nearly half of the NPV distribution falls in the negative quadrant even though its mean NPV is barely positive, small 250 observation samples will always be inadequate. In this situation, which can be easily identified a-priori, a search for the optimum number of *additional* cases needed to augment the small sample is recommended. Variations in the size of the second sample of 20% or 30% around the optimum number of extra observations needed are not likely to have much effect on expected total loss (Schlaifer 1961, p. 337). So even if a second sample is necessary, money can be saved by only taking 70% to 80% of the recommended optimal number of additional surveys.

Like our worked example, many prospective investments fall somewhere in the middle ground between can't miss and borderline proposals. The existence of this grey area makes it risky to rely exclusively on any particular rule of thumb, be it for small, medium or large samples. But, in general, given an initial expectation for WTP and a service flow outcome so the time pattern and magnitude of gross benefits is held constant, the more costly the project the larger the sample that will be needed to justify it. This paper has tried to show how to make that general rule operational.

The literature on benefits transfer and meta analysis has been skeptical about the value of using accumulated past experience to estimate the benefits of new projects (Brouwer and Spaninks 1999). The real value of this kind of information has largely been ignored because researchers have focused mainly on the degree of correspondence between predictions of WTP generated from past studies and the actual mean WTP results from field work, working under the *as if* presumption that prior information would be used to replace new sampling.

This focus might be misplaced. Prior information need not be regarded purely as a substitute for new *in situ* CV survey sampling. The two are complementary because combining good prior predictions of WTP with actual survey samples can save a good deal of new sampling effort and money. The synthesis of past CV results to make accumulated contingent valuation WTP information transportable to new situations might pay off, but only if the status quo were to involve the systematic use of an optimal Bayesian sample size protocol under the handicap of total ignorance or diffuse priors.

To date, international lending institutions have not systematically followed reliable protocols for selecting CV survey sample sizes in their appraisal of prospective investments, and they are not alone. In this operating environment, new information has little value beyond its immediate contribution to the specific decision at hand, which is to economically justify a given project. The WTP data is used once and then forgotten. But this information could become more valuable if sample sizes were chosen in the future that take account of the expected opportunity loss the actual investor might incur. Then, there would be a good reason to take a longer range view about the value of information.

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ANNEX 1

Data and Formulae for Nonparametric Estimates of Mean WTP and Its Variance

Table A1.1 The Small Sample Data

Bid Group j	Bid R$ [b_j]	Total "No" Answers [N_j]	Total Cases [$Total_j$]
j=0	0	na	none
j=1[a]	2	35	98
j=2	5	34	54
j=3	12	39	51
j= M =4	20	37	47
j=M +1=5	> 20	na	none
Column Totals:		145	250

Note:
a. The first two bid groups (R$ 0.50 and R$2.00) were pooled at R$2.00 to preserve monotonicity.

Three Nonparametric Measures of the Mean and the Variance of the Mean

There are three nonparametric estimators of the mean: (1) a lower bound measure that understates mean WTP (the Turnbull mean, see Haab and McConnell, 1997); (2) an intermediate measure (Kriström's mean, see Kriström 1990 and Boman *et al.* 1999) and (3) an upper bound measure that overstates mean WTP (the Paasche mean, see Boman *et al.* 1999). The logic behind all three nonparametric estimators is the same. The proportion of "No" answers at each bid level b_j provides a discrete stepwise approximation to the cumulative distribution function. The mean E(b) of a continuous random variable x with a cumulative distribution function F(b)[31] and probability density function f(b) – which is the first derivative of F(b) w.r.t. b – is given by:

(1) $E(b) = \int b\, f(b)\, db$

The problem is to use a discrete approximation to compute:

(2) $E(b) = E(WTP) \approx \sum_{J} b_j\, f(b_j)$

where the range of b is from zero to some upper limit b_{max} that forces F(b) close to 1.0 because the bid is so high that almost all respondents would be unwilling to pay that amount for the environmental improvement.

Haab and McConnell's lower bound Turnbull mean sets each b_j to the lower bound of the bid interval (i.e. the first interval runs from zero to the lowest bid offered so b_j at j equals zero is set to 0, etc). The intermediate and upper bound means are obtained by simply redefining the point of evaluation, b, in each

[31] To obtain the mean from the survival function, 1-F(x), the same reasoning developed below also applies.

interval to some fraction κ times the lower bound plus (1-κ) times the upper bound of the interval, where $0 \le \kappa \le 1$. Kriström's intermediate mean sets κ to ½ (the mid point of the interval) while the upper bound mean sets κ to 0. While Boman *et al.* (1999) try to put all three measures on a consistent symbolic footing, there are errors in their notation for the means and, unfortunately, their variance formulas are incorrect.[32] Below, all three measures are recast in Haab and McConnell's notation, which is conceptually correct.

Table A1.2. Formulae for Nonparametric Means and Their Variances

Measure	Mean[a]	Variance of Mean
Lower Bound	$\sum_{j=1}^{M+1} b_{j-1} p_j$	$\sum_{j=1}^{M+1} (b_{j-1})^2[V(F_j) + V(F_{j-1})] - 2\sum_{j=1}^{M} (b_j b_{j-1})V(F_j)$
Inter-mediate	$\sum_{j=1}^{M+1} [\kappa b_{j-1} + (1-\kappa) b_j] p_j$	$\sum_{j=1}^{M+1} [\kappa b_{j-1} + (1-\kappa)b_j]^2[V(F_j) + V(F_{j-1})] - 2\sum_{j=1}^{M} [\kappa b_{j-1} + (1-\kappa)b_j] \bullet [\kappa b_j + (1-\kappa) b_{j+1}]V(F_j)$
Upper Bound	$\sum_{j=1}^{M+1} b_j p_j$	$\sum_{j=1}^{M+1} (b_j)^2[V(F_j) + V(F_{j-1})] - 2\sum_{j=1}^{M} (b_j b_{j+1})V(F_j)$

Notes:

a. The probability density in bid group j, p_j, equals the difference between the estimates of cumulative density in the current and preceding bid groups, $F_j - F_{j-1}$, where, letting N_j represent the number of "No" responses and Y_j the "Yes" responses in group j, $F_j = N_j / (N_j + Y_j)$. There are j = 1...M distinct bids specified in the survey. The bid j = M+1 is the ultimate bid level that the researcher must *assume*. It presumably drives F_j to 1.0.

b. The variance of each proportion F_j is equal to $[F_j \bullet (1 - F_j)] \div (N_j + Y_j)$.

c. The parameter κ is assumed by the researcher to form a weighted average of the lower and upper bound bids in any interval. Kriström's mean uses κ = 0.5, but any value of κ between zero and one is admissable. If κ = 0 the Turnbull lower bound mean results, and κ = 1 returns the Paasche upper bound mean of Boman *et al.* 1999.

Our preferred measure of the mean that is used in the main text sets κ to 0.75, which is more conservative (lower) than Kriström's intermediate mean. Table 3 below provides the mechanics of our intermediate mean and variance calculation. Calculation of the rest of the means and variances proceeds analogously.

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[32] The Bowman *et al.* (1999) variance formulas incorrectly treat the bid, not the cell proportions, as a random variable and are inconsistent with the respective expected value formulas because they were not derived from them using the fundamental rules pertaining to the variance of a sum of random variables. Instead, an inappropriate textbook formula was forced to stand in. We discovered this discrepancy by comparing the variances of the lower bound means produced using the Haab and McConnell formula and the Bowman formula. The variance from the latter was roughly double the former. We then ran 20000 trials of a Monte Carlo simulation in Crystal Ball letting each cell proportion at each trial involve a draw from a binomial distribution with parameters defined as the number of observations in each bid cell and the probability of refusal. The empirical results independently confirmed the correctness of the Haab and McConnell variance formula. Our formulas for the variances of the intermediate and upper bound means were derived by extending the Haab and McConnell formula to these situations and were also successfully validated by Monte Carlo simulation.

Table A1.3. Calculation of Intermediate Nonparametric Mean and Variance of Mean Assuming = 0.75.

Bid Group j	Bid	Weighted Bid bwt_j = 0.75	Total "No" Answers	Total Cases	Cumulative Distribution	Probability Density	Bid Group j Variance	Square of Weighted Bid	Product of Adjacent Weighted Bids	Product of Adjacent Group Variances	Variance Term #1	Variance Term #2	E(WTP)
	[b_j]	[b_{j-1} + (1- b_j]	[N_j]	[$N_j + Y_j$]	[F_j=N_j / $Total_j$]	$p_j = F_j - F_{j-1}$	$V(F_j)$	bwt_j^2	$bwt_j \bullet bwt_{j+1}$	$V(F_j) - V(F_{j-1})$	$bwt_j^2 \bullet [V(F_j) - V(F_{j-1})]$	$-2 \bullet (bwt_j \bullet bwt_{j+1}) \bullet V(F_j)$	$bwt_j \bullet p_j$
j=0	0		na	none	0.000000	na	0.000000	…	…	…	…	…	
j=1	2	0.50	35	98	0.357143	0.357143	0.002343	0.2500	1.3750	0.002343	0.000586	-0.006443	0.18
j=2	5	2.75	34	54	0.629630	0.272487	0.004318	7.5625	18.5625	0.006661	0.050375	-0.160322	0.75
j=3	12	6.75	39	51	0.764706	0.135076	0.003528	45.5625	94.5000	0.007847	0.357506	-0.666802	0.91
j=4	20	14.00	37	47	0.787234	0.022528	0.003564	196.0000	350.0000	0.007092	1.389995	-2.494630	0.32
j=M+1=5	40	25.00	na	0	1.000000	0.212766	0.000000	625.0000	0.0000	0.003564	2.227348	0.000000	5.32
	Column Totals:		145	250		1.000000					4.025811	-3.328198	7.47
												Mean:	7.47
												Variance of the Mean (Term#1 + Term #2):	0.697613
												Standard Error of the Mean (Square Root of Variance):	0.835233

ANNEX 2

Approximations to Indicate Whether More Sampling is Needed and the Size of ΔN

Schlaifer (1961) relates the need for more sampling to the values of his *essential parameters of the problem of sample size*, labeled Z_0 and the previously defined D_0, and provides a nomogram (Figure 21.4, p. 332) that indicates whether it is worth taking a second sample, depending on the values of these parameters. The essential parameter Z_0 is a function of the marginal contribution to NPV of a change in WTP (i.e. β), the marginal costs of sampling (i.e. k_s) the population standard deviation of WTP (i.e. σ, approximated by the standard deviation, s, from the first sample) and the standard error of the mean WTP posterior to taking a first small sample (i.e. $\sigma_1(\tilde{\mu})$):

$$Z_0 = [\sigma_1(\tilde{\mu}) / \sigma] \bullet [\beta\sigma / k_s]^{1/3}$$

Since many readers may not have easy access to Schlaifer's book and the decision nomogram, we fit a logit probability model with a second-order index function to 197 pairs of D_0 and Z_0 points read from his Figure 21.4, coding the dependent variable as 1 if the nomogram recommended "Sample before Acting", and as 0 if it recommended "Act without Sampling". The model fit was reasonably good (pseudo R^2 of 0.80) with 182 correct predictions and 15 false predictions. As a substitute for the Schlaifer's figure, a decision to take an additional sample should be made if the predicted probability from the model is equal to or greater than 0.5:

$$\text{Probability}_{\text{Sample}} = 1/ \{1 + exp\ [2.2061 - 1.1255\ Z_0 - 4.6102\ D_0 + 0.\ 0066\ (Z_0)^2 + 4.8539\ (D_0)^2\ ]\} \geq 0.5\ ?$$

If the answer from evaluating the probability model (or Schlaifer's Figure 21.4) is "Sample before Acting" it will be necessary to search for the optimum, ΔN^*. A good starting value for the grid search over ΔN can be found from a rough approximation to the optimum, $\Delta N_{\text{Approx.}}$ using another simplification from (Schlaifer 1961, pp. 334-335):

$$\Delta N^* \approx \Delta N_{\text{Approx}} = [(\beta\sigma/k_s)^{1/3}]^{1/2} \bullet [½\ Z_0 / P_N\ (D_0)]$$

where P_N (D_0) is the probability density of the standard normal density function evaluated at D_0. The optimum size ΔN^* of the addition to the original small sample (N_0 = 250) either be found through trial and error by constructing crude fixed-step size grid search in the neighborhood of the initial guess, ΔN_{Approx}.or by calling an optimization routine like Excel's Solver after specifying $\Delta N_{\text{Approx.}}$ as the starting value.

ANNEX 3

Spreadsheet Formula Layout for the Optimum Sample Size Calculation: Tietê Case at Baseline Under Diffuse Prior Information (1998 Brazilian Reals)

Row #	COLUMN A: Labels	COLUMN B: Labels	COLUMN C: Formulas	RESULT	COMMENT
1-3	**I. Form Priors**				
4	*Prior Mean and Standard Deviation of WTP*				
5	Prior Mean	$E_0(\tilde{\mu}) = \hat{\mu}$	0.01*828	$8.28	Guess the mean WTP.
6	Prior error @50%	e	4	$4.00	Guess the variation in the mean covering a ±50% interval.
7	Prior error UL@50%	$\hat{\mu} + e$	C5+C6	$12.28	Find the upper limit of the interval.
8	Prior error LL @50%	$\hat{\mu} - e$	C5-C6	$4.28	Find the lower limit of the interval
9	Prior Upper Alt. U @+25%	$[\mu - E_0(\tilde{\mu})] / \sigma_0(\tilde{\mu})$	NORMINV(0.75,0,1)	0.6745	Find the Standard Normal z statistic value for each tail outside the interval (i.e. each contains 25%)
10	Prior Standard Deviation of Population Mean	$\sigma_0(\tilde{\mu})$	(C7-C5)/C9	$5.9304	Find the standard deviation of population mean WTP implied by the prior, based on the error limits, e.
11	Prior Variance of Population Mean	$\sigma_0^2(\tilde{\mu})$	C10^2	$35.1697	Find the variance of mean WTP implied by the prior.
12-13	**II. Get Posterior Distribution from Normal Prior and Sampling Distributions, Sampling Variance Known**				
14-15	*Sample Data*				
16	Initial (or First) Sample Size	N_0	250	250	Input value. Number of cases in initial small sample.
17	Expected Value of Sample Mean	$E(\bar{x}) = \mu$	7.47	$7.47	Input value. Calculate(Nonparametric) Mean WTP
18	Sample Variance	s^2	C19^2	$174.40	Calculate variance of WTP from sample estimate of standard deviation of WTP immediately below.
19	Sample Standard Deviation	s	C16^0.5*C21	$13.21	Calculate sample estimate of standard deviation of WTP using standard error and square root of sample size, 250.
20	Variance of Sample Mean	$\sigma^2(\bar{x}) = s^2 / N_0$	0.449	$0.70	Input value. Calculate variance of sample Mean WTP.
21	Standard Error of Sample Mean	$\sigma(\bar{x}) = s / N_0^{1/2}$	0.670	$0.84	Input value. Calculate standard error of sample Mean WTP as square root of variance of mean WTP.
22	*Posterior Calculation*				
23	Posterior Mean	$E_1(\tilde{\mu})$	(C5*1/C11+C17*1/C20) / (1/C11+1/C20)	$7.49	Compute posterior mean as a weighted average of prior and sample means based on quantity of information provided by each. See Rows 27 through 29 below.
24	Posterior Standard Error of Mean	$\sigma_1(\tilde{\mu})$	(1/(1/C11+1/C20))^.5	$0.83	Compute posterior standard error of mean as a weighted average of prior and sample standard errors based on quantity of information provided by each. See Rows 27 through 29 below.

Row #	COLUMN A: Labels	COLUMN B: Labels	COLUMN C: Formulas	RESULT	COMMENT
25	Posterior Variance of Mean	$\sigma_1^2(\tilde{\mu})$	C24 ^2	$0.68	Compute as square of posterior standard error.
26	*Quantity of Information*				
27	In Sample Mean	$I_{\bar{x}}$	1/C20	1.43	Relative Information content.
28	In Prior Mean	I_0	1/C11	0.03	Relative Information content.
29	In Posterior Mean	I_1	1/C24^2	1.46	Pooled information content
30	Check	$I_1 = I_0 + I_{\bar{x}}$	C27 + C28	1.46	
31-32	**III. Expected Profit After First Small Sample (i.e. Based on Posterior from II Above)**				
33	Linear Profit Function Intercept	α	-594653984	-$594,653,984	Input data for intercept of linear relation between NPV and WTP, i.e. NPV=α+β*WTP. Calculate outside from data generated by deterministic cost-benefit analysis model.
34	Linear Profit Function Slope	β	100,988,487	$100,988,487	Input data for slope of linear relation between NPV and WTP. Calculate outside from data generated by deterministic cost-benefit analysis model.
35	Expected Profit (NPV)	$\alpha + \beta \bullet E_1(\tilde{\mu})$	C33+C34*C23	$161,738,697	Expected NPV at posterior baseline mean WTP of $5.83.
36	Break Even Value of WTP	$\alpha \div - \beta$	C33/-C34	$5.89	Value of WTP that sets expected NPV to zero, given posterior mean WTP.
37	Standardized Loss	$\lvert D \rvert = \lvert \mu_b - E_1(\tilde{\mu}) \rvert / \sigma_1(\tilde{\mu})$.	ABS(C36-C23)/C24	1.94	Standardized distance between break-even WTP and posterior baseline mean.
38	Unit Normal Loss Integral	$L_N(D)$	NORMDIST(C37,0,1,0) -C37* (1-NORMDIST(C37,0,1,1))	0.010051	Unit Normal Loss Integral Factor (Schlaifer, 1961, Table IV, p. 370)
39	Expected Loss of Optimal Terminal Action	ELTA = EVPI = $\beta \bullet \sigma_1(\tilde{\mu}) \bullet L_N(\lvert D \rvert)$	C34*C24*C38	$839,505	Expected toss of making an optimal "go" or "no-go" investment decision at this point <u>without</u> any additional sampling (i.e. based only on the initial priors and the original small sample N=250 cases).
40-41	**IV. Optimal New Sample Size (Depends on Data in Rows 46 to 64)**				
42	Optimal New Sample Size	ΔN	Insert Approximate Trial Size to Initialize Oprimization from C83 (2683) and then Optimize	**2243**	**Size of a hypothetical second sample to augment the initial sample of N=250. To find an optimum, iterate over alternative values of N to find the sample size that maximizes the Expected Net Gain from a second sample (ENGS(N) in Row 64 below.**
43-45	**V. Expected Value of Information from a New Sample vs. Cost of Sampling**				
46	*Current Prior Set Former Posterior=New Prior*				
47	Current Prior for Mean=Posterior From III	$E_1(\tilde{\mu})$	C23	$7.49	Repeat of previously computed posterior value for new set of calculations. It now becomes a <u>prior</u> value in this step.
48	Current Prior for Standard Error of Mean	$\sigma_1(\tilde{\mu})$	C24	$0.83	Repeat. Former posterior in III now a prior.
49	Current Prior for Variance of Mean	$\sigma_1^2(\tilde{\mu})$	C25	$0.68	Repeat. Former posterior in III now a prior.
50	Variance of (Population) Mean at New	$\sigma^2(\bar{x}) = \sigma^2 / \Delta N^2$	C19^2/C42	$0.08	Key step. Standard error of the mean

Row #	COLUMN A: Labels	COLUMN B: Labels	COLUMN C: Formulas	RESULT	COMMENT
	Sample Size, Given Population Sigma Assumed Known and ≈ s				of the new sample of arbitrary size N=50. Used below to get revised posteriors in Rows 52 and 53.
51	*New Posteriors*				
52	Preposterior Reduction in Variance of Mean from N	$\sigma^2(\tilde{E}_1)$	C49* (C49 / (C49+C50))	\$0.61	Calculate updated change in variance due to a second sample of size N using posterior from first sample as a prior and the new sample estimate from Row 50.
53	Preposterior Reduction in Std Error of Mean from N	$\sigma(\tilde{E}_1)$ or σ^*	C52^0.5	\$0.78	Square root of change in variance in Row 52
54	|D| Absolute Value of Prior Standardized Loss from above	$\|D\| = \|\mu_b - E_1(\tilde{\mu})\| / \sigma_1(\tilde{\mu})$	C37	1.94	Repeat from above. Uses posterior 1 as new prior with 0 subscript
55	$\|D_E\|$ Absolute Value of Change in Standardized Loss due to ΔN	$\|D_E\| = \|\mu_b - E_1(\tilde{\mu})\| / \sigma(\tilde{E}_1)$	ABS(C36-C47)/C53	2.04	Uses New Posterior Standard Error of Mean to calculate standardized difference between break-even WTP and mean posterior to first sample of N=250.
56	Unit Normal Loss Integral	$L_N(D_E)$	NORMDIST(C55,0,1,0) - C55*(1-NORMDIST(C55,0,1,1))	0.00755	Unit Normal Loss Integral Factor for D_E (Schlaifer, 1961, Table IV, p. 370)
57	Expected Value of Sample Information	EVSI (ΔN)	C34*C53*C56	\$597,647	Expected value of information in new optimal sample of ΔN* = 2243
58	Unconditional Expected Terminal Loss	UETeL(N)	C39-C57	\$241,859	Terminal Loss after a new sample of ΔN=2243 is taken. Equal to ELTA before an additional sample (ie. at ΔN=0) minus the EVSI(N)
59	*Sampling Costs*				
60	Fixed Cost	K_s	0	0	Set to zero to simplify. Include actual value here.
61	Marginal=Variable Cost	k_s	89	\$89	Input data. Example estimate is in 1998 Brazilian Reals(R\$)
62	Total Sample Cost		C60+C61*C42	\$199,680	Multiply marginal sample cost by N and add to fixed cost.
63	*Expected Net Gain from a Second Sample*				
64	Expected Net Gain from a Second Sample of Size N	ENGS(ΔN)	C57-C62	\$397,967	**EVSI((N) minus the total cost of taking an additional sample of size (N). This is the Objective to optimize over alternative values of (N). Use a grid search (see text) or, more efficiently, SOLVER in EXCEL, setting the TARGET CELL as C64, equal to MAX; by changing the (N) cell, C42, subject to the constraint that C42 is ≥ a small positive number (e.g. 0.001)**

Row #	COLUMN A: Labels	COLUMN B: Labels	COLUMN C: Formulas	RESULT	COMMENT
65-67	**VI. Addendum: Decide if Additional Sample is Necessary and Compute Approximate ΔN for Optimization Starting Value (See Annex I)**				
68	*D Value Standardized Loss*	D	**C54**	**1.94**	**The First Essential parameter.** From Above
69	*Intermediate Components for Calculating Z_0*				
70	*First Component*				
71	$\sigma_1(\tilde{\mu})$ From Small Sample N_0		C48	$0.83	From Above
72	σ guess from sample data = s		C19	$13.21	From Above
73		$\sigma_1(\tilde{\mu}) / \sigma$	C71/C72	0.06	
74	*Second Component*				
75		k_t	C34	$100,988,487	From Above
76		$\mathbf{k_s}$	C61	$89	From Above
77		$(k_t \quad k_s)^{1/3}$	((+C75*C72)/C76)^0.33	245.19	
78	*Z Value*	$\mathbf{Z_0}$	**C77*C73**	**15.36**	**The Second Essential Parameter**
79		*	((1/C78* 0.5)* (NOR MDIST (C68,0,1 ,0)))^0.5	0.0446	Crude approximation of the optimal ratio of: $n/((k_t \quad k_s)^{1/3})^2$ from Schlaifer
80		$((k_t \quad k_s)^{1/3})^2$	C77 ^2	60,116	Denominator in ratio of $\eta = \Delta N / ((k_t \quad k_s)^{1/3})^2$ See Row 83.
81	Probability an Additional Sample Should be Taken	Logit Probability Model Approximation	+1/(1+exp(2.206+C78*-1.1255+C68*-4.6102+C78^2*0.006596+C68^2*4.8539)	0.99	Logit function fit to data from Schlaifer's Sample Decision Figure 21.4
82	Take an Additional Sample Before Acting?		**IF(C81>0.5,"YES","NO")**	Probably **Yes**	**Result from evaluating Logit model**
83	Quick Approximate Optimal Acting?" is "NO"	(Ignore if Answer to "Sample Before	**C79*C80**	**2683**	**Approximate solution for N. Use in C42 above to initialize grid search or SOLVER optimization.**