Inter-American Development Bank

The Integrated Economic-Environmental Modeling (IEEM) Platform

IEEM Platform Technical Guides: IEEM Mathematical Statement

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Environment, Rural Development and Risk Management Division

TECHNICAL NOTE Nº
IDB-TN- 01842

March 2020

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Cataloging-in-Publication data provided by the
Inter-American Development Bank
Felipe Herrera Library
Banerjee, Onil.
The Integrated Economic-Environmental Modeling (IEEM) Platform: IEEM Platform technical guides: IEEM mathematical statement / Onil Banerjee, Martin Cicowiez.
p. cm. — (IDB Technical Note ; 1842)
Includes bibliographic references.
1. Environmental economics-Mathematical models. 2. Environmental policy. 3. Environmental auditing. 4. Economic development-Environmental aspects. I. Cicowiez, Martin. II. Inter-American Development Bank. Environment, Rural Development and Risk Management Division. III. Title. IV. Series.
IDB-TN-1842

JEL Codes: D58 Computable and Other Applied General Equilibrium Models; Q56 Environment and Development • Environment and Trade • Sustainability • Environmental Accounts and Accounting • Environmental Equity • Population Growth.

Keywords: Integrated Economic-Environmental Modeling; IEEM; Public policy and investment analysis; ex-ante economic analysis; cost benefit analysis; dynamic computable general equilibrium modeling; natural capital valuation; ecosystem services; System of Environmental Economic Accounting; SEEA.

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The Integrated Economic-Environmental Modeling (IEEM) Platform

IEEM Platform Technical Guides: IEEM Mathematical Statement

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Acknowledgements

This work was funded by the Inter-American Development Bank.

Abstract

This technical guide presents the Integrated Economic-Environmental Modeling (IEEM) Platform, a recursive dynamic computable general equilibrium model developed at the Inter-American Development Bank for the medium and long run analysis of public policies and investments. IEEM pays special attention to the modeling of the interaction between the economy and the environment with the integration of the System of Environmental-Economic Accounting. This document presents a discursive description of IEEM, followed by model equations and variables.

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Contents

1. Introduction

This document presents the Integrated Economic-Environmental Modeling (IEEM) Platform, a recursive dynamic computable general equilibrium (CGE) model developed at the Inter-American Development Bank (IDB) for medium and long run public policy and investment analysis (Banerjee et al. 2016, Banerjee et al. 2019a, Banerjee et al. 2019b). IEEM pays special attention to modeling the interaction between the economy and the environment with the integration of information from the System of Environmental-Economic Accounting (SEEA). The document presents a discursive description of IEEM followed by a presentation of model equations and variables.

2. A Discursive Presentation of IEEM

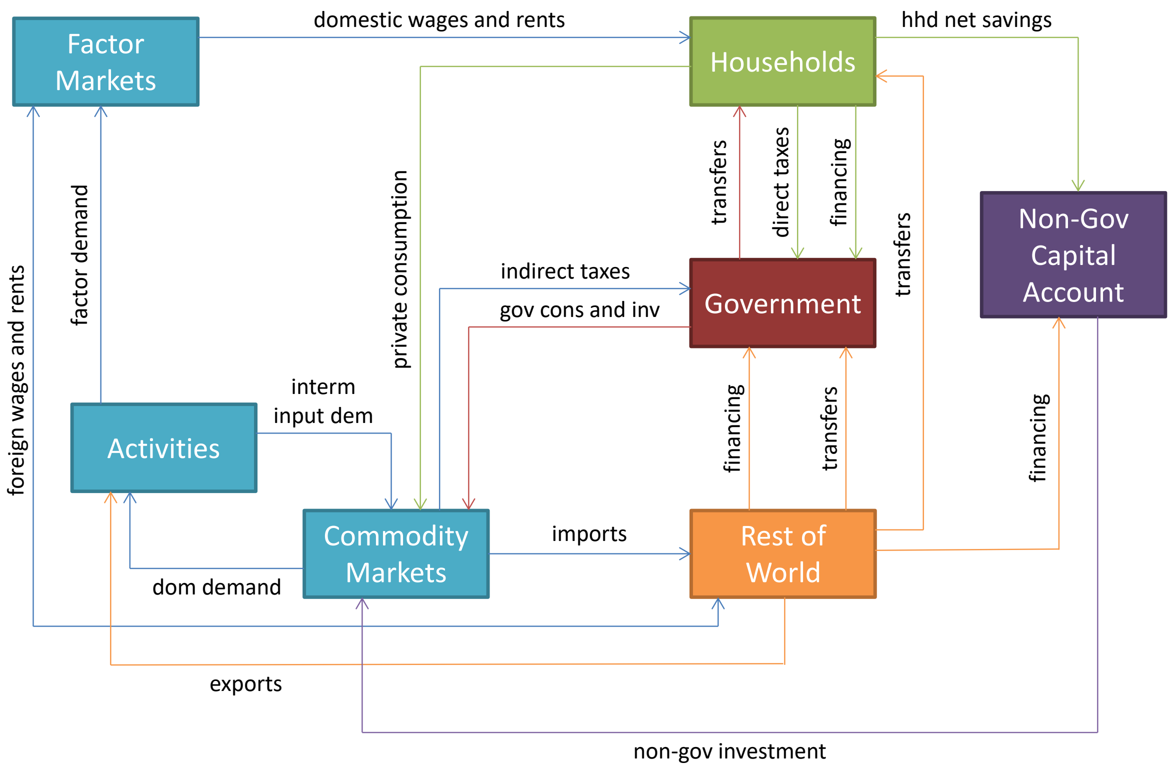
IEEM is a CGE model that integrates environmental information associated with natural capital, organized under the first international statistical standard for environmental-economic accounting, The System of Environmental-Economic Accounting (SEEA; United Nations, European Commission et al. 2014). For example, the SEEA links information on land use, non-renewable natural resources extraction, and emissions to the consumption of energy goods and services reported in the System of National Accounts. IEEM extends conventional CGE analysis to the study of phenomena that impact the economy and the environment.

In addition, IEEM pays special attention to the modeling of sectors that use natural resources (or natural capital), both from the supply side and from the demand side. IEEM further innovates on a conventional CGE approach with (a) a microsimulation model to more accurately estimate impacts on poverty and inequality, and (b) modeling of changes in land use and land cover enabling estimation of the effects of public policy and investment on not only provisioning ecosystem services but also regulating ecosystem services (e.g., pollination and erosion mitigation, among others; Banerjee et al. In press).

2.1 IEEM Overview

IEEM has some relatively standard features common to most CGE models (see, for example, Lofgren et al. 2002 and Robinson 1999) and others that make it particularly useful for assessing the policy and investment impacts on the economy and the environment. Figure 1 summarizes the main economic flows captured by IEEM. Arrows represent the flow of income, goods and factors of production. In general, CGE models including IEEM are concerned with the real side of the economy; they exclude monetary aspects such as inflation. IEEM and other CGE models focus on capturing changes in the way a country's economic resources are allocated.

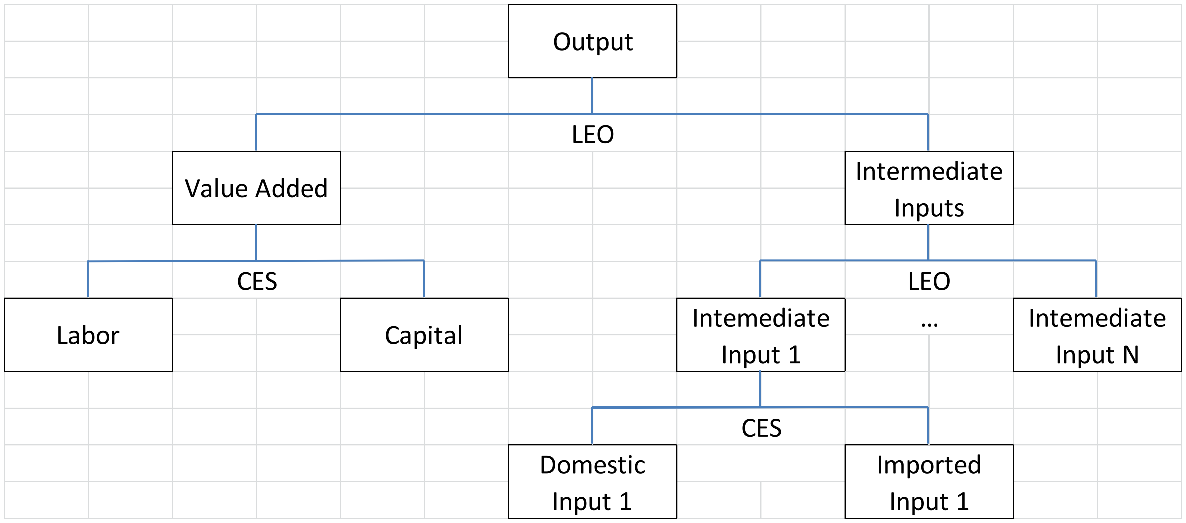
Figure 1. The circular flow of income in IEEM.



Source: Authors' own elaboration.

The productive sectors are represented by activities that maximize profits in competitive markets. The production technology, in its simplest form, is summarized in Figure 2. First, value-added and intermediate inputs are combined in fixed proportions. Value-added in turn is generated by combining primary factors of production (labor, capital and, depending on the activity, natural capital). Intermediate inputs may come from domestic supply or imported from the rest of the world. Activities can produce one or more products, while each product can be produced by more than one activity. The output of each good or service can be destined to the domestic market and/or exported to the rest of the world.

Figure 2. The production function.



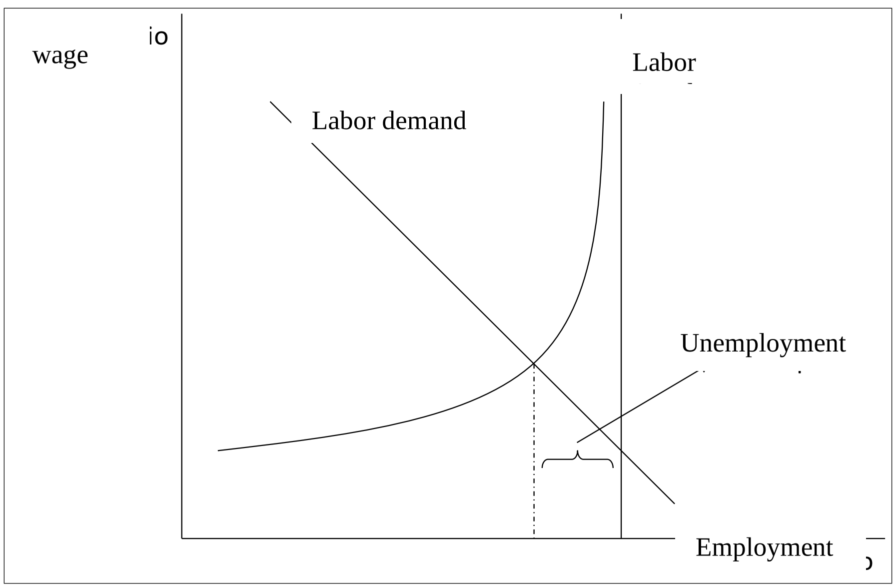
Note: CES is constant elasticity of substitution and LEO is Leontief fixed proportions. Source: Authors' own elaboration.

Typically, IEEM identifies families/households, enterprises, government, and the rest of the world as institutional sectors. Households obtain their income from the productive factors they own, as well as from the transfers they receive from the other institutions included in the model. Households allocate their income to buy the goods and services they consume, save, pay direct taxes and make transfers to the other institutions. The government receives tax revenues, it both consumes and supplies goods and services, it makes transfers to households, and it borrows. The rest of the world demands exports and supplies imports. The model identifies eight types of taxes, namely: household income tax, activity tax, consumption tax, value added tax, export tax, import tax, factor income tax, as well as taxes on the use of factors of production. Marketing and transport margins are explicitly modeled. It is assumed that the corresponding marketing and transport margins required to move a good from the producer to the consumer are demanded in fixed proportions.

In terms of foreign trade, goods and services are assumed to differ according to the country of origin (Armington 1969). Thus, trade can be modeled in two directions, in other words, the same good or service can be imported and exported simultaneously. The combination of national and imported products is carried out at the border of the modeled country. Thus, the national and imported composition of the product is the same regardless of the destination of the product. The imperfect substitutability between imports and domestic products is implemented through a CES (Constant Elasticity of Substitution) function. On the production side, a symmetrical assumption is made: exports are an imperfect substitute for sales to the domestic market. This imperfect transformation is implemented through a CET (Constant Elasticity of Transformation) function. In addition, the modeled country is considered small, therefore, it takes as a given the international prices of the products that it trades with the rest of the world.

In the labor market it is assumed that there is unemployment generated by a wage curve (Figure 3), which establishes a negative relationship between the wage level and unemployment rate (see Blanchflower and Oswald 1994). In all cases, labor is perfectly mobile between sectors. On the other hand, capital, once installed, is immobile between sectors.

Figure 3. The labor market with unemployment.



Source: Authors' own elaboration.

2.2 IEEM Dynamics

IEEM is a recursive dynamic model, that is, economic agents such as households and firms are considered to be myopic, so their expectations are stationary. Consequently, economic agents expect future prices to be identical to the prices of the current period. There are four sources of dynamics in the IEEM: capital accumulation, labor force growth, natural resources supply growth, and changes in factor productivity. At the beginning of each period, sectoral capital stocks are modified based on the investment of the previous period. On the other hand, the endowments of other productive factors grow exogenously. The investment and capital stocks of each period differ between public and private sectors.

In general, conventional CGE analysis does not generate estimates of distributive impacts with any meaningful level of disaggregation. To overcome this limitation, IEEM incorporates a microsimulation model to estimate the scenario impacts on poverty and inequality. Specifically, IEEM results for per capita income (or expenditure) for each of the representative households identified in the Social Accounting Matrix (SAM) are used to modify per capita income of each of the households that are recorded in the most recent Household Income and Expenditure Survey for the modeled country.

In terms of the database, the main source of information to calibrate IEEM and CGE models in general is a SAM. To simplify, a SAM is a square matrix that records all transactions made in an economy (sub-national, national or global) in a given year. In the case of IEEM, the SAM is complemented with natural capital data to capture the two-way interactions between the economy and environment. Banerjee et al. (2019c) describe in detail the construction of an environmentally extended SAM with information from Guatemala.

3. Mathematical Description of IEEM

This section presents the mathematical description of IEEM.

3.1 Variables

Table 1 presents the notational principles used in this mathematical description. Tables 2 to 5 define model sets, variables, Latin-letter parameters, Greek-letter parameters, respectively

Table 1. Notation used.

Item	Notation	Example
Sets or subscripts	Lowercase Latin letters as subscripts of variables and parameters	Check rows below
Endogenous variables	Uppercase Latin letters (without top bar) *	$QG_{c,t}$
Exogenous variables**	Uppercase Latin letters with top bar *	$\overline{QFS}_{f,t}$
Parameters**	Lowercase Latin letters * or lowercase Greek letters (with or without supra indexes)	$ica_{c,a}$; ρ_c^q

* The variables and parameters names related to prices, quantities and factorial remuneration begin with P, Q and WF, respectively. ** The distinction between exogenous variables and parameters is that the latter always have exogenous values while the former can be endogenous under alternative assumptions.

Table 2. Sets or subscripts.

Name	Description
$ac \in AC$	Global set; SAM accounts and other items
$a \in A$	Activities (i.e., productive sectors or industries)
$a \in AREG$	Activities with price regulation
$c \in C$	Commodities (i.e. goods and services)
$c \in C1(\subset C)$	Utility function of upper level commodities
$c \in C2(\subset C)$	Utility function of lower level commodities
$c \in CEN(\subset C)$	Energy products
$c \in CT(\subset C)$	Commodities linked to distribution margins; usually trade and transportation
$c \in CWAT(\subset C)$	Water as a product
$d \in D(ISND \cup FCAP \cup A \cup TAC)$	Domestic demand (or types of demand) institutions (for consumption), investment by capital type, activities, and distribution margins
$f \in F$	Factors
$f \in FVA(\subset F)$	Factors that receive part of the value-added (in SAM)
$f \in FCAP(\subset F)$	Capital factors
$f \in FCAPG(\subset FCAP, \not\subset FVA)$	Not remunerated government capital factors
$f \in FCAPNG(\subset FCAP, \subset FVA)$	Capital factors that receive part of the value-added
$f \in FLAB(\subset FVA)$	Labor factors
$f \in FLAND(\subset FVA)$	Land factors
$f \in FOTH(\subset FVA, \not\subset FLAB, \not\subset FCAP \not\subset FLAND)$	Other factors that receive part of the value-added
$f \in FWAT$	Water as a factor
$f \in FNCAPV1(\subset FVA)$	Non-capital factors at the upper level of production
$f \in FMOB(\subset FVA)$	Mobile factors between sectors
$f \in FNMOB(\subset FVA)$	Immobile factors between sectors
$f \in FUEND(\subset FVA)$	Factors with endogenous unemployment rate
$g \in GHG$	Greenhouse gases
$i \in INS$	Institutions
$i \in INSD(\subset INS)$	Domestic institutions
$i \in INSDNG(\subset INSD)$	Domestic institutions (not government)
$h \in H(\subset INSDNG)$	Households
$i \in INSNH(\subset INS)$	Non-household institutions
$i2 \in INS2$	govz and ngovz
$t \in T$	Time (simulation years)
$t \in TMIN(\subset T)$	Baseline year (first simulation period)

Table 3. Variables.

Name	**Description**
$CALTFP_t$	Total Factor Productivity (TFP) index to generate base scenario
$CALTFPA_{a,t}$	TFP index by activity to generate base scenario
CPI_t	Consumer price index
$DKA_{f,a,t}$	Change in capital stock *f* installed in activity *a*
$DKINS_{i,f,t}$	*i* Institution´s investment in capital stock *f*
DPI_t	Domestic products price index
EG_t	Government current expenditure
$EH_{h,t}$	Consumption spending for household h
$EMI_{ghg,ac,ac',t}$	Greenhouse gas (GHG) emissions from the *ac* use by the *ac* ’demandant
EXR_t	Exchange rate (local currency per unit of the rest of the world currency)
FDI_t	Foreign direct investment (in foreign currency)
$FDISCAL_t$	Adjustment factor for FDI_t
$FPRDA_{f,a,t}$	Productivity index for f factor in activity a
$GDPMP_t$	Nominal GDP at market prices
$GOVRECGDP_{acgovrec,t}$	acgovrec government income as a share of GDP
$GOVSPNDGDP_{acgovspnd,t}$	acgovspnd government expenditure as a share of GDP
$IADJ_{i2,f,t}$	Public investment adjustment factor in capital stock *f* and Private investment in capital stock *f*
$IADJ2_{i2,t}$	Total public investment adjustment factor and total private investment
$INVVALG_t$	Government investment value
$LABPARTRAT_t$	Ratio between labor force and working-age population
$MPS_{i,t}$	Marginal propensity to save of households and enterprises
$MPSSCAL_t$	Adjustment factor $MPS_{i,t}$
$NDFG_t$	Government net domestic financing
$NFFG_t$	Government net foreign financing (in foreign currency)
$NFFNGSCAL_t$	Adjustment factor for $NFFG_t$
$NFFNG_t$	Households and enterprises net foreign financing (in foreign currency)
$NGOVPAYGDP_{acngovpay,t}$	Household and enterprises payments as a share of GDP
$PA_{a,t}$	Product price of the activity *a*
$PAREAL_{a,t}$	Real commodity price of activity *a*
$PDD_{c,t}$	Demand price for Commodity *c* produced and consumed domestically
$PDS_{c,t}$	Commodity *c* produced and consumed domestically
$PDSREAL_{c,t}$	Real supply price for commodity *c* produced and consumed domestically
$PE_{c,t}$	Export price for commodity *c*

$PEN_{a,t}$	Aggregate energy price for activity *a*
$PINTA_{a,t}$	Aggregate intermediate input price for activity *a*
$PK_{f,t}$	Replacement cost per unit of capital stock *f*
$PKEN_{a,t}$	Capital-energy aggregate price for activity *a*
$PLANDWAT_{a,t}$	Land-water aggregate price for activity *a*
$PLL_{a,t}$	Labor-other factors aggregate price for activity *a*
$PM_{c,t}$	Import price for commodity *c*
$POP_{ac,t}$	Population of *ac* (representative households and working-age population)
$PQD_{c,d,t}$	Domestic and imported aggregate demand price of commodity *c* demanded by *d* (activities and institutions)
$PQS_{c,t}$	Domestic and imported aggregate supply price of commodity *c* demanded by *d* (activities and institutions)
$PQSREAL_{c,t}$	Domestic and imported aggregate real supply price of commodity *c* demanded by *d* (activities and institutions)
$PVA_{a,t}$	Value-added price for activity *a*
$PWATA_{a,t}$	Water added price for activity *a*
$PX_{c,t}$	Producer price for commodity *c*
$QA_{a,t}$	of activity *a*
$QD_{c,t}$	Quantity of commodity *c* produced and consumed domestically
$QDEFOR_{t}$	Deforestation (change in forest land)
$QE_{c,t}$	Quantity of exports for commodity *c*
$QEN_{a,t}$	Quantity of aggregate energy for activity *a*
$QF_{f,a,t}$	Quantity demanded of factor *f* from activity *a*
$QFINS_{i,f,t}$	Endowment of factor *f* for institution *i*
$QFINSINIT_{i,f,t}$	Endowment of factor *f* for institution *i* at the beginning of the period
$QFMIGR_{f,f\prime,t}$	Factorial migration from *f* to *f'*
$QLANDUSE_{ac,t}$	Quantity of land in alternative uses
$QFS_{f,t}$	Factor *f* supply
$QG_{c,t}$	Quantity of commodity *c* consumed (provided) by the government
$QGSCAL_{t}$	Adjustment factor $QG_{c,t}$
$QH_{c,h,t}$	Quantity of commodity *c* consumed by the household *h*
$QINT_{c,a,t}$	Quantity of commodity *c* used as an intermediate input for activity *a*
$QINTA_{a,t}$	Quantity of aggregate intermediate inputs for activity *a*
$QINV_{c,t}$	Quantity of investment demanded for commodity *c*
$QKEN_{a,t}$	Quantity of aggregate capital-energy for activity *a*
$QLABSCAL_{t}$	Labor supply adjustment factor to impose share rate
$QLANDWAT_{a,t}$	Quantity aggregated of land-water for activity *a*
$QLL_{a,t}$	Quantity of labor-other factors aggregate for activity *a*

$QM_{c,t}$	Quantity of imports of commodity *c*
$QNGO_{c,t}$	Quantity of commodity *c* consumed (provided) by the Non-profit institutions serving households (NPISH)
$QNGOSCAL_{c,t}$	Adjustment factor $QNGO_{c,t}$
$QQ_{c,t}$	Quantity supplied for commodity *c*
$QT_{c,t}$	Quantity demanded of trade and transport for commodity *c*
$QTRSTF_{c,t}$	Quantity of commodity *c* consumed by international tourists
$QVA_{a,t}$	Quantity of value-added in activity *a*
$QWATA_{a,t}$	Quantity aggregate water for activity *a*
$QX_{c,t}$	Quantity produced of commodity *c*
$REGPA_{a,t}$	Price regulation for activity *a*
$REGPQS_{c,t}$	Domestic and imported supply price regulation for commodity *c*
$REXR_{t}$	Real exchange rate
$RGDPFC$	Real GDP at factor cost (at constant prices for the base year)
$RGDPMP_{t}$	Real GDP at buyer prices (at constant prices for the base year)
$SAVF_{t}$	Savings from the rest of the world (in foreign currency)
$SAVINS_{i,t}$	Savings of households and enterprises
$SHIF_{i,f,t}$	for institution *i* in the income of factor *f*
$SUBCT_{t}$	Total expenditure on consumption subsidies
$SURPG_{t}$	Government surplus
$TFP_{a,t}$	Sectoral TFP index
$TRNSFR_{ac,i,t}$	Transfers from institution *i* to *ac*
$TRNSFRSCAL_{a,t}$	$TRNSFR_{ac,i,t}$ adjustment factor
$UERAT_{f,t}$	Unemployment rate for factor *f*
$WALRAS_{t}$	Walras Law compliance verification variable; It is satisfied if value is zero.
$WF_{f,t}$	Wage of factor *f*
$WFAVG_{f,t}$	Average wage of factor *f*
$WFDIST_{f,a,t}$	Wage distortion factor for factor *f* in activity *a*
$WFRAT_{f,f\prime a,t}$	Ratio between wage of factor *f* 'and wage of factor *f*
$YF_{f,t}$	Income of factor *f*
YG_{t}	Government current revenue
$YIF_{i,f,t}$	Income of institution *i* from factor *f*
$YI_{i,t}$	Income of households and business

Table 4. Latin letters parameters

Name	Description
$capcomp_{c,f}$	Quantity of commodity *c* per unit of capital *f*
$cwts_{c,h}$	Weight of commodity c in consumption basket of household *h*
$depr_{f,t}$	Depreciation rate of capital *f*
drf_t	Change in international reserves (foreign currency)
$ddkins_{i2,f,t}$	Exogenous change in real gross fixed capital formation for *govz* and *ngovz*
$dkinsb_{i2,f,t}$	Exogenous component in real gross fixed capital formation for *govz* and *ngovz*
$dwts_c$	Weight of commodity *c* in the DPI_t (producer price index with $PDS_{c,t}$)
$iadj01_f$	Investment adjustment factor /parameter 0-1 to make government investment more flexible
$ica_{c,a}$	Intermediate consumption of commodity *c* per unit of aggregate intermediate input for activity *a*
$icd_{c,c'}$	Commodity *c* as trade and transport input per unit of commodity *c*' produced and sold domestically
$ice_{c,c'}$	Commodity *c* as trade and transport input per exported product unit of *c'*
$icm_{c,c'}$	Commodity c as trade and transport input per imported product unit of *c'*
$iemi_{ghg,ac,ac',t}$	Emissions per unit of commodity *c* used or factor *f* per demander ac'
$ifa_{f,a,t}$	Factor *f* demand per unit produced of activity *a*
$inta_a$	Aggregate intermediate inputs per activity *a* unit
iva_a	Value-added per activity *a* unit
$mpsb_{i,t}$	Marginal propensity to save for households and enterprises
$mpk_{a,f,t}$	Marginal product of capital *f* in activity *a*
$pqdist_{c,d,t}$	Commodity *c* price distortion factor for demander *a*
$pwe_{c,t}$	Commodity *c* export price (in foreign currency)
$pwm_{c,t}$	Commodity *c* import price (in foreign currency)
$qdeforb_t$	Deforestation (change in forest land)
$qdstk_{i2,c,t}$	Stock variation
$qfsexog_{f,t}$	Factor *f* exogenous supply
$qgb_{c,t}$	Exogenous quantity of commodity *c* consumed by the household *h*
$qheff_{c,h,t}$	Commodity *c* consumption efficiency parameter of household *h*
$qngob_{c,ngo,t}$	Exogenous quantity of commodity *c* consumed (provided) by the NPISH
$qtrstfb_{c,t}$	Exogenous quantity of commodity *c* consumed by international tourists
$shii_{i,i'}$	Share of institution *i* in post-tax income and institution i ´savings
$shland_{f,t}$	Shares of land types
$shyregpa_{ac,a,t}$	Share of Institution or factor in the income by price regulation of activity *a*
$shyregpqs_{ac,c,t}$	Share of Institution or factor in the income by Domestic and imported aggregate supply price regulation of commodity *c*
$subc_{c,d,t}$	Consumption subsidy rate of commodity *c* for demander *d*
$ta_{a,t}$	Output tax rate for the activity *a*

$te_{c,t}$	Export tax rate of commodity *c*
$tf_{f,t}$	Income tax rate of factor *f*
$tfact_{f,a,t}$	Tax rate of the use of factor *f* in activity *a*
$tfp01_{a}$	Parameter [0-1] that determines the growth rate of the sector TFP when GDP growth rate is imposed
$tfpexog_{t}$	Exogenous component TPF
$tm_{c,t}$	Import tariff rate for commodity *c*
$tq_{c,t}$	Rate of sales tax for commodity *c*
$trnsfrb_{i,ac,t}$	Exogenous component for transfers from ins to *ac*
$tvac_{c,d,t}$	Rate of Value-added tax for the consumption of Commodity *c* from demander *d*
$ty_{i,t}$	Household and enterprises income tax rate

Table 5. Greek letters parameters.

Name	Description
$\theta_{a,c}$	Commodity *c* output per activity *a* unit
$\delta_{f,a}^{va}$	CES production value-added function share parameter for factor *f* in activity *a*
φ_{a}^{va}	Scale/shift parameter in the CES production value-added function of activity *a*
σ_{a}^{va}	Elasticity of substitution in the CES production value-added function in activity *a*
ρ_{a}^{va}	CES production value-added function exponent of activity *a*
δ_{a}^{ken}	Share parameter for the Aggregate KEN in CES aggregate function for activity *a (LL + KEN + LANDWAT*)
δ_{a}^{ll}	Share parameter for the Aggregate LL in CES aggregate function for activity a *(LL + KEN + LANDWAT*)
$\delta_{a}^{landwat}$	Share parameter of the Aggregate LANDWAT in CES aggregate function for activity *a* (LL + KEN + LANDWAT)
δ_{a}^{k}	Share parameter of the Aggregate K in CES aggregate function for activity *a* (KEN)
δ_{a}^{en}	Share parameter of the Aggregate EN in CES aggregate function for activity *a* (KEN)
φ_{a}^{ken}	Scale/shift parameter in CES aggregate function for activity *a* (KEN)
σ_{a}^{ken}	Elasticity of substitution in CES aggregate function for activity *a* (KEN)
ρ_{a}^{ken}	Exponent in CES aggregate function activity *a* (KEN)
$\gamma_{f,a}^{ll}$	Share parameter of the Aggregate LL in CES aggregate function for activity *a* (LL)
φ_{a}^{ll}	Scale/shift parameter in CES aggregate function for activity *a* (LL)
σ_{a}^{ll}	Elasticity of substitution in CES aggregate function for activity *a* (LL)
ρ_{a}^{ll}	Exponent in CES aggregate function for activity *a* (LL)
$\gamma_{c,a}^{en}$	Share parameter of the Aggregate *E* in *CES* aggregate function for activity *a* (EN)
φ_{a}^{en}	Scale/shift parameter as a function of *CES* aggregate activity *a* (EN)

σ_a^{en}	Elasticity of substitution as a function of CES aggregate activity *a* (EN)
ρ_a^{en}	Exponent in CES aggregate function for activity *a* (EN)
$\delta_{f,a}^{land}$	Share parameter of the aggregate LAND in CES as a function for activity *a* (LANDWAT)
δ_a^{wat}	Share parameter of the Aggregate WAT in CES aggregate function for activity *a* LANDWAT
$\varphi_a^{landwat}$	Scale/shift parameter in CES aggregate function for activity *a* (LANDWAT)
$\sigma_a^{landwat}$	Elasticity of substitution as a function of CES aggregate activity a (LANDWAT)
$\rho_a^{landwat}$	Exponent in CES aggregate function for activity *a* (LANDWAT)
$\gamma_{ac,a}^{wat}$	Share parameter in CES aggregate function for activity *a* WAT (product or factor)
φ_a^{wat}	Scale/shift parameter in CES aggregate function for activity *a* WAT (product or factor)
σ_a^{wat}	Elasticity of substitution as a function of CES aggregate activity *a* (product or factor)
ρ_a^{wat}	Exponent in CES aggregate function for activity *a* WAT (product or factor)
δ_c^{dd}	Share parameter for domestic output in domestic and imported CES aggregate function for commodity *c*
δ_c^{m}	Share parameter for imports in domestic and imported CES aggregate function for commodity *c*
φ_c^{q}	Scale/shift parameter in domestic and imported CES aggregate function for commodity *c*
σ_c^{q}	Elasticity of substitution in domestic and imported CES aggregate function for commodity *c*
ρ_c^{q}	Exponent in domestic and imported CES aggregate function for commodity *c*
δ_c^{ds}	Share parameter for domestic products in CET function for commodity *c*
δ_c^{e}	Share parameter for exported products in CET function for commodity c
φ_c^{x}	Scale/shift parameter in CET function for commodity c
σ_c^{x}	Elasticity of substitution in CET function for commodity c
ρ_c^{x}	Exponent in CET function for commodity c
$\alpha_{i,t}^{sav}$	Intercept in function of households and enterprises savings
$\beta_{c,h}$	Share parameter in LES function for household *h* consumption for commodity *c*
$\gamma_{c,h}^{min}$	Minimum consumption in LES function for household *h* consumption for commodity *c*
$\delta_{c,h}^{qh}$	Share parameter in CES function for lower level household consumption utility function
$\varphi_{c,h}^{qh}$	Scale parameter in CES function for lower level household consumption utility function
$\sigma_{c,h}^{qh}$	Elasticity of substitution in CES function for lower level household consumption utility function
$\rho_{c,h}^{qh}$	Exponent in CES function for household consumption lower level utility function
η_f^{wf}	Elasticity of remuneration of factor *f* regarding the unemployment rate in the wage curve

η_f^{migr}	Elasticity of migration of factor *f* regarding the ratio of average remuneration
η_f^{qfs}	Elasticity of supply of factor *f* regarding its average remuneration
κ_f	Sensitivity of new capital *f* to the differences in return rates between activities

IEEM equations are organized into the 14 blocks. For presentation purposes, the mathematical presentation makes some simplifications regarding the options available in the GAMS code of the model.

3.2 Equations: Production Function and Prices

First, we describe IEEM's production function. Equations PF1 and PF2 determine the use of value-added and intermediate inputs as fixed proportions of the output level, respectively. As follows, IEEM extends the concept of value-added to allow substitution between output factors and selected intermediate inputs. For instance, IEEM allows replacing natural pastures with feedlots in livestock production.

PF1	$QVA_{a,t} = iva_a \cdot QA_{a,t}$	$t \in T$ $a \in A$
PF2	$QINTA_{a,t} = inta_a \cdot QA_{a,t}$	$t \in T$ $a \in A$

3.2.1 Value-Added-Energy-Water.

The value-added (VA) production function nests several CES functions whose components are described below. In this presentation of IEEM it is assumed that the extended value-added function includes the following components: primary factors of production (e.g., labor, capital and natural resources), energy, and water. Energy is combined with capital $QKEN_{a,t}$ while land is combined with water $QLANDWAT_{a,t}$. Then Equation PF3, combines these two aggregates with other factors of production (i.e., labor and others) to produce extended VA. The last term of equation PF3 refers to the potential (and typically positive) effect that the public infrastructure stock can have on total factor productivity; the parameter $mpk_{a,f,t}$ measures the marginal product of the capital stock *f* over the VA of activity *a*. The magnitude $QFINS_{i,f,t} - QFINS_{i,f,t}^0$ is the difference in the capital stock *f* with respect to that recorded in the baseline scenario. Equations PF4-PF6 are the First Order Conditions (FOC) which, given their prices, determine the optimal demands of the aggregates $QKEN_{a,t}$, $QLL_{a,t}$ and $QLANDWAT$ respectively.

PF3	$QVA_{a,t} = TFP_{a,t} \cdot \varphi_a^{va} \left(\delta_a^{ken} \cdot QKEN_{a,t}^{-\rho_a^{va}} + \delta_a^{ll} \cdot QLL_{a,t}^{-\rho_a^{va}} + \delta_a^{landwat} \cdot QLANDWAT_{f,a,t}^{-\rho_a^{va}}\right)^{\frac{-1}{\rho_a^{va}}} + \sum_{f \in FCAP} \sum_{i \in INS} mpk_{a,f,t}\left(QFINS_{i,f,t} - QFINS_{i,f,t}^{0}\right)$	$t \in T$ $a \in A$
PF4	$PKEN_{a,t} = PVA_{a,t} \cdot QVA_{a,t} \left(\delta_a^{ken} \cdot QKEN_{a,t}^{-\rho_a^{va}} + \delta_a^{ll} \cdot QLL_{a,t}^{-\rho_a^{va}} + \delta_a^{landwat} \cdot QLANDWAT_{f,a,t}^{-\rho_a^{va}}\right)^{-1} \delta_a^{ken} \cdot QKEN_{a,t}^{-\rho_a^{va}-1}$	$t \in T$ $a \in A$
PF5	$PLL_{a,t} = PVA_{a,t} \cdot QVA_{a,t} \left(\delta_a^{ken} \cdot QKEN_{a,t}^{-\rho_a^{va}} + \delta_a^{ll} \cdot QLL_{a,t}^{-\rho_a^{va}} + \delta_a^{landwat} \cdot QLANDWAT_{f,a,t}^{-\rho_a^{va}}\right)^{-1} \delta_a^{ll} \cdot QLL_{a,t}^{-\rho_a^{va}-1}$	$t \in T$ $a \in A$
PF6	$PLANDWAT_{a,t} = PVA_{a,t} \cdot QVA_{a,t} \left(\delta_a^{ken} \cdot QKEN_{a,t}^{-\rho_a^{va}} + \delta_a^{ll} \cdot QLL_{a,t}^{-\rho_a^{va}} + \delta_a^{landwat} \cdot QLANDWAT_{f,a,t}^{-\rho_a^{va}}\right)^{-1} \delta_a^{landwat} \cdot QLANDWAT_{a,t}^{-\rho_a^{va}-1}$	$t \in T$ $a \in A$

3.2.2 Capital-Energy

Equations PF7-PF9 determine the capital-energy composition of the aggregate $QKEN_{a,t}$ described above. The CES production function that combines capital factor with the energy aggregate $QEN_{a,t}$ appears in equation PF7. The equations PF8 and PF9 are the FOCs that determine the demands of capital and the energy aggregate, respectively. The equations include the exogenous variable $\overline{FPRDA}_{f,a,t}$ that allows modeling changes in the productivity level of capital.

PF7	$QKEN_{a,t} = \varphi_a^{ken} \left(\delta_a^{k}\left(\overline{FPRDA}_{f,a,t} \cdot QF_{f,a,t}\right)^{-\rho_a^{ken}} + \delta_a^{en} \cdot QEN_{a,t}^{-\rho_a^{ken}}\right)^{\frac{-1}{\rho_a^{ken}}}$	$t \in T$ $f \in FCAPNG$ $a \in A$

PF8	$$WFA_{f,a,t} = PKEN_{a,t} \cdot QKEN_{a,t}\left(\delta_a^k\left(\overline{FPRDA}_{f,a,t} \cdot QF_{f,a,t}\right)^{-\rho_a^{ken}} + \delta_a^{en} \cdot QEN_{a,t}^{-\rho_a^{ken}}\right)^{-1} \delta_a^k \cdot QF_{f,a,t}^{-\rho_a^{ken}-1} \cdot \overline{FPRDA}_{f,a,t}^{-\rho_a^{ken}}$$	$t \in T$ $f \in FCAPNG$ $a \in A$
PF9	$$PEN_{f,a,t} = PKEN_{a,t} \cdot QKEN_{a,t}\left(\delta_a^k\left(\overline{FPRDA}_{f,a,t} \cdot QF_{k,a,t}\right)^{-\rho_a^{ken}} + \delta_a^{en} \cdot QEN_{a,t}^{-\rho_a^{ken}}\right)^{-1} \delta_a^{en} \cdot QEN_{f,a,t}^{-\rho_a^{ken}-1}$$	$t \in T$ $f \in FCAPNG$ $a \in A$

3.2.3 Other Factors

Equations PF10 and PF11 are applied to the productive factors that appear in the upper level of the production function (i.e., the contents in the set $FNCAPVA1$), typically, labor and other non-land natural resources). Equation PF10 is the CES function that adds to the individual factors in $QLL_{a,t}$. Equation PF11 is the FOC that determines the quantity of every individual factor used. Again, the exogenous variable $\overline{FPRDA}_{f,a,t}$ allows changes in the productivity of each factor considered individually. Equation PF12 is applied to productive factors that are demanded in a constant proportion with respect to the production level. Therefore, there is a relationship of constant proportionality between factor supply and the level of production of an activity.

PF10	$$QLL_{a,t} = \varphi_a^{ll}\left(\sum_{f \in FNCAPVA1} \gamma_a^k\left(\overline{FPRDA}_{f,a,t} \cdot QF_{f,a,t}\right)^{-\rho_a^{ll}}\right)^{\frac{-1}{\rho_a^{ll}}}$$	$t \in T$ $a \in A$
PF11	$$WFA_{f,a,t} = PLL_{a,t} \cdot QLL_{a,t}\left(\sum_{f \in FNCAP} \gamma_a^k\left(\overline{FPRDA}_{f,a,t} \cdot QF_{k,a,t}\right)^{-\rho_a^{ll}}\right)^{-1} \gamma_a^k QF_{f,a,t}^{-\rho_a^{ll}-1} \cdot \overline{FPRDA}_{f,a,t}^{-\rho_a^{ll}}$$	$t \in T$ $f \in FNCAPVA1$ $a \in A$
PF12	$$QF_{f,a,t} = \frac{ifa_{f,a,t}}{\left(1 + fleo01_{f,a}\left(\frac{TFP_{a,t}}{TFP_a^{00}} - 1\right)\right)} QA_{a,t}$$	$t \in T$ $f \in FLEO$ $a \in A$

3.2.4 Energy

Equation PF13 defines the aggregate of energy $QEN_{a,t}$ as a CES function that combines the different energy commodities contained in the CEN set. Equation PF14 is the FOC that determines the intermediate demand of each particular energy commodity. In practice, this formulation allows considering the case in which the same productive sector can substitute between alternative sources of energy (e.g., natural gas versus electricity).

PF13	$QEN_{a,t} = \varphi_a^{en}\left(\sum_{c\in CEN}\gamma_{c,a}^{en}\cdot QINT_{c,a,t}^{-\rho_a^{en}}\right)^{\frac{-1}{\rho_a^{en}}}$	$t \in T$ $a \in A$
PF14	$PQD_{c,a,t} = PEN_{a,t}\cdot QEN_{a,t}\left(\sum_{c\in CEN}\gamma_{c,a}^{en}\cdot QINT_{c,a,t}^{-\rho_a^{en}}\right)^{-1}\gamma_a^{en} \cdot QINT_{c,a,t}^{-\rho_a^{en}-1}$	$t \in T$ $c \in CEN$ $a \in A$

3.2.5 Land-Water

Equations PF15-PF17 model the aggregate of water and land that typically appears in the production functions for agricultural sectors. Equation PF15 uses a CES function to combine the factors identified as land and the aggregate of commodities and/or factors identified as water $QWATA_{a,t}$. As shown below, IEEM allows for the existence of imperfect mobility between different types of land. Equations PF16 and PF17 are the FOCs that determine the demands for land and the aggregate of water, respectively.

PF15	$QLANDWAT_{a,t} = \varphi_a^{landwat}\left(\sum_{f\in FLAND}\delta_{f,a}^{land}\left(\overline{FPRDA}_{f,a,t}\cdot QF_{f,a,t}\right)^{-\rho_a^{landwat}} + \delta_{f,a}^{wat}\cdot QWATA_{a,t}^{-\rho_a^{landwat}}\right)^{\frac{-1}{\rho_a^{landwat}}}$	$t \in T$ $a \in A$

PF16	$$WFA_{f,a,t} = PLANDWAT_{a,t} \cdot QLANDWAT_{a,t} \left(\sum_{f \in FLAND} \delta_{f,a}^{land} \left(\overline{FPRDA}_{f,a,t} \cdot QF_{f,a,t} \right)^{-\rho_a^{landwat}} + \delta_{f,a}^{wat} \cdot QWATA_{a,t}^{-\rho_a^{landwat}} \right)^{-1} \delta_{f,a}^{land} \cdot QF_{f,a,t}^{-\rho_a^{landwat}-1} \cdot FPRDA_{f,a,t}^{-\rho_a^{landwat}}$$	$t \in T$ $f \in FLAND$ $a \in A$
PF17	$$PWATA_{a,t} = PLANDWAT_{a,t} \cdot QLANDWAT_{a,t} \left(\sum_{f \in FLAND} \delta_{f,a}^{land} \left(\overline{FPRDA}_{f,a,t} \cdot QF_{f,a,t} \right)^{-\rho_a^{landwat}} + \delta_{f,a}^{wat} \cdot QWATA_{a,t}^{-\rho_a^{landwat}} \right)^{-1} \delta_{f,a}^{wat} \cdot QWATA_{a,t}^{-\rho_a^{landwat}-1}$$	$t \in T$ $a \in A$

3.2.6 Water

Equations PF18-PF20 determine the composition of the composite of water sources $QWATA_{a,t}$. The different water sources are combined using the CES function in equation PF17. Generally, productive activities can extract water from the environment (equation PF18; e.g. rainfed agriculture) and/or from other productive activities (equation PF19; e.g. irrigated agriculture).

PF18	$$QWATA_{a,t} = \varphi_a^{wat} \left(\sum_{f \in FWAT} \gamma_{f,a}^{wat} \left(\overline{FPRDA}_{f,a,t} \cdot QF_{f,a,t} \right)^{-\rho_a^{wat}} + \sum_{c \in CWAT} \gamma_{c,a}^{wat} \cdot QINT_{c,a,t}^{-\rho_a^{wat}} \right)^{\frac{-1}{\rho_a^{wat}}}$$	$t \in T$ $a \in A$

PF19	$WFA_{f,a,t} = PWATA_{a,t} \cdot QWATA_{a,t} \left(\sum_{f \in FWAT} \gamma_{f,a}^{wat} \left(\overline{FPRDA}_{f,a,t} \cdot QF_{f,a,t} \right)^{-\rho_a^{wat}} + \sum_{c \in CWAT} \gamma_{c,a}^{wat} \cdot QINT_{c,a,t}^{-\rho_a^{wat}} \right)^{-1} \gamma_{f,a}^{wat} \cdot QF_{f,a,t}^{-\rho_a^{wat}-1} \cdot \overline{FPRDA}_{f,a,t}^{-\rho_a^{wat}}$	$t \in T$ $f \in FWAT$ $a \in A$
PF20	$PQD_{c,a,t} = PWATA_{a,t} \cdot QWATA_{a,t} \left(\sum_{f \in FWAT} \gamma_{f,a}^{wat} \left(\overline{FPRDA}_{f,a,t} \cdot QF_{f,a,t} \right)^{-\rho_a^{wat}} + \sum_{c \in CWAT} \gamma_{c,a}^{wat} \cdot QINT_{c,a,t}^{-\rho_a^{wat}} \right)^{-1} \gamma_{c,a}^{wat} \cdot QINT_{c,a,t}^{-\rho_a^{wat}-1}$	$t \in T$ $c \in CWAT$ $a \in A$

3.2.7 Other Production

This block of equations completes the production function. Equation PF21 completes the presentation of the extended value-added, defining total factor productivity. For simplicity, this presentation only identifies an exogenous component of total factor productivity given by the parameter $tfpexog_{a,t}$. The variable $CALTFP_t$ becomes endogenous when the user imposes a GDP trajectory, typically when generating a baseline projection. Equation PF22 determines the intermediate demand of each commodity by each of the productive activities. In this case, it is assumed that intermediate inputs are combined in fixed proportions; alternatively, a Leontief-type production function is used. Equation PF23 converts the production of activities into commodities output. For that purpose, a yield coefficient $\theta_{a,c}$ is used to indicate the output produced from commodity *c* per unit produced of activity *a*.

PF21	$TFP_{a,t} = tfpexog_{a,t}(1 + tfp01_a \cdot \overline{CALTFP}_t)$	$t \in T$ $a \in A$
PF22	$QINT_{c,a,t} = ica_{c,a,t} \cdot QINTA_{a,t}$	$t \in T$ $c \in C$ $a \in A$
PF23	$QX_{c,t} = \sum_{a \in A} \theta_{a,c} \cdot QA_{a,t}$	$t \in T$ $c \in C$

3.2.8 Output Prices

Prices directly related to the output of goods and services are defined in this block of equations. Equations PR1 and PR2 define the remuneration of mobile and sector-specific productive factors between sectors, respectively. The parameter $tfact_{f,a,t}$ is the tax rate for the use of factor f by activity a. IEEM can include an institution linked to social security that collects similar taxes that are explicitly defined as social security contributions. In turn, this institution can provide goods and services and/or make transfers to other institutions, typically households. Equation PR3 defines the price of value-added as the difference between the activity income and the cost of intermediate inputs. Finally, equation PR4 defines the price of the aggregate for intermediate inputs defined above.

PR1	$WFA_{f,a,t} = WF_t \cdot \overline{WFDIST}_{f,a,t} \cdot \left(1 + tfact_{f,a,t}\right)$	$t \in T$ $f \in FMOB$ $a \in A$
PR2	$WFA_{f,a,t} = \overline{WF}_t \cdot WFDIST_{f,a,t} \cdot \left(1 + tfact_{f,a,t}\right)$	$t \in T$ $f \in FNMOB$ $a \in A$
PR3	$PVA_{a,t} \cdot QVA_{a,t} = PA_{a,t}\left(1 - ta_{a,t} - REGPA_{a,t}\right)QA_{a,t} - PINTA_{a,t} \cdot QINTA_{a,t}$	$t \in T$ $a \in A$
PR4	$PINTA_{a,t} = \sum_{\substack{c \in C \\ \left(\substack{c \notin CEN \\ c \notin CWAT}\right)}} PQD_{c,a,t} \cdot ica_{c,a}$	$t \in T$ $a \in A$
PR5	$PA_{a,t} = \sum_{c \in C} \theta_{a,c} \cdot PX_{c,t}$	$t \in T$ $a \in A$

3.2.9 Mining Production Function

The mining sector can be modeled with a production function that assumes an inverse relationship between factor productivity and the remaining subsoil resource stock. Thus, the smaller the remaining stock is, the more expensive it becomes to extract. In other words, IEEM makes it possible to assume that mineral products extraction is limited by the remaining stock of extractive resources, which is not characteristic of a conventional CGE. The MIN1 equation is the production function of the mining sector. In this case, a variable of total factor productivity $TFPA_{a,t}$, specific to mining activity is added and defined below. The MIN2 equation links the Total Factor Productivity (TFP) of mining sectors with the extractive resources stock remaining in the subsoil. In particular, the marginal cost of extraction is assumed to increase as the stock $QSTKMIN_{a,t}$ decreases. IEEM also allows the extension of the MIN3 equation for the inclusion of new mineral discoveries that could more than compensate for the extraction of $QA_{a,t}$

MIN1	$QA_{a,t} = TFPA_{a,t} \cdot \varphi_a^{va} \left(\sum_{f \in F} \delta_{f,a}^{va} \cdot QF_{f,a}^{-\rho_a^{va}} \right)^{\frac{-1}{\rho_a^{va}}}$	$t \in T$ $a \in AMIN$
MIN2	$TFPA_{a,t} = QSTKMIN_{a,t}^{\eta_a^{min}} \varphi_a^{min}$	$t \in T$ $a \in AMIN$
MIN3	$QSTKMIN_{a,t} = QSTKMIN_{a,t-1} - QA_{a,t}$	$t \in T$ $a \in AMIN$

3.2.10 Fishery Production Function

The fishery sector can be modeled using a production function that incorporates bioeconomic elements. Particularly, it can be assumed that the stock of the fishing resource grows according to a logistic function, which is widely used for modeling this type of phenomena. Mathematically, the production function for the fishing sector is composed of two parts. The FHS1 equation determines the fishing stocks evolution $QB_{a,t}$ as a function of their intrinsic growth rate ($grwfsh_a$), the carrying capacity of the environment $kfsh_a$ and the catch volume $QA_{a,t}$.

A classic catch function widely used in bioeconomic analysis is described below. In particular, the catch volume is modeled as:

$$QA_{a,t} = q_a \cdot QB_{a,t} \cdot E_{a,t}$$

Where $q_a$is a (exogenous) catch coefficient, $QB_{a,t}$is the stock of the fishery resource, and $E_{a,t}$ is the fishing effort. In IEEM, equations FSH2 and FSH3 represent the catch function. In these equations, effort is replaced by a value-added production function similar to that used above. In practice, different aquatic species interact with each other through predator-prey relations, for example, and more complex production functions can be considered.

FSH1	$QB_{a,t} = QB_{a,t-1} + \left[grwfsh_a \cdot QB_{a,t-1}\left(1 - \frac{QB_{a,t-1}}{kfsh_a}\right)\right] - QA_{a,t}$	$t \in T$ $a \in AFSH$
FSH2	$QA_{a,t} = TFPA_{a,t} \cdot \varphi_a^{va}\left(\sum_{f \in F} \delta_{f,a}^{va} \cdot QF_{f,a}^{-\rho_a^{va}}\right)^{\frac{-1}{\rho_a^{va}}}$	$t \in T$ $a \in AFSH$
FSH3	$TFPA_{a,t} = q_a \cdot QB_{a,t}$	$t \in T$ $a \in AFSH$

3.3 Domestic and International Trade

IEEM models domestic and international trade in the standard way as with most CGE models. In particular and following the Armington assumption (Armington 1969), goods and services are assumed to differ according to the country of origin. Thus, domestic and imported commodities are imperfect substitutes on the demand side. In turn, domestic and foreign sales are also imperfect substitutes from the producer's perspective. In other words, changing the destination of domestic production has an associated cost.

The domestic prices for imports and exports are determined in equations TR1 and TR2, respectively. In both cases, trade policies (i.e., import tariffs and export taxes) and distribution costs (i.e., trade and transportation) are included. In the latter case, it is the cost of moving goods (a) from the producer to the port of departure, and (b) from the port of entry to the consumer, for exports and imports, respectively. In equations TR1 and TR2, these costs are proportional to the quantities imported and exported, (i.e., see coefficients $icm_{c',c}$ and $ice_{c',c}$).

Equation TR3 is similar to TR1 and TR2, but it refers to national commodities that are sold domestically. In this case, only distribution costs are applied. Typically, domestic products sales excluding imports are not taxed. Equations TR4-TR5 determine the domestic/imported composition of each of the goods and services consumed. Equation TR4 replaces equation TR4 for single-origin products, and in this case, equation TR5 is excluded from the model. Equations TR6 and TR7 determine the supply and demand prices of each product. IEEM identifies taxes and subsidies on sales of products and value-added-type taxes (VAT). Subsidies may differ between demanders of a particular product.

Equations TR8-TR10 model the choice of sales to domestic and foreign markets that producers of the modeled country face. Equation TR8 is a CET (Constant Transformation Elasticity) function that disaggregates the total output of each commodity $QX_{c,t}$ into the destined volumes for the domestic ($QD_{c,t}$) and foreign markets ($QE_{c,t}$). Equation TR9 shows that sales to both markets are determined based on their relative prices; ceteris paribus, an increase in the world price of exports of commodity *c* will increase the ratio $QE_{c,t}/QD_{c,t}$.

Equation TR8 replaces equation TR9 for products that are sold in solely one of the markets, and in this case, equation TR9 is excluded from the model. Equation TR10 determines the price of domestic output as an average of the prices received in each of the two markets mentioned. Lastly, equation TR11 determines the demand for commodities linked to distribution margins that typically correspond to marketing and transport services.

IEEM also allows (a) to differentiate between business partners as destination (origin) for exports (imports), (b) to assume that domestic sales are exogenous while exports are determined residually, and (c) to assume that exports are exogenous while domestic sales are determined residually.

TR1	$PM_{c,t} = (1 + tm_{c,t})EXR_t \cdot pwm_{c,t} + \sum_{c' \in CT} PQD_{c',tacm,t} \cdot icm_{c',c}$	$t \in T$ $c \in C$
TR2	$PE_{c,t} = (1 - te_{c,t})EXR_t \cdot pwe_{c,t} - \sum_{c' \in CT} PQD_{c',tace,t} \cdot ice_{c',c}$	$t \in T$ $c \in C$
TR3	$PDD_{c,t} = PDS_{c,t} + \sum_{c' \in CT} PQD_{c',tacd,t} \cdot icd_{c',c}$	$t \in T$ $c \in C$
TR4	$QQ_{c,t} = \varphi_c^q \left(\delta_c^m \cdot QM_{c,t}^{-\rho_c^q} + \delta_c^{dd} \cdot QD_{c,t}^{-\rho_c^q}\right)^{\frac{-1}{\rho_c^q}}$	$t \in T$ $c \in C$
TR4'	$QQ_{c,t} = QM_{c,t} + QD_{c,t}$	$t \in T$ $c \in C$
TR5	$\frac{QM_{c,t}}{QD_{c,t}} = \left(\frac{PDD_{c,t}}{PM_{c,t}} \cdot \frac{\delta_c^m}{\delta_c^{dd}}\right)^{\sigma_c^q}$	$t \in T$ $c \in C$
TR6	$PQS_{c,t} \cdot QQ_{c,t} = PDD_{c,t} \cdot QD_{c,t} + PM_{c,t} \cdot QM_{c,t}$	$t \in T$ $c \in C$
TR7	$PQD_{c,d,t} = PQS_{c,t}(1 + tq_{c,t})(1 - subc_{c,d,t})(1 + tvac_{c,d,t})$	$t \in T$ $c \in C$ $d \in D$
TR8	$QX_{c,t} = \varphi_c^x \left(\delta_c^e \cdot QE_{c,t}^{\rho_c^x} + \delta_c^{ds} \cdot QD_{c,t}^{\rho_c^x}\right)^{\frac{1}{\rho_c^x}}$	$t \in T$ $c \in C$

TR8'	$QX_{c,t} = QE_{c,t} + QD_{c,t}$	$t \in T$ $c \in C$
TR9	$\frac{QE_{c,t}}{QD_{c,t}} = \left(\frac{PE_{c,t}}{PDS_{c,t}} \cdot \frac{\delta_c^{ds}}{\delta_c^{e}}\right)^{\sigma_c^x}$	$t \in T$ $c \in C$
TR10	$PX_{c,t} \cdot QX_{c,t} = PDS_{c,t} \cdot QD_{c,t} + PE_{c,t} \cdot QE_{c,t}$	$t \in T$ $c \in C$
TR11	$QT_{c,t} = \sum_{c\prime \in C} icd_{c,c\prime} \cdot QD_{c\prime,t} + \sum_{c\prime \in C} icm_{c,c\prime} \cdot QM_{c\prime,t} + \sum_{c\prime \in C} ice_{c,c\prime} \cdot QE_{c\prime,t}$	$t \in T$ $c \in CT$

3.4 Factor Income

The equations in this group determine the functional distribution of income (i.e., factor income) and the distribution of factor income among institutional sectors. Equation F1 uses factor endowments to calculate each institution share in the total income for each factor. Equations F2 and F3 define the factor income of the mobile and sector-specific factors, respectively. Factors of production can receive income from the rest of the world, for example, from the remittance of profits linked to foreign direct investment made by the modeled country in past periods. Equation F4 uses the variable $SHIF_{i,f,t}$ defined in equation F1 to distribute factor income among institutional sectors. It is relevant to mention that, as it will be shown later, the distribution of factor income among institutional sectors evolves endogenously in IEEM. For example, if the rural population is reduced at the same time as the urban population increases, the same would happen with the factor income of both representative households.

F1	$SHIF_{i,f,t} = \frac{QFINS_{i,f,t}}{\sum_{i\prime \in INS} QFINS_{i\prime,f,t}}$	$t \in T$ $i \in INS$ $f \in F$
F2	$YF_{f,t} = \sum_{a \in A} WF_{f,t} \cdot \overline{WFDIST}_{f,a,t} \cdot QF_{f,a,t} + trnsfr_{f,row,t} \cdot EXR_t$	$t \in T$ $f \in FMOB$
F3	$YF_{f,t} = \sum_{a \in A} \overline{WF}_{f,t} \cdot WFDIST_{f,a,t} \cdot \overline{QF}_{f,a,t} + trnsfr_{f,row,t} \cdot EXR_t$	$t \in T$ $f \in FNMOB$
F4	$YIF_{i,f,t} = SHIF_{i,f,t} \cdot YF_{f,t}\left(1 - tf_{f,t}\right)$	$t \in T$ $i \in INS$ $f \in F$

3.5 Institutional Sectors: Non-Government institutions

Equations I1 defines the income of domestic non-government institutions (i.e, households, enterprises, non-profit institutions serving households (NPISH), among others) as the sum of (a) their factor income, and (b) the transfers received from other institutions in the model. Equations I2 and I3 determine the marginal propensity to save and the savings of domestic non-government institutions, respectively. The savings function has an autonomous component $\alpha_{i,t}^{sav} \cdot \overline{CPI}_t$ and another component that is a function of disposable income. Thus, it is possible to calibrate IEEM imposing a marginal propensity to save that does not depend on the observed level of savings; for example, the observed level of savings in the SAM that is used to calibrate the model may be negative but the marginal propensity to save that is imposed exogenously ($mpsb_{i,t}$) will be typically positive.

Equation I4 determines household consumption expenditure. This is the net disposable income after transfers to other institutions. Equations I5-I7 represent the two-level household demand system. In equation I5, set C1 contains the commodities that appear in the upper level of the household utility function. In equation I6, set C2 contains the products that appear in the lower level of the household utility function. For instance, food can be an element of C1 while fruits, vegetables and meats could be elements of C2. Equation I7, which corresponds to the first order condition of a CES function, determines the demand for household consumption for each of the products included in the C2 set. Equation I8 establishes the equality between income and expenditure of the NPISH; these institutions allocate their income to pay direct taxes, provide goods and services, make transfers to other institutions, and saving. The NPISH demand different products in constant proportions (equation I9). Finally, equation I10 of this block determines the demand, exogenous in this presentation, of foreign tourists. Again, it is assumed that foreign tourists demand goods and services in fixed proportions. IEEM also allows to identify social security as a private institution that collects social security contributions. However, social security transactions are typically modeled as transactions made by the government.

I1	$$YI_{i,t} = \sum_{f \in F} YIF_{i,f,t} + \sum_{i\prime \in INS} TRNSFR_{i,i',t}$$	$t \in T$ $i \in INSDNG$
I2	$$MPS_{i,t} = mpsb_{i,t} \cdot \overline{MPSSCAL}_t$$	$t \in T$ $i \in INSDNG$
I3	$$SAVINS_{i,t} = \alpha_{i,t}^{sav} \cdot \overline{CPI}_t + MPS_{i,t}(1 - ty_{i,t})YI_{i,t}$$	$t \in T$ $i \in INSDNG$
I4	$$EH_{h,t} = YI_{h,t}(1 - ty_{h,t}) - SAVINS_{h,t} - \sum_{i \in INS} TRNSFR_{i,h,t}$$	$t \in T$ $h \in H$
I5	$$QH_{c,h,t} = qhmin_{c,h,t} + \frac{\delta_{c,h}^{les}}{PQD_{c,h,t}} \left(EH_{h,t} - \sum_{c\prime \in C1} PQD_{c\prime,h,t} \cdot qhmin_{c\prime,h,t} \right)$$	$t \in T$ $c \in C1$ $h \in H$
I6	$$QH_{c,h,t} = \varphi_{c',h}^{qh} \left(\sum_{c' \in MC1C2(c,c\prime)} \delta_{c\prime,h}^{qh} \left(qheff_{c',h} \cdot QH_{c\prime,h}^{qh} \right)^{-\rho_{c,h}^{qh}} \right)^{\frac{-1}{\rho_{c,h}^{qh}}}$$	$t \in T$ $c \in C1$ $h \in H$
I7	$$QH_{c,h,t} = \sum_{c' \in MC1C2(c\prime,c)} \left(\frac{PQD_{c',h,t}}{PQD_{c,h,t}} \right)^{\sigma_{c',h}^{qh}} \cdot \left(\delta_{c,h}^{qh} \right)^{\sigma_{c',h}^{qh}} \cdot \left(qheff_{c,h} \cdot \varphi_{c',h}^{qh} \right)^{\sigma_{c',h}^{qh} - 1} \cdot QH_{c\prime,h,t}$$	$t \in T$ $c \in C2$ $h \in H$
I8	$$PQD_{c,ngo,t} \cdot QNGO_{c,ngo,t} = YI_{ngo,t}(1 - ty_{ngo,t}) - SAVINS_{ngo,t} - \sum_{i \in INS} TRNSFR_{i,ngo,t}$$	$t \in T$ $c \in C2$
I9	$$QNGO_{c,ngo,t} = qngob_{c,ngo,t} \cdot QNGOSCAL_{ngo,t}$$	$t \in T$ $c \in C$
I10	$$QTRSTF_{c,trst,t} = qtrstb_{c,trst,t} \cdot \overline{QTRSTSCAL}_{ngo,t}$$	$t \in T$ $c \in C$

3.6 Institutional Sectors: Government

The equations in this block represent the government income, expenditure and budget. Equations G1-G8 compute the income of each of the tax instruments identified in IEEM. In all cases, tax collection is computed as the sum of the products between tax rates and tax bases. The Equation G9, similar to the former ones, calculates total government expenditure on subsidies for the consumption of goods and services. Equation G10 is the government's current revenue, composed of (a) the sum of tax collections, (b) the transfers received from other institutions, (c) its factor income, and (d) incomes (or expenses) associated with the presence of regulated prices. Equation G11 is the consumption (or provision) of goods and services by the government as an institutional sector. Typically, the products that appear in this equation are public administration services and public health and education. Equation G12 is total government expenditure, which includes the following elements: (a) goods and services provision, (b) transfers to other institutions, and (c) consumption subsidies. Equation G13 defines the government surplus as the difference between current income and total expenditure (i.e., current and capital). Finally, equation G14 is the government's capital account that shows how the public deficit $-SURPG_t$ is financed. In this presentation, it is assumed that foreign financing $NFFG_t$ is exogenous while the government budget is balanced by domestic borrowing $NDFG_t$.

In this presentation, VAT is modeled as a tax imposed on selected demanders, usually households, while intermediate consumption is excluded. Alternatively, IEEM allows the modeling of VAT as a tax with fiscal credits and debits. Specifically, producers can receive a fiscal credit for their purchases of intermediate inputs while they generate a fiscal debit (or obligation) for their sales.

G1	$YTAXDIR_t = \sum_{i \in INSDNG} ty_{i,t} \cdot YI_{i,t}$	$t \in T$
G2	$YTAXFAC_t = \sum_{f \in F} tf_{f,t} \cdot YF_{f,t}$	$t \in T$
G3	$YTAXACT_t = \sum_{a \in A} ta_{a,t} \cdot PA_{a,t} \cdot QA_{a,t}$	$t \in T$
G4	$YTAXFACT_t = \sum_{f \in FMOB, a \in A} tfact_{f,a,t} \cdot WF_t \cdot \overline{WFDIST}_{f,a,t} + \sum_{f \in FNMOB, a \in A} tfact_{f,a,t} \cdot \overline{WF}_t \cdot WFDIST_{f,a,t}$	$t \in T$
G5	$YTAXCOM_t = \sum_{c \in C} tq_{c,t} \cdot PQS_{c,t} \cdot QQ_{c,t}$	$t \in T$
G6	$YTAXIMP_t = \sum_{c \in C} tm_{c,t} \cdot EXR_t \cdot pwm_{c,t} \cdot QM_{c,t}$	$t \in T$
G7	$YTAXEXP_t = \sum_{c \in C} te_{c,t} \cdot EXR_t \cdot pwe_{c,t} \cdot QE_{c,t}$	$t \in T$

G8	$YTAXVAT_t = \sum_{c\in C,a\in A}(1-subc_{c,a,t})PQS_{c,t}(1+tq_{c,t})tvac_{c,a,t} \cdot QINT_{c,a,t} + \sum_{c\in C,h\in H}(1-subc_{c,h,t})PQS_{c,t}(1+tq_{c,t})tvac_{c,h,t} \cdot QH_{c,h,t} + \sum_{c\in C,h\in NGO}(1-subc_{c,ngo,t})PQS_{c,t}(1+tq_{c,t})tvac_{c,ngo,t} \cdot QNGO_{c,ngo,t} + \sum_{c\in C}(1-subc_{c,gov,t})PQS_{c,t}(1+tq_{c,t})tvac_{c,gov,t} \cdot QG_{c,t} + \sum_{c\in C}(1-subc_{c,trst,t})PQS_{c,t}(1+tq_{c,t})tvac_{c,trst,t} \cdot QTRSTF_{c,t} + \sum_{c\in C,f\in FCAP}(1-subc_{c,f,t})PQS_{c,t}(1+tq_{c,t})tvac_{c,f,t} \cdot capcomp_{f,c} \cdot \sum_{i2\in INS2} DKINS_{i2,f,t} + \sum_{c\in C}(1-subc_{c,dstk,t})PQS_{c,t}(1+tq_{c,t})tvac_{c,dstk,t} \cdot \sum_{i2\in INS2} qdstk_{i2,c,t} + \sum_{c\in CT,c'\in C}(1-subc_{c,tacd,t})PQS_{c,t}(1+tq_{c,t})tvac_{c,tacd,t} \cdot icd_{c,c'} \cdot QD_{c',t} + \sum_{c\in CT,c'\in C}(1-subc_{c,tacm,t})PQS_{c,t}(1+tq_{c,t})tvac_{c,tacd,t} \cdot icm_{c,c'} \cdot QM_{c',t} + \sum_{c\in CT,c'\in C}(1-subc_{c,tace,t})PQS_{c,t}(1+tq_{c,t})tvac_{c,tace,t} \cdot ice_{c,c'} \cdot QE_{c',t}$	$t \in T$

G9	$SUBCT_t = \sum_{c\in C,a\in A} subc_{c,a,t} \cdot PQS_{c,t}(1+tq_{c,t}) \cdot QINT_{c,a,t} + \sum_{c\in C,h\in H} subc_{c,h,t} \cdot PQS_{c,t}(1+tq_{c,t}) \cdot QH_{c,h,t} + \sum_{c\in C,h\in NGO} subc_{c,ngo,t} \cdot PQS_{c,t}(1+tq_{c,t}) \cdot QNGO_{c,ngo,t} + \sum_{c\in C} subc_{c,gov,t} \cdot PQS_{c,t}(1+tq_{c,t}) \cdot QG_{c,t} + \sum_{c\in C} subc_{c,trst,t} \cdot PQS_{c,t}(1+tq_{c,t}) \cdot QTRSTF_{c,t} + \sum_{c\in C,f\in FCAP} subc_{c,f,t} \cdot PQS_{c,t}(1+tq_{c,t}) \cdot capcomp_{f,c} \cdot \sum_{i2\in INS2} DKINS_{i2,f,t} + \sum_{c\in C} subc_{c,dstk,t} \cdot PQS_{c,t}(1+tq_{c,t}) \cdot \sum_{i2\in INS2} qdstk_{i2,c,t} + \sum_{c\in CT,c\prime\in C} subc_{c,tacd,t} \cdot PQS_{c,t}(1+tq_{c,t}) \cdot icd_{c,c\prime} \cdot QD_{c\prime,t} + \sum_{c\in CT,c\prime\in C} subc_{c,tacm,t} \cdot PQS_{c,t}(1+tq_{c,t}) \cdot icm_{c,c\prime} \cdot QM_{c\prime,t} + \sum_{c\in CT,c\prime\in C} subc_{c,tace,t} \cdot PQS_{c,t}(1+tq_{c,t}) ice_{c,c\prime} \cdot QE_{c\prime,t}$	$t \in T$
G10	$YG_t = YTAXDIR_t + YTAXFAC_t + YTAXFFAC_t + YTAXACT_t + YTAXVAT_t + YTAXCOM_t + YTAXIMP_t + YTAXEXP_t + \sum_{i\in INS} TRNSFR_{gov,i,t} + \sum_{f\in F} YIF_{gov,f,t} + \sum_{a\in AREG} shryregpa_{gov,a,t} \cdot REGPA_{a,t} \cdot PA_{a,t} \cdot QA_{a,t}$	$t \in T$
G11	$QG_{c,t} = qgb_{c,t} \cdot \overline{QGSCAL}_t$	$t \in T$ $c \in C$
G12	$EG_{c,t} = \sum_{c\in C} PQD_{c,gov,t} \cdot QG_{c,t} + \sum_{i\in INS} TRNSFR_{i,gov,t} + SUBCT_t$	$t \in T$
G13	$SURPG_t = YG_t - EG_t - INVVALG_t$	$t \in T$

G14	$-SURPG_t = EXR_t \cdot \overline{NFFG}_t + NDFG_t$	$t \in T$

3.7 Institutional Sectors: Rest of the World

The rest of the world is represented through the balance of payments. Specifically, equations RW1 and RW2 correspond to the current account and the capital account of the balance of payments, respectively. Equation RW1 has, on its left-hand side (right-hand side), the foreign exchange inflows (outflows). In other terms, the income and expenses of the rest of the world are shown to the right-hand side and left-hand side of the equality, respectively. The saving of the rest of the world $SAVF_t$ goes to foreign government financing $NFFG_t$, foreign non-government financing $NFFNG_t$ and foreign direct investment $FDI_{f,t}$. In this presentation, these three terms are exogenous.

RW1	$\sum_{c \in C} pwe_{c,t} \cdot QE_{c,t} + \frac{\sum_{i \in INSD} TRNSFR_{i,row,t}}{EXR_t} + \frac{\sum_{i \in F} TRNSFR_{f,row,t}}{EXR_t} + \frac{\sum_{c \in C} PQD_{c,trst,t} \cdot QTRSTF_{c,trst,t}}{EXR_t} + SAVF_t = \sum_{c \in C} pwm_{c,t} \cdot QM_{c,t} + \frac{\sum_{i \in INSD} TRNSFR_{row,i,t}}{EXR_t} + \frac{\sum_{f \in F} YIF_{row,f,t}}{EXR_t}$	$t \in T$
RW2	$SAVF_t = \overline{NFFG}_t + \overline{NFFNG}_t + \sum_{f \in FCAP} \overline{FDI}_{f,t}$	$t \in T$

3.8 Transfers

Equations TR1-TR6 calculate transfers between institutional sectors (TR1-TR5) and from the rest of the world to the factors of production of the modeled country (TR6). Transfers from households, NPISH and enterprises to the other institutions are calculated as an exogenous proportion $shii_{i,i\prime}$ of their income net of direct taxes and savings (equation TR1). The other transfers are modeled as an exogenous amount adjusted for the nominal exchange rate or the consumer price index depending on which institutions are involved. In particular, all transfers to/from the rest of the world are adjusted by the nominal exchange rate.

TR1	$TRNSFR_{i,i',t} = shii_{i,i'}\left(YI_{i',t}\left(1 - ty_{i',t}\right) - SAVINS_{i',t}\right)$	$t \in T$ $i \in INS$ $i' \in INSDNG$
TR2	$TRNSFR_{i,gov,t} = trnsfrb_{i,gov,t} \cdot CPI_t$	$t \in T$ $i \in INS$
TR3	$TRNSFR_{row,gov,t} = trnsfrb_{row,gov,t} \cdot EXR_t$	$t \in T$
TR4	$TRNSFR_{gov,row,t} = trnsfrb_{gov,row,t} \cdot EXR_t$	$t \in T$
TR5	$TRNSFR_{i,row,t} = trnsfrb_{i,row,t} \cdot EXR_t$	$t \in T$ $i \in INS$
TR6	$TRNSFR_{f,row,t} = trnsfrb_{f,row,t} \cdot EXR_t$	$t \in T$ $f \in F$

3.9 Equilibrium Conditions

The equations in this group define equilibrium conditions. Equation E1 is a wage curve which establishes a negative relationship between wage level and unemployment rate (Blanchflower and Oswald 1994). Equation E2 is the equilibrium condition in the factor markets for factors that are mobile between sectors: the net factor supply (left-hand side) equals demand (right-hand side). Equation E3 is the equilibrium condition for the goods and services markets. Again, the total supply ($QQ_{c,t}$) includes both domestic output and imports. It is relevant to note that, international tourists who travel to the modeled country to consume goods and services are included among the sources of demand. In other terms, it is an export type in which consumers from the demander country travel to the supply country's territory.

Equation E4 transforms the different capital stock investments into goods and services demand. In order to do this, the matrix $capcomp_{f,c}$ is used. This matrix indicates the composition by product of each capital stock modeled. For instance, public investment usually has relatively a more important construction services component than does private investment. Equation E5 establishes the equality between private investment and its financing. There are three financing sources for private investment: household and enterprises savings net from government loans, external debt, and foreign direct investment.

Equation E6 defines the consumer price index, which is used as the model numeraire. Finally, equation E7 defines the activities' price with price regulation as the ratio between the nominal price and the nominal exchange rate. Equation E7 applies to activities whose price becomes exogenous while the variable $REGA_{a,t}$ defined above becomes endogenous.

E1	$\frac{\frac{WF_{f,t}}{\overline{CPI}_t}}{\frac{WF_f^{00}}{\overline{CPI^{00}}}} = \left(\frac{UERAT_{f,t}}{UERAT_f^{00}}\right)^{\eta_f^{wf}}$	$t \in T$ $f \in FUEND$
E2	$QFS_{f,t}(1 - UERAT_{f,t}) = \sum_{a \in A} QF_{f,a,t,}$	$t \in T$ $f \in FMOB$
E3	$\sum_{a \in A} QINT_{c,a,t} + \sum_{h \in H} QH_{c,h,t} + QNGO_{c,ngo,t} + QG_{c,t} + QINV_{c,t} + \sum_{i2 \in INS2} qdstk_{i2,c,t} + QT_{c,t} + QTRSTF_{c,trst,t} = QQ_{c,t}$	$t \in T$ $c \in C$
E4	$QINV_{c,t} = \sum_{i2 \in INS2} \sum_{f \in FCAPNG} capcomp_{f,c} \cdot DKINS_{i2,f,t}$	$t \in T$ $c \in C$
E5	$\sum_{f \in FCAPNG} PK_{f,t} \cdot DKINS_{ngov,f,t} + \sum_{c \in C} PQD_{c,dstk,t} \cdot qdstk_{ngov,c,t} + WALRAS_t = \sum_{i \in INSDNG} SAVINS_{i,t} - NDFG_t + EXR_t \cdot \overline{NFFINS}_t + \sum_{f \in FCAPNG} EXR_t \cdot \overline{FDI}_{f,t}$	$t \in T$
E6	$\sum_{c \in C, h \in H} PQD_{c,h,t} \cdot cwts_{c,h} = \overline{CPI}_t$	$t \in T$
E7	$PAREAL_{a,t} = \frac{PA_{a,t}}{EXR_t}$	$t \in T$ $a \in AREG$

3.10 Dynamics: Investment by Destination

This group of equations determines the investment (i.e., the new capital stock) that each productive activity receives. For this purpose, differences between public and private capital stocks are noted. Equation D1 calculates the unit cost of capital. For it, the matrix $capcomp_{f,c}$ (see above) is multiplied by the price of each of the demanded products for investment (e.g., construction and machinery and equipment). Equation D2 determines the government's investment in the different capital stocks. In this presentation, it is assumed that public investment is exogenous. Equation D3, similar to the previous one, determines the investment of the non-government institutions (i.e., households, NPISH and enterprises). In this case, it is assumed that private investment is endogenously adjusted to match the available savings. Equation D4 calculates the government nominal investment from adding gross fixed capital formation and the variation in government stocks.

Equations D5-D7 determine the evolution of private capital stocks used for the different productive activities. Equation D5 calculates the average capital rate of return $WFAVG_{f,t}$ as the ratio between the total rents received by the capital factor and the capital stock. Equation D6 determines how the new capital is distributed (i.e., the sum of the investments made by institutions) among the different productive activities. Equation D6 assumes that activities with a higher (lower) than the average rate of return will receive a relatively higher (lower) proportion of the new capital. In addition, the allocation of new capital depends on the capital distribution of the previous period. The importance of each of the mentioned elements depends on the value assigned to the parameter κ; for example, if κ is equal to zero, only the distribution of capital in the previous period matters to determine how the new capital is allocated among sectors. Equation D7 determines the evolution of sectoral capital.

D1	$PK_{f,t} = \sum_{c \in C} capcomp_{f,c} \cdot PQD_{c,f,t}$	$t \in T$ $f \in FCAP$
D2	$DKINS_{gov,f,t} = dkinsb_{gov,f,t}\left(1 + \overline{ISCAL2}_{gov,t} \cdot iadj01_f\right)$	$t \in T$ $f \in FCAP$
D3	$DKINS_{ngov,f,t} = dkinsb_{ngov,f,t}\left(1 + ISCAL2_{ngov,t} \cdot iadj01_f\right)$	$t \in T$ $f \in FCAP$
D4	$INVVALG_t = \sum_{f \in FCAP} DKINS_{gov,f,t} \cdot PK_{f,t} + \sum_{c \in C} qdstk_{gov,c,t} \cdot PQD_{c,dstk,t}$	$t \in T$
D5	$WFAVG_{f,t} = \frac{\sum_{a \in A} WF_{f,t} \cdot WFDIST_{f,a,t} \cdot QF_{f,a,t}}{\sum_{a \in A} QF_{f,a,t}}$	$t \in T$ $f \in FCAPNG$
D6	$DKA_{f,a,t} = \sum_{i2 \in INS} DKINS_{i2,f,t} \cdot \frac{QF_{f,a,t-1}}{\sum_{a' \in A} QF_{f,a',t-1}}\left(1 + \kappa\left(\frac{WF_{f,t} \cdot WFDIST_{f,a,t}}{WFAVG_{f,t}} - 1\right)\right)$	$t \in T$ $f \in FCAPNG$ $a \in A$
D7	$QF_{f,a,t} = QF_{f,a,t-1}\left(1 - deprcap_f\right) + DKA_{f,a,t-1}$	$t \in T$ $f \in FCAPNG$ $a \in A$

3.11 Dynamics: Factor Supply and Endowments

Equations F1-F14 define the evolution of factor supply and institutions' factor endowments. In order to do this, IEEM distinguishes between households and other institutions. In particular, the size of each representative household is taken as a function of population projections, which are assumed to be exogenous for this presentation. That is, factor endowments are determined as a function of the growth rates of factor supplies and the population growth rates. Equations F1-F3 determine the evolution of factor stocks. In order to allow imperfect factor mobility, IEEM distinguishes between factor supplies and endowments at the beginning (i.e., before moving between sectors) and at the end of each period (i.e., after moving between sectors).

Equation F1 refers to the factors whose supply evolves exogenously such as labor and other factors different than land and capital. Equation F2 allows modeling the supply of productive land (i.e, that used in agriculture or forest plantations) as a function of (a) an exogenous component, (b) an endogenous component dependent on the land rent, and (c) a component dependent on deforestation. Specifically, increases in deforestation of non-productive forest areas can be translated, in part, into increases in the available area for agricultural (or forest plantation) production. Equation F3 determines the supply of private capital as a function of the remaining capital from the previous period and the investment also of the previous period. Equation F4 calculates the stock of public capital held by the government. The parameter $dqfins_{gov,f,t}$ allows simulating scenarios in which the public capital stock is suddenly reduced, for example, due to a natural disaster. Clearly, such a shock will have repercussions on the rest of the economy through its effect on TFP.

Equations F5 and F6 refer to the evolution of land endowments and other factors other than labor and private capital (equation F5) and other institutions (equation F6). Equation F5 imposes that the factor endowment of households evolves according to (a) the factor growth rate, and (b) the population growth rate for each household. Specifically, the per capita endowment of each household remains constant. Equation F6 imposes that other institutions' factor endowment evolves according factor supply. Equations F7 and F8 are similar to others used above, but these are related to labor. In this case, equation F9 enables keeping the labor participation rate constant which is defined as the ratio between the labor force and working-age population. The variable

$QLABSCAL_{f,t}$ is endogenously determined such that the participation rate is fixed at the exogenously imposed values. Also, equations F10 and F11 are similar to equations F5 and F6 but refer to the institutions' private capital stocks, specifically, households in the case of equation F10 and all other institutions in the case of equation F11. Equation F12 calculates the total supply for each factor except capital, as the sum of the corresponding institutional endowments. Finally, it is important to highlight that the endowments defined in this group of equations are computed before the movement of factors between sectors or segments of the factor markets.

F1	$QFSINIT_{f,t} = QFS_{f,t-1} \cdot \frac{qfsexog_{f,t}}{qfsexog_{f,t-1}}$	$t \in T$ $f \in (FLAB \cup FOTH)$
F2	$QFSINIT_{f,t} = QFS_{f,t-1} \cdot \frac{qfsexog_{f,t}}{qfsexog_{f,t-1}} + \left[QFS_f^{00} \left(\frac{\frac{WFAVG_{f,t}}{CPI_t}}{\frac{WFAVG_f^{00}}{CPI^{00}}} \right)^{\eta_f^{qfs}} - QFS_f^{00} \right] + shrland_{f,t} \cdot QDEFOR_{t-1}$	$t \in T$ $f \in FLAND$
F3	$QFS_{f,t} = QFS_{f,t-1}(1 - deprcap_f) + \sum_{i2 \in INS2} DKINS_{i2,f,t-1}$	$t \in T$ $f \in FCAPNG$
F4	$QFINS_{gov,f,t} = QFINS_{gov,f,t-1}(1 - deprcap_f) + \sum_{i2 \in INS2} DKINS_{i2,f,t-1} + dqfins_{gov,f,t}$	$t \in T$ $f \in FCAPG$
F5	$QFINSINIT_{h,f,t} = \frac{QFINS_{h,f,t-1} \cdot \frac{\overline{POP}_{h,t}}{\overline{POP}_{h,t-1}}}{\sum_{h\prime \in H} QFINS_{h\prime,f,t-1} \cdot \frac{\overline{POP}_{h\prime,t}}{\overline{POP}_{h\prime,t-1}}} \sum_{h\prime \in H} QFINS_{h\prime,f,t-1} \cdot \frac{QFSINIT_{f,t}}{QFSINIT_{f,t-1}}$	$t \in T$ $h \in H$ $f \in (FLAND \cup FOTH)$
F6	$QFINSINIT_{i,f,t} = QFINS_{i,f,t-1} \cdot \frac{QFSINIT_{f,t}}{QFSINIT_{f,t-1}}$	$t \in T$ $i \in INSNH$ $f \in (FLAND \cup FOTH)$

F7	$QFINSINIT_{h,f,t} = \frac{QFINS_{h,f,t-1} \cdot \frac{\overline{POP}_{h,t}}{\overline{POP}_{h,t-1}}}{\sum_{h\prime \in H} QFINS_{h\prime,f,t-1} \cdot \frac{\overline{POP}_{h\prime,t}}{\overline{POP}_{h\prime,t-1}}} \sum_{h\prime \in H} QFINS_{h\prime,f,t-1} \cdot \frac{QFSINIT_{f,t}}{QFSINIT_{f,t-1}} \cdot QLABSCAL_t$	$t \in T$ $h \in H$ $f \in FLAB$
F8	$QFINSINIT_{i,f,t} = QFINS_{i,f,t-1} \cdot \frac{QFSINIT_{f,t}}{QFSINIT_{f,t-1}}$	$t \in T$ $i \in INSNH$ $f \in FLAB$
F9	$LABPARTRAT_t = \sum_{i \in INS, f \in FLAB} \frac{QFINSINIT_{i,f,t}}{\overline{POP}_{agelab,t}}$	$t \in T$
F10	$QFINS_{h,f,t} = \frac{QFINS_{h,f,t-1} \cdot \frac{\overline{POP}_{h,t}}{\overline{POP}_{h,t-1}}}{\sum_{h\prime \in H} QFINS_{h\prime,f,t-1} \cdot \frac{\overline{POP}_{h\prime,t}}{\overline{POP}_{h\prime,t-1}}} \sum_{h\prime \in H} QFINS_{h\prime,f,t-1} \cdot \frac{QFS_{f,t}}{QFS_{f,t-1}}$	$t \in T$ $i \in INS$ $f \in CAPNG$
F11	$QFINS_{i,f,t} = QFINS_{i,f,t-1} \cdot \frac{QFS_{f,t}}{QFS_{f,t-1}}$	$t \in T$ $i \in INSNH$ $f \in FCAPNG$
F12	$QFS_{f,t} = \sum_{i \in INS} QFINS_{i,f,t}$	$t \in T$ $f \in F$ $f \notin FCAP$

3.12 Dynamics: Factor Mobility

The equations in this section allow factors to be modeled as imperfectly mobile between different segments of the same factor market. For instance, it allows modeling migration of workers between urban and rural areas or the formal and informal segments of the labor market. Also, this formulation allows modeling transitions of land between alternative uses. Consequently, an advantage of our approach over other alternatives (e.g., CET function) is that the physical units of modeled factors are preserved at the same time that factors are imperfectly mobile between sectors/segments.

Equation FM1 computes the expected remuneration ratio between two factors *(f and f')* for which migration is allowed. Equation FM2 computes the factor movements from *f* to *f´* as a function of (a) the wage ratio defined in FM1, and (b) the elasticity $\eta^{migr}_{f,f\prime}$. Equation FM3 imposes the restriction that migration cannot be negative. Equation FM4 computes the factor endowment that does not migrate (i.e., moves from *f* to the same *f* or, in other words, remains where it is located) as the difference between the initial endowment and the migration to other factors. This equation is necessary because of the way in which the factor endowment of the institutions in the FM5 equation is computed. That is to say, equation FM5 calculates the total endowment of factor f as the sum of the movements from *f* 'to *f*.

FM1	$WFRAT_{f,f',t} = \dfrac{WFAVG_{f',t}\left(1 - UERAT_{f',t}\right)}{WFAVG_{f,t}\left(1 - UERAT_{f,t}\right)}$	$t \in T$ $f, f' \in MFFP(F,F)$
FM2	$QFMIGR_{i,f,f',t} = QFINSINIT_{i,f,t}\left(\dfrac{WFRAT_{f,f',t}}{WFRAT^{0}_{f,f',t}}\right)^{\eta^{migr}_{f,f'}} - QFINSINIT_{i,f,t}$	$t \in T$ $i \in INS$ $f, f' \in MFFP(F,F)$
FM3	$QFMIGR_{i,f,f',t} \geq 0$	$t \in T$ $i \in INS$ $f, f' \in MFFP(F,F)$
FM4	$QFMIGR_{i,f,f,t} = QFINSINIT_{i,f,t} - \sum_{f' \in MFFP(F,F)} QFMIGR_{i,f,f',t}$	$t \in T$ $i \in INS$ $f \in FNCAP$
FM5	$QFINS_{i,f,t} = \sum_{f' \in MFFP(F,F)} QFMIGR_{i,f',f,t}$	$t \in T$ $i \in INS$ $f \in FNCAP$

3.13 Dynamics: Land Use

The equations in this block compute the number of hectares allocated to every possible land use. In particular, IEEM distinguishes between productive forest uses (forest plantations), non-productive forest uses (unmanaged natural forest), agriculture (crops and pastures), and other uses. To simplify this presentation, it is assumed that deforestation is exogenous (equation T1). Then, equation T2 defines the forest area at the beginning of a given period as the available forest area at the beginning of a previous period minus deforestation also from the previous period. The area with forest plantations is equal to the land used in forestry activities. Equation T3 defines the non-productive forest area as the difference between the total forest area and the area with forest plantations. Equations T5 and T6 define the areas for crops and livestock activities, respectively. Equation T7 computes the total area devoted to agricultural activities. Finally, it is worth mentioning that land (and other factors) can move between alternative uses depending on the assumptions made about it (see section on imperfect factor mobility).

T1	$QDEFOR_t = qdeforb_t$	$t \in T$
T2	$QLANDUSE_{fortot,t} = QLANDUSE_{fortot,t-1} - QDEFOR_t$	$t \in T$
T3	$QLANDUSE_{forplant,t} = \sum_{f \in FFOR, a \in A} QF_{f,a,t}$	$t \in T$
T4	$QLANDUSE_{fornprod,t} = QLANDUSE_{fortot,t} - QLANDUSE_{forplant,t}$	$t \in T$
T5	$QLANDUSE_{livestock,t} = \sum_{f \in FLAND, a \in ALIV} QF_{f,a,t}$	$t \in T$
T6	$QLANDUSE_{crops,t} = \sum_{f \in FLAND, a \in ACRO} QF_{f,a,t}$	$t \in T$
T7	$QLANDUSE_{agr,t} = QLANDUSE_{crops,t} + QLANDUSE_{livestock,t}$	$t \in T$

3.14 Emissions

In this block of equations, greenhouse gas (GHG) emissions are calculated based on the use, both intermediate and final, of polluting products or factors. The equations EM1, EM2 and EM3 compute the emissions related to the use of specific products by productive activities, households and government, respectively. Usually, these are energy products such as fossil fuels or firewood. Equation EM4 computes the emissions linked to the use of particular productive factors. For instance, emissions linked to the size of the livestock stock. Finally, the EM5-EM7 equations compute the emissions of activities and households and government that cannot be linked to the use of any particular product or factor. In practice, the last three equations can be used in case there is not enough available information to implement the other equations of this module. Particularly, when we only know the total emissions volume of activities and households.

EM1	$EMI_{ghg,c,a,t} = iemi_{ghg,c,a,t} \cdot QINT_{c,a,t}$	$t \in T$ $ghg \in GHG$ $c \in C$ $a \in A$
EM2	$EMI_{ghg,c,h,t} = iemi_{ghg,c,h,t} \cdot QH_{c,h,t}$	$t \in T$ $ghg \in GHG$ $c \in C$ $h \in H$
EM3	$EMI_{ghg,c,gov,t} = iemi_{ghg,c,gov,t} \cdot QG_{c,t}$	$t \in T$ $ghg \in GHG$ $c \in C$
EM4	$EMI_{ghg,f,a,t} = iemi_{ghg,f,a,t} \cdot QF_{f,a,t}$	$t \in T$ $ghg \in GHG$ $f \in F$ $a \in A$
EM5	$EMI_{ghg,oth,a,t} = iemi_{ghg,oth,a,t} \cdot QA_{a,t}$	$t \in T$

		$ghg \in GHG$ $a \in A$
EM6	$EMI_{ghg,oth,h,t} = iemi_{ghg,oth,h,t} \sum_{c \in C} PQD^{00}_{c,h,t} \cdot QH_{c,h,t}$	$t \in T$ $ghg \in GHG$ $a \in A$
EM7	$EMI_{ghg,oth,gov,t} = iemi_{ghg,oth,gov,t} \sum_{c \in C} PQD^{00}_{c,gov,t} \cdot QG_{c,t}$	$t \in T$ $ghg \in GHG$

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